

*Erkenntnis*, 1998, Vol. 49, 1998:1-20

**Hypothetico-Deductivism: The Current State of Play;  
The Criterion of Empirical Significance: Endgame**

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**Abstract:** Any precise version of H-D needs to handle various problems, most notably, the problem of selective confirmation: Precise formulations of H-D should not have the consequence that where S confirms T, for any T', S confirms T&T'. It is the perceived failure of H-D to solve such problems that has led John Earman to recently conclude that H-D is "very nearly a dead horse". This suggests the following state of play: H-D is an intuitively plausible idea that breaks down in the attempt to give it a precise formulation. Indeed I think that fairly captures the view among specialists in the field of confirmation theory. Here I argue that the truth about H-D is largely the reverse: H-D can be given a precise formulation that avoids the longstanding technical problems, however, it relies on a fundamentally unsound philosophical intuition. The bulk of this paper involves reviewing the problems affecting previous attempts at giving precise formulations of H-D and displaying some recent versions that can handle these problems. It then briefly explains why the basic intuition behind H-D is itself unsound, namely, because H-D involves a tacit assumption of inductive scepticism. Finally, the historical relation between H-D and the positivists' quest for a criterion of empirical significance will be reconsidered with the surprising result that having glossed H-D as fundamentally unsound it is concluded that a sound version of the criterion of empirical significance is now available. The demarcation criterion, the positivists' philosopher's stone that serves to separate claims with empirical significance from claims lacking empirical significance having finally been found, it is argued that we should regard empirical significance as just one among a variety of virtues and not follow the positivists in taking it to be a *sin qua non* for all meaningful statements.

## 1. Introduction: The Perceived State of Play

Clark Glymour, in summing up various strategies for relating evidence to theory, says the following concerning hypothetico-deductivism, hereafter H-D,

At one time, anyway, the most popular version of the evidential connection took it to depend on a deductive connection going in the opposite direction. Evidence confirms a theory if the evidence can be deduced, in an appropriate way, from the theory. It may, far all I know, still be the most popular version. (Glymour 1980, p.12)

Any precise version of H-D, besides capturing the general insight of H-D, that one confirms a theory by testing its consequences, needs to handle the following problems,

(1) The problem of selective confirmation, or, tacking by conjunction: Precise formulations of H-D should not have the consequence that where S confirms T, for any T' such that T&T' is contingent then S confirms T&T';

(2) The problem of pseudo confirmations, or, tacking by addition: Precise formulations H-D should not have the consequence that where S confirms T, for any S' such that SvS' is contingent, SvS' confirms T;

and

(3) The problem of instance confirmation: Precise formulations of H-D should allow that instances confirm their generalizations.

Its is the perceived failure of H-D to solve such problems that has lead John Earman to recently conclude that H-D is "very nearly a dead horse" (Earman 1992, p.63-64). Earman cites Glymour (1980) where the emphasis is squarely on H-D's perceived inability to handle the problem of selective confirmation.<sup>1</sup> It is interesting to juxtapose Glymour's observation that H-D may still be the most popular account of confirmation with Earman's nearly-a-dead-horse-verdict. The difference here is not merely a reflection of the 12 years that separate Glymour's observation from Earman's

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<sup>1</sup> Glymour (1980a) makes further objections to H-D. Gemes (1993) offers replies to these objections.

verdict. While Earman is particularly concerned with the technical criticisms that have long plagued all attempts to give a precise formulation to H-D, Glymour's observation is, I suspect, based more on the fact that generally philosophers, perhaps leaving aside specialists in confirmation theory, are still largely sympathetic to H-D's central claim that confirmation of a theory comes through testing its consequences. This suggests the following state of play: H-D is an intuitively plausible idea that breaks down in the attempt to give it a precise formulation. Indeed I think that fairly captures the view among specialists in the field of confirmation theory. Here I will argue that the truth about H-D is largely the reverse: H-D can be given a precise formulation that avoids the longstanding technical problems such as (1), (2) and (3), however, it relies on a fundamentally unsound philosophical intuition. The bulk of this paper will be concerned with reviewing the problems affecting previous attempts at giving precise formulations of H-D and displaying some recent versions that can handle these problems. I will then try to briefly explain why I think the basic intuition behind H-D is itself unsound. Finally, the historical relation between H-D and the positivists' quest for a criterion of empirical significance will be reconsidered with the surprising result that having glossed H-D as fundamentally unsound it is concluded that a sound version of the criterion of empirical significance is now available.

## 2. Canonical Formulations of H-D and their Problems

Simple canonical versions of H-D such as  
 (H-D1)  $S$  confirms  $T$  iff  $S$  and  $T$  are contingent and  $T \vdash S$ ,  
 fail to address any of problems (1), (2) or (3). However, Carl Hempel (Hempel 1965, p.26-7), a founding father of H-D, and, more recently, Paul Horwich (Horwich 1983, p.58), have suggested the following alternative version of H-D,  
 (H-D2)  $S$  confirms  $T$  iff  $S$  and  $T$  are contingent and for some  $S'$  and  $S''$ ,  $S \vdash S' \& S''$ ,  
 $S' \& T \vdash S''$ , and  $S' \vdash S''$

Hempel introduced his version of (H-D2) under the name "the prediction-criterion of confirmation". While (H-D2) fails to address problems (1) or (2) it successfully deals with (3), producing the result, for instance, that 'Ra&Ba' confirms '(x)(Rx ⊃ Bx)'. More generally (H-D2) has the consequence that where αc&βc is an instance of the universal generalization (x)(αx ⊃ βx), αc&βc confirms (x)(αx ⊃ βx). However, as noted in Gemes (1990), (H-D2) also produces the disastrous result that 'Ra&~Ba' confirms '(x)(Rx ⊃ Bx)' since,

$$\begin{aligned} Ra\&\sim Ba \not\vdash & \vdash ((x)(Rx \supset Bx) \supset (Ra\&\sim Ba)) \& (Ra\&\sim Ba), \text{ and} \\ ((x)(Rx \supset Bx) \supset (Ra\&\sim Ba)) \& (x)(Rx \supset Bx) & \vdash Ra\&\sim Ba, \text{ and} \\ ((x)(Rx \supset Bx) \supset (Ra\&\sim Ba)) & \not\vdash (Ra\&\sim Ba). \quad [\alpha \not\vdash \beta \text{ means } \beta \text{ is not a} \\ & \text{consequence of } \alpha] \end{aligned}$$

Nor will it do to merely supplement (H-D2) with the condition that S be consistent with T, since this would still leave us with a version of H-D that has the consequence that for any contingent S and T such that S is consistent with T, S confirms T, since in this case it is still true that  $S \not\vdash (T \supset S)\&S$ ,  $T\&(T \supset S) \vdash S$ , and  $(T \supset S) \vdash S$ .<sup>2</sup>

More commonly, advocates of H-D troubled by problem (3) have offered formulations of H-D which make use of the notion of relative confirmation, such as

$$(H-D3) \text{ S confirms T relative to background evidence b iff } (T\&b) \text{ is contingent and } (T\&b) \vdash S \text{ and } b \vdash S.$$

While (H-D3) does not have the consequence that, for example, 'Ra&Ba' confirms '(x)(Rx ⊃ Ba)' it does have the consequence that 'Ba' confirms '(x)(Rx ⊃ Ba)' relative to 'Ra'. This has been deemed a close enough surrogate to instance confirmation. However (H-D3) does not address problems (1) and (2). Indeed, until recently, no formal version of H-D has come close to successfully addressing these problems.

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<sup>2</sup> For a fuller discussion of these issues see Gemes (1990).

Gerhard Schurz has lately produced a precise formulation of H-D which successfully handles (1) and (2). Furthermore, with a little modification along lines already suggested by Schurz, it can handle problem (3).

### 3. Schurz's New Formulation

Before we give Schurz's formulation of H-D we need to get some preliminary definitions on the table. Basically Schurz's strategy is to determine whether S confirms T by first constructing canonical versions of S and T and then making sure that the deduction of S from T involves no extraneous elements. Thus where  $T \vdash S$ , and hence  $T \& T' \vdash S$ , this does not make for S confirming T&T' because T' is extraneous to the derivation of S. Thus where  $T \vdash S$ , and hence  $T \vdash S \vee S'$ , S∨S' does not confirm T because S∨S' is entailed by T irrespective of the nature of S'. To capture this formally Schurz introduces the notions of conclusion relevant deductions, premise relevant deductions and relevant consequence elements as follows:

Where  $\alpha \vdash \beta$ ,  $\alpha \vdash \beta$  is conclusion relevant deduction iff no predicate in  $\beta$  is such that the replacement of some of its occurrences in  $\beta$  by any other predicate of the same arity results in a  $\beta'$  such that  $\alpha \vdash \beta'$ . (Cf. Schurz 1991, p. 409-411).<sup>3</sup>

Where  $\alpha \vdash \beta$ ,  $\alpha \vdash \beta$  is a premise relevant deduction iff (i) there is no single occurrence of a predicate in  $\alpha$  such that its replacement in  $\alpha$  by any other predicate of the same arity results in an  $\alpha'$  such that  $\alpha' \vdash \beta$  and (ii) and there are no predicate occurrences in  $\alpha$  such that they are replaceable by other predicates of the same arity resulting in an  $\alpha'$  such that  $\alpha' \not\vdash \alpha$ . (Cf. Schurz 1991, p. 421-422).

$\beta$  is a relevant consequence element of  $\alpha$  iff  $\alpha \vdash \beta$  is a conclusion relevant deduction and there exists no finite set S of conclusion relevant consequences of

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<sup>3</sup> So, for instance,  $(x)Fx \vdash FavGb$  is not a conclusion relevant deduction since the replacement of 'G' in 'FavGb' by any other predicate does not lead to the disruption of the deduction.

$\alpha$  such that  $\beta$  is logically equivalent to the conjunction of the members of  $S$  and for each member  $\phi$  of  $S$ ,  $\phi$  is shorter than  $\beta$ . (Cf. Schurz 1994, p. 184)<sup>4</sup>

We are now in a position to give Schurz's new definition of H-D:

(H-D4)  $S$  confirms  $T$  iff for some  $S'$  and  $T'$  which are equivalent to  $S$  and  $T$ ,  $S'$  is a conjunction of relevant consequence elements of  $S$ ,  $T'$  is a conjunction of relevant consequent elements of  $T$ , and  $T'$  and  $S'$  are contingent and  $T' \vdash S'$  and the deduction  $T' \vdash S'$  is a conclusion and premise relevant deduction. (Cf. Schurz, *ibid.*, and Gemes 1994, p.175).

(H-D4) does not allow that where  $S$  confirms  $T$ , for any  $T'$  if  $T \& T'$  is contingent then  $S$  confirms  $T \& T'$ , because in such cases  $T \& T' \vdash S$  is not a premise relevant deduction.

(H-D4) does not allow that where  $S$  confirms  $T$ , for any  $S'$  such that  $S \vee S'$  is contingent,  $S \vee S'$  confirms  $T$ , because, in such cases  $T \vdash S \vee S'$  is not a conclusion irrelevant deduction. Thus (H-D4) successfully handles problems (1) and (2). However, as it stands, (H-D4) does not allow for instance induction. In order to deal with problem (3) Schurz suggests (H-D4) be supplemented with the following principle

(S.C.) If  $S$  confirms  $T$  and  $S^*$  logically implies  $S$  and is consistent with  $T$ , then  $S^*$  confirms  $T$  (Schurz 1991, p. 394 and 1994, p. 183-4)

The idea here is that while (H-D4) does not entail that, for instance, ' $Ra \& Ba$ ' confirms ' $(x)(Rx \supset Bx)$ ', it does entail that ' $Ra \supset Ba$ ' confirms ' $(x)(Rx \supset Bx)$ ', and hence by (S.C.), since ' $Ra \& Ba$ ' logically implies ' $Ra \supset Ba$ ' and is consistent with ' $(x)(Rx \supset Bx)$ ', ' $Ra \& Ba$ ' confirms ' $(x)(Rx \supset Bx)$ '. Now, in fact, strictly speaking, (S.C.) and (H-D4) are incompatible. (H-D4) has as a consequence that ' $Ra \supset Ba$ ' confirms ' $(x)(Rx \supset Bx)$ ' and ' $Ra \& Ba$ ' does not confirm ' $(x)(Rx \supset Bx)$ ', while (S.C.) has the consequence that if ' $Ra \supset$

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<sup>4</sup> Thus ' $Fa \supset Gb$ ' is not a relevant consequent element of ' $Fa \& (Fa \supset Gb)$ ' since ' $Fa \& (Fa \supset Gb)$ ' is equivalent to the conjunction of the members of the set  $\{ 'Fa', 'Gb' \}$  and each member of the set is shorter than ' $Fa \supset Gb$ '. The recourse to relevant consequence elements is needed otherwise Schurz's definition of H-D, (H-D5) below, would have the consequence that ' $Gb$ ' confirms ' $Fa \& (Fa \supset Gb)$ ' since  $Fa \& (Fa \supset Gb) \vdash Gb$  is a premise and conclusion relevant deduction. For more on this see Gemes (1994) and Schurz (1994).

Ba' confirms '(x)(Rx  $\supset$  Bx)' then so does 'Ra&Ba'. This follows since (H-D4) gives both necessary and sufficient conditions for confirmation. We could render (H-D4) and (S.C.) compatible by construing (H-D4) as giving only sufficient conditions for confirmation, replacing 'E confirms H iff' in (H-D4) with 'E confirms H if'. However I think Schurz is best understood as suggesting that (H-D4) incorporate (S.C.) in the following manner

(H-D5) S confirms T iff S is consistent with T and for some S' and T', S' is a consequence of S and T is equivalent to T', S' is a conjunction of relevant consequent elements of S, T' is a conjunction of relevant consequent elements of T, and T' and S' are contingent and T'  $\vdash$  S' and the deduction T'  $\vdash$  S' is a conclusion and premise relevant deduction.

(H-D5) does handle problems (1), (2) and (3), that is to say, it disallows tacking by conjunction and disjunction and allows for instance confirmation.

One of the problems with (H-D5) is that it loses some of the deductive flavor of more conventional versions of H-D. Indeed, to some degree, it seems to be more in line with an inductivist conception of confirmation. For instance, it allows that 'Fb' confirms 'Fa'. This follows since 'Fb' is consistent with 'Fa', '( $\exists$ x)Fx' is a consequence of 'Fb', 'Fa' is a one conjunct conjunction of relevant consequence elements of 'Fa', '( $\exists$ x)Fx' is a one conjunct conjunction of relevant consequence elements of '( $\exists$ x)Fx', and Fa  $\vdash$  ( $\exists$ x)Fx is a conclusion and premise relevant deduction. On the other hand, (H-D5) goes some way beyond induction since it also allows that 'Fb&~Fc&~Fd&~Fe' confirms both 'Fa' and '~Fa' since 'Fb&~Fc&~Fd&~Fe' entails both '( $\exists$ x)Fx' and '( $\exists$ x)~Fx'.

(H-D5) also has the very strange result that 'Ra&~Rb&~Bb' confirms both '(x)(Rx  $\supset$  Bx)' and '~(x)(Rx  $\supset$  Bx)'. 'Ra&~Rb&~Bb' confirms '(x)(Rx  $\supset$  Bx)' since it is consistent with '(x)(Rx  $\supset$  Bx)' and it entails 'Rb  $\supset$  Bb' and both 'Rb  $\supset$  Bb' and '(x)(Rx  $\supset$  Bx)' are conjunctions of their own relevant consequence elements and the deduction (x)(Rx  $\supset$  Bx)  $\vdash$  Rb  $\supset$  Bb is premise and conclusion relevant. 'Ra&~Rb&~Bb' confirms '~(x)(Rx  $\supset$  Bx)' since it is consistent with '~(x)(Rx  $\supset$  Bx)' and it entails '( $\exists$ x)Rx&(x)~Bx' and both '( $\exists$

$x)Rx \& (\exists x) \sim Bx$ ' and ' $\sim(x)(Rx \supset Bx)$ ' are conjunctions of their own relevant consequence elements and the deduction  $\sim(x)(Rx \supset Bx) \vdash (\exists x)Rx \& (\exists x)\sim Bx$  is premise and conclusion relevant.

Generally the consistency principle,

(C.P.) If E confirms H then it does not confirm  $\sim H$ ,<sup>5</sup>

is taken as a adequacy condition for any definition of confirmation. Moreover, H-D, which in all its earlier formulations requires H to entail contingent E if E is to confirm H, is generally taken to satisfy (C.P.). This is good reason to conclude that (H-D5) is not an adequate representation of H-D.<sup>6</sup>

Above we saw that in order to handle problem (3), the problem of instance confirmation, Schurz recommends (H-D4) be supplemented with the Principle of Strengthening the Confirmans, (S.C.). But strictly speaking (H-D4) and (S.C.) are inconsistent. To incorporate (S.C.) within a version of something like (H-D4) we constructed (H-D5). However (H-D5) runs foul of the consistency principle (C.P.). In order to handle problem (3) without recourse to a definition embodying principle (S.C.) we can construct the following version of (H-D4) which like the old (H-D3) makes use of the notion of relative confirmation,

(H-D6) S confirms T relative to B iff S is consistent with T and B and for some S' and (T&B)', S' is equivalent to S, (T&B)' is equivalent to (T&B), S' is a conjunction of

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<sup>5</sup> This is not to be confused with the much more contentious principle that if E confirms H and H' is inconsistent with H then E does not confirm H'. It is one thing to say that there are cases where evidence confirms both a claim and some rival claim and quite another to say there is evidence that confirms a claim and its negation.

<sup>6</sup> Note, (H-D4) when combined with (S.C.) produces these same strange results. This follows even where (H-D4) is construed as given only sufficient rather than necessary and sufficient conditions for confirmation.



relevant consequence elements of S, (T&B)' is a conjunction of relevant consequent elements of (T&B), and (T&B)' and S' are contingent and (T&B)'  $\vdash$  S' and the deduction (T&B)'  $\vdash$  S' is a conclusion and premise relevant deduction.

(H-D6) handles problems (1) and (2), and, while it does not produce (H-D5)'s result that, for instance, 'Ra&Ba' confirms '(x)(Rx  $\supset$  Bx)', it produces the surrogate result that 'Ba' confirms '(x)(Rx  $\supset$  Bx)' relative to 'Ra'. (H-D6) does not yield the inductivist result that 'Fb' confirms 'Fa' nor does it have (H-D5)'s strange consequences that 'Fb&~Fc&~Fd&~Fe' confirms both 'Fa' and '~Fa' and 'Ra&~Rb&~Bb' confirms both '(x)(Rx  $\supset$  Bx)' and '~(x)(Rx  $\supset$  Bx)'.

(H-D6) does however have the result that '( $\exists$ x)Fx' confirms 'Fa' relative to any tautology. Furthermore, while (H-D6) has the desirable result that evidence 'Ha&Rb&Wc&Dd' confirms '(x)(Hx  $\supset$  Rx)&(x)(Wx  $\supset$  Dx)' relative to the background claim 'Ra&Hb&Dc&Wd', it also produces the strange result that the same evidence relative to the same background claim confirms '(x)(Hx  $\supset$  Rx)&(x)(Wx  $\supset$  Dx)&(x)(Hx  $\supset$  Wx)&(x)(Rx  $\supset$  Dx)'. The later result follows since '(x)(Hx  $\supset$  Rx)&(x)(Wx  $\supset$  Dx)&(x)(Hx  $\supset$  Wx)&(x)(Rx  $\supset$  Dx)&(Ra&Hb&Dc&Wd)' is equivalent to '(x)(Hx  $\supset$  Wx)&(x)(Rx  $\supset$  Dx)&(x)(Wx  $\supset$  Rx)&Ra&Hb&Dc&Wd' which is a conjunction of its own relevant consequence elements and the deduction (x)(Hx  $\supset$  Wx)&(x)(Rx  $\supset$  Dx)&(x)(Wx  $\supset$  Rx)&Ra&Hb&Dc&Wd  $\vdash$  Ha&Rb&Wc&Dd is a conclusion and premise relevant deduction. So, for instance, finding that a is a human and b is rational and c is water and d is H<sub>2</sub>O confirms the claim that being human and being rational are (materially) equivalent and being water and being H<sub>2</sub>O are (materially) equivalent and being human and being water are (materially) equivalent and being rational and being H<sub>2</sub>O are (materially) equivalent relevant to the background claim a is rational and b is human and c is H<sub>2</sub>O and d is water.

Concerning these cases there are two natural responses. The first response is to say that the intuitive notion of hypothetico-deductive confirmation is committed to the claim that these are actual cases of confirmation. Having made this response one may

then go on defend H-D by arguing that they do in fact constitute genuine cases of confirmation, or, alternatively one might argue that they show that H-D itself is fundamentally flawed. The second response is to say that H-D is not committed to these being genuine cases of confirmation and hence that (H-D6) is not a fully adequate representation of H-D.

(H-D6) has the result, that 'Ba' does not confirm 'Ra  $\supset$  Ba' relative to 'Ra'. The problem here is that the relevant consequent elements of 'Ra&(Ra  $\supset$  Ba)' are 'Ra' and 'Ba' and the deduction Ra&Ba  $\vdash$  Ba is not premise relevant. Yet, according to (H-D6), while 'Ba' does not confirm 'Ra  $\supset$  Ba' relative to 'Ra', 'Ra&Ba' does confirm 'Ra  $\supset$  Ba' relative to 'Ra'. Finally, (H-D6) has the result where there is no logical equivalent (T&B) of (T&B) such that the deduction (T&B)  $\vdash$  S is premise relevant S cannot confirm T relative to B. Thus, for instance, 'Fa' does not confirm '(x)(Fx)' relative to 'Fb'. Generally, this suggests that (H-D6) does not make for a very useful definition of relative confirmation since we often want to consider cases where the background evidence B contains surplus information which renders the derivation of the alleged confirming evidence E from the conjunction of T and B is a non-premise relevant deduction.

Here one might best respond by simply claiming that it is better for a definition of confirmation to be a too exclusive than too inclusive.

#### 4. Gemes' New Formulation

Despite the above mentioned problems I think Schurz's general strategy for expressing H-D is on the right track. That is to say, I endorse his approach of looking for relevance considerations in the deduction of S from T to determine if S H-D confirms T. In particular, to find whether an axiom A of a theory T is confirmed by S we need to find a canonical version T' of T and then see if A is a content part of those axioms of T'

necessary for the derivation of S from T'. To do this we need first to get an account of what the content of a theory is and what counts as a natural axiomatization. Elsewhere I have suggested the following account of content and natural axiomatization,

$\alpha$  is part of the content of  $\beta$  iff  $\alpha$  and  $\beta$  are contingent,  $\beta \vdash \alpha$ , and every  $\alpha$ -relevant model of  $\alpha$  has an extension which is a  $\beta$ -relevant model of  $\beta$ , (Gemes 1997, but see also Gemes 1994b).<sup>7</sup>

T' is a natural axiomatization of T iff (i) T' is a finite set of wffs such that T' is logically equivalent to T, (ii) every member of T' is a content part of T' and (iii) no content part of any member of T' is entailed by the set of the remaining members of T'. (Gemes 1993, p.483)

These definitions pave the way for the following definition of H-D,

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<sup>7</sup> To define the notion of a relevant model we first need to define the notion of a relevant atomic wff. An atomic wff  $\phi$  is relevant to wff  $\alpha$  iff for some partial substitutional interpretation P, every full substitutional interpretation P' that is an extension of P is a model of  $\alpha$  and each such extension of P assigns the same value to  $\phi$ , and there is no proper substitutional sub-interpretation P'' of P, such that for every full substitutional interpretation of P''' that is an extension of P'', P''' is a model of  $\alpha$ . For any wff  $\alpha$ , a  $\alpha$ -relevant model M of  $\alpha$  is a distribution of truth values to all those atomic wffs relevant to  $\alpha$  (and to no other atomic wffs), such that  $\alpha$  is true on M given a substitutional reading of  $\alpha$ . Thus the only 'Fa'-relevant model of 'Fa' is that partial interpretation which assigns T to the atomic wff 'Fa' and makes no other assignments to atomic wffs. The only '(x)Fx'-relevant model of '(x)Fx' is that partial interpretation which assigns T to every atomic wff of the form 'Fc' where c is any individual constant of the language in question, and makes no other assignment to atomic wffs. So the only 'Fa'-relevant model of 'Fa' can clearly be extended to a '(x)Fx'-relevant model of '(x)Fx', so 'Fa' is a content part of '(x)Fx'. 'FavGb', on the other hand, is not a content part of '(x)Fx' since that 'FavGb'-relevant model of 'FavGb' which assigns T to 'Gb' and makes no other assignment to atomic wffs can not be extended to a '(x)Fx'-relevant model of '(x)Fx' since no such model makes any assignment to 'Gb'. For more on this see Gemes (1997).

(H-D7) S hypothetico-deductively confirms axiom A of theory T relative to background evidence b iff S and b are content parts of (T&b), and there is no natural axiomatization, n(T), of T such that for some subset s of the axioms of n(T), S is a content part of (s&b) and A is not a content part of (s&b). (Gemes (1993), p.486)

(H-D7) yields the equivalent of a non-relative version of H-D by simply letting b be any tautology. (H-D7) solves the problem of tacking by conjunction since where the content of axiom A is necessary for the derivation of S from background evidence b, it does not follow that the content of the conjunction of A and arbitrary A' will also be necessary for the derivation of S from background b. That is to say, in the specific terms of (H-D7), where A needs to be part of the content of s&b if S is to be part of the content of s&b it does not follow that A&A' needs to be part of the content of s&b if S is to be part of the content of s&b. (H-D7) solves the problem of tacking by disjunction, since, where S is a content part of T, it does not follow that for arbitrary S', SvS' is part of the content of (T&b). (H-D7) solves the problem of instance confirmation analogously to the method of (H-D3) since it produces the result that, for instance, 'Ba' confirms axiom '(x)(Rx  $\supset$  Bx)' relative to 'Ra'. (H-D7) generally has the result that where  $\alpha c \& \beta c$  is an instance of the universal generalization (x)( $\alpha x \supset \beta x$ ) then  $\beta c$  confirms (x)( $\alpha x \supset \beta x$ ) relative to  $\alpha c$ . (H-D7) does not produce an analog of (H-5)'s inductivist result that 'Fa' confirms 'Fb', that is to say, it does not have the result that 'Fa' confirms 'Fb' (relative to any tautology t). Nor does it have the results that 'Fb&~Fc&~Fd&~Fe' confirms both 'Fa' and '~Fa' (relative to any tautology) and 'Ra&~Rb&~Rb' confirms both '(x)(Rx  $\supset$  Bx)' and '~(x)(Rx  $\supset$  Bx)' (relative to any tautology). It does not have (H-D6)'s consequence that 'Ha&Rb&Wc&Dd' confirms '(x)(Hx  $\supset$  Rx)&(x)(Wx  $\supset$  Dx)&(x)(Hx  $\supset$  Wx)&(x)(Rx  $\supset$  Dx)' relative to the background claim 'Ra&Hb&Dc&Wd'. The point here is that there is a natural axiomatization of '(x)(Hx  $\supset$  Rx)&(x)(Wx  $\supset$  Dx)&(x)(Hx  $\supset$  Wx)&(x)(Rx  $\supset$  Dx)', namely that which has the three axioms '(x)(Hx  $\supset$  Rx)' and '(x)(Wx  $\supset$  Dx)' and '(x)(Rx  $\supset$  Wx)' which allows for the derivation of 'Ha&Rb&Wc&Dd' from 'Ra&Hb&Dc&Wd' using only a proper

subset of the three axioms, namely the first two axioms, which does not include  $'(x)(Hx \supset Rx) \& (x)(Wx \supset Dx) \& (x)(Hx \supset Wx) \& (x)(Rx \supset Dx)'$  as a content part. (H-D7) does not have (H-D6)'s consequence that  $'(\exists x)Fx'$  confirms  $'Fa'$  since  $'(\exists x)Fx'$  is not a content part of  $'Fa'$ . Finally, (H-D7), unlike (H-D6), does have the result that  $'Ba'$  confirms  $'Ra \supset Ba'$  relative to  $'Ra'$  and  $'Fa'$  confirms  $'(x)Fx'$  relative to  $'Fb'$ .<sup>8</sup>

##### 5. The Problem with H-D's core Intuition

While I take (H-D7) to capture the spirit of H-D better than any of (H-D1)-(H-D6) does, this is not to say that I am wholly sympathetic to that spirit. While being H-D confirmed by some available evidence certainly counts in favor of a claim, speaking plainly, the problem with H-D is that it is not sufficiently inductivist. This charge may come as a surprise since it is often assumed that H-D involves some tacit form of inductivism, that H-D, for instance, allows that were  $'Fa'$  confirms  $'(x)Fx'$  then that confirmation spills over to other parts of  $'(x)Fx'$  that extent beyond  $'Fa'$ . In fact, canonical formulations of H-D such as (H-D1) expressly have the consequence that  $'Fa'$  does not confirm such parts of  $'(x)Fx'$  as  $'Fb'$ , since  $Fa \not\vdash Fb$ . Canonical forms of H-D typically demand a deductive link between a hypothesis and any confirmatory evidence for it. Since, by definition, there is no deductive link between the deductively untested parts of a hypothesis and such evidence it cannot confirm those parts. Though it is

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<sup>8</sup> (H-D7) has it's own minor foibles. For instance, like (H-D5), and (H-D6), (H-D7) has the consequence that  $'FavFb'$  does not H-D confirm  $'(x)Fx'$ . However in this business it is better to be a little exclusive than too inclusive. Furthermore, recasting (H-D7) in a recursive form, allowing a recursion clause such as 'If both S and S' hypothetico-deductively confirm axiom A of theory T relative to background evidence b then SvS' hypothetico-deductively confirms axiom A of theory T relative to background evidence b, would yield a version of (H-D7) which allowed that  $'FavFb'$  confirms  $'(x)Fx'$ .

often unnoticed, H-D confirmation is in fact to some extent similar to Popperian corroborationism. It allows that a claim may be tested by finding some consequence of it to be true but it does not allow that this confirms the deductively untested parts of the claim. Similarly, according to Popper, 'Fa' corroborates '(x)Fx' but does not confirm 'Fb'. It is worth recalling that H-D, like Popper's corroborationism, was originally developed as an alternative to genuine inductivist accounts of confirmation. The underlying rationale behind versions of H-D such as (H-D1) and Popper's corroborationism is that 'Fa' confirms '(x)Fx' because it eliminates a as a potential falsifier of '(x)Fx'. But there is much more to confirmation than merely cutting the untested content of a hypothesis. In particular, there is the real confirmation that comes when evidence actually confirms the deductively untested parts of a hypothesis.<sup>9</sup> Of course, it is open to the advocate of H-D to claim that H-D confirmation is just one brand of confirmation. Thus where the ambitious (H-D1)-(H-D6) offers necessary and sufficient conditions for confirmation, the more cagey (H-D7) speaks only of necessary and sufficient condition for hypothetico-deductive confirmation. The advocate of (H-D7) can say that while 'Fa' hypothetically deductively confirms '(x)Fx' (relative to any tautology) and does not hypothetico-deductively confirm 'Fb', 'Fa' still confirms 'Fb' in some non hypothetico-deductive way. But then much of our interest would deservedly shift to the question of what exactly is this other way.<sup>10</sup>

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<sup>9</sup> For more on this see Gemes (1996a).

<sup>10</sup> It is worth noting that Hempel actually abandoned the predictive theory of confirmation not because he thought it was open to counter-examples but because it provided only a sufficient but not necessary condition for confirmation (see Hempel 1965, p.27). This is not to say that Hempel clearly recognized that H-D accounts are flawed in that they did not allow that evidence can confirm a hypothesis deductively untested by that evidence.

## 6. The Criterion of Empirical Significance

In his "Studies in the Logic of Confirmation", Hempel after presenting the Prediction-Criterion appends the following footnote

The following quotations from A.J. Ayer's book Language, Truth and Logic (London, 1936) formulate in a particularly clear fashion the conception of confirmation as successful prediction (although the two are not explicitly defined by definition): "...the function of an empirical hypothesis is to enable us to anticipate experience. Accordingly, if an observation to which a given proposition is relevant conforms to our expectations,...that proposition is confirmed (loc. cit. pp. 142-3, [1971, p. 130]); "it is the mark of a genuine factual proposition ...that some experiential propositions can be deduced from it in conjunction with certain other premises without being deducible from those other premises alone." (loc. cit. p. 26, [1971, p. 52]). (Hempel 1965, p. 27).

This quotation is interesting in that the two quotation it cites as precursors of the prediction-criterion point in slightly different philosophical directions. The first quotation from Ayer, while initially talking of the function of an empirical hypothesis, concludes by giving the nucleus of the H-D account of the confirmation of empirical propositions, and, indeed, the quoted text is preceded by a direct posing of the question "What is the criterion by which we test the validity of an empirical proposition?" (Ayer 1971, p.131). The second quotation is explicitly addressed to the problem of what constitutes the mark of a genuinely factual proposition. We may take it that for Hempel and Ayer the empirical and the factual are two sides of the same coin. Now the question of how one confirms a factual/empirical proposition and the question of how one identifies a genuine factual/empirical proposition need not be seen as one and the same question. For instance, switching to the more congenial idiom of statements rather than the idiom of propositions, one might claim that while the mark of a genuine empirical statement is that it together with certain other statements implies some observation statement not implied by the other statements alone, nevertheless a statement, with or without the presence of certain other statements, need not entail a given observation statement to

be confirmed by that statement. Despite the positivists' inclination, the question of empirical significance and the question of confirmation do not have to be identified as one and the same question.

The search for a criterion of empirical significance did not much outlive the failings of the numerous attempts to give a formal criterion of empirical significance. Perhaps this has something to do with the grandiose names and aspirations these attempts paraded under; the search for a criterion of meaningfulness, the search for a criterion of cognitive significance. Certainly the imperialistic ambitions that spawned that search, the desire to overthrow the evil empire of metaphysics, have long been deemed philosophically incorrect. At any rate, while the search for the criteria of empirical significance has all but been abandoned as a completely degenerate project, the hypothetico-deductive account of confirmation that was spawned along with it has better weathered the test of time. H-D has shown a remarkable ability to live on despite the continued failures to give a precise version, while the positivists' account of empirical significance pretty much died with the failure of the attempts to formulate it precisely. But once we have separated the question of empirical significance from the question of confirmation we may well ask ourselves whether in fact the question of what a given statement entails, the question of whether it together with other statements entails an observation statement, is not more relevant to the question of its empirical significance than to the question of its confirmation. If this is so the historical verdict needs to be reversed; the positivistic H-D can be rejected as a degenerate research program while the attempt to give an account of empirical significance deserves closer attention.

Indeed, I believe, the machinery of (H-D7) can with a little work be turned into a workable criterion of empirical significance which pretty much captures the intuition behind the second quotation from Ayer, the intuition that a sentence is significant within



a theory if it adds to the observational consequences of the theory as a whole.<sup>11</sup> To make good that intuition one needs an account of a natural representation of a theory, otherwise one finds oneself in the bind famously faced by Ayer, that is, the bind of having to recognize that even that Heideggerian *bete noir* 'The nothing nothings' is empirically significant within the theory consisting of the two claims 'The nothing nothings' and 'If the nothing nothings then Sydney has a harbor bridge'.<sup>12</sup> Now the notion of a natural axiomatization used in (H-D7) provides the needed notion of a natural representation. The theory consisting of the two claims 'The nothing nothings'

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<sup>11</sup> Where Ayer's talk of "certain other propositions" seems to be pointing to the combination of, say, a universal generalization with an instantiation of its antecedent clause, my talk of a sentence "within a theory" points to a much more inclusive background. Perhaps some readers will feel that in this shift I obscure one of the main reasons why the positivists' project of finding a criterion of empirical significance has been abandoned, that reason being the wide scale acceptance of Quine's holistic strictures against the notion of a single sentence having its own fund of empirical (experiential) consequences. While there is some truth in this claim it is worth noting that Ayer in *Language Truth and Logic* (see Ayer 1972, p.125) and Hempel in his "Empiricist Criteria of Cognitive Significance" (see Hempel 1965, p. 112) and his much earlier "On the Logical Positivists Theory of Truth" made due recognition of holism. The triumph of holism does not account for the demise of the project of searching for a criterion of empirical significance though it may well be part of the account of the abandonment of the Carnapian project of attempting to reduce statements about material objects to statements about experience.

<sup>12</sup> 'The Nothing nothings' was of course the principle target of Carnap's "The Elimination of Metaphysics", though in Ayer (1971, p. 59) there is a little rough handling of Heidegger's Nothing. Ayer (1971, p. 15 and p. 49) is more explicitly concerned with 'The Absolute is lazy' and the claim 'The Absolute enters into, but is incapable of, evolution and progress' which Ayer attributes to Bradley.

and 'If the nothing nothings then Sydney has a harbor bridge' is naturally represented as the theory consisting of the conjunction of the two claims 'The nothing nothings' and 'Sydney has a harbor bridge', and in this formulation 'The nothing nothings' does nothing to add to the empirical content of the total theory. The point here is that before we see if a sentence adds to the empirical content of a theory and hence is an empirically significant part of the theory we must first determine what counts as a natural axiomatization of the theory. Once we have a clear notion of what counts as a natural axiomatization of the theory we then see if there is some natural axiomatization which contains a subset of axioms which are empirically equivalent to the theory but do not contain the sentence in question as a content part. This insight may be generalized into the following account of empirical significance

(E.S.) Axiom A of theory T is an empirically significant part of T iff for any natural axiomatization  $T(n)$  of T there is no subset S of  $T(n)$  such that S is empirically equivalent to T and A is not a content part of S.

In keeping with tradition, I will not say much about what exactly counts as empirical equivalence. Typically, this is glossed by assuming a division of the vocabulary of the language in question into a set of theoretical terms and observational terms, and, hence, theoretical statements and observational statements, and defining two theories expressed in the language as being empirically equivalent iff they entail the same set of observation statements. However, I suggest, we need a broader notion of empirical equivalence, perhaps something along the following lines: Two theories are empirically equivalent iff they bear the same probabilistic relations to any given observation statement. This would go some way to accommodating statistical hypotheses which often do not deductively entail any observational consequences even in the context of a wider theory.

Now suppose the above account of empirical significance holds. Does that mean it's time to once again declare open season on Heideggerians, Hegelians, and metaphysicians of all stripes? I think that would be a little too hasty a conclusion. It is true that being empirically significant is a virtue, and it may well be true that adding, for instance, Heideggerian claims to our current beliefs (or, in the case of converts, subtracting Heideggerian claims from their current beliefs) does not change the empirical content of those beliefs. But, pace the Positivists' suggestion, there are other virtues besides that of being empirically significant (and/or analytically true) and before we relegate, Heideggerianism, Hegelianism, Metaphysics, et al, to the dustbin of history we need to get a clearer notion of what these other virtues are and which theories partake of them.<sup>13</sup>

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<sup>13</sup> This is not to say that the positivists did not recognize other virtues such as that of simplicity. Rather the point is that they tended to count being empirically significant or analytic as a necessary condition for being worthy of any credence. For more on the notion of there being a range of theoretical virtues and for an account of some of these virtues see Gemes (1994a).

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