Ordering Selection Operators Under Partial Ignorance

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ABSTRACT

Optimising queries in real-world situations under imperfect conditions is still a problem that has not been fully solved. We consider finding the optimal order in which to execute a given set of selection operators under partial ignorance of their selectivities. The selectivities are modelled as intervals rather than exact values and we apply a concept from decision theory, the minimisation of the maximum regret, as a measure of optimality. The associated decision problem turns out to be NP-hard, which renders a brute-force approach to solving it impractical. Nevertheless, by investigating properties of the problem and identifying special cases which can be solved in polynomial time, we gain insight that we use to develop a novel heuristic for solving the general problem. We also evaluate minmax regret query optimisation experimentally, showing that it outperforms a currently employed strategy of optimisers that uses mean values for uncertain parameters.

Categories and Subject Descriptors

H.2.4 [**Information Systems**]: Database Management—*Systems*

Keywords

query optimisation, decision theory, minmax regret

1. INTRODUCTION

Although query optimisation in database management systems (DBMSs) has been a topic of research for decades, there are still important unresolved issues. In his recent blog post [20], Guy Lohman highlights errors made in estimating cardinalities as a crucial factor. These kinds of errors cause optimisers to generate query execution plans that are way off the target in terms of efficiency. Consequently, an optimiser should try to avoid potentially bad plans rather than strive for an optimal plan based on unreliable information.

For typical workloads, a DBMS can compile statistical data over time to obtain a fairly accurate picture. For instance, estimating the selectivities of simple predicates on base relations in a relational database is fairly well understood and can be done quite accurately [12, 14]. However, the situation changes once systems are confronted with very unevenly distributed data values or predicates that are complex.

Trying to estimate selectivities in dynamic settings, such as data streams [28], or in non-relational contexts, such as XML databases [26, 30], also poses challenges. It may even be impossible to obtain any statistical data, because the query is running on remote servers [29]. Detailed information may also not be available because a user issues an atypical ad-hoc query or utilises parameter markers in a query. We propose to use techniques from decision theory for making decisions under ignorance[1], meaning that we know what the alternatives and their outcomes are, but we are unable to assign concrete probabilities to them [25].

In our approach we propose to build a robust query optimiser that is aware of the unreliability of database statistics and considers this during optimisation. When executing a query, the DBMS encounters a particular instance of concrete parameter values: we call this a *scenario*. The problem is that, during the prior optimisation step, the optimiser does not know which scenario the DBMS will face during plan execution. Additionally, it is highly unlikely that there is a single execution plan that will yield the optimal cost for every potential scenario. Consequently, our goal is to choose a query execution plan that performs reasonably well regardless of the scenario it encounters. More specifically, we try to minimise the difference between the cost of a plan p and the cost of the optimal plan when p is executed under its worst-case scenario. This is called *minmax regret optimisation* (MRO), which is a well-known technique for making decisions under ignorance. Previous work on query optimisation has considered measures of robustness for query plans [3, 4, 21], but not in terms of MRO.

In this paper, we focus on the selection operator σ, an operator common to many data querying languages. Selection is sometimes called a filter operator in contexts such as data stream processing [2, 5] and sensor networks [10], where there is renewed interest in improving the efficiency of processing these operators. A very common setting is determining the order in which to apply a set of commutative filters to a stream or a set of data items, e.g. tuples of a relation, so as to keep the processing costs to a minimum.

There are well-known techniques for ordering selection operators to filter out as many tuples as possible as early as possible at the lowest possible cost [13]. However, these techniques rely on having accurate values for the operators' selectivities, i.e., the percentage of tuples passing a filter, and their processing costs (per tuple). Getting

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[1]Sometimes these are also called decisions under uncertainty. We refer to them as decisions under ignorance to distinguish them from probability-based methods.

the estimation of selectivities (and/or costs) wrong can lead to high overall costs for the pipelined execution.

Our technique is based on using *intervals* rather than exact values for describing selectivities, aiming at generating query plans that are minmax regret optimal. However, identifying such plans, even for selection ordering, turns out to be NP-hard. As a result, we leave the investigation of further operators for future work and focus first on finding a good heuristic for MRO selection ordering.

Intervals can provide a useful way to model selectivities when exact values are unknown or hard to compute. For example, Babu et al. [4] compute intervals from single-point estimates in order to model levels of uncertainty regarding the accuracy of estimates, based on how such estimates were derived. Moerkotte et al. [23] consider histograms which guarantee a maximum multiplicative error (called the q-error) for cardinality estimates. Given such an estimate, the true cardinality (selectivity) can easily be modelled by an interval, as we show in Section 2.

For another situation in which interval selectivities arise, consider estimating the selectivities of string predicates which perform substring matching using SQL `like`, a problem known to be difficult [6]. As an example, let us consider a database in which email messages are stored in a relation `emails`, with attributes such as `sender`, `subject` and `body` (the textual contents of the email). Assume that many queries use selection predicates such as `subject like '%invest%'`, so the database maintains indexes on words and on 2-grams (say) of words which allow it also to provide selectivities for these.

Although the database maintains an index on words, the selectivity for the word 'invest' will be an underestimate for the selectivity of `subject like '%invest%'` since the strings 'reinvest' and 'investigation' (and many others) also match this predicate. Even if we are able to enumerate all words containing the string 'invest', we do not know how to combine their individual selectivities into a single selectivity. Instead we can use an interval selectivity with the exact match as a lower estimate. As the upper estimate, we can use the minimum selectivity of all the 2-grams of 'invest' since any string containing 'invest' must contain all of its 2-grams as well.

EXAMPLE 1. *As a concrete example, consider the following query on the Enron email data*[2]*:*

```
select sender
from   emails
where  body like '%action%' and
       body like '%like%' and
       subject like '%use%';
```

Let us denote the three predicates by A, L and U (for 'action', 'like' and 'use'). The interval selectivities for the three predicates, as computed using the method proposed above and explained in more detail in Section 7, are $[0.03, 0.68]$ *for* A*,* $[0.17, 0.27]$ *for* L *and* $[0.0008, 0.06]$ *for* U*. Even if we consider only the upper and lower bounds of these intervals, they give rise to 8 possible scenarios. No single plan (order) is optimal for all 8 scenarios, so the best we can do is find the plan which minimises the maximum regret. This plan corresponds to the order* UAL*. The maximum regret for this plan arises in the scenario when* U *has its maximum selectivity, while* A *and* L *have their minimum selectivities (in this case, the predicates* U *and* A *should be swapped to get the optimal order).*

In the case of the above query, our heuristic finds the minmax regret optimal solution. By way of contrast, an alternative heuristic such as that which takes the midpoints of the intervals and produces an optimal ordering based on those, produces the plan ULA*. This plan has a maximum regret which is 44% worse than the minmax regret optimal plan.* ◇

[2] `http://www.cs.cmu.edu/~./enron/`

We should mention that the technique of using intervals can be applied to other approximate or error-tolerant queries as well. All we need is the selectivity for an exact query as the lower bound and the selectivity for a query that determines a candidate set with false positives as the upper bound.

Our contributions in this paper are as follows:

- We formalise the problem of optimal selection ordering under partial ignorance, i.e., when selectivities are given as intervals.
- We identify a number of properties of the problem, including that (i) only *extreme* scenarios (i.e., in which each operator takes on its minimum or maximum selectivity) need to be considered, (ii) operators which *dominate* others (i.e., both their maximum and minimum selectivities are smaller) must appear before the dominated ones in any optimal plan, and (iii) the decision version of the problem is NP-hard.
- We investigate a number of special cases in which selection ordering under partial ignorance can be solved in polynomial time. Along the way, we also identify other important properties of scenarios in MRO selection ordering.
- Based on our findings we develop efficient optimisation heuristics, which we evaluate experimentally, using synthetic data, the Enron email data, and the Star Schema Benchmark (SSB) [27]. The experiments demonstrate the benefit of using minmax regret optimisation, in some cases halving the deviation from the optimal plan compared to conventional techniques.

The remainder of this paper is organised as follows. We start by reviewing related work on selection ordering and optimisation techniques in the next section. In Section 3, we formalise the problem of selection ordering under partial ignorance, using minmax regret optimisation as the criterion for optimality. Various properties of the problem, including NP-hardness, are identified in Section 4. Section 5 presents some special cases of the problem which can be solved in polynomial time. Our heuristic algorithm is given in Section 6, with its experimental evaluation presented in Section 7. Finally, we conclude in Section 8.

2. BACKGROUND AND RELATED WORK

We assume we are given a set $S = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ of selection operators, or equivalently a conjunctive predicate $p_1 \wedge p_2 \wedge \cdots p_n$. The *selectivity* s_i of operator σ_i or predicate p_i is the fraction of tuples that satisfy the operator or predicate. Associated with each operator s_i is also a cost c_i, which is the cost per tuple of evaluating the operator.

Most database systems keep statistics allowing them to estimate the selectivity for single attributes fairly accurately. For the joint selectivity of multiple attributes, much early work and many systems make the *attribute value independence* (AVI) assumption. This assumes that the selectivity of a set of operators $\{\sigma_{i_1}, \sigma_{i_2} \ldots \sigma_{i_m}\}$ is equal to $s_{i_1} \times s_{i_2} \times \cdots \times s_{i_m}$. If instead a system stores (some) joint selectivities (it is infeasible for it to store all of them), we can use the AVI assumption to "fill in the gaps" or use the estimation approach advocated in [22].

2.1 Selection Ordering

Assuming we have accurate values for the selectivity s_i and cost c_i of selection operator σ_i, we can calculate the *rank* r_i of σ_i:

$$r_i = (s_i - 1)/c_i \tag{1}$$

Given a set of selection operators, sorting and executing them in non-decreasing order of their ranks results in the minimal expected pipelined processing cost [18] under the AVI assumption. Clearly, the computation of the ranks and the sorting can be done in polynomial time. A similar argument applies if a query uses a conjunction of predicates on the same relation, and query evaluation uses a simple table scan. In such a case, the optimiser should test the predicates in the order which minimises the total number of tests. Basically, ordering selection operators optimally is a solved problem, but only when given exact values for the s_i and c_i.

Similar optimisation problems have been studied in the context of sequential testing. Here the goal is to find faulty components as quickly as possible by testing them one by one. Each component has a probability of working correctly and a cost for testing it. One of the earliest proposed solutions [15] relies on ranking the components and then ordering them by their ranks, very similar to the selection ordering described above.

2.2 Optimising under Uncertainty

In the following, we review different approaches for dealing with uncertain parameters during query optimisation. A common approach of many optimisers is to use the mean or modal value of the parameters and then find the plan with least cost under the assumption that this value remains constant during query execution, an approach called Least Specific Cost (LSC) in [7]. As Chu et al. point out in [7], if the parameters vary significantly, this does not guarantee finding the plan of least expected cost.

An alternative is to use probabilistic information about the parameters fed into the database optimiser, an approach known as Least Expected Cost (LEC) [7]. (A discussion regarding the circumstances under which LEC or LSC is best appears in [8].) In decision-theoretic terms, we are making decisions under risk, maximising the expected utility. However, probability distributions for the possible parameter values are needed to make this approach work, whereas in our case we do not have these prerequisites.

In parametric query optimisation several plans can be precompiled and then, depending on the query parameters, be selected for execution [11]. However, if there is a large number of optimal plans, each covering a small region of the parameter space, this becomes problematic. First of all, we have to store all these plans. In addition, constantly switching from one plan to another in a dynamic environment (such as stream processing) just because we have small changes in the parameters introduces a considerable overhead. In order to amend this, researchers have proposed reducing the number of plans at the cost of slightly decreasing the quality of the query execution [9]. Our approach can be seen as an extreme form of parametric query optimisation by finding a single plan that covers the whole parameter space.

Another approach to deal with the lack of reliable statistics is adaptive query processing, in which an execution plan is re-optimised while it is running [2, 4, 16, 21]. It is far from trivial to determine at which point to re-optimise and adaptive query processing may also involve materialising large intermediate results. More importantly, this means modifying the whole query engine; in our approach no modifications of the actual query processing are needed. A gentler approach is the incremental execution of a query plan [24]. Deciding on how to decompose a plan into fragments and putting them together is still a complex task, though.

Estimates based on intervals arise explicitly in [4] and implicitly in [23]. As mentioned in the Introduction, Babu et al. [4] use intervals to model uncertainty in the accuracy of a single-point estimate. Uncertainty is represented by a value from 0 (none) to 6 (very high). Upper and lower bounds for the single-point estimate are then calculated using the estimate and the uncertainty value. During optimisation, only three scenarios, those using the low estimates, the exact estimates and the high estimates, are considered, rather than all scenarios as in our approach. Moerkotte et al. [23] study histograms which provide so-called q-error guarantees. Given an estimate $\hat{s}$ for s, the q-error of $\hat{s}$ is $\max(s/\hat{s}, \hat{s}/s)$. An estimate is q-acceptable if its q-error is at most q. So if an estimate $\hat{s}$ is q-acceptable, the true value s lies in the interval $1/q \times \hat{s} \leq s \leq q \times \hat{s}$, but there is no knowledge about any distribution within the interval. The authors of [23] show that these histograms can be implemented efficiently in real-world systems such as SAP HANA.

Notions of robustness in query optimisation have been considered in [3, 4, 21]. Babcock and Chaudhuri [3] use probability distributions derived from sampling as well as user preferences in order to tune the predictability (or robustness) of query plans versus their performance. For Markl et al. [21], robustness means not continuing to execute to completion a query plan which is found to be suboptimal during evaluation; instead re-optimisation is performed. On the other hand, Babu et al. [4] consider a plan to be robust only if its cost is within e.g. 20% of the cost of the optimal plan. None of these papers consider robustness in the sense of MRO. Moreover, these techniques need additional statistical information to work.

2.3 Optimising under Ignorance

Minmax regret optimisation (MRO) has been applied to a number of optimisation problems where some of the parameters are (partially) unknown [1]. The complexity of the MRO version of a problem is often higher than that of the original problem. Many optimisation problems with polynomial-time solutions turn out to be NP-hard in their MRO versions [1].

One example is minimising the *total flow time* (TFT), in which n jobs are scheduled on a single machine [17]. The *flow time* of a job is the sum of its processing time and the time it has had to wait before starting execution. The total flow time is the sum of the flow times of all n jobs. This scheduling problem can be solved in polynomial time given exact job lengths (by sorting the jobs in non-decreasing order of their processing times [19]), but becomes NP-hard in its MRO variant [19]. Researchers have developed approximation algorithms for the problem; for example, a 2-approximation algorithm, bounding the approximate solution to be no more than twice the optimal solution, is proposed in [17].

Among all MRO problems, TFT is the one closest to the problem we are investigating. However, there are substantial differences: the formula for computing the cost of a schedule is much simpler for TFT, and the approach chosen to obtain a 2-approximation does not guarantee a bound for MRO selection ordering, as we show in Section 4.

3. SELECTION ORDERING MRO

In this section we give a formal definition of the generalised selection ordering problem with partially defined selectivities. The exact costs of selection operators can also be unknown, but for the moment we restrict ourselves to partially defined selectivities.

3.1 Basic Definitions

We start out with definitions for selection operators with interval selectivities and basic properties.

DEFINITION 1. *Given a set $S = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ of selection operators, each has a selectivity s_i and a cost c_i. Each selectivity is defined by a closed interval: for $1 \leq i \leq n$, $s_i = [\underline{s}_i, \overline{s}_i]$ with $\underline{s}_i, \overline{s}_i \in [0, 1]$ and $\underline{s}_i \leq \overline{s}_i$. For $1 \leq i \leq n$, $c_i \in \mathbf{R}^+$ represents the cost of σ_i for processing an input tuple.*

Depending on their selectivity intervals selection operators may relate to each other in a special way. Later on we exploit this property in order to optimise selection orders.

DEFINITION 2. *Given two selection operators $\sigma_i, \sigma_j \in S$, we say that σ_i* dominates *σ_j if $\underline{s}_i \leq \underline{s}_j$ and $\overline{s}_i \leq \overline{s}_j$. The set S of operators is called* dominant *if for each pair $\sigma_i, \sigma_j \in S$ it is the case that either σ_i dominates σ_j or σ_j dominates σ_i.*

Later on, it will be helpful to consider a special case of dominant sets of operators.

DEFINITION 3. *Given two selection operators $\sigma_i, \sigma_j \in S$, we say that σ_i* strictly dominates *σ_j if $\overline{s}_i \leq \underline{s}_j$. A* strictly dominant *set is defined analogously to a dominant set.*

If for two selection operators $\sigma_i, \sigma_j \in S$, neither σ_i dominates σ_j nor σ_j dominates σ_i, then σ_i and σ_j form a *nested* pair of operators. So, operator σ_i is *nested* in σ_j if $\underline{s}_j < \underline{s}_i$ and $\overline{s}_i < \overline{s}_j$.

EXAMPLE 2. *Let $S = \{\sigma_1, \sigma_2, \sigma_3\}$ be a set of selection operators, with selectivities $s_1 = [.2, .8]$, $s_2 = [.3, .5]$ and $s_3 = [.1, .4]$. Operator σ_3 dominates both σ_1 and σ_2, but does not strictly dominate either of them. Because σ_2 is nested in σ_1, the set S is not dominant.* ◇

DEFINITION 4. *An assignment of a concrete value to each of the n selectivities is called a* scenario *and is defined by a vector $x = (s_1, s_2, \ldots, s_n)$, with $s_i \in [\underline{s}_i, \overline{s}_i]$.*

Every time we actually run a query, we encounter one scenario. However, during the optimisation step we are unaware of which scenario we will face. The set of all possible scenarios can be described by $X = \{x \mid x \in [\underline{s}_1, \overline{s}_1] \times [\underline{s}_2, \overline{s}_2] \times \cdots \times [\underline{s}_n, \overline{s}_n]\}$. There are certain scenarios we are particularly interested in:

DEFINITION 5. *A scenario $x_{ext} = (s_1, s_2, \ldots, s_n)$ is called an* extreme scenario *if, for each $1 \leq i \leq n$, s_i is equal to either $\underline{s}_i$ or $\overline{s}_i$.*

Let π^n be the set of all possible permutations over $1, 2, \ldots, n$. For $\pi_j \in \pi^n$, $\pi_j(i)$ denotes the i-th element of π_j.

DEFINITION 6. *A query execution plan p_j is a permutation $\sigma_{\pi_j(1)}, \sigma_{\pi_j(2)}, \ldots, \sigma_{\pi_j(n)}$ of the n selection operators. The set of all possible query execution plans is given by*

$$P = \{p \mid p = \sigma_{\pi(1)}, \sigma_{\pi(2)}, \ldots, \sigma_{\pi(n)} \text{ such that } \pi \in \pi^n\}.$$

The cost of evaluating plan p_j under a given scenario x is

$$\begin{aligned} \text{Cost}(p_j, x) &= \Omega(c_{\pi(1)} + s_{\pi(1)} c_{\pi(2)} + s_{\pi(1)} s_{\pi(2)} c_{\pi(3)} \\ &\quad + \cdots + \prod_{i=1}^{n-1} s_{\pi(i)} c_{\pi(n)}) \\ &= \Omega \left(\sum_{i=1}^{n} \left(\prod_{j=1}^{i-1} s_{\pi(j)} \right) c_{\pi(i)} \right) \end{aligned} \quad (2)$$

Ω is the cardinality of the relation on which we execute the selection operators. Currently we make the AVI assumption that the selection predicates are stochastically independent. Extending our approach to situations in which (some) joint selectivities are known is a topic for future work.

EXAMPLE 3. *Recall the set $S = \{\sigma_1, \sigma_2, \sigma_3\}$ of selection operators from Example 2, with selectivities $s_1 = [.2, .8]$, $s_2 = [.3, .5]$ and $s_3 = [.1, .4]$. There are 8 extreme scenarios for this example, one being given by scenario $x_1 = (\underline{s}_1, \underline{s}_2, \underline{s}_3) = (.2, .3, .1)$. One the the 6 possible plans for S is given by plan $p_1 = \sigma_1\sigma_2\sigma_3$. Assuming that Ω and each cost c_i is set to 1, we can calculate the cost of plan p_1 under scenario x_1, Cost(p_1, x_1), using Equation (2) as follows:*

$$\textit{Cost}(p_1, x_1) = (1 + .2 + .2 \times .3) = 1.26$$

◇

Let $p_{opt(x)}$ stand for the query execution plan having the minimal cost for scenario x, and let $\pi_{opt(x)}$ be the permutation of the selection operators for this plan. Since we are facing multiple scenarios, the criterion for evaluating the optimality of a plan p_j is different to the one used in the classical selection ordering problem. We utilise minmax regret optimisation to determine the quality of a plan.

3.2 Minmax Regret Optimisation

Below we define the regret for a plan given a scenario, the maximal regret for a plan, and finally the problem of finding a plan that minimises the maximal regret.

DEFINITION 7. *Given a plan p and a scenario x, the absolute* regret *$\gamma(p, x)$ of p for x is:*

$$\gamma(p, x) = \textit{Cost}(p, x) - \textit{Cost}(p_{opt(x)}, x) \quad (3)$$

where $p_{opt(x)}$ is the optimal plan for scenario x. The maximal regret of a plan is the regret for its worst-case scenario and is simply defined as $\max_{x \in X}(\gamma(p, x))$.

DEFINITION 8. *Given the set P of all possible execution plans and the set X of all possible scenarios, minimising the maximal regret is done as follows (where $R(P, X)$ is the optimal regret):*

$$R(P, X) = \min_{p \in P}(\max_{x \in X}(\gamma(p, x)))$$

Given a set S of selection operators, let $P(S)$ denote the set of possible plans for S and $X(S)$ denote the set of possible scenarios for S. Then the minmax regret optimisation *problem for S, which we denote $MRO(S)$, is to find a plan whose maximum regret matches $R(P(S), X(S))$. For simplicity and when there is no confusion, we also use $MRO(S)$ to denote a plan which minimises $R(P(S), X(S))$.*

EXAMPLE 4. *Recall once again the set $S = \{\sigma_1, \sigma_2, \sigma_3\}$ of selection operators from Examples 2 and 3, with selectivities $s_1 = [.2, .8]$, $s_2 = [.3, .5]$ and $s_3 = [.1, .4]$. For simplicity, assume that all operators have the same cost 1 and that the relation has cardinality $\Omega = 1$ (so to get the real costs, the numbers in Table 1 just have to be multiplied by the true cardinality). To find the plan which minimises the maximum regret, we can perform an exhaustive enumeration of all possible execution plans under every possible scenario. We show later in Theorem 1 that it is sufficient to consider only the extreme scenarios since the worst case scenario for any plan is always an extreme one. Hence, if there are n operators, we need to consider $n!$ different execution plans under each of 2^n extreme scenarios. For our example, Table 1 shows the 48 regret values for the 6 possible plans under each of 8 extreme scenarios.*

For example, recall from Example 3 that the cost of the first plan $p_1 = \sigma_1\sigma_2\sigma_3$ under scenario $x_1 = (\underline{s}_1, \underline{s}_2, \underline{s}_3) = (.2, .3, .1)$ is 1.26. The optimal plan $p_{opt(x)}$ for any scenario x is one in which the operators are in non-decreasing order of their selectivities.

	$\underline{s}_1$ $\underline{s}_2$ $\underline{s}_3$	$\underline{s}_1$ $\underline{s}_2$ $\overline{s}_3$	$\underline{s}_1$ $\overline{s}_2$ $\underline{s}_3$	$\underline{s}_1$ $\overline{s}_2$ $\overline{s}_3$	$\overline{s}_1$ $\underline{s}_2$ $\underline{s}_3$	$\overline{s}_1$ $\underline{s}_2$ $\overline{s}_3$	$\overline{s}_1$ $\overline{s}_2$ $\underline{s}_3$	$\overline{s}_1$ $\overline{s}_2$ $\overline{s}_3$	Max Regret
$\sigma_1\sigma_2\sigma_3$	0.14	0	0.18	0.02	0.91	0.62	**1.05**	0.6	1.05
$\sigma_1\sigma_3\sigma_2$	0.1	0.02	0.1	0	**0.75**	0.7	0.73	0.52	0.75
$\sigma_2\sigma_1\sigma_3$	0.24	0.1	0.48	0.32	0.41	0.12	**0.75**	0.3	0.75
$\sigma_2\sigma_3\sigma_1$	0.21	0.16	**0.43**	0.42	0.2	0	0.4	0.1	0.43
$\sigma_3\sigma_1\sigma_2$	0	0.22	0	0.2	0.05	**0.3**	0.03	0.12	**0.3**
$\sigma_3\sigma_2\sigma_1$	0.01	0.26	0.03	**0.32**	0	0.1	0	0	0.32

Table 1: The regret for each plan under each scenario in Example 4.

Therefore, the optimal plan for scenario x_1 is $p_{opt(x_1)} = \sigma_3\sigma_1\sigma_2$ and its cost is:

$$\mathit{Cost}(p_{opt(x_1)}, x_1) = (1 + .1 + .1 \times .2) = 1.12$$

The regret of plan p_1 under scenario x_1 using Equation (3) is:

$$\begin{aligned}\gamma(p_1, x_1) &= \mathit{Cost}(p_1, x_1) - \mathit{Cost}(p_{opt(x_1)}, x_1)\\ &= 1.26 - 1.12 = 0.14\end{aligned}$$

In order to find the minmax regret solution, the maximum regret of each plan needs to be found. For plan p_1, the maximum regret is 1.05 *which occurs in scenario $(\overline{s}_1, \overline{s}_2, \underline{s}_3)$, its worst-case scenario. The maximum regret for each plan is shown in bold face in Table 1.*

Finally, we are looking for the plan with the smallest maximum regret (i.e., the smallest value in the last column of Table 1). As a result the minmax regret solution, $MRO(S)$, is plan $\sigma_3\sigma_1\sigma_2$, which has the best performance among all plans when confronted with their worst-case scenarios. ◇

In the above example, it is interesting to consider which scenario gives rise to the maximum regret for each plan. Note that for each plan its worst-case scenario is one in which the operators in some initial sequence in the plan each take on their maximum selectivity followed by the remaining operators taking on their minimum selectivity. We call such a scenario a *max-min* scenario.

DEFINITION 9. *Let p be the plan $\sigma_{\pi(1)}, \sigma_{\pi(2)}, \ldots, \sigma_{\pi(n)}$. A scenario for p is called a* max-min *scenario if there is a $0 \leq k \leq n$ such that for all $1 \leq i \leq k$, $s_{\pi(i)} = \overline{s}_{\pi(i)}$, and for all $k+1 \leq i \leq n$, $s_{\pi(i)} = \underline{s}_{\pi(i)}$.*

So the first k operators in p take on their maximum selectivity, while the rest take on the minimum. Note that for a plan p with n operators, there are $n+1$ max-min scenarios. Max-min scenarios are the only scenarios considered by the max-min heuristic we develop in this paper. However, it is important to state that, in general, the worst-case scenario for a plan may not be a max-min scenario.

4. PROPERTIES OF MRO

Before presenting algorithms for solving the MRO selection ordering problem, we identify some of its important properties[3].

In order to determine the worst-case scenario of a plan, i.e., the scenario for which a plan exhibits its largest regret, we only have to check extreme scenarios.

THEOREM 1. *The worst-case scenario for any query plan p is always an extreme scenario.*

[3]The proofs of results in this section and the next are published in a technical report available at `http://arxiv.org/abs/1507.08257`.

We can determine the relative order two operators have to be in to minimise the maximal regret if one operator dominates the other.

THEOREM 2. *If σ_a dominates σ_b, then there exists a plan p minimising the maximal regret in which σ_a precedes σ_b.*

EXAMPLE 5. *Recall from Example 4 the set $S = \{\sigma_1, \sigma_2, \sigma_3\}$ of selection operators, with selectivities $s_1 = [.2, .8]$, $s_2 = [.3, .5]$ and $s_3 = [.1, .4]$. Because σ_3 dominates σ_1 and σ_2, in the minmax regret solution, i.e. plan $\sigma_3\sigma_1\sigma_2$, σ_3 precedes σ_1 and σ_2. As a result of domination, in this example we would only have to consider two plans when searching for the minmax regret solution.* ◇

For the TFT problem, Kasperski used the simple heuristic of sorting jobs in non-decreasing order according to the midpoints of their intervals, yielding a 2-approximation [17]. This approach does not guarantee a bound for MRO selection ordering; as shown below, the quality of the solution can become arbitrarily bad.

Given $2n+1$ operators, the first n operators have the selectivities $\underline{s_i} = 0$ and $\overline{s_i} = 1$ $(1 \leq i \leq n)$, while the next n operators have the selectivities $\underline{s_i} = \overline{s_i} = 0.5 + \epsilon$ $(n+1 \leq i \leq 2n)$ for some small ϵ. The final operator has a constant selectivity of 1 to guarantee that it will always be in last position, meaning that its selectivity will not impact any further steps.

The midpoint heuristic will order the operators in exactly this way, from 1 to $2n+1$. Clearly, the worst-case scenario for this plan is when s_i is set to 1 for $1 \leq s_i \leq n$. In the optimal plan for this scenario, the operators σ_i with $n+1 \leq i \leq 2n$ will be executed first.

The regret of this plan is computed as follows:

$$\begin{array}{ccccccccc} & 1 & + & 1^2 & \ldots & + & 1^n & + & f(n) \\ - & (0.5+\epsilon) & - & (0.5+\epsilon)^2 & \ldots & - & (0.5+\epsilon)^n & - & g(n) \end{array}$$

where $f(n)$ and $g(n)$ stand for the cost of the remaining operators in the plan. A lower bound for this expression is the following, since $f(n) \geq g(n)$ (see Lemma 3 below):

$$n - n(0.5 + \epsilon)$$

With increasing n and small values for ϵ, this expression can get arbitrarily large.

LEMMA 3. *Given a query plan p and a scenario x, we have the following relationship between the summands in $\mathit{Cost}(p, x)$ and $\mathit{Cost}(p_{opt(x)}, x)$, where $p_{opt(x)}$ is the optimal plan for scenario x:*

$$\prod_{j=1}^{k} s_{\pi(j)} \geq \prod_{j=1}^{k} s_{\pi_{opt(x)}(j)} \text{ for all } k \text{ with } 1 \leq k \leq n-1$$

In common with other MRO problems, we can show that the decision problem for general $MRO(S)$, which we call MINMAX REGRET, is NP-hard. In this version, we are given a set $S = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ of selection operators each having unit cost, as well as a set $X = \{X_1, X_2, \ldots, X_m\}$ of scenarios, where each scenario X_j specifies a selectivity s_{ij} for each operator σ_i, $1 \leq j \leq m$ and $1 \leq i \leq n$.

MINMAX REGRET: given a set S of n selection operators, a set X of m scenarios, and a real number R, is there a plan whose maximum regret is less than R?

We can show that MINMAX REGRET is NP-hard by reducing a version of SET COVER to it.

THEOREM 4. MINMAX REGRET *is NP-hard.*

5. SOME POLYNOMIAL-TIME CASES

In this section we show that, for sets of selection operators S satisfying certain properties, $MRO(S)$ can be found in polynomial time. In particular, we look at dominant operators, which can easily be ordered correctly, and their combination with constant operators, i.e., operators for which we can obtain exact selectivity values. As before, we assume that the cost of each operator is one.

Let S be a set of selection operators such that the selectivity of each operator can be estimated accurately (i.e., each selectivity is constant). Then, as mentioned in Section 2.1, $MRO(S)$ can be found by sorting the operators in non-decreasing order of their rank given by Equation (1). Given our assumption that each operator has cost one, finding $MRO(S)$ reduces to sorting the operators in non-decreasing order of their selectivity alone.

Recall from Section 3.1 the definition of a dominant set S of operators. Given a dominant set S of operators, it follows from Theorem 2 that the minmax regret solution is one where the operators are sorted in non-decreasing order according to their minimum (or maximum) selectivity value. (Note that a set of constant operators is a special case of a dominant set of operators.) We therefore have:

COROLLARY 1. *If S is a dominant set of n operators, then $MRO(S)$ can be solved in $O(n \log n)$ time.*

When we include nested operators (recall the definition from Section 3.1), the problem becomes much more difficult. As a step in the direction of solving the general problem, we consider below the simple case of a strictly dominant set of operators (also defined in Section 3.1) along with a single constant operator nested within one of the non-constant operators. If S is a strictly dominant set of operators, then the plan $MRO(S)$ has zero regret under all scenarios. This is because all operators in $MRO(S)$ will be in the same position as in the corresponding optimal plan under all scenarios.

Let S be a strictly dominant set which includes a constant operator σ_c nested within one of the non-constant operators, say σ_i. In this case, we know how to place the dominant operators relative to each other in $MRO(S)$ but we need to determine the position of σ_c in $MRO(S)$. Since $\underline{s}_i \leq s_c \leq \overline{s}_i$, the constant operator σ_c should be either immediately before or immediately after σ_i in $MRO(S)$. Interestingly, the correct position for σ_c depends only on the midpoint of the selectivity s_i of σ_i.

PROPOSITION 1. *Let S be a strictly dominant set of n operators such that $MRO(S) = (\sigma_1, \ldots, \sigma_n)$. Let σ_c be an operator with constant selectivity s_c such that $\underline{s}_i \leq s_c \leq \overline{s}_i$, for some $1 \leq i \leq n$, and $S' = S \cup \{s_c\}$. In $MRO(S')$, σ_c is placed between (1) σ_{i-1} and σ_i if $s_c \leq (\underline{s}_i + \overline{s}_i)/2$, or (2) σ_i and σ_{i+1} if $s_c \geq (\underline{s}_i + \overline{s}_i)/2$.*

Note that if $s_c = (\underline{s}_i + \overline{s}_i)/2$, then σ_c can be placed either between σ_{i-1} and σ_i or between σ_i and σ_{i+1} in $MRO(S')$.

Proposition 1 can be generalised to the case in which each non-constant operator has at most one constant operator nested within it. An interesting observation about the situation described in Proposition 1 is that the worst-case scenario is a max-min scenario.

PROPOSITION 2. *Let S be a strictly dominant set of n operators such that $MRO(S) = (\sigma_1, \ldots, \sigma_n)$. Let σ_c be an operator with constant selectivity s_c such that $\underline{s}_i \leq s_c \leq \overline{s}_i$, for some $1 \leq i \leq n$, and $S' = S \cup \{s_c\}$. The scenario $(\overline{s}_1, \ldots, \overline{s}_{j-1}, s_c, \underline{s}_j, \ldots, \underline{s}_n)$, in which either σ_{j-1} or σ_j is equal to σ_i, is a worst-case scenario for $MRO(S')$.*

6. MAX-MIN HEURISTIC

Computing the regret of every selection ordering for every possible scenario makes the brute-force algorithm infeasible, since there are $n!$ different orderings and 2^n scenarios, given n operators. So in order to find an efficient heuristic, we have to significantly reduce the number of orderings and scenarios. While doing so, we want to leverage the insights gained from our theoretical investigation.

Let us first look at the number of possible scenarios. As we have seen in the previous section, max-min scenarios seem to play a special role when it comes to the maximum regret of a given plan p. Intuitively this makes sense, as in an optimal plan many of the operators σ_i located towards the beginning of p with selectivities $\overline{s_i}$ will trade places with operators σ_j located towards the end of p with selectivities $\underline{s_j}$. Consequently, there tends to be a large difference between the plan p and an optimal plan for a max-min scenario, leading to a substantial (if not maximal) regret for p. So in our heuristic we aim to generate plans that perform well for max-min scenarios. This reduces the number of scenarios we have to consider from 2^n to $n + 1$.

We now turn to determining the order of the selection operators. There are two well-known basic methods for doing this (efficiently). The first one is constructing a plan by combining partial plans in a way that leads to an optimised execution order. Very often putting the partial plans together requires using a heuristic to solve a combinatorial problem. The second method is to quickly create a complete plan (e.g., by using a simple heuristic) and then try to improve the plan by rewriting it (e.g., by swapping or removing and re-inserting operators). In our approach we wanted to have both options available, so we decided to develop different variants. The complexity of our heuristic shows slight differences depending on the variant we use; however, the algorithms we apply all have polynomial complexity.

Our max-min heuristic algorithm, $H(p, q)$, which is in fact a template for a number of algorithms, is shown as Algorithm 1. It is parameterised by two inputs: p, a (possibly empty) starting plan, and q, an order in which to process operators. Clearly, to generate a complete plan the union of p and q has to contain all the operators. If the intersection of p and q is empty, our algorithm is similar to insertion sort: in turn, we consider each operator in q and place it into p at the position that minimises the regret over all max-min scenarios. If an operator in q is already present in p, then we remove it from p before re-inserting it. This is equivalent to moving an operator to a different position. Again we determine the position minimising the regret over all max-min scenarios.

Algorithm 1: $H(p, q)$

```
foreach operator t from the sequence q do
    if t is in p then remove t from pAssume p currently
    comprises i operators;
    foreach position j, 1 ≤ j ≤ i + 1, in p do
        Temporarily insert t in position j in p;
        foreach max-min scenario for p do
            Calculate the regret of plan p;
            Store the maximum regret for position j;
    Choose as the final position for t in p that which minimises
    the maximum regret;
Return p;
```

It is clear that the max-min heuristic runs in polynomial-time. For each partial plan comprising i operators, we consider $i + 1$ possible positions for the next operator. In each of these positions, we con-

sider $i+2$ max-min scenarios. Calculating the regret of a plan with n operators can be done in time $O(n \log n)$. Hence the algorithm described above has an overall complexity of $O(n^4 \log n)$ (in the worst case $i = n$ for every execution of the outer loop). However, by computing costs incrementally when an operator moves position and one max-min scenario moves to the next, we can implement the heuristic to run in time $O(n^3)$.

EXAMPLE 6. *Recall from Example 5 the set $S = \{\sigma_1, \sigma_2, \sigma_3\}$ of selection operators, with selectivities $s_1 = [.2, .8]$, $s_2 = [.3, .5]$ and $s_3 = [.1, .4]$. Consider our max-min heuristic algorithm, $H(p, q)$, with initial plan $p = \sigma_3\sigma_1$ and remaining operator $q = \sigma_2$. Since p consists of two operators, σ_2 should be checked in three positions: before σ_3, after σ_1 and between them. For each position and resulting plan, the regret is calculated under all max-min scenarios, of which there are four in this example.*

As an example, consider the plan in which σ_2 is placed between σ_3 and σ_1. The regret will be calculated for the scenarios $(\underline{s}_3, \underline{s}_2, \underline{s}_1)$, $(\overline{s}_3, \underline{s}_2, \underline{s}_1)$, $(\overline{s}_3, \overline{s}_2, \underline{s}_1)$ and $(\overline{s}_3, \overline{s}_2, \overline{s}_1)$. The maximum regret for this plan is 0.3 which occurs in scenario $(\overline{s}_3, \overline{s}_2, \underline{s}_1)$.

Finally, the solution will be the plan with the smallest maximum regret, which happens to be $\sigma_3\sigma_2\sigma_1$. As a matter of fact, the solution returned by the max-min heuristic is the same as the actual minmax regret solution, as was shown in Example 4. ◇

In the following two subsections, we consider various criteria for choosing an initial plan and for ordering the remaining operators.

6.1 Choosing an Initial Plan

Even though we can run our heuristic with an empty initial plan p, i.e., building a solution by inserting all operators one by one, often it makes sense to start with a prebuilt partial plan.

One particular and important case is that of dominant operators. Given a set S of operators, if we can identify a subset $S' \subseteq S$ of dominant operators, we know that we can find an optimal solution p' for S' quickly and that the relative order of the operators in p' will not change in any optimal plan for S (see Theorem 2). Thus, taking p' as the initial plan when calling $H(p, q)$ makes good sense. However, there may be different ways to choose S', as in general there may be more than one such dominant set. If we have more than one option, we can use the following criteria to make a decision: choose the subset S' (1) with the maximum cardinality or (2) whose operators have the largest total width. While the intuition in choosing the largest subset is clear, the motivation for maximising the width may not be so evident: the wider an interval, the more impact it has on the solution, so it is more important to slot wide intervals into the correct positions. As we often encountered several subsets sharing the same maximum cardinality, we introduced a tie-breaker: choose the subset S' (3) with the maximum cardinality whose total width is greatest. In our experiments, we found that this third approach gave the best overall results.

EXAMPLE 7. *Recall from Example 6 the set $S = \{\sigma_1, \sigma_2, \sigma_3\}$ of selection operators, with selectivities $s_1 = [.2, .8]$, $s_2 = [.3, .5]$ and $s_3 = [.1, .4]$. Set S has two dominant subsets: $S_1 = \{\sigma_1, \sigma_3\}$ and $S_2 = \{\sigma_2, \sigma_3\}$. Both obviously satisfy criterion (1) above, being of maximum cardinality. However, if we use criterion (2), namely the set which has operators with the largest total selectivity width, then we will choose S_1 since its total width is 0.9 while that of S_2 is 0.5. S_1 would also be chosen according to criterion (3).*

After choosing the preferable subset, we need to produce initial plan p by sorting the operators in nondecreasing order of their minimum (or maximum) selectivities. Therefore, $p = \sigma_3\sigma_1$ when S_1 is chosen, while $p = \sigma_3\sigma_2$ if S_2 is chosen. ◇

Having an initial plan allows us to combine our algorithm with other heuristics. We can take the output of another algorithm as our initial plan p and then refine this result by running $H(p, q)$ on it. Moreover, we can use the output of $H(p, q)$ as input for another iteration of our own heuristic.

6.2 Ordering Criteria

Since our algorithm makes only a single pass over all the operators when (re-)inserting them into the plan, the order in which operators are considered may have a significant impact on the final outcome. For example, when inserting selections into an empty initial plan, operators considered earlier are tested in fewer positions relative to each other compared to those considered later.

We have considered two different ordering criteria in our experiments: interval *midpoint* (denoted by M) and interval *width* (denoted by W). Given a selectivity interval $s = [\underline{s}, \overline{s}]$, the midpoint of s is $(\underline{s} + \overline{s})/2$ while the width of s is $\overline{s} - \underline{s}$. In each case, operators can be ordered by non-decreasing (denoted $+$) or non-increasing (denoted $-$) values. Overall, the ordering criteria are denoted by $M+$, $M-$, $W+$ and $W-$. So, for example, $W+$ stands for operators being considered in non-decreasing order of their selectivity interval width.

7. EXPERIMENTAL RESULTS

We evaluated the max-min heuristic experimentally, measuring the impact of different parameters on its performance. We also implemented the brute-force algorithm for finding optimal solutions in order to evaluate how well the heuristic performs.

A commodity PC, with 8 GB RAM, Intel Core i5 processor running at 3.19 GHz and Windows 7 Enterprise (64-bit), was used to perform the experiments. The minmax regret brute-force algorithm and max-min heuristic were implemented in Java and compiled with the Eclipse IDE (Juno release), which is JDK compliant and uses the JavaSE-1.7 execution environment. The Star Schema Benchmark (SSB) queries were run on a simulation platform written in Ruby 1.9.3.

7.1 Generating Test Data

We first generated a synthetic data set to investigate the performance of our heuristic. Each test case corresponded to a set of k selection operators, with k ranging from 2 to 10, and for each k we generated a hundred different sets. While $k = 2$ is not hard to solve, it was included for verification purposes (any heuristic has to be able to find the optimal plan for this simple case). Ten operators was the upper limit we were able to solve optimally, checking $10! \cdot 2^{10}$ (≈ 3.7 billion) different costs for each test case. For each set of selection operators we determined the lower and upper bounds of their selectivity intervals by generating $2k$ uniformly distributed random numbers between 0 and 1.

For real-world data, we used the Enron email data set, as introduced in Example 1. Once again, test queries used from 2 to 10 operators/predicates. For each $n \in [2, 10]$, 20 queries were generated, each with one predicate on `subject` and $n-1$ predicates on `body`. The 20 queries were generated by randomly selecting from 40 keywords for `subject` and 45 keywords for `body`, and were checked to ensure that each returned a nonempty answer.

We also evaluated minmax regret optimisation using a version of SSB with data skew [27] (SSB itself is a variation of the TPC-H benchmark). We generated benchmark data with a scaling factor of 1, meaning that the central facts table, *lineorder*, contains 6,000,197 tuples, and joined all dimensional tables to the *lineorder* table. We then randomly picked from two to ten attributes from a subset of all available attributes to generate queries. Queries basically consist of

a conjunctive predicate whose clauses are made up of the selected attributes compared to a random value taken from the attribute's domain, using a less-than or greater-than operator. The following predicate is an example generated in our experiments:
`orderKey < 2964443 and linenumber > 5 and quantity < 29`.

7.2 Parameters

For the synthetic and Enron data sets, we looked at the effects of the ordering criteria and the choice of initial plan on the quality of our heuristic. Additionally, we investigated the impact of running our heuristic multiple times, using the output of one phase as the initial plan of the next phase.

We measure the performance of our heuristic by defining the *regret ratio* $\lambda(S)$, which is the regret computed by $H(p, q)$ divided by the optimal regret. More formally, given a set S of selection operators, let us denote the set of possible plans by $P(S)$ and the set of possible scenarios by $X(S)$. Recall from Section 3.1 that $R(P(S), X(S))$ then denotes the optimal regret. Then

$$\lambda(S) = \frac{R(H(p,q), X(S))}{R(P(S), X(S))}$$

We only calculate $\lambda(S)$ using the above formula when the optimal minmax regret is non-zero. As mentioned in Section 5, the optimal minmax regret is zero only when S forms a strictly dominating set. For such cases, our max-min heuristic always finds the optimal minmax regret solution, so we define $\lambda(S)$ to be one.

In view of having multiple test cases per number of selection operators, we calculate the *average regret ratio* and the *worst regret ratio* (simply the maximum value of $\lambda(S)$).

For the Enron data, we calculated selectivity intervals for the `like` predicates as described in the Introduction. For each selected keyword, we ran queries to find the minimum selectivity (given by exact matches of the keyword) and the maximum selectivity (given by the minimum selectivity of all 2-grams of the keyword). This gave rise to a range of intervals: those with small values such as $[0.0004, 0.01]$ for keyword 'progress' in the `subject`, those with larger values such as $[0.6, 0.7]$ for 'you' in the `body`, and those with a big range such as $[0.07, 0.6]$ for 'price' in the `body`.

For the Star Schema Benchmark we created some very rudimentary histograms by dividing the domain of an attribute into equal-sized ranges, counting the number of tuples that fall into each range. We do not keep any further information on the distribution of tuples within each range of a histogram. For example, Figure 1 shows the histogram for the attribute ordtotalprice, consisting of 20 ranges each covering roughly 18,000 different values, e.g., bucket #1 covers the range from 1 to 17,673.

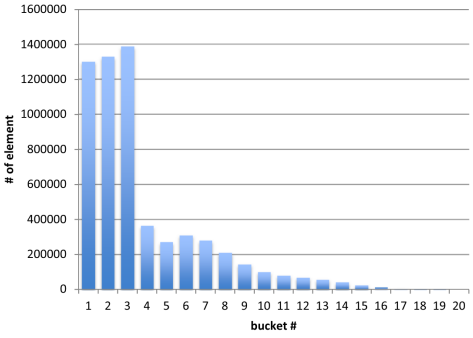


Figure 1: Histogram for attribute ordtotalprice.

This basic information allows us to determine intervals for the selectivities of selection operators. For a "less than" / "greater than" operator, we know that all histogram ranges exclusively covering smaller/larger values have to be included fully. However, for the range the predicate value falls into, we do not know precisely how many elements will be selected. In extreme cases, none or all of the elements satisfy the predicate, giving us the lower and upper bound for the selectivity. Example 8 illustrates this with concrete values.

EXAMPLE 8. *Given the histogram for attribute X below and the predicate $X < 126$, we can compute the lower bound and upper bound for the selectivity as follows: lower bound* = $\frac{200}{1000} = 0.2$, *upper bound* = $\frac{200+100}{1000} = 0.3$. ◇

Range	# of elements
1-100	200
101-200	100
201-300	400
301-400	300

Many sophisticated query optimisation techniques, such as least expected cost (LEC), assume that they have access to probability distributions of parameter values. LEC needs this to be able to compute utilities [7]. However, in our case we only have very rudimentary statistics, since we do not know anything about the distribution of attribute values within a range. The best we can do is to fall back on the assumption of uniform distribution, approximating the distribution using a mean value (this is also what least specific cost (LSC) optimisation would do in this case). For example, applying this method to the numbers given in Example 8 would yield a selectivity of 0.225 for the predicate $X < 126$. We compare our minmax regret optimisation technique to a mean-value-based approach using SSB data. Additionally, we do a comparison with a simple midpoint heuristic, i.e., sorting the intervals in non-decreasing order of their midpoint.

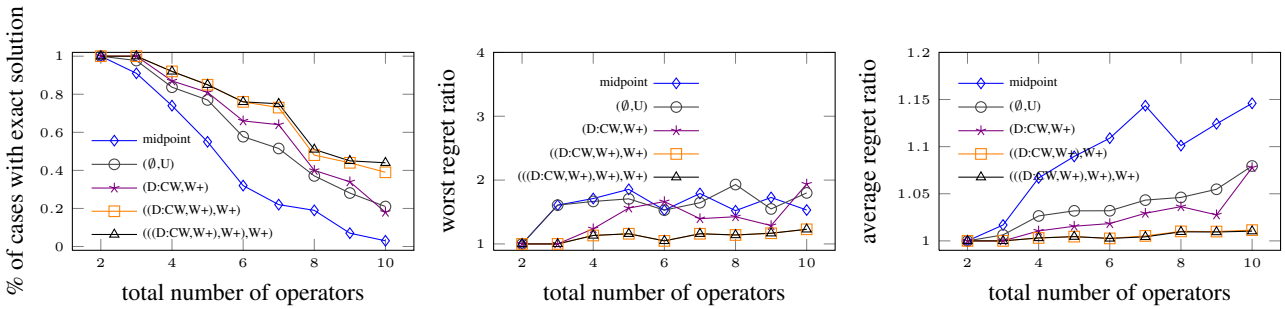
7.3 Results

First we present the results obtained studying the different variants of the max-min heuristic on the synthetic and Enron data, and then move on to the Star Schema Benchmark results.

7.3.1 Synthetic and Enron Data Sets

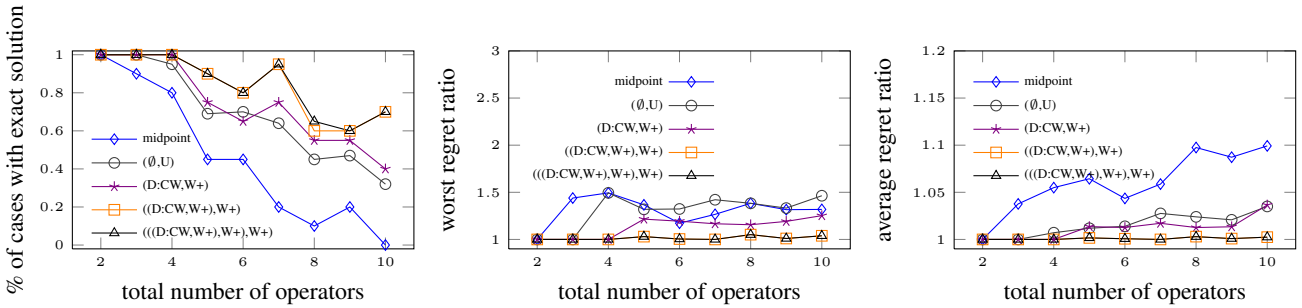
We experimented with a number of operator ordering criteria and initial plans for the max-min heuristic. These included starting with an empty initial plan ($\emptyset$), considering random operator ordering (U), ordering by midpoint (M- and M+) and ordering by width (W- and W+). We briefly summarise the findings of our experiments here. Overall, the W+ ordering (non-decreasing width) performed best with an overall average regret ratio of 1.03 and an overall worst regret ratio of 1.94. W- was often even worse than a random order, while M+ and M- sometimes generated plans whose regret ratio was above 3. We also ran a midpoint heuristic that simply ordered the intervals in non-decreasing order of their midpoints (not going through all max-min scenarios). The midpoint heuristic was often worse than running the max-min heuristic with a random order.

While W+ ordering performs better than the M+, M-, and W- max-min heuristics and the midpoint heuristic, it is still not significantly better than the random ordering. In a second phase of our evaluation we seeded our heuristic with an initial plan. The results for initial plan D:CW with operator ordering W+ were best (D:CW stands for the largest subset of dominant operators, and, in case of a tie, the one with the greatest total width of the operators) in terms of the percentage of exact solutions and the average regret ratio. The



(a) Percentage of exact solutions (b) Worst regret ratio (c) Average regret ratio

Figure 2: Results for synthetic data set.



(a) Percentage of exact solutions (b) Worst regret ratio (c) Average regret ratio

Figure 3: Results for Enron data set.

results for the worst case regret ratio were rather inconclusive, so we tried to improve on this by running multiple phases of our heuristic.

Figures 2(a), (b) and (c) show the results for running our heuristic multiple times. This means that we take the output of running one phase of our heuristic and use it as the initial plan for the next phase. The figures show the results for starting off by running (D:CW, W+) first and then executing two more phases.

As can be seen, this variant clearly outperforms the baseline algorithm (∅,U), the midpoint heuristic, and the other variants in all respects. For example, for 10 operators, the worst regret ratio is less than 1.23 and the average ratio is approximately 1.01, compared to approximately 1.94 and 1.08, respectively, for running only a single phase of the heuristic. Moreover, running one additional phase improves the quality of the generated plan significantly, but running another phase makes almost no difference.

The results on the Enron data set (Figure 3) showed similar trends, but were more impressive in every respect. The two- and three-phase variants of the max-min heuristic found the minmax optimal solution in 84% of cases, had a worst regret ratio of only 1.05, and an average regret ratio of less than 1.001. By contrast, the midpoint heuristic had a worst regret ratio of over 1.49, an average of 1.06, and did not find single minmax optimal solution with 10 operators.

To highlight how bad a poor choice of selectivity can be, we also tested using the minimum selectivity values of the intervals (as would be done if estimates were based simply on the selectivity of the keywords themselves). This produced a worst case regret ratio of *almost 30 for only 5 operators*.

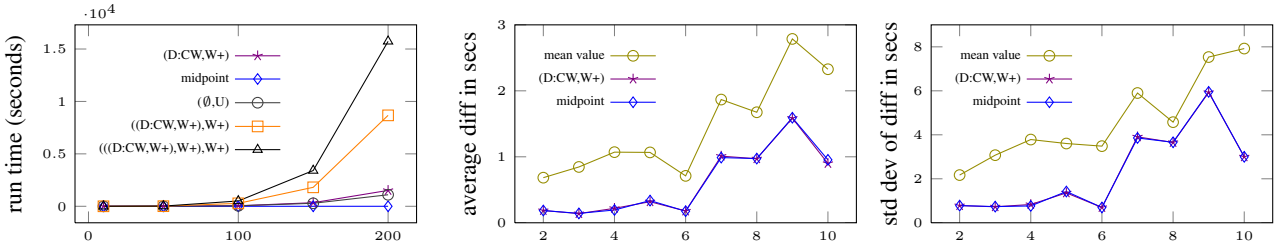
Figure 4(a) shows the run time of the W+ ordering variant (single and multiple phases) together with the baseline algorithm (∅,U) when generating plans for up to 200 operators using the synthetic data set (the run times on the Enron data set were similar). Unsurprisingly, the variants midpoint, (∅,U), and (D:CW,W+) have the fastest run times, as they only sort a set of operators or execute a single operator insertion phase. Furthermore, it can be clearly seen that the additional run time of (((D:CW,W+),W+),W+) does not pay off, since it produces plans that are only marginally better than those of ((D:CW,W+),W+).

7.3.2 Star Schema Benchmark

We optimised the generated SSB queries using minmax regret optimisation, a mean-value-based approach, and also computed the optimal execution plan using exact selectivities, which means that we are comparing actual query plan costs rather than regret ratios.

Figure 4(b) shows the results for the average difference in costs between the query execution plans generated by different methods and the optimal plan (every data point in the diagram averages the measurement obtained by running 100 different queries). We only include two variants of minmax regret optimisation, (D:CW,W+) and the simple midpoint heuristic, as for SSB no major differences were discernible between the different variants in terms of the quality of the query plans. Surprisingly, the midpoint heuristic, although not very good at optimising the regret ratio, seems to produce efficient query execution plans. Considering the fact that all queries had an average run time between 60 and 80 seconds, the numbers shown in Figure 4(b) may not seem like a big difference. However, this shows that minmax regret optimisation delivers better plans than a mean-value-based approach.

More important is the robustness of the approaches, i.e., how good are they in avoiding bad plans? Figure 4(c) shows the standard deviation of the cost difference to the optimal plan, illustrating that the mean-value-based approach is more erratic than minmax regret optimisation. The most extreme case for all SSB queries was a mean-value-optimised plan more than doubling the run time of the optimal plan (from 60s to 135s), while for minmax regret optimisation the very worst plan added roughly 50% more to the cost of the optimal plan (from 60s to 92s).



(a) Run time of the heuristic (synthetic data set) (b) Average difference in cost to opt (c) Std dev of difference in cost to opt

Figure 4: Results on run time (synthetic data set) and difference in cost to optimal (SSB).

8. CONCLUSION

We have investigated query optimisation under partial ignorance, in particular ordering selection operators optimally if their selectivities are defined by an interval rather than an exact value. The strategy we employed, minmax regret optimisation (MRO), is considered to be a pessimistic approach compared to other techniques from decision theory. In our opinion this makes it well-suited to query optimisation in database systems, which should be about avoiding bad plans rather than finding the best one. There is one major drawback, though: selection ordering under MRO becomes NP-hard. However, we have shown that special cases can be solved efficiently and that heuristics can quickly find good solutions.

For future work we plan to extend our approach to costs described by intervals and relative regret, i.e., considering the ratio of the cost of a plan to the optimal plan for a scenario rather than the difference. Also interesting are other operators, such as joins, whose ordering is heavily influenced by selectivities as well and suffers from similar issues: it is hard to obtain exact values. Further topics we would like to tackle are finding approximation algorithms with proven bounds and modelling correlation of query predicates. Nevertheless, we think this is an important first step in discovering new approaches for making query optimisers more robust and one of our medium term goals is to build a general framework for query optimisation under partial ignorance.

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