Bisecting the Mental Number Line in Near and Far Space

Matthew R. Longo¹ and Stella F. Lourenco²

¹Institute of Cognitive Neurosciences, University College London
²Department of Psychology, Emory University

Address Correspondence to:
Matthew R. Longo
Institute of Cognitive Neurosciences
University College London
17 Queen Square
London WC1N 3AR, United Kingdom
m.longo@ucl.ac.uk

Word count: 3,355
Numerical Attention and Distance

Abstract

Much evidence suggests that common posterior parietal mechanisms underlie the orientation of attention in physical space and along the mental number line. For example, the small leftward bias (pseudoneglect) found in paper-and-pencil line bisection is also found when participants “bisect” number pairs, estimating (without calculating) the number midway between two others. For bisection of physical lines, pseudoneglect has been found to shift rightward as lines are moved from near space (immediately surrounding the body) to far space. We investigated whether the presentation of stimuli in near or far space also modulated spatial attention for the mental number line. Participants bisected physical lines or number pairs presented at four distances (60, 120, 180, 240 cm). Clear rightward shifts in bias were observed for both tasks. Furthermore, the rate at which this shift occurred in the two tasks, as measures by least-squares regression slopes, was significantly correlated across participants, suggesting that the transition from near to far space induced a common modulation of lateral attention in physical and numerical space. These results demonstrate a tight coupling between number and physical space, and show that even such prototypically abstract concepts as number are modulated by our on-line interactions with the world.

Keywords: Mental number line, Near space, Peripersonal space, spatial attention
Introduction

Numbers are commonly conceptualized with the metaphor of the *mental number line*, smaller numbers on the left and larger numbers on the right. An increasing body of evidence supports the theory that numerical information is represented spatially (e.g., Dehaene, Bossini, & Giraux, 1993; Fischer, Castel, Dodd, & Pratt, 2003; Göbel, Calabria, Farnè, & Rossetti, 2006; Loetscher, Bockisch, & Brugger, 2008a; Loftus, Nicholls, Mattingley, & Bradshaw, 2008; Longo & Lourenco, 2007a; Zorzi, Priftis, & Umiltà, 2002; for reviews see Hubbard, Piazza, Pinel, & Dehaene, 2005; de Hevia, Valler, & Girelli, 2008; Umiltà, Priftis, & Zorzi, 2009). For example, Zorzi and colleagues (2002) asked patients with left hemi-spatial neglect following a posterior parietal lesion to ‘bisect’ numerical intervals by indicating the number midway between two others, without overtly computing the correct answer. These patients showed a ‘rightward’ bias when bisecting number pairs (i.e., overestimating the true midpoint of the numerical interval), analogous to their rightward bias when bisecting physical lines. More recently, Pia and colleagues (2009) reported a patient with right neglect following a left hemisphere stroke who showed a leftward bias both when bisecting physical lines and numbers. Similarly, the small leftward bias (*pseudoneglect*) found in healthy adults on line bisection (Bowers & Heilman, 1980; Jewell & McCourt, 2000) also appears for number bisection (Göbel, et al., 2006; Loftus, Nicholls, Mattingley, & Bradshaw, 2008; Loftus et al., 2008, 2009; Longo & Lourenco, 2007a; Lourenco & Longo, 2009b), and, importantly, is correlated across the two tasks (Longo & Lourenco, 2007a). Together, these data suggest that common mechanisms, likely in posterior parietal regions, underlie directional attention in physical and numerical space.

Recently, Loetscher, Schwarz, Schubiger, and Brugger (2008b) found that turning one’s head to the left or right modulated random number generation; specifically, right head turning
Numerical Attention and Distance

led to larger numbers being generated more frequently than did left head turning. These results provide an intriguing demonstration of a relation between attention to numerical information and the position and orientation of body parts with respect to each other. However, it is unknown whether, in addition to the internal spatial **configuration** of the body, attention along the mental number line is also affected by the external spatial **relations** of the body to objects in the world. Numerous studies have found that lateral attention in physical space is different in the near space immediately surrounding the body than in more distant, far space. When participants bisect lines with a laser pointer in near space, the same leftward bias found on paper-and-pencil tasks is observed; as lines are moved farther away, however, this bias gradually shifts rightward (Longo & Lourenco, 2006, 2007b; Varnava, McCarthy, & Beaumont, 2002). Here we tested whether viewing distance also affects spatial attention to mental representations of number, producing rightward shifts in attentional bias with increasing distance. To the extent that common mechanisms of spatial attention operate along physical lines as well as the mental number line, modulation of lateral attention by presenting stimuli in near or far space should produce comparable modulation of bisection biases for physical lines and the mental representation of numbers.

Participants bisected physical lines and number pairs presented at four distances between 60 and 240 cm. We quantified the effect of distance on spatial attention by regressing rightward bias on distance for physical line bisection and number bisection for each participant. The y-intercept of these regressions provides a measure of lateral attentional bias at hypothetical distance 0 cm, analogous to paper-and-pencil responses; the slope of these regressions provides a measure of how bias changes with increasing distance. Thus, pseudoneglect should manifest itself as a negative intercept in each task, and a rightward shift with increasing distance should
manifest itself as a positive slope. Furthermore, there is strong test-retest correlation of these slopes for physical line bisection (Longo & Lourenco, 2007b) indicating consistent individual differences not only in lateral attentional biases (cf. Levy, Heller, Banich, & Burton, 1983), but in their modulation by viewing distance. Thus, if the transition from near to far space has a common effect on both tasks, the rightward shift with increasing distance as indexed by the regression slopes should be correlated as well.

Experiment 1

Methods

Twenty-five students at Emory University (15 female), between 18 and 23 years, participated. All participants had normal or corrected-to-normal vision. Participants were on average right handed as assessed by the Edinburgh Inventory ($M$: 63.65, range: -68.42 – 100), and received payment or course credit for their participation. Procedures were approved by the local ethics committee.

Participants bisected physical lines and number pairs in sequential blocks, counterbalanced across participants. Physical lines were printed on legal-size (8.5” x 14”) sheets of white paper, and measured 1 mm in height and either 10, 20, or 30 cm in length. Sheets were suspended from the wall with a pair of paper clips, at a height of 145.3 cm. Participants bisected 60 lines, five of each length at each of four distances (60, 120, 180, and 240 cm). A laser pointer, constantly activated, was attached to the head of a tripod, with the height of the tripod adjusted for each participant’s comfort. On all trials, the tripod was positioned to the participant’s right and equally far from the wall as his/her feet. Participants used their right hand to move the head of the tripod to bisect the line with the laser beam. Responses were marked by an experimenter (blind to the hypotheses of the study), who, until then, remained behind the
participant. Two coders measured bisection responses off-line, never disagreeing by more than 0.25 mm. Mean percent deviations were calculated for each participant at each distance in each condition.

Participants were instructed to bisect each numerical interval by estimating the number midway between the two presented numbers, without explicitly computing the answer. Number pairs were presented on a computer monitor using a custom MATLAB script (Mathworks, Natick, MA). The font size of the numbers was increased in proportion to increases in distance, such that visual angle size was held constant across conditions (0.645° in height). Participants made untimed verbal responses, but were instructed to go quickly, giving the answer that seemed immediately intuitive. There were 128 trials, in four blocks of 32 trials each. Within each block, there were eight trials at each distance, counterbalanced for left/right position of the smaller and larger numbers in each pair.

Number pairs were generated by randomly selecting numbers between 11 and 99, with the constraint that the interval between numbers be 11 or greater and not a multiple of 10. The same 128 number pairs were used for all participants, but were randomly assigned to the different distances for each participant. The range of smaller numbers was 11 – 83, and for larger numbers it was 28 – 98. The interval size between the numbers was, on average, 33.7 (range: 11 – 75).

Results and Discussion

We quantified effects of distance on spatial attention by regressing rightward bias on distance for physical lines and number pairs for each participant. As indicated above, the intercept of these analyses provides a measure of attentional bias at hypothetical distance zero and the slope provides a measure of change in bias with increasing distance.
Figure 1 shows the raw data for physical line (Figure 1a) and number (Figure 1b) bisection. Analyses of intercepts show overall leftward biases (pseudoneglect) for physical line bisection ($M$: -1.00 mm), $t(24) = -2.23, p < .05, d = .45$, and for number bisection ($M$: -1.66), $t(24) = -6.29, p < .0001, d = 1.26$, consistent with previous findings (Göbel et al., 2006; Loftus et al., 2009; Longo & Lourenco, 2007a; Lourenco & Longo, 2009b). These biases were correlated across physical line and number bisection, $r(24) = .63, p < .001$ (see Figure 2a), replicating our previous finding of consistent individual differences in bias across these tasks (Longo & Lourenco, 2007a).

Of particular interest here is whether shifts in attentional bias, well documented for physical line bisection (Longo & Lourenco, 2006, 2007b; Lourenco & Longo, 2009a; McCourt & Garlinghouse, 2000; Varnava et al., 2002), also occur for number bisection. Consistent with previous results, bias for bisection of physical lines showed a significant rightward shift with distance, mean $\beta = .81$ mm / meter, $t(24) = 4.18, p < .0005, d = .84$ (see Figure 1a). A similar rightward shift was observed for bisection of number pairs, mean $\beta = .37$ / meter, $t(24) = 2.75, p < .02, d = .55$ (See Figure 1b). Furthermore, there was a significant correlation between the slopes on the two tasks, $r(24) = .57, p < .005$ (see Figure 2b).

We previously found that the extent of pseudoneglect for number pairs increased with the magnitude of the pairs to be bisected (Longo & Lourenco, 2007a; Lourenco & Longo, 2009b). While effects of distance and of numerical magnitude should be independent, since number pairs were randomly assigned to distances for each participant, an additional analysis was conducted to confirm the independence of these effects. We ran multiple linear regressions for each participant, regressing numerical bias simultaneously on both viewing distance and the mean of the bisected numbers. Consistent with the above analyses, significant independent effects were
obtained of both distance, mean $\beta = .36 / \text{meter}$, $t(25) = 2.81$, $p < .01$, $d = .56$, and numerical magnitude, mean $\beta = -.038$, $t(24) = -4.68$, $p < .0001$, $d = .94$.

These results suggest that the perception of stimuli at near vs. far distances has similar effects on lateral attention in physical and numerical space. However, one potential alternate explanation for the apparent effect of viewing distance on numerical bisection concerns the physical size of the digit stimuli. Since angular size was held constant across distance, the actual physical size of digits was larger in far than near space. Given that the physical size of digits is known to affect numerical processing (e.g., Henik & Tzelgov, 1982; Goldfarb & Tzelgov, 2005), it is possible that the rightward shift with distance for number stimuli was an artifact of differences in their physical size, rather than the result of attentional shifts along the mental number line. The strong correlation between the regression slopes for physical and numerical bisection, however, suggests that the effect for numerical bisection does not result from an artifact which would not affect physical line bisection. Nevertheless, to definitively address this issue, we conducted a second experiment in which the physical size of the number digits was held constant across distances.

Experiment 2

Methods

Twenty-six students at Emory University (21 female), between 18 and 24 years, participated. All participants had normal or corrected-to-normal vision. Participants were on average right handed as assessed by the Edinburgh Inventory ($M$: 70.96, range: -55.50 – 100), and received course credit for their participation. Data from an additional participant was excluded as responses on an unacceptably large proportion of trials (19%) were outside the range of the numbers to be bisected.
Methods were identical to Experiment 1 with three exceptions. First, only the numerical bisection task was administered. Second, the physical size of the number stimuli was held constant (16 mm in height) across distances. Third, rather than each participant receiving the same 128 number pairs, the same 256 numbers were randomized into pairs for each participant to ensure that nothing about the specific set of number pairs contributed to the effect. Across participants, the mean interval size between the numbers was 36.8 (range: 35.3 – 38.4).  

**Results and Discussion**

Figure 3 shows the raw data for numerical bisection. Analysis of intercepts showed a clear overall leftward bias (pseudoneglect; $M$: -2.64), $t(25) = -8.03$, $p < .0001$, $d = 1.57$. As in Experiment 1, a significant rightward bias with viewing distance was found, mean $\beta = .72 \text{ / meter}$, $t(25) = 4.81$, $p < .0001$, $d = .94$, demonstrating that the rightward shift found in Exp 1 was not an artifact of differences in the absolute size of stimuli presented at different distances.

![Insert Figure 3 here]

For completeness, we also conducted a multiple regression analysis regressing bisection bias simultaneously on both viewing distance and the mean of the bisected numbers. Consistent with the above results and Exp 1, significant independent effects were found for both viewing distance, mean $\beta = .45 \text{ / meter}$, $t(25) = 5.03$, $p < .0001$, $d = 1.01$, and numerical magnitude, mean $\beta = -.071$, $t(25) = -6.31$, $p < .0001$, $d = 1.25$.

Thus, across the two experiments, clear rightward shifts in bias for numerical bisection were observed regardless of whether visual angular size (Exp 1) or actual physical size (Exp 2) was held constant across distance, analogous to our previous findings for physical line bisection.
Numerical Attention and Distance

(Longo & Lourenco, 2006; Lourenco & Longo, 2009a). These results demonstrate that viewing distance modulates attention in numerical space independent of known effects of stimulus size on number representation (e.g., Henik & Tzelgov, 1982; Goldfarb & Tzelgov, 2005).

General Discussion

Viewing distance modulates attention in both physical and numerical space. Rightward shifts in bias were observed with increasing viewing distance for bisection both of physical lines and numerical intervals. Furthermore, the magnitude of this shift in the two tasks was correlated across participants, suggesting that common mechanisms underlie the representation of spatial extent, whether of physical lines or the mental number line. These results demonstrate that attention to numerical representations – like attention to physical spatial stimuli – varies as a function of distance. Whereas the findings of Loetscher and colleagues (2008b) show that attention in numerical space is affected by the internal configuration of the body, the present findings show that it is also modulated by the relation between the body and the external world.

What accounts for the shifts in lateral attention in physical and numerical space as a function of viewing distance? The parietal lobes of each hemisphere direct attention contralaterally (Corbetta, Shulman, Miezin, & Petersen, 1995). Nevertheless, models of lateral attention generally propose overall rightward directional bias, either because the rightward orienting tendency of the left parietal lobe is stronger than the leftward orienting tendency of the right parietal lobe (Kinsbourne, 1987; Làdavas, Del Pesce, & Provinciali, 1987), or because the right parietal lobe has an ipsilateral orienting tendency in addition to its (stronger) contralateral tendency (Mesulam, 1981). Several studies have found that processing information in near space is related to areas in the right parietal lobe (e.g., Bjoertomt, Cowey, & Walsh, 2002; Fierro et al., 2000; Fink et al., 2000; Previc, 1998), specifically the angular and supramarginal gyri.
Numerical Attention and Distance

(Bjoertomt, Cowey, & Walsh, 2009). Thus, the more strongly near space representations are activated, the greater the resulting leftward shift in attention. We suggest that this accounts for the leftward bias observed in near space, when physical lines are closer to the viewer (Longo & Lourenco, 2006; Varnava et al., 2002). On this account, pseudoneglect would result from the leftward attentional bias induced by the stimulus being in near space, which is, on average, slightly stronger than the baseline rightward attentional bias. That pseudoneglect would result from a slight asymmetry in the magnitude of two spatial attentional biases, which may vary independently across participants, could also account in large part for the variability seen across participants (cf. Jewell & McCourt, 2000).

There are three basic effects seen for physical line bisection in near and far space: (1) a slight leftward bias in near space (pseudoneglect), (2) a rightward shift in bias with increasing distance, and (3) an overall rightward bias in far space. The present results are consistent with several recent studies in showing pseudoneglect for number bisection in near space (Göbel et al., 2006; Loftus et al., 2008, 2009; Longo & Lourenco, 2007a; Lourenco & Longo, 2009b), and replicate the correlation between pseudoneglect for number pairs and physical lines in near space we observed previously (Longo & Lourenco, 2007a). Most importantly, the novel contribution of this study is to show that the modulation of attentional bias as a function of distance occurs for numerical space as well (i.e., ‘rightward’ shifts in bias associated with increases in viewing distance). Furthermore, the magnitude of this effect in physical and numerical space is significantly correlated across participants. Thus, not only are baseline biases on paper-and-pencil tasks correlated in physical space and the mental number line (Longo & Lourenco, 2007a), a common modulation of these biases is also induced depending on the location of
Numerical Attention and Distance

stimulus presentation. These findings provide dramatic support for the proposal that common attentional mechanisms operate in physical space and for the mental number line.

Several studies have shown the perceiving magnitude in one dimension (e.g., numerosity, size, duration) can alter representations in other dimensions (e.g., Casasanto & Boroditsky, 2008; Cohen, Hansel, & Sylvestre, 1953; Oliveri et al., 2008; Xuan, Zhang, He, & Chen, 2007). Thus, it is possible that perceiving stimuli at longer distances could prime some generic representation of ‘more’ and hence prime larger numerical responses. While the present data do not directly rule out this hypothesis, the strong correlation between the rightward slopes seen for the two tasks argues against this interpretation, given that it would apply only to the numerical task.

One difference between physical and numerical space is that the overall rightward bias in far space seen for physical lines was not observed for number bisection, in which leftward biases were observed at all distances. This is consistent with previous observations that pseudoneglect is more robust for numbers than for physical lines (Göbel et al., 2006; Longo & Lourenco, 2007a). For example, we previously (Longo & Lourenco, 2007a) found that although biases for paper-and-pencil physical and number bisection were strongly related, participants were significantly more likely to show an overall leftward bias for number bisection than physical line bisection. Such findings are consistent with recent claims that in addition to striking similarities, there are also dissociations between directional attention as it relates to physical lines and the mental representation of numbers (e.g., Doricchi, Guariglia, Gasparini, & Tomaiuolo, 2005; Loetscher & Brugger, 2007), which, at least under some accounts, could result from additional right posterior parietal cortex recruitment during numerical processing (e.g., Chochon, Cohen, van de Moortele, & Dehaene, 1999; Knops, Nuerk, Sparing, Foltys, & Willmes, 2006; Le Clec’H et al., 2000).
Another possibility for the lack of an overall rightward bias for number bisection is that eye movements may affect performance on physical line bisection. Pseudoneglect is stronger and more robust for brief tachistoscopic presentation of pre-bisected lines (landmark task) than for manual bisection (Jewell & McCourt, 2000). Furthermore, while manual bisection shows a clear slight-left to larger-right bias as stimuli are moved from near to far space (this study, Longo & Lourenco, 2006, 2007b; Lourenco & Longo, 2009a; Varnava et al., 2002), landmark judgments appear to show a transition from a larger-left bias to a smaller left bias (Bjoertomt et al., 2002; McCourt & Garlinghouse, 2000). This pattern suggests that scanning of lines via eye-movements produces an overall rightward shift, apparently independent of viewing distance, consistent with known effects of scanning on bisection biases (e.g., Chokron et al., 1998; McCourt & Olafson, 1997). While number bisection is known to induce eye movements (Loetscher et al., 2008a), these are uninformative about the true midpoint, unlike physical line bisection in which they may be informative. Indeed, the pattern we observed in the present study for number bisection (strong left bias in near space, smaller left bias in far space) mirrors previous findings from tachistoscopic landmark tasks.

The distinction between near and far space has long been known to affect attention in physical space (e.g., Berti & Frassinetti, 2000; Cowey, Small, & Ellis, 1994; Halligan & Marshall, 1991; Longo & Lourenco, 2006). The present results demonstrate that this distinction also affects attention along the mental number line. This is consistent with recent results of Zäch and Brugger (2008) who found that participants perceived time moving faster on clocks imagined to be in far than near space. Such findings suggest that the distinction between near and far space may be relevant to a broad range of mental representations. Previous findings have shown that numerical cognition involves spatial representations. The present results reveal a
more fundamental relation than has heretofore been described, showing that numerical cognition is also modulated by our on-line spatial *interactions* in the world.
References


Cowey, A., Small, M., & Ellis, S. (1994). Left visuo-spatial neglect can be worse in far than in


Acknowledgments

The authors would like to thank Daniel Chen, Shaina Gordon, Shazeen Hussaini, Austin Sena, Sharmin Shariff, Kirsten Skillrud, Sonia Rao, and, especially, Dede Addy, Edmund Fernandez, and Paul Pfeilschiefter for help with recruiting and testing of participants as well as with coding data in these experiments.
Footnotes

1) This procedure resulted in some trials in which the interval size between the numbers to be bisected was very small. Trials in which the interval size was less than 11 were removed from analyses, as such intervals were not used in Exp 1 nor in our previous studies (Longo & Lourenco, 2007a; Lourenco & Longo, 2009b) given obvious concerns about ceiling effects on performance. On average, 11.4% of trials were thus removed (range: 7.0 – 15.6%).
Figure Captions

Figure 1: Effect of viewing distance on physical line bisection (left panel) and numerical bisection (right panel) in Experiment 1. Bias for physical lines indicates the mean distance (in mm) between the true midpoint and participants’ response; positive values indicate rightward bias, negative values leftward bias (pseudoneglect). Bias for numerical bisection indicates the mean difference between participants’ responses and the true mean of the number pairs to be bisected; positive values indicate overestimation, negative values underestimation (pseudoneglect). Error bars are one SEM.

Figure 2: Scatterplots showing relations between least-squares regression parameters, y-intercepts (left panel) and slopes (right panel), for physical line bisection and numerical bisection in Experiment 1.

Figure 3: Effect of viewing distance on numerical bisection in Experiment 2. Error bars are one SEM.