State Dependence in Labor Market Fluctuations: Evidence, Theory, and Policy Implications

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August 2018
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Abstract

This paper documents state dependence in labor market fluctuations. Using a Threshold Vector-Autoregression model, we establish that the unemployment rate, the job separation rate and the job finding rate exhibit a larger response to productivity shocks during periods with low aggregate productivity. A Diamond-Mortensen-Pissarides model with endogenous job separation and on-the-job search replicates these empirical regularities well. The transition rates into and out of employment embed state dependence through the interaction of reservation productivity levels and the distribution of match-specific idiosyncratic productivity. State dependence implies that the effect of labor market reforms is different across phases of the business cycle. A permanent removal of layoff taxes is welfare enhancing in the long run, but it involves distinct short-run costs depending on the initial state of the economy. The welfare gain of a tax removal implemented in a low-productivity state is 4.9 percent larger than the same reform enacted in a state with high aggregate productivity.

JEL Classification: E24, E32, J64, C11.

Keywords: Search and Matching Models, State Dependence in Business Cycles, Threshold Vector Autoregression.

*We would like to thank Paul Beaudry, Anton Braun, Alex Clymo, Juan Dolado, Renato Faccini, Roger Farmer, Jesus Fernandez-Villaverde, Chris Foote, Shigeru Fujita, Francesco Furlanetto, Gabriele Galati, Masashige Hamano, Peter Ireland, Philipp Kircher, Michael Krause, Federico Mandelman, Monika Merz, Ryan Michaels, Steve Millard, Paulo Santos Monteiro, John Muellbauer, Morten Ravn, Ricardo Reis, Juan Rubio-Ramirez, Michal Rubaszek, Frank Schorfheide, Shigenori Shiratsuka, Harald Uhlig, Thijs Van Rens, Yang Yu, Tao Zha and seminar participants at the Asian Development Bank Institute, Bank of England, Bank of Japan, Birmingham University, De Nederlandsche Bank, European Central Bank, European University Institute, Federal Reserve Board, Federal Reserve Bank of Atlanta, Federal Reserve Bank of Boston, Federal Reserve Bank of Philadelphia, International Monetary Fund, Keio University, Narodowy Bank Polski, Norges Bank, University of Bern, University of Cologne, University of Oxford, University of Sheffield, University of Sophia, University of Surrey, University of Tokyo, University of York, University of Warwick, University of Waseda, CEPR-ECB annual conference on labor markets, Bank of Italy-Bocconi-Bicocca-Cattolica Workshop on Macroeconomic Dynamics, the Royal Economic Society (Sussex), the Econometric Society Meetings of the European Economic Association (Geneva), Essex Search and Matching Workshop and SaM Workshop at the University of Cambridge for extremely valuable comments. Francesco Zanetti would like to thank the Bank of Japan and its staff for hospitality and support while some of this work was completed. The present project was partially supported by the John Fell Fund. Please address correspondence to Carlo Pizzinelli or Francesco Zanetti, University of Oxford, Department of Economics, Manor Road, Oxford, OX1 3UQ, UK; Emails: Carlo.Pizzinelli@economics.ox.ac.uk or Francesco.Zanetti@economics.ox.ac.uk.
1 Introduction

Numerous studies, starting with Neftci (1984), show that macroeconomic fluctuations differ across phases of the business cycle. This paper builds on this strand of research and identifies systematic changes in the cyclical properties of labor market variables that are linked to the state of aggregate productivity. Movements in the unemployment rate, job separation rate, and job finding rate are considerably larger in periods of low aggregate productivity. A Threshold Vector Autoregression (TVAR) model, which identifies the effect of productivity shocks and allows for two distinct regimes of aggregate productivity, establishes that the shocks have a significantly larger effect on the unemployment rate, the job separation rate, and the job finding rate in periods of low aggregate productivity.

To explain these findings and assess the policy implications, we develop a Diamond-Mortensen-Pissarides (DMP) search model with endogenous job separation and on-the-job search (OJS). The primary driver of state dependence is the interaction between the threshold and distribution of individual productivity. In a state with low aggregate productivity, the firm retains profits by setting a high reservation threshold to yield a positive surplus in the match, dismissing jobs with individual productivity below the reservation threshold. Under standard assumptions, the high threshold lies in a region of the match-specific distribution with a high density of jobs. An exogenous movement in aggregate productivity that changes the threshold generates large shifts in the job separation rate and the unemployment rate. Conversely, in a state with high aggregate productivity, the firm sets a low threshold for efficient matches that is associated with a low density of jobs. An equivalent change in productivity produces limited movements in job separation and unemployment rates. By the same mechanism, a state with low aggregate productivity entails larger fluctuations in the share of new matches that fail to turn into employment, increasing the volatility of the job finding rate.

Our baseline model replicates observed business cycle fluctuations and generates powerful state dependence in the job separation rate and job finding rate, which exhibit larger response to productivity shocks in periods of low aggregate productivity. Large movements in job transition probabilities in periods of low aggregate productivity jointly contribute to the state depen-
dence of the unemployment rate, whose volatility is 18 percent larger in periods of aggregate productivity below median. Robustness analysis, which considers alternative calibrations and specifications of the baseline model excluding OJS and endogenous separations, shows that the calibration of individual productivity is critical to generate state dependence in labor market variables together with plausible aggregate fluctuations. OJS contributes to state dependence by amplifying movements in the measure of job seekers.

To illustrate the mechanism underpinning state dependence and the importance for the effect of labor market reforms, we enrich the baseline model with “wasteful” layoff taxes levied on the firm for the termination of existing jobs but averted on new matches that fail to turn into jobs. The tax increases the surplus of incumbent matches that continue into the next period since they forego the tax payment, but it reduces the surplus of new matches in the prospect of paying the layoff tax if the job terminates in the future. The tax raises the threshold of individual productivity that makes new matches profitable, therefore discouraging OJS. The pool of job seekers diminishes, and the firm’s recruiting costs for establishing a profitable match rise, leading to a decrease in hiring. Overall, the layoff tax considerably reduces the job finding rate and increases the pool of workers subject to job separation by discouraging OJS. These complementary forces generate a rise in the unemployment rate.

We use the model with layoff taxes to assess whether an unexpected permanent removal of the tax generates distinct transitional dynamics and welfare effects when implemented in states with low and high aggregate productivity. In the long run, the elimination of the tax generates a fall in the unemployment rate and a rise in output that is welfare-enhancing regardless of the initial states of aggregate productivity. In the short run, however, the reform generates sharp differences in the transitional dynamics of labor market variables across initial states. The unemployment rate gradually declines in the state with high aggregate productivity whereas in the low-productivity state, it suddenly contracts. These temporary differences disappear after four quarters, but they produce significant welfare differences. The tax removal raises the

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1 We focus on layoff taxes since Cacciatore and Fiori (2016) show that they are effective policies in reducing inefficiencies of unemployment fluctuations. In addition, an array of studies shows that they are powerful in affecting labor market outcomes (see, for example, Campolmi and Faia (2011), Zanetti (2011a) and references therein).
surplus of establishing a job relation, inducing firms to post vacancies and workers to search on the job. The considerable rise in search efforts by firms and workers generates temporary welfare losses caused by the deadweight costs of matching frictions. Because the rise in vacancies and OJS following the tax removal is larger in high-productivity states, the short-run costs of the reform are greater in a high-productivity state than in a low-productivity state. The total discounted welfare gain of a tax removal enacted in the state with low aggregate productivity is 4.9 percent larger than the same reform in the state with high-aggregate productivity.

Our analysis relates to empirical and theoretical studies on the asymmetry of labor market fluctuations over the business cycle. On the empirical side, the works by Neftci (1984), Altissimo and Violante (2001), Panagiotidis and Pelloni (2007), Barnichon (2012), Abbrittti and Fahr (2013), Barattieri et al. (2014), Caggiano et al. (2014), and Benigno et al. (2015) show that unemployment and wages fluctuate differently across phases of the business cycles. Compared to these studies, we establish that state dependence in labor market fluctuations is linked to the level of aggregate productivity, and we extend the analysis to job transition rates. On the theoretical side, our work is related to studies that develop structural models to investigate asymmetric dynamics of the labor market. Unlike Sedlăček (2014) and Kohlbrecher and Merkl (2016), who focus on job creation, we study nonlinearities allowing for interaction between the job finding rate and the separation rate, a choice that is empirically supported by the findings of the TVAR model. Ferraro (2016) shows that employment cycles are characterized by large skewness and develops a search model with permanent worker heterogeneity in productivity to explain the finding. Our version of the DMP model hinges on heterogeneity in match-specific productivity, which allows for job-to-job transitions. We focus on OJS building on the work of Fujita and Ramey (2012) that shows it is critical to deliver a realistic performance of the DMP model. Our analysis also establishes the importance of OJS for timing in the implementation of structural reforms. Petrosky-Nadeau and Zhang (2017) show that a standard DMP model with exogenous job separation generates state dependence via the inherent nonlinearities of the policy function for market tightness. We contribute to this strand of research by developing a framework that embeds nonlinearities in both the job separation and job finding rates.

Finally, this paper is related to the growing body of literature that explores the state-
dependent effect of labor market reforms on aggregate fluctuations. In the context of search and matching models, Zanetti (2009), Poilly and Wesselbaum (2014), Cacciatore et al. (2015), Jung and Kuester (2015) and Cacciatore and Fiori (2016) study the transition dynamics of loosening employment protection and workers bargaining power without distinguishing between reforms enacted during different phases of the business cycle. Eggertsson et al. (2014) and Cacciatore et al. (2016) consider the impact of a reduction in employment protection enacted at the zero lower bound of monetary policy or during a large recession. Differing from our analysis, they find that implementing a reform during recessions exacerbates the economic downturn and unemployment increases in the short run. In our model, the effect of labor market reforms is primarily driven by the job finding rate and job-to-job transitions while the reaction of job separation is muted due to the offsetting increases in OJS. Unlike these studies, our analysis abstracts from changes in aggregate demand that may result from labor reforms and which diminish the benefits of reforms during downturns.

The remainder of the paper is structured as follows. Section 2 presents the empirical findings. Sections 3 and 4 outline the model and discuss the main mechanisms generating state dependence in labor market fluctuations, respectively. Section 5 describes the calibration and presents the main results. Section 6 performs a series of robustness checks on the calibration and alternative specifications of the model. Section 7 assesses differences in the implementation of labor market reforms. Section 8 concludes.

2 Empirical evidence

This section isolates systematic differences in fluctuations of labor market variables linked to the state of aggregate productivity. Comovements of unemployment and job transition rates with average labor productivity are stronger in periods of low aggregate productivity, resulting in a larger volatility of labor market variables. We assess the evidence descriptively and formally through a structural TVAR model. We use quarterly series for the (un)employment rate, the job finding rate, the job separation rate, output, hours, and labor productivity over the period.
1950:I-2014:IV. To extract the cyclical component of variables, we use an HP filter with a smoothing parameter equal to 1,600.

2.1 Descriptive evidence

Figure 1 plots quarterly growth rates for the unemployment rate, job separation rate, and job finding rate against the quarterly growth rate of labor productivity for periods in which the level of productivity is above (left panels) and below (right panels) the median value. The elasticity coefficients are larger in periods of low productivity, suggesting that the comovement between changes in labor market variables and changes in productivity is stronger in periods of low productivity. This first pass of the data outlines systematic differences in the variability of the unemployment rate that are linked to the state of productivity. In the subsequent analysis, we use a more formal statistical method to isolate significant changes in the cyclical properties of labor market variables across distinct states of productivity.

Table 1 shows the standard deviation of the unemployment rate, the separation rate, the job finding rate, the employment rate, output, and productivity, across periods in which the initial level of productivity is below (Column 1) or above (Column 2) the median value of its cyclical component and the ratio of the standard deviations of each variable in periods when productivity is below and above the median value (Column 3). Column (4) reports the $P$-value on the statistical significance of the differences in volatilities, as proposed by Levene (1960). The table reports the standard deviation of variables in levels (top panel), quarterly growth rates (middle panel), and yearly growth rates (bottom panel). The standard deviations of most of the variables in levels are 20 to 30 percent larger in periods of productivity below the median.

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3 Appendix A.1 provides details on the data sources. Appendix B shows that results are robust to using a smoothing parameter equal to 10.$^5$

4 Specifically, the figure plots $\Delta x_t = x_t - x_{t-1}$ when productivity at time $t-1$ is above or below the median. The median value of the HP-filtered series is 0, which is very close to 0, suggesting that the two states can be interpreted as periods in which productivity is below or above trend. The elasticity coefficients are computed with a univariate regression.

5 In the online appendix we show that, using the steady-state unemployment rate approximation proposed by Shimer (2012), both the separation and job finding rates contribute to the larger volatility of unemployment in times of low productivity.

6 The null hypothesis of the test is that the variances of the two samples are the same. Therefore, lower $P$-values provide stronger evidence for the rejection of the null.

7 For the levels, we take the log of all variables before we apply the HP filter except for the separation and job finding rates. Growth rates are approximated by log differences.
Figure 1: Labor market fluctuations and labor productivity across phases of the business cycle.

Note. The figures plot quarterly growth rates of the unemployment rate, the separation rate, and the job finding rate against quarterly growth rates of labor productivity. The left and right panels consider periods in which the starting level of productivity is respectively above and below the historical median of its cyclical component. The red solid line represents the best fit from a least squares regression, with the slope coefficient reported in each plot.

The job finding rate exhibits a more limited difference across states of productivity with a ratio across variances equal to 1.07. The P-value for all entries is below 0.1, suggesting strong statistical significance in systematic differences in the variability of labor market variables across states of aggregate productivity. Entries in growth rates show similar results, with unemployment and employment rates showing particularly large differences between the two states. Only the separation rate has a P-value above 0.1.

To ensure results are robust, we undertake a series of robustness checks, reported in Ap-

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8If we use first differences of the level, instead of the log, the difference in the separation rate volatility also reaches statistical significance.
Table 1: Standard deviation for different states of productivity

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>$\sigma_{p&lt;\text{Median}}$</th>
<th>$\sigma_{p&gt;\text{Median}}$</th>
<th>$\sigma_{p&lt;50}$/$\sigma_{p&gt;50}$</th>
<th>$P$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Levels</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Unemployment</td>
<td>0.1511</td>
<td>0.1197</td>
<td>1.26</td>
<td>0.029</td>
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<tr>
<td>Job Finding Rate</td>
<td>0.0397</td>
<td>0.0335</td>
<td>1.19</td>
<td>0.082</td>
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<tr>
<td>Separation Rate</td>
<td>0.0020</td>
<td>0.0013</td>
<td>1.53</td>
<td>0.000</td>
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</tr>
<tr>
<td>Employment Rate</td>
<td>0.0095</td>
<td>0.0075</td>
<td>1.27</td>
<td>0.016</td>
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<tr>
<td>Output</td>
<td>0.0215</td>
<td>0.0167</td>
<td>1.29</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0092</td>
<td>0.0071</td>
<td>1.31</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td><strong>Quarterly Growth Rates</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0829</td>
<td>0.0401</td>
<td>2.06</td>
<td>0.00</td>
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<tr>
<td>Job Finding Rate</td>
<td>0.0566</td>
<td>0.0444</td>
<td>1.27</td>
<td>0.03</td>
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<tr>
<td>Separation Rate</td>
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<td>0.0497</td>
<td>1.16</td>
<td>0.20</td>
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<tr>
<td>Employment Rate</td>
<td>0.0051</td>
<td>0.0023</td>
<td>2.15</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.0153</td>
<td>0.0090</td>
<td>1.71</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0099</td>
<td>0.0074</td>
<td>1.34</td>
<td>0.00</td>
<td></td>
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<tr>
<td><strong>Yearly Growth Rates</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.2302</td>
<td>0.1651</td>
<td>1.39</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>0.1435</td>
<td>0.1023</td>
<td>1.40</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Separation Rate</td>
<td>0.0912</td>
<td>0.0855</td>
<td>1.07</td>
<td>0.33</td>
<td></td>
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<tr>
<td>Employment Rate</td>
<td>0.0147</td>
<td>0.0091</td>
<td>1.62</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.0361</td>
<td>0.0274</td>
<td>1.32</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0191</td>
<td>0.0168</td>
<td>1.14</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

Note. The data is quarterly over the period 1950:I-2014:IV. The series of the (un)employment rate, output, and productivity are in logs. Series are HP-filtered with a smoothing parameter equal to 1,600. Growth rates are log differences of quarterly averages. In columns (1)-(3), $\sigma_{p<\text{Median}}$ ($\sigma_{p>\text{Median}}$) represents the standard deviation of the variable for the productivity state below (above) the median, and its ratio. Column (4) reports the $P$-value for the statistical test by Levene (1960) against the null hypothesis of the two variances being equal.

Appendix B: In Table B.2, we show that results continue to hold if we use a smoothing parameter for the HP filter equal to $10^5$—as suggested by Shimer (2005)—or set the regimes based on the productivity series by Fernald (2014). We show that state dependence is robust to two specific sub-periods: the Great Moderation (1984-2007) and the full pre-Great Recession period (1950-2007). Finally, we show that results hold if we consider more marked cases of low and high productivity, using as thresholds the 25th and 75th percentiles of average labor productivity, respectively, thus excluding observations in second and third quartiles. We further assess the sensitivity of results to defining low- and high-productivity states using alternative variables: yearly growth rates of productivity, NBER recession dates, quarterly growth rates of productivity (both a four quarter moving average and in its raw series), and output. Tables B.1 and B.3 in the Appendix show that alternative definitions for the thresholds produce classifications of

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For all of the series except the NBER recessions dates, the threshold is based on the median value of the variable. For the recession dates, we base the low state as the quarters of economic recession.
regimes that are similar to those in the benchmark analysis and that results continue to hold. These findings point to large and systematic differences in the variability of labor market variables across states of aggregate productivity. Since labor productivity itself exhibits a larger volatility in states with low aggregate productivity, in the next subsection we assess the systematic differences in labor market fluctuations using a Threshold Vector-Autoregression (TVAR) model that isolates the response of variables to shocks controlling for the state of productivity.

2.2 The Threshold Vector Autoregression model

The TVAR model, based on the original study by Chen and Lee (1995), allows the VAR parameters to vary across an aggregate state of the economy. The switching mechanism is based on the value of one of the endogenous variables being above or below a threshold, and unlike Markov-switching models, the parameter change is endogenous to the dynamics of the VAR process. The reduced-form model can be expressed as follows:

\[ Z_t = \xi_t \left\{ c_1 + \sum_{k=1}^{K} B_{k,1} Z_{t-k} + \Sigma_1^{1/2} v_t \right\} + (1 - \xi_t) \left\{ c_2 + \sum_{k=1}^{K} B_{k,2} Z_{t-k} + \Sigma_2^{1/2} v_t \right\} , \]

where \( Z_t \) is the vector of \( N \) observed variables, \( c_1 \) and \( c_2 \) are constant coefficients, \( B_{k,1} \) and \( B_{k,2} \) are coefficients of the VAR, \( \Sigma_{k,1} \) and \( \Sigma_{k,2} \) are covariance matrices, and \( v_t \) is the error term. Switches across regimes are governed by the indicator variable \( \xi_t \in \{0, 1\} \), which is equal to 1 if labor productivity in period \( t - 1 \), \( \tilde{z}_{t-1} \) is below the threshold \( z^* \), otherwise it is equal to zero:

\[ \xi_t = 1 \text{ if } \tilde{z}_{t-1} < z^*, \text{ otherwise } \xi_t = 0. \]

Under conjugate priors for VAR parameters and conditional on the value of the threshold \( z^* \), the posterior distribution of the VAR coefficient vector is a conditional Normal-Wishart

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\(^{10}\)Table B.3 shows that the larger volatility of labor market variables holds for alternative definitions of the threshold based on the level and growth rate of output, NBER recessions, and quarterly and yearly growth rates of productivity. Results based on the series of productivity growth rates (yearly and quarterly) fail to generate large state dependence in the unemployment rate and job finding rate. This is explained by the fact that growth rates of productivity are not necessarily indicative of whether the aggregate level of productivity is high or low. Table B.1 shows that there are several quarters in which the level of productivity is above median, but the growth rate of productivity is below median.
distribution, and we obtain draws with the Gibbs sampler. Since the posterior distribution of \( z^* \) conditional on the VAR parameters is unknown, we use a Metropolis-Hastings step to obtain the posterior distribution (see Chen and Lee (1995); Chen (1998); Lopes and Salazar (2006)). Appendix C.1 provides details on priors.

The variables of interest are labor productivity, unemployment rate, job separation rate, and job finding rate. To be consistent with the related literature, particularly Balleer (2012) and Canova et al. (2012), we include average hours worked and Fernald’s measure of labor productivity. We use eight lags in the TVAR model (i.e., \( k = 8 \)), and one-quarter lag of labor productivity as the variable determining the state \( \xi_t \). All variables are in logs and HP-filtered using a smoothing parameter equal to 1,600. The median of the posterior of the threshold \( z^* \) is equal to 0.12, and the mean is 0.11. These values are fairly close to 0, and hence to the median of the unconditional distribution of productivity.

To identify technology shocks, we follow the medium-run, maximum variance scheme proposed by Uhlig (2004) and assume that the productivity shock explains the majority of the forecast error variance of labor productivity at business cycle frequencies (i.e. over the horizon of 0 to 40 quarters). Appendix C.2 provides details on the identification scheme.

To assess whether responses to technology shocks are significantly different across states, Figure 2 plots the linear Impulse Response Functions (IRFs) produced for each regime separately. The identified productivity shock has a larger effect on the unemployment rate, job separation rate, and job finding rate when labor productivity is below the threshold (second row) and differences across states are statistically significant (third row). The estimated variance of the productivity shock is similar across regimes, as shown in the entries in the first column of the figure, and therefore, the variance of productivity explained by the identified productivity shock is sub-

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\[ ^{11} \] The results are robust to excluding hours. Robustness checks to the TVAR model are reported in the online appendix.

\[ ^{12} \] We set the number of lags in the benchmark model equal to eight to facilitate comparison of the benchmark identification scheme with the identification that uses long-run restrictions as in Balleer (2012) and Canova et al. (2012) (results are reported in the online appendix). As it is established in those studies and it is further discussed in Erceg et al. (2005) and Ravenna (2007), a sufficiently large number of lags is needed to mitigate identification bias that arises from the (lag) truncation of the VAR. The online appendix shows that results continue to hold with fewer lags.

\[ ^{13} \] Figure C.2 in the Appendix plots prior and posterior distributions for the threshold \( z^* \).

\[ ^{14} \] The online appendix shows that the results are robust when the productivity shock is identified using long-run identification restrictions (see sections 3.2 and 3.3.4).
stantially identical across states. This observation holds true despite the unconditional variance of productivity in Table 1 being larger in periods of low productivity.\footnote{Figure C.3 in the Appendix reports the fraction of forecast error variance of each endogenous variable that is explained by productivity shocks. Productivity shocks explain more than double of the forecast error variance of all variables at horizons between one and 12 quarters in periods of low aggregate productivity.}

Figure 2: Benchmark Model: Impulse Responses

Note. The solid blue line represents the pointwise median IRF, and the shaded area is the corresponding 16th and 84th percentiles of the posterior distribution. Horizontal axes report quarters, vertical axes report percentage deviations from the trend. The third row displays the posterior distribution of the difference between impulse responses for low (first row) and high (second row) states of productivity. The red line in the second row is the pointwise median from the Low Productivity Regime.

The analysis reveals strong and statistically significant state dependence in the response of labor market variables to the productivity shock. The IRFs are obtained using regime-specific coefficient matrices, which assume that the system remains in the current regime. To ensure that results are robust to the possibility for the system to switch across regimes of productivity, we compute simulation-based Generalized IRFs, as developed in Koop et al. (1996).\footnote{Appendix C.3 describes computation.} The approach computes IRFs at different points of the business cycle, accounting for potential transitions across regimes that may influence the dynamic response of variables to the
technology shock.

Figure 3: Generalised Impulse Responses: Low- versus High-Productivity Starting Point

Note. The solid line represents the pointwise median IRF, and the shaded area is the corresponding 16th and 84th percentiles of the posterior distribution. The horizontal axes are in quarters; the vertical axes are in percentage deviations from the trend. The third row displays the posterior distribution of the difference between Low- (first row) and High- (second row) Productivity Regime Impulse Responses. The red line in the second row is the pointwise median from the Low Productivity Regime. The terms Low- and High-Productivity Regime refer now to the starting point of the economy (initial conditions/histories) when the shock “hits” the economy.

Figure 3 compares generalized IRFs at a starting point of productivity below the estimated threshold (Low Productivity Regime) against responses with a starting point of productivity above the estimated threshold (High Productivity Regime). Different from responses in Figure 2, the reaction of variables to technology shocks in Figure 3 accounts for the possibility that the current regime may change depending on the size and sign of the shock. At the peak of response (roughly four to five quarters), the reaction of the unemployment rate is twice as large as at the low productivity regime compared to the high productivity regime. Similarly, responses of the job separation rate and job finding rate are three times and twice as large in the state of low productivity, respectively. Overall, these findings establish that fluctuations in
the unemployment rate, the job separation rate and the job finding rate exhibit statistically
different responses to technology shocks across states of aggregate productivity.\footnote{The online
appendix shows that the results are robust to the use of a shorter number of lags (Section 3.3.1),
looser priors (Section 3.3.2), different measures of labour productivity (Section 3.3.3), and different
identification schemes (Section 3.3.4).}

3 The model

This section lays out a DMP search and matching model with endogenous job separation
and OJS. The main features of the model as similar to those in \cite{mortensen1994search}, \cite{merz1999}, \cite{krause2007labor}, \cite{thomas2009}, \cite{fujita2012}, and \cite{mueller2017}. It differs from these studies in allowing for the separation of
newly-established jobs and, in the last section, by introducing layoff taxes on the termination
of existing jobs while workers can search on the job similar to \cite{sedlac2014}.

Economic environment and timing. A continuum of households of mass one and a con-
tinuum of firms operate in a discrete time environment. Households supply labor to firms
inelastically. Matching frictions in the labor market prevents full employment. Firms pay a
fixed cost for each vacancy posted to recruit new workers. However, in a given period, nei-
ther are all vacancies filled nor are all job-seekers hired. Employed workers produce a single
consumption good, whose price is normalized to one, and they may search for a new job while
employed. In every period $t$, production by a single worker depends on aggregate labor produc-
tivity, $a_t$, and individual, idiosyncratic productivity, $x$. For each $a_t$, there is a reservation level
of individual productivity $x^r(a_t)$, below which jobs are not mutually efficient and are dismissed.
Similarly, there is a level of individual productivity $x^s(a_t)$, below which employed workers find
it efficient to pay a fixed cost to search for other jobs.\footnote{Below, we use the following notation: when $x$ has a time subscript, it refers to an aggregate variable (e.g. $x^r_t$ is the individual productivity threshold that applies to all firms given an aggregate productivity level $a_t$). Without a subscript, it refers to any individual productivity level for a match independent of aggregate states.}

Within each period $t$, the timing of events is as follows. At the start of period, firms
post vacancies that are matched with job seekers by the end of the period. Employed workers

produce and may search for a new job within the period. At the end of the period, a fraction of employed workers is exogenously separated, and another fraction of employed workers obtains a new draw of individual productivity. At the beginning of the next period $t + 1$, aggregate productivity $a_{t+1}$ and the individual productivity $x$ of each worker are observed. Each firm converts profitable matches into jobs, and each worker that searches on the job decides whether to move to a new firm or remain in the current job.

The matching function. A matching function encapsulates search frictions in the labor market. In each period $t$, the constant-returns-to-scale matching function establishes the number of matches between job seekers and vacancies:

$$m_t = m(u_t + \psi_t, v_t) = \gamma(u_t + \psi_t)^{1-\eta}v_t^\eta, \quad (3)$$

where $u_t$ is unemployment, $\psi_t$ is the mass of OJS workers, $v_t$ are vacancies, and $0 < \eta < 1$. The sum of employed OJS workers and unemployed workers forms the number of job searchers. The probability for a job seeker to match a vacancy and for a vacancy to be filled can be expressed in terms of the “labor market tightness,” defined as the ratio of vacancies to job seekers, $\theta_t = v_t/(u_t + \psi_t)$. The probability of a job seeker to find a suitable vacancy is $p(\theta_t) = m(u_t + \psi_t, v_t)/(u_t + \psi_t) = m(1, v_t/(u_t + \psi_t))$ and the probability for the firm to find a suitable worker is $q(\theta_t) = m((u_t + \psi_t), v_t)/v_t = m((u_t + \psi_t)/v_t, 1)$.\footnote{As we discuss below, due to individual productivity shocks, $p(\theta_t)$ and $q(\theta_t)$ cannot be interpreted as the job finding and job filling probabilities, respectively. We therefore refer to them as the “contact” probabilities for workers and firms, respectively. Also note that labor market tightness includes also OJS workers and hence differs from the empirically observable vacancy/unemployment ratio.}

Production and matched workers. Each firm manufactures a unique final good by hiring labor. Each hired worker produces $a_t x$ units of output. Aggregate productivity $a_t$ follows the auto regressive process:

$$\ln a_{t+1} = \rho \ln a_t + \epsilon_{t+1}, \quad (4)$$

where $\epsilon \sim N(0, \sigma^2)$ and $\|\rho\| < 1$. During each period $t + 1$, an existing worker maintains the previous individual productivity level with probability $(1 - \lambda)$, and with probability $\lambda$, the
worker receives a new productivity drawn from the constant distribution \( F(x) \) over the domain \([x_L, x_H]\). Job seekers matched in period \( t \) also receive a productivity value from the same distribution in the beginning of period \( t + 1 \).

**Job separation and job creation.** During each period \( t \), total job separations comprise exogenous and endogenous terminations. Existing workers are separated from their jobs with the exogenous probability of \( s < 1 \). Given aggregate productivity \( a_t \), the firm establishes a threshold of individual productivity \( x^r(a_t) \), below which existing matches are mutually inefficient. All workers whose individual productivity satisfies \( x < x^r(a_t) \) are dismissed whereas if \( x \geq x^r(a_t) \), the job relation continues in the next period.

**On-the-job search.** A worker may search for a new job at the cost \( k^s \). An employed job searcher is matched to a firm from the same pool as the unemployed job seekers and therefore is subject to the same matching frictions. Once matched, the worker receives an idiosyncratic \( x \) from the distribution \( F(x) \) as any other newly-matched job seeker. If the draw of individual productivity is below the reservation threshold, the match is discontinued and the employed job searcher stays in the original job. Also, as any existing worker, the job searcher who remains with her current firm draws a new individual productivity with probability \( \lambda \) and faces exogenous job separation. Each firm applies the same separation threshold to employed and unemployed job seekers.\(^{21}\)

**Recursive formulation.** Four value functions solve the model: the value of unemployment \((U)\), the value of a vacancy \((V)\), the joint value of a match \((M)\), and the joint surplus of a match \((S)\). The joint surplus of a match is split in constant proportions through Nash bargaining for wages, assigning the fraction \( \phi \) of the joint match surplus to the worker and the fraction \( 1 - \phi \)

\(^{21}\)This simplifying assumption abstracts from the fact that the actual outside option for employed job seekers is their current employment contract rather than unemployment. This simplification avoids the issue of heterogeneity in wage bargaining and hence the fact that new wages depend on the value of \( x \) for the current contract and the value of \( x \) from the previous employer. These dynamics would substantially complicate the aggregation for the solution of the model because the entire distribution of \( x \) over employed workers, which is history-dependent, would become a relevant state variable for firms’ decisions. Within the microeconomic literature, the details of wage bargaining from on-the-job search have been considered by Postel-Vinay and Robin (2004) and Shimer (2006), among others. See also Gottfries (2018) for a recent contribution.
to the firm. The value of unemployment is:

\[ U(a_t) = b + \beta E_t \left[ U(a_{t+1}) + p(\theta_t)\phi \int_{x_L}^{x_H} S(a_{t+1}, x')dF(x') \right]. \]  

(5)

Equation (5) shows that the value of unemployment is equal to the opportunity cost of working (i.e., the flow value of unemployment b) and the expected benefits that finding a job brings in the next period. In period \( t + 1 \), the prospective worker encounters a suitable vacancy with probability \( p(\theta_t) \), and if the match is mutually profitable, the worker gains a fraction \( \phi \) of the total surplus on top of the value of staying unemployed. Otherwise the job seeker remains unemployed, gaining the continuation value \( U(a_{t+1}) \).

The value of an open vacancy is:

\[ V(a_t) = -k + \beta E_t \left[ V(a_{t+1}) + q(\theta_t)(1 - \phi) \int_{x_L}^{x_H} S(a_{t+1}, x')dF(x') \right]. \]  

(6)

Equation (6) shows that the present value of an open vacancy is equal to the fixed cost of posting the vacancy \( k \) and the expected benefits that the vacancy brings in the next period. In period \( t + 1 \), the firm finds a prospective worker with probability \( q(\theta_t) \), and if the match is profitable, the firm gains a fraction \( (1 - \phi) \) of the total surplus. Otherwise, the vacancy remains open, giving the firm a continuation value \( V(a_{t+1}) \). In equilibrium, the free-entry condition leads firms to post vacancies until their expected value is equal to zero in each period (i.e. \( V(a_t) = 0 \), for all \( t \)). This equilibrium condition applied to equation (6) yields the job-creation condition:

\[ \frac{k}{q(\theta_t)} = (1 - \phi)\beta E_t \left[ \int_{x_L}^{x_H} S(a_{t+1}, x')dF(x') \right]. \]  

(7)

Equation (7) shows that the expected cost of a match (left side of the equation) is equal to the expected benefit that the match brings into the firm if the job is established (right side of the equation). With this formulation, the problem can be recast in terms of choosing a given market tightness \( \theta(a_t) \) for a level of aggregate productivity.

For each given vector of \((a_t, x)\), an employment relationship is established if the match is mutually efficient, and therefore, the joint value of establishing a job relation is greater than
the value of the outside options (i.e., the individual values from separation). Thus, the joint value of a firm-worker match is:

\[ M(a_t, x) = \max \left\{ M^{n,c}(a_t, x), M^{s,c}(a_t, x), U(a_t) + V(a_t) \right\}, \quad (8) \]

where \( M^{n,c}(a_t, x) \) is the joint value of a continued match without OJS, \( M^{s,c}(a_t, x) \) is the joint value of the continued match with OJS, and \( U(a_t) + V(a_t) \) is the joint value of the outside option.

The joint value of a continued match without OJS is:

\[ M^{n,c}(a_t, x) = a_t x + \beta \mathbb{E}_t \left\{ U(a_{t+1}) + V(a_{t+1}) \right\} + (1 - s) \left[ (1 - \lambda) S(a_t, x) + \lambda \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right]. \quad (9) \]

Equation (9) shows that the joint value of a continued match is equal to production plus the expected continuation value of the work relationship. Meanwhile the joint value of a continued match while searching on the job is

\[ M^{s,c}(a_t, x) = a_t x - k^s + \beta \mathbb{E}_t \left\{ U(a_{t+1}) + V(a_{t+1}) \right\} \]

\[ + \left[ 1 - p(\theta_t) F(x_{t+1}) \right] (1 - s) \left[ (1 - \lambda) S(a_{t+1}, x) \right. \]

\[ + \lambda \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \left[ + p(\theta_t) \phi \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right] \}. \quad (10) \]

where \( F(x_{t+1}) = (1 - F[x^r(a_{t+1})]) \).

The last term uses the fact that \( \int_{x'(a_{t+1})}^{x_H} S(a_{t+1}, x') dF(x') = \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \), which represents the expected surplus that may accrue to the worker if she is matched with another firm and the match is continued. This event materializes with probability \( p(\theta_t) F(x_{t+1}) \), and encompasses all of the values of \( x \) above the reservation threshold \( x_{t+1}^r \).

The joint surplus of a match equals the value of a match, \( M \), net the outside option for the worker, \( U \), and the firm, \( V \) (i.e. \( S = M - U - V \)). Thus, the value function for the joint surplus
of a continuing match is:

$$S(a_t, x) = \max\{S^{n,c}(a_t, x), S^{s,c}(a_t, x), 0\}, \quad (11)$$

where $S^{n,c}(a, x)$ is surplus of the match when the job relation continues without OJS and $S^{s,c}(a, x)$ is the surplus of a continued match with OJS. The surpluses are defined as follows:

$$S^{n,c}(a_t, x) = a_t x - b + \beta E_t \left\{ (1 - s) \left[ (1 - \lambda) S(a_{t+1}, x) + \lambda \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right] ight. - \left. p(\theta_t) \phi \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right\}, \quad (12)$$

$$S^{s,c}(a_t, x) = a_t x - k^s - b + \beta E_t \left\{ \left[ 1 - p(\theta_t) F(x_{t+1}^r) \right] (1 - s) \left[ (1 - \lambda) S(a_{t+1}, x) ight. ight. \\
\left. \left. + \lambda \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right] \right\}. \quad (13)$$

A worker searches while on the job if $S^{s,c}(a_t, x) \geq S^{n,c}(a_t, x)$. The presence of OJS introduces a threshold $x^S(a_t)$, below which it is efficient to search on the job. For values of the threshold $x^r(a_t) < x^S(a_t) < x_H$, it is efficient to incur in the search costs for all $x \in (x^r(a_t), x^S(a_t)]$. Substituting equations (12) and (13) into the condition $S^{s,c}(a_t, x) \geq S^{n,c}(a_t, x)$ yields

$$k^s \leq \beta E_t \left\{ -p(\theta_t) F(x_{t+1}^r) (1 - s) \left[ (1 - \lambda) S(a_{t+1}, x) + \lambda \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right] ight. + \left. p(\theta_t) \phi \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right\}. \quad (14)$$

With equality, equation (14) determines the efficient threshold under which workers engage in OJS. Intuitively, the cost of searching has to be smaller than the increase in the continuation value coming from the possibility of finding a new match.

Finally, for a given aggregate state $a_t$, the individual productivity threshold for exogenous separations is the value of $x$, which makes the joint surplus of continuing a match equal to zero,
such that

\[ S^{a,t}(a_t, x^t(a_t)) = 0. \]  

(15)

Labor flows depend on the distribution of \( x \) across employed matches. The distribution of individual productivity among employed workers is history dependent: \( G_t(x) = Pr(X < x | a^t) \), where \( a^t \) represents the history of aggregate productivity shocks \( \{a_0, a_1, ..., a_t\} \) realized up to time \( t \). The conditional distribution is determined by the measure of employed workers over individual productivity, \( e_t(x) \), which follows a law of motion determined by flows between unemployment and employment and within employment. For those workers whose individual productivity is in the OJS interval \((x^t_t, x^t_s)\):

\[
e_{t+1}(x) = p(\theta_t)[1 - e_t(x_H)][F(x) - F(x^t_{t+1})] + p(\theta_t)[F(x) - F(x^t_{t+1})]e_t(x^t_t)
+ (1 - s)\{ \lambda[F(x) - F(x^t_{t+1})]e_t(x_H) - p(\theta_t)F(x^t_{t+1})e_t(x^t_t) \}
+ (1 - \lambda)[e_t(x) - e_t(x^t_{t+1})]\left[1 - p(\theta_t)F(x^t_{t+1})\right]. \]  

(16)

For the non-searching workers with \( x > x^t_s \):

\[
e_{t+1}(x) = p(\theta_t)[1 - e_t(x_H)][F(x) - F(x^t_{t+1})] + p(\theta_t)[F(x) - F(x^t_{t+1})]e_t(x^t_t)
+ (1 - s)\{ \lambda[F(x) - F(x^t_{t+1})]e_t(x_H) - p(\theta_t)F(x^t_{t+1})e_t(x^t_t) \}
+ (1 - \lambda)[e_t(x) - e_t(x^t_t) + (1 - p(\theta_t)F(x^t_{t+1}))[e_t(x^t_t) - e_t(x^t_{t+1})]]. \]  

(17)

**Gross flows and transition rates.** Gross flows from employment to unemployment represent the total mass of workers separated from a job between two periods:

\[
EU_{t+1} = s\left[e_t(x_H) - p(\theta_t)F(x^t_{t+1})e_t(x^t_t)\right]
+ (1 - s)\left\{ \lambda F(x^t_{t+1})e_t(x_H) - p(\theta_t)F(x^t_{t+1})e_t(x^t_t) \right\}
+ (1 - \lambda)e_t(x^t_{t+1})\left[1 - p(\theta_t)F(x^t_{t+1})\right]. \]  

(18)

\(^{22}\)As \( S(a_t, x) \) is monotonically increasing in \( x \), the individual productivity threshold \( x^t(a_t) \) is unique, and \( S(a_t, x) > 0 \ \forall \ x > x^t(a_t) \). Appendix 1D.1 provides a detailed discussion.
The job separation rate is then defined as the probability that an employed worker in period \( t \) is not employed in period \( t + 1 \): 
\[
SR_t = EU_{t+1}/[e_t(x_H)].
\]
Similarly, the gross unemployment to employment (UE) flow is the total mass of workers who start a new job from unemployment:
\[
UE_{t+1} = u_t p(\theta_t)(1 - F(x_{t+1}^r)),
\]
and the job finding rate (JFR) is defined as the probability that an unemployed worker in period \( t \) is not unemployed in period \( t + 1 \):
\[
JFR_t = UE_{t+1}/u_t = p(\theta_t)(1 - F(x_{t+1}^r)).
\]  
(19)

The job-to-job rate (JJR) is measured as the ratio of gross employment to new employment (EE) flows over total employment:
\[
JJR_{t+1} = EE_{t+1}/e_t(x_H) = \frac{e_t(x_s^a)p(\theta_t)\bar{F}(x_{t+1}^r)}{e_t(x_H)}.
\]  
(20)

4 **Mechanisms for state-dependent fluctuations**

The distinct responses of labor market variables over states of aggregate productivity is generated by the effect of changes in the individual productivity threshold \( x^r(a) \) on the distributions of individual productivity for newly-established matches, \( F(x) \), and for continuing jobs, \( G(x) \). \footnote{To simplify notation, we drop the time index from the distributions. Given the timing assumption in the model \( G(x) \) refers to \( G_{t-1}(x) \) whereas \( F(x) \) does not vary with time.} Figure 4 shows an illustrative probability density function for the \( x \) of new matches (i.e. \( F'(x) \)) and incumbent ones (i.e. \( G'(x) \)) in red and blue, respectively. The difference between the two distributions is that \( G(x) \) has zero mass below the individual productivity threshold \( x_s^a \) since jobs with productivity lower than the threshold are terminated whereas \( F(x) \) is continuous and twice differentiable since the productivity of new jobs is positively defined across the whole domain of individual productivity. \footnote{We make the standard assumption that \( F(x) \) is continuous, twice differentiable, and unimodal since it proxies the wage distribution in the data, as examined in \cite{Moscarini2005}.} For both distributions, workers whose productivity is below the searching threshold \( x_s^a \) and above \( x^r(a) \) search on the job.
Figure 4: Distribution for $F'(x)$ and $G'(x)$

Note. The figure shows the p.d.f. for $F(x)$ (labelled $F'(x)$, red line) and $G(x)$ (labelled $G'(x)$, blue line).

Figure 5: States of aggregate productivity and the job separation rate

Panel (a): State with high productivity

Panel (b): State with low productivity

Note. An increase of the threshold of individual productivity from $\tilde{x}_0^r$ to $\tilde{x}_1^r$ generates a larger response in the job separation rate in states with low aggregate productivity than an equivalent increase of the threshold of individual productivity from $x_0^r$ to $x_1^r$ in states with high-aggregate productivity. The shaded area shows the mass of jobs sensitive to job separation in response to the change in the threshold.

Movements in the individual productivity threshold generate distinct responses in the job separation rate in relation to the state of aggregate productivity. Panel (a) in Figure 5 shows the initial productivity threshold $x_0^r$ on the distribution for continuing jobs $G'(x)$ that is associated with a high level of aggregate productivity. In response to a fall in aggregate productivity, the individual productivity threshold increases from $x_0^r$ to $x_1^r$, leading to a rise in the job separation rate equal to shaded area A. Panel (b) shows the effect of an equivalent fall in aggregate productivity from an initially low level of aggregate productivity. In this instance, the individual productivity threshold is high and located in the domain of the distribution with

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25Given that the surplus is increasing in both $x$ and $a$, equation \( \text{[15]} \) implies that a high level of aggregate productivity is associated with a low individual productivity threshold. Assuming that the threshold always lies below the distribution mode, a lower threshold is located in a region of the distribution associated with lower density.
high density. The same fall in aggregate productivity increases the individual productivity threshold from $\bar{x}_r^0$ to $\bar{x}_r^1$, leading to a rise in the job separation rate equal to shaded area B, which is larger than area A. Thus, the effect of a shock on the mass of jobs exposed to movements in the individual productivity threshold differs across levels of aggregate productivity, and the response of the job separation rate to the aggregate productivity shock is larger when aggregate productivity is low. By the same principle, the job finding rate may exhibit stronger responses in states with low aggregate productivity.\(^\text{26}\)

5 Model simulation and quantitative results

This section presents the calibration of the model and the quantitative results. It first compares simulated moments in the model against those in the data. It then investigates the extent to which the model replicates the observed changes in the magnitude of fluctuations at distinct states of aggregate productivity. Finally, it presents generalized impulse response functions to isolate the dynamic responses of labor market variables in different states of aggregate productivity.

5.1 Calibration

To allow the theoretical framework to embed state-dependent dynamics, we solve the model non-linearly iterating over the policy function on a discretized state space, following the approach in Tauchen (1986). Appendix D.3 outlines the solution procedure.

We calibrate the model at a monthly frequency. The discount factor $\beta$ is set equal to 0.953\(^{\text{1/12}}\), as in Shimer (2005). The cost of posting a vacancy $\kappa$ is set equal to 0.17 to match the derived calculations on costs of a job opening based on survey results cited in Barron and Bishop (1985) and Barron et al. (1997).\(^\text{27}\) The flow value of unemployment $b$ is set to equal 0.71, as in Hall and Milgrom (2008), which is between the value of 0.4 in Shimer (2005) and the value of 0.95 in Hagedorn and Manovskii (2008). The elasticity of the matching

\(^{\text{26}}\)Appendix D.2 reports a similar graphical representation of the mechanism underpinning the distinct response of the job finding rate at different levels of productivity.

\(^{\text{27}}\)As in Fujita and Ramey (2012), the value is derived from a calculation of the costs based on survey results cited in the above papers.
Table 2: Parameter values for the baseline model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy cost</td>
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</tr>
<tr>
<td>$\kappa_s$</td>
<td>OJS cost</td>
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<tr>
<td>$b$</td>
<td>Flow value of unemployment</td>
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<td>$\eta$</td>
<td>Elasticity of matching with respect to vacancies</td>
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<td>$\gamma$</td>
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<tr>
<td>$\phi$</td>
<td>Worker’s bargaining power</td>
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<tr>
<td>$s$</td>
<td>Exogenous job separation rate</td>
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<tr>
<td>$\lambda$</td>
<td>Arrival rate of individual productivity shocks</td>
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<tr>
<td>$x_L$</td>
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<tr>
<td>$x_H$</td>
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<td>$\mu_x$</td>
<td>Mean of log individual productivity shocks</td>
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</tr>
<tr>
<td>$\sigma_x$</td>
<td>Standard deviation of individual productivity shocks</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>Persistence parameter of aggregate productivity</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of aggregate productivity</td>
<td>0.0068</td>
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</table>

function with respect to vacancies $\eta$ is set equal to 0.5 in the range of empirical estimates in Petrongolo and Pissarides (2001). To satisfy the Hosios (1990) condition, which ensures that the equilibrium of the decentralized economy is Pareto efficient, we assume that the elasticity of labor market tightness with respect to vacancies is equal to the firm’s bargaining power, $(1 - \phi)$ i.e., $\eta = (1 - \phi) = 0.5$. The parameter of match efficiency $\gamma$ is set equal to 0.47 to match the empirical average job finding rate of 0.45, as in Hagedorn and Manovskii (2008). The exogenous separation probability $s$ is set equal to 0.022 to match the average job separation rate of 0.03. The standard deviation of individual productivity $\sigma_x$ is set to equal 0.13, to match the empirical quarterly standard deviation of the HP-filtered log separation rate of 0.055. The probability of receiving a new individual productivity shock $\lambda$ is set to 0.05 to match a quarterly autocorrelation of the separation rate of 0.63, as in Fujita and Ramey (2012). The distribution of individual productivity shocks is a truncated log-normal density function with the lower bound equal to zero (i.e. $x_L = 0$) and the upper bound set to have less than one percent of the mass of the distribution above it (i.e. $x_H = 1.55$). The mean of the log distribution of individual productivity $\mu_x$ is set to -0.087 to normalize the long-run average productivity in the economy to one. The cost of searching on the job $k^s$ is set to equal 0.128 to match the mean monthly job-to-job transition rate of 3.2, calculated from the CPS data by Moscarini and Thomsson (2007). The autoregressive parameter $\rho$ and the standard deviation $\sigma$ of the aggregate productivity process are set to equal to 0.973 and 0.0068, respectively, to match the autocorrelation and standard
deviation of HP-filtered log labor productivity at quarterly frequency, as in Hagedorn and Manovskii (2008). Table 2 summarizes the calibration of parameters. Table D.1 in Appendix D.3 shows that the simulated target moments are close to the empirical counterparts in the data.

5.2 Business cycle statistics and dynamic responses

Table 3 compares the standard deviation and the correlation coefficient with productivity for selected variables in the data (top panel) against the corresponding statistics in the simulated model (bottom panel). The moments are based on a set of 1,000 simulations of the same length as the empirical data. The model accurately reproduces the standard deviation of output 0.021 in the data. The simulated standard deviation of unemployment and the job finding rate of 0.101 and 0.063 are approximately 70 percent of those in the data (0.137 and 0.089, respectively). The simulated standard deviation of vacancies, equal to 0.040, is 30 percent of the value of 0.138 in the data. Likewise, the simulated standard deviation of vacancy-to-unemployment ratio (v/u) equal to 0.137 is approximately half the value in the data.

Table 3: Labor market statistics in the data and the model

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>p</th>
<th>U</th>
<th>JFR</th>
<th>SR</th>
<th>E</th>
<th>V</th>
<th>V/U</th>
<th>Y</th>
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<td>0.232</td>
<td>0.394</td>
<td>0.316</td>
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<tr>
<th></th>
<th>Model</th>
<th>p</th>
<th>U</th>
<th>JFR</th>
<th>SR</th>
<th>E</th>
<th>V</th>
<th>V/U</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ_x</td>
<td>0.013</td>
<td>0.101</td>
<td>0.063</td>
<td>0.055</td>
<td>0.008</td>
<td>0.040</td>
<td>0.137</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>Corr(pt,xt)</td>
<td>1.000</td>
<td>-0.970</td>
<td>0.978</td>
<td>-0.951</td>
<td>0.929</td>
<td>0.927</td>
<td>0.991</td>
<td>0.991</td>
<td></td>
</tr>
</tbody>
</table>

Note. The table reports cyclical statistics for average labor productivity (p), the unemployment rate (U), the job finding rate (JFR), the job separation rate (SR), the employment rate (E), vacancies (V), the V/U ratio (V/U), and output (Y). The simulated moments are computed as a mean of 1,000 simulations of 1,380 monthly periods. After discarding the first 600 observations in each simulation, the remaining series are aggregated at quarterly frequency with the same length of the observed series for the period 1950:I-2014:IV.

The model replicates accurately the sign of the correlation coefficient of the variables with productivity. The unemployment rate and the job separation rate are negatively correlated.

---

28 The relatively larger fluctuations of the job finding rate compared to vacancies indicate that in the model, the productivity threshold $x_r$ rather than posting vacancies is the main channel through which firms adjust their recruiting decisions. Firms adjust $x_r$ to determine endogenous separation, which also affects the expected value of new matches. The adjustment in reservation productivity mitigates the fluctuations in the expected surplus of new matches and hence dampens the response of vacancies.
with productivity whereas the rest of the variables are positively correlated with productivity. The correlations in the model are larger than those in the data since productivity shocks are the only exogenous source of aggregate fluctuations. It is worth noting that the negative correlation between unemployment and vacancy fluctuations yields an empirically consistent, negatively sloped Beveridge Curve.

Table 4 shows the standard deviations of simulated labor market variables in levels (columns 1-3), quarterly growth rates (columns 4-6) and yearly growth rates (columns 7-9) associated with productivity below and above its median value. Columns (1) and (2) show that the standard deviations of the simulated variables are larger in periods of productivity below the median value. Column (3) reports the ratio of the standard deviations for the variables when productivity is below and above the median value. All variables have larger fluctuations in periods with low aggregate productivity, with the ratios in Column (3) ranging from 1.16 to 1.59. A similar result holds for the statistics relative to quarterly and yearly growth rates (columns 4-6 and 7-9, respectively).

**Table 4: Standard deviation of simulated variables for different states of productivity**

<table>
<thead>
<tr>
<th></th>
<th>Levels</th>
<th>Quarterly Growth Rates</th>
<th>Yearly Growth Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\sigma_{p&lt;\text{Median}})</td>
<td>(\sigma_{p&gt;\text{Median}})</td>
<td>(\sigma_{p&lt;50})</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0947</td>
<td>0.0812</td>
<td>1.18</td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>0.0291</td>
<td>0.0272</td>
<td>1.16</td>
</tr>
<tr>
<td>Separation Rate</td>
<td>0.0018</td>
<td>0.0014</td>
<td>1.34</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0.0077</td>
<td>0.0049</td>
<td>1.59</td>
</tr>
<tr>
<td>Output</td>
<td>0.0185</td>
<td>0.0161</td>
<td>1.16</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0111</td>
<td>0.0114</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Note. Entries are averages of 1,000 simulations over 1,380 monthly periods. After discarding the first 600 observations in each simulation, the remaining series are aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV. \(\sigma_{p<\text{Median}}\) represents the standard deviation of the variable for the productivity state below (above) the median.

To investigate the extent to which the dynamic responses of labor market variables are different across states with high and low aggregate productivity, Figure 6 plots generalized Impulse Response Functions (IRFs) for the separation rate (top panels), the job finding rate (middle panels), and the unemployment rate (bottom panels) to a positive productivity shock equal to one quarterly standard deviation (solid line) together with the 5th-95th percentiles.

---

29By construction, the volatility of productivity does not vary over the cycle.
The left panels report the response of the variables in states in which productivity is above the 90th percentile of its distribution while the right panels consider an initial productivity level below the 10th percentile.

The top panel in Figure 6 shows that the response of the job separation rate is more than twice as large in the state with low productivity compared to the state with high productivity. As discussed in Section 4, the mechanism that generates these distinct dynamics is straightforward. The different responses with respect to the state with aggregate productivity originate from the effect of shifts in the reservation threshold for individual productivity on the job separation rate. In the state with high aggregate productivity, the threshold is low and located in a region of individual productivity distribution with low density. Aggregate shocks that move the threshold displace a limited number of workers and therefore have a limited effect on the job separation rate. By contrast, in the state with low aggregate productivity, the threshold of individual productivity is high and located in a region of the distribution of individual productivity with high density. Thus, an identical aggregate productivity shock that moves the threshold similarly displaces a larger fraction of workers, thereby generating a large shift in the job separation rate.

The middle panel in Figure 6 shows that the response of the job finding rate is 20 percent larger in the state with low productivity compared to the state with high productivity. The reservation productivity, $x^r$, is the important driver of state dependence. For a given contact probability, $p(\theta_t)$, the proportion of matches turning into jobs is $1 - F(x^r_{t+1})$. For the mechanism outlined in Section 4, a high value of $x^r$ in periods of low aggregate productivity generates large fluctuations in $(1 - F(x^r_{t+1}))$ in reaction to productivity shocks, leading to overall more volatility in the job finding rate.

The bottom panel in Figure 6 shows that the response of the unemployment rate is almost twice as large in the state with low productivity compared to the state with high productivity. Appendix D.4 describes the computational method to derive generalized IRFs. In the online appendix we analyze asymmetries in the variables’ responses with respect to the sign of the shock. Figure D.2 in the Appendix shows that the response of the matching probability, $p(\theta_t)$, to the technology shock is similar across states of the business cycle. Thus, the state dependence of the job finding probability is produced by the probability of a contact turning into a formed match.
In the model, the unemployment rate results from changes in the job separation rate and the job finding rate. The analysis shows that the state dependence in the job transition rate jointly generates the large response of the unemployment rate in periods of low aggregate productivity.

Overall, the model generates large state dependence in labor market fluctuations that explains the larger volatility of the unemployment rate in periods of low aggregate productivity. The model attributes this reaction to the joint interaction of job creation and job separation.

6 Alternative calibrations and reduced models

In this section we investigate whether state dependence in the baseline model is robust to: (i) different calibration choices and (ii) alternative structures of the model that exclude OJS and assume exogenous separations, respectively. Each subsection briefly outlines the calibration
and presents main results, leaving supplementary material to Appendix E. The exercise points to the importance of the calibration of the individual productivity shock, $\sigma_x$, and the relevance of OJS to generate state dependence.\textsuperscript{33}

### 6.1 Alternative calibration of baseline model

The baseline calibration replicates the absolute volatility of the job separation rate, which is a targeted moment, but it generates limited volatility in vacancies and the vacancy-to-unemployment ratio ($v/u$), an important statistics for labor market fluctuations. As an alternative calibration strategy, we follow the approach in Krause and Lubik (2007) and target the volatility of job destruction relative to the volatility of employment (i.e., $\sigma_{SR}/\sigma_e$). Over the sample period 1950-2014 used in the baseline calibration, the ratio is 6.5, which is close to the value of 7 in Krause and Lubik (2007) for the shorter sample period 1964-2002.\textsuperscript{34} The value of $\sigma_x$ in the alternative calibration is equal to 0.105, which is approximately 20 percent smaller than the baseline calibration. The resulting distribution of individual productivity is narrower and steeper, and movements in the threshold of individual productivity generate strong state dependence.

Table 5: State-dependent volatility for the model with OJS and endogenous separations, under the alternative calibration.

<table>
<thead>
<tr>
<th></th>
<th>Levels</th>
<th>Quarterly Growth Rates</th>
<th>Yearly Growth Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{p&lt;\text{Median}}$</td>
<td>$\sigma_{p&gt;\text{Median}}$</td>
<td>$\sigma_{p&lt;50}$</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0668</td>
<td>0.0486</td>
<td>1.39</td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>0.0226</td>
<td>0.0198</td>
<td>1.23</td>
</tr>
<tr>
<td>Separation Rate</td>
<td>0.0011</td>
<td>0.0007</td>
<td>1.62</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0.0054</td>
<td>0.0032</td>
<td>1.70</td>
</tr>
<tr>
<td>Output</td>
<td>0.0167</td>
<td>0.0148</td>
<td>1.13</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0116</td>
<td>0.0019</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note. Entries are averages of 1,000 simulations over 1,380 monthly periods. After discarding the first 600 observations in each simulation, the remaining series are aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV. $\sigma_{p<\text{Median}}$ represents the standard deviation of the variable for the productivity state below (above) the median.

Table 5 shows the standard deviations of simulated labor market variables across distinct

\textsuperscript{33} An additional set of robustness checks to the calibration of the model calibration is contained in the online appendix.

\textsuperscript{34} Tables E.1, E.2, E.3 in Appendix E report the parameters, the targets, the business cycle moments of this calibration.
states of aggregate productivity for the alternative calibration. Differences in the standard deviations of labor market variables across productivity states are larger relative to the baseline calibration. Figure 7 plots generalized IRFs for the model with the alternative calibration and shows that both the job separation rate and job finding rate exhibit a large response to the shock in low-productivity states relative to high-productivity states. These rates have an overall smaller response of variables in both regimes compared to the baseline calibration. The same result holds for simulated standard deviations of the unemployment rate and job transition probabilities (see Table E.3), which are approximately 40 percent smaller relative to those in the benchmark calibration (see Table 4). Overall, the exercise shows that alternative calibration strategies involve a tradeoff between the degree of state dependence and the overall magnitude of the volatility in labor market variables.

Figure 7: Generalized IRFs: model with OJS and endogenous separations, under the alternative calibration.

Note. The solid line represents the mean IRF value in each period. The shaded area represents the 5th and 95th percentiles of the IRF values. Responses of the variables in periods with high (low) aggregate productivity are in left (right) panels.
6.2 Model without OJS

Abstracting from OJS, we define the match surplus by equations (11) and (12). Targets for the calibration remain those in the baseline model, described in section 5.1. Table 6 shows standard deviations of simulated labor market variables, and Figure 8 shows generalized IRFs. Exclusion of OJS decreases the degree of state dependence. The model then generates a low overall volatility and an empirically implausible, upward-sloped Beveridge Curve (see last row of Table E.6), consistent with the findings in Fujita and Ramey (2012).

Table 6: State-dependent volatility for the model with endogenous separations.

<table>
<thead>
<tr>
<th></th>
<th>Levels</th>
<th>Quarterly Growth Rates</th>
<th>Yearly Growth Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>σ_p&lt;Median</td>
<td>σ_p&gt;Median</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0651</td>
<td>0.0618</td>
<td>1.06</td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>0.0149</td>
<td>0.0146</td>
<td>1.03</td>
</tr>
<tr>
<td>Separation Rate</td>
<td>0.0019</td>
<td>0.0015</td>
<td>1.27</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0.0046</td>
<td>0.0035</td>
<td>1.32</td>
</tr>
<tr>
<td>Output</td>
<td>0.0150</td>
<td>0.0142</td>
<td>1.06</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0105</td>
<td>0.0109</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Note. Entries are averages across 1,000 simulations over 1,380 monthly periods. After discarding the first 600 observations in each simulation, the remaining series are aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV.

The pro-cyclicality of OJS is important to generate state dependence. In periods of low aggregate productivity, a limited mass of workers search on the job, and fewer searching workers find a new job while being employed. Because workers that search on the job are at the lower end of the distribution of idiosyncratic productivity, the option of finding a new match serves as a way to escape endogenous job separation. A fall in OJS activity implies that a larger fraction of workers are at risk of job separation, which is critical to generate state dependence.

By abstracting from OJS, the calibration of the system requires the distribution of individual productivity, $x$, to retain long tails to generate a plausible volatility in the job separation rate. In such a case, the value for $\sigma_x$ that matches the data is equal to 0.30, which is twice as large as the value in the baseline calibration. The wider and flatter density function reduces the state dependence generated by movements of the threshold, since the mechanism described in Section 4 is quantitatively reduced.

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35 See Appendix E. Parameter values, calibration targets, and cyclical moments are in Tables E.4, E.5 and E.6 respectively.

29
Figure 8: Generalized IRFs for the model without OJS.

Note. The solid line represents the mean IRF value in each period. The shaded area represents the 5th and 95th percentiles of the IRF values. Responses of the variables in periods with high (low) aggregate productivity are in left (right) panels.

6.3 Model with exogenous separations

As a final robustness check, we assess the extent to which the DMP model with exogenous separations generates state-dependent fluctuations. Appendix E presents the details of the model. To abstract from endogenous job separation, we assume that workers have the same level of match-specific productivity (i.e., $x = 1$) and that the unique source of state dependence stems from the nonlinearity in the firm’s choice with respect to market tightness.

Shimer (2005) shows that the prototype DMP model with exogenous separations fails to generate large and plausible fluctuations in labor-market variables. In the ensuing debate, Hagedorn and Manovskii (2008) establish that the issue resolves by calibrating the outside option of working, $b$, and the Nash bargaining parameter, $\phi$, to generate a small surplus for the firm, which produces volatile profits and a large elasticity of job creation. In line with these

See Hornstein et al. (2011) for a critical discussion on the approach and Zanetti (2011b) for a general extension of the issue in the presence of labor market institutions.
Table 7: State-dependent volatility for the model with exogenous separations, under the Hagedorn and Manovskii (2008) calibration.

<table>
<thead>
<tr>
<th>Levels Quarterly Growth Rates</th>
<th>Yearly Growth Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>σp&lt;Median</td>
<td>σp&gt;Median</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0882</td>
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<tr>
<td>Job Finding Rate</td>
<td>0.0141</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0.0074</td>
</tr>
<tr>
<td>Output</td>
<td>0.0181</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

Note. Entries are averages of 1,000 simulations over 1,380 monthly periods. After discarding the first 600 observations in each simulation, the remaining series are aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV.

Figure 9: Generalized IRFs: exogenous separation model with the Hagedorn and Manovskii (2008) calibration.

Note. The solid line represents the mean IRF value in each period. The shaded area represents the 5th and 95th percentiles of the IRF values. Responses of variables in periods with high (low) aggregate productivity are in left (right) panels. For comparison with the other calibrations, the weekly job finding rate is converted into its monthly equivalent before computing the IRF.

findings, we calibrate this version of the model using the approach in Hagedorn and Manovskii (2008). Table 7 shows that the model replicates accurately state-dependent fluctuations in the job finding rate and the unemployment rate. The IRFs in Figure 9 show that the 10th percentile of productivity, the response of the job finding rate is 30 percent larger than at the 90th percentile. These results are in line with those in Petrosky-Nadeau and Zhang (2017), who show that the calibration in Hagedorn and Manovskii (2008) generates strong nonlinearities,
especially during large recessions.

7 State dependence and labor market reforms

This section enriches the model with labor market protection in the form of a layoff tax levied on the dismissal of established jobs. The analysis investigates the effect of the tax on the long-run equilibrium of the model and assesses whether a permanent tax removal in states with low or high aggregate productivity generates substantial differences in transitional dynamics and welfare.

7.1 Introducing layoff taxes

The model described in Section 3 is enriched with a “wasteful” layoff tax, \( \tau \), that the firm must pay to cover administrative costs and layoff procedures whenever a worker is (endogenously or exogenously) separated.\(^{38}\) Firms whose workers move to another job with OJS do not incur the layoff tax. Layoff taxes are not levied on the separation of newly-established matches, and therefore, the joint value of an employment relationship for new matches (indexed by \( N \)) and continuing matches (indexed by \( O \)) is distinct. Appendix F reports the derivation of surplus functions under the layoff tax.

The total surplus equals the value of establishing a match net of outside options to the worker and the firm. Thus, the value functions for joint surpluses for new and continuing jobs are:

\[
S_N(a_t, x) = \max[S_{N,c}(a_t, x), 0], \quad (21)
\]

\[
S_O(a_t, x) = \max[S_{O,c}(a_t, x), 0], \quad (22)
\]

where \( S_{N,c}(a, x) \) and \( S_{O,c}(a, x) \) represent the total surpluses in case the worker and the firm establish a new match or continue an existing job relationship, respectively, accounting for the

\(^{38}\)See Fella (2007), Ljungqvist (2002), Postel-Vinay and Turon (2014), and Cozzi and Fella (2016) for a discussion on the role of employment protection measures in matching models.
optimal choice of OJS. The total surpluses for new and old matches, without and with OJS are:

\[
S_{N,n}(a_t, x) = a_t x - b + \beta E_t \left\{ (1 - s) \left[ -\tau + (1 - \lambda) S_{O}(a_{t+1}, x) + \lambda \int_{x_L}^{x_H} S_{O}(a_{t+1}, x') dF(x') \right] \\
- p(\theta_t) \phi \int_{x_L}^{x_H} S_{N}(a_{t+1}, x') dF(x') \right\}, \tag{23}
\]

\[
S_{N,s}(a_t, x) = a_t x - k^s - b + \beta E_t \left\{ \left[ 1 - p(\theta) F(x_{r+1}^x) \right] (1 - s) \left[ -\tau + (1 - \lambda) S_{O}(a_{t+1}, x) \right] + \lambda \int_{x_L}^{x_H} S_{O}(a_{t+1}, x') dF(x') \right\}, \tag{24}
\]

\[
S_{O,n}(a_t, x) = S_{N,n}(a_t, x) + \tau, \tag{25}
\]

\[
S_{O,s}(a_t, x) = S_{N,c}(a_t, x) + \tau. \tag{26}
\]

Equations (23) and (24) show that the surpluses of newly-established job relations (with or without OJS) are reduced by the expected layoff tax if the job is dismissed in the future. Equations (25) and (26) show that the surpluses for existing job relations (with or without OJS) entail an intertemporal tradeoff between the benefit of avoiding the tax if the job is not severed in the present period \( t \) and the cost of having to pay the layoff tax if the worker is dismissed in the future. Important for our analysis, the difference in the surpluses for newly-hired and existing workers generates distinct thresholds of individual productivity. The reservation productivity at which new matches become inefficient is higher than the reservation threshold for existing workers because firms are not discouraged from discontinuing the newly-formed match at time \( t \) as they would be for incumbent workers. Consequently, the firm retains existing workers with individual productivity in the range of \( x \in (x_{r,O}(a), x_{r,N}(a)) \) but does not hire new matches with individual productivity in the same interval. Within this range of individual productivity, it is inefficient to pay layoff taxes to dismiss existing workers, but it is efficient to refuse new matches to which layoff taxes do not apply.

A worker’s decision to search on the job is influenced by the prospects of obtaining a successful match. The productivity threshold that applies to this expectation, and hence to the decision to search on the job, is \( x_{r,N}(a) \) since employed and unemployed job seekers are identical to the hiring firm. Additionally, the cutoff level for OJS is the same across new and
incumbent workers. Appendix F outlines the laws of motion of employment in the presence of layoff taxes.

Figure 10: Thresholds for separation, job creation, and OJS, over match-specific productivity and aggregate productivity in the baseline model and the model with layoff taxes

Note. The x-axis reports the level of aggregate productivity \( a \). The y-axis reports the level of individual productivity \( x \). The red lines report the thresholds for the baseline case, where \( x^* \) is the OJS threshold and \( x^r \) is the reservation productivity level for both job separations and formation of new matches. The blue lines report the thresholds for the model with employment protection with \( \tau = 0.15 \), where \( x^r_\tau \) is the OJS threshold, \( x^r_{\tau,O} \) is the reservation productivity level for incumbent workers, and \( x^r_{\tau,N} \) is the threshold for the formation of new matches. We set the value for the layoff tax equal to 15 percent of average monthly productivity (\( \tau = 0.15 \)), which makes the tax approximately equivalent to 5 percent of average quarterly wages, as in Llosa et al. (2014). Figure 10 shows how the layoff tax changes the relevant productivity thresholds for incumbent workers, \( x^r_{\tau,O} \), new jobs, \( x^r_{\tau,N} \), and OJS, \( x^S \). The reservation productivity for firing incumbent workers in the presence of the layoff tax (\( x^r_{\tau,O} \)) is slightly lower compared to the case of no layoff tax (\( x^r \)): for an incumbent match, the tax creates a small wedge between the surplus in the baseline model and the model with layoff cost. For new matches, however, the layoff tax increases the reservation threshold \( x^r_{\tau,N} \). At the same time,

\[ \text{39} \]

Given that the tax (and future taxes) enter linearly into both the surplus from searching and not searching, as seen in equations (23)-(26), the threshold level for OJS \( x^* \) that satisfies \( S^{N,a}(a_t,x) = S^{N,s}(a_t,x) \) is the same as that which satisfies \( S^{O,a}(a_t,x) = S^{O,s}(a_t,x) \).

\[ \text{40} \]

An explanation for why the threshold for incumbent workers only decreases by a small amount, while that of new workers rises by almost the entire value of the tax, can be given by a steady-state version of the model.
the threshold for OJS ($x_r^t$) decreases. For a given level of market tightness, a firm’s stricter productivity requirements induce the formation of fewer matches. As a result, the highest level of match-specific productivity at which workers decide to search for another job also decreases. For low levels of aggregate productivity, the OJS threshold is sufficiently low to be located below the threshold for new matches, implying that no new match chooses OJS. Overall, the levels and slope of the thresholds imply that the tax reduces the mass of OJS workers, and especially during times of low productivity.

7.2 The long-run effect of layoff taxes

Table 8 compares the long-run values of labor market variables in the version of the model without (Column 1) and with (Column 2) the layoff tax equivalent to 15 percent of average monthly productivity ($\tau = 0.15$). The tax increases the long-run unemployment rate by two percentage points from 6.8 percent to 8.7 percent, as a result of the large drop in the job finding rate and the broadly constant rate of job separation. The effect of the tax on the long-run value of the job separation rate is limited. The reason is straightforward. While introducing the tax slightly decreases the efficiency threshold for continuing jobs and therefore decreases endogenous separation, it also increases the efficiency threshold for new jobs and therefore discourages OJS. Overall, OJS decreases from 6.6 percent to 3.4 percent, leaving a greater fraction of workers subject to job separation. These two opposing forces offset each other, and the job separation rate effectively remains unchanged.

The tax lowers the job finding rate from 44.5 percent to 32.4 percent. The fall in the job finding rate originates from the increase in the threshold of efficient matches for new hires, $x_r^N(a)$, and the fall in vacancies. The higher productivity threshold for new matches discourages on-the-job search and consequently reduces the total number of job seekers, increasing the search costs per vacancy filled accrued to firms, which react by decreasing vacancy postings. Thus, the reduction in OJS amplifies the contractionary effect of the layoff tax on the job finding rate and generates a large rise in the unemployment rate.

There, the wedge between the surplus functions for new workers in the no-tax and the tax economies is close to $-\bar{s}\beta\tau/(1 - (1 - \bar{s})\beta) = -0.128$, using a steady state separation rate, $\bar{s} = 0.03$. Meanwhile for incumbent workers, the wedge is $\tau - \bar{s}\beta\tau/(1 - (1 - \bar{s})\beta) = 0.0215$. 35
Table 8: The long-run effects of a layoff tax

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Layoff tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau = 0$</td>
<td>$\tau = 0.15$</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.068</td>
<td>0.087</td>
</tr>
<tr>
<td>Job finding rate</td>
<td>0.445</td>
<td>0.324</td>
</tr>
<tr>
<td>Separation Rate</td>
<td>0.030</td>
<td>0.030</td>
</tr>
<tr>
<td>Job-to-job rate</td>
<td>0.032</td>
<td>0.012</td>
</tr>
<tr>
<td>On-the-job search</td>
<td>0.066</td>
<td>0.034</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0.932</td>
<td>0.913</td>
</tr>
<tr>
<td>Vacancies</td>
<td>0.179</td>
<td>0.158</td>
</tr>
<tr>
<td>V/U</td>
<td>2.832</td>
<td>1.919</td>
</tr>
<tr>
<td>Productivity</td>
<td>1.000</td>
<td>1.004</td>
</tr>
</tbody>
</table>

Note. The table shows the long-run averages of labor market variables in the baseline model ($\tau = 0$) and in the alternative model with layoff taxes ($\tau = 0.15$).

The model involves a positive relation between layoff taxes and the unemployment rate in the long run. There is no established consensus on the effect of layoff taxes on the unemployment rate.\(^{41}\) While empirical evidence shows that employment protection legislation reduces job separation and creation, the final effect on unemployment depends on which of the two effects prevail.\(^{42}\) Similar results hold on the theoretical side where most recently, Baley et al. (2018) show how even the same model results in different conclusions depending on the calibration of parameters. Only a limited number of studies considers the impact of employment protection legislation on OJS and job-to-job transitions. While Boeri (1999) finds descriptive evidence suggesting lower job-to-job transitions for countries with larger employment protection legislation, more recently Bassanini and Garnero (2013) show that higher employment protection legislation reduces job-to-job transitions within the same industry in OECD countries. Postel-Vinay and Turon (2014) develop a model with on-the-job search, minimum wages, and severance packages that generates a negative relation between layoff taxes and the unemployment rate. The result originates from the different setup of wage bargaining and the use of severance payments by incumbent firms that induces workers to accept outside offers. Our version of the DMP model uses a standard Nash sharing of surplus and layoff taxes that negatively affect job-to-job transitions by discouraging both the formation of new matches and on-the-job search.

\(^{41}\)See Nickell et al. (2005) and Boeri et al. (2015) for extensive reviews of both empirical and theoretical works.

\(^{42}\)See studies by Messina and Vallanti (2007), Kugler and Pica (2008), Bassanini and Duval (2009), and Haltiwanger et al. (2014) for a discussion of the issues.
7.3 Short-run effects and welfare of layoff tax removal

This section investigates whether the timing of an unexpected, permanent removal of the layoff tax enacted in alternative states of aggregate productivity is critical for transitional dynamics and welfare. We assume that the removal of the tax only applies to newly formed matches from both unemployment and employment so that incumbent workers are protected from the reform. We compare the effect of the reform enacted at a level of labor productivity below the 5\textsuperscript{th} percentile and above the 95\textsuperscript{th}. Appendix F.2 provides details on the simulation procedure.

Figure 11 shows transitional paths for the unemployment rate (solid line) that result from the removal of the layoff tax in the low productivity state (right panel) and the high productivity state (left panel), together with the 10\textsuperscript{th}-90\textsuperscript{th} interval of the differences across states (shaded area). The dashed line shows the long-run difference between the unemployment rate in the economy with the tax eliminated in the first period and the unemployment rate in the economy with the tax always in place. In the long run, the tax removal leads to a similar fall in the average unemployment rate of approximately 2.2 percentage points across states with initially low and high aggregate productivity since both economies converge to the equilibrium over the long run. In the first period of the reform, the unemployment rate falls by approximately one percentage point in both initial states of aggregate productivity. However, in subsequent periods, the transitional dynamics of the unemployment rate differ significantly. When productivity is high, the reform causes the unemployment rate to gradually decrease towards its long-run equilibrium whereas in the state with low productivity, the decline of unemployment is sharp, and the change in the unemployment rate remains below its long-run equilibrium for a protracted number of periods.

To interpret these marked differences, we consider the transitional dynamics of the job

\footnote{This assumption, however, is not crucial because, as we show below, the main channel of adjustment is job creation. The results for the version in which all workers are affected by the reform are available upon request.}

\footnote{The transition path of unemployment from the outset of the reform at time $t$ is computed as $TP(u_t|e_0(x), \{a_t\}_0^T) = u_{t+1}^R - u_{t+1}^\tau$, where the superscripts $R$ and $\tau$ indicate the reform and tax scenario, respectively, assuming an initial state $e_0(x)$ and a productivity sequence $\{a_t\}_0^T$.}

\footnote{The reported long-run difference of -2.3 is slightly different from the value reported in Table 8 since the value in the table is computed using a set of short simulations equivalent to the 1950-2014 period, as explained in Appendix D.3. Meanwhile, the dotted line in the figures is the long-run mean difference between the two economies taken as the average difference after 25 quarters.}

37
Figure 11: Transition path of the unemployment rate to a permanent elimination of the layoff tax

Note. The solid blue lines represent the average difference in the unemployment rate (in percentage points) of the economy where at time 0 the layoff tax was abolished from that of the same economy where the tax persists, drawn from 2,000 simulations of the model. The shaded grey area represents the 5th-95th percentile interval of the differences. The dashed red line represents the long-run average difference between the two economies.

finding rate, the job separation rate, the job-to-job transition rate, and vacancies after the tax removal across states of aggregate productivity, which are reported in Appendix F.3. In both high- and low-productivity states, the reform fails to provoke a spike in job separations. The reason is straightforward. While removing the tax raises the individual productivity threshold and therefore increases endogenous job separation, it also increases OJS, which enables workers to avoid endogenous separation by moving to a new job. However, the rise in vacancies in periods of the high-productivity state is more than twice as large, yielding a larger spike in the job finding rate and job-to-job transitions. At peaks of the business cycle, firms receive greater incentive to recruit from the reform. As shown in the diagram in Figure 10, the gap between \( x^r_{r,N} \) and \( x^r \) is larger for higher values of \( a \). However, since the initial unemployment is higher in the low-productivity state as a result of the high job separation rate, even a small rise in the job finding rate yields a large fall in unemployment in the state with low productivity, as shown in the right entry in Figure 11.

How do these sharp differences in the dynamic responses of labor-market variables influence welfare across states of aggregate productivity? The tax removal unambiguously increases
welfare in the long run since firms stop paying the wasteful tax. Resource allocation becomes efficient because firms terminate the low-productive jobs they had retained to avoid the payment of the layoff tax, and they recruit high-productivity workers whose hiring was prevented in anticipation of payment of the tax in future periods.

In the short run, however, the timing of the reform is critical for welfare since the transitional dynamics of labor market variables are notably different across distinct states of aggregate productivity.46 To investigate the relevance in the timing of structural reforms, we proxy welfare with the flow value of the economy that comprises output and the flow value of unemployment net of hiring costs, OJS costs, and layoff taxes.47 It is straightforward to derive the welfare gain of the tax removal by subtracting the flow value of the economy with the layoff tax from the flow value of the economy without the tax in each period:

$$\Delta FV_t = \left( a_t \int_{x_L}^{x_H} x \ de^R_t(x) + bu^R_t - kv^R_t - k^s\phi^R_t \right) - \left( a_t \int_{x_L}^{x_H} x \ de^\tau_t(x) + bu^\tau_t - kv^\tau_t - k^s\phi^\tau_t - \tilde{EU}^\tau_t \right) \tag{27}$$

where the superscript $R$ indicates variables in the economy with tax reform, the superscript $\tau$ indicates variables in the economy with the layoff tax still in place, and $\tilde{E}U$ indicates the endogenous separations. Equation (27) tracks the welfare change of the tax removal during each period $t$.

Figure 12 plots equation (27) for the initial 25 quarters across 2,000 simulations and shows the net welfare gains from the removal of the layoff tax for the state with high- and low-aggregate productivity (left and right panels, respectively). A tax removal generates a contemporaneous welfare loss approximately twice as large in the state with high productivity compared to the state with low aggregate productivity. When the reform is enacted in the state with high-aggregate productivity, the welfare loss is equal to a reduction of approximately 2.8 units in the flow value of the economy, compared to the reduction of approximately 1.2 units.


47See Ljungqvist and Sargent (2012) for a similar approach to approximate welfare.
Figure 12: Transition path of the aggregate flow value to a permanent elimination of the layoff tax

Note. The solid blue lines represent the average difference in the aggregate flow value of the economy where at time 0, the layoff tax was abolished from that of the same economy where the tax persists from 2,000 simulations of the model. The shaded grey area represents the 10th-90th percentile interval of the differences. The dashed red line represents the long-run average difference between the two economies.

48 Welfare losses are short lived across different states of aggregate productivity as they disappear after four quarters once the economy reaches its long-run equilibrium. Over the long run, the welfare gain from the reform is equivalent to approximately 0.3 welfare units, as indicated by the dashed red line. We use equation (27) to compute an overall measure of welfare gain from the tax removal by deriving the present discounted gain from the tax removal (i.e. the weighted discounted sum of future flow values, \( \sum_0^\infty \beta^t \mathbb{E}_0 \{ \Delta FV_t \} \)). We find that the flow value associated with the implementation of the tax removal during periods with low productivity is 4.9 percent larger than the value associated with the same reform enacted in states with high aggregate productivity.

The analysis reveals that state dependence in labor market fluctuations is critical for welfare and that the tax removal involves sharp welfare losses in initial periods of the reform. Thereafter, the benefits quickly outweigh the costs. To identify the sources of welfare losses in the initial periods of the tax removal, Figure 13 documents for the first eight quarters of the transition the difference in the flow values into its components, namely production plus the value of

\[ 48 \text{The flow value of welfare is implicitly normalized by marginal utility of consumption, which is equal to 1 since preferences are linear in consumption.} \]
Figure 13: Transition path of the components of aggregate flow value after a permanent elimination of the layoff tax

Note. Each line represents the average difference of the variable from the economy where at time 0 the layoff tax was abolished from that of the same economy where the tax persists. The solid blue lines represent the average difference in output plus unemployment value, the red dashed line represents the average difference in total vacancy costs, the dash-dot yellow line represents the average difference in OJS costs, and the dotted purple line represents the difference in firing cost (a positive value implying a fall in firing costs). All lines are averages from 2,000 simulations of the model. For clarity, we focus on the first eight quarters of the transition.

unemployment, vacancy costs, OJS costs, and firing costs. Each line represents the average difference across simulation between the economy where the layoff tax is removed and the economy where the layoff tax is kept in place. The main difference across high- and low-productivity initial states lies in the vacancy costs and the OJS costs, which are initially larger for transitions that start in periods of high productivity. Intuitively, when productivity is high, labor markets tighten and OJS rises sharply. As both firms’ recruiting efforts and voluntary job search are costly, the short-run costs of the reform are high despite the immediate fall in unemployment.

Overall, the analysis shows that tax removal involves important short-run tradeoffs mainly related to the deadweight losses of search costs that are considerably larger in states with high-aggregate productivity. The timing of labor market reforms is critical. The tax elimination in states with low productivity involves lower short-run welfare losses than during states with high-aggregate productivity. Over the long run, the labor market reform is welfare-enhancing across states of aggregate productivity.
8 Conclusion

This paper isolates important state dependence in labor market fluctuations over the business cycle. The unemployment rate and its transition rates exhibit a stronger comovement with labor productivity in periods of low-aggregate productivity. A DMP model with endogenous job separation and on-the-job search captures this state dependence through the interaction between the distribution of match-specific productivity and the reservation threshold of firms for efficient matches, replicating empirical regularities accurately. Our application to layoff costs establishes critical differences of labor market reforms enacted in distinct states of the economy for the transitional dynamics of labor market variables and welfare.

The analysis may be extended in several dimensions. In particular, since the DMP framework is a key building block of richer models, it would be relevant to study the interaction of the DMP asymmetries with frictions from other sides of the economy. To this end, the mechanism generating state dependence may be recast in a comprehensive model that accounts for a broader range of real and nominal rigidities needed to replicate several business cycle properties in the data. The more general framework may unveil important interactions between state dependence of labor market dynamics and a broad set of macroeconomic variables. This research will prove challenging, however, because it requires a non-linear solution to a complex model. It also would be interesting to use the framework to study the design of optimal labor market reforms. Future work could extend the analysis to determine the optimal provision of labor market reforms at different states of productivity. The analysis can be further extended to assess a wide range of labor market institutions (e.g. unemployment benefits and hiring subsidies, among others) to provide a comprehensive appraisal of the welfare implications of alternative labor market reforms. These investigations remain outstanding tasks for future research.
9 Bibliography


Appendix

A Data appendix

A.1 Data sources

The analysis uses the following time series: real Gross Domestic Product (GDP), average labor productivity, the unemployment rate, vacancies, the job finding rate, and the separation rate. Real GDP is the non-farm business output as provided by the Bureau of Labor Statistics (BLS), while labor productivity is output per worker in the non-farm business sector. Both series were downloaded from Federal Reserve Bank of St. Louis Database (FRED). Unemployment is also provided by the BLS via FRED. The monthly job separation and job finding probabilities are computed following the continuous-time adjustment proposed by Shimer (2012). While we leave the details to the original paper, the essence of continuous time adjustment is to estimate the transition probabilities between unemployment and employment as discrete time probabilities derived from continuous-time hazard rates that are assumed to be constant within each month. This method controls for the bias of simultaneously estimating two related discrete probabilities. We used the original series provided on Rober Shimer’s web page from 1950 to 2007, and extend them using the BLS data until 2014. The monthly series are then averaged over the respective quarters.
## B Robustness checks

Table B.1: Cross-tabulation of low and high states across alternative threshold definitions with the baseline threshold.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>State</th>
<th>Average Labor Productivity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Below Median</td>
<td>Above Median</td>
<td>All</td>
</tr>
<tr>
<td><strong>HP-Filter $10^5$</strong></td>
<td>Below Median</td>
<td>80%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>Above Median</td>
<td>20%</td>
<td>80%</td>
</tr>
<tr>
<td><strong>Fernald Measure</strong></td>
<td>Below Median</td>
<td>76%</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>Above Median</td>
<td>24%</td>
<td>76%</td>
</tr>
<tr>
<td><strong>NBER Recessions</strong></td>
<td>Recession</td>
<td>93%</td>
<td>7%</td>
</tr>
<tr>
<td></td>
<td>Recovery</td>
<td>41%</td>
<td>59%</td>
</tr>
<tr>
<td><strong>Yearly Growth Rates</strong></td>
<td>Below Median</td>
<td>70%</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>Above Median</td>
<td>30%</td>
<td>70%</td>
</tr>
<tr>
<td><strong>Output Quarterly Growth Rates</strong></td>
<td>Below Median</td>
<td>69%</td>
<td>31%</td>
</tr>
<tr>
<td></td>
<td>Above Median</td>
<td>31%</td>
<td>69%</td>
</tr>
<tr>
<td><strong>Log Output</strong></td>
<td>Below Median</td>
<td>67%</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td>Above Median</td>
<td>33%</td>
<td>67%</td>
</tr>
<tr>
<td><strong>Quarterly Growth Rates (4Q-MA)</strong></td>
<td>Below Median</td>
<td>69%</td>
<td>31%</td>
</tr>
<tr>
<td></td>
<td>Above Median</td>
<td>31%</td>
<td>69%</td>
</tr>
<tr>
<td><strong>Quarterly Growth Rates</strong></td>
<td>Below Median</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td></td>
<td>Above Median</td>
<td>40%</td>
<td>60%</td>
</tr>
</tbody>
</table>

Note. For each alternative regime definition and for both the low- and high-productivity states, the table reports the percent of quarters in which ALP (i.e. the baseline regime definition) indicates a low state (i.e. productivity below median) or a high one (i.e. productivity above median). The alternative definitions are presented in descending order based on their correlation with ALP, based on Table ???. The alternative definitions are: ALP using an HP-filter weight of $10^5$, ALP based on the factor-intensity adjusted measure of Fernald (2014), NBER recession dates, yearly growth rates of productivity (computed as 4-quarter log differences), quarterly growth rates (computed as log differences) both using a 4-quarter moving average and in their raw values.
Table B.2: Robustness of the variables’ standard deviation to different time samples, to an HP-weight of $10^5$, and to using the labor productivity series by Fernald (2014).

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Sample: Pre Great Recession</th>
<th>Sample: Great Moderation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p&lt; 50th percentile</td>
<td>p&gt; 50th percentile</td>
<td>σ&lt;50</td>
</tr>
<tr>
<td><strong>Levels</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.1511</td>
<td>0.1197</td>
<td>1.26</td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>0.0397</td>
<td>0.0335</td>
<td>1.19</td>
</tr>
<tr>
<td>Separation Rate</td>
<td>0.0020</td>
<td>0.0013</td>
<td>1.53</td>
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<tr>
<td>Employment Rate</td>
<td>0.0095</td>
<td>0.0075</td>
<td>1.27</td>
</tr>
<tr>
<td>Output</td>
<td>0.0215</td>
<td>0.0167</td>
<td>1.29</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0092</td>
<td>0.0071</td>
<td>1.31</td>
</tr>
<tr>
<td><strong>Growth rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0829</td>
<td>0.0401</td>
<td>2.06</td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>0.0566</td>
<td>0.0444</td>
<td>1.27</td>
</tr>
<tr>
<td>Separation Rate</td>
<td>0.0576</td>
<td>0.0497</td>
<td>1.16</td>
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<tr>
<td>Employment Rate</td>
<td>0.0051</td>
<td>0.0023</td>
<td>2.15</td>
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<td>Output</td>
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<tr>
<td>Productivity</td>
<td>0.0099</td>
<td>0.0074</td>
<td>1.34</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p&lt; 50th percentile</td>
<td>p&gt; 50th percentile</td>
<td>σ&lt;50</td>
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<td><strong>Level</strong></td>
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<td>0.1966</td>
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<td>0.0014</td>
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<td>1.11</td>
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<td>0.0108</td>
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<td>0.0101</td>
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Note. The table reports the standard deviation of labor market variables, both in levels and growth rates, across states of low and high labor productivity for the baseline case and for a battery of robustness checks. The checks include: considering only the pre-Great Recession and the Great Moderation periods, using an HP-filter weight of $10^5$ for labor productivity and the other variables in levels, using the 25th and 75th percentiles of productivity as thresholds, and using the factor-intensity adjusted measure of labor productivity by Fernald (2014). The ratios with a value above 1 are reported in bold font.
Table B.3: Standard deviation of labor market variables using alternative definitions of low- and high-productivity regimes.

<table>
<thead>
<tr>
<th>Threshold: NBER Recessions</th>
<th>Threshold: Log output</th>
<th>Threshold: Output Quarterly Growth Rate</th>
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</thead>
<tbody>
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<td></td>
<td>Recession</td>
<td>Recovery</td>
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<td>Level</td>
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<td></td>
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<td>0.0427</td>
<td>0.0355</td>
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<td>Separation Rate</td>
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<td>0.0013</td>
</tr>
<tr>
<td>Employment Rate</td>
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<td>0.0080</td>
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<tr>
<td>Output</td>
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<td>Growth rate</td>
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<td></td>
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<td>Output</td>
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<tr>
<td>Productivity</td>
<td>0.0124</td>
<td>0.0083</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Threshold: Yearly Growth Rate of Productivity</th>
<th>Threshold: Quarterly Growth Rate of Productivity (4Q-MA)</th>
<th>Threshold: Quarterly Growth Rate of Productivity</th>
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</thead>
<tbody>
<tr>
<td>p&lt; 50\text{th} percentile</td>
<td>p&gt; 50\text{th} percentile</td>
<td>p&lt; 50\text{th} percentile</td>
</tr>
<tr>
<td>Level</td>
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<td></td>
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<td>Productivity</td>
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<td>0.0101</td>
</tr>
<tr>
<td>Growth rate</td>
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<td></td>
</tr>
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<td>Unemployment</td>
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</tr>
<tr>
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<td>0.0584</td>
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<td>Employment Rate</td>
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</tr>
<tr>
<td>Output</td>
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<td>0.0115</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0093</td>
<td>0.0090</td>
</tr>
</tbody>
</table>

Note. The table reports the standard deviations across low and high productivity states using different definitions of the two regimes. The alternative definitions are: NBER recession dates, yearly growth rates of productivity (computed as 4-quarter log differences), quarterly growth rates (computed as log differences) both using a 4-quarter moving average and in their raw values. The ratios with a value above 1 are reported in bold font.
C  TVAR: details and additional figures

C.1 Priors

The formulation of priors follows Banbura et al. (2010) and the same prior moments have been used for the parameters in both regimes. To be precise, it is assumed that the prior distribution of the VAR parameter vector has a Normal-Wishart conjugate form

\[ b|\Sigma \sim N(b_0, \Sigma \otimes \Omega_0), \Sigma \sim IW(v_0, S_0). \] (28)

where \( b \) is obtained by stacking the columns of the matrix of the autoregressive coefficients \( B \).

The prior moments of \( b \) are given by

\[ E[(B_k)_{i,j}] = \begin{cases} \delta_i & i = j, k = 1 \\ 0 & \text{otherwise} \end{cases}, \quad Var[(B_k)_{i,j}] = \lambda \sigma_i^2 / \sigma_j^2, \]

and as it is explained by Banbura et al. (2010) they can be constructed using the following dummy observations

\[
Y_D = \begin{pmatrix}
\frac{\lambda}{\delta_1} \sigma_1 & \ldots & \frac{\lambda}{\delta_N} \sigma_N \\
0 & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & 0
\end{pmatrix}
\quad \text{and} \quad
X_D = \begin{pmatrix}
J_K \otimes diag(\sigma_1, \ldots, \sigma_N) \\
0_{N \times NK} \\
\vdots \\
0_{1 \times NK}
\end{pmatrix}
\] (29)

where \( J_K = diag(1, 2, \ldots, K) \) and \( diag \) denotes the diagonal matrix. The prior moments of (28) are just functions of \( Y_D \) and \( X_D \), \( B_0 = Y_D X_D' (X_D X_D')^{-1}, \Omega_0 = (X_D X_D')^{-1}, S_0 = (Y_D - B_0 X_D) (Y_D - B_0 X_D)' \) and \( v_0 = T_D - NK \). Finally, the hyper-parameter \( \lambda \) controls the tightness of the prior.

The values of the persistence – \( \delta_i \) – and the error standard deviation – \( \sigma_i \) – parameters of the AR(1) model are obtained from its OLS estimation (as in Mumtaz and Zanetti (2012) and Mumtaz and Zanetti (2015)). Sensitivity analysis reveals that the results are robust to different selections of VAR lags. As we work with a five variables VAR and eight lags, we follow Canova et al. (2012) and select a value for \( \lambda \) that implies fast lag decay towards zero (\( \lambda = 0.25 \)). It is shown in the next section that the results are robust when “looser” priors are used.

Finally, \( z^* \) is assumed to be normally distributed with zero mean and standard deviation \( \sigma_{z^*} \) calibrated to deliver an MCMC acceptance rate between 20% and 40%.

C.2 Max Variance VAR Shock Identification Method

Following Uhlig (2004), the benchmark identification scheme employed in this study amounts to finding the shock that explains most of the variation of the (adjusted for utilisation) labor productivity (Fernald, 2014) series over the business cycle frequencies. The scheme consists in finding an orthonormal rotation matrix \( Q \) of the orthogonalized shocks that maximizes the sum of the forecast error variance of labor productivity from horizons 0 to \( h \). The scheme is applied separately for the observation in each regime. Therefore, for simplicity we drop the regime specific notation \( i = 1, 2 \) in the following description. For a regime-specific VAR model,
the conventional moving-average (MA) notation is
\[ Z_t = \sum_{j=0}^{\infty} \tilde{B}_j v_{t-j}, \]

where \( v_t \)'s are reduced-form shocks, \( \tilde{B}_0 = I_N \), and \( \tilde{B}_j = \sum_{s=0}^{j} \tilde{B}_{s-j} B_j \) (with \( B_j \) being the reduced-form coefficients, such that \( B_j = 0 \) for \( j > K \)). Using the MA representation, the forecast error variance at horizon \( h \) is
\[ \Sigma_h = \sum_{j=0}^{h-1} \tilde{B}_j A A' \tilde{B}_j' \]

where \( \Sigma_v = AA' \) is the variance-covariance matrix of reduced form shocks, and \( A \) is a lower-triangular matrix obtained through a Choleski decomposition. Assume (without loss of generality) that labor productivity is the first series in the VAR and that the shock driving labor productivity between 0 and 40 quarters is ordered first. In this case the identification is achieved by finding the first column of matrix \( Q \) that solves the following maximisation problem:
\[ \arg \max_{Q_1} e_1' \left[ \sum_{h=0}^{H} \sum_{j=0}^{h-1} \tilde{B}_j A Q_1 Q_1' A' \tilde{B}_j' \right] e_1 \]

such that \( Q_1 Q_1' = 1 \) and \( e_1 \) is a \( N \)-by-1 vector with 1 in the first entry and 0 in all the others. As shown by Uhlig (2004), identification of the productivity shock only requires finding the first column of \( Q \) (i.e. \( Q_1 \)). Moreover, the maximization can be re-written as an eigenvalue eigenvector problem and a solution can be easily obtained. The identification of the productivity shock under this scheme requires no further restrictions.

C.3 Generalized Impulse Response Functions

The generalized impulse responses presented in below and in Section ?? are calculated using parallel computing technology (MATLAB Distributed Computing Server/Parallel Computing Toolbox on 32 cores). Similar to Koop et al. (1996), the exact simulation steps are as follows:

1. We draw \( 1 \times 48 \) structural shocks \( \omega_t \) from the standard normal distribution (where \( t = 1, \ldots, 40 \))
2. We simulate the model using the shocks from step 1, we denote the simulated data by \( y_t \)
3. We simulate the model using again from step 1 but now we increase the value of the structural shock of interest in period 1 by an amount \( x \), namely
\[ \tilde{\omega}_{j,1} = \omega_{j,1} + x \]

where \( x = 1, -1, 2 \) and \( -2 \). We denote the data obtained from this simulation by \( \tilde{y}_t \)
4. Steps 1 to 3 are repeated histories \( Th \) (all starting points in the data)
5. The GIRF is calculated as follows

\[
GIRF = \frac{1}{Th} \sum_{i=1}^{Th} (\tilde{y}_t^i - y_t^i)
\]  

(31)

Steps 1-5 are implemented for all posterior draws.

In the regime-specific IRFs, the response of the economy does not take into account:

- the current state of the economy
- the probability that the economy might switch to a different regime
- the size and sign of the shock
C.4 Additional figures from the TVAR

Figure C.1: Labor productivity and regimes based on the median of the posterior of the threshold.

Note. The blue line plots the HP-filtered series of log labor productivity. The red line, switching from 0 to -1 represents the current regime as identified by the median of the threshold’s posterior distribution.

Figure C.2: Historical decomposition of fluctuations explained by productivity shocks from the TVAR
Figure C.3: Forecast Error Variance Decomposition from the TVAR

Note. The blue line reports the median fraction of the forecast error variance each variable that is explained by the identified productivity shocks. The shaded area comprises the 16th-84th percentile range. In the second row, the red line reports the fraction from the low-productivity regime for comparison.
D Model: additional information

D.1 Uniqueness of productivity threshold \( x^r(a_t) \)

Assuming the threshold for OJS \( x^s(a) \) lies above the threshold \( x^r(a) \), and setting \( S^s(a, z) = 0 \), we can rearrange (13) into

\[
b + k^s = a_t x_t^r + \beta \mathbb{E}_t \left\{ (1 - s) \left( 1 - p(\theta_t) F(x_{t+1}^r) \right) \left( 1 - \lambda \right) S(a_{t+1}, x) \right. \\
+ \left. \lambda \int_{x_L}^{x_H} S(a_{t+1}, x') dF(x') \right\}
\]

The first term on the RHS is continuous, strictly increasing, and bounded below in \( x \) over the support \([x_L, x_H]\). For a given \( \theta_t \), the term \( \left( 1 - p(\theta_t)(1 - F(x_{t+1}^r)) \right) \) is continuous and strictly increasing in \( x \). By the properties of \( S(a, x) \), the second term on the RHS is bounded below by 0 and is continuous and weakly increasing in \( x \). Therefore, the RHS is bounded below by 0, strictly increasing and continuous in \( x \). These conditions are sufficient for the uniqueness of the value \( x^r(a_t) \). Furthermore, as the LHS of the equation is constant, and \( S(a, x) \) is increasing in \( a \), then \( x^r(a) \) must be decreasing in \( a \).

The proof easily extends to the two cutoffs in the model with layoff taxes, as the layoff tax merely imply adding the tax terms to the LHS.

D.2 Job finding rate and asymmetry with respect to the state of the economy: a graphic example

Figure D.1: Illustrative diagram of the mechanism driving asymmetries with respect to state of the economy in the job finding rate.

Figure D.1 shows the effect of a movement in the individual productivity threshold on the p.d.f. for new workers \( F'(x) \) at different levels of productivity. The figure shows that the mass of jobs sensitive to movements in the individual productivity threshold depends on the location in the support of the distribution of individual productivity shocks. The closer the productivity threshold is to the mode of the density function, the larger the increase in the mass of non-formed matches generated by a rise in the reservation threshold. For this reason, the mass of new matches affected by movements in the reservation threshold are larger when the threshold is already high (i.e. when productivity is low). The figure shows that for the same increase in the reservation threshold from \( x_0^r \) to \( x_1^r \) and from \( \tilde{x}_0^r \) to \( \tilde{x}_1^r \), respectively, the increase in the cumulative density function is lower when the threshold is low (shaded area A) compared
to when it is high (shaded area B). Thus, the effect of a shock on the mass of new matches exposed to movements in the threshold is different across different levels of productivity. Note that this mechanism holds to the extent that \( x''(a) \) lies in the region of the domain of \( x \) where \( F''(x) > 0 \). In such cases, changes in the individual productivity threshold affect a larger proportion of workers when the equilibrium individual productivity threshold is higher.

D.3 Solution of the model and targeted moments

We solve the model nonlinearly using an iterative procedure. To generate an accurate solution, we set the number of grid points of the state space to 45 for the discretized AR(1) process \( a_t \), following the approach described in Tauchen (1986), and 800 for the individual productivity \( x \). The iteration starts with a guess for the policy function of the market tightness \( \theta^{(0)}(a) \). Using the guess, we compute the match surplus for all values of \( x \) and \( a \) and derive the individual productivity threshold \( x'(a) \) and the OJS threshold \( x^*(a) \). Using these results, \( \theta^{(1)}(a) \) is computed through the free entry condition and used again to compute the surplus. The process is repeated until the norm of \( ||\theta^{(0)}(x) - \theta^{(1)}(x)|| \) is below a chosen critical value. Using the result from Pissarides (2000), under a linear production function, hiring and layoff decisions do not depend on the aggregate level of employment or on the distribution of individual productivity. Hence, the only relevant state variable for the policy function is the aggregate productivity factor.

To obtain business cycle statistics, we run 1,000 simulations of the model. Each simulation comprises 1,380 monthly periods. After discarding the first 600 periods, we take quarterly averages of all the simulated series to create a time series of the same length as the data. For each simulation we compute the relevant business cycle moments after taking logs and HP-filtering the series. The simulated business cycle statistics are the average of each moment across the simulations.

Table D.1: Model targets

<table>
<thead>
<tr>
<th></th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job finding rate - mean</td>
<td>0.45</td>
<td>0.442</td>
</tr>
<tr>
<td>Separation rate - mean</td>
<td>0.03</td>
<td>0.031</td>
</tr>
<tr>
<td>Separation rate - standard deviation</td>
<td>0.055</td>
<td>0.055</td>
</tr>
<tr>
<td>Separation rate autocorrelation</td>
<td>0.63</td>
<td>0.647</td>
</tr>
<tr>
<td>Job-to-job rate - mean</td>
<td>0.032</td>
<td>0.032</td>
</tr>
<tr>
<td>Productivity - mean</td>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>Productivity - standard deviation</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>Productivity - autocorrelation</td>
<td>0.757</td>
<td>0.769</td>
</tr>
</tbody>
</table>

Note. The moments are computed as the means of 1,000 simulations of 1,380 monthly periods. After discarding the first 600 observations in each simulation, the remaining series are aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV.

D.4 Computation of Generalized IRFs for the model

We resort to numerical simulations of the model to produce the response of the variables at different points in the state space. To implement the computation, it is critical to establish the starting points for the IRFs, which we describe below together with the procedure used to compute the IRFs.

\(^{49}\)The value we use is \( 10^{-8} \).
Figure D.2: Generalized IRFs for the job contact probability $p(\theta)$

Note. The solid line represents the mean IRF value in each period. The shaded area represents the 5th and 95th percentiles of the IRF values. Responses of the variables in periods with high (low) aggregate productivity are in left (right) panels. Units on the y-axis are percentage points.

After obtaining the firm’s policy functions for job separation and job finding rates, we simulate the model for 10,000 monthly periods by generating a random sequence of the Markov process for productivity and then computing the relevant policy variables and state variables. We then obtain the stationary distribution of labor productivity and compute the 10th and 90th percentiles. We split the simulated time series into two samples. The first sample includes all of the periods in which productivity is equal to 10th percentile: this is the “bad-times” sample. The second sample includes all period in which productivity is equal to the 90th percentile: this is the “good-times” sample. For each observation in a given sample, we collect the following variables: $u_t$, $a_t$, $e_t(x)$ and $e_{t-1}(x)$. We compute four IRFs as combinations of the following conditions: productivity is either “high” or “low”, and the economy is hit by either a positive or a negative one-standard deviation productivity shock.

Each IRF is obtained through a series of 1,000 simulations. As a starting point for each simulation, we draw a random observation from the relevant sample (either “bad” or “good”) with replacement: $\{u_0, a_0, e_0(x), e_{-1}(x)\}$. We then simulate a continuous Markov path of productivity from the preset starting value $a_0$ to $a_T$, where $T = 90$ months. We compute corresponding values of $\{u_t, e_t(x), sr_t, jfr_t, jjr_t\}$, where $sr_t$ is the separations rate, $jfr_t$ the job finding rate, and $jjr_t$ is the job-to-job transition rate. We compute the “alternative” history in which the path of productivity is initially hit by a further positive or negative shock in the initial period: i.e. $\{\tilde{a}_t\}$ such that $\tilde{a}_0 = a_0 + \sigma$ or $\tilde{a}_0 = a_0 - \sigma$. The corresponding variables under the alternative aggregate productivity path are $\{\tilde{u}_t, \tilde{e}_t(x), \tilde{sr}_t, \tilde{jfr}_t, \tilde{jjr}_t\}$. Because the value of $\tilde{a}_t$ does not fall on one of the notes of the discretized grid, we compute the paths of all variables using a linear interpolation of the policy functions. After taking quarterly averages, the IRF for a given variable is computed as the difference between the values under the alternative and the baseline history: e.g. $du_q = \tilde{u}_q - u_q$ for the unemployment response, where $q$ represents a one-quarter period. The 10th, 50th, and 90th percentile IRFs are calculated as the relevant percentiles of $du_t$ (or any other variable) across all simulations at each $t = 0, T$. 

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D.5 Generalized IRFs for the job contact probability

E Details of alternative calibration and alternative models

E.1 Details and additional results of alternative calibration of baseline model

Table E.1: Parameters for the baseline model under the alternative calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa^s$</td>
<td>OJS cost</td>
<td>0.113</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Matching function efficiency parameter</td>
<td>0.415</td>
</tr>
<tr>
<td>$s$</td>
<td>Exogenous job separation rate</td>
<td>0.028</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Arrival rate of individual productivity</td>
<td>0.045</td>
</tr>
<tr>
<td>$\mu_x$</td>
<td>Mean of log individual productivity</td>
<td>-0.055</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>Standard deviation of individual</td>
<td>0.105</td>
</tr>
</tbody>
</table>

Table E.2: Targets for the model with OJS and endogenous separations, under the alternative calibration.

<table>
<thead>
<tr>
<th>Target Model</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job Finding Rate - mean</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Separation Rate - mean</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma_{SR}/\sigma_E$</td>
<td>6.5</td>
<td>6.34</td>
</tr>
<tr>
<td>Separation rate - autocorrelation</td>
<td>0.63</td>
<td>0.649</td>
</tr>
<tr>
<td>Job-to-Job Rate - mean</td>
<td>0.032</td>
<td>0.031</td>
</tr>
<tr>
<td>Productivity - mean</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table E.3: Long run means and standard deviations for the model with OJS and endogenous separations, under the alternative calibration.

<table>
<thead>
<tr>
<th>Data</th>
<th>p</th>
<th>U</th>
<th>JFR</th>
<th>SR</th>
<th>V</th>
<th>V/U</th>
<th>Y</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $X_t$</td>
<td>0.9994</td>
<td>0.0651</td>
<td>0.4518</td>
<td>0.0301</td>
<td>0.1735</td>
<td>2.789</td>
<td>0.9347</td>
<td>0.9342</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>0.014</td>
<td>0.0638</td>
<td>0.0444</td>
<td>0.0293</td>
<td>0.0468</td>
<td>0.1052</td>
<td>0.0183</td>
<td>0.0046</td>
</tr>
<tr>
<td>corr($p_t, X_t$)</td>
<td>1</td>
<td>-0.9469</td>
<td>0.968</td>
<td>-0.9153</td>
<td>0.9156</td>
<td>0.9804</td>
<td>0.9948</td>
<td>0.915</td>
</tr>
</tbody>
</table>

E.2 Details and additional results for the model without OJS

Table E.4: Parameters for the model with endogenous separation and no OJS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Matching function efficiency parameter</td>
<td>0.42</td>
</tr>
<tr>
<td>$s$</td>
<td>Exogenous job separation rate</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Arrival rate of individual productivity</td>
<td>0.36</td>
</tr>
<tr>
<td>$x^H$</td>
<td>Upper bound of individual productivity shocks</td>
<td>1.9</td>
</tr>
<tr>
<td>$\mu_x$</td>
<td>Mean of log individual productivity</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>Standard deviation of individual</td>
<td>0.3</td>
</tr>
</tbody>
</table>

50 We start with an initial 200 observations that are then discarded.

51 It is important to note that the plotted IRFs therefore do not represent a specific and unique response, but simply the percentiles of the distribution of responses in period $t$. 

59
Table E.5: Targets for the model with endogenous separations.

<table>
<thead>
<tr>
<th>Target Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Job Finding Rate - mean</td>
<td>0.45</td>
<td>0.47</td>
</tr>
<tr>
<td>Separation Rate - mean</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Separation Rate - standard deviation</td>
<td>0.055</td>
<td>0.054</td>
</tr>
<tr>
<td>Separation Rate - autocorrelation</td>
<td>0.63</td>
<td>0.637</td>
</tr>
<tr>
<td>Productivity - mean</td>
<td>1</td>
<td>0.988</td>
</tr>
</tbody>
</table>

Table E.6: Long run means and standard deviations for the model with endogenous separations.

<table>
<thead>
<tr>
<th>Data</th>
<th>p</th>
<th>U</th>
<th>JFR</th>
<th>SR</th>
<th>V</th>
<th>V/U</th>
<th>Y</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $X_t$</td>
<td>1.001</td>
<td>0.062</td>
<td>0.471</td>
<td>0.030</td>
<td>0.091</td>
<td>1.496</td>
<td>0.939</td>
<td>0.937</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>0.013</td>
<td>0.072</td>
<td>0.029</td>
<td>0.054</td>
<td>0.025</td>
<td>0.050</td>
<td>0.017</td>
<td>0.0047</td>
</tr>
<tr>
<td>corr($p_t, X_t$)</td>
<td>1</td>
<td>-0.9707</td>
<td>0.963</td>
<td>-0.9538</td>
<td>-0.791</td>
<td>0.997</td>
<td>0.9966</td>
<td>0.9539</td>
</tr>
</tbody>
</table>

E.3 Details and additional results for the model with exogenous separations

The DMP model with exogenous separations is defined by the surplus function, $S(a_t)$ which only has aggregate productivity as an argument:

$$S(a_t) = a_t - b + \beta E\left\{(1 - s - p(\theta_t))S(a_{t+1})\right\}.$$  

The free entry condition becomes $k = q(\theta_t)\beta(1 - \phi)E\left\{S(a_{t+1})\right\}$.  

Hagedorn and Manovskii (2008) calibrate the flow value of unemployment $b$ and the worker’s bargaining power $\phi$ (hence departing from the Hosios condition) to match the mean ratio of wages to productivity and the elasticity of wages with respect to productivity. Based on Hornstein et al. (2011), these values are 0.97 and 0.5, respectively. This strategy is meant to solve the so called “Shimer puzzle” of low volatility in the job finding rate. A high value of $b$, combined with a low bargaining power for workers, implies that firms derive very small but highly volatile profits from the employment relationship and are therefore highly sensitive to fluctuations in aggregate productivity.

Table E.7: Parameters for the model with exogenous separation, with the Hagedorn and Manovskii (2008) calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9991</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy cost</td>
<td>0.17</td>
</tr>
<tr>
<td>$b$</td>
<td>Flow value of unemployment</td>
<td>0.93</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of matching with respect to vacancies</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Matching function efficiency paramete</td>
<td>0.083</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Worker’s bargaining power</td>
<td>0.062</td>
</tr>
<tr>
<td>$s$</td>
<td>Exogenous job separation rate</td>
<td>0.0094</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence parameter of aggregate productivity</td>
<td>0.9895</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of aggregate productivity shock</td>
<td>0.0034</td>
</tr>
</tbody>
</table>
Table E.8: Targets for the model with exogenous separations, with the Hagedorn and Manovskii (2008) calibration.

<table>
<thead>
<tr>
<th>Target Model</th>
<th>Mean Job Finding Rate</th>
<th>Mean Separation Rate</th>
<th>$\epsilon_{w,p}$</th>
<th>Mean wage/productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>0.139</td>
<td>0.0094</td>
<td>0.5</td>
<td>0.97</td>
</tr>
<tr>
<td>Model</td>
<td>0.138</td>
<td>0.0094</td>
<td>0.5</td>
<td>0.968</td>
</tr>
</tbody>
</table>

Table E.9: Long run means and standard deviations for the model with exogenous separations, with the Hagedorn and Manovskii (2008) calibration.

<table>
<thead>
<tr>
<th>Data</th>
<th>p</th>
<th>U</th>
<th>JFR</th>
<th>SR</th>
<th>E</th>
<th>V</th>
<th>V/U</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean X_t</td>
<td>0.9994</td>
<td>0.0671</td>
<td>0.1382</td>
<td>0.0094</td>
<td>0.1781</td>
<td>2.8666</td>
<td>0.9326</td>
<td>0.9328</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>0.014</td>
<td>0.0936</td>
<td>0.1073</td>
<td>0</td>
<td>0.1318</td>
<td>0.2141</td>
<td>0.0205</td>
<td>0.0071</td>
</tr>
<tr>
<td>corr($p_t, X_t$)</td>
<td>1</td>
<td>-0.9217</td>
<td>0.9783</td>
<td>0</td>
<td>0.9397</td>
<td>0.9809</td>
<td>0.9868</td>
<td>0.883</td>
</tr>
</tbody>
</table>

F   Model with layoff taxes: additional details

F.1 Derivations

The value of new and old matches are distinct and defined as

$$M^N(a_t, x) = \max[M^{c,s}(a_t, x), M^{c,n}(a_t, x), U(a_t) + V(a_t)],$$  \hspace{1cm} (32)

$$M^O(a_t, x) = \max[M^{c,s}(a_t, x), M^{c,n}(a_t, x), U(a_t) + V(a_t) - \tau],$$  \hspace{1cm} (33)

where $M^N(a_t, x)$ and $M^O(a_t, x)$ denote the joint value for new and existing workers, respectively, which account for on-the-job searchers. $M^{c,s}(a_t, x)$ and $M^{c,n}(a_t, x)$ are the joint values for continuing the job relationship with or without OJS, respectively, and are defined as:

$$M^{c,n}(a_t, x) = a_t x + \beta E_T \{ U(a_{t+1}) + V(a_{t+1})$$  
$$+ (1 - s) \left[-\tau + (1 - \lambda)S^O(a_t, x) + \lambda \int_{x_L}^{x_H} S^O(a_{t+1}, x')dF(x') \right],$$  \hspace{1cm} (34)

$$M^{c,s}(a_t, x) = a_t x - \kappa_s + \beta E_T \{ U(a_{t+1}) + V(a_{t+1})$$  
$$+ \left(1 - p(\theta)F(x'_{t+1})\right)(1 - s) \left[-\tau + (1 - \lambda)S(a_{t+1}, x)$$  
$$+ \lambda \int_{x_L}^{x_H} S^O(a_{t+1}, x')dF(x') \right] + p(\theta)\phi \int_{x(a_{t+1})}^{x_H} S^O(a_{t+1}, x')dF(x') \right\}.$$  \hspace{1cm} (35)

The surplus functions directly follow from the above equations as in the baseline model. The measure of employed workers with individual productivity below $x$ has the following
law of motion. For those workers whose individual productivity is in the OJS interval \( [x_{t}^r, x_t^s] \):

\[
e_{t+1}(x) = p(\theta_t)(1 - e_t(x_H))(F(x) - F(x_{t+1}^N)) + p(\theta_t)(F(x) - F(x_{t+1}^N))e_t(x_t^s)
\]

\[
+ (1 - s) \left[ \lambda(F(x) - F(x_{t+1}^O)) \left( e_t(x_H) - p(\theta_t)F(x_{t+1}^N)e_t(x_t^s) \right)
\]

\[
+ (1 - \lambda) \left( e_t(x) - e_t(x_{t+1}^O) \right) \left( 1 - p(\theta_t)F(x_{t+1}^N) \right) \right],
\]

where \( F(x) = 1 - F(x) \). For the non-searching workers, with \( x > x_t^s \):

\[
e_{t+1}(x) = p(\theta_t)(1 - e_t(x_H))(F(x) - F(x_{t+1}^N)) + p(\theta_t)(F(x) - F(x_{t+1}^N))e_t(x_t^s)
\]

\[
+ (1 - s) \left[ \lambda(F(x) - F(x_{t+1}^O)) \left( e_t(x_H) - p(\theta_t)F(x_{t+1}^N)e_t(x_t^s) \right)
\]

\[
+ (1 - \lambda) \left( e_t(x) - e_t(x_{t+1}^O) \right) + (1 - p(\theta_t)F(x_{t+1}^N))(e_t(x_t^s) - e_t(x_{t+1}^O)) \right].
\]

### F.2 Layoff tax removal: additional details

Incumbent workers remain covered by firing costs for endogenous separations. For simplicity, we assume that incumbent workers who search on the job and are covered by the layoff tax can only transition to new jobs that are not covered by the tax.\(^{52}\)

As the tax does not apply to new workers, the previous distinction between new and old workers turns into a distinction between workers hired pre- and post-reform. Once the reform is implemented, let the superscript \( R \) denote workers hired in the post-reform period, who are not covered by the tax, and the superscript \( \tau \) indicate workers who are still covered by the tax.

\[
S^{\tau,n}(a_t, x) = a_t x - b + \beta \mathbb{E}_t \left\{ (1 - s) \left[ -\tau + (1 - \lambda)S^\tau(a_{t+1}, x) + \lambda \int_{x_L}^{x_H} S^\tau(a_{t+1}, x')dF(x') \right] 
\]

\[
- p(\theta_t) \phi \int_{x_L}^{x_H} S^R(a_{t+1}, x')dF(x') \right\}, (36)
\]

\[
S^{\tau,s}(a_t, x) = a_t x - k^s - b + \beta \mathbb{E}_t \left\{ \left[ 1 - p(\theta)(1 - F(x_{t+1}^r)) \right] (1 - s) \left[ -\tau + (1 - \lambda)S^\tau(a_{t+1}, x) 
\]

\[
+ \lambda \int_{x_L}^{x_H} S^\tau(a_{t+1}, x')dF(x') \right]\right\}, (37)
\]

\[
S^{R,n}(a_t, x) = a_t x - b + \beta \mathbb{E}_t \left\{ (1 - s) \left[ (1 - \lambda)S^R(a_{t+1}, x) + \lambda \int_{x_L}^{x_H} S^R(a_{t+1}, x')dF(x') \right]
\]

\[
- p(\theta_t) \phi \int_{x_L}^{x_H} S^R(a_{t+1}, x')dF(x') \right\}, (38)
\]

\(^{52}\)The case in which “protected” OJS workers can search for “protected” jobs is complicated by the fact that firms would have to take into account the share of searching workers who are “protected” and would have a separate bargaining process, involving distinct reservation levels.
\[ S^R_s(a_t, x) = a_t x - k^s - b + \beta \mathbb{E}_t \left\{ \left[ 1 - p(\theta)(1 - F(x_{t+1}^R)) \right] (1 - s) \left[ (1 - \lambda) S^R(a_{t+1}, x) \right. \\
\left. + \lambda \int_{x_L}^{x_H} S^R(a_{t+1}, x') dF(x') \right] \right\}, \] (39)

Note that OJS workers of both types only accept jobs that are above the reservation level of non-protected matches \( x^{R,r} \). Also, \( S^{\tau,n} \) involves a term with respect to \( S^R_{t,n} \) within its continuation value. Meanwhile, the post-reform surplus are essentially those of the baseline no-tax case.

The free-entry condition is based on the expected surplus of matches that are not covered by employment protection.

We use a long simulation of the model economy with the tax in place to obtain a distribution of state variables \( \{e_{t-1}(x), a_t\} \) when labor productivity is at the 10th and 90th percentiles, representing the trough and peak of the productivity cycle. From these distributions we draw (with replacement) a sample of 2,000 initial starting points. For each starting point we simulate the ensuing path of aggregate productivity through a random sequence of exogenous innovations for 25 quarters (75 months). For each sampled technology series we then compute the path of the economy under two scenarios. In the first scenario there are no changes to the economy, and the tax is expected to remain in place forever. In the second scenario there is an unannounced permanent elimination of the tax in period 0. Specifically, in the first case the path of the economy is computed using the firms’ policy function for the model with tax, while in the second case the policy function used from period 0 onward is the solution for the baseline model without the tax. This alternative scenario is interpreted as the “structural reform” case.
Figure F.1: Transition path of the separation rate, job finding rate, and job-to-job rate to a permanent elimination of the layoff tax.

(a) Separation rate

(b) Job finding rate

(c) Job-to-job rate

(d) Vacancies

Note. The solid blue lines represent the average difference in the respective variable for the economy where at time 0 the layoff tax was abolished from that of the same economy where the tax persists from 2,000 simulations of the model. The shaded grey area represents the 10th-90th percentile interval of the differences. The dashed red line represents the long-run average difference between the two economies.