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Real Rigidities and Optimal Stabilization at the Zero Lower Bound in New Keynesian Economies

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Abstract

The paper re-visits the literature on real rigidities in New Keynesian models in the context of an economy at the zero lower bound. It identifies strategic interaction among price- and wage-setting agents in the economy as an important determinant of both optimal policy and economic dynamics in deep recessions. In particular, labour market segmentation is shown to have a significant influence on the length of the forward commitment to keep interest rates at zero, the magnitude of the fiscal policy responses as well as inflation volatility in the economy under optimal policy.

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1. Introduction

Keeping the nominal interest rate at zero even after the natural rate has recovered to positive values, enacting an increase in government spending or, more controversially, introducing tax increases have all been discussed as viable stabilization policy strategies in New Keynesian economies subject to deep recessions. This paper studies the extent to which the desirability of such strategies is affected by the nature of interaction among firms and households in the product and labour markets. It highlights that both the optimal length of time spent at the zero bound, the strength of policy responses, and the magnitude of observed macroeconomic outcomes under optimal policy (such as inflation rates) are significantly affected by the degree of strategic complementarity in price- and wage-setting. We show that the structure of the labour market (in particular, whether or not labour markets are segmented) has a profound effect on both optimal policy and macroeconomic outcomes.

The importance strategic complementarity between price- or wage-setters has received considerable attention in the context of the ability of New Keynesian models to replicate observed persistence in the real economy following monetary policy shocks.[1] However, the literature on stabilization policy at the zero lower bound has so far largely ignored the implications of strategic interaction in price and wage setting for policy under the specific circumstances presented to policy makers by the presence of this nonlinearity.[2] Exploring the interaction between strategic complementarity and optimal policy at the zero lower bound is important, as seemingly innocuous assumptions about market structure or structural parameters often taken in the literature have non-trivial implications

[1] See Edge (2002), Woodford (2003, Chapter 3) or Ascari (2003), for example.

[2] Most recently, Eggertsson and Singh (2016) discuss sector specificity of labour markets in a paper on zero-lower-bound issues. They concentrate on the analytical usefulness of this assumption, and do not explore the implications for policy or economic dynamics.

for the way we should think about 'good' policy and what is desirable to achieve in terms of outcomes in the economy.

In this paper, we study a New Keynesian setup in which prices and/or wages are sticky, and the labour market can be either non-segmented (or economy-wide) or segmented (sector-specific). Government spending is valued and the income tax policy is determined endogenously. This economy is subject to a large fundamental shock as a result of which optimally set nominal interest rates hit the zero lower bound.[3] While Correia et al. (2013) have shown that a sufficiently rich set of policy instruments can completely circumvent the liquidity trap problem, and may even enable policy makers to implement the first best outcome, in this paper we study a world in which solutions that are costless in welfare terms are ruled out. In addition to setting the tax on wage income and government spending, the authorities can use forward commitment concerning interest rates to stabilize the economy in a liquidity trap.[4] Eggertsson and Woodford (2006) and Nakata (2011) have also studied a simultaneous determination of optimal monetary and fiscal policy in a deep recession but did not consider in greater depth the role of wage stickiness and, in particular, strategic complementarities. Christiano et al. (2009) and Christiano (2010), whilst including wage stickiness, only examined the functioning of ad hoc (tax) policies and concentrated on the implied real economy effects.

[3]The policy prescriptions obtained in our framework are standard given that the source of the downturn in our model is also standard—a shock to the rate of time preference of agents. Schmitt-Grohé and Uribe (2013) and Mertens and Ravn (2014) have questioned the usefulness of such conventional policy advice if the cause of the severe downturn in the economy is that expectations are not well-anchored. We believe this discussion is beyond the scope of the intended contribution of this paper.

[4]Such a set of policy tools better reflects the policy decisions implemented by central banks and governments around the developed world in the wake of the most recent severe recession. See, for example, European Commission (2009) or Council of Economic Advisers (2010). Only the United Kingdom have on a one-off basis implemented a policy concerning the general VAT rate that is vaguely in line with Correia et al. (2013).

We find that the optimal response in the inflation rate to a given large shock varies by as much as one order of magnitude depending on whether we assume sector-specific labour markets or an economy-wide labour market. We also highlight the importance of key parametric assumptions for policy and outcomes at the zero lower bound. In particular, we show that depending on the nature of nominal wage adjustment in the economy, a linear production technology may justify relatively high expected inflation or—at the other extreme—strict price-level targeting strategies. The differences are smaller with a concave production function.

Intuitively, in the presence of nominal price and/or wage rigidities (à la Calvo), features that force price- or wage-optimizers to consider more carefully the potential adverse implications of demand reallocations for their profits act to supress relative price adjustment following shocks. It matters, in particular, if real wage changes are seen as an economy-wide phenomenon or something that affects particular industries only. In the latter case, sectoral price determination needs to take into account the consequences of price-change-induced demand re-allocation for sectoral wages (and profits). An implication of this is more caution in price re-optimization, less inflation volatility and more volatility in real variables. The latter is manifested in both larger response magnitudes and longer duration of adjustment following shocks. Such considerations are, understandably, exacerbated by factors such as the intersectoral substitutability of different types of products and labour, and the nature of the production technology. This is something we demonstrate in the paper too.

The optimal policy response in the presence of conditions for dampened adjustment is to act more forcefully. In terms of monetary policy action, the commitment to keep interest rates at zero lasts longer after the zero bound ceases to bind once real rigidities are taken into account. Krugman (1998) famously

argued that monetary policy is not ineffective in a liquidity trap as long as it is able to affect inflation expectations. Expectations of higher inflation lower the current real interest rate and act to stimulate demand even if the short-term nominal interest rate is stuck at zero. It has been shown in the context of standard New Keynesian models that the monetary policy consistent with such evolution of prices involves a commitment to keep the nominal interest rate at zero for some time after the zero bound ceases to bind. Eggertsson and Woodford (2003), Jung et al. (2005) and Adam and Billi (2006) have shown this formally, whilst arguing for very modest rates of expected inflation. Our paper demonstrates that the optimal duration of the commitment to keep interest rates at zero as well as the implied inflation rates vary considerably depending on the assumed degree of strategic complementarity in price and wage setting decisions. Contrary to what one might expect, longer forward commitment does not translate into larger inflation responses. It merely mitigates their absence. The point of a stronger monetary policy response is primarily to engineer a larger boom in the real economy in the future which reduces desired savings and stimulates demand in the short run. This is consistent with a thought experiment in Werning (2012) who examined the case of a simple economy in a liquidity trap with artificially fixed prices. We show that such a simple exercise is a close approximation of optimal dynamics in a sticky-price sticky-wage New Keynesian economy with a linear production technology.

When real rigidities are stronger, other tools in the conventional stabilization toolbox are applied more forcefully too: the desired short-term government spending expansion is larger and the government must commit itself to greater cuts in the future. A policy strategy of 'leaning against the wind' in which government spending is first raised and then cut whilst the nominal interest rate is at the zero bound has been proposed by Gertler (2003) due to its impact on the natural rate

of interest. Nakata (2011) and Werning (2012) have shown this to be a feature of optimal policy in a liquidity trap. Werning (2012) argues that the mentioned strategy is almost entirely 'opportunistic' and the motivation for it has little to do with stabilization.[5] We provide evidence supporting this view too. Since we study optimal policy from a timeless perspective, in line with Schmidt (2013), we do not find large gains in terms of the stability of nominal or real variables as a result of the deployment of government spending.

The idea that an income tax hike is desirable at the zero bound due to its effect on (expected) inflation and the real interest rate has been discussed in Benigno (2009), Eggertsson (2011) and Nakata (2011). In Correia et al. (2013), tax policy is best thought of as a price stabilization tool given its impact on the marginal cost in the economy. In our model, we also observe gains in price stability once tax policy is activated in addition to the other tools in the policy maker's toolbox. Overall, the budgetary impact of stabilization measures is close to zero in the short term.

We also examine the state-dependency of our results as in Burgert and Schmidt (2014). We find that higher initial indebtedness tends to amplify the differences across economies with different labour market structures. In particular, the optimal inflation response is even larger in the economy with economy-wide labour markets relative to the alternatives considered when initial debt is high. Tax policy deployed more forcefully bears the brunt of the initial adjustment in debt. This can be an increase or a cut depending on where the economy starts relative to its steady state.

The rest of the paper is organized as follows. Section 2 introduces different versions of a baseline model that form the basis for our analysis of the design

[5] In a public finance context, 'opportunistic' policy makers will seek to increase the provision of public goods when the marginal rate of transformation between public and private goods falls.

of optimal monetary and fiscal policies in a liquidity trap. This model is parameterized and solved using the nonlinear method explained in great detail in Nakata (2011). The results of the numerical exercise are presented and related to the existing literature in Section 4. Section 5 concludes.

2. The model

This section describes a model of an economy with sticky nominal wages and prices akin to Benigno and Woodford (2005) which builds on Erceg et al. (2000). The government authorities in our economy set the interest rate, government spending and the distortive labor income tax rate to stabilize the economy. Shocks to the discount factor are the only source of disturbance in the model, and we examine the economy's adjustment under perfect foresight along a deterministic path following a single large innovation to the discount factor. If this innovation was small, it could be fully offset by a cut in the nominal interest rate, and other policy instruments would not play a role in stabilizing the economy.

Whilst the model is closer to the widely used medium-scale setups than the more common simple stylized frameworks in terms of its complexity, it should still be thought of only as a relatively tractable environment for the study of policy interactions. The quantitative results from this model are especially subject to this caveat. The main lessons concerning policy coordination should, however, apply more generally, as the circumstances we examine are implicit in all larger-scale models.

2.1. The discount factor shock

An exogenous shock to the discount factor of agents, representing a change in their preferences in terms of consumption and savings, is used to capture the idea of a severe demand-led contraction in the economy.

As in Nakata (2011), we assume that the discount factor at time $t+s$ is defined as $\beta\delta_s$, i.e. δ_s shows the relative difference between discount factors at time $t+s$ and $t+s+1$. The following assumptions about the discount factor shock hold in the model

$$\begin{aligned}
\delta_0 &= 1, \\
\delta_1 &= 1+\varepsilon_{\delta,1}, \\
\delta_s &= 1+\rho_\delta\left(\delta_{s-1}-1\right) \text{ for } s \geqslant 2.
\end{aligned}$$

The discount factor shock is realized before optimization decisions are made. It holds that $\varepsilon_{\delta,1} > 0$ and the shock persists, but decays with the time at the rate $0 < \rho_\delta < 1$.

2.2. Households and the labour market

There is a continuum of monopolistically competitive households located on the unit interval $[0,1]$. Those of type j choose private consumption of a final good $C_t(j)$ and holdings of one-period risk-free nominal government bond $B_t(j)$ to maximize welfare given by

$$E_t \sum_{s=0}^{\infty} \beta^s \prod_{k=0}^{s} \delta_k \left[\frac{C_{t+s}(j)^{1-\chi_C}}{1-\chi_C} - \chi_{N,0} \frac{N_{t+s}\left(j\right)^{1+\chi_{N,1}}}{1+\chi_{N,1}} + \chi_{G,0} \frac{G_{t+s}^{1-\chi_{G,1}}}{1-\chi_{G,1}} \right]$$

subject to the constraint

$$P_{t+s}C_{t+s}(j) + \frac{B_{t+s}(j)}{R_{t+s}} \leqslant (1-\tau_{n,t+s})W_t\left(j\right)N_{t+s}(j) + B_{t+s-1}(j) - T_{t+s}^{LS} + D_{t+s} \quad (2.1)$$

The variable P_t is a price of a final good, R_t stands for the gross nominal return on the bond, while $\tau_{n,t}$ is the labor income tax rate. T_t^{LS} refers to the lump sum taxes (transfers) that may be paid by (to) the households. The profits generated by monopolistically competitive firms are transferred to households in the form of

lump-sum dividends D_t. This maximization exercise yields the Euler equation

$$C_t^{-\chi_C} = E_t \beta \delta_t R_t C_{t+1}^{-\chi_C} \Pi_{t+1}^{-1}, \tag{2.2}$$

where $\Pi_t = P_t/P_{t-1}$ is price inflation. The Euler equation is not indexed by the households, as we assume completeness of insurance market against idiosyncratic shocks and that the initial holdings of assets are the same across households.[6] Therefore, $C_t(j) = C_t$ for all j and t.[7]

Households of type j supply a differentiated labor service $N_t(j)$ at a wage rate $W_t(j)$. There is a perfectly-competitive employment agency that aggregates the supplied differentiated labor in an index according to the standard Dixit-Stiglitz formula

$$N_t = \left[\int_0^1 N_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

in which ε is the elasticity of substitutions between differentiated labour. The perfectly-competitive employment agency sells aggregated labour to producers of final goods at an aggregate wage index W_t. The agency chooses $N_t(j)$ to maximize nominal profits $W_t N_t - \int_0^1 W_t(j) N_t(j)$, taking the wage rate $W_t(j)$ and the aggregate price index W_t as given. In optimum, the employment agency's demand for type-j labour is given by

$$N_t(j) = N_t \left[\frac{W_t(j)}{W_t} \right]^{-\varepsilon}. \tag{2.3}$$

The aggregate wage index is then given by

$$W_t = \left[\int_0^1 W_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}.$$

[6] An implication of the former is that the exact distribution of shares across firms does not matter. Hence, we do not specify dividends D in detail.

[7] Notice here that if δ is small enough, it can be fully offset by a change in R, leaving the rest of the economy unaffected.

To introduce wage stickiness, the model assumes a system of staggered wage contract for the households: households of a certain type are able to change their wages with probability $1-\xi_w$ at any given period of time. Whenever the households are allowed to re-optimize their wage, they choose optimal W_t^* to maximize expected discounted sum of utilities, taking into account that they may not be allowed change the wage rate, subject to the demand for labor equation and the budget constraint. For simplicity, we do not consider wage indexation. The households thus choose the wage rate to maximize

$$E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \prod_{k=0}^{s} \delta_k \left[\frac{C_{t+s}^{1-\chi_C}}{1-\chi_C} - \chi_{N,0} \frac{N_{t+s}(j)^{1+\chi_{N,1}}}{1+\chi_{N,1}} + \chi_{G,0} \frac{G_{t+s}^{1-\chi_{G,1}}}{1-\chi_{G,1}} \right]$$

subject to (2.1) and (2.3). This problem gives us the wage setting equation

$$(w_t^*)^{1+\varepsilon\chi_{N,1}} = \frac{\varepsilon}{\varepsilon-1} \frac{N_{n,t}}{N_{d,t}}, \tag{2.4}$$

where $w_t^* = W_t^*/W_t$ with

$$N_{n,t} = \chi_{N,0} N_t^{1+\chi_{N,1}} + E_t \beta \delta_t \xi_w \left(\Pi_{t+1}^w\right)^{\varepsilon\left(1+\chi_{N,1}\right)} N_{n,t+1}, \tag{2.5}$$

$$N_{d,t} = w_t N_t C_t^{-\chi_C} (1-\tau_{n,t}) + E_t \beta \delta_t \xi_w \left(\Pi_{t+1}^w\right)^{\varepsilon-1} N_{d,t+1}. \tag{2.6}$$

We have defined $\Pi_t^w = W_t/W_{t-1}$ and $w_t = W_t/P_t$. Given our wage setting mechanism, the evolution of the aggregate wage index follows

$$1 = (1-\xi_w)(w_t^*)^{1-\varepsilon} + \xi_w (\Pi_t^w)^{\varepsilon-1}. \tag{2.7}$$

2.3. Firms

There is a continuum of intermediate differentiated goods indexed i. Firms operating in sector i use a linear production technology to produce output

$$Y_t(i) = N_t(i)^{1/\alpha}. \tag{2.8}$$

with $\alpha \geqslant 1$. The price of an intermediate good i is $P_t(i)$. The representative final goods producer that operates in a perfectly competitive environment sells Y_t which is an aggregate of $Y_t(i)$ according to

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{2.9}$$

in which θ is the elasticity of substitutions between the differentiated intermediate products. The representative final goods producing firm sells its product to the consumers at a price P_t. It chooses the quantity of each differentiated good to maximize its profit $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$. As a result, demand for intermediate good i is given by

$$Y_t(i) = Y_t \left[\frac{P_t(i)}{P_t} \right]^{-\theta}. \tag{2.10}$$

The aggregate price index is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}.$$

Price adjustment is assumed to be staggered too. It is assumed that in any given period, the intermediate goods producing firms operating in a given sector are able to re-optimize their price with a probability $1 - \xi_p$. Whenever the firms are able to re-optimize their price, they choose the optimal P_t^* to maximize expected discounted sum of profits subject to the demand for their product defined in equation (2.9). The problem of the firms is thus

$$\max_{P_t^*} E_t \sum_{s=0}^{\infty} (\xi_p \beta)^s \prod_{k=0}^{s} \delta_k \left[P_{t+s}^* Y_{t+s}(i) - W_{t+s} Y_{t+s}(i)^{\alpha} \right]$$
$$s.t. \ (2.10).$$

The solution for the optimal price is given by

$$(p_t^*)^{1+\theta(\alpha-1)} = \frac{\theta}{\theta-1} \frac{C_{n,t}}{C_{d,t}}, \tag{2.11}$$

where $p_t^* = P_t^*/P_t$ with

$$C_{n,t} = \alpha w_t Y_t^{\alpha} C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta\alpha} C_{n,t+1}, \tag{2.12}$$

$$C_{d,t} = Y_t C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta-1} C_{d,t+1}. \tag{2.13}$$

The dynamic of the aggregate price index follows

$$1 = \left(1 - \xi_p\right) \left(p_t^*\right)^{1-\theta} + \xi_p \Pi_t^{\theta-1}. \tag{2.14}$$

2.4. Government

Monetary and fiscal authorities coordinate their action to maximize social welfare. The monetary branch of the central government sets the nominal interest rate R_t, and is constrained by the zero lower bound

$$R_t \geqslant 1 \text{ for all } t. \tag{2.15}$$

The fiscal authority sets the tax rate $\tau_{n,t}$ and decides about government spending G_t. The government flow budget constraint tracking the evoution of debt is then given by

$$\frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} - \tau_{n,t} w_t N_t + G_t - T_t^{LS}. \tag{2.16}$$

2.5. Further equilibrium conditions

Given the intermediate goods producing firms' production function (2.8), the demand for intermediate goods (2.10), and the labor market clearing condition $N_t = \int_0^1 N_t(i)\, di$, it can be shown that

$$s_t Y_t^{\alpha} = N_t \tag{2.17}$$

where

$$s_t = \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta\alpha} di = \left(1 - \xi_p\right) \left(p_t^*\right)^{-\theta\alpha} + \xi_p \Pi_t^{\theta\alpha} s_{t-1} \tag{2.18}$$

stands for price dispersion. The resource constraint is given by

$$C_t + G_t = Y_t. \tag{2.19}$$

An important equilibrium condition is the identity describing the evolution of real wages in the economy

$$\frac{w_t}{w_{t-1}} = \frac{\Pi_t^w}{\Pi_t}. \tag{2.20}$$

Chugh (2006) highlights the importance of this identity in generating endogenous persistence in a sticky-price, sticky-wage economy.

2.6. Alternative versions of the model

We consider two versions of this model in which wages will be flexible but the steady state is the same as in the economy set out above. These versions are distinct in one crucial aspect: labour market segmentation. This has a key implication for the price determination in the economy and ultimately for the degree of sluggishness in the response of the real economy to shocks and policy action.

2.6.1. No labour market segmentation

When intermediate goods producing firms hire labour from the economy-wide market, their pricing decision still affects the demand for the differentiated goods produced by these firms but they consider the economy-wide real wage rate as being unaffected by their decision. This significantly increases the sensitivity of prices to shocks and accelerates real adjustment following disturbances and policy action. This version of the model is the same as the one presented in the sections above with ξ_w set to zero and relative wages set to one at all times. In the presence of perfect insurance against idiosyncratic risk, we also need not consider differentiated types of labour and write N_t and W_t instead of their sector-specific values in the households' problem.

2.6.2. Segmented labour markets

In this version of the model, which is close to the setup of Eggertsson and Woodford (2003) and Adam and Billi (2006), the intermediate goods producing firms internalize the consequence of their pricing decision for demand for the specific type of good, and the subsequent implications for the sectoral wage rate through the demand for sector-specific labour. The firms' problem gets modified in a fundamental way. We now have firms choosing the optimal price to maximize

$$\max_{P_t^*} E_t \sum_{s=0}^{\infty} \left(\xi_p \beta\right)^s \prod_{k=0}^{s} \delta_k \left[P_{t+s}^* Y_{t+s}(i) - W_{t+s}(i) Y_{t+s}(i)^{\alpha}\right]$$
$$s.t. \ (2.10)$$

and the definition of the real wage rate coming from the household problem.[8] Following Woodford (2003), by symmetry between i and j, we can write

$$W_t(i) = \frac{\chi_{N,0} Y_t^{\alpha\chi_{N,1}} \left(\frac{P_t^*}{P_t}\right)^{-\theta\alpha\chi_{N,1}}}{(1-\tau_{n,t}) C_t^{-\chi_C}}.$$

Equations (2.11) to (2.13) now become

$$(p_t^*)^{1+\theta\left[\alpha\left(1+\chi_{N,1}\right)-1\right]} = \frac{\theta}{\theta-1}\frac{C_{n,t}}{C_{d,t}},$$

where $p_t^* = P_t^*/P_t$ with

$$\begin{aligned} C_{n,t} &= \alpha Y_t^{\alpha\left(1+\chi_{N,1}\right)} C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta\alpha\left(1+\chi_{N,1}\right)} C_{n,t+1}, \\ C_{d,t} &= Y_t C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta-1} C_{d,t+1}. \end{aligned}$$

[8] The introduction of sector-specificity raises questions about wage formation. In order to avoid the need to consider monopsony in the labour market, we are implicitly assuming that there are many firms and many households in each sector in the economy, i.e. a 'double continuum' of firms and households, as explained in Woodford (2003, Chapter 3). Hence, we have used the plural form 'firms' and 'households' of a certain type throughout the text.

2.7. The policy problems

We shall consider the alternative versions of the model with different elements of the policy maker's toolbox switched on and off. In all cases, the objective will be to find sequences of endogenous variables that maximize an unweighted average of welfare across households

$$W_t = E_t \sum_{s=0}^{\infty} (\beta)^s \prod_{k=0}^{s} \delta_k \left[\frac{{C_{t+s}}^{1-\chi_C}}{1-\chi_C} - \chi_{N,0} \frac{{N_{t+s}}^{1+\chi_{N,1}}}{1+\chi_{N,1}} m_{t+s} + \chi_{G,0} \frac{G_{t+s}^{1-\chi_{G,1}}}{1-\chi_{G,1}} \right],$$

where

$$\begin{aligned} m_t &= \int_0^1 \left[\frac{W_t(j)}{W_t} \right]^{-\varepsilon\left(1+\chi_{N,1}\right)} dj \\ &= (1-\xi_w)\left(w_t^*\right)^{-\varepsilon\left(1+\chi_{N,1}\right)} + \xi_w \left(\Pi_t^w\right)^{\varepsilon\left(1+\chi_{N,1}\right)} m_{t-1} \end{aligned} \tag{2.21}$$

is a measure of wage dispersion. This is equal to one for all t when wages are flexible. Moreover, in the case of the flexible-wage economy with economy-wide labour markets, the disutility of labour supply is expressed as

$$\chi_{N,0} \int \frac{N_t(j)^{1+\chi_{N,1}}}{1+\chi_{N,1}} dj = \frac{\chi_{N,0}}{1+\chi_{N,1}} Y_t^{\alpha\left(1+\chi_{N,1}\right)} s_t^{SLM}$$

with $s_t^{SLM} = \int \left[\frac{P_t(i)}{P_t} \right]^{-\theta\alpha\left(1+\chi_{N,1}\right)} di$.

We shall be looking for policies that are optimal from a timeless perspective (Woodford, 2003). In other words, we will be solving for time-invariant policy rules assuming that preferences in the initial period are augmented so that the policy maker does not take advantage of the fact that there had been no expectations formed about the initial outcomes. The equilibrium conditions and the first-order conditions for each version of the model are listed in the Appendix.

3. Parameterization and solution

We parameterize the model with values commonly used in the literature.[9] We refer to the model under this parameterization as our 'baseline' case. The discount factor β is assumed to be 0.99. The discount factor shock $\varepsilon_{\delta,1}$ is set to 0.02 to make sure the economy hits the zero bound. The persistence of the innovation ρ_δ is 0.9. Thus, to determine when the natural rate of interest exceeds zero, one needs to check at what quarter the product of $\beta\delta_t$ falls below 1. For the parameters of the shock process, the discount factor and the persistence, the natural rate of interest is above zero from t>7.[10] We assume preferences are logarithmic in government spending, set χ_C to 1/6 and the inverse Frisch elasticity of labour supply to 1.[11] The preference parameters $\chi_{N,0}$ and $\chi_{G,0}$ are set to 1 and 0.2 respectively. This parameterization implies that steady-state government spending is close to 20 percent of steady-state output and the steady-state public debt is at 50 percent of annualized GDP. The elasticity of susbstitution for goods θ is set to 11. We follow Chugh (2006) in setting the elasticity of substitution in the labour market ε to 21. The measure of price stickiness ξ_p is 0.75 implying an average four-quarter duration or price contracts. The same value is used to parameterize the duration of wage contracts when wages are sticky.[12] The production function is assumed to be linear in labour in the baseline case (as in Nakata, 2011 or Fernandez-Villaverde et al., 2015).

When conducting sensitivity tests, the steady-state of our model is going to

[9] The parameter values are summarized in Table A.1 in the appendix.

[10] Werning (2012) shows this need not be equivalent to the point in time when the zero bound stops binding, as the optimal interest rate reaction function may involve other terms that are non-zero at the zero bound in addition to the natural rate. We only have a numerical solution for the interest rate, and so cannot be more precise here.

[11] The value for χ_C was also used in Jung et al. (2005), Nakata (2011), and is close to the estimate of Rotemberg and Woodford (1997).

[12] In the flexible-wage case, we set this parameter to zero but retain imperfect competition in the labour market so that the flexible-wage and sticky-wage economies are easier to compare.

change. We maintain comparability by ensuring that in all cases the steady-state debt-to-GDP ratio remains at 50 percent of GDP.

Given that we consider an event in which the economy departs far from its steady state, and an inequality constraint becomes binding, we solve the model in its non-linear form. We use the procedure described in detail in Nakata (2011), which embeds the modified Newton method of Juillard et al. (1998) into a shooting algorithm. As shown in Nakata (2011) there are significant accuracy gains from using a nonlinear solution relative to piecewise linear methods. Fernandez-Villaverde et al. (2015) also argue in favour of explicitly considering nonlinearities.[13]

4. Results

There are two ways of dissecting our results. We shall look at comparisons across different labour market arrangements. At the same time, we can understand a lot about the intuition behind the various policy interventions and the transmission of policy decisions by inspecting the same economy under different policies and parameterizations. In this section, we draw conclusions from both of these approaches. First, we look at the baseline model. Later, we demonstrate the robustness of our intuition by conducting sensitivity tests.

4.1. The model under baseline parameterization

Our first set of results is shown in Figure A.1 in which impulse response functions to a discount factor shock for three types of economic settings are compared (a model with flexible wages and an economy-wide labour market, a model with flexible wages and segmented labour markets labelled 'SLM', and a model with

[13] For the sake of balance, we mention that Eggertsson and Singh (2016) tend to downplay the importance of the differences arising from approximation accuracy in a model similar to ours.

sticky wages). Here, we assume that monetary policy is the only available tool to stabilize the economy and that the fiscal solvency constraint is satisfied through lump-sum taxes.

The model's simulation suggests that the optimal inflation volatility at the zero lower bound is significantly affected by the nature of the labour markets in the economy. When labour market sector specificity is introduced into the model with flexible wages, the optimal inflation response to a given shock drops by as much as one order of magnitude. In the model with sticky wages, inflation volatility drops even further. For all practical purposes, the dynamic of the optimal sticky-wage economy is the same as the dynamic of an economy without price or nominal wage inflation. The figure also shows that in all three models, it is optimal to keep the nominal interest rate at zero even after the zero bound ceases to bind. This result is in line with the earlier literature. However, our key contribution is to show that it is optimal to keep the interest rate at zero for even longer when labour markets are segmented or nominal wage rigidities are present. This policy—as we demnonstrate—is associated with smaller rather than larger expected inflation in the economy.

The intuition for this result is best understood in the context of the literature that investigates the ability of New Keynesian models to generate realistic degrees of output persistence (Edge, 2002, Ascari, 2003, Woodford, 2003, Chapter 3). Changing the nature of the labour markets affects the way price-setting firms and/or wage-setting households respond to a demand contraction or expansion in important ways. In a flexible-wage economy, a shock with negative implications for demand implies lower demand for labour and hence a downward pressure on real wages (and, by implication, on marginal cost). In an economy with flexible wages and economy-wide labour markets, optimizing firms would reflect the effect on wages to a large extent in their pricing decision. They would consider the fact

that they may not be able to change the price soon as demand recovers (and in fact overshoots) which will limit the extent to which prices drop. The firms also consider that lowering prices induces substitution of demand from goods for which prices remain unchanged. They can, however, recruit additional labour at the prevailing economy-wide real wage rate, which is a cost unrelated to the industry they operate in. With labour market segmentation, the firms need to consider that price-change-induced intersectoral re-allocation of demand needs to be met by hiring additional labour from a specific market with a specific wage rate. This will be affected by the need to meet the extra demand. This additional wage effect—which would eat into firms' profits—introduces an element of caution in price setting. As a consequence, there is a dampened price response in the first place, and a more persistence in the adjustment of the real economy. This mechanism explains the much more subdued inflation dynamic under segmented labour markets as depicted in Figure A.1. With sticky nominal wages, marginal cost adjusts sluggishly by construction. In addition, optimizing households consider the broader implications of their wage decisions. Cutting (raising) wages too much, whilst other households keep wages constant, might re-allocate demand towards (from) their speciality, and the welfare cost of labour supply increases on the margin. This reinforces wage stability already introduced via staggered wage-setting. Overall, wages and, as a consequence, prices react little to shocks.

The downside of such price stability is that it imples a higher path for real interest rates. In the absence of significant expected inflation, the future real boom would be more subdued ceteris paribus and discounted more heavily. Relatively low wealth implies lower consumption in the short run.

The monetary policy maker mitigates subdued price inflation by keeping the interest rate at zero for longer. This induces a greater real economy boom. It is in this sense that we argue, following Werning (2012), that monetary policy in

the liquidity trap is geared towards generating an expected real economy boom rather than inflation per se.

Overall, we still observe a significantly larger drop in output and consumption under segmented labour markets and nominal wage rigidity than with economy-wide labour markets. However, an optimizing policy maker will be comfortable with achieving more price stability at the expense of larger consumption (and output) volatility. This is because strategic complementarity affects not only the degree to which marginal cost pressures translate into price movements (the slope of the Phillips curve) but also the relative costliness of inflation and output variability from a welfare perspective. With more strategic complementarity, a flatter Phillips curve implies that there is larger misallocation of resources arising from a given rate of inflation.

Introducing government spending as a policy tool does not change the overall picture markedly, which can be inferred from Figure A.2. This is consistent with Schmidt (2013). In fact, as argued in Werning (2012), government spending policy may have little to do with stabilization in the economy and instead be driven by public finance considerations. In an economy with depressed private demand, the marginal disutility of labour is low. An 'opportunistic' policy maker deciding on the optimal amount of public spending within the period following the Samuelson rule will observe a small marginal rate of transformation between the public and private goods (a drop in the relative price of public good), and will seek to increase provision of valued G. The reverse holds in boom time.

To get a feel of the relative contribution of such considerations for government spending policy, we do the following comparison. We take the output dynamic from the optimal economy without fiscal variables in use as given, and ask ourselves the question: What would an optimizing policy maker driven purely by public finance considerations do if he/she saw output dynamic from the

economy stabilized only by monetary policy? Our objective is thus to find $G^{PF} = \arg\max_G(C, C+G, G)$ with C taken as given.[14] We then compare the result of this exercise with the optimal dynamic of G in the various versions of our New Keynesian economies with tax rates fixed. We report the results in Figure A.3. In line with Werning (2012), it is clear that the time profile as well as the magnitude of the response in G is very well explained by public finance considerations. One effect of such leaning against the wind via G is that output becomes more stable but by not too much. Figure A.5 which depicts optimal dynamics in the sticky-wage economy under different policy options makes this point clear.

Finally, we add tax policy to our set of policy instruments used to stabilize the economy (see Figure A.4). The tax in our economy is labour income tax levied on household earnings. This tax directly affects marginal cost, and therefore, is an effective instrument deployed to deliver the desired evolution of prices. This view of the role of tax policy is the same as in Correia et al. (2013). In our model, taxes generally rise in the short-term which is consistent with the demand-side considerations found in the literature. In particular, Benigno (2009), Eggertsson (2011) and Nakata (2011) sought to justify tax increases through their impact on (expected) inflation and the real interest rate. This is in turn different from Bils and Klenow (2008) who concentrated on the income effect of a tax cut, which is the reasoning probably closest to the philosophy behind similar real-world stimulus measures. In our model, taxes lean against the wind: they counteract the dynamic of marginal cost resulting primarily in a more stable inflation rate. This is best seen in the case of the sticky wage economy, as shown in Figure A.5, but the intuition is valid in our flexible wage economies as well. In a sticky-wage economy,

[14] We adjust the preferences of the policy maker with respect to G so that in the steady state, he would choose the G/Y ratio that prevails in the steady state of our economies.

with only a fraction of wage-setters reacting to tax policy (affecting the net gains from employment), tax policy needs to act more robustly to achieve the desired aggregate outcome.

The overall budgetary impact of stabilization measures is close to zero initially and public debt gradually falls towards a new lower steady state level. It is a feature of the model that there is a continuum of steady states indexed by tax rates with a corresponding debt level. As in Nakata (2011), the welfare-maximizing tax rate is negative (eliminating the distortions to the steady state), and the corresponding steady state features a higher output level, more government spending and government holding net assets. Following the shock, the economy moves into a steady state located closer to such an outcome.[15]

4.2. Sensitivity analysis

In this section, we explore the sensitivity of our results to parameters that determine the degree of strategic complementarity in the economy. By looking at parameters that drive the extent to which (downward) marginal cost pressures arising from the a shock with severe demand implications are reflected in price- and wage-setting decisions of optimizing firms and households, we can verify if the intuition set out in the previous section is correct. Finally, we check how the results are affected if the economy has an inherited public debt level significantly above and below the steady-state level.[16]

[15] The optimal debt dynamic would likely differ in a model with a different role for government expenditures, given that the zero-rate interest policy would likely affect public sector investment decisions, for example, were they included in the model. Nevertheless, the model is relevant for real-world considerations in the sense that it shows that stabilization and reduction of debt levels towards a lower efficient level can go hand in hand.

[16] The results are quite predictably sensitive to parameters driving nominal rigidity. When the degree of wage stickiness is lowered (from four to two quarters on average), inflation volatility increases somewhat, and real volatility drops. Also, interest rates are kept at zero for only one period longer than otherwise (two periods in the baseline calibration). However, the main intuition still holds, and the quantitative impact is moderate.

4.2.1. Concave production function

The link between the shape of the production function and strategic complementarity is subtle. When the production function is no longer linear, changes in the amount of labour are no longer proportional to the changes in demand for production. Even in a non-segmented labour market, a profit-maximizing price setter has to consider the situation that his production costs will be more-than-proportionately affected if additional demand comes his way as a result of re-setting prices, whilst others keep theirs unchanged. This, again, introduces caution into the price setting. As a consequence, we observe reduced price volatility and increased output response in the economy with a non-segmented labour market. The peak of the inflation response drops by almost a half of what it was with a linear production function and the time spent at the zero bound lengthens in this economy to 10 periods versus the 9 periods in the baseline version (see Figure A.6). This result confirms that real rigidity—whether induced by a particular labour market structure or other factors such as the shape of the production function—is an important determinant of the magnitude of the desired inflation response to shocks at the zero bound, and the time spent at the zero lower bound.

Somewhat counterintuitively, in the sticky-wage model, inflation volatility increases moderately when production function is modelled to be concave. Inflation now behaves similarly to the economy with segmented labour markets. The reason for this dynamic can be traced back to what happens in the labour market in the flexible price and wage version of our economy. The natural level of output in the economy is determined as the intersection between labour supply and labour demand functions in a (Y, w) plane (equations (2.4) and (2.11) with the left-hand sides equal to one and the ξ's equal to zero). With a linear production function, the demand function is horizontal at a level determined by

the steady-state markup. The labour supply function is upward sloping. If a shock affects labour supply, the equilibrium (natural) real wage rate will stay unaffected. With a concave production function, labour supply still slopes upwards. The labour demand schedule, however, becomes downward sloping in the (Y, w) plane. Marginal cost now depends on the quantity of production and the equilibrium real wage rate thus must fall when output (labour supply) increases.

In the full version of the model, it is a feature of our economy that a future boom is generated to stabilize the economy in the short term. In this boom, labour supply needs to expand, and real wages need to fall as they loosely track the natural rate. A mild inflation facilitates this adjustment. This is shown in Figure A.7. The role of inflation in facilitating real wage adjustment in an economy with sticky nominal wages has been highlighted in Schmitt-Grohé and Uribe (2005).[17]

4.2.2. Degree of competition

With lower substitutability across sectors, one would expect strategic complementarity to play a smaller role in the price setting decision. Firms should not be wary of bold moves, as sizeable demand shifts from or to sectors where prices are not re-optimized happen less easily. If our story about strategic complementarity is true, we should expect larger swings in inflation, smaller volatility in real variables, and a shorter time spent at the zero lower bound. Figure A.8 confirms the intuition. It shows that the optimal economy with a concave production function reverts back towards our baseline model with linear production technology in terms of policies and outcomes once the degree of competition (elasticity of subsitution in the goods market) is lowered.

[17] The shape of the production function may indeed be one of the main contributing factors to the opposite findings concerning optimal inflation volatility by Chugh (2006).

4.2.3. Other parameters

In addition to the parameters reported above, we also checked the sensitivity with respect to the elasticity of substitution in the labour market ε and the elasticity of labour supply (the inverse of which is $\chi_{N,1}$). The results confirm the intuition conveyed above but in comparison with the analysis of different forms of the production function, the sensitivity to changes in the elasticity was less pronounced for plausible values of parameters. This is consistent with Ascari (2003) who makes a similar point.

The intertemporal elasticity of substitution χ_C affects the model in a variety of ways, making sensitivity tests less straightforward. It affects the transmission of the shock and monetary policy in the model, and the wealth effect of labour supply (and hence the slope of the Phillips curve). A shock of a given magnitude has smaller real consequences as before and policy action has to be more forceful to have impact. In our sensitivity analysis, we have increased the magnitude of the shock so that the depth of the contraction is similar to the one observed in the flexible-wage economy non-segmented labour markets above. The key messages from our paper survive this modification. The differences across specifications, however, become relatively small both in terms of policy and outcomes. Hence, we conclude that the characterization of what constitutes 'good' policy in a New Keynesian setup at the zero lower bound is most robust when the intertemporal elasticity of substitution is relatively small.[18]

4.2.4. Initial level of debt

Burgert and Schmidt (2014) demonstrated that inherited debt level matters for both monetary and fiscal policy at the zero lower bound. We examined the state-

[18] For the sake of brevity, the results from these exercises are not displayed here but are available upon request from the authors.

dependency of dynamics in our baseline economy by considering the following two cases. In the 'high debt' scenario, the initial level of public debt was set at twice the steady-state level of debt, i.e. at 100 percent of GDP. In the 'low debt' scenario, the inherited indebtedness was half of the steady-state level of debt. As in Burgert and Schmidt (2014), we find that the magnitude of the inflation response is increasing, the initial increase in government spending is falling, and the initial response in the tax rate is increasing in the level of inherited debt. Their conclusions obtained under discretionary policy thus carry over into an economy with time-consistent policy of the 'timeless perspective' type. In line with much of the New Keynesian literature (see, for instance, Benigno and Woodford, 2004 or Schmitt-Grohé, S. and M. Uribe, 2004), the initial deviation of debt from its steady-state level is never fully undone.[19] This is a manifestation of intertemporal smoothing of welfare in tax and government spending policy.

As regards the interaction between the initial level of debt and labour market structure, our results show that higher initial indebtedness tends to amplify the differences observed across economies with different structures when it comes to inflation volatility, in particular. In the economy with non-segmented labour markets, a larger inflation response (a deeper fall in the real interest rate)—the consideration behind which is to a great extent fiscal (directly and indirectly through the tax rate)—enables a smoother adjustment in real variables. In line with that, government spending barely moves (there is a slight contraction). Overall, as shown in Figure A.9, we see debt level falling well below its initial level, and stabilizing at a level that is much higher than the calibrated steady-state level.

[19] In the case of low inherited debt, the debt-to-GDP ratio falls further, for the same reason as debt falls below its steady-state level in the baseline economy. To economize on space, we do not display this case.

5. Conclusions

We have shown that the optimal length of the forward commitment concerning interest rates at the zero bound and key outcomes such as the magnitude of expected inflation or the depth of the recession under optimal policy depend crucially on the assumed degree of real rigidity in the model. In addition to simple parametric assumptions, more fundamental structural assumptions about the nature of the labour market play an important role in this regard. Labour market segmentation and the presence of staggered wage adjustment were shown to have particularly significant consequences for the type of policy one might wish to implement in an economy hit by a large shock that depresses demand. In those circumstances, it is a good idea for governments to lean against the wind in two different ways. First, an increase in government spending when output is low (and vice versa) stabilizes output (and prices) but this policy can be justified almost wholly with reference to static public finance considerations. Second, an increase in taxes when output is low (and vice versa) stabilizes prices via their impact on marginal cost. The results interact in interesting ways with the initial conditions in the economy. With higher inherited debt, fiscal sustainability considerations matter more for monetary and tax policy and the explained differences across market structures grow larger.

The emphasis in the paper is on theory and intuition. Nevertheless, it should be of interest to modellers working with medium-scale models in which sticky wages are a standard feature. Different estimations of such models often yield diametrically different parameter estimates. Our paper highlights that such shifts in parameter values need not be inoccuous modifications of the setup but may require a different way of thinking about policy, particularly at the zero lower bound.

There is a lot more work to be done in the broadest sense to build better models to study economic cycles and their welfare consequences. The smallest departure from the present setup would be to have a model with a better account of the welfare costs of unemployment or financial market failures. Nevertheless, our paper allows the reader to have a better understanding of how market structures matter for macroeconomic policy and outcomes.

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A. Appendix

A.1. Figures and Tables

Table A.1: Baseline parameter values

Notation	Description	Value
β	discount factor	0.99
χ_C	coefficient of relative risk aversion	1/6
$\chi_{N,0}$	leisure preference parameter	1
$\chi_{N,1}$	inverse Frisch elasticity of labour supply	1
$\chi_{G,0}$	government spending preference parameter	0.25
$\chi_{G,1}$	government spending preference parameter	1
θ	elasticity of substitution in the goods market	11
ε	elasticity of substitution in the labour market	21
ξ_p	probability of no price adjustment	0.75
ξ_w	probability of no wage adjustment	0.75
α	production function parameter	1
$\varepsilon_{\delta,1}$	shock to the discount factor	0.02
ρ_δ	persistence of the shock	0.90

A.2. Optimality conditions

This section lists the equilibrium conditions defining the evolution of the optimal economy under the various scenarios we consider.[20] When fiscal policy instruments are held constant, the respective first-order condition from the Ramsey problem is replaced by an equation that holds the value of the variable constant at its initial steady-state level. Moreover, when nominal interest rates are the only tool used, we assume that lump-sum taxes are available to satisfy the solvency constraint. The conditions are time-invariant due to the fact that we automatically include the terms that appear as a result of the penalty terms added to the objective function under the timeless perspective approach. Such penalty terms summarize

[20] The ω's are Lagrange multipliers associated with the constraints of the Ramsey problem.

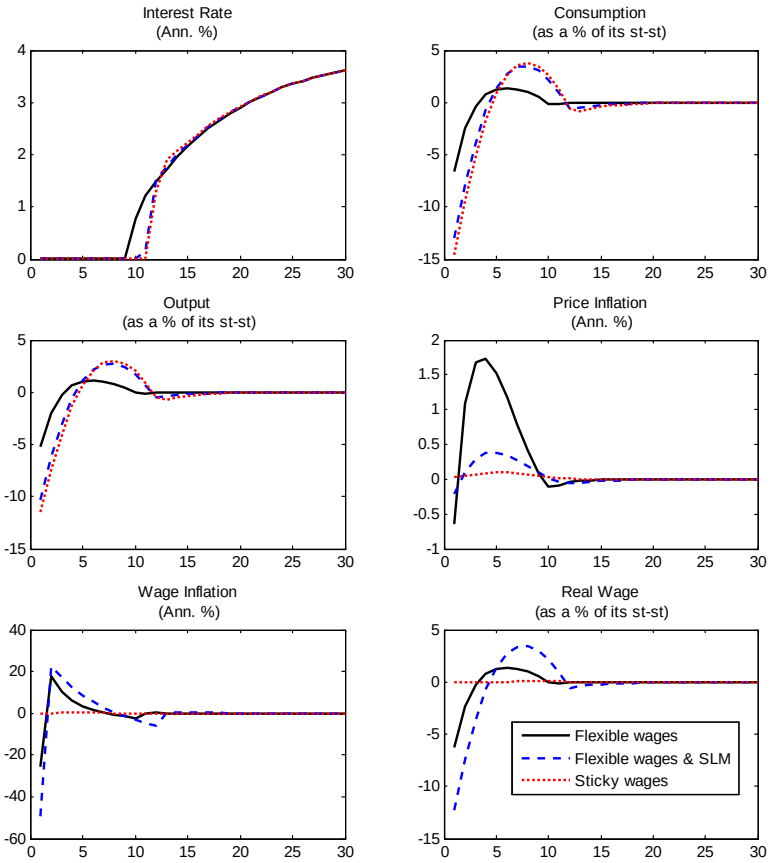

Figure A.1: Optimal response of the baseline economy with fiscal variables held constant and solvency issues ignored (SLM denotes segmented labour markets)

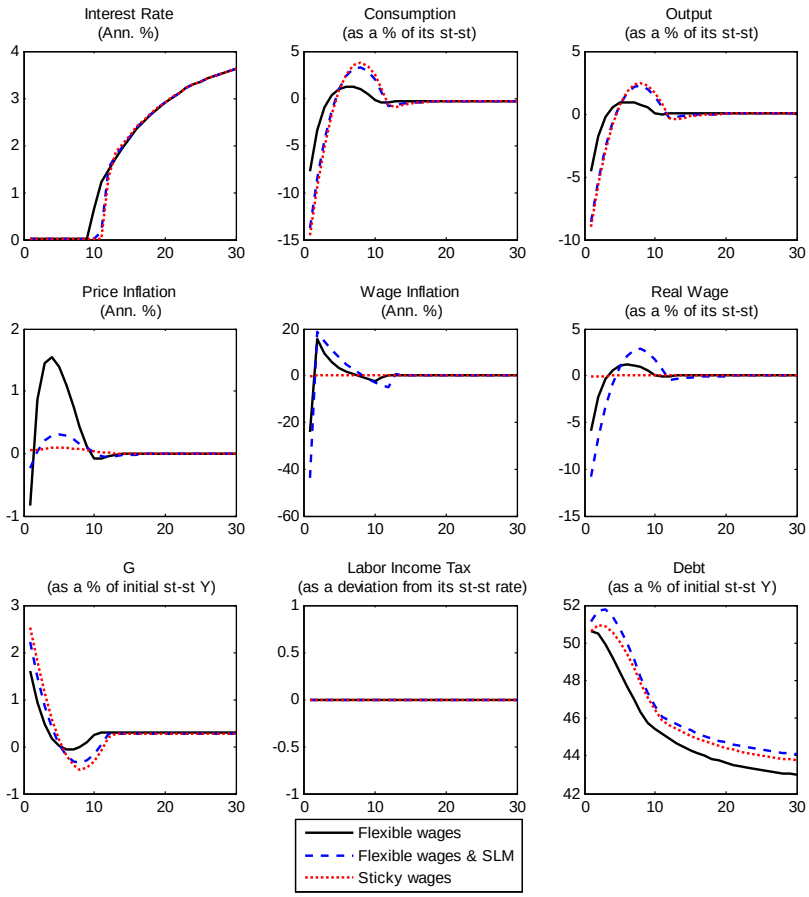

Figure A.2: Optimal dynamics in economies in which the tax rate is held constant

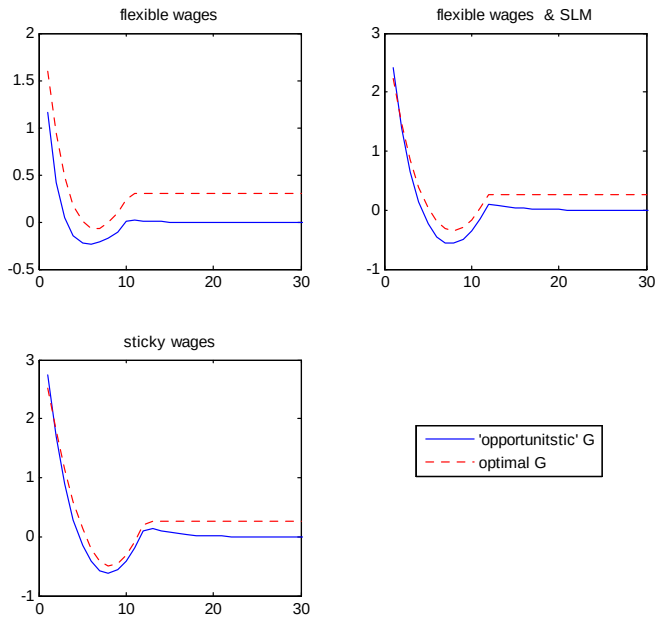

Figure A.3: 'Opportunistic' versus optimal G

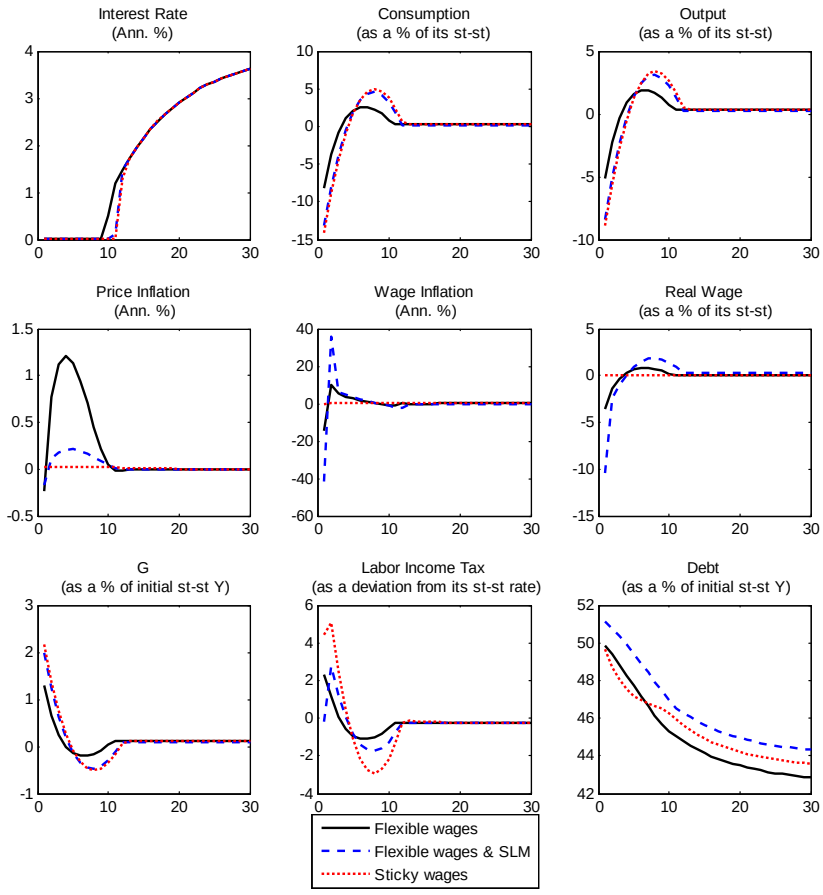

Figure A.4: Optimal dynamics with all policy instruments switched on

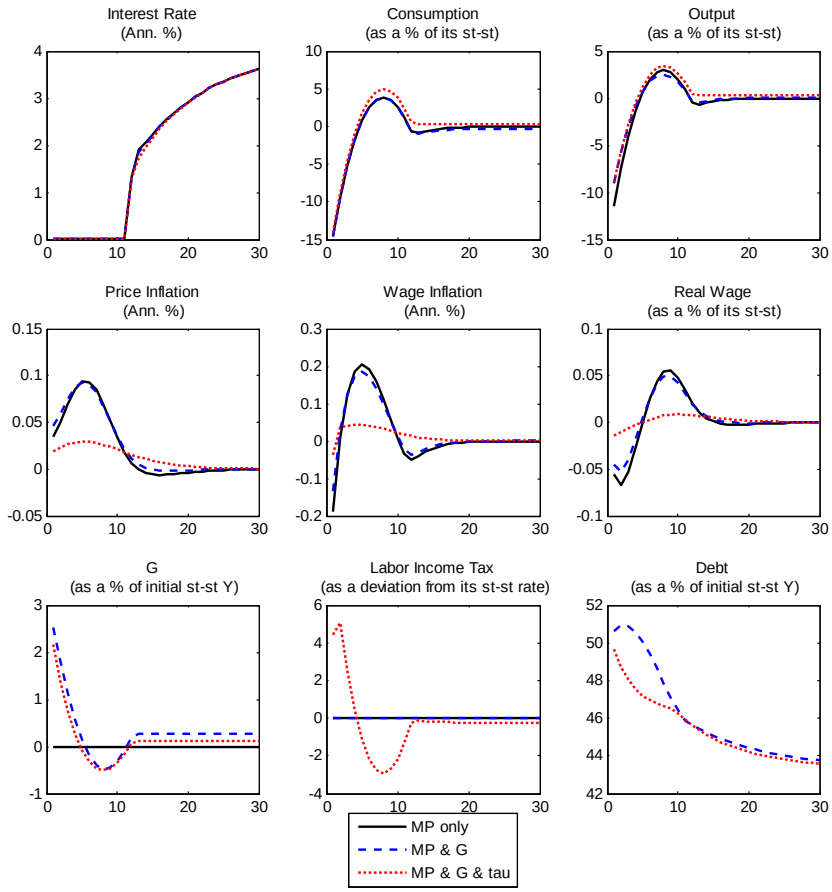

Figure A.5: The sticky-wage economy under various policy scenarios

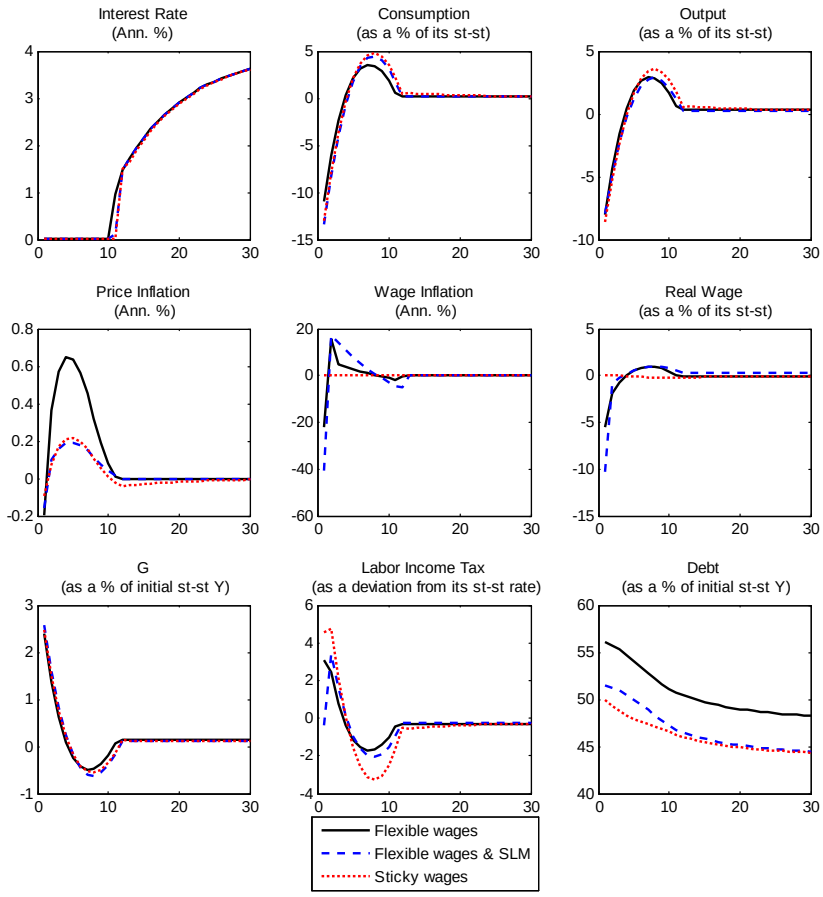

Figure A.6: Optimal dynamics when production function is concave

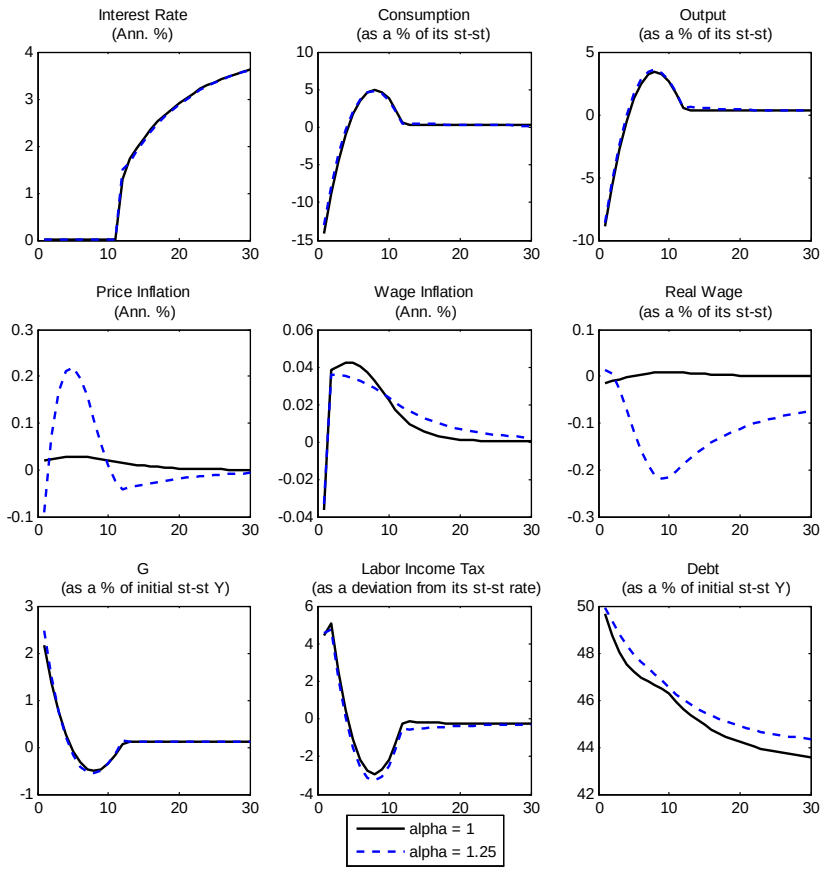

Figure A.7: Optimal dynamics in the sticky wage economy with different production functions

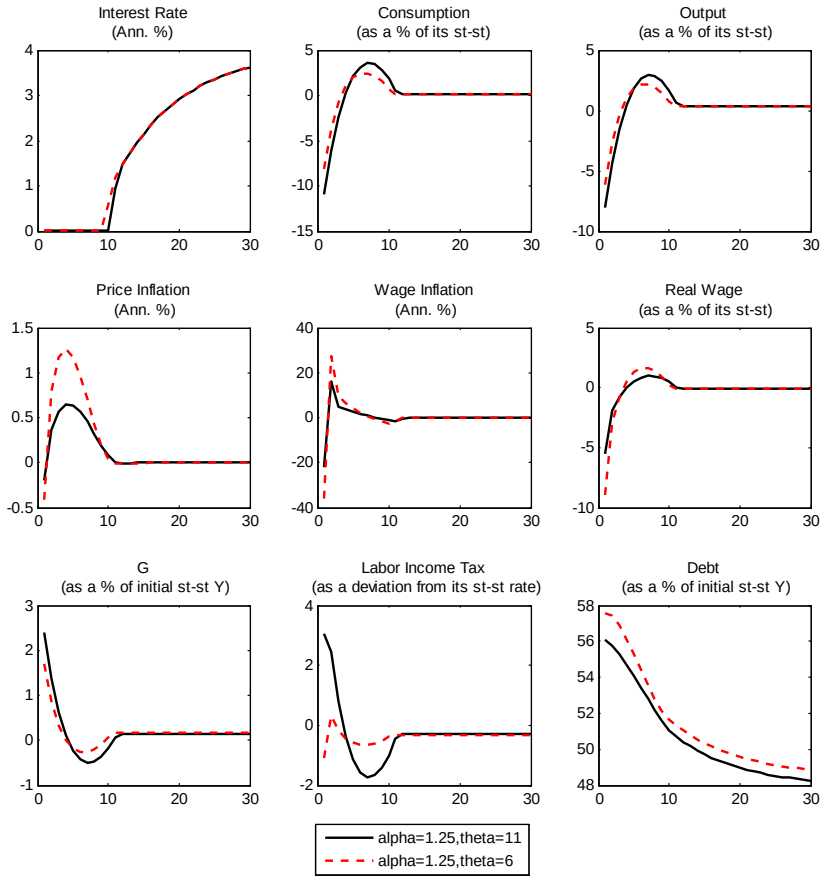

Figure A.8: Optimal dynamics in the flexible-wage economy with economy-wide labour markets under different degrees of product market competition

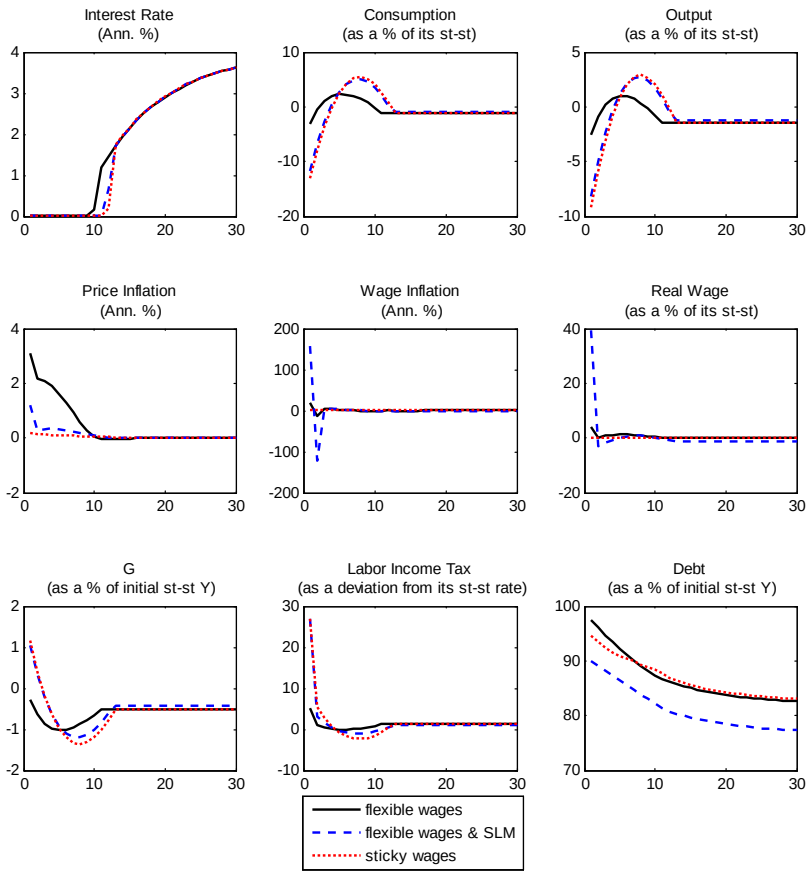

Figure A.9: Optimal dynamics when initial public debt is twice the steady-state value

the commitments concerning the initial period that a policy maker would adhere to should he be implementing policies he would have set for the current period in the distant past. See Benigno and Woodford (2012) for details.

A.2.1. Flexible wage economy

$$
\begin{array}{ll}
\omega_1: & C_t^{-\chi_C} = E_t \beta \delta_t R_t C_{t+1}^{-\chi_C} \Pi_{t+1}^{-1} \\
\omega_2: & (1-\tau_{n,t})\, w_t = \frac{\varepsilon}{\varepsilon-1} \chi_{N,0} N_t^{\chi_{N,1}} C_t^{\chi_C} \\
\omega_3: & s_t = \left(1-\xi_p\right)\left(p_t^*\right)^{-\theta\alpha} + \xi_p \Pi_t^{\theta\alpha} s_{t-1} \\
\omega_4: & 1 = \left(1-\xi_p\right)\left(p_t^*\right)^{1-\theta} + \xi_p \Pi_t^{\theta-1} \\
\omega_5: & \left(p_t^*\right)^{1+\theta(\alpha-1)} = \frac{\theta}{\theta-1} \frac{C_{n,t}}{C_{d,t}} \\
\omega_6: & C_{n,t} = \alpha w_t Y_t^{\alpha} C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta\alpha} C_{n,t+1} \\
\omega_7: & C_{d,t} = Y_t C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta-1} C_{d,t+1} \\
\omega_8: & s_t Y_t^{\alpha} = N_t \\
\omega_9: & C_t + G_t = Y_t \\
\omega_{10}: & \frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} - \tau_{n,t} w_t N_t + G_t \ (\text{-}T_t^{LS} \text{ when monetary policy only}) \\
\omega_{11}: & R_t \geqslant 1 \\
C: & C_t^{-\chi_C} - \omega_{1,t} \chi_C C_t^{-\chi_C-1} + \omega_{1,t-1} \chi_C R_{t-1} C_t^{-\chi_C-1} \Pi_t^{-1} - \omega_{2,t} \frac{\varepsilon}{\varepsilon-1} \chi_C \chi_{N,0} N_t^{\chi_{N,1}} C_t^{\chi_{C-1}} \\
 & + \omega_{6,t} \chi_C \alpha w_t Y_t^{\alpha} C_t^{-\chi_{C-1}} + \omega_{7,t} \chi_C Y_t C_t^{-\chi_{C-1}} - \omega_{9,t} = 0 \\
Y: & \omega_{6,t} \alpha^2 w_t Y_t^{\alpha-1} C_t^{-\chi_C} - \omega_{7,t} C_t^{-\chi_C} + \omega_{8,t} \alpha s_t Y_t^{\alpha-1} + \omega_{9,t} = 0 \\
N: & -\chi_{N,0} N_t^{\chi_{N,1}} - \omega_{2,t} \frac{\varepsilon}{\varepsilon-1} \chi_{N,0} \chi_{N,1} N_t^{\chi_{N,1-1}} C_t^{\chi_C} - \omega_{8,t} - \omega_{10,t} \tau_{n,t} w_t = 0 \\
w: & \omega_{2,t} (1-\tau_{n,t}) - \omega_{6,t} \alpha Y_t^{\alpha} C_t^{-\chi_C} - \omega_{10,t} \tau_{n,t} N_t = 0 \\
\Pi: & \omega_{1,t-1} R_{t-1} C_t^{-\chi_C} \Pi_t^{-2} - \omega_{3,t} \theta \alpha \xi_p \Pi_t^{\theta\alpha-1} s_{t-1} - \omega_{4,t} (\theta-1) \xi_p \Pi_t^{\theta-2} \\
 & - \omega_{6,t-1} \theta \alpha \xi_p \Pi_t^{\theta\alpha-1} C_{n,t} - \omega_{7,t-1} (\theta-1) \xi_p \Pi_t^{\theta-2} C_{d,t} - \omega_{10,t} \frac{b_{t-1}}{\Pi_t^2} = 0 \\
s: & \omega_{3,t} - E_t \omega_{3,t+1} \beta \delta_t \xi_p \Pi_{t+1}^{\theta\alpha} + \omega_{8,t} Y_t^{\alpha} = 0 \\
p^*: & \omega_{3,t} \theta \alpha \left(1-\xi_p\right)\left(p_t^*\right)^{-\theta\alpha-1} + \omega_{4,t} (\theta-1) \left(1-\xi_p\right)\left(p_t^*\right)^{-\theta} \\
 & + \omega_{5,t} \left[1+\theta(\alpha-1)\right]\left(p_t^*\right)^{\theta(\alpha-1)} = 0 \\
C_n: & -\omega_{5,t} \frac{\theta}{\theta-1} C_{d,t}^{-1} + \omega_{6,t} - \omega_{6,t-1} \xi_p \Pi_t^{\theta\alpha} = 0 \\
C_d: & \omega_{5,t} \frac{\theta}{\theta-1} \frac{C_{n,t}}{C_{d,t}^2} + \omega_{7,t} - \omega_{7,t-1} \xi_p \Pi_t^{\theta-1} = 0 \\
R: & \omega_{1,t} \beta \delta_t C_{t+1}^{-\chi_C} \Pi_{t+1}^{-1} + \omega_{10,t} \frac{b_t}{R_t^2} + \omega_{11,t} = 0 \\
G: & \chi_{G,0} G_t^{-\chi_{G,1}} - \omega_{9,t} + \omega_{10,t} = 0 \\
\tau_n: & -\omega_{2,t} - \omega_{10,t} N_t = 0 \\
b: & -\omega_{10,t} R_t^{-1} + E_t \omega_{10,t+1} \beta \delta_t \Pi_{t+1}^{-1} = 0
\end{array}
$$

A.2.2. Flexible wage economy with segmented labour markets

$$\omega_1: \quad C_t^{-\chi_C} = E_t \beta \delta_t R_t C_{t+1}^{-\chi_C} \Pi_{t+1}^{-1}$$

$$\omega_2: \quad (p_t^*)^{1+\theta\left[\alpha\left(1+\chi_{N,1}\right)-1\right]} = \frac{\theta}{\theta-1}\frac{\varepsilon}{\varepsilon-1}\frac{C_{n,t}}{C_{d,t}}$$

$$\omega_3: \quad C_{n,t} = \frac{\alpha\chi_{N,0}Y_t^{\alpha\left(1+\chi_{N,1}\right)}}{(1-\tau_{n,t})} + E_t\beta\delta_t\xi_p\Pi_{t+1}^{\theta\alpha\left(1+\chi_{N,1}\right)}C_{n,t+1}$$

$$\omega_4: \quad C_{d,t} = Y_t C_t^{-\chi_C} + E_t\beta\delta_t\xi_p\Pi_{t+1}^{\theta-1}C_{d,t+1}$$

$$\omega_5: \quad 1 = \left(1-\xi_p\right)\left(p_t^*\right)^{1-\theta} + \xi_p\Pi_t^{\theta-1}$$

$$\omega_6: \quad s_t = \left(1-\xi_p\right)\left(p_t^*\right)^{-\theta\alpha\left(1+\chi_{N,1}\right)} + \xi_p\Pi_t^{\theta\alpha\left(1+\chi_{N,1}\right)}s_{t-1}$$

$$\omega_7: \quad C_t + G_t = Y_t$$

$$\omega_8: \quad \frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} - \frac{\tau_{n,t}}{(1-\tau_{n,t})}\frac{\varepsilon}{\varepsilon-1}\chi_{N,0}Y_t^{\alpha\left(1+\chi_{N,1}\right)}C_t^{\chi_C}s_t + G_t \ (\text{-}T_t^{LS} \text{ when monetary policy only})$$

$$\omega_9: \quad R_t \geqslant 1$$

$$C: \quad C_t^{-\chi_C} - \omega_{1,t}\chi_C C_t^{-\chi_C-1} + \omega_{1,t-1}\chi_C R_{t-1}C_t^{-\chi_C-1}\Pi_t^{-1} + \omega_{4t}\chi_C Y_t C_t^{-\chi_{C-1}}$$
$$-\omega_{7,t} - \omega_{8,t}\frac{\varepsilon}{\varepsilon-1}\chi_C\frac{\tau_{n,t}}{(1-\tau_{n,t})}Y_t^{\alpha\left(1+\chi_{N,1}\right)}C_t^{\chi_{C-1}}s_t = 0$$

$$Y: \quad -\chi_{N,0}\alpha Y_t^{\alpha\left(1+\chi_{N,1}\right)-1}s_t - \omega_{3,t}\frac{\alpha^2\left(1+\chi_{N,1}\right)\chi_{N,0}Y_t^{\alpha\left(1+\chi_{N,1}\right)-1}}{(1-\tau_{n,t})} - \omega_{4,t}C_t^{-\chi_C} + \omega_{7,t}$$
$$-\omega_{8,t}\frac{\varepsilon}{\varepsilon-1}\frac{\tau_{n,t}}{(1-\tau_{n,t})}\left[\alpha\left(1+\chi_{N,1}\right)\right]Y_t^{\alpha\left(1+\chi_{N,1}\right)-1}C_t^{\chi_C}s_t = 0$$

$$\Pi: \quad \omega_{1,t-1}R_{t-1}C_t^{-\chi_C}\Pi_t^{-2} - \omega_{3,t-1}\theta\alpha\left(1+\chi_{N,1}\right)\xi_p\Pi_t^{\theta\alpha\left(1+\chi_{N,1}\right)-1}C_{n,t}$$
$$-\omega_{4,t-1}\left(\theta-1\right)\xi_p\Pi_t^{\theta-2}C_{d,t} - \omega_{5,t}\left(\theta-1\right)\xi_p\Pi_t^{\theta-2}$$
$$-\omega_{6,t}\theta\alpha\left(1+\chi_{N,1}\right)\xi_p\Pi_t^{\theta\alpha\left(1+\chi_{N,1}\right)-1}s_{t-1} - \omega_{8,t}\frac{b_{t-1}}{\Pi_t^2} = 0$$

$$s: \quad -\chi_{N,0}\frac{\alpha Y_t^{\alpha\left(1+\chi_{N,1}\right)}}{\left(1+\chi_{N,1}\right)} + \omega_{6,t} - E_t\omega_{6,t+1}\beta\delta_t\xi_p\Pi_{t+1}^{\theta\alpha\left(1+\chi_{N,1}\right)}$$
$$-\omega_{8,t}\frac{\varepsilon}{\varepsilon-1}\frac{\tau_{n,t}}{(1-\tau_{n,t})}\chi_{N,0}Y_t^{\alpha\left(1+\chi_{N,1}\right)}C_t^{\chi_C} = 0$$

$$p^*: \quad \omega_{2,t}\left\{1+\theta\left[\alpha\left(1+\chi_{N,1}\right)-1\right]\right\}\left(1-\xi_p\right)\left(p_t^*\right)^{\theta\left[\alpha\left(1+\chi_{N,1}\right)-1\right]} - \omega_{5,t}\left(\theta-1\right)\left(1-\xi_p\right)\left(p_t^*\right)^{-\theta}$$
$$+\omega_{6,t}\theta\alpha\left(1+\chi_{N,1}\right)\left(1-\xi_p\right)\left(p_t^*\right)^{-\theta\alpha\left(1+\chi_{N,1}\right)-1} = 0$$

$$C_n: \quad -\omega_{2,t}\frac{\theta}{\theta-1}\frac{\varepsilon}{\varepsilon-1}C_{d,t}^{-1} + \omega_{3,t} - \omega_{3,t-1}\xi_p\Pi_t^{\theta\alpha\left(1+\chi_{N,1}\right)} = 0$$

$$C_d: \quad \omega_{2,t}\frac{\theta}{\theta-1}\frac{\varepsilon}{\varepsilon-1}\frac{C_{n,t}}{C_{d,t}^2} + \omega_{4,t} - \omega_{4,t-1}\xi_p\Pi_t^{\theta-1} = 0$$

$$R: \quad \omega_{1,t}\beta\delta_t C_{t+1}^{-\chi_C}\Pi_{t+1}^{-1} + \omega_{8,t}\frac{b_t}{R_t^2} + \omega_{9,t} = 0$$

$$G: \quad \chi_{G,0}G_t^{-\chi_{G,1}} - \omega_{7,t} + \omega_{8,t} = 0$$

$$\tau_n: \quad -\omega_{3,t}\alpha - \omega_{8,t}\frac{\varepsilon}{\varepsilon-1}C_t^{\chi_C}s_t = 0$$

$$b: \quad -\omega_{8,t}R_t^{-1} + E_t\omega_{8,t+1}\beta\delta_t\Pi_{t+1}^{-1} = 0$$

A.2.3. Sticky-wage economy

$$
\begin{aligned}
\omega_1: &\quad C_t^{-\chi_C} = E_t \beta \delta_t R_t C_{t+1}^{-\chi_C} \Pi_{t+1}^{-1} \\
\omega_2: &\quad s_t = \left(1-\xi_p\right)\left(p_t^*\right)^{-\theta\alpha} + \xi_p \Pi_t^{\theta\alpha} s_{t-1} \\
\omega_3: &\quad 1 = \left(1-\xi_p\right)\left(p_t^*\right)^{1-\theta} + \xi_p \Pi_t^{\theta-1} \\
\omega_4: &\quad \left(p_t^*\right)^{1+\theta(\alpha-1)} = \tfrac{\theta}{\theta-1}\tfrac{C_{n,t}}{C_{d,t}} \\
\omega_5: &\quad C_{n,t} = \alpha Y_t^{\alpha} w_t C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta\alpha} C_{n,t+1} \\
\omega_6: &\quad C_{d,t} = Y_t C_t^{-\chi_C} + E_t \beta \delta_t \xi_p \Pi_{t+1}^{\theta-1} C_{d,t+1} \\
\omega_7: &\quad s_t Y_t^{\alpha} = N_t \\
\omega_8: &\quad C_t + G_t = Y_t \\
\omega_9: &\quad \tfrac{b_t}{R_t} = \tfrac{b_{t-1}}{\Pi_t} - \tau_{n,t} w_t N_t + G_t \ (\text{-}T_t^{LS} \text{ when monetary policy only}) \\
\omega_{10}: &\quad \left(w_t^*\right)^{1+\varepsilon\chi_{N,1}} = \tfrac{\varepsilon}{\varepsilon-1}\tfrac{N_{n,t}}{N_{d,t}} \\
\omega_{11}: &\quad N_{n,t} = \chi_{N,0} N_t^{1+\chi_{N,1}} + E_t \beta \delta_t \xi_w \left(\Pi_{t+1}^{w}\right)^{\varepsilon\left(1+\chi_{N,1}\right)} N_{n,t+1} \\
\omega_{12}: &\quad N_{d,t} = w_t N_t C_t^{-\chi_C}(1-\tau_{n,t}) + E_t \beta \delta_t \xi_w \left(\Pi_{t+1}^{w}\right)^{\varepsilon-1} N_{d,t+1} \\
\omega_{13}: &\quad 1 = \left(1-\xi_w\right)\left(w_t^*\right)^{1-\varepsilon} + \xi_w \left(\Pi_t^{w}\right)^{\varepsilon-1} \\
\omega_{14}: &\quad m_t = \left(1-\xi_w\right)\left(w_t^*\right)^{-\varepsilon\left(1+\chi_{N,1}\right)} + \xi_w \left(\Pi_t^{w}\right)^{\varepsilon\left(1+\chi_{N,1}\right)} m_{t-1} \\
\omega_{15}: &\quad \tfrac{w_t}{w_{t-1}} = \tfrac{\Pi_t^{w}}{\Pi_t}
\end{aligned}
$$

$$
\begin{aligned}
C: \quad & C_t^{-\chi_C} - \omega_{1,t}\chi_C C_t^{-\chi_C-1} + \omega_{1,t-1}\chi_C R_{t-1} C_t^{-\chi_C-1}\Pi_t^{-1} + \omega_{5t}\chi_C \alpha w_t Y_t^{\alpha} C_t^{-\chi_{C-1}} \\
& +\omega_{6,t}\chi_C Y_t C_t^{-\chi_{C-1}} - \omega_{8,t} + \omega_{12,t}\chi_C w_t N_t C_t^{-\chi_C}(1-\tau_{n,t}) = 0 \\
Y: \quad & -\omega_{5,t}\alpha^2 w_t Y_t^{\alpha-1} C_t^{-\chi_C} - \omega_{6,t} C_t^{-\chi_C} + \omega_{7,t}\alpha s_t Y_t^{\alpha-1} + \omega_{8,t} = 0 \\
N: \quad & -\chi_{N,0} N_t^{\chi_{N,1}} m_t - \omega_{7,t} - \omega_{9,t}\tau_{n,t} w_t - \omega_{11,t}\chi_{N,0}\left(1+\chi_{N,1}\right) N_t^{\chi_{N,1}} \\
& -\omega_{12,t} w_t C_t^{-\chi_C}(1-\tau_{n,t}) = 0 \\
w: \quad & -\omega_{5,t}\alpha Y_t^{\alpha} C_t^{-\chi_C} - \omega_{9,t}\tau_{n,t} N_t - \omega_{12,t} w_t C_t^{-\chi_C}(1-\tau_{n,t}) \\
& +\omega_{15,t} w_{t-1}^{-1} - E_t \beta \delta_t \omega_{15,t+1} \tfrac{w_{t+1}}{w_t^2} = 0 \\
\Pi: \quad & \omega_{1,t-1} R_{t-1} C_t^{-\chi_C} \Pi_t^{-2} - \omega_{2,t}\theta\alpha\xi_p \Pi_t^{\theta\alpha-1} s_{t-1} - \omega_{3,t}(\theta-1)\xi_p \Pi_t^{\theta-2} \\
& -\omega_{5,t-1}\theta\alpha\xi_p \Pi_t^{\theta\alpha-1} C_{n,t} - \omega_{6,t-1}(\theta-1)\xi_p \Pi_t^{\theta-2} C_{d,t} - \omega_{9,t}\tfrac{b_{t-1}}{\Pi_t^2} + \omega_{15,t}\tfrac{\Pi_t^w}{\Pi_t^2} = 0 \\
s: \quad & \omega_{2,t} - E_t \omega_{2,t+1}\beta\delta_t \xi_p \Pi_{t+1}^{\theta\alpha} + \omega_{7,t} Y_t^{\alpha} = 0 \\
p^*: \quad & \omega_{2,t}\theta\alpha\left(1-\xi_p\right)\left(p_t^*\right)^{-\theta\alpha-1} + \omega_{3,t}(\theta-1)\left(1-\xi_p\right)\left(p_t^*\right)^{-\theta} \\
& +\omega_{4,t}\left[1+\theta(\alpha-1)\right]\left(p_t^*\right)^{\theta(\alpha-1)} = 0 \\
C_n: \quad & -\omega_{4,t}\tfrac{\theta}{\theta-1} C_{d,t}^{-1} + \omega_{5,t} - \omega_{5,t-1}\xi_p \Pi_t^{\theta\alpha} = 0 \\
C_d: \quad & \omega_{4,t}\tfrac{\theta}{\theta-1}\tfrac{C_{n,t}}{C_{d,t}^2} + \omega_{6,t} - \omega_{6,t-1}\xi_p \Pi_t^{\theta-1} = 0 \\
w^*: \quad & \omega_{10,t}\left(1+\varepsilon\chi_{N,1}\right)\left(w_t^*\right)^{\varepsilon\chi_{N,1}} - \omega_{13,t}(1-\xi_w)(1-\varepsilon)\left(w_t^*\right)^{-\varepsilon} \\
& +\omega_{14,t}(1-\xi_w)\varepsilon\left(1+\chi_{N,1}\right)\left(w_t^*\right)^{\left[-\varepsilon\left(1+\chi_{N,1}\right)-1\right]} = 0 \\
\Pi^w: \quad & -\omega_{11,t-1}\xi_w\varepsilon\left(1+\chi_{N,1}\right)\left(\Pi_t^w\right)^{\varepsilon\left(1+\chi_{N,1}\right)-1} N_{n,t} - \omega_{12,t-1}\xi_w(\varepsilon-1)\left(\Pi_t^w\right)^{\varepsilon-2} N_{d,t} \\
& -\omega_{13,t-1}\xi_w(\varepsilon-1)\left(\Pi_t^w\right)^{\varepsilon-2} - \omega_{14,t}\xi_w\varepsilon\left(1+\chi_{N,1}\right)\left(\Pi_t^w\right)^{\varepsilon\left(1+\chi_{N,1}\right)-1} m_{t-1} \\
& -\omega_{15,t}\Pi_t^{-1} = 0 \\
N_n: \quad & -\omega_{10,t}\tfrac{\varepsilon}{\varepsilon-1}\tfrac{1}{N_{d,t}} + \omega_{11,t} - \omega_{11,t-1}\xi_w\left(\Pi_t^w\right)^{\varepsilon\left(1+\chi_{N,1}\right)} = 0 \\
N_d: \quad & \omega_{10,t}\tfrac{\varepsilon}{\varepsilon-1}\tfrac{N_{n,t}}{N_{d,t}^2} + \omega_{12,t} - \omega_{12,t-1}\xi_w\left(\Pi_t^w\right)^{\varepsilon-1} = 0 \\
m: \quad & -\chi_{N,0}\tfrac{{N_t}^{1+\chi_{N,1}}}{1+\chi_{N,1}} + \omega_{14,t} - E_t\beta\delta_t\omega_{14,t+1}\xi_w\left(\Pi_{t+1}^w\right)^{\varepsilon\left(1+\chi_{N,1}\right)} = 0 \\
R: \quad & \omega_{1,t}\beta\delta_t C_{t+1}^{-\chi_C}\Pi_{t+1}^{-1} + \omega_{9,t}\tfrac{b_t}{R_t^2} + \omega_{16,t} = 0 \\
G: \quad & \chi_{G,0} G_t^{-\chi_{G,1}} - \omega_{8,t} + \omega_{9,t} = 0 \\
\tau_n: \quad & -\omega_{9,t} + \omega_{12,t} C_t^{-\chi_C} = 0 \\
b: \quad & -\omega_{9,t} R_t^{-1} + E_t\omega_{9,t+1}\beta\delta_t\Pi_{t+1}^{-1} = 0
\end{aligned}
$$