Corrigendum

Involution Products in Coxeter Groups

J. Group Theory 14 (2011), no.2, 251 - 259

S.B. Hart and P.J. Rowley

In [1], Theorem 2.4 states a well-known result on Coxeter groups which gives conditions under which the stabilizer of a nonzero vector is a proper parabolic subgroup. However the hypothesis of this result is incorrectly stated in our paper: it holds for finite Coxeter groups but is not true in general for infinite Coxeter groups. We are grateful to an anonymous referee of a subsequent paper for pointing this out. As a consequence, the proof of Theorem 1.1 in [1], which uses Theorem 2.4, is incomplete. Here we complete the proof of Theorem 1.1 without recourse to Theorem 2.4.

Theorem 1.1 states that if $X$ is a strongly real conjugacy class of a Coxeter group $W$ (not necessarily finite), then there exists $w_* \in X$ such that $e(w_*) = 0$. That is to say, there are involutions $\sigma, \tau$ of $W$ such that $w_* = \sigma \tau$ and $\ell(w) = \ell(\sigma) + \ell(\tau)$. At the point in the proof where Theorem 2.4 is used, we have established that $zy$ is an element of $X$ where $z$ and $y$ are involutions with the following properties. First, $y$ is the central involution of some standard parabolic subgroup $W_J$ of $W$. The involution $z$ has the property that $\ell(gzg^{-1}) \geq \ell(z)$ for all $g \in W_J$. It follows that if $\ell(zr) < \ell(z)$ for any $r \in J$, then $rzr = z$ and $z \cdot \alpha_r = -\alpha_r$.

Now let $K = \{ r \in J : \ell(zr) < \ell(z) \}$. From the above we know that $z \cdot \alpha_r = -\alpha_r$ for all $r \in K$. If $K$ is nonempty then, as $\Phi_K^+ \subseteq N(z)$, $\Phi_K^+$ is finite. Therefore $W_K$ has a unique longest element $w_K$, which is an involution, and $N(w_K) = \Phi_K^+$. If $K = \emptyset$ we set $w_K = 1$. In all cases, since $y$ is central in $W_J$ and $w_K \in W_J$, we see that $w_Ky = yw_K$ is an involution. Moreover $zr = rz$ for all $r \in K$, and thus $zw_K$ is also an involution. Let $\sigma = zw_K$ and $\tau = w_Ky$. Then $\sigma \tau = zy \in X$. Moreover $z$ and $y$ both act as $-1$ on $\Phi_K^+$. Thus, by Lemma 2.2,

$$N(\sigma) = N(z) \setminus [-z \cdot N(w_K)] = N(z) \setminus N(w_K)$$

and

$$N(\tau) = N(y) \setminus [-y \cdot N(w_K)] = N(y) \setminus N(w_K) = \Phi_J^+ \setminus N(w_K).$$

Consider $r \in J$. If $r \in K$, then $\alpha_r \in N(w_K)$ and so $\alpha_r \notin N(z) \setminus N(w_K) = N(\sigma)$. On the other hand if $r \in J \setminus K$ then by definition of $K$, $\alpha_r \notin N(z)$ and hence $\alpha_r \notin N(\sigma)$, which is after all a subset of $N(z)$. Hence for all $r \in J$ we have $\alpha_r \notin N(\sigma)$. This implies that $N(\sigma) \cap \Phi_J^+ = \emptyset$, because every positive root in $\Phi_J^+$ is a positive linear combination of some $\alpha_r, r \in J$. But $N(\tau) \subseteq \Phi_J^+$ and therefore $N(\sigma) \cap N(\tau) = \emptyset$. Hence, by Lemma 2.2, $\ell(\sigma \tau) = \ell(\sigma) + \ell(\tau)$. Setting $w_* = \sigma \tau$ we have $w_* \in X$ and $e(w_*) = 0$, so completing the proof of Theorem 1.1.

□

References