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Abstract

This paper investigates two elements of agency costs, namely the wealth-transfer and the value destruction problems, associated with the equity-conversion and writedown CoCo bonds. By focussing on the costs as those stemming from the deviation from absolute priority rule (DAPR), we derive the expressions for the CoCo bonds and show that both agency costs are aggravated under these structures. We demonstrate this by studying the case of Banca Monte dei Paschi bail-out in 2013. We argue that the replacement of government bail-out by bond-holder bail-in is akin to replacing moral hazard for agency costs, and that by encouraging bail-in structures the regulator prioritises the reduction of the former while ignoring the aggravation of the latter.

JEL Classification: D82; G21; G28; G32

Keywords: CoCo bond; bail-in; agency cost; incentives

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1. Introduction

The new financial regulation, namely Basel III, will have a strong impact not only on the nature of the banking business,[1] but especially on the capital structure of the banks. Amongst the new Basel III features,[2] the new style of subordinate debt stands out the most: the CoCo (Contingent Convertible) bonds. This is an intricate product which is becoming in vogue in a low yielding environment, as investors rush into high yield instruments, and banks take advantage of it by issuing a "cheap" (relative to the cost of equity of the banks) equity-like instruments that helps bolstering the capital and leverage ratios.[3] However, the lack of standardisation[4] and its complex nature means that its impact on banks' behaviour is not yet well understood.[5]

The aim of this paper is to investigate the main shortcomings of the new financial regulation in general, and the CoCos in particular, which is the overlook of the impact on the agency costs. Here two elements of agency costs are investigated. The first is the *wealth-transfer problem*, where the equityholders have an incentive to take on riskier projects because of the long option position held by them, and "sold" by the guarantors - the government in the government bail-out case and the bondholders in the bail-in cases. Higher volatility of the projects' values means higher option value, leading to wealth being transferred from the guarantors to the equityholders.[6] The second is

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[1] Banks retracting from high ROE but high RWA businesses, such as investment banking, structured credit, etc.

[2] E.g. Leverage Ratio, Net Stable Funding Ratio, etc.

[3] See for example, "CoCo Bond Feeding Frenzy Sends Yields Tumbling", *The Financial Times*, March 26, 2014.

[4] Such as equity conversion ratio, permanent/temporary writedowns/offs, high/low trigger and the embedded equity option (for equity conversion CoCos).

[5] See for example, "Regulators must act on coco bond risks", *The Financial Times*, May 7, 2014.

[6] Basically, the buyer of an option is then able to determine the volatility of the underlying asset. If this was possible in a financial market, then it would be an illegal market manipulation.

the *value destruction problem*, where in a falling solvency scenario the equityholders are tempted to "gamble-for-ressurection", i.e. sacrifice value for higher volatility. We investigate these as the unintended consequences of the deviation from absolute priority rules (DAPR).[7] Under the absolute priority rule (APR), bondholders should not bear losses until equityholders have been wiped out. Since the banks are systemic entities, the regulator has so far mostly rescued troubled banks ("bail-out") to minimise market disruptions and deposit runs, which have historically exacerbated the intrinsic moral hazard of the banking industry. To tackle this, the new financial regulation advocates for the bondholders to assume losses ("bail-in") on a going concern basis (hence "deviation" from APR). We agree that bail-in should replace bail-out as investors and not taxpayers should bear the losses of their bad investments. From the shareholders' standpoint there is not much difference between the two: when the regulator deems the bank's point of non-viability (PONV) in a bail-out (the point at which a bail-out is triggered) at the level where the CoCo triggers are set (7%) in a bail-in, the equityholders take the first loss down to 7% Core Tier 1 (CT1), for either taxpayers (bail-out) or bondholders (bail-in) to subsequently take any further losses to replenish the CT1 back to the pre-crisis level. Our view is that in the bail-out there is an embedded moral hazard problem, while in the bail-in there are agency costs that arises from the violation of the priority rule. The new bail-in set-up therefore simply replaces the former with the latter.

So far the literature has focused on CoCo styles (Albul, Jaffee and Tchistyi, 2013; Calomiris and Herring, 2011; Flannery, 2009; Glasserman and Nouri, 2012), triggers

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[7]The DAPR has been magnified more recently by the introduction of the Maximum Distributible Amount (MDA) that kicks in when the CT1 falls below the Combined Buffer (comprised by the Conservation, Countercyciclical and Systemic Buffer) which forces the bank to suspend coupons and dividends through a ratchet structure. The new Additionial Tier 1 (AT1) CoCos are coming with no dividend pusher / stopper, meaning that banks can suspend CoCos coupons whilst still paying out dividends in the event of the CT1 falling below the combined buffer.

(Prescott, 2011) and valuation (Pennachi, 2010). Others such as Berg and Kaserer (2011) have highlighted the potential perverse incentives that can dissuade shareholders from taking preemptive actions in a falling solvency scenario. They indirectly touch upon the agency cost of CoCo to discuss the potential over-investment problem that can stem from the increasing vega of the embedded option. They derive expressions for "Convert-to-surrender CoCo" and "Convert-to-steal CoCo" which are essentially the CoCo and the Writedown bonds investigated here. However they assume immediate full conversion of the bonds, while we allow for partial conversions and investigate the structures as those specifically implementing DAPR. Eberhart and Senbet (1993) also investigates the role of APR violation, but they assume the wealth-transfer to be a constant proportion of the firm value. Here we derive explicitly the values of the DAPR-induced wealth-transfer. This allows us to analytically demonstrate, first, that the vega of the equityholders' position increases with the level of CoCo bond as a proportion of the total debt (the wealth-transfer problem), and second, that "gamble-for-ressurection" is a compelling option for the bank (the value destruction problem). We argue that the latter leads to the bank straying from the Capital Market Line (CML) in the Capital Asset Pricing Model, in order to pursue low Sharpe Ratio projects at the expense of bondholders. This, we demonstrate, in a case study of the 2013 bail-out of Banca Monte dei Paschi di Siena. Finally, our analysis extends Moraux and Navatte's (2009) work on "admissible" debt-to-equity swap (DES), and show that the CoCo structure is a non-admissible DES. This is because its terms of restructuring are pre-set in advance, preventing bondholders from seeking a swap that somewhat compensates them for the forgiven debt related losses. This is shown to lead to higher agency costs for CoCo bail-in compared to the traditional DES. Overall, unlike the existing literature, our stress is on

the point that by encouraging bail-in structures, the regulator prioritises the reduction of moral hazard while ignoring the aggravation of the agency costs.

The paper is organised as follows. In Section 2, we describe comprehensively the four structures under comparison: no bail-out/bail-in, government bail-out, bail-in with CoCo bonds and with Writedown bonds. We show that in the bail-in cases, the DAPR inherent in the structures can be valued as "condor-like" structures[8] held by the equityholders. This allows us to derive expressions for the prices of the bail-in bonds. In Section 3, we dwell on the wealth-transfer element of the agency cost demonstrated by the rising vega of the equityholders' positions. We also identify the regulators' problem of a trade-off between banks' safety and the wealth-transfer agency cost. In Section 4, we show that bailing-in increases the value destruction incentive of the equityholders, represented by the falling delta-vega ratio of the equityholders' positions. In Section 5, we compare the CoCo bail-in structure with the traditional DES. In Section 6, we demonstrate our results in Section 4 by studying the case of Banca Monte dei Paschi bail-out in 2013. Finally in Section 7, we give concluding remarks.

2.   Comparison of Structures

We investigate the agency costs in the following four cases:

1. No bail-out/bail-in
2. Government bail-out
3. Bail-in with CoCo bonds
4. Bail-in with Writedown bonds

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[8] A *condor* is created by a combination of either a bull call spread with a bear call spread, or a bull put spread with a bear put spread.

Much of the analysis will be focussed on the CoCo bond. We consider a simple firm financed by equity capital and discount bonds with maturity T. The total face value of the bonds is F, which may include CoCo (face value F_C) or Writedown (F_W) bonds. The face value of the remaining straight bond is F_B. Therefore the firm can either have $F = F_B$ (no bail-out/in or government bail-out), $F = F_B + F_C$ (CoCo bail-in) or $F = F_B + F_W$ (Writedown bail-in). The total asset value at time T is $V_T = D_T + E_T$, where D_T and E_T represent the stakeholders' (i.e. the debtholders' and the equityholders') shares of V_T. The bail-in bonds trigger at the capital ratio of τ. In line with the reality, we set a minimum capital ratio of $\underline{E}$ which the regulators insist on after a trigger has occurred. Similarly, in a bail-out where the government ends up with some preference shares, we set a minimum common equity floor of $\underline{E_C}$.

In this section, for each of the above cases we derive the stakeholders' payoffs at time T as a function of V_T, and their present values using the Merton (1974) framework given the asset volatility, σ. The derivation details are given in Appendix A, where, for demonstration purpose, we consider a representative firm with the following parameter values when relevant: $F = 90$, $F_C = 20$, $F_W = 20$, $\tau = 7\%$, $\underline{E} = 10\%$ and $\underline{E_C} = 5\%$.

2.1. No Bail-out / Bail-in

It is well established in the literature that, without any possibilities of a bail-out/in the equityholders hold a long call option at strike price F, while the bondholders' position is the bond minus a put option of the same strike price. Their payoffs are then,

$$PayOff_D^N = \min\left[V_T, F\right]$$
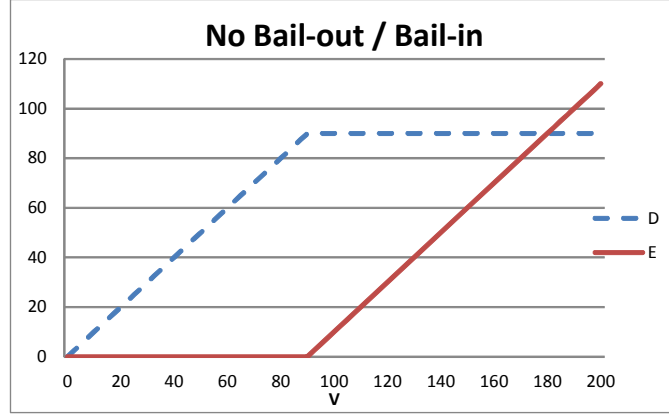$$PayOff_E^N = \max\left[V_T - F, 0\right].$$

(1)

Figure 1: No Bail-out / Bail-in: $F = 90$

Fig.1 depicts the payoffs for the example with $F = 90$ as analysed in Appendix A.1.

The Merton valuation of the debt and equity holdings at time $t = 0$ are,

$$V_D^N = Fe^{-rT} - P(F)$$
$$V_E^N = C(F)$$

(2)

where $C(K)$ and $P(K)$ are the prices of call and put options with strike price K,

$$C(K) = V_0 N(d_1(K)) - Ke^{-rT}N(d_2(K))$$
$$P(K) = -V_0 N(-d_1(K)) + Ke^{-rT}N(-d_2(K))$$
$$\text{with } d_1(K) = \frac{\ln\left(\frac{V_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \ , \ d_2(K) = d_1 - \sigma\sqrt{T}$$

(3)

and r is the risk-free rate, T is the bond's time to maturity and σ is the asset volatility.

2.2. Government Bail-out

The government bail-out comes in twofold. First, the balance sheet is restored to maintain the minimum capital ratio $\underline{E}$. With the debtholders fully protected at their face value F, the balance sheet has a floor level of $\frac{F}{1-\underline{E}}$. When the firm's value V_T falls below
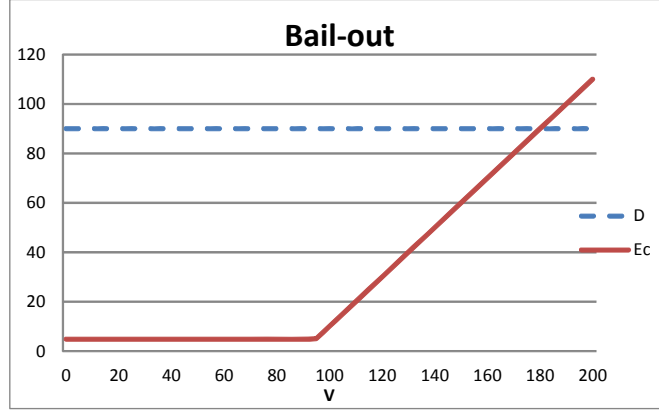
7

Figure 2: Government Bail-out: $F = 90$, $\underline{E} = 10\%$, $\underline{E_C} = 5\%$

this, a bail-out is initiated in the form of an injection of preference shares held by the government. Secondly for values of V_T below $F + \frac{F}{1-\underline{E}}\underline{E_C}$, the government also ensures a minimum common equity floor of $\underline{E_C}$. With the equityholders no longer bearing the loss, the taxpayers provide the shortfall $V_T - \left(F + \frac{F}{1-\underline{E}}\underline{E_C}\right)$. The payoffs at T for the debtholders, the equityholders and the government for its preference shares are,

$$
\begin{aligned}
PayOff_D^{BO} &= F \\
PayOff_E^{BO} &= \max\left[V_T - F, \frac{F}{1-\underline{E}}\underline{E_C}\right] \\
PayOff_{E_P}^{BO} &= \max\left[\frac{F}{1-\underline{E}} - V_T, 0\right] - \max\left[\left(F + \frac{F}{1-\underline{E}}\underline{E_C}\right) - V_T, 0\right].
\end{aligned}
\tag{4}
$$

The payoffs of the debtholders and the equityholders for the example firm in Appendix A.2 are depicted in Fig.2. The present values of the debt and equity holdings are,

$$
\begin{aligned}
V_D^{BO} &= Fe^{-rT} \\
V_E^{BO} &= \frac{F\underline{E_C}}{1-\underline{E}}e^{-rT} + C\left(F + \frac{F\underline{E_C}}{1-\underline{E}}\right),
\end{aligned}
\tag{5}
$$

with $C(K)$ as given in Eq.(3). When $\underline{E_C} = 0$ we recover V_E^N in Eq.(2) for the equity-holders, while the debtholders still benefit from the government guarantee.

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2.3. Bail-in with CoCo Bonds

We now investigate the CoCo bond bail-in structure in detail. In contrast to the common practice in the literature, here we allow partial conversion of the bond. The debt F consists of F_C of CoCo bond and F_B of straight bond. The bail-in triggers when the equity capital ratio, $\frac{V_T - F}{V_T}$, falls below the trigger rate τ, or when $V_T \leq \frac{F}{1-\tau}$. Initially, as with the government bail-out, the bail-in ensures that the minimum capital ratio $\underline{E} \geq \tau$ is reattained. For $\frac{F_B}{1-\underline{E}} \leq V_T \leq \frac{F}{1-\tau}$ this means that, of $(1 - \underline{E}) V_T$ of the liability, $(1 - \underline{E}) V_T - F_B$ remains as the unconverted portion of the CoCo bond. Thus $F_C - [(1 - \underline{E}) V_T - F_B]$ of the CoCo bond is converted into $E_D = (\underline{E} - \tau) V_T$ of equity held by the debtholders. This results in their loss of $(\underline{E} - \tau) V_T - \{F_C - [(1 - \underline{E}) V_T - F_B]\} = F - (1 - \tau) V_T$. For $V_T < \frac{F_B}{1-\underline{E}}$, the minimum capital ratio $\underline{E}$ is unattainable even with a full conversion of the CoCo bond. Assuming no outsider capital injection or forced conversion of the straight bond,[9] for $\frac{F_B}{1-\tau} \leq V_T < \frac{F_B}{1-\underline{E}}$ the entire CoCo bond is converted into $E_D = (1 - \tau) V_T - F_B$ of equity held by the debtholders, while the equityholders maintain $E_C = \tau V_T$ of equity. For V_T below $\frac{F_B}{1-\tau}$, the CoCo bondholders are totally wiped out $(E_D = 0)$ and the equityholders' holding is written down. Finally when $V_T < F_B$ the firm becomes insolvent and the straight bond holders become the residual claimant. The details are given in the Appendix A.3. The payoffs, which for debtholders is the total of straight and CoCo bonds plus their equity, are,

$$PayOff_D^C = \min\left[F, (1 - \tau) V_T\right] + \max\left[F_B - (1 - \tau) V_T, 0\right] - \max\left[F_B - V_T, 0\right]$$

$$PayOff_E^C = \max\left[V_T - F, \tau V_T\right] - \max\left[F_B - (1 - \tau) V_T, 0\right] + \max\left[F_B - V_T, 0\right].$$

(6)

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[9]In reality burden sharing may occur, where the straight bonds are forced to convert or written down to attain the minimum capital ratio $\underline{E}$.
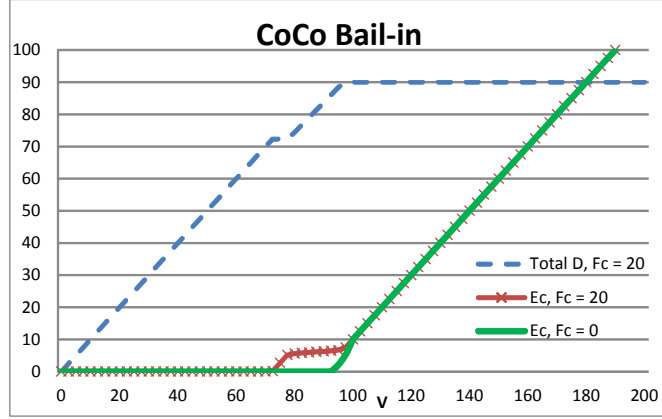
Figure 3: Bail-in with $\tau = 7\%$, $\underline{E} = 10\%$, $F = 90$ and (a) $F_C = 20$, (b) $F_C = 0$

Fig.3 shows the payoffs for our example firm. Their present values are,

$$
\begin{aligned}
V_D^C &= Fe^{-rT} - (1-\tau)P\left(\tfrac{F}{1-\tau}\right) + (1-\tau)P\left(\tfrac{F_B}{1-\tau}\right) - P(F_B) \\
V_E^C &= \tau V_0 + (1-\tau)C\left(\tfrac{F}{1-\tau}\right) - (1-\tau)P\left(\tfrac{F_B}{1-\tau}\right) + P(F_B).
\end{aligned}
\tag{7}
$$

We recover V_D^N and V_E^N when $\tau = F_C = 0$. V_E^C derived in Eq.(7) differs from the expression derived for "Convert-to-surrender CoCo" in Berg and Kaserer (2011) in two ways. First, they assume 100% conversion of the CoCo bond when triggered. Here we allow partial conversion. Second, they assume the whole liability to be CoCo bonds, i.e. the equityholders are never wiped out for $V_T > 0$. Here our assumption of $F_C < F$ means that once the CoCo bond is wiped out, the normal absolute priority rule (APR) resumes where the equityholders' holdings are written down ahead of the straight bonds.

One way of viewing the CoCo bail-in effect is to regard the difference between V_E^C in Eq.(7) and V_E^N in Eq.(2) as the wealth-transfer induced by the introduction of deviation from absolute priority rule (DAPR). In Fig.3, this is the area between the E_C payoffs for $F_C = 0$ and $F_C = 20$. Eberhart and Senbet (1993) also investigates the role of APR violations in reducing agency conflicts between bondholders and shareholders. However
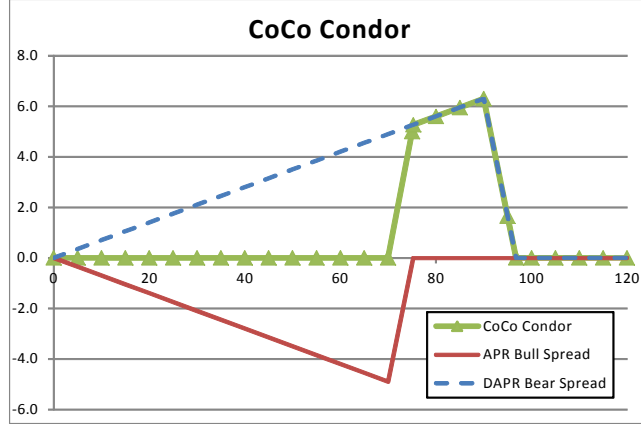
10

Figure 4: CoCo Condor with $F_B = 70$, $F_C = 20$ and $\tau = 7\%$

they assume the wealth-transfer to be a constant proportion of the firm value, and argue that when the firm is in distress the negative vega of the assumed wealth-transfer partly offsets the positive vega of the equityholders' position, hence mitigating the agency cost incentive. Here we are able to explicitly derive the amount of DAPR-induced wealth-transfer as $V_E^C - V_E^N$. To do this, first use the put-call parity relation to re-express V_E^C in Eq.(7) as,

$$V_E^C = C(F) + \left[(1-\tau)P\left(\frac{F}{1-\tau}\right) - P(F)\right] - \left[(1-\tau)P\left(\frac{F_B}{1-\tau}\right) - P(F_B)\right]. \quad (8)$$

Comparing this to the no bail-out/in V_E^N in Eq.(2), we can first see that the equityholders payoff is improved by a bear spread-like protection, $(1-\tau)P\left(\frac{F}{1-\tau}\right) - P(F)$. This represents the DAPR induced by the introduction of the CoCo bond. The bull spread-like structure $-\left[(1-\tau)P\left(\frac{F_B}{1-\tau}\right) - P(F_B)\right]$ then reinstates the APR once the CoCo bond is wiped out. Together they create a "condor-like" structure, which we will call the "*CoCo condor*", depicted in Fig.4. Below we argue that for values of F_C sufficiently large, the vega of this CoCo condor increases as the firm approaches its distress level, hence fur-

ther aggravating the wealth-transfer incentive. Note it may be likely that in reality, in the extreme case that the CoCo bonds are totally wiped out, the regulator will step in and enforce either the writedown of the remaining straight bonds or a debt-to-equity swap. In such cases the APR bull spread does not exist, and instead of the condor-like structure we have a *CoCo bear spread* reflecting just the DAPR.

As shown in Fig.5 the payoff of the CoCo bond has a discontinuous drop at τ,

$$
\begin{aligned}
PayOff_{F_C} =\ & F_C - \frac{\underline{E} - \tau}{1 - \underline{E}} F \chi_{V_T < \frac{F}{1-\tau}} \\
& - (1 - \underline{E}) \max\left[\frac{F}{1-\tau} - V_T, 0\right] + (1 - \underline{E}) \max\left[\frac{F_B}{1 - \underline{E}} - V_T, 0\right],
\end{aligned}
$$

where $\chi_{V_T < \frac{F}{1-\tau}} = \begin{cases} 1 &, \quad V_T < \frac{F}{1-\tau} \\ 0 &, \quad V_T \geq \frac{F}{1-\tau} \end{cases}$ is an indicator function. Then,

Proposition 1 (CoCo Bond Price) *When the total bond issued by the firm is F, the price of the zero coupon CoCo bond with face value F_C, maturity T, trigger capital ratio τ and regulatory minimum capital ratio $\underline{E}$ is,*

$$
CC_0 = F_C e^{-rT} - \left(\frac{\underline{E} - \tau}{1 - \underline{E}} F\right) B_P\left(\frac{F}{1-\tau}\right) - (1 - \underline{E}) P\left(\frac{F}{1-\tau}\right) + (1 - \underline{E}) P\left(\frac{F_B}{1 - \underline{E}}\right)
$$

(9)

where $B_P(K)$ is the price of the binary put option with unit payout at strike K,

$$
B_P(K) = e^{-rT} N(-d_2(K)),
$$
(10)

$P(K)$ is the put option price given by Eq.(3).

We can immediately see from Eq.(9) that, in defining $\delta = \frac{V_T - F}{V_T} - \tau$ as the "*distance-*
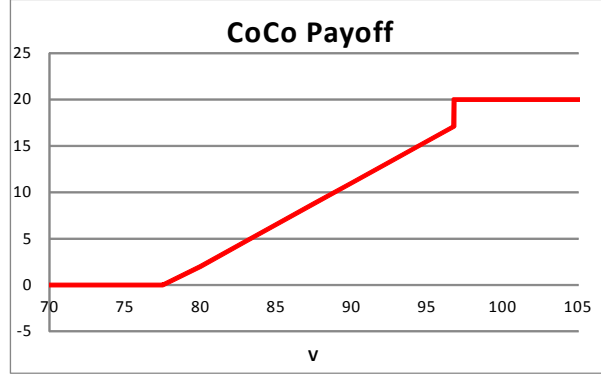
Figure 5: CoCo bond payoff with $\tau = 7\%$, $\underline{E} = 10\%$, $F_C = 20$ and $F = 90$

to-trigger", then $\frac{\partial CC_0}{\partial V_T} > 0$ (positive delta) and $\frac{\partial V_T}{\partial \delta} > 0$ imply $\frac{\partial CC_0}{\partial \delta} > 0$, suggesting that the larger the distance-to-trigger, the higher the value of the CoCo bond.

2.4.   Bail-in with Writedown Bonds

Finally, we consider Writedown bonds, where the debtholders hold face value F_B of straight bonds and F_W of the Writedown bonds, where $F = F_B + F_W$. As with the CoCo bail-in, the Writedown bail-in triggers once $V_T \leq \frac{F}{1-\tau}$. The entire Writedown bond is then writtendown/off (temporarily or permanently). The payoffs are, where for bondholders it is the total of straight and Writedown bonds,

$$PayOff_D^W = \min\left[V_T, F_B\right] + F_W \chi_{V_T > \frac{F}{1-\tau}}$$
$$PayOff_{EC}^W = \max\left[V_T - F_B, 0\right] - F_W \chi_{V_T > \frac{F}{1-\tau}}. \tag{11}$$

Again $\chi_{V_T > \frac{F}{1-\tau}}$ is an indicator function. Fig.6 depicts the payoffs of the example given in Appendix A.4. The present values of the debt and equity holdings are,

$$V_D^W = F_B e^{-rT} - P\left(F_B\right) + F_W B_C \left(\frac{F}{1-\tau}\right)$$
$$V_E^W = C\left(F_B\right) - F_W B_C \left(\frac{F}{1-\tau}\right) \tag{12}$$

13
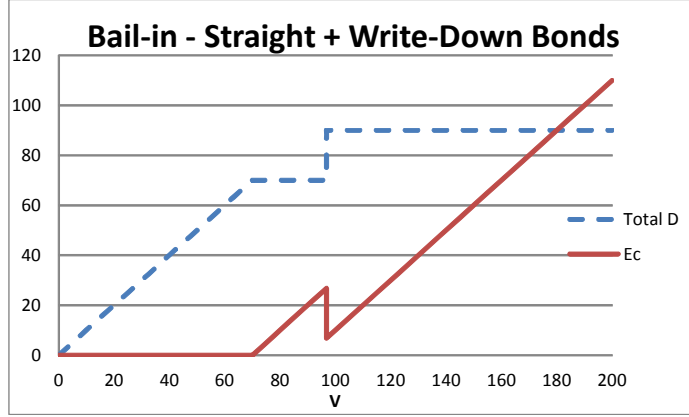
Figure 6: Write-down: $F_B = 70$, $F_W = 20$, $\tau = 7\%$ and $\underline{E} = 10\%$.

where $B_C(K)$ is the unit binary call option with strike price K, given by,

$$B_C(K) = e^{-rT} N\left(d_2(K)\right). \tag{13}$$

V_E^W in Eq.(12) is the same as the expression for "Convert-to-steal CoCo" in Berg and Kaserer (2011), with the additional specification that the trigger point is at $\frac{F}{1-\tau}$.

Analogous to the CoCo bail-in analysis, V_E^W can be re-expressed as,

$$V_E^W = C(F) + F_W B_P\left(\frac{F}{1-\tau}\right) - [P(F) - P(F_B)]. \tag{14}$$

As before the extra option positions compared to V_E^N in Eq.(2) can be interpreted as the *DAPR binary put* and the *APR bull spread*, resulting in a *Writedown condor* position as shown in Fig.7.[10] We argue below that the positive vega of this condor for sufficiently large values of F_W aggravates the agency costs. Again in reality it may be likely that the regulators would enforce writedown of the remaining straight bond once the Writedown

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[10]The fact that here the DAPR structure is a put binary option rather than a bear spread, as was in the CoCo bail-in case, stems from the fact that we assume a 100% writedown of the Writedown bond once it is triggered. In contrast we assumed a partial conversion with the CoCo bond.
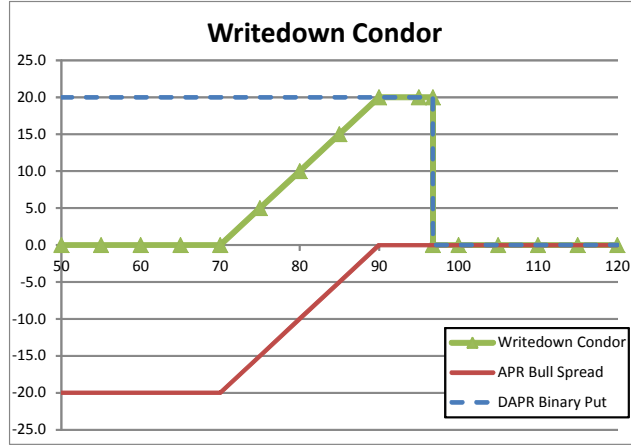
Figure 7: Writedown Condor with $F_B = 70$, $F_W = 20$ and $\tau = 7\%$

bond is wiped out. In this case the APR bull spread would be forfeited, resulting in a *Writedown binary put* rather than the condor above.

The Writedown bond payoff is that of a binary call:

$$PayOff_W = F_W \chi_{V_T > \frac{F}{1-\tau}}.$$

Proposition 2 (Writedown Bond Price) *When the total bond issued by the firm is* F, *the price of the zero coupon Writedown bond with face value* F_W, *maturity* T *and trigger capital ratio* τ *is,*

$$WD_0 = F_W B_C \left(\frac{F}{1-\tau} \right).$$

where $B_C(K)$ *is the unit binary call option price given by Eq.(13).*

3.   Agency Cost: Wealth-Transfer Problem

We distinguish two types of agency costs associated with the over-investment problems in bail-out / in structures:

1. *Wealth-transfer problem.* This is when the equityholders have an incentive for higher risk-taking, normally represented by the vega of their option position.

2. *Value-destruction.* Eberhart and Senbet (1993) state, "Risk-shifting can enhance equity value even when higher risk projects are of lower value, implying that investment decisions can be distorted away from firm value maximisation." When negative NPV projects are still beneficial to the equityholders due to their convex payoff and the project's higher volatility, the reduction in the firm's total value represents this type of agency cost.

We investigate these in turn in this section and the next. For the purpose of the technical analyses, we assume $r > \frac{\sigma^2}{2}$ for the remainder of the paper.

As common in the literature (e.g. Eberhart and Senbet, 1993; Berg and Kaserer, 2011), we investigate the vega of the equityholder position as a measure of their incentive to take on riskier projects. The vegas for each of the above cases are,

$$
\begin{aligned}
Vega_E^N &= V_0\sqrt{T}N'\left(d_1\left(F\right)\right) \\
Vega_E^{BO} &= V_0\sqrt{T}N'\left(d_1\left(F + \frac{FE_C}{1-E}\right)\right) \\
Vega_E^C &= V_0\sqrt{T}\left[\left(1-\tau\right)N'\left(d_1\left(\frac{F}{1-\tau}\right)\right) - \left(1-\tau\right)N'\left(d_1\left(\frac{F_B}{1-\tau}\right)\right) + N'\left(d_1\left(F_B\right)\right)\right] \\
Vega_E^W &= V_0\sqrt{T}N'\left(d_1\left(F_B\right)\right) + \frac{1}{\sigma}d_1\left(\frac{F}{1-\tau}\right)F_W e^{-rT}N'\left(d_2\left(\frac{F}{1-\tau}\right)\right)
\end{aligned}
\tag{15}
$$
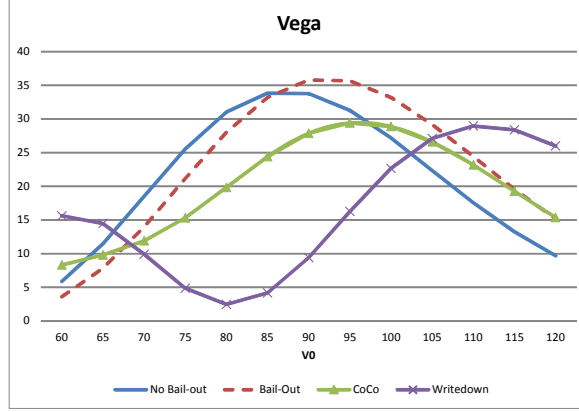
Figure 8: Vega vs V_0 when $F = 90$, $F_C = F_W = 20$, $\tau = 7\%$ and $\underline{E} = 10\%$

where $N'(d_1(K)) = \frac{1}{\sqrt{2\pi}} e^{-\frac{[d_1(K)]^2}{2}}$ for strike K. These are depicted in Fig.8. The graph compares the incentives for the equityholders to take on riskier projects at different values of V_0 between the four structures. Several observations can be made:

1. *With no bail-out or bail-in, the incentive for higher risk taking increases as the firm's asset value falls towards the critical value F.*

   In Fig.8 the critical value for no bail-out/in is $F = 90$. This basically states that the vega of a call option increases as the option approaches at-the-money (ATM). More technically, the no bail-out/in vega curve is quasi-concave, as e^{-x^2} is a quasi-concave function and $d_1(F)$ is a monotonically increasing function of V_0. Its maximum occurs at,

$$\frac{\partial Vega_E^N}{\partial V_0} = -\frac{1}{\sigma} d_2(F) N'(d_1(F)) = 0 \Leftrightarrow V_0 = F e^{-\left(r - \frac{\sigma^2}{2}\right)T}. \tag{16}$$

   $\frac{\partial Vega}{\partial V_0}$, sometimes called the *vanna*, is negative for $V_0 > F e^{-\left(r - \frac{\sigma^2}{2}\right)T}$, or $Vega_E^N$ increases as V_0 falls towards F (and unambiguously so for $r > \frac{\sigma^2}{2}$). The wealth-transfer happens when the equityholders choose higher σ projects, resulting in an

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increase in the value of their call option, and an equal fall in the value of the debtholders' position due to the rise in the value of their short put option.

2. *For the asset value above the trigger point, the risk-taking incentive is higher for the government bail-out case than for the no bail-out/in case.*

In Fig.8 the trigger point for the government bail-out is $\frac{F}{1-\underline{E}} = 100$. Compare the vegas for the bail-out and no bail-out/in cases: $Vega_E^{BO} > Vega_E^N$ if and only if,

$$N'\left(d_1\left(F + \frac{FE_C}{1-\underline{E}}\right)\right) > N'\left(d_1\left(F\right)\right) \Leftrightarrow V_0 > F\left(1 + \frac{E_C}{1-\underline{E}}\right)^{\frac{1}{2}} e^{-\left(r+\frac{\sigma^2}{2}\right)T}.$$

The trigger point $\frac{F}{1-\underline{E}}$ is strictly greater than $F\left(1 + \frac{E_C}{1-\underline{E}}\right)^{\frac{1}{2}} e^{-\left(r+\frac{\sigma^2}{2}\right)T}$,[11] and hence the statement is proved. Intuitively the existence of the minimum capital ratio E_C shifts higher the strike price of the equityholders' call option by a factor $\left(1 + \frac{E_C}{1-\underline{E}}\right)$. Hence for values $V_0 \geq \frac{F}{1-\underline{E}}$ the call option is closer to ATM than in the no bail-out/in case, resulting in a higher vega. Note here with the debtholders' holdings protected at $V_D^{BO} = Fe^{-rT}$, the wealth-transfer is from the government (and ultimately the taxpayers) to the equityholders.

3. *For the asset value above the trigger point the risk-taking incentive increases with the level of CoCo bond: the incentive is lower than that of the no bail-in/out case for low levels of the CoCo bond, and higher for high levels of the CoCo bond.*

In Fig.8, the trigger point for the CoCo bail-in is $\frac{F}{1-\tau} = 96.77$. As was shown in Section 2.3, issuing of CoCo bond results in an additional long CoCo condor

---

[11] It suffices to show that $\frac{F}{1-\underline{E}} > F\left(1 + \frac{E_C}{1-\underline{E}}\right)^{\frac{1}{2}}$. This is true when $\frac{1}{1-\underline{E}} > \left(\frac{1-\underline{E}+E_C}{1-\underline{E}}\right)^{\frac{1}{2}} \Leftrightarrow 1 > \left(1 - \underline{E} + E_C\right)\left(1 - \underline{E}\right)$, which is true as $\underline{E} \geq E_C$.
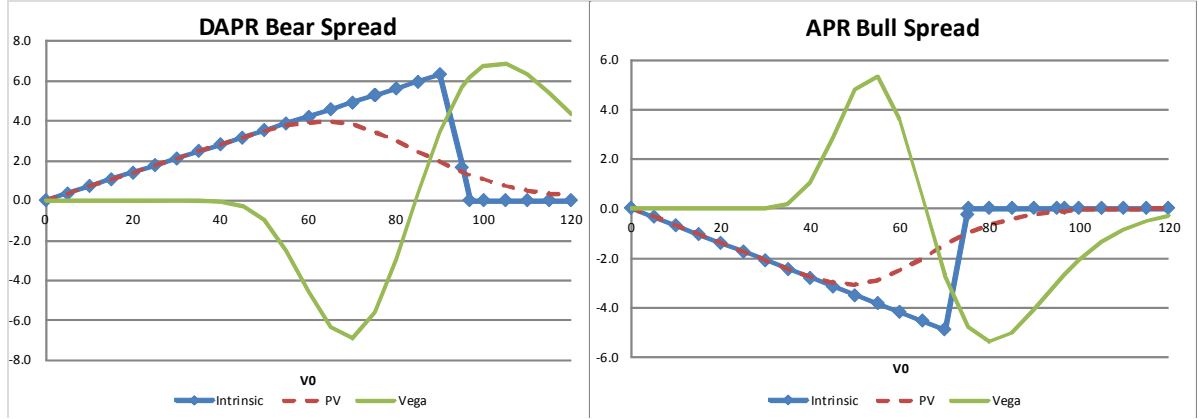
Figure 9: (a) DAPR Bear Spread and (b) APR Bull Spread, with $F_B = 70$, $F_C = 20$ and $\tau = 7\%$

position for the equityholders. As explained this condor is constructed by a DAPR bear spread and an APR bull spread. Fig.9 shows the vega of these respectively. The strike prices of the DAPR bear spread are $\frac{F}{1-\tau} = 96.77$ (i.e. the trigger point) and $F = 90$, while those of the APR bull spread are $\frac{F_B}{1-\tau} = 75.27$ and $F_B = 70$. As depicted, at the trigger point the vega of the DAPR bear spread is positive, while that of the APR bull spread is negative. When the level of CoCo bond F_C is small (i.e. F_B is closer to F), latter more than offsets the former, making the vega of the CoCo condor close to the trigger point negative. This makes the net vega of the equityholders' position (call option plus the CoCo condor) less than that in the no bail-out/in case. At a higher level of CoCo bond the bull spread is more out-of-the-money (OTM), resulting in the CoCo condor vega becoming positive at and above the trigger point, as depicted in Fig.10, and increasingly so as F_C is increased. This implies a higher incentive for risk-taking for the equityholders than in the no bail-out/in case. In summary, there is therefore *a trade-off between the safety of the banks, achieved by a higher proportion of the debt made up by CoCo bonds,*
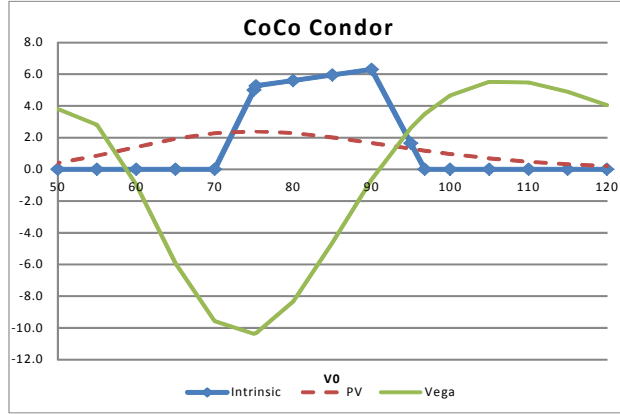
Figure 10: CoCo Condor with $F_B = 70$, $F_C = 20$ and $\tau = 7\%$

and the higher agency cost, which is the incentive for higher risk-taking. This is an issue we believe so far has not been considered by the regulators. Finally note, if in reality, the regulator forces writedown of straight bonds when the CoCo bonds are wiped out, then with no APR bull spread the vega increase, and hence the higher agency cost, at and above the trigger point is unambiguous for all values of F_C.

4. *For the asset value above the trigger point the risk-taking incentive increases with the level of Writedown bond: the incentive is lower than that of the no bail-in/out case for low levels of the Writedown bond, and higher for high levels of the bond.*

This is analogous to the CoCo case. As shown in Fig.11 the vega of the DAPR put binary is positive around and above the trigger point ($\frac{F}{1-\tau} = 96.77$) while that of the APR bull spread is negative. When F_W is small the latter dominates the former making the net vega of the Writedown condor negative at the trigger point, while for larger F_W the net vega becomes positive as depicted in Fig.12. This again implies a trade-off between the bank's safety and the higher agency cost which the
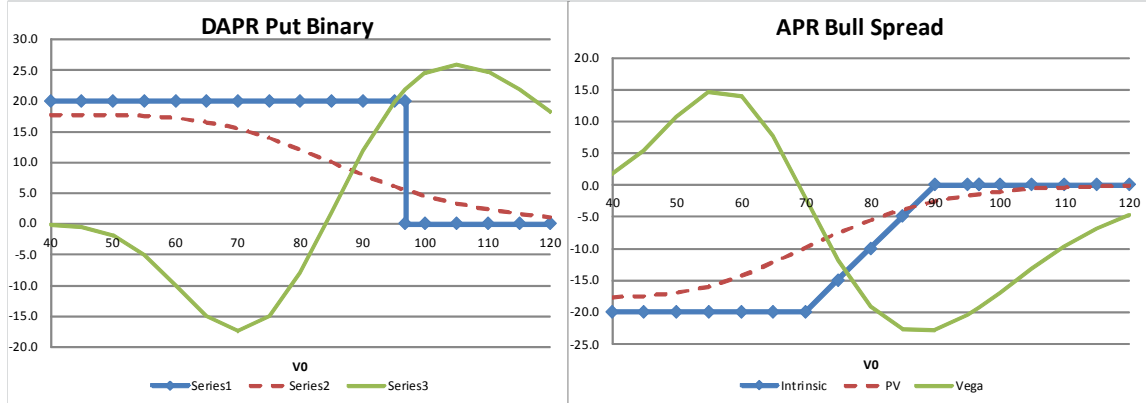
Figure 11: (a) DAPR Put Binary and (b) APR Bull Spread, with $F_B = 70$, $F_W = 20$ and $\tau = 7\%$

regulator must consider. As with the CoCo case, if the regulator enforces violation of APR beyond that provided by the Writedown bond, then the increase in the agency cost is again unambiguous for all values of F_W.

To conclude, with sufficiently large CoCo or Writedown bonds the incentive for higher risk-taking is increased, exacerbating the wealth-transfer element of the agency cost.
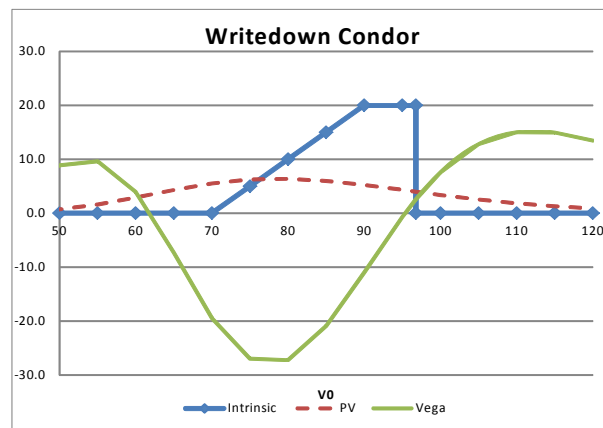
Figure 12: Writedown Condor with $F_B = 70$, $F_W = 20$ and $\tau = 7\%$

4. Value Destruction

Value destruction agency cost occurs when the equityholders do not follow value maximisation for the firm. This is a Principal-Agent problem where the interest of the decision makers (the equityholders) does not align with that of the firm.

To investigate this, let there be a discrete set of projects defined by their expected outcome $E\left[V_T^i\right]$ and the return volatility σ^i. Let the market price of risk be λ. Then the present value of each project is,

$$V_0^i = e^{-r^i T} E\left[V_T^i\right] \text{ , where } r^i = r^f + \lambda \sigma^i \tag{17}$$

where r^i is the required rate of return of project i and r^f is the risk-free rate. Under value maximisation the firm would choose project m such that,

$$V_0^m = \max\left\{V_0^i\right\}.$$

On the other hand, under no bail-out the equityholders project m^N such that,

$$V_0^{m^N} = \max\left\{V_E^N\left(V_0^i\right)\right\}, \text{ where } V_E^N\left(V_0^i\right) = C\left(V_0^i, F, \sigma^i\right)$$

with $C\left(.\right)$ as given in Eq.(2), where the arguments now also specify the underlying asset value and its volatility as well as its strike price. When $m^N \neq m$, $V_0^{m^N} < V_0^m$, and hence there is value destruction.

The value destruction problem arises from the fact that the firm value depends on the spot value (estimated as the present value in Eq.(17)), but not on the asset volatility

beyond its effect on the required rate of return r^i, while for the equityholders their value increases with higher σ. Value destruction results when the reduction in the equityholders' value due to the lower spot asset value (the delta effect) is more than offset by the increase in the value due to the higher volatility (the vega effect). The degree of this effect can therefore be represented by the relative size of the two, which we denote η:

$$\eta = \frac{\Delta}{Vega}.$$

The smaller the η of the structure, the more likely there will be value destruction.

We now make the following observations:

1. *For the asset value above the trigger point, the value destruction is more likely for the government bail-out case than for the no bail-out case.*

   Compare the delta of the two structures:

   $$\Delta_E^N = N\left(d_1\left(F\right)\right) > \Delta_E^{BO} = N\left(d_1\left(F + \frac{FE_C}{1 - \underline{E}}\right)\right), \forall V_0. \qquad (18)$$

   We have already established that for $V_0 > \frac{F}{1-\underline{E}}$, $Vega_E^N < Vega_E^{BO}$. Hence $\eta^N > \eta^{BO}$ above the trigger point.

2. *The value destruction is more likely for CoCo bail-in than for no bail-out for sufficiently high levels of CoCo bond.*

   Given the CoCo condor

   $$CCcdr = \left[(1-\tau)\,P\left(\frac{F}{1-\tau}\right) - P\left(F\right)\right] - \left[(1-\tau)\,P\left(\frac{F_B}{1-\tau}\right) - P\left(F_B\right)\right], \qquad (19)$$

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the delta of this is,

$$\Delta_{CCcdr} = \left[N\left(-d_1\left(F\right)\right) - \left(1-\tau\right) N\left(-d_1\left(\frac{F}{1-\tau}\right)\right) \right]$$
$$- \left[N\left(-d_1\left(F_B\right)\right) - \left(1-\tau\right) N\left(-d_1\left(\frac{F_B}{1-\tau}\right)\right) \right]. \qquad (20)$$

The sufficient condition for $\frac{\partial}{\partial K}\left[N\left(-d_1\left(K\right)\right) - \lambda N\left(-d_1\left(\frac{K}{\lambda}\right)\right) \right] < 0$ when $\lambda \in (0,1)$

is that $V_0 > \frac{K}{\lambda}$ (see Appendix B). Thus for $V_0 > \frac{F}{1-\tau}$, $\Delta_{CCcdr} < 0$. This can also

be seen from the downward-sloping PV curve in Fig.10. Hence $\Delta_E^C < \Delta_E^N$ above

the trigger point. As $Vega_E^C > Vega_E^N$ for sufficiently large values of F_C (shown

before), this implies that $\eta^N > \eta^C$ for those values of F_C. Intuitively, CoCo bail-in

aggravates the value destruction agency cost through two channels: by decreasing

the benefit to the equityholders of a higher firm value (decreased delta), while also

increasing their benefit of a higher volatility (increased vega). However, where the

regulator enforces DAPR beyond that provided by CoCo, the value destruction

problem is worse than in no bail-out/in case for all values of F_C unambiguously.

3. *The value destruction can be large for Writedown bail-in for larger face values of*

   Writedown bonds.

Given the Writedown condor

$$WDcdr = F_W B_P\left(\frac{F}{1-\tau}\right) - \left[P\left(F\right) - P\left(F_B\right)\right], \qquad (21)$$

the delta of this is,

$$\Delta_{WDcdr} = -\frac{F_W e^{-rT}}{V_0 \sigma \sqrt{T}} N'\left(-d_2\left(\frac{F}{1-\tau}\right)\right) + \left[N\left(-d_1\left(F\right)\right) - N\left(-d_1\left(F_B\right)\right)\right]. \quad (22)$$
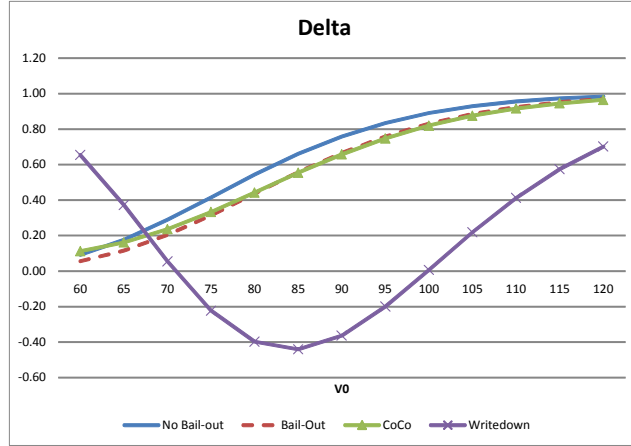
Figure 13: Delta vs V_0 when $F = 90$, $F_C = F_W = 70$ and $\tau = 7\%$

In Appendix C we prove that this is negative for $V_0 > \frac{F}{1-\tau}$, which is also shown in Fig.10 by the downward-sloping PV curve. Hence $\Delta_E^W < \Delta_E^N$ above the trigger point. As we know that $Vega_E^W > Vega_E^N$ for sufficiently large F_W, this implies that $\eta^N > \eta^W$ for those values of F_W. Thus Writedown bail-in also aggravates the value destruction agency cost via both decreasing delta and increasing vega. Furthermore, in this case for larger values of F_W, Δ_E^W can turn negative (see Fig.13 where $F_W = 70$). This is an extreme outcome where the equityholders are better off destroying the value of the firm even without the benefit from higher volatility.

To conclude, not only do introduction of CoCo or Writedown bonds increase the incentive for wealth-transfer by increasing the vega of the equityholders' position as described in Section 3, we have established in this section that it also increases the incentive for value destruction by decreasing the delta, hence aggravating the delta-vega ratio η (i.e. higher incentive to sacrifice value in order to "gamble-for-resurrect"). This result may be observable in reality if banks deliberately opt for low Sharpe Ratio investments: a case study showing this is given in Section 6.

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5.   CoCo Bond as Non-admissible Debt-to-Equity Swap

In the above sections we concluded that the CoCo and Writedown bond bail-in structures have inherently higher agency costs than under no bail-out/in. However, in reality banks are rarely allowed to become insolvent, with restructuring occurring long before the asset value is allowed to fall below F. Here we consider one of those possibilities, the debt-to-equity swap (DES), and compare this with the CoCo bail-in structure.

We begin with a detailed investigation of the DES structure. The framework is an extension of Moraux and Navatte (2009) (MN here onwards). As before a firm is financed by equity and a zero coupon bond with maturity T and face value F. Under normal APR the payoffs at T for the debt and equityholders are similar to Eq.(1), with an additional feature of bankruptcy costs of a proportion $1 - \beta \in (0, 1]$ of the asset value V_T. Then the debtholders' payoff when there is no DES is,

$$PayOff_D^N = \begin{cases} F, & \text{if } V_T \geq F \\ \beta V_T, & \text{if } V_T < F \end{cases}.$$

The existence of the bankruptcy cost means that there is a gain from restructuring, which the stakeholders can share. In a DES the debtholders rescue the equityholders by: (i) extending the existing debt by further s years to $S = T + s$ at rate r, and (ii) forgiving an amount $A \in [0, F]$ of the debt while receiving a proportion $\theta \in [0, 1]$ of the firm's equity in exchange. As extreme examples, $\theta = 0$ with $A > 0$ means that the debtholders forgive part or all of the debt with no equity in return, while $\theta = 1$ means that they expropriate current equityholders. As with MN we assume that debtholders control the financial restructuring but with no intent to take over the firm, i.e. $\theta < 1$. In

contrast to MN, which assumes that the DES only kicks in when $V_T < F$, we assume that DES is enforced by the regulator at the point of non-viability (PONV), which reflects the reality more. This level is further assumed to be the same as the bail-in trigger point τ in the previous sections. Hence DES is implemented when $V_T \leq \frac{F}{1-\tau}$. Post-DES, at the new bond maturity S the face value of the debt is $(F - A) e^{rs}$.[12] For $V_T > \frac{F}{1-\tau}$ when there is no DES, the debt is assumed to roll over to S with the new face value Fe^{rS}. For simplicity we assume there to be no debt restructuring at S irrespective of the asset value V_S at time S.[13] The table below summarises the stakeholders' holdings at S for the different cases:

$$
\begin{array}{|c|c|}
\hline
V_T \text{ range} & V_S \text{ range and the payoffs} \\
\hline
\left(\frac{F}{1-\tau}, \infty\right] & \begin{array}{l} V_S \geq Fe^{rs} : D_S = Fe^{rs}, E_S = V_S - Fe^{rs} \\ V_S < Fe^{rs} : D_S = \beta V_S, E_S = 0 \end{array} \\
\hline
\left[0, \frac{F}{1-\tau}\right] & \begin{array}{l} V_S \geq (F-A) e^{rs} : \begin{array}{l} D_S = (F-A) e^{rs} + \theta \left[V_S - (F-A) e^{rs}\right] \\ E_S = (1-\theta) \left[V_S - (F-A) e^{rs}\right] \end{array} \\ V_S < (F-A) e^{rs} : D_S = \beta V_S, E_S = 0. \end{array} \\
\hline
\end{array}
\tag{23}
$$

The expected present value at time T of the stakeholders' payoffs at time S can now be calculated. When there is no DES at T, these are,

$$
\begin{aligned}
D_T^{NoDES} &= e^{-rs} \widehat{E}_T \left[Fe^{rs} \chi_{V_S \geq Fe^{rs}} + \beta V_S \chi_{V_S < Fe^{rs}} \right] \\
&= \beta V_T N \left(-d_1 \left(Fe^{rs}\right)\right) + FN \left(d_2 \left(Fe^{rs}\right)\right) \\
E_T^{NoDES} &= e^{-rs} \widehat{E}_T \left[(V_S - Fe^{rs}) \chi_{V_S \geq Fe^{rs}} \right] \\
&= V_T N \left(d_1 \left(Fe^{rs}\right)\right) - FN \left(d_2 \left(Fe^{rs}\right)\right),
\end{aligned}
\tag{24}
$$

---

[12] This differs from MN, who seem to assume that the repayment for the remaining debt $F - A$ is simply postponed until S with zero interest cost. The problem with this assumption is that, *ceteris paribus*, the debtholders would always want an immediate redemption.

[13] This can be an extension to this analysis. Especially in the case that $V_T \geq \frac{F}{1-\tau}$ at T, it seems reasonable to allow DES at S when $V_S < \frac{Fe^{rs}}{1-\tau}$, even if we rule out repeated restructuring for the case of $V_T < \frac{F}{1-\tau}$ and $V_S < \frac{(F-A)e^{rs}}{1-\tau}$.

where $\widehat{E}_T\,[\,]$ is the risk-neutral expectation taken at time T, χ is again the indicator function and as before,

$$d_1\left(K\right) = \frac{\ln\left(\frac{V_T}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)s}{\sigma\sqrt{s}} \text{ and } d_2\left(K\right) = d_1\left(K\right) - \sigma\sqrt{s}. \tag{25}$$

Similarly when DES is triggered,

$$
\begin{aligned}
D_T^{DES} &= e^{-rs}\widehat{E}_T\left[\{(F-A)\,e^{rs} + \theta\,[V_S - (F-A)\,e^{rs}]\}\,\chi_{V_S \geq (F-A)e^{rs}} + \beta V_S \chi_{V_S < (F-A)e^{rs}}\right] \\
&= \beta V_T + (\theta - \beta)\,V_T N\left(d_1\left((F-A)\,e^{rs}\right)\right) + (1-\theta)\,(F-A)\,N\left(d_2\left((F-A)\,e^{rs}\right)\right) \\
E_T^{DES} &= e^{-rs}\widehat{E}_T\left[(1-\theta)\,[V_S - (F-A)\,e^{rs}]\,\chi_{V_S \geq (F-A)e^{rs}}\right] \\
&= (1-\theta)\,[V_T N\left(d_1\left((F-A)\,e^{rs}\right)\right) - (F-A)\,N\left(d_2\left((F-A)\,e^{rs}\right)\right)].
\end{aligned}
\tag{26}
$$

Note as a special case, when $s \to \infty$, $D^{DES} = \beta V_T + (\theta - \beta)\,V_T$, i.e. for a very long term investment the creditors are incited to swap debt for equity only when $\theta > \beta$.

The restructuring problem is a choice of three parameters: the amount of debt forgiven, A, the proportion of equity received, θ, and the term to maturity of the rescheduled bond, s. With the debtholders controlling the financial restructuring, their problem is that of maximising their wealth with the choice of (A, θ, s). The equityholders are always better off with the DES for $\theta < 1$, as they will receive a strictly positive claim instead of the zero value that would result from bankruptcy.

First consider the socially optimal outcome. This is where the total present value of the firm is maximised:

$$\max_{A,s} e^{-rs}\widehat{E}_T\left[V_S\right] = \beta V_T + (1-\beta)\,V_T N\left(d_1\left((F-A)\,e^{-rs}\right)\right). \tag{27}$$

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The second term is the net gain of restructuring, which would then be shared between the stakeholders. Eq.(27) is monotonically increasing in A for any given s, and thus the first-best is attained at $A^* = F$. This is intuitive: a strictly positive value of $F - A$ implies a strictly positive probability of insolvency at the new maturity S; given the non-zero restructuring cost β this reduces the present value of the firm; hence the firm value is maximised at $A = F$ where the future insolvency probability is zero. Then comparing Eq.(27) with D_T^{DES} in Eq.(26) reveals that the socially optimal outcome can be implemented by the choice $\theta^* = 1$. Thus any equilibrium outcome with $\theta^* \in (0, 1)$ would be second-best. However,

Proposition 3 *When the debtholders have the full bargaining power, their optimal strategy is the full takeover of the firm, $(A, \theta) = (F, 1)$.*

Proof. Consider the following derivatives of D_T^{DES}:

$$
\begin{aligned}
\frac{\partial D_T^{DES}}{\partial \theta} &= V_T N \left(d_1 \left((F - A) e^{rs} \right) \right) - (F - A) N \left(d_2 \left((F - A) e^{rs} \right) \right) > 0 \ \forall \theta \forall A \\
\frac{\partial D_T^{DES}}{\partial A} &= \frac{(1-\beta)}{\sigma \sqrt{s}} N' \left(d_2 \left((F - A) e^{rs} \right) \right) - (1 - \theta) N \left(d_2 \left((F - A) e^{rs} \right) \right).
\end{aligned}
\tag{28}
$$

The monotonicity of $\frac{\partial D_T^{DES}}{\partial \theta}$ is not surprising; whilst A determines the future default probability, and hence the present value of the firm, θ simply determines the stakeholders' shares of it. Therefore for any levels of A the debtholders would prefer the full transfer $\theta^* (A) = 1$.[14] The second derivative suggests that there are two opposing effects of an increase in A on the value of D_T^{DES}: an increase in A increases the total value of the firm by decreasing the probability of future default, but it also decreases the face value of the debt holding. Now suppose that the optimal outcome is (A^*, θ^*) with $A^* < F$ and

---

[14]Similarly, $\frac{\partial E_T^{DES}}{\partial \theta} < 0 \ \forall \theta \forall A$ implies that the equityholders would always prefer $\theta^* (A) = 0$.
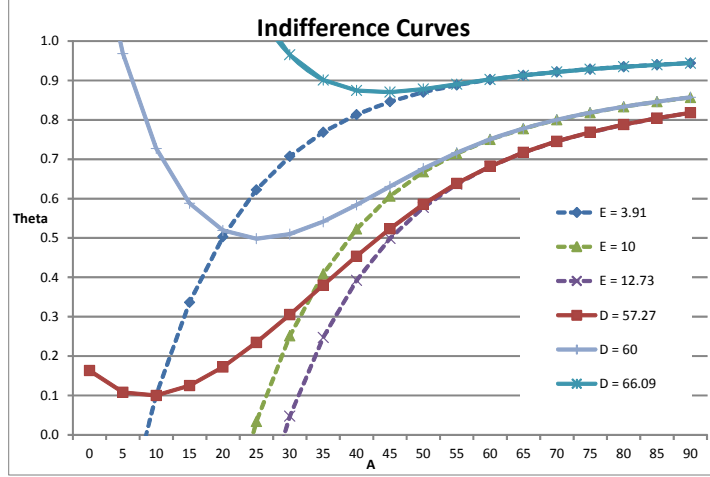
Figure 14: Indifference Curves for Equityholders and Debtholders: $V_T = 70$, $F = 90$, $r = 6\%$, $\sigma = 20\%$, $s = 2$ and $\beta = 0.8$

$\theta^* < 1$. However this outcome is not stable, as we know that $\theta^*(A) = 1 \ \forall A$. At $\theta = 1$, $\frac{\partial D_T^{DES}}{\partial A} > 0$, and the debtholders will optimally choose $A = F$ where $D_T^{DES} = V_T$. This is the only stable outcome. ∎

Note the choice of s is then irrelevant. This is the socially optimal outcome discussed above. This result can also be seen diagrammatically by the use of indifference curves (ICs). Consider the following derivatives of E_T^{DES}:

$$\begin{aligned}
\frac{\partial E_T^{DES}}{\partial \theta} &= -\left[V_T N\left(d_1\left((F-A)\,e^{rs}\right)\right) - (F-A)\,N\left(d_2\left((F-A)\,e^{rs}\right)\right)\right] < 0 \ \forall\theta\forall A \\
\frac{\partial E_T^{DES}}{\partial A} &= (1-\theta)\,N\left(d_2\left((F-A)\,e^{rs}\right)\right) > 0 \ \forall\theta\forall A.
\end{aligned} \tag{29}$$

Thus for equityholders there is a simple trade-off between A and θ. Then on a $\theta - A$ plane, their ICs are upward-sloping. For debtholders, their ICs are downward-sloping for lower values of A where $\frac{\partial D_T^{DES}}{\partial A} > 0$, while they turn upward-sloping for larger values of A. Examples of these ICs are depicted in Fig.14, where the solid curves are the debtholders' ICs and the dashed curves are those of the equityholders. The equityholders and the

debtholders are better off with lower and higher ICs, respectively. The turning point in the debtholders' ICs is where the value of their holding is maximised for given levels of θ.[15] The diagram shows the tangency points of the ICs to be at $A = F$. There is a continuum of such Pareto efficient equilibria, and the choice of the final outcome depends on the relative bargaining power of the stakeholders. With full bargaining power the debtholders optimise at $D_T^{DES} = V_T$, attained by the choices $(A^*, \theta^*) = (F, 1)$, as stated in Proposition 3. On the other hand if the equityholders had the full bargaining power, it would result in D_T^{DES} being driven down to the debtholders' outside option, which is default without DES when they would receive βV_T. Note that this minimum value for the debtholders ensures that $\theta^* \geq \beta$ for all cases of DES.

In reality, however, we observe partial forgiveness $A < F$. In order to achieve this we introduce a cost term $C(\theta)$ for the debtholders of higher control of the firm, with $C' > 0$, $C'' > 0$, $C(0) = 0$ and $\lim_{\theta \to 1} C(\theta) = \infty$. This reflects the bondholders' reluctance to take over the firm.[16] Here, we let $C(\theta) = \frac{k\theta}{1-\theta}$ for some constant $k > 0$. We will see below that he inclusion of the cost term results in the debtholders choosing an outcome that is less than the socially optimal. The debtholders' value under DES is now,

$$D_T^{DES} = \beta V_T + (\theta - \beta) V_T N (d_1 ((F - A) e^{rs})) + (1 - \theta) (F - A) N (d_2 ((F - A) e^{rs})) - \frac{k\theta}{1 - \theta},$$

(30)

We define the admissibility of a DES under full debtholder bargaining power as follows:

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[15] For example for $\theta = 0.1$, the maximum value for the debtholders is that of the IC tangent to the horizontal line $\theta = 0.1$. On the diagram this is shown to be $D_T^{DES} = 57.27$.

[16] Bondholders may not be interested in taking over banks/companies, because, not only they lack the company and industry expertise and know-how of shareholders, but also their mandate to invest in low yielding, low volatile instruments may mean that they would be forced to sell their equity position.

Definition 4 (DES Admissibility) *The DES structure is **admissible** if the parameters* (A^*, θ^*, s^*) *maximise the value of debtholders' holding:*

$$(A^*, \theta^*, s^*) = \arg\max_{A,\theta,s} D_T^{DES} \text{ subject to } D_T^{DES} \geq \beta V_T \tag{31}$$

where $A \in [0, F]$, $\theta \in [0, 1]$ *and* $s \geq 0$, *and* D_{DES}^T *is given by Eq.(30).*[17]

This problem can be solved in two stages: first, find the optimal (A^*, θ^*) given s, and second, find s^* with the maximum D_T^{DES}. In focussing on the first stage, the first-order conditions are:

$$\frac{\partial D_T^{DES}}{\partial \theta} = V_T N(d_1) - (F - A) N(d_2) - \frac{k}{(1-\theta)^2} = 0$$

$$\frac{\partial D_T^{DES}}{\partial A} = \frac{(1-\beta)}{\sigma\sqrt{s}} N'(d_2) - (1-\theta) N(d_2) = 0.$$

Solving the simultaneous equations yields the optimal A^* as the solution to,

$$V_T N(d_1((F-A)e^{rs})) - (F-A) N(d_2((F-A)e^{rs})) - \frac{k\sigma^2 s}{(1-\beta)^2} \left[\frac{N(d_2((F-A)e^{rs}))}{N'(d_2((F-A)e^{rs}))} \right]^2 = 0 \tag{32}$$

---

[17] *In their definition of admissibility, MN has the condition that the received portion of equity exactly covers the amount of the face value of the debt forgiven,*

$$
\begin{aligned}
A &= e^{-rs} \widehat{E}_T \left[\theta \left[V_S - (F - A) e^{rs} \right] \chi_{V_S \geq (F-A)e^{rs}} \right] \\
&= \theta \left[V_T N(d_1((F-A)e^{rs})) - (F-A) N(d_2((F-A)e^{rs})) \right] = f(A, s).
\end{aligned}
$$

They argue that this condition may be viewed as an equilibrium condition: for debtholders, the amount A forgiven is the maximum for a given portion θ of equity received, while for equityholders, it is the minimum amount acceptable. We believe that this is not the case. In their Table 1 they simulate the optimal (A^*, s^*) for $\theta = 0.5$ and different values of β and V_T. Then for $\beta = 0.8$ and $V_T = 30$, their admissible (A^*, s^*) are computed as $(1.25, 2.48)$. In this case $f(1.25, 2.48) = 1.25$, i.e. the PV of the equity received (in their set-up - see footnote 12) equals the forgiven amount, as assumed. Then $D_T^{DES} = 24.07 + 1.25 = 25.32$, where 24.07 is the PV of the restructured bond. However for $A = 7.04$, it can be computed that $f(7.04, 2.48) = 2.25$ and $D_T^{DES} = 23.27 + 2.25 = 25.52$. In other words, the debtholders are able to increase their value by forgiving A higher than the PV of the equity received. The equityholders are also better off as E_T^{DES} increases from 1.25 to 2.25, and this is therefore a Pareto-improving agreement. As such, MN's admissibility condition does not yield a Pareto efficient outcome. Intuitively, by increasing A the debtholders are able to increase the value of their equity holding more than the loss in the value of the remaining debt.

and θ^* given by,

$$\theta^* = 1 - \frac{(1-\beta)\, N\left(d_2\left((F-A)\, e^{rs}\right)\right)}{\sigma\sqrt{s}\, N'\left(d_2\left((F-A)\, e^{rs}\right)\right)} < 1. \tag{33}$$

Then $A = F$ can no longer be the solution as the the last term goes to ∞ in Eq.(32).

Compare now this DES scheme with the CoCo bail-in structure. To do so, we introduce future default risk at time S in our CoCo set-up. We assume the following for simplicity: (i) when the CoCo bonds are partially (but not wholly) converted, i.e. $V_T \in \left(\frac{F_B}{1-\underline{E}}, \frac{F}{1-\tau}\right]$, then the remaining CoCo bonds are replaced by straight bonds of the same face value; and (ii) when the firm is insolvent even after full CoCo conversion, i.e. $V_T \in [0, F_B)$, the regulator enforces DES to rescue the bank. Then the payoffs at S are,

V_T range	V_S range and the payoffs
$A: \left(\frac{F}{1-\tau}, \infty\right]$	$V_S \geq F e^{rs} : D_S = F e^{rs}, E_S = V_S - F e^{rs}$ $V_S < F e^{rs} : D_S = \beta V_S, E_S = 0$
$B: \left(\frac{F_B}{1-\underline{E}}, \frac{F}{1-\tau}\right]$	$V_S \geq (1-\underline{E}) V_T e^{rs} : \begin{aligned} D_S &= (1-\underline{E}) V_T e^{rs} + \frac{\underline{E}-\tau}{\underline{E}} \left[V_S - (1-\underline{E}) V_T e^{rs}\right] \\ E_S &= \frac{\tau}{\underline{E}} \left[V_S - (1-\underline{E}) V_T e^{rs}\right] \end{aligned}$ $V_S < (1-\underline{E}) V_T e^{rs} : D_S = \beta V_S, E_S = 0$
$C: \left[\frac{F_B}{1-\tau}, \frac{F_B}{1-\underline{E}}\right]$	$V_S \geq F_B e^{rs} : \begin{aligned} D_S &= F_B e^{rs} + \frac{(1-\tau)V_T - F_B}{V_T - F_B} \left(V_S - F_B e^{rs}\right) \\ E_S &= \frac{\tau V_T}{V_T - F_B} \left(V_S - F_B e^{rs}\right) \end{aligned}$ $V_S < F_B e^{rs} : D_S = \beta V_S, E_S = 0$
$D: \left[F_B, \frac{F_B}{1-\tau}\right)$	$V_S \geq F_B e^{rs} : D_S = F_B e^{rs}, E_S = V_S - F_B e^{rs}$ $V_S < F_B e^{rs} : D_S = \beta V_S, E_S = 0$
$E: [0, F_B)$	$V_S \geq (F_B - A)\, e^{rs} : \begin{aligned} D_S &= (F_B - A)\, e^{rs} + \theta\left[V_S - (F_B - A)\, e^{rs}\right] \\ E_S &= (1-\theta)\left[V_S - (F_B - A)\, e^{rs}\right] \end{aligned}$ $V_S < (F_B - A)\, e^{rs} : D_S = \beta V_S, E_S = 0.$

$$\tag{34}$$

Each range of V_T represents the following cases as described in detail in Appendix A.3: A, no CoCo trigger; B, partial CoCo trigger; C, full CoCo triggered; D, remaining equity written down; and E, the firm is insolvent. Then,

Proposition 5 *CoCo bond bail-in is a non-admissible DES.*

Proof. Compare the CoCo payoff for the debtholders in Table (34) with that of DES
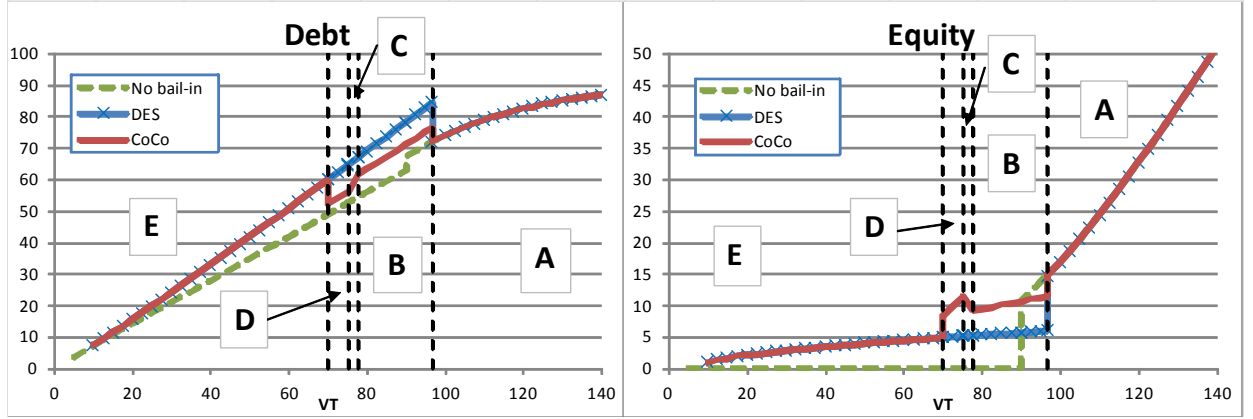
Figure 15: PV at T for Debtholders and Equityholders: $F = 90$, $F_C = 20$, $r = 6\%$, $\sigma = 20\%$, $s = 2$ years, $\beta = 0.7$, $k = 1$, $\tau = 7\%$ and $\underline{E} = 10\%$.

in Table (23). The two are equivalent for the two end cases of $V_T \geq \frac{F}{1-\tau}$ (no bail-in / no DES) and $V_T < F_B$ (DES in both). When $\frac{F_B}{1-\underline{E}} \leq V_T < \frac{F}{1-\tau}$, the DES payoff is equivalent to the CoCo payoff when $A = F - (1 - \underline{E}) V_T$ and $\theta = \frac{\underline{E}-\tau}{\underline{E}}$. Similarly when $\frac{F_B}{1-\tau} \leq V_T \leq \frac{F_B}{1-\underline{E}}$, the two are equivalent when $A = F_C$ and $\theta = \frac{(1-\tau)V_T - F_B}{V_T - F_B}$. Finally when $F_B \leq V_T < \frac{F_B}{1-\tau}$, the two payoffs are equal when $A = F_C$ and $\theta = 0$. This means that the CoCo payoff is within the (unconstrained) feasible set of possible DESs, but for $F_B \leq V_T < \frac{F_B}{1-\tau}$, it gives the debtholders zero value, which is below their outside option of βV_T. Hence the CoCo payoffs cannot be the admissible DES. ∎

Basically, in contrast to the DES, in CoCo bail-in the bondholders are unable to negotiate A or θ as these are both pre-defined at the CoCo bond inception. Note the Writedown bond is the worst case scenario where A and θ are pre-set at $(F, 0)$.

Fig.15 graphs the present values (PV) at T of the stakeholders' payoffs for both admissible DES and CoCo structures. The regions A to E are those specified in Table (34). The admissible DES PV is simulated by solving Eq.(32) numerically for the optimal A^* for each V_T, which is then substituted in Eq.(33) to compute θ^*. As shown,

the debtholders are able to attain higher values by their choice of A and θ than under the CoCo structure. For the equityholders their PV is higher under CoCo than under DES, with the difference again represented by a condor-like structure. This again implies higher vega and lower delta for the CoCo structure than with the DES at and above the trigger point. We therefore conclude that wealth-transfer and value destruction agency costs are higher under CoCo bail-in than for a traditional DES.

6. Case Study: Banca Monte dei Paschi, 2013

In Section 4, we established that the introduction of CoCo or Writedown bail-in would aggravate the value destruction agency cost by incentivising banks to opt for investments which have high volatility but low present value. Under the standard Capital Asset Pricing Model framework, this "gamble-for-resurrection" means an investment away from the Capital Market Line that represents the efficient trade-off between returns and volatility. In practical terms this results in the lowering of the Sharpe Ratio of the investment portfolio. Here we outline one such case of Banca Monte dei Paschi di Siena (BMPS). The world's oldest surviving bank was bailed out by the Italian government in 2013, on account of, amongst many things, their flawed investment in BTPs (Italian Sovereign Bonds) that took place through their Santorini vehicle during 2010. As we discuss below, we see this as a "gamble-for-ressurection" that did not pay off, which, in turn, was attempted as a result of losses from earlier trades.

In Fig.16 we plot some of the listed equity investments of BMPS between 2003-2010.[18] Through the cycle, the portfolio risk-return profile is balanced with the average

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[18]This is limited to listed equity investments as there is no disclosure on fixed income investments. The proxy market portfolio constitutes of an equally weighted allocation between European and Fixed
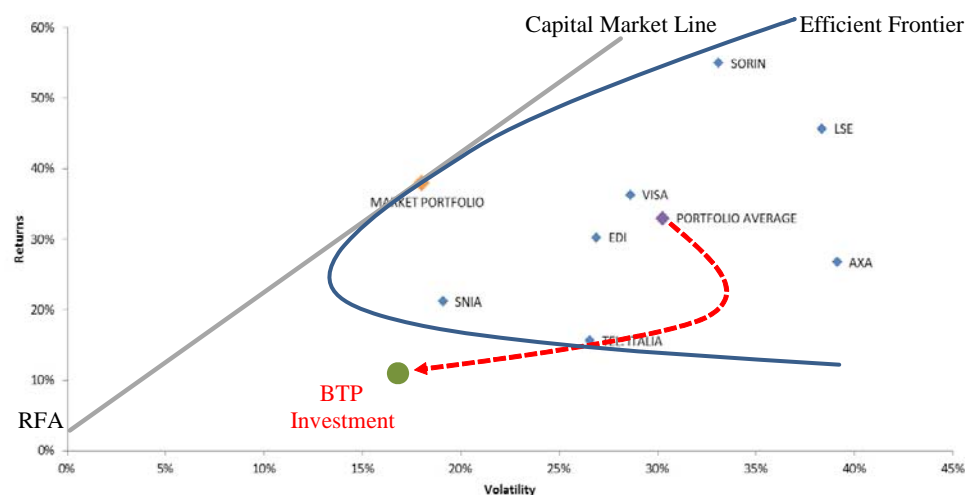
Figure 16: BMPS Investments, 2003-2010. Source: Bloomberg, BMPS Annual Accounts

Sharpe Ratio of 1.0, albeit with a significant dispersion. Due to the 2008 financial crisis, the Intesa San Paolo equity investment loss (€375mn) emerged in 2010. Consequently, BMPS entered into an agreement with Deutsche Bank that involved, amongst other things, the purchase of €2.75bn BTPs. We suspect that the bank was hoping to plug in their deficit hole by targeting above 12-13% return in the BTPs bet. A 40% fall in the Sharpe Ratio between their initial investment portfolio and the BTPs investment highlights the highly risky profile of this bet.

Through the 2003-2010 Cycle Sharpe Ratio	
AXA	0.5x
EDI	1.1x
LSE	1.2x
SNIA	1.0x
TEL. ITALIA	0.2x
VISA	1.3x
SORIN	1.5x
Average	1.0x
2010 BTP Targeted Sharpe Ratio	
BTP	0.6x

Using 10 YR German Bund Yearly Returns

Sharpe Ratios

Income Equities, Risk Free Asset (Bund) and Gold.
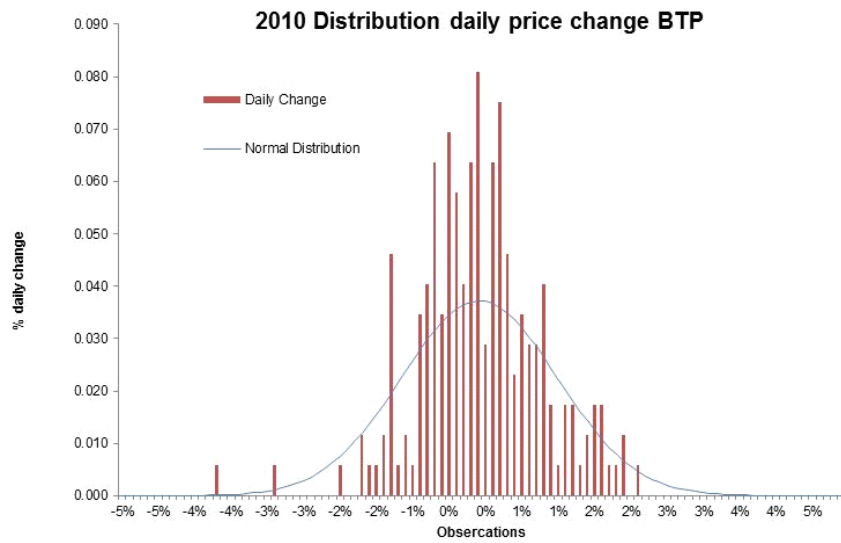
Figure 17: BTP Distribution, 2010. Source: Bloomberg

Moreover, BTPs were (and still are) volatile investment with annualised standard deviation of 15% through the cycle, and thus the volatility adjusted return ratio was very poor as shown below.

2010 BTP Performance		BTP Through the Cycle Performance	
Return	6.6%	Return	0.0%
Vol	15.0%	Vol	14.6%
RFA Return	39.7%	RFA Return	1.1%
Sharpe Ratio (excl RFA)	-2.2x	Sharpe Ratio (excl RFA)	-0.1x
Vol Adjusted Return Ratio	0.4x	Vol Adjusted Return Ratio	0.0x
Skewness	-1.11	Skewness	-1.35
Kurtosis	12.9	Kurtosis	11.4

BTP Performance

Further, the Gaussian distribution of the daily price change of BTP through 2010 displays negative skewness and fat tails (kurtosis of more than 3.5) as shown in Fig.17. However this did not seem to dissuade BMPS from investing in this speculative trade. It seems that BMPS thought that the rise in the BTP yields throughout 2008-9 would retrace back to the pre-crisis 3% level. It did not happen, and once Italy was engulfed
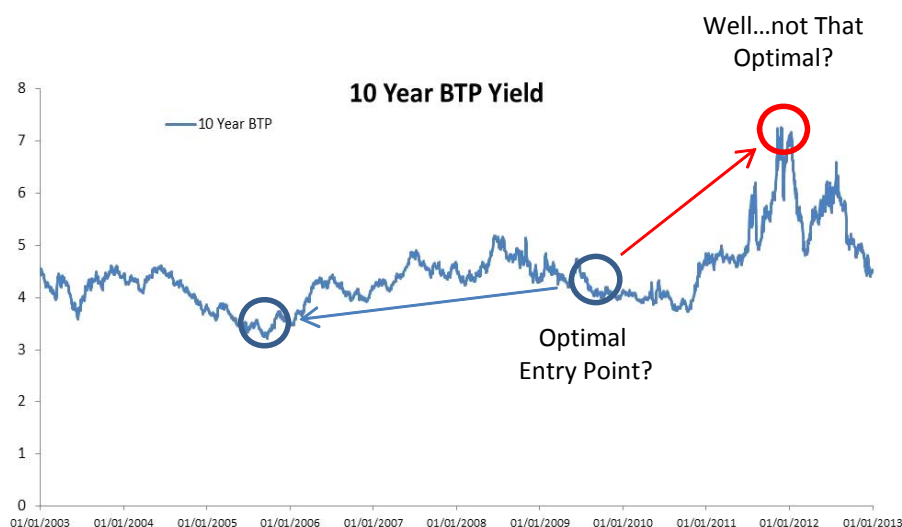
Figure 18: 10 year BTP Yield. Source: Bloomberg

by the sovereign crisis the BTP yields surged to above 7% (Fig.18). As a consequence the speculative bet brought down the bank, which was finally bailed out by the Italian government. We view this as an example of "gamble-for-resurrection" which led the bank to pursue low Sharpe Ratio investments in order to survive. This paper has shown that the incentive problem will not be different between this government bail-out structure and the more recent CoCo bail-in structures.[19] The put that was "sold" by the government in the past is now sold by the creditors; this is just a swap of ultimate bank rescuers from the taxpayers to the bondholders. The point is that the problem has not been eradicated; in the process, the moral hazard problem has been replaced by the agency costs, as analysed extensively in this paper.

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[19]It is very plausible that the incentive problem will be higher under CoCo bail-in since bondholders, and not taxpayers will prop up the bank, which mitigates the negative knock on impact on the bank's franchise.

7. Concluding Remarks

The new financial regulation has been articulated to dampen moral hazard and to minimise the chances of another financial crisis that could jeopardise again the integrity of the banking system. However in reality, the regulator is "swapping" bail-out for bail-in, which, as discussed in this paper, is in essence a replacement of moral hazard with agency costs. If the burden of an ailing bank fell to the taxpayers in the past, it will now fall to the bondholders who will be required to be very mindful about the investments they own in a bank. Historically, apart from the very few cases where the bank was fully nationalised (e.g. Bankia in 2012; SNS in 2013), the equityholders would simply suffer dilution (e.g. Lloyds and ING, both in 2008), or, in many cases, were unaffected with the injection of new equity in the form of preference shares with CT1 qualification (Goldman Sachs, Morgan Stanley, etc.). Under the new bail-in regime, the equityholders take the first losses up to the CoCo trigger point where bondholders get writtendown/off or converted into a non-admissible DES, whilst there is still at least 7.0% of assets in equity. This going-concern DAPR accentuates the agency costs that the bail-in structure is introducing into the banking industry.

It is, moreover, possible that the new bail-in structure may even aggravate the moral hazard problem. One could argue that the equityholders have more incentives to "gamble for resurrection" when the wealth extraction comes from other investors (creditors) rather than taxpayers, as the media scrutiny, and hence the reputational impact, would likely be lower. Further, bail-in may not result in restrictions on dividends or bankers' compensations as there would be with taxpayer bail-out. Whilst these are issues not analysed in this paper, they enhance our case that the new financial regulation may not

alleviate the incentive problems as aimed.

Traditional Corporate Finance literature has underscored the detrimental effects of agency costs on the relationships between bondholders and equityholders, especially due to the limited investment of the latter. Higher equity advocated by some (e.g. Admati *et al.*, 2013) does not attenuate the problem when the equityholders enjoy the implicit put of the bail-in-able balance sheet. Higher capital costs on risky investments (Risk-Weighted Asset inflation) could potentially make banks safer, but banks are volatile institutions with Non-Performing Loans and speculative trading that makes the business unpredictable. Equityholders are aware of this and they will exploit the opportunity to deviate from the Capital Asset Line in the Capital Asset Pricing Model, pursuing low Sharpe Ratio "bets" and speculate with the DAPR offered by the bondholders' put. The aggravation of this agency cost will trespass the bank's balance sheet to penetrate into the asset management industry (as the major owners of the bank's debt), and ultimately into the real economy. This latter point will be explored in detail in subsequent papers. In this paper we focussed on several aspects that arise from our view within this new bail-in world. Wealth-transfer and value destruction are two consequences of the dominance of equityholders in their private "game" against bondholders. This is even more pronounced when bondholders do not have the chance to steer the restructuring to attain a fair agreement that partially compensates their losses, as would do in an admissible DES. To conclude, the new regulations do not tackle the intrinsic moral hazard of the banking industry; instead they are "solutions" that yield new unintended consequences. Unfortunately the inherent problems continue to lurk behind the scenes of an industry.

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Appendix

A. Derivations of the Payoffs

A.1. No Bail-out / Bail-in

The long call option of the equityholders and the short put position of the debtholders, both at strike price F, lead to the following payoffs for the stakeholders,

V_T	$[0, F)$	$[F, F + E)$	$[F + E, \infty]$
D	V_T	F	F
E	0	$V_T - F$	$V_T - F$
Total Firm	V_T	V_T	V_T
Notes	Capital wiped out, debt write down	Capital write down	No write down

The summary payoffs are as given in Eq.(1). For example in the case that $F = 90$, the payoffs for different values of V_T are,

V_T	80.0	82.5	85.0	87.5	90.0	92.5	95.0	97.5	100.0	102.5	105.0
D	80.0	82.5	85.0	87.5	90.0	90.0	90.0	90.0	90.0	90.0	90.0
E	0	0	0	0	0	2.5	5.0	7.5	10.0	12.5	15.0
Total Firm	80.0	82.5	85.0	87.5	90.0	92.5	95.0	97.5	100.0	102.5	105.0

This is depicted in Fig.1.

A.2. Government Bail-Out

As described in the main text the government bails out the bank to restore its balance sheet, such that its minimum capital ratio $\underline{E}$ is maintained. The debtholders are fully

protected at their face value F, leading to a balance sheet floor of $\frac{F}{1-\underline{E}}$. When V_T is below this level the bail-out is triggered, with an injection of preference shares by the government. For values of V_T below $F+\frac{F}{1-\underline{E}}\underline{E_C}$, the government also ensures a minimum common equity floor of $\underline{E_C}$, with the taxpayers providing $V_T - \left(F + \frac{F}{1-\underline{E}}\underline{E_C}\right)$ to prop up both the equityholders' common equity and the debtholders' holdings. The stakeholders' payoffs at T are then, where E_C is the common equity held by the equityholders and E_P is the preference shares owned by the government,

V_T	$\left[0, F+\frac{F}{1-\underline{E}}\underline{E_C}\right)$	$\left[F+\frac{F}{1-\underline{E}}\underline{E_C}, \frac{F}{1-\underline{E}}\right)$	$\left[\frac{F}{1-\underline{E}}, F + E_0\right)$	$[F + E_0, \infty]$
D	F	F	F	F
E_C	$\frac{F}{1-\underline{E}}\underline{E_C}$	$V_T - F$	$V_T - F$	$V_T - F$
E_P	$\left(\underline{E}-\underline{E_C}\right)\frac{F}{1-\underline{E}}$	$\frac{F}{1-\underline{E}} - V_T$	0	0
Total Firm	$\frac{F}{1-\underline{E}}$	$\frac{F}{1-\underline{E}}$	V_T	V_T
Tax payers	$-\left[\left(F+\frac{F}{1-\underline{E}}\underline{E_C}\right)-V_T\right]$	0	0	0
Capital ratio	$\underline{E}$	$\underline{E}$	$\left[\underline{E}, \frac{E_0}{F+E_0}\right]$	$\left[\frac{E_0}{F+E_0}, \infty\right]$
Common equity ratio	$\underline{E_C}$	$\left[\underline{E_C}, \underline{E}\right]$	$\left[\underline{E}, \frac{E_0}{F+E_0}\right]$	$\left[\frac{E_0}{F+E_0}, \infty\right]$
Notes	Bail-out to attain minimum capital ratio and minimum common equity	Bail-out to attain minimum capital ratio	Equity capital write down	No write down

The payoffs are summarised in Eq.(4).

Consider for example the outcome $V_T = 85$. Assume that the minimum capital ratio of $\underline{E} = 10\%$, and the minimum common equity floor of $\underline{E_C} = 5\%$. With $F = 90$, in the absence of a bail-out the equityholders are wiped out, while the bondholders also lose 5. With the bail-out the government provides 15 to restore the balance sheet to $\frac{90}{1-0.10} = 100$. The bondholders receive 5 of this and the equityholders receive 5. The government ends up with 5 of the preference shares to ensure the capital ratio of 10%,

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resulting in the net cost of -10 to the taxpayers. This and other examples are given below:

V_T	85.0	87.5	90.0	92.5	95.0	97.5	100.0	102.5	105.0
D	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
E_C	5.0	5.0	5.0	5.0	5.0	7.5	10.0	12.5	15.0
E_P	5.0	5.0	5.0	5.0	5.0	2.5	0	0	0
Total Firm	100.0	100.0	100.0	100.0	100.0	100.0	100.0	102.5	105.0
Tax payers	-10.0	-7.5	-5.0	-2.5	0	0	0	0	0
Capital ratio	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	12.2%	14.3%
Common equity ratio	5.0%	5.0%	5.0%	5.0%	5.0%	7.5%	10.0%	12.2%	14.3%

The payoff graphs are depicted in Fig.2.

A.3. Bail-in with CoCo Bonds

As explained, the bail-in triggers when $\frac{V_T - F}{V_T} \leq \tau \Leftrightarrow V_T \leq \frac{F}{1-\tau}$, which ensures the minimum capital ratio $\underline{E}$. This is done by converting $F_C - [(1 - \underline{E}) V_T - F_B]$ of the CoCo bond into $E_D = (\underline{E} - \tau) V_T$ of equity held by the debtholders, resulting in their loss of $F - (1 - \tau) V_T$. For V_T below $\frac{F_B}{1-\underline{E}}$, all of the CoCo bond is used up and hence $\underline{E}$ is not attainable, though the equityholders still hold equity level of τV_T. For V_T below $\frac{F_B}{1-\tau}$, even this is not attainable and the equityholders' holding is written down. Finally when $V_T < F_B$, with the capital totally wiped out, the debtholders become the residual claimants. The payoffs are thus,

V_T	$[0, F_B)$	$\left[F_B, \frac{F_B}{1-\tau}\right)$	$\left[\frac{F_B}{1-\tau}, \frac{F_B}{1-\underline{E}}\right]$	$\left(\frac{F_B}{1-\underline{E}}, \frac{F}{1-\tau}\right]$	$\left(\frac{F}{1-\tau}, F+E_0\right)$	$[F+E_0, \infty]$
D	V_T	F_B	F_B	F_B	F_B	F_B
$CoCo$	0	0	0	$(1-\underline{E})V_T - F_B$	F_C	F_C
E_D	0	0	$(1-\tau)V_T - F_B$	$(\underline{E}-\tau)V_T$	0	0
E_C	0	$V_T - F_B$	τV_T	τV_T	$V_T - F$	$V_T - F$
Total Debt	V_T	F_B	$(1-\tau)V_T$	$(1-\tau)V_T$	F	F
Total Firm	V_T	V_T	V_T	V_T	V_T	V_T
Capital ratio	0	$[0,\tau]$	$[\tau, \underline{E}]$	$\underline{E}$	$\left[\tau, \frac{E_0}{F+E_0}\right]$	$\left[\frac{E_0}{F+E_0}, \infty\right]$
Notes	Capital wiped out, debt-holders residual claimants	Equity capital write down	CoCo wholly triggered, $\underline{E}$ unattainable	CoCo partially triggered	Equity capital write down, no trigger	No write down

The summary payoffs are as given in Eq.(6).

As an example, consider the case where there are $F_B = 70$ of the straight bond, $F_C = 20$ of the CoCo bond and the initial equity of $E_0 = 10$. The minimum capital ratio and the trigger point are again $\underline{E} = 10\%$ and $\tau = 7\%$. For $V_T < F_B + F_C + E_0 = 100$, only the equity capital is written down if $V_T > \frac{F}{1-\tau} = \frac{90}{1-0.07} = 96.77$. Below this point CoCo bail-in occurs. For example at $V_T = 80$, the bail-in is triggered at the equity capital level of $E_C = V_T \times \tau = 80 \times 0.07 = 5.6$. The equityholders therefore bear the first $10 - 5.6 = 4.4$ of the total loss of 20 of the firm. The remaining 15.6 of the loss is bourne by the CoCo bondholders, whose $F_C - [(1 - \underline{E})V_T - F_B] = 20 - (0.9 \times 80 - 70) = 18$ of the CoCo bond is converted into $(\underline{E} - \tau)V_T = (0.1 - 0.07) \times 80 = 2.4$ of equity, held by the CoCo bond holders. The total equity capital of $E_C + E_D = 5.6 + 2.4 = 8$ satisfies the minimum capital ratio of $\underline{E} = 10\%$. The total position of the bondholders is $72 + 2.4 = 74.4$. This is shown below in the column for $V_T = 80$. When $V_T = \frac{F_B}{1-\underline{E}} = \frac{70}{1-0.1} = 77.78$, the

trigger point of the equity capital is $77.78 \times 0.07 = 5.44$. To maintain $\underline{E} = 10\%$, the firm has to reduce its total debt to $0.9 \times 77.78 = 70$ which equals the face value of its straight bond. Thus the whole of CoCo bond is converted to $(0.1 - 0.07) \times 77.78 = 2.33$ of the debtholders' equity. For V_T lower than $\frac{F_B}{1-\tau} = \frac{70}{1-0.07} = 75.27$, according to the absolute priority rule (APR) the equityholders' remaining capital is written down to below $\tau = 7\%$. For V_T less than $F_B = 70$ the firm becomes insolvent.

V_T	70.0	75.0	75.27	77.78	80.0	85.0	90.0	95.0	96.77	97.5	100.0	105.0
D	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
$CoCo$	0.0	0.0	0.0	0.0	2.0	6.5	11.0	15.5	20.0	20.0	20.0	20.0
E_D	0.0	0.0	0.0	2.33	2.4	2.55	2.7	2.85	0.0	0.0	0.0	0.0
E_C	0.0	5.0	5.27	5.44	5.6	5.95	6.3	6.65	6.77	7.5	10.0	15.0
Total Debt	70.0	70.0	70.0	72.33	74.4	79.05	83.7	88.35	90.0	90.0	90.0	90.0
Total Firm	70.0	75.0	75.0	77.78	80.0	85.0	90.0	95.0	96.77	97.5	100.0	105.0
Capital ratio	0%	6.7%	7.0%	10%	10%	10%	10%	10%	7.0%	7.7%	10%	14.3%

Fig.3 depicts these payoffs.

A.4. Bail-in with Writedown Bonds

For Writedown bonds, the bail-in triggers once $\frac{V_T - F}{V_T} \leq \tau \Leftrightarrow V_T \leq \frac{F}{1-\tau}$ when the entire Writedown bond is written down/off.

V_T	$[0, F_B)$	$\left[F_B, \frac{F}{1-\tau}\right]$	$\left(\frac{F}{1-\tau}, F\right)$	$[F, \infty]$
D	V_T	F_B	F_B	F_B
F_W	0	0	F_W	F_W
E_C	0	$V_T - F_B$	$V_T - F$	$V_T - F$
Total Debt	V_T	F_B	F	F
Total Firm	V_T	V_T	V_T	V_T
Notes	Capital wiped out, debt written down	Writedown triggered	Equity capital written down, no trigger	No write down

The summary payoffs are given in Eq.(11).

As an example let there be $F_B = 70$ of the straight bond, $F_W = 20$ of the Writedown bond and $E = 20$ of the initial equity capital. The minimum capital ratio and the trigger point are again $\underline{E} = 10\%$ and $\tau = 7\%$. The equity capital is then written down until V_T hits $V_T = \frac{70+20}{1-0.07} = 96.77$. Below this point the bail-in is triggered and the entire 20 of the Writedown bond is converted to equity and writtendown / off. There is, therefore, a discontinuity at this point for the payoffs of both the debtholders and the equityholders.

V_T	60.0	65.0	70.0	75.0	80.0	85.0	90.0	95.0	100.0	105.0
D	60.0	65.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
F_W	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	20.0
E_C	0.0	0.0	0.0	5.0	10.0	15.0	20.0	25.0	10.0	15.0
Total Debt	60.0	65.0	70.0	70.0	70.0	70.0	70.0	70.0	90.0	90.0
Total Firm	60.0	65.0	70.0	75.0	80.0	85.0	90.0	95.0	100.0	105.0

Fig.6 shows the payoffs of this example.

B. Proof of $\frac{\partial}{\partial K}\left[N\left(-d_1\left(K\right)\right) - \lambda N\left(-d_1\left(\frac{K}{\lambda}\right)\right)\right] < 0$ for

$V_0 > \frac{K}{\lambda}$

Noting that $\frac{\partial}{\partial K}N\left(-d_1\left(K\right)\right) = \frac{1}{K\sigma\sqrt{T}}N'\left(-d_1\left(K\right)\right)$,

$$\frac{\partial}{\partial K}\left[N\left(-d_1\left(K\right)\right) - \lambda N\left(-d_1\left(\frac{K}{\lambda}\right)\right)\right] = \frac{1}{K\sigma\sqrt{T}}\left[N'\left(-d_1\left(K\right)\right) - \lambda N'\left(-d_1\left(\frac{K}{\lambda}\right)\right)\right].$$

This is negative if and only if,

$$\exp\left(-\frac{d_1^2\left(K\right)}{2}\right) < \lambda\exp\left(-\frac{d_1^2\left(\frac{K}{\lambda}\right)}{2}\right)$$

$$\Leftrightarrow d_1^2\left(K\right) > d_1^2\left(\frac{K}{\lambda}\right) - 2\ln\lambda.$$

Now $d_1\left(\frac{K}{\lambda}\right) = d_1\left(K\right) + \frac{\ln\lambda}{\sigma\sqrt{T}}$ and hence,

$$\Leftrightarrow d_1^2\left(K\right) > d_1^2\left(K\right) + \frac{2\ln\lambda}{\sigma\sqrt{T}}d_1\left(K\right) + \frac{(\ln\lambda)^2}{\sigma^2 T} - 2\ln\lambda$$

$$\Leftrightarrow 0 > \frac{\ln\lambda}{\sigma\sqrt{T}}\left[2\left(d_1\left(K\right) - \sigma\sqrt{T}\right) + \frac{\ln\lambda}{\sigma\sqrt{T}}\right].$$

As $d_1\left(K\right) - \sigma\sqrt{T} = d_2\left(K\right)$ and when $0 < \lambda < 1$, $\ln\lambda < 0$,

$$\Leftrightarrow 0 < d_2\left(K\right) + \frac{\ln\lambda}{2\sigma\sqrt{T}} = d_2\left(K\right) + \frac{\ln\lambda}{\sigma\sqrt{T}} - \frac{\ln\lambda}{2\sigma\sqrt{T}} = d_2\left(\frac{K}{\lambda}\right) - \frac{\ln\lambda}{2\sigma\sqrt{T}}$$

$$\Leftrightarrow \frac{\ln\lambda}{2\sigma\sqrt{T}} < d_2\left(\frac{K}{\lambda}\right).$$

For $0 < \lambda < 1$ then the sufficient condition is that $d_2\left(\frac{K}{\lambda}\right) > 0 \Leftrightarrow V_0 > \frac{K}{\lambda}\exp^{-\left(r-\frac{\sigma^2}{2}\right)T}$, which in turn is sufficiently satisfied for $V_0 > \frac{K}{\lambda}$ when $r > \frac{\sigma^2}{2}$.

C. Proof of Eq.(22) being Negative

First note that, at $F_W = 0$, $\Delta_{WDcdr} = 0$. Investigate what happens when F_W increases while keeping F constant,

$$\frac{\partial}{\partial F_W} \Delta_{WDcdr} = -\frac{e^{-rT}}{V_0 \sigma \sqrt{T}} N'\left(-d_2\left(\frac{F}{1-\tau}\right)\right) + \frac{1}{F_B \sigma \sqrt{T}} N'\left(-d_1\left(F_B\right)\right).$$

Now note the following property of the Black-Scholes put option pricing formula:

$$P_0\left(K\right) = -S_0 N\left(-d_1\left(K\right)\right) + Ke^{-rT} N\left(-d_2\left(K\right)\right) \Rightarrow S_0 N'\left(-d_1\left(K\right)\right) = Ke^{-rT} N'\left(-d_2\left(K\right)\right).$$

Applying this here, $\frac{\partial}{\partial F_W}\Delta_{WDcdr} < 0$ if and only if,

$$\left(\frac{1-\tau}{F}\right) N'\left(-d_1\left(\frac{F}{1-\tau}\right)\right) > \frac{1}{F_B} N'\left(-d_1\left(F_B\right)\right).$$

Analyse this:

$$\Leftrightarrow \quad -\left(d_1\left(\frac{F}{1-\tau}\right)\right)^2 + \left(d_1\left(F_B\right)\right)^2 > 2\ln\left(\frac{F}{1-\tau}\frac{1}{F_B}\right)$$

$$\Leftrightarrow \quad \left[\ln\left(\frac{V_0}{F_B}\right)\right]^2 - \left[\ln\left(\frac{V_0\left(1-\tau\right)}{F}\right)\right]^2 + 2\left(r+\frac{\sigma^2}{2}\right)T\ln\left(\frac{F}{1-\tau}\frac{1}{F_B}\right) > 2\sigma^2 T\ln\left(\frac{F}{1-\tau}\frac{1}{F_B}\right)$$

$$\Leftrightarrow \quad \left[\ln\left(\frac{V_0^2\left(1-\tau\right)}{F_B F}\right)\right] + 2\left(r+\frac{\sigma^2}{2}\right)T > 2\sigma^2 T$$

$$\Leftrightarrow \quad V_0 > \left(\frac{F}{1-\tau}F_B\right)^{\frac{1}{2}} e^{-\left(r-\frac{\sigma^2}{2}\right)T}.$$

This is certainly satisfied for $V_0 > \frac{F}{1-\tau}$ when $r > \frac{\sigma^2}{2}$. Hence $\Delta_{WDcdr} < 0$ unambiguously $\forall F_W$ for V_0 above the trigger point.

D.   CoCo Bond as Debt-to-Equity Swap

The present values at T of the payoffs described in (34) are,

V_T range	PV at T
$\left(\frac{F}{1-\tau}, \infty\right]$	$D_T = e^{-rs}\widehat{E}_T\left[Fe^{rs}\chi_{V_S \geq Fe^{rs}} + \beta V_S \chi_{V_S < Fe^{rs}}\right]$
	$E_T = e^{-rs}\widehat{E}_T\left[(V_S - Fe^{rs})\chi_{V_S \geq Fe^{rs}}\right]$
$\left(\frac{F_B}{1-\underline{E}}, \frac{F}{1-\tau}\right]$	$D_T = e^{-rs}\widehat{E}_T\left[\left\{(1-\underline{E})V_Te^{rs} + \frac{\underline{E}-\tau}{\underline{E}}\left[V_S - (1-\underline{E})V_Te^{rs}\right]\right\}\chi_{V_S \geq (1-\underline{E})V_Te^{rs}}\right.$
	$\left. + \beta V_S \chi_{V_S < (1-\underline{E})V_Te^{rs}}\right]$
	$E_T = e^{-rs}\widehat{E}_T\left[\frac{\tau}{\underline{E}}\left[V_S - (1-\underline{E})V_Te^{rs}\right]\chi_{V_S \geq (1-\underline{E})V_Te^{rs}}\right]$
$\left[\frac{F_B}{1-\tau}, \frac{F_B}{1-\underline{E}}\right]$	$D_T = e^{-rs}\widehat{E}_T\left[\left\{F_Be^{rs} + \frac{(1-\tau)V_T - F_B}{V_T - F_B}\left(V_S - F_Be^{rs}\right)\right\}\chi_{V_S \geq F_Be^{rs}} + \beta V_S \chi_{V_S < F_Be^{rs}}\right]$
	$E_T = e^{-rs}\widehat{E}_T\left[\frac{\tau V_T}{V_T - F_B}\left(V_S - F_Be^{rs}\right)\chi_{V_S \geq F_Be^{rs}}\right]$
$\left[F_B, \frac{F_B}{1-\tau}\right)$	$D_T = e^{-rs}\widehat{E}_T\left[F_Be^{rs}\chi_{V_S \geq F_Be^{rs}} + \beta V_S \chi_{V_S < F_Be^{rs}}\right]$
	$E_T = e^{-rs}\widehat{E}_T\left[(V_S - F_Be^{rs})\chi_{V_S \geq F_Be^{rs}}\right]$
$[0, F_B)$	$D_T = e^{-rs}\widehat{E}_T\left[\left\{(F_B - A)e^{rs} + \theta\left[V_S - (F_B - A)e^{rs}\right]\right\}\chi_{V_S \geq (F_B - A)e^{rs}}\right.$
	$\left. + \beta V_S \chi_{V_S < (F_B - A)e^{rs}}\right]$
	$E_T = e^{-rs}\widehat{E}_T\left[(1-\theta)\left[V_S - (F_B - A)e^{rs}\right]\chi_{V_S \geq (F_B - A)e^{rs}}\right].$

These can be evaluated as the following:

V_T range	PV at T
$\left(\frac{F}{1-\tau}, \infty\right]$	$D_T = \beta V_T N\left(-d_1\left(Fe^{rs}\right)\right) + FN\left(d_2\left(Fe^{rs}\right)\right)$
	$E_T = V_T N\left(d_1\left(Fe^{rs}\right)\right) - FN\left(d_2\left(Fe^{rs}\right)\right)$
$\left(\frac{F_B}{1-\underline{E}}, \frac{F}{1-\tau}\right]$	$D_T = \beta V_T + \left(\frac{\underline{E}-\tau}{\underline{E}} - \beta\right)V_T N\left(d_1\left((1-\underline{E})V_Te^{rs}\right)\right) + \frac{\tau(1-\underline{E})}{\underline{E}}V_T N\left(d_2\left((1-\underline{E})V_Te^{rs}\right)\right)$
	$E_T = \frac{\tau}{\underline{E}}\left[V_T N\left(d_1\left((1-\underline{E})V_Te^{rs}\right)\right) - (1-\underline{E})V_T N\left(d_2\left((1-\underline{E})V_Te^{rs}\right)\right)\right]$
$\left[\frac{F_B}{1-\tau}, \frac{F_B}{1-\underline{E}}\right]$	$D_T = \beta V_T + \left[\frac{(1-\tau)V_T - F_B}{V_T - F_B} - \beta\right]V_T N\left(d_1\left(F_Be^{rs}\right)\right) + \frac{\tau V_T}{V_T - F_B}F_B N\left(d_2\left(F_Be^{rs}\right)\right)$
	$E_T = \frac{\tau V_T}{V_T - F_B}\left[V_T N\left(d_1\left(F_Be^{rs}\right)\right) - F_B N\left(d_2\left(F_Be^{rs}\right)\right)\right]$
$\left[F_B, \frac{F_B}{1-\tau}\right)$	$D_T = \beta V_T N\left(-d_1\left(F_Be^{rs}\right)\right) + F_B N\left(d_2\left(F_Be^{rs}\right)\right)$
	$E_T = V_T N\left(d_1\left(F_Be^{rs}\right)\right) - F_B N\left(d_2\left(F_Be^{rs}\right)\right)$
$[0, F_B)$	$D_T = \beta V_T + (\theta - \beta)V_T N\left(d_1\left((F_B - A)e^{rs}\right)\right) + (1-\theta)(F_B - A)N\left(d_2\left((F_B - A)e^{rs}\right)\right)$
	$E_T = (1-\theta)\left[V_T N\left(d_1\left((F_B - A)e^{rs}\right)\right) - (F_B - A)N\left(d_2\left((F_B - A)e^{rs}\right)\right)\right].$