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Abstract

Numerous studies have examined the effect on credit spreads of renegotiation. These studies have generally focussed on the impact on spread levels in general, and not on how renegotiation influences the relative pricing of senior versus junior debt claims. In this paper, we show that the scope for sequential renegotiation may reduce and even eliminate the premium for debt seniority. Our analysis also explains why companies may engage in repeated Chapter 11 bankruptcy filings (a phenomenon commonly referred to as Chapter 22).

Keywords: real options, bankruptcy, debt service, absolute priority, bargaining (JEL: G13, G33, G34)

Introduction

Influential studies by Franks and Torous (1989), (1994) and Asquith, Gertner and Scharfstein (1990) demonstrated, by detailed analysis of the experience of distressed companies, how the renegotiation of debt claims, either in formal bankruptcy proceedings like Chapter 11 or in restructurings outside formal bankruptcy, may lead to deviations from absolute priority.

These insights stimulated a series of theoretical papers studying how credit spreads are affected when equity-holders can (i) make take-it-or-leave-it offers to reduce the contractually agreed coupon (Anderson and Sundaresan (1997), Mella-Barral and Perraudin (1998)), (ii) make take-it-or-leave-it offers to write off debt principal (Mella-Barral (1998)), or (iii) bargain on the coupon (Fan and Sundaresan (2000)).[1] All of these papers focuss on conflicts of interest between equity- and debt-holders and resulting Absolute Priority Violations (APV) that benefit equity-holders.

Recent evidence suggests that conflicts of interest between different debtor classes may have become relatively more important. In particular, there seems to have been a 'secular decline' in the frequency of equity-related APV. Bharath, Panchapegesan and Werner (2007) find the frequency of APV in favor of equity holders has declined from 22% in the period $1991 - 2005$ to 9% during $2000 - 2005$. Also Bris, Welch and Zhu (2006) find fewer APR violations (12%) compared to earlier studies (their dataset includes Chapter 11 and Chapter 7 cases during the period $1995 - 2001$). The same figure on APV (12%) is found in Ayotte and Morrison (2009). At the same time, the

[1] Our paper is a contribution to a large literature in which the pricing implications of different types of strategic behavior by parties to debt contracts are examined. Relevant studies apart from those already mentioned include Leland (1994), Leland and Toft (1996), Hege and Mella-Barral (2000), Lambrecht (2001), Hege and Mella-Barral (2005), Acharya, Huang, Subrahmanyam, and Sundaram (2006), Hennessy, Hackbarth and Leland (2007), Broadie, Chernov and Sundaresan (2007) and Davydenko and Strebulaev (2007).

output of restructuring seems to be more and more driven by conflicts amongst senior and junior creditors rather than creditors versus equity holders (again, see Ayotte and Morrison (2009)).

In this paper, we provide a theoretical analysis of debt-holder conflicts and how they affect valuation and restructuring within a regime that allows for Chapter 11 bankruptcies. We show how the possibility of renegotiating the claims of different classes of debt sequentially may: (i) limit the effectiveness of seniority provisions by excluding senior creditors from renegotiations, (ii) reverse the relative positions of senior and junior debt-holders, even leading to situations in which the credit spreads on senior debt exceed those on junior debt issued by the same firm and (iii) still lead to cases of APV when the equity holders have sufficient bargaining power.

In modeling bankruptcy, we aim to match the main features of the Chapter 11 of the US Bankruptcy Code. We follow Brown (1989) who emphasizes that in a Chapter 11 filing those claim-holders left 'unimpaired' by a plan lose their veto power. These provisions greatly enhance the equity-holders' strategic behavior in bankruptcy in that they may renegotiate with one creditor at a time in a 'private renegotiation' while excluding unimpaired creditors.

This possibility has strong empirical support. LoPucki and Whitford (1993) and LoPucki (2004) find that senior secured classes are typically left unimpaired or, when impaired, the impairment involves all classes. In the dataset used by LoPucki and Whitford (1993) we could analyse the treatment of each class for 31 Chapter 11 cases. Out of 31 cases, in 19 cases, at least one creditor's class was unimpaired, most frequently a secured class.

The framework we employ is a model of a levered firm with two classes of perpetual debt. In bankruptcy, equity-holders may sequentially bargain with holders of the two

debt classes. Our analysis yields a unique equilibrium, the nature of which depends on the parameters of the problem, but in which firms restructure their debt twice.

This pattern of restructuring thresholds explains why companies emerging from bankruptcy often re-enter Chapter 11 within a few years, a phenomenon commonly referred to as "Chapter 22". According to Gilson (1997) and Hotchkiss (1995) respectively, 25 percent and 32 percent of firms file for bankruptcy or restructure their debts a second time. LoPucki's Bankruptcy Research Database (BRD) shows that companies emerging from Chapter 11 (between $2000 - 2005$) refiled within 5 years in 25.23% of the cases.

This puzzling empirical regularity has been attributed to different factors. Hotchkiss (1995) links Chapter 22 to inefficiencies in the Bankruptcy Code while Aggarval (1995) suggests as explanation inefficiencies in renegotiation due to coordination problems. Kahl (2002) attributes Chapter 22 to incomplete information, suggesting the firm's viability is imperfectly known at the first bankruptcy and that a second restructuring helps creditors to make better informed decisions.

Unlike these authors, our explanation of the Chapter 22 phenomenon relies on structural aspects of Chapter 11. In line with Brown (1989), who recognizes the importance of veto power in restructuring, our first restructuring is filed by the equity holders in a strategic fashion. When bankruptcy occurs a second time, the firm value is lower, reducing the liquidation threat of 'strong' creditors who will then make more concessions.

In particular, for reasonable levels of bankruptcy costs (not excessively high) and junior debt face value (not negligibly small), the senior debt will be unimpaired at the first bankruptcy threshold and will be restructured only if the firm value falls sufficiently, triggering a second bankruptcy proceeding. As mentioned, such equilibria

are rather common in Chapter 11 reorganizations where, typically, senior secured classes are left unimpaired or, when impaired, the impairment involves all classes (see LoPucki and Whitford (1993) and LoPucki (2004)).

Furthermore, such equilibria may entail greater deviations from Absolute Priority. The intuitive explanation is that renegotiation with the senior creditor is delayed until the firm value is low and hence seniority provisions are less valuable.

The paper is organized as follows. Section I described features of Chapter 11 that we aim to capture in our formal modeling. Section II describes the model. Section III looks at implications for credit spreads. Section IV concludes.

I Key Features of Chapter 11

In this section, we set out important aspects of Chapter 11 that have the potential to influence the allocation of value among securities holders. An alternative, exhaustive discussion of the Chapter 11 rules is provided by Kordana and Posner (1999).

1. **Timing of bankruptcy.** The timing of a Chapter 11 bankruptcy is generally determined by equity-holders who in almost all cases initiate the process. Chapter 11 allows for involuntary bankruptcy too. However, the Court will dismiss a creditors's bankruptcy petition if the debtor has not failed to pay its debts when due.[2]

2. **First proposal.** Equity-holders have the exclusive right to propose a first reorganization plan.[3] The shareholders' right of first proposal lasts 120 days (plus 60 days for securing acceptance of the plan) and can be extended by the

[2]Bankruptcy Code, Section 303, Paragraph (h)(1).
[3]Bankruptcy Code, Section 1121.

Court. The debtors' ability to extend the exclusivity period has been generally used as leverage in the negotiating process. Under the 2005 Act,[4] the exclusivity period cannot be extended beyond a maximum of 18 months.[5]

3. **Acceptance of plan**. A plan must be approved by each impaired class (for these purposes, equity-holders are considered to be always impaired), with approval by at least two-thirds in amount and more than one-half in number within each class.[6]

4. **Impairment rules.** A creditor cannot reject a plan if he or she receives cash equal to the face value of his or her claim or if the plan calls for no scaling down of the coupon payment scheduled in the existing contract. In this case, the creditor is said to be unimpaired[7] by the plan and loses veto power.[8] The 'impairment' rule allows equity holders to negotiate with one creditor at a time.

5. **Cram down.** Chapter 11 allows for confirmation by the Court of non-consensual plans –so called 'crammed-down' plans. That is, at the request of the plan's proponent, the Court may confirm a plan in spite of rejection by some impaired class as long as: i) at least one impaired class has accepted the plan and ii) impaired rejecting classes receive at least what they would receive in a Chapter 7 liquidation.[9] As reported by Lopucki and Whitford (1990), restructuring rarely

[4]The Bankruptcy Code has been amended a number of times, with most recent amendments set out with "The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005" (shortly, the 2005 Act) effective since October 17, 2005.

[5]Bankruptcy Code, Section 1121, Paragraph (d)(2).

[6]Bankruptcy Code, Section 1126 Paragraph (c)

[7]Bankruptcy Code, Section 1124(1). "A class of claims or interests is impaired under a plan unless, with respect to each claim or interest of such class, the plan leaves unaltered the legal, equitable, and contractual rights to which such claim or interest entitles the holder of such claim or interest".

[8]Bankruptcy Code, Section 1126.

[9]Bankruptcy Code, Section 1129, Paragraph (b)(1)).

involves non-consensual plans, however.[10]

6. **Subsequent proposals.** There are no specific agenda rules concerning subsequent proposals in the bankruptcy code. Once the equity holders' proposal has been rejected, the rules become unbiased towards different players. In Chapter 11, when multiple plans are filed and accepted by the voting classes, the Court shall decide which plan to confirm on the grounds of 'the preferences of creditors and equity security holders'.[11] A certain degree of discretion is granted at this stage and the outcome of the negotiation may depend on the ability of the players to propose reorganization plans and influence the court.

7. **Liquidation.** If no agreement is reached the case is converted into Chapter 7 and the company is liquidated. In Chapter 7, APR is always followed. In a study by Bris, Welch and Zhu (2006) in about half of their Chapter 7 liquidations (30 cases), secured creditors receive nothing and unsecured creditors receive nothing in 95% of the cases.[12]

[10] In a sample of 43 Chapter 11 reorganizations, they find that in no case was a plan confirmed without approval of all debt classes. Not even seems cram-down to be a strategic threat to speed up acceptance of a plan. According to Lopucki and Whitford (1990), bankruptcy practitioners would rarely suggest a cram-down strategy. Rather than for strategic reason, they argue that cram-down is accounted for in Chapter 11 because when a class receives nothing under a plan, that class is deemed to reject –even if no actual disagreement arises– and no vote is cast for that class (Bankruptcy Code, Section 1126, Paragraph (g)). Thus cram-down is an expedient to avoid non-confirmation of plans which are unanimously agreed on.

[11] Bankruptcy Code, Section 1129, Paragraph (c).

[12] Their study reports APV in Chapter 11 but not in Chapter 7 because, as they state, in Chapter 7 APR is always followed.

II The Model

A The Form of the Game

Before describing our assumptions and model mathematically, it may assist the reader if we describe the basic form of the game we formulate. Recall that Chapter 11 rules imply that, in an initial period, equity-holders make an offer which is adopted if it is accepted by every impaired class. If an impaired class rejects the offer, any class may file a plan. If no plan is accepted, the firm is liquidated and claim-holders are paid using the proceeds.

In our game theoretic representation of Chapter 11, equity-holders make a take-it-or-leave it offer to creditors of a reduction in debt service. If creditors reject this, cooperative bargaining ensues in which any party may propose a plan. If agreement is not reached in the cooperative bargaining, the firm experiences a value-reducing transfer of ownership and the stake-holders are paid in strict order of priority. Figure 1 shows the extensive form representation of the game between different stake-holder groups.

As we demonstrate below, there is a unique equilibrium in our game in which the equity-holders' offer is accepted. This offer effectively allocates value between stake-holders in a way that is influenced by the cooperative bargaining that occurs if the offer is rejected and the cooperative bargaining allocation is itself influenced by the payoffs that stake-holders would obtain if the firm were liquidated through scrapping or value-reducing transfer of ownership.

B Assumptions

Our basic modeling assumptions are as follows.

Assumption 1

1. *Agents are risk neutral and may freely borrow and lend at a constant interest rate, r.*

2. *The firm consists of a claim to a cash flow process, p_t, which follows a geometric Brownian motion:*

$$dp_t = \mu p_t dt + \sigma p_t dW_t \ .$$

3. *The firm has issued two classes of perpetual debt consisting of: a senior class paying a coupon b_s and with a face value of $F_s = b_s/r$ and a junior class paying b_j with face value $F_j = b_j/r$. We also assume that*[13] *$F_s > \gamma$.*

4. *At any time, the firm may be scrapped through a piecemeal liquidation that yields a monetary amount γ.*

Let V_t denote the total firm value. Our model contains no frictions and hence the sum of claim values will always equal total firm value. We abstract from frictions because our focus is on how renegotiation affects the split of value between different claim-holders. Standard arguments imply:

$$\begin{aligned} V(p_t, \underline{p}) &= \frac{p_t}{r-\mu} + \left(\gamma - \frac{\underline{p}}{r-\mu}\right)\left(\frac{p_t}{\underline{p}}\right)^{\lambda} \qquad (1) \\ \text{where} \quad \lambda &= \frac{-(\mu - \sigma^2/2) - \sqrt{(\mu - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2} \ , \end{aligned}$$

[13] If $F_s \leq \gamma$ the senior creditor is fully secured at any level of the cash flow p_t and the senior creditor will never be restructured in bankruptcy. This assumption allows us to focus on cases where also the senior creditor is restructured in bankruptcy (see footnote in Proposition 1).

where $\underline{p}$ is a trigger level at which the firm is scrapped. The total value of the firm is maximized if the firm is scrapped when p_t first hits the trigger level:

$$\underline{p} = \frac{\lambda}{\lambda - 1}\gamma(r - \mu). \tag{2}$$

In some circumstances, we will consider cases in which the the firm is sold as a going concern prior to a piecemeal liquidation or scrapping. In this case, we suppose that new equity-holders operate the firm on a pure equity basis without issuing new debt. The sale price available through such a "going-concern disposal," denoted $V_L(p_t)$, is assumed to equal:

$$V_L(p_t) = \alpha V(p_t) + (1 - \alpha)\gamma \ . \tag{3}$$

The assumption that liquidation occurs via such a sale of assets is in the spirit of Mella-Barral (1999). The difference $V(p_t) - V_L(p_t)$ captures the implicit cost of a transfer-of-ownership liquidation. The reason is that a liquidation sale might involve a partial dismantlement of the technology, with, for instance, possible "loss of human capital, know-how, and competitive edge," Mella-Barral (1999). Furthermore, the above specification resembles that of Leland (1994) and Leland and Toft (1996)[14] and equals their formulation when the scrapping value, γ, equals zero. It is also in line with Mella-Barral and Perraudin (1997) in that the value obtained when ownership changes is always below the maximum firm value $V(p_t)$ but never below the scrapping value of the firm. Their bankruptcy costs become zero as the state variable, p_t, approaches the optimal shutting down trigger, which is also true in our case.[15]

[14]Their formulation is $V_L = \alpha V$ in our notation.

[15]The difference $V(p_t) - V_L(p_t) = (1 - \alpha)(V(p_t) - \gamma)$, which may be thought of as bankruptcy costs, converges to zero when p_t tends to the optimal shutting down trigger $\underline{p}$.

Note that, in the game we formulate below, value-reducing transfers of ownership of the type just described only occur along "off-equilibrium" paths. Hence, while they influence the allocation of value through affecting the "outside offers" that creditors obtain if bargaining breaks down, they do not influence total firm value.

Our assumptions regarding bankruptcy may be listed as:

Assumption 2

1. *Equity-holders precipitate bankruptcy*[16] *by ceasing to meet operating losses, including coupon payments, by further equity injections.*[17]

2. *For simplicity, we treat each class of claimants as a single agent. This precludes holdout problems.*

3. *Filing a plan is costless.*[18]

4. *A restructuring plan consists of:*

   (i) *An allocation of value $V(p_t)$ for each claim-holder group for each level of the state variable. It is implemented by a set of variable instantaneous coupon payment functions $b_i(p_t) \leq b_i$ with $i = s, j$. The letters 's' and 'j' denote the senior and junior claim-holder respectively. The coupon payment functions are consistent with the value allocations.*[19]

[16]We abstract from agency conflicts between managers and shareholders. See Lambrecht and Myers (2008) for an analysis of default decision with rent maximizing managers. However, their study focusses on optimal capital structure with one layer of (safe or risky) debt, while ours focusses on pricing two layers of debt with exogenous capital structure.

[17]Over the period 2000-2010, 97.23% of large, public company bankruptcy cases which have been confirmed were voluntary cases (see LoPucki's Bankruptcy Research Dataset). Also, the assumption that firms enter bankruptcy voluntarily follows the approach of many recent studies including Leland (1994) and Mella-Barral and Perraudin (1997).

[18]One could in principle introduce exogenous costs for filing a plan following Brown; but it would complicate the analysis without changing the basic thrust of our argument.

[19]We focus on plans involving reductions in coupon payments to one or both creditor classes. This

(ii) Creditors receiving $b_i(p_t) < b_i$ (for $i = j, s$) are impaired under the plan and are allowed to vote and reject the plan. Classes in receipt of their contractually specified coupon payment, b_i, are referred to as unimpaired by the plan and are deemed to have accepted the plan.

5. *Equity holders are always treated as impaired and allowed to vote on a plan.*

6. *The equity-holders propose the first restructuring plan by making a take-it or leave-it offer to creditor.*

7. *If the plan initially proposed by the equity-holders is rejected by at least one impaired class, without time delay, the restructuring process moves into a second stage where a set of plans are simultaneously voted on by impaired stake-holders. Each plan is the outcome of Nash bargaining amongst impaired classes with disagreement payoffs equal to their liquidation payoffs.*

8. *If in the second stage no plan receives unanimous approval, the firm is liquidated.*[20]

9. *Absolute priority holds in liquidation.*

The second stage of restructuring in our model with voting on Nash bargaining allocations captures two essential features of chapter 11. First, as argued by legal scholars, in Chapter 11 judicial discretion is granted in many circumstances (see Schwartz (2002)) resulting in a 'refereed' bargaining system. We allow for exogenous asymmetries between parties (which might reflect the ability of claim-holders to influence the

is in line with Perraudin and Mella-Barral (1997). Alternative approaches to implementing a plan include permanent reduction of face value, as examined in Mella-Barral (1999), or debt-equity swaps such as those employed in Fan and Sundaresan (2000).

[20]We implicitly assume that only consensual plans (i.e. plans accepted by impaired creditors) are confirmed, thereby abstracting from cram down possibilities.

court) by using a Nash asymmetric bargaining.[21]

Second, rather than letting all players bargain together in a Nash cooperative framework, we emphasize the importance of the impairment rule during the entire restructuring process, which allows for the exclusion of a class from the bargaining table.

Note that there are many different ways of modeling the second stage of restructuring. Brown (1989) assumes that only a finite number of proposals is allowed and the bankruptcy court determines the order in which proposals can be voted. Each proposal might rank first, second, or third in the agenda with same probability.

C Bargaining

Throughout this section we assume that the firm has entered bankruptcy. In the next section, we analyse the optimal choice of the equity holder to enter bankruptcy. Moreover, we restrict the analysis of this section to the case when the firm is in bankruptcy at a level of the cashflow p_t such that $V_L(p_t) \leq F_s$. We discuss in the Appendix the case in which $V_L(p_t) > F_s$.[22]

We begin by solving the cooperative game. Assumptions 2.4, 2.5 and 2.7 imply that there are three possible bargaining allocations depending on whether agents are unimpaired and hence excluded from bargaining. These are:

(i) All stake-holders are impaired and each class receives a Nash bargaining share

[21] According to Welch (1997), such asymmetry can be explained as a reflection of different organization skills and reputations. Welch argues that a bank, unlike bond-holders, may benefit from having a reputation for "tough behavior," which may discourage other borrowers from attempting opportunistic renegotiation. Our Nash axiomatic approach is also in line with Fan and Sundaresan (2000) and with Aivazian and Callen (1983) who argue that a cooperative approach is the best representation of formal renegotiation supervised by the Court.

[22] In this case, as we show in Appendix, after a first restructuring takes place, the new equilibrium is such that the firm is always restructured when $V_L(p_t) = F_s$, which reconnects to the case discussed here.

of the assets denoted by P_e, P_j and P_s for the equity, junior and senior class respectively;

(ii) The junior class is impaired (along with the equity holders -always impaired) and the senior is unimpaired; the equity holder and the junior class receive a Nash bargaining share, denoted as $P_{e,j}$, $P_{j,e}$ while the senior class continues receiving the full coupon b_s with claim value denoted by S.

(iii) The senior class is impaired (along with the equity holders) and the junior is unimpaired; the equity holder and the senior class receive a Nash bargaining share, denoted as $P_{e,s}$, $P_{s,e}$ while the junior class continues receiving the full coupon b_j with claim value denoted by J.

In what follows, we (i) formalize the three different types of Nash bargaining allocations and (ii) analyze how agents will vote between these three possibilities.

If the state variable p_t falls far enough, we expect that in equilibrium, even if the plan adopted is type (ii) (only junior initially impaired) or type (iii) (only senior initially impaired), eventually both debt classes will be impaired.[23] Let p_s denote the level of the state variable at which senior debt is impaired in a type (ii) plan, and p_j denotes the trigger level for junior impairment in a type (i) plan.

Let the *absolute* bargaining power of class $i = e, j, s$ be denoted x_i . When one class is unimpaired, the *relative* bargaining powers are $\eta_i = x_i/(x_i+x_e)$ for $i = j, s$ and $1 - \eta_i$ for the equity holders. When all classes are impaired, the relative bargaining powers are: $\xi_i = x_i/(x_e + x_j + x_s)$ with $i = e, j, s$.

The allocations received by the security-holder groups under the three possible

[23]We expect both classes to be impaired ultimately as p_t falls since, by assumption, $F_s > \gamma$, and hence as p_t moves towards $\underline{p}$ (and, therefore, $V(p_t)$ approaches the scrap value γ) the level of cash flow is insufficient to continue paying the full contractual coupon to either creditor class.

plans may be expressed as follows:

1. Equity and junior debt impaired and senior unimpaired, with allocation:

$$\begin{aligned} \textit{Equity claim} &= P_{e,j} &\equiv\ & (1-\eta_j)(V(p_t)-S(p_t,p_s)), \\ \textit{Junior debt claim} &= P_{j,e} &\equiv\ & \eta_j(V(p_t)-S(p_t,p_s)), \\ \textit{Senior debt claim} &= S(p_t,p_s) \end{aligned} \tag{4}$$

2. Equity and senior debt impaired and junior unimpaired, with allocation:

$$\begin{aligned} \textit{Equity claim} &= P_{e,s} &\equiv\ & (1-\eta_s)(V(p_t)-J(p_t,p_j)-V_L(p_t)), \\ \textit{Senior debt claim} &= P_{s,e} &\equiv\ & \eta_s(V(p_t)-J(p_t,p_j)-V_L(p_t))+V_L(p_t), \\ \textit{Junior debt claim} &= J(p_t,p_j) \end{aligned} \tag{5}$$

3. Equity and both debt classes impaired, with allocation:

$$\begin{aligned} \textit{Equity claim} &= P_e &\equiv\ & \xi_e(V(p_t)-V_L(p_t)), \\ \textit{Junior debt claim} &= P_j &\equiv\ & \xi_j(V(p_t)-V_L(p_t)), \\ \textit{Senior debt claim} &= P_s &\equiv\ & \xi_s(V(p_t)-V_L(p_t))+V_L(p_t). \end{aligned} \tag{6}$$

To understand which plan is accepted at different levels of the state variable, p_t, it is helpful to calculate the following threshold levels:

$$\begin{aligned} p_s^* &= \arg\max P_{j,e}(p_t,p_s) = \arg\max P_{e,j}(p_t,p_s) \\ &= \arg\min S(p_t,p_s) = \frac{\lambda}{\lambda-1}\frac{F_s-\gamma(1-\alpha_{\xi_s})}{\alpha_{\xi_s}}(r-\mu) \end{aligned} \tag{7}$$

where $\alpha_{\xi_s} \equiv \xi_s(1-\alpha)+\alpha$.

$$\begin{aligned} p_j^* &= \arg\max P_{s,e}(p_t,p_j) = \arg\max P_{e,s}(p_t,p_j) \\ &= \arg\min J(p_t,p_j) = \frac{\lambda}{\lambda-1}\frac{F_j+\gamma\xi_j(1-\alpha)}{\xi_j(1-\alpha)}(r-\mu) \end{aligned} \tag{8}$$

The following lemma reveals how, for different levels of the state variable, p_t, each debt class views the choice between being jointly impaired (along with the other debt class) or being impaired alone.

Lemma 1 *Junior debt holders prefer: i) to be impaired alone rather than jointly with senior debt holders for any $p_t > p_s^*$ and ii) to be jointly impaired with senior creditors for $p_t \leq p_s^*$. Similarly, senior debt holders prefer: i) to be impaired alone rather than jointly with junior debt holders for any $p_t > p_j^*$ and ii) to be jointly impaired with junior creditors for $p_t \leq p_s^*$.*

Proof: See Appendix. □

When the firm is in bankruptcy there are three possible plans the equity holders can vote, $P_{e,j}$, $P_{e,s}$ and P_e. The preferences of the equity holders over the three possible plans are given by the following lemma.

Lemma 2 *If $p_s^* < p_j^*$, for $p_t > p_j^*$, $P_{e,j} > P_{e,s} > P_e$ and for $p_t \in (p_s^*, p_j^*]$, $P_{e,j} > P_{e,s} = P_e$. While if $p_j^* < p_s^*$, for $p_t > p_s^*$, $P_{e,s} > P_{e,j} > P_e$ and for $p_t \in (p_j^*, p_s^*]$, $P_{e,s} > P_{e,j} = P_e$. For $p_t \leq \min\{p_s, p_j\}$ $P_{e,j} = P_{e,s} = P_e$.*

Proof: See Appendix. □

The preferences of debt and equity holders over the different plans derived from Lemmas 1 and 2 are summarized in Table 1 and illustrated in Figure 2. In the figure, we show the value of junior, senior debt (top diagram) and equity values (bottom

diagram) under the different plans when $p_j^* > p_s^*$. In the table, we set out how stake-holder groups rank the different plans for different levels of the state variable, p_t.

Table 1: Securities-holder Voting

Case 1: $p_s^* < p_j^*$									
	$p_t \geq p_j^*$			$p_s^* \leq p_t < p_j^*$			$p_t \leq p_s^*$		
	S	J	E	S	J	E	S	J	E
Senior impaired, junior not	yes	$--$	no	no	$--$	no	no	$--$	no
Junior impaired, senior not	$--$	yes	yes	$--$	yes	yes	$--$	no	no
Both impaired	no	no	no	yes	no	no	yes	yes	yes
Case 2: $p_j^* < p_s^*$									
	$p_t \geq p_s^*$			$p_j^* \leq p_t < p_s^*$			$p_t \leq p_j^*$		
	S	J	E	S	J	E	S	J	E
Senior impaired, junior not	yes	$--$	yes	yes	$--$	yes	no	$--$	no
Junior impaired, senior not	$--$	yes	no	$--$	no	no	$--$	no	no
Both impaired	no	no	no	no	yes	no	yes	yes	yes

In the next sub-section, we combine the implications of Lemmas 1 and 2 as summarized in Table 1 and an analysis of the optimal bankruptcy trigger in order to infer results on equilibrium.

D Equilibrium

In the following proposition, we describe the equilibrium in our model and show that, for a given set of model parameters, it is unique.

Proposition 1 *If $p_s^* < p_j^*$, no restructuring occurs for $p_t > p_{bj}$, the junior debt alone is restructured for $p_t \in [p_s^*, p_{bj}]$ and both debt classes are restructured together when*

$p_t < p_s^*$.[24]

Alternatively, if $p_s^* > p_j^*$, no restructuring occurs for $p_t > p_{bs}$, the senior debt alone is restructured for $p_t \in [p_j^*, p_{bs}]$ and both debt classes are restructured together when $p_t < p_j^*$.

The bankruptcy triggers, p_{bs} and p_{bj}, in the two cases equal:

$$\begin{aligned} p_{bj} &= \lambda/(1-\lambda)\ (\eta_j F_s + F_j)/\eta_j\ (r-\mu) && if \quad p_s^* < p_j^* \\ p_{bs} &= \lambda/(1-\lambda)\ (\eta_s F_j + F_s - \gamma(1-\alpha_{\eta_s}))/\alpha_{\eta_s}(\ r-\mu) && if \quad p_s^* > p_j^* \end{aligned} \tag{9}$$

where $\alpha_{\eta_s} \equiv \eta_s(1-\alpha)+\alpha$. When $p_s^* < p_j^*$, then $p_s^* < p_{bj} < p_j^*$. When, $p_j^* < p_s^*$, then $p_j^* < p_{bs} < p_s^*$.

Proof: Equilibrium consists of (i) a plan that, for a given level of the state variable, p_t, is acceptable to any stake-holders that are impaired at that level of p_t, and (ii) a trigger level for bankruptcy, p_b that maximizes equity value.

From Table 1, it is evident that, if $p_s^* < p_j^*$, in the cooperative game, there is only one plan that, for all levels of the state variable, is accepted by all impaired stake-holders, namely the plan in which junior debt-holders are impaired for $p_s^* < p_t \leq p_j^*$ and both debt-holder groups are impaired for $p_t < p_s^*$. Similarly, when $p_j^* \leq p_s^*$, there is again a single plan that is accepted by all impaired stake-holders, namely the plan in which senior debt-holders are impaired for $p_j^* < p_t \leq p_s^*$ and both debt-holder groups are impaired for $p_t \leq p_j^*$.

By assumption, equity-holders determine the timing of bankruptcy by ceasing to pay the contractual coupon. Solving for the trigger level that maximizes the value of equity in the two cases ($p_s^* < p_j^*$ and $p_j^* < p_s^*$), we obtain the bankruptcy triggers stated in the proposition (for derivations, see the Appendix). □

[24]Note that the assumption $F_s > \gamma$ guarantees that p_s^* is greater than $\underline{p}$.

In the following proposition, we characterize those parameter values which are associated with the two possible outcomes, initial restructuring of the senior debt or initial restructuring of the junior debt.

Proposition 2 *$p_j^* \lesseqgtr p_s^*$ if and only if*

$$\frac{F_s - \gamma}{\alpha_{\xi_s}} \gtreqless \frac{F_j}{\xi_j(1-\alpha)}. \tag{10}$$

The intuition behind this result is simple. When the equity holder precipitate bankruptcy, they would benefit from reducing coupon payments to the creditor whose face value is relatively high compared to the overall face value F. However, the benefit of impairing the creditor with higher face value must be weighed against the 'strength' of that creditor at the negotiating table; that is, the ability to extract a valuable package of concessions in renegotiation. This depends on the liquidation value of the firm, the priority of the claim, and the creditor's bargaining power. The higher α and γ, the bigger the firm liquidation value, which, in turn, strengthens the bargaining position of the senior creditor while weakening that of the junior creditor.

Therefore, when the firm liquidation value is sufficiently high, the equity holders and the junior creditor agree on a plan which leaves the senior creditor unimpaired and share the surplus generated by excluding a strong senior creditor from the negotiating table. The senior creditor is impaired (along with the junior one) only when the liquidation threat of the senior creditor becomes less valuable. This occurs if the state variable falls sufficiently low, at a level p_s^*. This type of equilibrium, which, according to Proposition 2, occurs when $p_s^* < p_j^*$ is depicted in Figure 3 where we illustrate debt and equity values and the state-dependent coupon payments (derived in Appendix) to each debt class.

We discuss the implications of our model in the next section and we conclude this section by highlighting how our results differ from those of Brown (1989).

First, in Brown's study senior classes are always fully secured and (legally as well as economically) unimpaired,[25] which implies that the spread of senior classes is always zero. Unlike Brown, we show that senior creditor can be left unimpaired even if her/his liquidation value is below face value ($V_L < F_s$). This is because the dynamic nature of our model allows us to distinguish between legal un-impairment and economic un-impairment. We can have economic impairment despite legal un-impairment (achieved by unaltering contractual coupon payments).

Second, even if V_L is greater than senior face value (therefore the senior class should be virtually fully secured[26]), not only the senior spread is positive (because $S(p_t)$ is always below F_s) and but also bankruptcy renegotiation contributes to increasing the senior spread.[27] In other words, the senior creditor would be better off if the firm was liquidated. Moreover, and most importantly this entails APV in favor of junior classes.

Third, our model also allows for un-impairment of junior classes (when first bankruptcy occurs). The reason why is that the scope of un-impairment in our paper is broader than in Brown's study. Brown uses un-impairment to limit the payoff to senior classes. We use un-impairment not only to limit payoff to senior creditors (for instance when the firm liquidation value is above senior face value), but also to exclude 'strong' creditors from the negotiating table. Therefore, the set of strategy is expanded compared to Brown, because the strong creditor might be the junior

[25]Brown refers to senior classes as to classes whose liquidation value is sufficient to fully repay the claim value. In our paper seniority is a pre-bankruptcy provision, not endogenous to the bankruptcy/liquidation outcome.

[26]This case should be more directly comparable to Brown's study.

[27]A detailed discussion of the contribution of renegotiation on the senior spread is presented in the next section.

creditor and not necessarily the senior one. Unlike Brown, our model can deliver (depending on parameter values, see Proposition 2) an equilibrium where the junior creditor is left unimpaired when the first bankruptcy occurs and is impaired along with the senior in a second bankruptcy (with declined assets value).

III Credit spreads and priority violation

A Priority and renegotiation premia

In this section, we expand on the implications of the model for the pricing of debt. The key point is that the opportunity to reschedule claims sequentially allows the equity holders to delay renegotiation with the 'strong' creditor until the firm value is sufficiently low and hence the liquidation threat is less effective.

We shall show that the difficulty of enforcing seniority may have a substantial impact on spreads. To this effect, we follow Mella-Barral and Perraudin (1997) in defining the percentage contribution of sequential renegotiation to the senior credit spread as:

$$rp_s = \frac{\min\{V_L, F_s\} - S}{F_s - S} \tag{11}$$

During restructuring, if a creditor can threaten to liquidate, one would expect the renegotiation premium to be negative. That is, the possibility of rescheduling debt should reduce spreads for levels of the state variable for which restructuring occurs.

Our analysis shows that, with sequential renegotiation, this conclusion still holds for the junior creditors. Whether junior or senior debt is rescheduled first, the renegotiation premium to the junior creditor is always negative.[28]

[28] If the junior creditor is left unimpaired, his liquidation payoff at the first bankruptcy threshold p_{bs} is equal to $\max\{V_L - F_s, 0\} = 0$ (because when the senior is impaired first $V_L(p_{bs}) < F_s$).

In contrast, however, if senior claim-holders are left unimpaired in restructuring, by losing their veto power, senior debt-holders lose a valuable outside option. Depending on whether junior creditors or they are first impaired, the renegotiation premium of senior creditors may be positive or negative.

Our results are summarized in Figure 4, which shows the renegotiation premium rp_s (evaluated at a particular level of the state variable, p_t) as a function of the senior debt service flow, b_s. In the figure, b_s varies from 0 to 0.25 while the total coupon payment, $b_s + b_j$, is held constant at 0.25).

B Spread reversals between senior and junior spread

Comparison of the credit spreads of the senior and the junior creditors (i.e., $CS_s = \frac{b_s}{S} - r$ and $CS_j = \frac{b_j}{J} - r$) yields a further interesting implication of our model. In Figure 5, we show the two spreads, as functions of b_s.[29] The spreads are evaluated at the same level of p_t but for three different level of total debt face value, namely $b1 = 0.25$, $b2 = 0.30$ and $b3 = 0.35$. Note that for all three levels of total debt face value, for a high enough level of senior face value, the senior spreads exceed those for junior debt.

One may derive a simple sufficient condition that rules out spread reversals for all levels of the state variable. To do this, we wish to compare the junior spread and the spread of the senior claim when the latter is 'stripped' of the scrapping value γ. We refer to the 'stripped claim' as the senior 'unsecured' claim. One may decompose the

Therefore, as J is positive, the renegotiation premium to the junior creditor is negative (that is, $rp_j = -J/(F_j - J) < 0$). If the junior is impaired first, at p_{bj}, his Nash bargaining share $P_{j,e}$ is always above his liquidation payoff and, again, rp_j is negative.

[29]The parameters employed are the same as those used in Figure 4.

senior spread in the following way:

$$\begin{aligned} CS_s &= r\frac{F_s - S}{S} \\ &= r\frac{(F_s - \gamma) - (S - \gamma)}{S} \cdot \frac{S - \gamma}{S - \gamma} \\ &= r\frac{\hat{F}_s - \hat{S}}{\hat{S}} \cdot \frac{\hat{S}}{S} \ . \end{aligned} \tag{12}$$

Here, $\hat{S} \equiv S - \gamma$ and $\hat{F}_s \equiv F_s - \gamma$. The first term in equation (12) denoted

$$\widehat{CS}_s = r\frac{\hat{F}_s - \hat{S}}{\hat{S}}, \tag{13}$$

represents the credit spread on the senior 'unsecured' claim. The second term, $\hat{S}/S$, captures the effect of the 'fully-secured' part of the claim on the overall credit spread CS_s.[30] Some simple algebra yields:

$$\widehat{CS}_s - CS_j = r\frac{\hat{F}_s J - \hat{S} F_j}{\hat{S} J}. \tag{14}$$

Using the above definitions, one may derive the following proposition:

Proposition 3 *The difference $\widehat{CS}_s - CS_j$ is positive if and only if the equilibrium impairment strategy is $\{p_j^*, p_{bs}\}$. $\widehat{CS}_s - CS_j$ is negative if and only if the impairment strategy is $\{p_s^*, p_{bj}\}$. The equality holds, and $\widehat{CS}_s - CS_j = 0$ when $p_j^* = p_s^*$.*

Proof: See Appendix. □

Note that Proposition 3 provides a sufficient condition for the case in which $CS_s < CS_j$ for any level of the state variable. In fact, the inequality $CS_s < CS_j$ rearranges into $\widehat{CS}_s \hat{S}/S < CS_j$, which holds if $\widehat{CS}_s < CS_j$ because the term $\hat{S}/S$ is always

[30] We employ the term 'fully-secured' to mean that S can never fall below γ. In fact, the bigger the fully-secured part of the claim the smaller the ratio $\hat{S}/S$ and the spread CS_s.

smaller than one. When the collateral γ tends to zero, the sufficient condition becomes also a necessary condition (because, by definition of $\widehat{CS}_s$, with $\gamma = 0$ then $\widehat{CS}_s = CS_s$). This result can be summarized in the following proposition.

Proposition 4 *If γ tends to zero, the difference $CS_s - CS_j$ is positive if and only if the impairment strategy is $\{p_j^*, p_{bs}\}$. $CS_s - CS_j$ is negative if and only if the impairment strategy is $\{p_s^*, p_{bj}\}$. When $p_j^* = p_s^*$, then $CS_s - CS_j = 0$.*

Proof: See Appendix. □

In general, low collateral value V_L (that is, low α and/or γ) makes reversal of the spreads more likely because the effective bargaining strength of the senior creditor decreases and that of the junior increases (with α and/or γ decreasing). When a senior creditor is impaired first, seniority is enforceable, however there is little value attached to it because the liquidation threat has little impact.

C Empirical Implications

In practice, senior secured creditors are rarely impaired by a Chapter 11 bankruptcy plan and when they are, the impairment typically involves all classes (see LoPucki (2004) and LoPucki and Whitford (1993)). In the dataset used by LoPucki and Whitford (1993) we could analyse the treatment of each class for 31 Chapter 11 cases.[31] Out of 31 cases, in 19 cases, at least one creditor's class was unimpaired, most frequently a secured class. Moreover, amongst these 19 cases, in 15 cases only senior or secured classes were unimpaired while more junior or unsecured classes were impaired, while in 4 cases an unsecured class was unimpaired with a more senior class impaired. In the remaining 12 of the total 31 cases, all classes were impaired. In particular, of these 12

[31] The data consisted of 43 cases, but we did not have sufficient information on 12 cases about un-impairment.

cases, in 6 cases the priority structure was flat, e.g. no senior/secured/subordinated debt existed, in 6 cases there were secured debt (4 cases) and senior/subordinated debt (2 cases). Hence, the most empirically relevant observed equilibrium involves the junior debt being impaired first. This is consistent with our model, where senior debt is impaired first only if junior face value is particularly small relative to senior face value.[32]

Most important, empirical evidence shows that a large number of firms restructure their debt a second time. Using a sample of 197 public companies over the period $1979 - 88$, Hotchkiss (1995) finds that 32% of firms restructure their debt a second time either in Chapter 11 or in a private workout. Similarly, according to Gilson (1997), almost 25 percent of firms file for bankruptcy or restructure their debts a second time. More recent evidence can be found from LoPucki's BRD. Over 111 large public companies emerging from Chapter 11 during the period $2000 - 2005$, 25.23% refiled within 5 years.

The idea that Chapter 11 rules are particularly disadvantageous to secured creditors is widely recognized by legal scholars. Bebchuck and Fried (1996) argue that Chapter 11 rules tend to redistribute value from secured to unsecured creditors and to equity holders and that secured creditors may receive less than what they would receive in a Chapter 7 liquidation. Also, the general idea of Absolute Priority Violation in Chapter 11 is supported by several empirical studies suggesting that junior creditors and equity holders receive non-zero distributions before secured and/or senior creditors are fully paid. Frank and Torous (1994) find 'positive deviation' from absolute priority benefiting junior creditors and equity holders while bank, senior and secured creditors exhibit 'negative deviation' (with secured debt bearing the largest

[32]In Figure 5 senior claims are impaired first if the junior face value is less than 12% of total face value.

deviation).

Violation of Absolute Priority in Chapter 11 reorganizations has been documented by Weiss (1990), Eberhart et al. (1990), Altman (1991), Fabozzi et al. (1993), Altman and Eberhart (1994), Franks and Torous (1994). In particular, the possibility of positive renegotiation premia is in line with Pulvino and Pidot (1997) who find that bonds with very high collateral ratios (which, they argue, in principle, should be immune to default risk) yield 160 basis points above highly-rated bond yields.[33]

D Legal context

One may note that our analysis is consistent with the fact that secured, unimpaired creditors sometimes engage in legal action to resist a restructuring. In a Chapter 11 proceeding, unimpaired creditors often file a number of objections to confirmation of a plan and/or motions to convert to Chapter 7 or to lift (debtor) from automatic stay. Motions and objections to confirmation typically address such issues as improper classification, treatment of classes[34] and lack of feasibility of the plan.[35] Even though objections by unimpaired creditors are quite common, courts typically hold that a creditor whose rights are unimpaired under the plan has no right to object to confirmation.[36]

Though there may be no legal impairment, the default risk of senior or secured

[33]Their study uses US Airline secured bond yields and collateral.

[34]In re Mirant Corporation, et al. (Bankruptcy Court, Northern District of Texas, case n. 03-46590-DML-11, 2005) the Court denies the motion by a senior creditor (unimpaired and, hence, not entitled to vote), who argues that the plan actually impairs the Senior Notes and so entitled to vote.

[35]In re: Tavern Motor Inn Inc. (Bankruptcy Court, District of Vermont, case n. 56 B.R. 446, 1985), an unimpaired secured creditor filed a motion to convert the case into Chapter 7. The creditor argued the plan (approved by impaired unsecured creditors) actually impairs his class and is *not feasible* because there is a "likelihood of liquidation or further financial reorganization". The motion was denied.

[36]See In re Wonder Corp. of America, Bankruptcy Court, District of Connecticut, case n.70 B.R. 1018, 1023, 1987.

debt is affected because of the possibility of further deterioration of asset value and the consequent need to restructure senior/secured classes too. According to the Code, a plan must be feasible in the sense of not being likely to be followed by liquidation or need of further financial reorganization.[37]

However, as repeatedly held by the Bankruptcy Courts, the concept of feasibility simply involves reasonable prospects of financial stability and success. It is not necessary that success be guaranteed, but only that there may be a reasonable expectation of success. The mere prospect of financial uncertainty cannot defeat confirmation on feasibility grounds since a guarantee of the future is not required.[38]

It is worth noting that the possibility that an unimpaired senior/secured creditor receives a value, S, inferior to his collateral, V_L, is not ruled out by the Bankruptcy Code. In particular, the Code restricts the application of what is known as the "best interest test" to *impaired creditors*.[39] This test requires that impaired creditors must receive at least what they would receive in a Chapter 7 liquidation. Therefore, as the best interest test does not apply to unimpaired classes[40] nothing prevents positive renegotiation premia in Chapter 11.

Last, it is worth mentioning that in formal bankruptcy, if a plan of reorganization is confirmed, even if a cross-default can be asserted, confirmation of the plan resolves and eliminates it. That is, when non-impaired debt contains cross-default provisions, cure of the cross-defaults can be accomplished even if other classes are impaired

[37]Bankruptcy Code, Section 1129, Paragraph (a)(11).

[38]See In re Drexel Burnham Lambert Group Inc. Bankruptcy Court, Southern District of New York, case n. 138 B.R. 723, 1992.

[39]Bankruptcy Code, Section 1129, Paragraph (a)(7).

[40]See Seatco, Inc., (Bankruptcy Court, Northern District of Texas, case n. 00-37332-BJH-11, 2001) where "Class 3 creditors are unimpaired under the Plan and the best interest test is not applicable to them". See also, Bankruptcy Reform Act 1978, House Report (n.95-595) stating "the court may confirm a plan over the objection of a class of secured claims if the members of that class are unimpaired".

under the plan (see in In re: Mirant Corporation, et al., Bankruptcy Court, Northern District of Texas, case n. 03-46590-DML-11, 2005). This point is obvious because it would be inconsistent with the purposes of Chapter 11 to allow cross-defaults to defeat confirmation. Therefore, it emerges from our study that the purpose of cross default clauses is that of preventing equity holders from renegotiation with other creditor classes outside of chapter 11.

IV Conclusion

We show in this paper that sequential bankruptcy (Chapter 22) is a structural feature of Chapter 11 which resolves potential conflicts amongst senior and junior creditors. Recent empirical findings suggests that the outcome of Chapter 11 restructuring seems more and more driven by these type of conflicts rather than by conflicts amongst creditors and equity holders. In particular, we show that by leaving the contractual features of some debt claims unchanged (legal un-impairment), a group of stakeholders (equity and junior classes or equity and senior classes) cooperate in order to reach an agreement where unimpaired classes have no veto power and lose their outside option to liquidate the firm.

First, we find that, for reasonable parameter values, it is more likely that the equity holder and the junior creditor agree on a plan which leaves the senior creditor unimpaired at the first bankruptcy threshold. This is consistent with empirical findings. Commonly senior/secured creditors are unimpaired while more junior classes and equity holders share in the bankruptcy distribution. A second bankruptcy is triggered only if the firm value drops sufficiently low, in which case also senior creditor is restructured along with junior one. The intuition behind this type of equilibrium is

simple. When the senior creditor is in a strong bargaining position (highly secured in the event of liquidation), more junior stake-holders strategically delay renegotiation with a strong senior class until the firm value is low and hence seniority provisions are less valuable. The strategic use of the impairment rule comes from the dynamic nature of our model. By continuing serving contractual obligations, we are able to distinguish between legal and economic un-impairment. Despite legally unimpaired, senior claims are economically impaired with regards to their priority level.

Moreover, this type of equilibrium contributes to increasing the senior spread to the extent where the senior creditor would be better off if the firm was liquidated. On the other hand, this entails larger APV in favor of junior classes.

Second, the scope of un-impairment in our paper is broad and not only serves to reduce the value of seniority. Our model also allows for a second type of equilibrium where junior classes are left unimpaired (at first bankruptcy). This occurs when junior face value is particularly small compared to total face value and/or junior creditors have strong bargaining power. The intuition here is that even if junior classes, unlike senior ones, have no effective liquidation threat, they might still be though negotiator in bankruptcy as bargaining power should not depend on the size of a claim. Moreover, (when the size of the junior claim is small) impairing a small claim (while leaving a relatively larger one unimpaired) might not be a viable way out of bankruptcy. In practice this equilibrium is relevant when banks or, more likely, vulture funds owns junior debt. Also, this type of equilibrium is consistent with the treatment of convenience claims (small unsecured claims placed in a separate class for administrative convenience) often left unimpaired in bankruptcy.

Interestingly, the second type of equilibrium, where the senior creditor is impaired first and the junior one later on, might result in reversal of senior and junior spreads.

The senior credit spread may be higher than the junior one. Reversal of the spreads does not occur in the first type of equilibrium where the senior creditor is unimpaired at the first Chapter 11 bankruptcy.

A Appendix with Proofs

Proof for Lemma 1

First, we prove that $P_{j,e}(p_t, p_s^*) \geq P_j(p_t)$ for $p_t \geq p_s^*$. Because p_s^* minimizes $S(p_t, p_s^*)$ (thus $S(p_t, p_s^*)$ smooth-pastes to $P_s(p_t)$ from below), for $p_t \geq p_s^*$ we can write

$$
\begin{aligned}
P_{j,e}(p_t, p_s^*) &= \eta_j(V(p_t) - S(p_t, p_s^*)) \geq \eta_j(V(p_t) - P_s(p_t)) = && (15)\\
&= \eta_j(P_e(p_t) + P_j(p_t)) && (16)\\
&= \frac{x_j}{x_e + x_j}\frac{x_e + x_j}{x_e + x_j + x_s}(V(p_t) - V_L(p_t)) && (17)\\
&= \xi_j(V(p_t) - V_L(p_t)) && (18)\\
&= P_j(p_t) && (19)
\end{aligned}
$$

Also, because $\partial S(p_t, p_s^*)/\partial p_t \mid_{p_s^*} = \partial P_s(p_t)/\partial p_t \mid_{p_s^*}$ (again, due to the fact that $p_s^* = \arg\min S(p_t, p_s^*)$) it is immediate to see, from a similar calculation, that $P_{j,e}(p_t, p_s^*)$ smooth-pastes to $P_j(p_t)$, that is

$$
\begin{aligned}
\frac{\partial P_{j,e}(p_t, p_s^*)}{\partial p_t} \mid_{p_s^*} &= \frac{\partial \eta_j(V(p_t) - S(p_t, p_s^*))}{\partial p_t} \mid_{p_s^*} && (20)\\
&= \frac{\partial \eta_j(P_e(p_t) + P_j(p_t))}{\partial p_t} \mid_{p_s^*} && (21)\\
&= \frac{\partial P_j(p_t)}{\partial p_t} \mid_{p_s^*} . && (22)
\end{aligned}
$$

Therefore, we conclude that $P_{j,e}(p_t, p_s^*)$ smooth-pastes to $P_j(p_t)$ from above. This means that for any $p_t \geq p_s^*$ the junior creditor prefers to bargain directly with the equity holders while leaving the senior creditor unimpaired. Moreover, the value of junior creditor is maximized if the senior creditor is impaired (along with the junior one) when the state variable hits the level p_s^*.

Similarly we can prove that $P_{s,e}(p_t, p_j^*) \geq P_s(p_t)$ for $p_t \geq p_j^*$. Given that p_j^* minimizes

$J(p_t, p_j^*)$, then $J(p_t, p_j^*)$ smooth-pastes to $P_j(p_t)$ from below. Therefore for $p_t \geq p_j^*$ we have

$$
\begin{aligned}
P_{s,e}(p_t, p_s^*) &= \eta_s(V(p_t) - J(p_t, p_j^*) - V_L(p_t)) + V_L(p_t) \geq && (23)\\
&\geq \eta_s(V(p_t) - P_j(p_t) - V_L(p_t)) + V_L(p_t) && (24)\\
&= \eta_s(P_e(p_t) + P_s(p_t) - V_L(p_t)) + V_L(p_t) && (25)\\
&= \eta_s\left(\frac{x_e(V(p_t) - V_L(p_t))}{x_e + x_j + x_s} + \frac{x_s(V(p_t) - V_L(p_t))}{x_e + x_j + x_s} + V_L(p_t) - V_L(p_t)\right) + V_L(p_t) && (26)\\
&= \xi_s(V(p_t) - V_L(p_t)) + V_L(p_t) && (27)\\
&= P_s(p_t) && (28)
\end{aligned}
$$

Furthermore, because p_j^* minimizes $J(p_t, p_j^*)$ (therefore $\partial J(p_t, p_j^*)/\partial p_t \mid_{p_j^*} = \partial P_j(p_t)/\partial p_t \mid_{p_j^*}$), we have that $P_{s,e}(p_t, p_j^*)$ smooth-pastes to $P_s(p_t)$, that is

$$
\begin{aligned}
\frac{\partial P_{s,e}(p_t, p_j^*)}{\partial p_t} \mid_{p_j^*} &= \frac{\partial(\eta_s(V(p_t) - J(p_t, p_j^*) - V_L(p_t)) + V_L(p_t))}{\partial p_t} \mid_{p_j^*} && (29)\\
&= \frac{\partial(\eta_s(P_e(p_t) + P_s(p_t) - V_L(p_t)) + V_L(p_t))}{\partial p_t} \mid_{p_j^*} && (30)\\
&= \frac{\partial P_s(p_t)}{\partial p_t} \mid_{p_j^*} . && (31)
\end{aligned}
$$

We conclude that $P_{s,e}(p_t, p_j^*)$ smooth-pastes to $P_s(p_t)$ from above. Therefore, for any $p_t \geq p_j^*$ the senior creditor prefers to be impaired alone and the value of his claim is maximized if the junior creditor is impaired when the state variable hits the level p_j^*. □

Proof for Lemma 2

First consider the case where $p_s^* < p_j^*$. We know that for $p_t \leq p_j^*$, $P_{e,s} = V(p_t) - P_{s,e}(p_t, p_j^*) - J(p_t, p_j^*)$ rearranges as $P_{e,s} = V(p_t) - P_s(p_t) - P_j(p_t) = P_e$. The reason for this is that p_j^* maximizes the equity value when the equity holder impair the senior creditor alone (see equation 8 second equality) and hence for $p_t < p_j^*$ it is optimal to impair both classes. Therefore $P_{s,e}(p_t, p_j^*)$ and $J(p_t, p_j^*)$ become equal to $P_s(p_t)$ and $P_j(p_t)$ respectively. Therefore

for $p_t \in (p_s^*, p_j^*]$ we can verify that $P_{e,j} > P_{e,s}$ as follows:

$$P_{e,s} = P_e = V(p_t) - P_s(p_t) - P_j(p_t) < V(p_t) - P_{j,e}(p_t, p_s^*) - S(p_t, p_s^*) = P_{e,j} \tag{32}$$

$$S(p_t, p_s^*) + P_{j,e}(p_t, p_s^*) < P_j(p_t) + P_s(p_t) \tag{33}$$

$$S(p_t, p_s^*) + \eta_j(V(p_t) - S(p_t, p_s^*)) < P_j(p_t) + P_s(p_t) \tag{34}$$

$$\eta_j V(p_t) + (1 - \eta_j) S(p_t, p_s^*) < P_j(p_t) + P_s(p_t) \tag{35}$$

because for $p_t > p_s^*$ $S(p_t, p_s^*) < P_s(p_t)$, then we can take an upper bound to the left hand side,

$$\eta_j V(p_t) + (1 - \eta_j) S(p_t, p_s^*) < \eta_j V(p_t) + (1 - \eta_j) P_s(p_t) = \tag{36}$$

$$= \eta_j (V(p_t) - P_s(p_t)) + P_s(p_t) = P_j(p_t) + P_s(p_t). \tag{37}$$

Furthermore, it follows from Lemma 1 (where $P_{j,e}(p_t, p_s^*)$ smooth-pastes to $P_j(p_t)$ at p_s^*) that $P_{e,j}$ smooth-pastes to P_e at the threshold level p_s^*, in particular

$$\frac{\partial P_e}{\partial p_t} |_{p_s^*} = \frac{\partial (V(p_t) - P_s(p_t) - P_j(p_t))}{\partial p_t} |_{p_s^*} = \tag{38}$$

$$= \frac{\partial (V(p_t) - S(p_t, p_s^*) - P_{j,e}(p_t, p_s^*))}{\partial p_t} |_{p_s^*} = \frac{\partial P_{e,j}}{\partial p_t} |_{p_s^*} . \tag{39}$$

Now, we consider the case where $p_j^* < p_s^*$. For $p_t \leq p_s^*$, by a similar argument to the previous case, we can rewrite $P_{e,j} = V(p_t) - P_{j,e}(p_t, p_s^*) - S(p_t, p_s^*)$ as $P_{e,j} = V(p_t) - P_s(p_t) - P_j(p_t) = P_e(p_t)$. Then for $p_t \in (p_j^*, p_s^*]$, we verify that

$$P_{e,j} = P_e = V(p_t) - P_s(p_t) - P_j(p_t) < V(p_t) - P_{s,e}(p_t, p_j^*) - J(p_t, p_j^*) = P_{e,s} \tag{40}$$

$$P_{s,e}(p_t, p_j^*) + J(p_t, p_j^*) < P_j(p_t) + P_s(p_t) \tag{41}$$

$$\eta_s (V(p_t) - J(p_t, p_j^*) - V_L(p_t)) + V_L(p_t) + J(p_t, p_j^*) < P_j(p_t) + P_s(p_t) \tag{42}$$

$$\eta_s (V(p_t) - V_L(p_t)) + V_L(p_t) + (1 - \eta_s) J(p_t, p_j^*) < P_j(p_t) + P_s(p_t) \tag{43}$$

because for $p_t > p_j^*$ $J(p_t, p_j^*) < P_j(p_t)$ then we can take an upper bound to the left hand side,

$$\eta_s(V - V_L) + V_L + (1 - \eta_s)J(p_t, p_j^*) < \eta_s(V - V_L) + V_L + (1 - \eta_s)P_j(p_t) = \tag{44}$$

$$= \eta_s(V - V_L - P_j(p_t)) + V_L + P_j(p_t) = P_s(p_t) + P_j(p_t). \tag{45}$$

Additionally, from Lemma 1 (where $P_{s,e}(p_t, p_j^*)$ smooth-pastes to $P_s(p_t)$ at p_j^*) that $P_{e,s}$ smooth-pastes to P_e at the threshold level p_j^*, in particular

$$\frac{\partial P_e}{\partial p_t} |_{p_j^*} = \frac{\partial(V(p_t) - P_s(p_t) - P_j(p_t))}{\partial p_t} |_{p_j^*} = \tag{46}$$

$$= \frac{\partial(V(p_t) - P_{s,e}(p_t, p_j^*) - J(p_t, p_j^*))}{\partial p_t} |_{p_j^*} = \frac{\partial P_{e,s}}{\partial p_t} |_{p_j^*} . \tag{47}$$

Last, it is immediate that for $p_t \leq \min\{p_s, p_j\}$ $P_{e,s} = P_{e,j} = P_e$ all claims are impaired collectively -regardless of which claim has been impaired first. That is, $P_{j,e}(p_t, p_s^*) = P_j(p_t)$, $S(p_t, p_s^*) = P_s(p_t)$, $P_{s,e}(p_t, p_j^*) = P_s(p_t)$ and $J(p_t, p_j^*) = P_j(p_t)$. □

Derivations for Proposition 1

Case 1). If $p_s^* < p_j^*$, agreed claim values in bankruptcy are $P_{j,e}(p_t, p_s^*)$ and $S(p_t, p_s^*)$. The equity holder maximizes the equity value by solving the following

$$\max_{p_b} E = \max_{p_b} V(p_t) - \left\{ F_j + [P_{j,e}(p_b, p_s^*) - F_j] \left(\frac{p_t}{p_b}\right)^\lambda \right\} - \left\{ F_s + [P_s(p_s^*) - F_s] \left(\frac{p_t}{p_s^*}\right)^\lambda \right\},$$

equivalently

$$\min_{p_b} F_j + [P_{j,e}(p_b, p_s^*) - F_j] \left(\frac{p_t}{p_b}\right)^\lambda$$

which yields

$$p_{bj} = \frac{\lambda}{1 - \lambda} \frac{\eta_j F_s + F_j}{\eta_j} (r - \mu). \tag{48}$$

Case 2). When $p_s^* > p_j^*$ the equity holder maximizes

$$\max_{p_b} E = \max_{p_b} V(p_t) - \left\{ F_j + [P_j(p_j^*) - F_j] \left(\frac{p_t}{p_j^*}\right)^\lambda \right\} - \left\{ F_s + [P_{s,e}(p_b, p_s^*) - F_s] \left(\frac{p_t}{p_b}\right)^\lambda \right\},$$

which yields

$$p_{bs} = \frac{\lambda}{1-\lambda} \frac{\eta_s F_j + F_s - \gamma(1-\alpha_{\eta_s})}{\alpha_{\eta_s}} (r-\mu) \tag{49}$$

with

$$\alpha_{\eta_s} = \eta_s(1-\alpha) + \alpha.$$

Furthermore, we can show that when $p_s^* \geq p_j^*$ then $p_{bs} \in [p_j^*, p_s^*]$. (A proof showing that $p_s^* < p_j^*$ if and only if $p_{bj} \in [p_s^*, p_j^*]$ follows the same line as the one below, therefore we omit it.) As pointed out in Proposition 2 it is immediate to find that $p_s^* \geq p_j^*$ when

$$(F_s - \gamma)\frac{1-\alpha}{\alpha_{\xi_s}} \geq \frac{F_j}{\xi_j}. \tag{50}$$

First, we can compare p_{bs} and p_s^*, and find what range of parameters guarantees that $p_{bs} < p_s^*$. We find that

$$p_{bs} = \frac{\lambda}{\lambda-1} \frac{\eta_s F_j + F_s - \gamma(1-\alpha_{\eta_s})}{\alpha_{\eta_s}} (r-\mu) < \frac{\lambda}{\lambda-1} \frac{F_s - \gamma(1-\alpha_{\xi_s})}{\alpha_{\xi_s}} (r-\mu) = p_s^* \tag{51}$$

rearranges into

$$F_j < (F_s - \gamma)\frac{\alpha_{\eta_s} - \alpha_{\xi_s}}{\eta_s \alpha_{\xi_s}}. \tag{52}$$

Substituting for $\alpha_{\eta_s} = \eta_s(1-\alpha) + \alpha$ and $\alpha_{\xi_s} = \xi_s(1-\alpha) + \alpha$ in the numerator yields

$$F_j < (F_s - \gamma)\frac{(\eta_s - \xi_s)(1-\alpha)}{\eta_s \alpha_{\xi_s}}, \tag{53}$$

and by the definition of $\eta_s = x_s/(x_s + x_e)$ and $\xi_s = x_s/(x_s + x_e + x_j)$ we have

$$F_j < (F_s - \gamma)\frac{\frac{x_s}{x_s+x_e} - \frac{x_s}{x_s+x_e+x_j}}{\frac{x_s}{x_s+x_e}}\frac{1-\alpha}{\alpha_{\xi_s}} = \tag{54}$$

$$= (F_s - \gamma)\frac{\frac{x_s}{x_s+x_e}\frac{x_j}{x_s+x_e+x_j}}{\frac{x_s}{x_s+x_e}}\frac{1-\alpha}{\alpha_{\xi_s}} = \tag{55}$$

$$= (F_s - \gamma)\frac{\frac{x_j}{x_s+x_e+x_j}(1-\alpha)}{\alpha_{\xi_s}} = \tag{56}$$

$$= (F_s - \gamma)\frac{\xi_j(1-\alpha)}{\alpha_{\xi_s}}, \tag{57}$$

which holds when $p_s^* \geq p_j^*$.

Second, we can compare p_{bs} and p_j^*, and check when $p_{bs} > p_j^*$, that is,

$$p_{bs} = \frac{\lambda}{\lambda - 1}\frac{\eta_s F_j + F_s - \gamma(1-\alpha_{\eta_s})}{\alpha_{\eta_s}}(r-\mu) > \frac{\lambda}{\lambda-1}\frac{F_j + \gamma(1-\alpha)\xi_j}{(1-\alpha)\xi_j}(r-\mu) = p_j^* \tag{58}$$

which by simple algebra rearranges

$$F_j < (F_s - \gamma)\frac{(1-\alpha)\xi_j}{\alpha_{\eta_s} - \eta_s(1-\alpha)\xi_j}. \tag{59}$$

Substituting for $\alpha_{\eta_s} = \eta_s(1-\alpha) + \alpha$ in the denominator yields

$$F_j < (F_s - \gamma)\frac{(1-\alpha)\xi_j}{\eta_s(1-\alpha) + \alpha - \eta_s(1-\alpha)\xi_j}. \tag{60}$$

By the definition of $\eta_s = x_s/(x_s+x_e)$ and $\xi_j = x_s/(x_s+x_e+x_j)$ the latter can be rearranged

as follows

$$F_j < (F_s - \gamma)\frac{(1-\alpha)\xi_j}{\eta_s(1-\alpha)(1-\xi_j)+\alpha} = \tag{61}$$

$$= (F_s - \gamma)\frac{(1-\alpha)\xi_j}{\frac{x_s}{x_s+x_e}(1-\alpha)\frac{x_e+x_s}{x_s+x_e+x_j}+\alpha} = \tag{62}$$

$$= (F_s - \gamma)\frac{(1-\alpha)\xi_j}{\frac{x_s}{x_s+x_e+x_j}(1-\alpha)+\alpha} = \tag{63}$$

$$= (F_s - \gamma)\frac{\xi_j(1-\alpha)}{\alpha_{\xi_s}}, \tag{64}$$

which holds when $p_s^* \geq p_j^*$.

□

Proof of Proposition 3

By the definitions of $\widehat{CS}_s$ and CS_j, the sign of $\widehat{CS}_s - CS_j$ is positive if and only if

$$\frac{F_s - \gamma}{F_j} > \frac{S - \gamma}{J}. \tag{65}$$

We prove that this inequality holds when $p_s^* > p_j^*$, that is, the senior creditor is impaired first.

When $p_t \in [\underline{p}, p_j^*]$, with $S = P_s = \xi_s(V - V_L) + V_L$ and $J = P_j = \xi_j(V - V_L)$ (by substituting for $V - V_L = (1-\alpha)(V-\gamma)$ and $V_L - \gamma = \alpha(V-\gamma)$) inequality 65 rearranges into

$$\frac{F_s - \gamma}{F_j} > \frac{\alpha_{\xi_s}}{\xi_j(1-\alpha)}. \tag{66}$$

which holds because $p_s^* > p_j^*$ (see Proposition 2).

When $p_t \in (p_j^*, p_{bs}]$, we know that $S = P_{s,e}$ and $J = F_j + (P_j(p_j^*) - F_j)(p_t/p_j^*)^\lambda$, therefore

inequality 65 becomes

$$\frac{F_s - \gamma}{F_j} > \frac{P_{s,e} - \gamma}{F_j + (P_j(p_j^*) - F_j)(p_t/p_j^*)^\lambda}. \tag{67}$$

Now notice that for any $p_t > p_j^*$ we have that $P_{s,e} < F_s + (P_s(p_j^*) - F_s)(p_t/p_j^*)^\lambda$ (because p_{bs} is selected by the equity holder to minimize $S(p_{bs}, p_j^*) = F_s + (P_{s,e}(p_j^*, p_{bs}) - F_s)(p_t/p_{bs})^\lambda$ then $S(p_{bs}, p_j^*) < S(p_j^*, p_j^*) = F_s + (P_s(p_j^*) - F_s)(p_t/p_j^*)^\lambda$). Then we can easily prove that inequality 67 holds by proving the following first inequality

$$\frac{F_s - \gamma}{F_j} > \frac{F_s + (P_s(p_j^*) - F_s)(p_t/p_j^*)^\lambda - \gamma}{F_j + (P_j(p_j^*) - F_j)(p_t/p_j^*)^\lambda} > \frac{P_{s,e} - \gamma}{F_j + (P_j(p_j^*) - F_j)(p_t/p_j^*)^\lambda}. \tag{68}$$

After some algebra, the first inequality rearranges again as

$$\frac{F_s - \gamma}{F_j} > \frac{P_s(p_j^*) - \gamma}{P_j(p_j^*)} = \frac{\alpha_{\xi_s}}{\xi_j(1-\alpha)}. \tag{69}$$

which again holds for $p_s^* > p_j^*$.

Similarly, when $p_t \in (p_{bs}, \infty)$ we can use again the fact $S = S(p_{bs}, p_j^*) < S(p_j^*, p_j^*)$ and show that the following first inequality

$$\frac{F_s - \gamma}{F_j} > \frac{F_s + (P_s(p_j^*) - F_s)(p_t/p_j^*)^\lambda - \gamma}{F_j + (P_j(p_j^*) - F_j)(p_t/p_j^*)^\lambda} > \frac{S - \gamma}{J}, \tag{70}$$

holds because it rearranges as

$$\frac{F_s - \gamma}{F_j} > \frac{P_s(p_j^*) - \gamma}{P_j(p_j^*)} = \frac{\alpha_{\xi_s}}{\xi_j(1-\alpha)}. \tag{71}$$

The proof runs similar in the opposite case when $p_j^* > p_s^*$, and the junior is impaired first. □

Proof of Proposition 4

Follows from proof of Proposition 3. □

B Debt service flow functions.

We derive here the debt service flow functions of senior and junior creditor in the two alternative scenarios where i) $p_b = p_{bs} > p_j^*$ and ii) $p_b = p_{bj} > p_s^*$. In order to avoid lengthy algebra we solve a general problem first. We know that debt values when restructured are equal to

$$S = \begin{cases} P_{s,e} = \eta_s(V(p_t) - J(p_t, p_j) - V_L(p_t)) + V_L(p_t) & \text{if j is unimpaired} \\ P_s = \xi_s(V(p_t) - V_L(p_t)) + V_L(p_t) & \text{if j is impaired} \end{cases} \tag{72}$$

$$J = \begin{cases} P_{j,e} = \eta_j(V(p_t) - J(p_t, p_j)) & \text{if s is unimpaired} \\ P_j = \xi_j(V(p_t) - V_L(p_t)) & \text{if s is impaired} \end{cases} \tag{73}$$

that is, restructured values are linear functions of V, V_L and, if the other class is unimpaired, also a linear function of the unimpaired debt class value S or J. Because V, V_L and unimpaired debt class J or S are all polynomial equations of degree λ, of the form $A_0 + A_1 p_t + A_2 P_t^\lambda$ (with A_0, A_1 and A_2 constant), also restructured values S and J (because linear functions of polynomial equations) are also polynomial equations of the form $D(p_t) = B_0 + B_1 p_t + B_2 P_t^\lambda$ (with B_0, B_1 and B_2 constant). Therefore the problem is to find a service flow function $b(p_t)$ which satisfies the no arbitrage condition

$$rD(p_t) = b(p_t) + \mu p_t D'(p_t) + \frac{\sigma^2}{2} p_t^2 D''(p_t) \tag{74}$$

$$\text{with } D(p_t) = B_0 + B_1 p_t + B_2 P_t^\lambda \tag{75}$$

where $D(p_t)$ is equal to J or S. By substituting for equation (75), D' and D'' into equation (74) and by rearranging, we have

$$r(B_0 + B_1 p_t) + rB_2 p_t^{\lambda} = b(p_t) + \mu p_t B_1 + \mu\lambda B_2 p_t^{\lambda} + \sigma^2/2\lambda(\lambda - 1)B_2 p_t^{\lambda}, \tag{76}$$

and because λ is the negative root of $r = \mu\lambda + \sigma^2/2\lambda(\lambda - 1)$, equation (76) becomes

$$r(B_0 + B_1 p_t) = b(p_t) + \mu p_t B_1, \tag{77}$$

which solved for $b(p_t)$ yields

$$b(p_t) = rB_0 + (r - \mu)p_t B_1. \tag{78}$$

Now we can derive the senior and junior debt service flow functions by using equation (78). With regards to the senior creditor, when impaired alone, after substituting for $V_L = \alpha V + (1 - \alpha\gamma)$, $V - V_L = (V - \gamma)(1 - \alpha)$, $J = F_j + (P_j(p_j^*) - F_j)(p_t/p_j^*)^{\lambda}$ and rearranging, the value $P_{s,e}$ rewrites as

$$S = P_{s,e} = (1-\alpha_{\eta_s})\gamma - \eta_s F_j + \alpha_{\eta_s}\frac{p_t}{r-\mu} - \eta_s(P_j(p_j^*) - F_j)\left(\frac{p_t}{p_j^*}\right)^{\lambda} + \alpha_{\eta_s}\left(\gamma - \frac{\underline{p}}{r-\mu}\right)\left(\frac{p_t}{\underline{p}}\right)^{\lambda}, \tag{79}$$

therefore by equation (75) we can set $B_0 = (1 - \alpha_{\eta_s})\gamma - \eta_s F_j$ and $B_1 = \alpha_{\eta_s}/(r - \mu)$ and by equation (78)

$$b_s(p_t) = (1 - \alpha_{\eta_s})r\gamma - \eta_s b_j + \alpha_{\eta_s} p_t. \tag{80}$$

When instead the senior creditor is impaired jointly with the junior one, P_s rearranges as

$$S = P_s = (1 - \alpha_{\xi_s})\gamma + \alpha_{\xi_s}\frac{p_t}{r-\mu} + \alpha_{\xi_s}\left(\gamma - \frac{\underline{p}}{r-\mu}\right)\left(\frac{p_t}{\underline{p}}\right)^{\lambda}. \tag{81}$$

Again, by equation (75), set $B_0 = (1 - \alpha_{\xi_s})\gamma$ and $B_1 = \alpha_{\xi_s}/(r - \mu)$ and equation (78) becomes

$$b_s(p_t) = (1 - \alpha_{\xi_s})r\gamma + \alpha_{\xi_s} p_t \tag{82}$$

Similarly, with regards to the junior creditor, when impaired alone (thus $S = F_s + (P_s(p_s^*) - F_s)(p_t/p_s^*)^\lambda$), one can rearrange $P_{j,e}$ as

$$J = P_{j,e} = -\eta_j F_s + \eta_j \frac{p_t}{r - \mu} - \eta_j (P_s(p_s^*) - F_s) \left(\frac{p_t}{p_s^*}\right)^\lambda + \eta_j \left(\gamma - \frac{\underline{p}}{r - \mu}\right) \left(\frac{p_t}{\underline{p}}\right)^\lambda, \tag{83}$$

by equation (75) $B_0 = -\eta_j F_s$ and $B_1 = \eta_j/(r - \mu)$ and thus equation (78) yields

$$b_j(p_t) = \eta_j (p_t - b_s). \tag{84}$$

Finally, when the junior creditor is impaired jointly with the senior one, P_j can be written as

$$J = P_j = \xi_j (1 - \alpha) \left(\frac{p_t}{r - \mu} - \gamma\right) + \xi_j (1 - \alpha) \left(\gamma - \frac{\underline{p}}{r - \mu}\right) \left(\frac{p_t}{\underline{p}}\right)^\lambda, \tag{85}$$

therefore with $B_0 = -\xi_j(1 - \alpha)\gamma$ and $B_1 = \xi_j(1 - \alpha)/(r - \mu)$, equation (78) yields

$$b_j(p_t) = \xi_j (1 - \alpha)(p_t - r\gamma). \tag{86}$$

C Appendix on Case $V_L(p_b) \geq F_s$

For large level of junior face value, renegotiation starts at some level of the state variable p_b such that $V_L(p_b) \geq F_s$. Therefore, the senior creditor would receive F_s if the firm is liquidated. In this case, senior creditor cannot be impaired (in the sense of being offered a service flow function, $b_s(p_t) < b_s$), because impairment would drive the senior value below his disagreement payoff F_s. Therefore, a reorganization plan (so that the firm is not liquidated) can only impair the equity and the junior claim and leave the senior unimpaired,

with allocation:

$P_{e,j} = \eta_e(V(p_t) - S(p_t, p_s))$

$P_{j,e} = \eta_j(V(p_t) - S(p_t, p_s) - (V_L(p_t) - F_s)^+) + (V_L(p_t) - F_s)^+$

$S(p_t, p_s)$

The discontinuity in the first derivative of $P_{j,e}$ can be dealt with by irreversibly writing-down junior face value when p_t hits the level $\hat{p}$, with

$$\hat{p} : V_L(\hat{p}) = F_s$$

.

After writing down junior debt, the new level of junior face value, say $\underline{F}_j$, must be such that the new junior debt is minimized at $p_t = \hat{p}$, that is $\hat{p}(\underline{F}_j) = \arg\min J(p_t, \hat{p})$ with $J(p_t, \hat{p}) = \underline{F}_j + (P_{j,e}(\hat{p}) - \underline{F}_j)(p_t/\hat{p})^\lambda$. Equivalently, one can say that $\underline{F}_j$ must satisfy the smooth-pasting condition

$$\frac{\partial J(p_t, \hat{p})}{\partial p_t} \mid_{\hat{p}} = \frac{\partial P_{j,e}(p_t)}{\partial p_t} \mid_{\hat{p}}$$

which yields

$$\hat{p} = \frac{\lambda}{\lambda - 1} \frac{\eta_j F_s + \underline{F}_j}{\eta_j} (r - \mu).$$

Solving for $\underline{F}_j$ gives:

$$\underline{F}_j = \eta_j \left(\frac{\lambda}{\lambda - 1} \frac{\hat{p}}{r - \mu} - F_s \right)$$

The service flow function, $b_j(p_t)$ is derived according to:

$$rJ(p_t) = b_j(p_t) + \mu p_t J'(p_t) + \frac{\sigma^2}{2} p_t^2 J''(p_t) \tag{87}$$

with

$$J(p_t) = P_{j,e} = \begin{cases} \eta_j(V(p_t) - S(p_t, p_s) - (V_L(p_t) - F_s)) + (V_L(p_t) - F_s) & if \quad p_t > \hat{p} \\ \eta_j(V(p_t) - S(p_t, p_s)) & if \quad p_t \leq \hat{p} \end{cases} \tag{88}$$

Equation 87 yields

$$b_j(p_t) = \begin{cases} b_j & if \quad p_t > p_b, \ \check{p}_t > \hat{p} \\ p_t[\eta_j(1-\alpha) + \alpha] + r\gamma(1-\alpha)(1-\eta_j) - b_s & if \quad \hat{p} < p_t \leq p_b, \ \check{p}_t > \hat{p} \\ \eta_j(p_t - b_s) & if \quad p_t \leq \hat{p} \\ r\underline{F}_j & if \quad p_t > \hat{p}, \ \check{p}_t \leq \hat{p} \end{cases} \tag{89}$$

where $\check{p}_t = \inf_{0 \leq k \leq t}\{p_t\}$.

The bankruptcy trigger p_b is derived a usual by minimizing junior debt value $J(p_t, \hat{p}) = F_j + (P_{j,e}(p_b) - F_j)(p_t/p_b)^\lambda$ (or by smooth-pasting $J(p_t)$ to $P_{j,e}(p_t)$ at $p_t = p_b$). This yields either an interior optimum,

$$p_b = \frac{\lambda}{\lambda - 1} \frac{F - \gamma(1-\alpha)(1-\eta_j)}{\eta_j(1-\alpha) + \alpha}(r - \mu)$$

or a corner solution, in which case junior face value is written down as soon as bankruptcy starts and $p_b = \hat{p}$.

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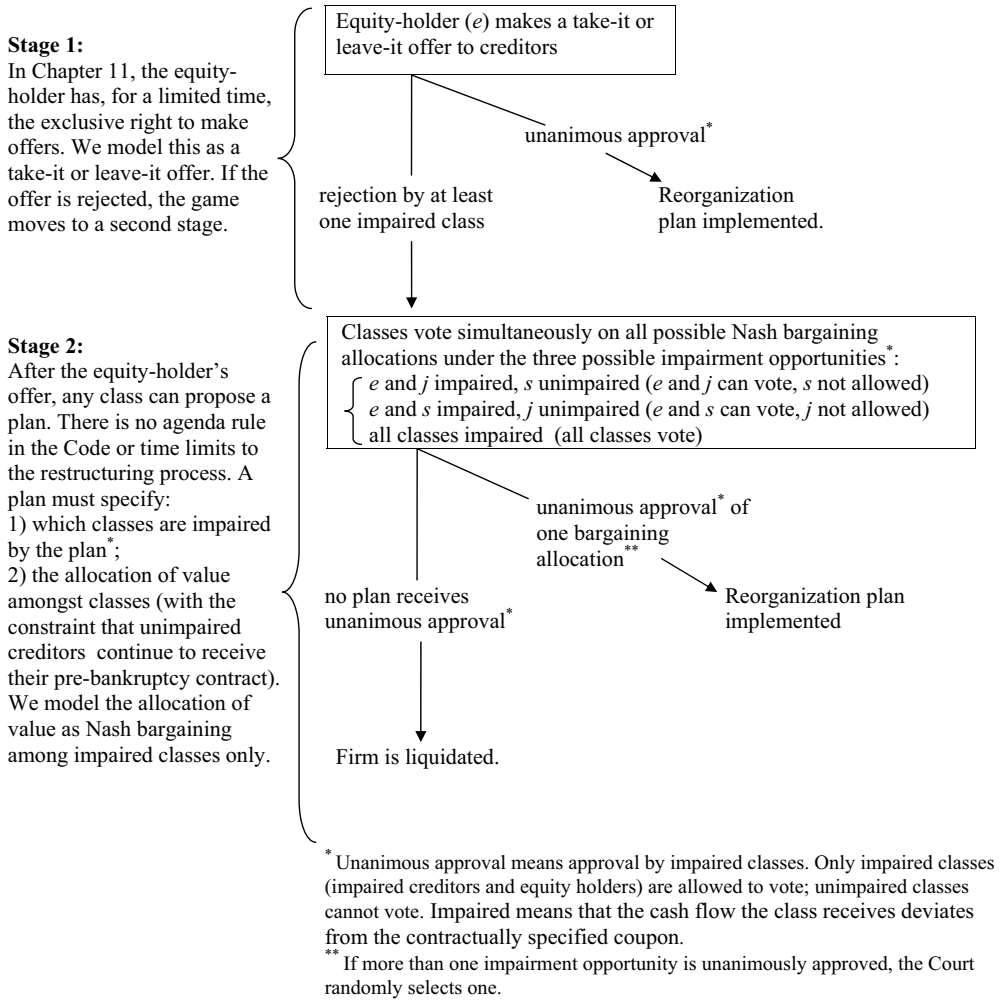

Figure 1: Extensive form game.

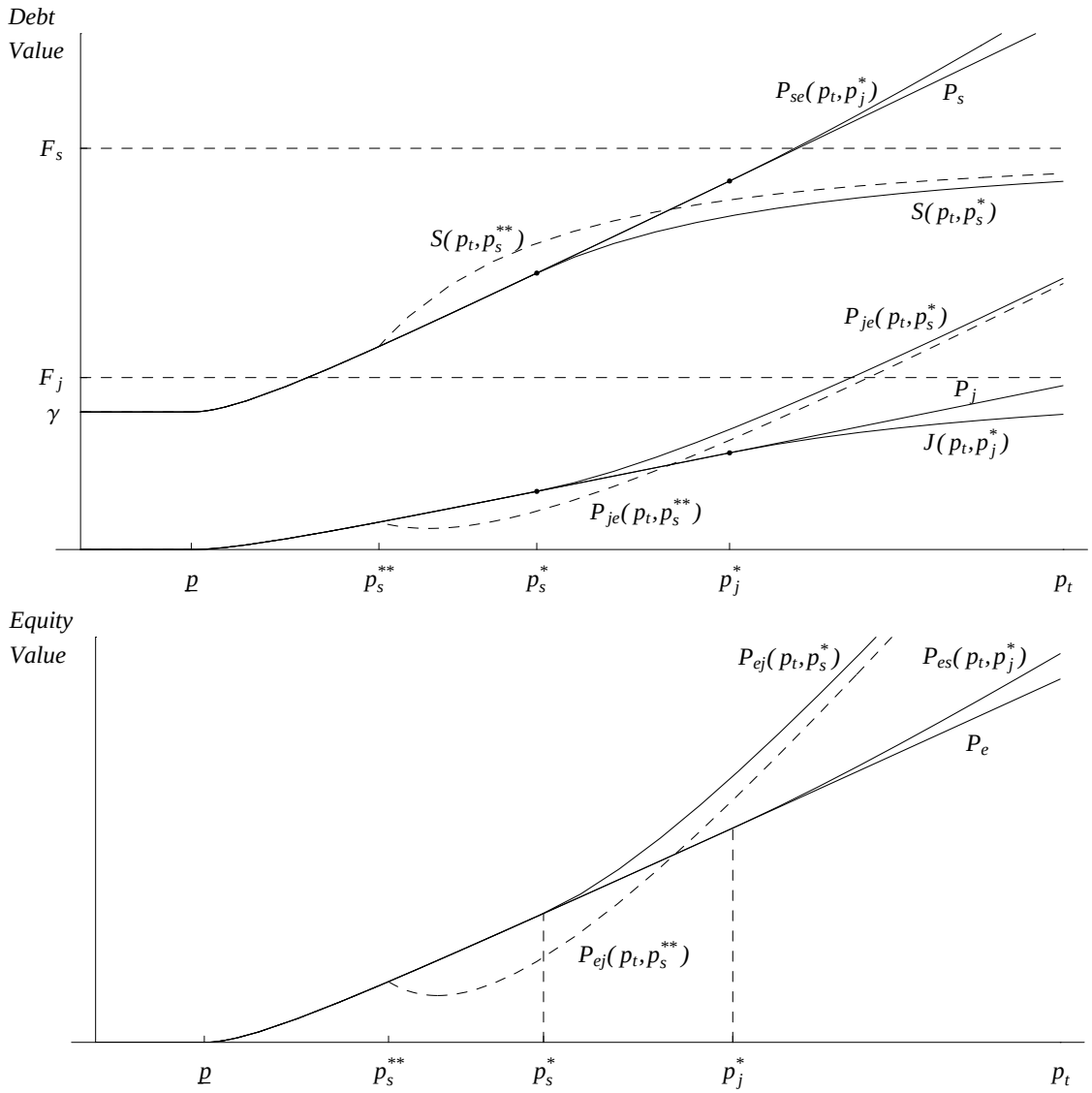

Figure 2: Debt values (top diagram) and equity values (bottom diagram) under the different Nash-allocations when $p_j^* > p_s^*$. In the top diagram, $P_{j,e}$ smooth pastes to P_j at p_s^* from above and $P_{s,e}$ smooth pastes to P_s at p_j^* from above. A sub-optimal threshold level p_s^{**} reduces the junior debt value $P_{j,e}$. Therefore, for any $p_t > p_s^*$ junior debt holders prefer to be impaired alone rather than jointly with senior debt holders. In the bottom diagram, the equity value is highest when the junior class is impaired first and the senior one is impaired along only when p_t falls to a level p_s^*. As well as the junior debt, the equity value declines if the senior class is impaired jointly with the junior at the sub-optimal threshold level p_s^{**} ($P_{e,j}(p_t, p_s^*) > P_{e,j}(p_t, p_s^{**})$). Therefore, the equity holders agree with the junior class on a Nash-allocation (with values $\{P_{e,j}, P_{j,e}, S(p_t, p_s^*)\}$ to e, j and s respectively) which leaves the senior class unimpaired for any $p_t > p_s^*$.

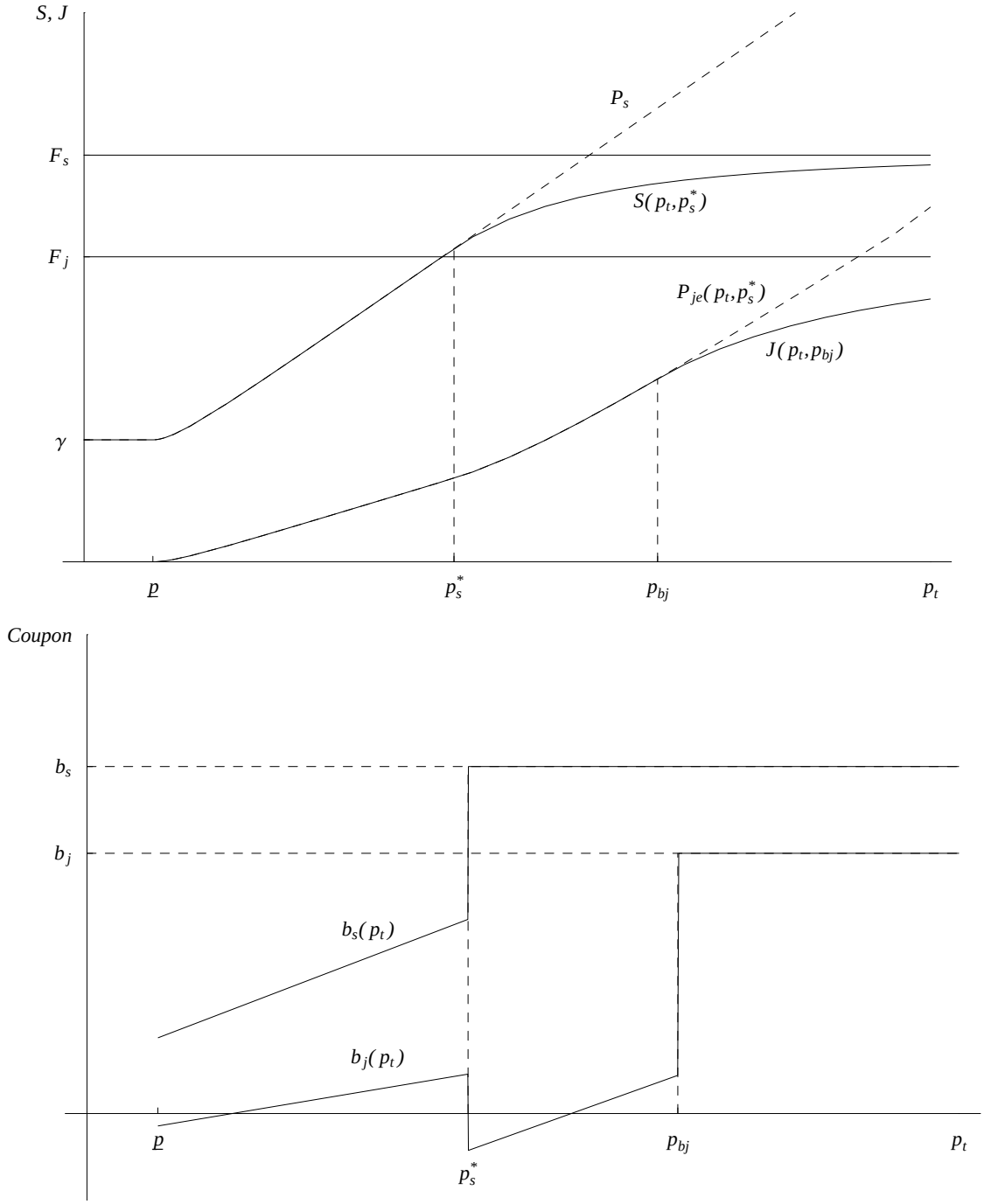

Figure 3: Equilibrium security values and coupons when $p_j^* > p_s^*$.

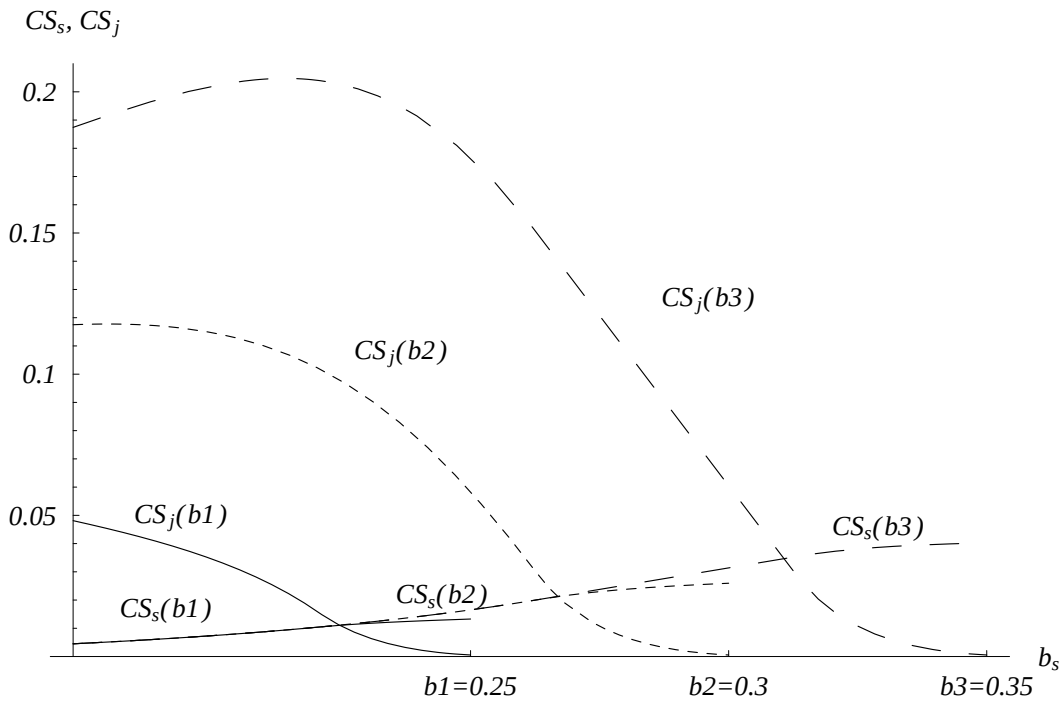

Figure 4: Senior renegotiation premium. The renegotiation premium, rp_s is shown as functions of the coupon $b_s \in [\gamma r, b = 0.25]$ with the total coupon, b, held constant. When the senior creditor is unimpaired, as in Panel A, the renegotiation premium can be positive or negative. It is positive when the priority violation more than offsets the benefits from collective renegotiation. If the senior creditor is impaired, as in Panel B, the renegotiation premium is always negative. The vertical line between Panel A and B corresponds to the level of b_s such that both creditors are jointly impaired. Baseline parameters are as follows: $r = 0.06$, $\mu = 0.02$, $\sigma = 0.15$, $c_e = c_s = c_j = 1$, $\gamma = 1$ and $\alpha = 0.6$.

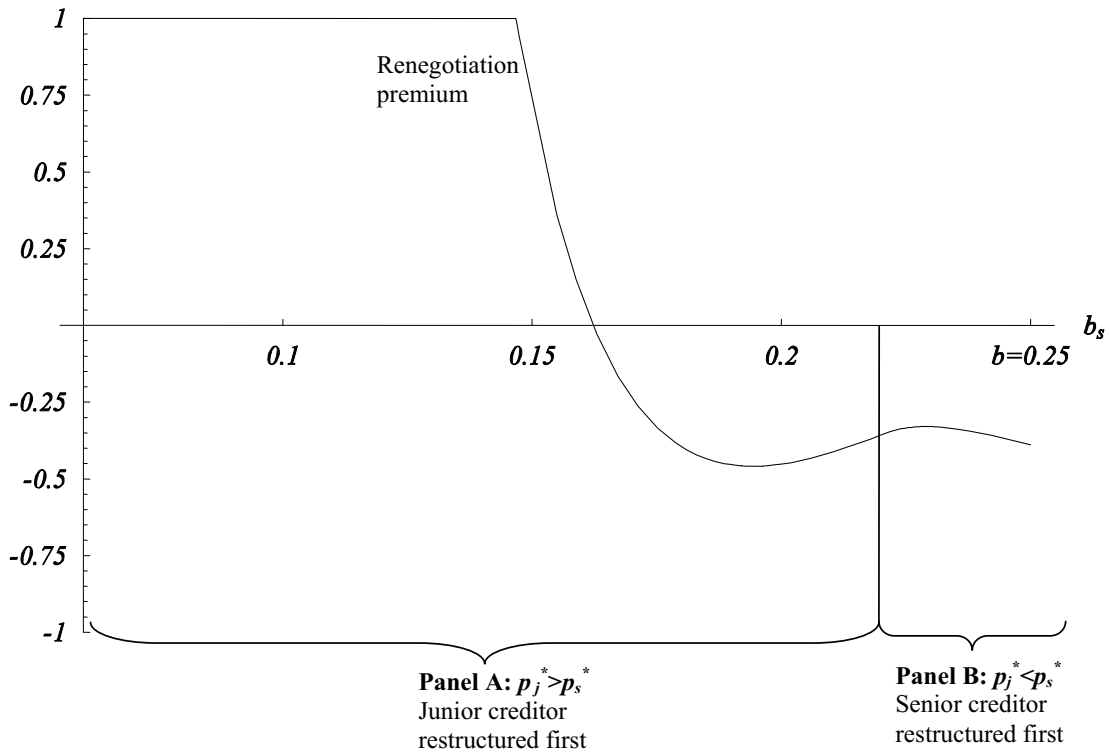

Figure 5: Senior and junior credit spreads for different level of total face value. The senior and junior credit spreads (CS_s and CS_j), as functions of b_s, are evaluated at different levels of total coupon b but at the same level of cash flows p_t (so they are directly comparable). The total coupon b takes values: $b1 = 0.25$, $b2 = 0.30$ and $b3 = 0.35$.