We estimate the effect of changes in demographic structure on long-term trends of key macroeconomic variables using a Panel VAR for 21 OECD economies from 1970-2014. The panel data variation assists the identification of demographic effects, while the dynamic structure, incorporating multiple channels of influence, uncovers long-term effects. We propose a theoretical model, relating demographics, innovation and growth, whose simulations match our empirical findings. The current trend of population ageing and low fertility is projected to reduce output growth, investment and real interest rates across OECD countries.

JEL: E32, J11

Keywords: population age profile, output growth, innovation, life-cycle, Kuznets cycles

Since the Great Recession, slow recovery and reducing productivity growth have prompted concern about longer run prospects for developed economies and the danger of secular stagnation. Debate has centred on the production of new ideas and the structural trends that may shape future economic conditions. While there has been disagreement on the future rate of production of new ideas (see Gordon (2012, 2014), Fernald and Jones (2014) and Brynjolfsson and McAfee (2011)), there is more agreement about the importance of structural trends. In particular demographic changes, through their impact on labour supply, are often seen as one of the ‘headwinds’ of the observed slowdown in macroeconomic performance. We argue that demography matters more generally for macroeconomic activity, altering demand, supply and the production of ideas.

Figure 1 shows how reduced fertility and increased longevity have changed the age profile of the advanced economies. The average proportion of the population
aged 60+ across our sample is projected to increase from 16 in 1970 to 29 percent in 2030, with most of the corresponding decline experienced in the 0 – 19 group. Kuznets (1960) stressed how changes in demographic structure on the population of producers, savers and consumers can affect the medium and long-term macroeconomic prospects (Kuznets cycles). Age structure matters since age groups differ in their (i) savings behaviour, according to the life-cycle hypothesis; (ii) productivity levels, according to the age profile of wages; (iii) labour input, the young and old tend not to work; (iv) contribution to innovation, with young and middle age workers contributing the most; and (v) investment opportunities, as firms target their different needs.

This paper provides an empirical and theoretical investigation of the effects of changes in demographic structure on longer-term macroeconomic trends. We find that ageing in advanced economies generates lower growth, lower investment and lower real interest rates. These effects can be traced both to variations in age related expenditures and savings and, most importantly, to changes in aggregate supply, in particular through a lower rate of innovation.

While the theoretical literature and most economic commentary on policy emphasise the importance of demographic structure, the econometric evidence for its importance is less compelling. There are a number of reasons for this. First, changes in demographic structure are low frequency phenomena, difficult to distinguish from other low frequency trends that dominate economic time series. Second, the vector of proportions in each age group is inevitably highly collinear, making precise estimation of their effect difficult. Hence it is common to impose strong restrictions on the age structure, for instance through the use a single variable, the dependency ratio. Finally, single-equation models explaining some dependent variable by a set of factors including demographics are unlikely
to capture the general equilibrium effects of demography. We address these issues by estimating a Panel VAR for a vector of six key macro variables, augmented by demographics, for 21 OECD economies over the period 1970-2014. Using six macro variables allows for multiple channels of influence. The panel variation helps identify a life cycle pattern in the effects of the slowly moving age profile on all the variables. Young and old dependants have a negative impact on economic activity while workers contribute positively. The dynamic structure of the VAR, allowing for the feedbacks through the variables of interest, reveals long-run effects that are stronger than short-run effects.

We use our long-run estimates and the United Nations (UN) population predictions to make conditional projections of the effect of demographic change for each country in our sample. The changes in age profile lead to a statistically and economically significant drop in investment, savings, growth and interest rates for most OECD economies. Trend output growth is expected to be reduced on average by 0.64 percent during the 2015-2025 decade.

If the main effect of ageing was through aggregate expenditure effects, such as the incentive to increase savings due to higher life expectancy, one would expect interest rates to fall and investment to increase. We find that both the real rate of interest and investment decrease as society ages, which indicates that a supply side effect, through the return on capital, must be in effect. To investigate this we extend our model to analyse the impact of demographics on innovative activities, proxied by the number of patent applications per capita. We find evidence that the production of patents are affected by the age structure of the society, with middle-aged workers (30-49) having a positive impact on patenting while dependants impact negatively. Thus, ageing may result in lower rates of innovation, and hence marginal productivity, reducing the rate of return of capital as suggested by Gordon (2012).

To help interpret the life cycle effects observed in the data, we develop a theoretical model of the impact of demography on the macroeconomy. The model has three generations (young, workers and retirees), investment in human capital, endogenous productivity and medium-term dynamics. It allows us to examine the longer-term interaction between demographic changes and savings, investment and innovation, which shape the evolution of output growth.

Using the UN population predictions we show that the theoretical model is able to replicate our empirical findings. While our model also contains an aggregate expenditure channel, the key mechanism in the model is the link between demographics and innovation. A change in the demographic profile towards an older society leads to a decline in innovation and the marginal product of capital, reducing investment and growth. This may explain why advanced economies find it difficult to revive economic activity despite low interest rates.

\footnote{Our aggregate expenditures channel effectively lowers potential output. Note that this is distinct to the aggregate demand externalities explored in the revived secular stagnation literature, which rely on nominal rigidities and constrained monetary policy altering the gap between output and its potential level.}
There is a large empirical and theoretical literature on the macroeconomic effects of demography. Empirical studies tend to focus on a single variable of interest, using 5-year average data and summarising demography by a single statistic (e.g. Higgins and Williamson (1997) and Bloom et al. (2007)), or using a low order polynomials in the parameters (e.g. Fair and Dominguez (1991) and Higgins (1998)). An exception is Favero and Galasso (2015) who use annual data and population weights to estimate single equation econometric models. We use annual data rather than 5-year averages, use a more granular representation of the demographic structure and, having a larger sample, we can estimate a panel VAR rather than a single equation, thus allowing for interactions between macro-variables that potentially capture general equilibrium effects.

On the theoretical side our paper is related to studies of the impact of demographies on the macroeconomy and interest rates (e.g. Krueger and Ludwig (2007), Carvalho et al. (2016) and Gagnon et al. (2016)). Our framework is distinctive in incorporating demographic heterogeneity in an endogenous growth model (following Romer (1990) and Comin and Gertler (2006)) and hence can analyze how changes in demographic structure affects innovation, productivity and growth. While comparing our projected impact of demographic changes in the next decades with the projections shown in Gagnon et al. (2016), we find that the link between demographics and productivity amplifies the effects of population changes on the interest rate. Finally, our work is also related to the Alvin Hanson’s recently popularised argument on whether mature economies are experiencing a long lasting stagnation due to permanently low demand. Most of this literature currently focuses on the effects of aggregate demand externalities in periods of financial deleveraging that may lead to prolonged periods of lower real rates of return (see Eggertsson and Mehrotra (2014) and Jimeno (2015)). By linking demographic changes to innovation and output growth, our results provide further indication that OECD economies are more likely to experience episodes where aggregate demand externalities together with supply side channels may lead to stagnation in the following decades.

The remainder of the paper is organised as follows. The econometric methodology is presented in Section I and the empirical results in Section II. The theoretical framework is presented in Section III and the simulation results in Section IV. Finally, Section V concludes.

I. Econometric model

We wish to discover the long-run impact of changes in the demographic structure, represented by a set of population shares \( W_t \), on a set of macroeconomic variables, \( Y_t \). The dynamic interactions among these economic variables are likely to be complicated. In Section III we present a theoretical general equilibrium model that allows for a range of these interactions, which we calibrate in terms of the deep parameters of the system. In principle, one might consider estimating a linearised version of this structural system. Identifying and directly estimating
such a system is likely to be difficult. Therefore we estimate a reduced form of the system and assume that conditional on the exogenous variables, it can be written as a VAR. We then compare the simulated predictions from our theoretical model of Section III with the predictions of the estimated empirical model.

In the Panel VAR we have a vector of macroeconomic variables $Y_{it}$ and of population shares $W_{it}$ for countries $i = 1, 2, ..., N$. We assume slope homogeneity across countries. While slope heterogeneity is undoubtedly important, we found that estimating heterogeneous slopes with relatively few degrees of freedom resulted in poorly determined parameters. Baltagi et al. (2000) show that the homogeneous estimators tend to have better forecasting properties. As a result, since our main aim is prediction, we assume slope homogeneity, but allow for intercept heterogeneity through $a_i$. Relying on the Schwarz Bayesian information criterion (SBC) a one-way fixed effect model with country intercepts was preferred for every equation to a two-way fixed effect model with country and year intercepts (two-way estimation results are discussed in the robustness section). The SBC of a first and second order VAR were similar and thus we select the more parsimonious model. Therefore, we estimate an augmented panel VARX(1) of the form:\footnote{The demographic variables show very low frequency variation relative to annual macroeconomic time-series and thus are assumed to be exogenous. A VARX with $W_{it}$ considered exogenous is appropriate when $Y_{i,t-1}$ do not Granger-cause $W_{it}$. We confirm the VARX specification in equation (1) is applicable, see Section II.A for details.}

\begin{equation}
Y_{it} = a_i + Ay_{i,t-1} + DW_{it} + u_{it},
\end{equation}

including two additional controls: lagged oil price, to allow for global shocks, and population growth, to capture the effects of demographic structure ($W_{it}$) rather than the population effect.\footnote{Implicitly we are assuming either that all the variables are stationary or that a flexible unrestricted VAR will capture stationary combinations by differencing or cointegrating linear combinations. Phillips and Moon (1999) and Coakley et al. (2006) suggest that spurious regression may be less of a problem in panels. Notice that we do not attempt to determine how $A$ and $D$ relate to structural (deep) parameters. Our primary empirical objective is to provide predictions of the long-run effect of the demographic variables and the same predictions would be obtained from any just identified structural model.}

The panel variability is crucial for the identification of the impact of demographics. The fixed effect (or within) estimator can be written in terms of deviations from the country means of the variables, so does not use the cross section (between country) variation. However, it benefits from the greater within variation that results from different countries entering the demographic transition at different times.

Having estimated $a_i$, $A$, and $D$, from the panel VAR, the long-run equilibrium for the system (ignoring population growth and oil prices) is then given by

\begin{equation}
Y^*_{it} = (I - A)^{-1} a_i + (I - A)^{-1} DW_{it},
\end{equation}
where the effect of the demographic variables is given by $D_{LR} = (I - A)^{-1} D$, which reflects both the direct effect of demographics on each variable and the feedback between the endogenous variables. Therefore, we can consider the effects of demography on savings to influence growth through the effect of savings on growth. We can isolate the long-run contribution of demography to each variable in each country by obtaining the demographic attractor for the economic variables at any moment in time

\[(3) \quad Y_{it}^D = (I - A)^{-1} DW_{it} = D_{LR} W_{it} \]

We denote each element in matrix $D_{LR}$, which is a function of parameters in matrices $A$ and $D$, $d_{ij}(A, D)$. In order to test whether each element or a sum of those elements in $D_{LR}$ are significantly different from zero (e.g. $H_0 : D_{LR}(i, j) = 0$ or $H_0 : d_{ij}(A, D) = 0$) we utilize a non-linear Wald test.\(^4\) Finally, it is important to distinguish between our long-run estimate and a long-run steady state. Our estimates provide a long-run forecast for the economic variables conditional on a particular vector of demographic shares after the completion of the endogenous adjustment of the economic variables and as such we are measuring the impact of demographics on the long-run trend of the key macroeconomic variables. However, as time passes the demographic structure might evolve towards a steady state demographic distribution. We do not model this process and thus are not providing explicitly an estimate of the effects of any demographic convergence to a steady state on the macroeconomy. Nonetheless, the UN projections used in our prediction exercise may embody some estimate of such convergence.

Our benchmark specification includes six endogenous variables: the growth rate of the real GDP, $g_{it}$; the share of investment in GDP, $I_{it}$; the share of personal savings in GDP, $S_{it}$; the logarithms of hours worked per capita, $H_{it}$; the real short-term interest rate, $rr_{it}$; and the rate of inflation, $\pi_{it}$. We denote the vector of these six variables as $Y_{it} = (g_{it}, I_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})'$. Lack of growth and investment, excess savings, reductions in labour supply and interest rates have all commonly been implicated in the secular stagnation debate and thus their inclusion is straightforward. The case for including inflation in a longer run model is less clear. But since our methodology relies on estimating the dynamic interactions among the macro variables where monetary policy is an active stabilisation tool, we include inflation to allow for these feedbacks. We discuss a series of alternative specifications, including a model without inflation, replacing output growth with per capita growth, replacing short-term with long-term interest rates and including net foreign assets. The main results of the benchmark specification

\(^4\)The Wald statistic is given by $d_{ij}(\hat{A}, \hat{D})^T[\hat{d}_{ij}'(\hat{A}, \hat{D})][\hat{V}(A, D)]^{-1}\hat{d}_{ij}'(\hat{A}, \hat{D}) \xrightarrow{D} \chi_2^2$, where $\hat{V}(A, D)$ is the estimated variance-covariance matrix and $\hat{d}_{ij}'(\hat{A}, \hat{D})$ is the gradient of function $d_{ij}(A, D)$.\
are robust to these variations.\textsuperscript{5}

We represent the age structure by the population shares of three groups: the young dependants, aged below 20; the working age population, aged 20 to 60; and the old dependants, aged 60 and over. We denote the share of age group $j = 1, 2, 3$ ($0 - 19$, $20 - 59$, $60+)$ in total population by $w_{jit}$. Since $\sum_{j=1}^{3} w_{jit} = 1$, there is exact collinearity if all the demographic shares are included. To deal with this, we restrict the coefficients to sum to 0, use $(w_{jit} - w_{3it})$ as explanatory variables and recover the coefficient of the oldest age group from $\delta_3 = -\sum_{j=1}^{2} \delta_j$. We denote the 2 element vector of $(w_{jit} - w_{3it})$ as $W_{it}$. This demographic representation is parsimonious and effectively reflect both the young and old dependency ratios often used in empirical studies on demographics.

We also consider a specification with 8 age groups ($0 - 9$, $10 - 19$, …, $70+$). Although a richer relationship between demographics and the macroeconomy is uncovered with a more granular representation of the age structure, the results confirm a strong pattern differentiating young dependants, workers and the old, supporting our benchmark model. Moreover, since our theoretical model has the same age groups (youngsters, workers and retirees), the three group specification provides a closer link between theory and empirics.

\section*{II. Estimation Results}

The annual dataset covers the period 1970-2014 for 21 OECD countries (see the online appendix for details). Population data were obtained from the World Population Prospects, the 2015 Revision (United Nations (2016)). Age compositions are calculated using the share of the \textit{de facto population} in the age group indicated in the total population. It is important to note that UN population data measures those living in a country, not just its citizens, thus reflecting immigration. We complement this with annual data on savings and investment rates, calculated from nominal GDP, hours worked, annual data on policy rates and the Consumer Price Index (CPI). Finally, real output growth rates are calculated from real GDP.

Table 1 shows our main parameter of interest, the $(I - A)^{-1} D = D_{LR}$ matrix: the long-term demographic effects for three age groups ($\beta$'s).\textsuperscript{6} We observe clear life-cycle patterns: workers contribute positively to growth, investment, savings,

\textsuperscript{5}In Section II.B. we extend our model with an additional variable, per capita patent applications, a proxy for innovation. However our benchmark econometric specification involves a trade-off between efficiency (precision of estimates) and bias (6 variables system against 7 variables system). Therefore, we start with the six variable model since the bias caused by the omitted variable, per capita patent applications, is very small, and we gain in terms of efficiency since the parameters are more precisely estimated.

\textsuperscript{6}Full estimates with robust standard errors of matrices $A$ and $D$ are shown in the online appendix. We note that hours worked, investment, savings and real rates are highly persistent and real output and inflation rate are moderately so. Granger causality tests suggest that all our endogenous variables are Granger causal (with two lags) for most of the other variables in the system; so we seem to be capturing the dynamic interactions between the main economic variables. The residual correlation matrix shows some strong contemporaneous correlations, possibly reflecting business cycle effects.
hours worked and interest rates and dependants, the old particularly, contribute negatively. Allowing for the dynamics and interactions strengthens the general impact of demographics; the long-run effects are much larger than the short-term effects. While in principle, statistical significance for each parameter is difficult to obtain given that each element in $D_{LR}$ is a function of 38 estimated parameters (matrix $A$ and a column of matrix $D$), most of the long term demographic structure parameters are precisely estimated. (11 out of 18 parameters). Furthermore, the joint test that $\beta_2, \beta_3 = 0$ is strongly rejected for all equations.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
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<tr>
<td>$g$</td>
<td>0.04</td>
<td>0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.28)</td>
<td>(0.04)</td>
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<td>$I$</td>
<td>0.13</td>
<td>0.08</td>
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<td></td>
<td>(0.01)</td>
<td>(0.48)</td>
<td>(0.02)</td>
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<td>(0.00)</td>
<td>(0.05)</td>
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<td>$rr$</td>
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<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.68)</td>
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</table>

Note: The p-values of the non-linear Wald Test (See footnote 4) with $H_0: D_{LR}(i,j) = 0$ is reported within brackets.

Table 1—Long-Run Demographic Impact Matrix - $D_{LR}$

Although the estimates indicate clear differences in the effect of different age groups on the macro variables, we are also interested in the net effects of demographic changes. To measure these we use the demographic predictions for each county ($W_{i,t+h}$) from the UN World Population Prospects, 2015 Revision, and our long-run demographic impact matrix ($D_{LR}$) to estimate the effect of expected demographic changes on the long-run macro trends (see equation (3)). Formally, we obtain $Y_{i,t+h} = D_{LR}(W_{i,t+h} - W_{i,t}) + Y_{i,t}$. Table 2 provides these conditional forecasts that incorporate the changes in demographic structure on average annual GDP growth (in percentage points). The first column shows the mean growth rate between 1970-2010 for each country (initial value at time $t - Y_{i,t}$). The second (third) column shows the projected long-run growth rate in 2015 (2025) incorporating the effects of demographic changes from 2010-2014 (2010-2024). The results suggest that in all countries in our sample, demographic changes over this decade depress long-term GDP growth. The magnitude of the drop is highly economically significant: for the US, for example, it is 0.65 percentage points and for Japan 0.42. The last column of Table 2 shows the probability
that the change in GDP growth, as a result of demographic changes, would be negative. This is less than 10 percent for all countries except for Sweden.

Whereas demography is quite predictable, being largely predetermined by known birth and death rates, economic variables are less so. Unrestricted VARs tend to have poor unconditional forecasting records, because they are over-parameterised, hence the widespread use of Bayesian VARs. Here we are using the VAR not to make a forecast but to make a conditional prediction of the effect of likely demographic change. We may not know what growth will be in 2025, but the estimates suggest that it will be lower than it would have been had the demographic structure not changed in the way the UN expects. In addition, VAR’s have been criticised for attributing implausibly large parts of the low frequency variation to deterministic components, (e.g. Giannone et al. (2017)) but here the low frequency drivers are the predictable exogenous changes in demographic structure. Finally, our conditional predictions do not take into account other possible counter balancing factors that drive long term trends, for instance technological progress associated with recent developments in robotics (Acemoglu and Restrepo (2017)).

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample Average (1970-2010)</th>
<th>Projected at 2015</th>
<th>Projected at 2025</th>
<th>Change ($\Delta g$) (2015-2025)</th>
<th>Prob($\Delta g &gt; 0$)</th>
</tr>
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<td>3.11</td>
<td>2.60</td>
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<td>-0.46</td>
<td>0.060</td>
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<td>1.93</td>
<td>-0.51</td>
<td>0.084</td>
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</table>

Table 2—Average Predicted Impact on GDP Growth (in percentage) by Country

We also present the projected effect of demographics on the trend of output growth, real rates, investment and savings for France, Italy, Japan, and the United States (the prediction for four additional countries is shown in the online appendix) for the period 2010 up until 2030 in Figure 2. As before, the initial point in the year 2010 is given by the observed sample average of the variable
for the period 1970-2010. Then at each year from 2011 to 2030 we depict the conditional forecasts \((Y_{t+k})\) for output growth, savings, investment and real rates.\(^7\)

We observe that demographic changes are expected to reduce long term growth and real interest rate in all OECD countries in our sample in the next two decades. Shaded areas (80 and 90 percent one sided tests) around conditional predictions of the macroeconomic variables in our dynamic system show that our estimates are reasonably precise; given the UN population projections all variables exhibit clear downward trends. In 2030 there is a higher than 90 percent chance that demographic changes lead to lower growth rates, investment and savings in all countries and 85-90 percent chance that real interest rates decrease.

Demographic changes are projected to reduce on average the ratio of investment over GDP by 2 percentage points, saving over GDP by 3 percentage points and output growth by 1.25 percentage points across countries in the OECD. The projections for real interest rate show a more substantiable fall. We note that in some of our robustness exercises, for instance when mortality and fertility trends are incorporated, the ageing effect on interest rate is dampened. In our structural model we also find that the implied fall in interest rates due to the predicted demographic changes is less pronounced. Therefore, we interpret the results in the benchmark estimation as a lower bound on the projected path of interest rates.

The depicted long term trends and their confidence bounds arise solely from our demographic attractor estimates; there will still be uncertainty associated with UN demographic predictions we do not account for and uncertainty coming from shocks to endogenous variables in the system, and hence the projections should not be interpreted as a general forecast. Nevertheless, our results show a strong positive association between real output growth, real rates and crucially investment rates, suggesting the secular stagnation hypothesis - the decline in real rates and real output - must be related to aggregate supply drivers and not only to aggregate demand externalities.

Finally, we perform a projection exercise focusing on the in sample period. We compare a prediction using our parameter estimates and the actual population changes from 1980 to 2015 with the long-term trends extracted from the data. We utilize a band pass filter that removes short and medium-term cycles (with frequency from 1 to 35 years) from the raw data.\(^8\) This should not be interpreted as a validation test of our empirical model since there are several factors that could be affecting long-term trends together with demographics. However, the analysis offers an insight as to whether demographics is a relevant driver of these long-term trends.

\(^7\)The first row of Figure 2 is effectively a graphical representation of the results of the conditional forecasts of output displayed in Table 2. For instance, in the case of the United States (top right corner), the initial point in 2010 is 2.85 percent, and in 2025, output growth is expected to fall to 1.87 percent.

\(^8\)Comin and Gertler (2006) define short-term cycles for frequencies of up to 8 years and medium-term cycles for the movements at frequencies from 8 to 50 years. Given the length of our time series we restrict medium-term cycles to 35 years.
trends. We find that the long-term projections based on demographic changes are well aligned with the observed trends across variables and countries, particularly being able to match the hump-shaped path of interest rates due to the effects of the baby-boomers; first entering the labour force in the 70’s and 80’s, pushing rates up, and having the opposite effect when approaching retirement.\(^9\) We also infer that the decreasing trends in investment and savings are influenced by age structure changes. Results are displayed in Figure 3.

A. Robustness

We analyse the robustness of our results by altering the benchmark model in various ways. Here we summarise them, all detailed results are given in the online appendix.\(^{10}\)

First we varied the exogenous controls. Our benchmark specification includes

\(^9\)In Aksoy et al. (2015) we look at the effect of increasing fertility for a decade and tracing the economy’s response as the entire demographic structure converges back to its starting point both in the theoretical and empirical models. This hump-shaped responses are obtained in both cases. The information content of demographics to explain interest rates is also stressed by Favero et al. (2016) while looking at bond yields and excess returns.

\(^{10}\)Aksoy et al. (2012) and Aksoy et al. (2015) estimate the model for the period up to the financial crisis (1970-2007). Results are qualitatively invariant to the sample used.
population growth as an exogenous control, but this will reflect trends in fertility and mortality rates. (See Favero and Galasso (2015) for a detailed discussion.) In an alternative specification we replace population growth with mortality and fertility rates calculated with the Lee and Carter (1992) procedure using age-specific fertility and mortality data. Our benchmark results are robust. The main difference in this specification is the impacts of the 60+ age group on real interest rates ($\beta_3$ increases from $-0.42$ to $0$). This effect is in line with the results discussed in Favero and Galasso (2015), who argue that controlling for mortality trends has a positive effect on interest rates projections. Nonetheless, given that the interest rate effect of the share of active population is strongly positive and significant, the demographic changes expected for most OECD economies would still generate a fall in interest rates. Under this specification the projected decrease in interest rates would be milder than in the benchmark model.\footnote{The coefficients of mortality and fertility are generally small and including them makes very little difference to the projections. The 60+ age group embeds two opposing drivers of the effects of demographics on interest rates. First, the share of 60+ individuals increases due to high life expectancy, which is responsible for pushing rates down. Second, the old tend to use up their savings pushing interest rates up. Including mortality trends introduces a control for the life expectancy effect and thus the parameter estimate of this age group reflects the second effect more strongly.} We conclude that varia-

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Demographics and Trends - In Sample Projection}
\end{figure}
tions in age structure reflect the two fundamental demographic shocks: fertility and mortality.

The results are largely unchanged if oil prices, used to control for global shocks, are removed. A more general control for global factors is to allow for time effects, though the SBC chose the one-way, country fixed effect model. Comparing the estimation results from the two way fixed effect model and the benchmark model reveals that long-term demographic effects are generally robust, maintaining the pattern of differentiation between workers and dependants. The relationship between investment and demographics becomes more uncertain ($\beta_1$ and $\beta_3$ are no longer significant). Nonetheless, ageing continues to lead to a drop in investment as $\beta_2 > 0$ is statistically different than $\beta_3 < 0$. We also observe that the 60+ age group no longer has a negative impact on real rates, similar to the specification with mortality and fertility trends.

Second we varied the vector of dependent variables ($Y_{it}$). The results are robust to excluding inflation, utilizing per capital output ($g_{pc}$) instead of output growth and using long-term (10 years) rates ($rr_{10}$) instead of short-term rates. The benchmark model only includes domestic variables. We add net foreign assets to GDP ratio ($nfa$) to the vector $Y_t$ to allow for international effects through current account dynamics. Due to data availability (see Lane and Milesi-Ferretti (2007)) the sample period is 1970-2011. Our current account proxy exhibits life cycle properties. Although parameter estimates are imprecise, net foreign assets to GDP are decreasing in working age population and increasing in young and old age groups.12 Our long term estimates and the demographic impact significance for other variables in the system are not qualitatively affected. Particularly, interest rate estimations are not significantly altered and hence although capital movements are driven by demographics, the presence of frictions might be preventing interest rate equalization across countries, preserving the heterogeneity in which our identification relies on.

Thirdly, we varied the demographic variables. Rather than three age groups we used 8 age groups $j = 1, 2, \ldots, 8$ (0−9, 10−19, ..., 70+) denoted $w_{j,i,t}$ and $W_{i,t}$ comprises the 7 element vector of $(w_{j,i,t} - w_{8,i,t})$. The general life cycle findings are confirmed with more granular age structure. Increases in the older age groups (60-69, 70+) are associated with reductions in all macroeconomic variables except inflation while working age groups (20-29, 30-39, 40-49, 50-59) impact positively on the macro variables other than inflation. Real rates are increased by the share in age groups 10-19, 20-29 and 30-39 and reduced by older groups, supporting our benchmark results. As more parameter estimates are needed, significance of each group effect is compromised. As demographic patterns affect the entire age structure, we also conduct joint significance test of the parameters for workers, dependants and their difference. Results confirm the life cycle pattern in the relationship between demographics and macroeconomic variables, suggesting that the parsimonious representation used in the benchmark model is preferable.

12The link between demographics and capital flows are also studied by Ferrero (2010).
Fourthly, we test for the weak exogeneity of $W_{it}$ (that is equivalent to check whether our set of macroeconomic variables does not Granger-cause the demographic structure). Although we find that the lags of some the macro variables significantly affect $W_{it}$, the estimated parameters were all small. For instance, the highest significant parameter estimate is 0.02, measuring the impact of the saving rate on the relative share of the young age group (0-19). That implies a one percentage point move in saving rates would lead to a change in that proportion from 10 to 10.02 percent in the short-run. We thus conclude that $Y_{i,t-1}$ do not significantly (economically) affect $W_{it}$. Demographics can therefore be included as an exogenous variable in the dynamic model of $Y_{i,t}$, as our benchmark VARX prescribes.

**B. Demographics and Innovation**

Our estimates indicate that ageing reduces growth, investment, and interest rates, suggesting that demographics influence aggregate supply. This accords with the literature. Kuznets (1960) discusses the relevance of population growth, and hence the presence of the young, for innovation. Feyrer (2007, 2008 and 2011) find a strong relationship between demographic structure and productivity, suggesting two potential channels: innovation and adoption of ideas through managerial and entrepreneurial activity. He shows that in the US innovators’ median age is stable around 48 over the 1975-95 sample period whereas median age of managers who adopt ideas is even lower. He argues that changes in the supply of workers may have an impact on the innovation rate. Similarly, Jones (2010) and Jones et al. (2014) consider the age distribution of scientists (Nobel Prize winners and great technological innovators) and development of ground-breaking ideas. They argue that the scientific or technological breakthroughs peak when scientists are in their late 30’s.

In order to account for possible dynamic interactions between demographic structure and innovation which in turn affect technological progress and aggregate supply, we re-estimate the model including an additional variable that proxies for R&D activity. We utilize the log number of residential patent applications per capita, ($R&D_{PA}$, see World Development Indictors - World Bank). Thus, the vector of endogenous variables is now given by $Y_{i,t} = (g_{i,t}, I_{i,t}, S_{i,t}, H_{i,t}, r_{i,t}, R&D_{PA}, \pi_{i,t})$.

Table 3 shows the estimated long-term demographic impact matrix $D_{LR} = (I - A)^{-1} D$. The introduction of innovation variable does not affect the main qualitative conclusions obtained in the benchmark estimation. Furthermore, we find that young and old have a negative effect on patenting whereas workers contribute positively. Using eight age groups confirms that the positive effect is concentrated in young and middle-aged workers, in line with the evidence in Jones (2010) and Feyrer (2008). Finally, using these long-term estimates and UN population predictions we find that the expected ageing in the next two decades may lead to a drop in per capita patent applications of 15 to 30 percent in our
sample of OECD countries.

III. Theoretical Model

In this section we propose a model which allows for demographic heterogeneity, life-cycle properties, innovation and endogenous productivity.

The economy consists of three sectors: production, innovation and households. The production sector comprises of a final good producer, whose inputs are differentiated goods provided by producers, who employ capital, labour and a composite intermediate good. The number of input producers is endogenous, determined through entry and exit.

The intermediate good aggregates an endogenous set of product varieties, created by a two stage innovation process involving product creation of prototypes through R&D and product adoption, converting the prototypes into intermediate goods.

Individuals, in the households sector, supply labour, accumulate assets and consume over their life-cycle. Finally, a zero expected profit financial intermediary facilitates the allocation of assets between the household and the production and innovation sectors.

A. Production

The final good producer combines inputs from $N^f_t$ firms, denoted by superscript $j$. Total output is thus given by

$$ Y_{c,t} = \left[ \int_0^{N^f_t} (Y_{c,t}^j)^{(1/\mu_t)} dj \right]^{\mu_t}, $$

Table 3—Long-Run Demographic Impact - Innovation
where $\mu_t$ denotes the mark-up of input firms. We assume $\mu_t = \mu(N_f^t)$, $\mu' (\cdot) < 0$ and that profits of intermediate good firms $\Pi_t(Y_{c,t}^J)$ must equate operating costs given by $\Omega \tilde{\Psi}_t$, where $\tilde{\Psi}_t$ is a scaling factor defined to ensure we obtain a balanced growth path (see below).

Each firm $j$ produces a specialised good using capital ($K^j_t$), labour ($L^j_t$) and an intermediate composite good ($M^j_t$). Production is given by

$$Y_{c,t}^j = \left[(U^j_t K^j_t \xi_t L^j_t)^{(1-\gamma_I)} \left[M^j_t\right]^{\gamma_I}\right].$$

where $U^j_t$ is the utilisation rate, $\gamma_I$ the intermediate good share, $\xi_t L_t$ denotes the effective labour units employed in production and $\alpha$ the capital share of added value. The intermediate composite good used by firm $j$ aggregates $A_t$ specialised goods such that

$$M^j_t = \left[\int_0^{A_t} (M^j_i)^{(1/\vartheta)} di\right]^{\vartheta}.$$

Each producer of specialized good $i$ acquires the right to market this good via the creation and adoption process. Total costs of production for firm $j$ are then given by

$$TC^j_t = W_t \xi_t L_t^j + (r^k_t + \delta(U^j_t))K^j_t + P^M_t M^j_t.$$

where $W_t$ is the wage, $\xi_t$ is the average effective unit of labour, $r^k_t$ is the rent of capital, $\delta(U^j_t)$ is the capital depreciation rate, with $\delta'(\cdot) > 0$, and $P^M_t$ is the price of the intermediate composite good.

Firms' optimization conditions determine the equilibrium wage, the rent of capital, the utilisation rate, the intermediate good composite and their price, the number of firms (through the entry condition), the mark-up and the depreciation rate. In equilibrium, the total production of final goods is given by

$$Y_{c,t} = (N_f^t)^{\mu_t-1} \left[(U_t K_t \xi_t L_t)^{(1-\gamma_I)} \left[M_t\right]^{\gamma_I}\right].$$

This condition embeds the two key assumptions that shape the dynamic responses in the production sector. Firstly, given that the mark-up $\mu_t$ is greater than one, as $N_f^t$ increases total production increases. Secondly, the intermediate good composite $M_t$ increases with the number of varieties $A_t$ and the number of input producers $N_f^t$. Thus, variations in aggregate supply may occur due to innovation effort determining the number of varieties $A_t$, which we look at closely next, and the incentives for entry, which is directly related to the level of aggregate demand.
B. R&D and Adoption

Innovation in intermediate good varieties is divided into two stages: R&D which invents prototypes and adoption which converts them into intermediate goods. This two step innovation process captures the lags between invention and adoption observed in practice, see Comin and Gertler (2006), and is quantitatively important for our results.

Let $Z^p_t$ be the stock of prototypes for innovator $p$, who at each period spends $S^p_t$ to invent $\varphi_t S^p_t$ new prototypes. Thus, $Z^p_{t+1}$ is given by

$$Z^p_{t+1} = \varphi_t S^p_t + \phi Z^p_t,$$

where $\phi$ is the product survival rate. In Comin and Gertler (2006) the productivity of new inventions $\varphi_t$ is assumed to be given by $\varphi^{CG}_t = \chi Z_t [\Psi_t(S_t)^{1-\rho}]^{-1}$, where $\chi$ is a scale parameter. Thus, it depends on the aggregate stock of prototypes ($Z_t$), such that there is a positive spillover as in Romer (1990), and on a congestion externality via the factor $[\tilde{\Psi}_t(S_t)^{1-\rho}]^{-1}$, such that a balanced growth path exists and the R&D elasticity of new technology creation in equilibrium is $\rho$. However, as Kremer (1993) discusses if each individual’s chance of being lucky or smart enough to invent something is independent of population size, then the number of individuals working relative to total population will be important to determine the aggregate growth rate of invented goods in an economy. Jones (2010), Jones et al. (2014) and Feyrer (2008) analyse the age profile of inventors/innovators and show that the frequency of inventions of middle-age individuals is substantially higher than the one observed for younger and older individuals. Derrien et al. (2017), exploiting the heterogeneity across regions in the US, show that a younger labour force innovates more. Finally, our estimates indicate that economies with a greater share of middle-age workers (30-49) generate more patents while economies with more dependants produce less patents. Consequently, innovation may be related to the demographic composition of the economy.

To reflect the role of demographics in innovation, we assume the productivity of invention is given by $\varphi_t \equiv (\Gamma_t^\rho)^\rho w^w \chi Z_t [\hat{\Psi}_t(S_t)^{1-\rho} N_t^{\rho w}]^{-1}$, where $\Gamma_t^\rho w$ is the stock of workers in the innovation sector and $\rho w$ controls their importance, which we calibrate using the data in Jones (2010) relating the production of ideas to age. As discussed in Jones (1995) and more recently Bloom et al. (2017), models of endogenous growth where an increase in the growth rate of the stock of workers employed in R&D (due to population growth) generates faster steady state output growth are inconsistent with the data. In order to avoid such feature and to ensure we obtain a balanced growth path, we include a measure of total population ($N_t$) in the congestion factor (as a robustness, we also consider a congestion measure

13Liang et al. (2014), looking at entrepreneurship show that a high proportion of old workers prevents young workers gaining the necessary knowledge to start up a new business, thus reducing entrepreneurship.
using the working age population \(- N^w_t\). If \(\rho_{yw} = 0\), the innovation process is equivalent to the one assumed in Comin and Gertler (2006). If \(\rho_{yw} > 0\), an economy that employs more of its young workers in research and development innovates more. We define \(\Gamma^w_t\) when we discuss the population dynamics below.

The stock of prototypes is then

\[
Z^p_{t+1} = (\Gamma^w_t)^{\rho_{yw}} \chi Z_t \left[ (\tilde{\Psi}_t) \rho(S_t)^{1-\rho} N^w_t \right]^{-1} S^p_t + \phi Z^p_t.
\]

We assume that innovators borrow \(S^p_t\) from the financial intermediary. Define \(J_t\) as the value of a prototype, then, innovator \(p\) will invest \(S^p_t\) until the marginal cost equates the expected gain. Thus,

\[
\phi E[J_{t+1}] = \frac{R_{t+1}}{\phi_t}.
\]

Where \(R_{t+1}\) is the interest rate.

Adopters \((q)\) obtain the rights to the technology from inventors and make an investment of \(\Xi_t\) to transform \(Z^q_t\) into \(A^q_t\), defined as the stock of converted goods ready to be sold to firms. This conversion process succeeds with probability \(\lambda_t = \lambda \left( \frac{A^q_t}{\Psi_t} \Xi_t \right)\) and \(\lambda'(\cdot) > 0\); thus more investment yields more adoptions. If unsuccessful, the good remains a prototype. Once converted it can be sold to firms with value

\[
V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1},
\]

where \(\Pi_{m,t} = (1 - 1/\varphi_t) \gamma_t \frac{Y_{c,t}}{\mu A_t}\) is the profit from selling an intermediate good to input firms. The value of a prototype is then

\[
J_t = \max_{\Xi_t} -\Xi_t + (R_{t+1})^{-1} \phi E_t [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}].
\]

The stock of prototypes at \(t\) is \((Z^q_t - A^q_t)\). Thus,

\[
A^q_{t+1} = \lambda_t \phi (Z^q_t - A^q_t) + \phi A^q_t.
\]

The expenditure in consumption goods of adopters, financed by borrowing, is \(\Xi_t(Z^q_t - A^q_t)\).

Conditions (11), (12) and (13) highlight the drivers of innovation. Firstly, as aggregate demand increases, the profit from selling new varieties \((\Pi_{m,t})\) increases and thus \(V_t\) also increases leading to a more successful rate of adoptions (higher \(\lambda_t\)). Consequently, holding productivity \((\phi_t)\) constant, the value of prototypes \(J_t\) increases, boosting the investment in product creation. Secondly, as the interest rate decreases, both the value of adopted goods \(V_t\) and prototypes \(J_t\) increase since these depend on future gains that are less heavily discounted. Finally,
the productivity of prototype creation ($\varphi_t$) is directly related to the share of young workers, and thus, demographic structure ultimately affects the evolution of the number of varieties ($A_t$). Finally, allowing for the lag between adoption and invention is important. If we set $J_t = V_t$, ignoring adoption, the impact of demographic changes is faster and more volatile than shown in the data.

C. Household Sector

There is a continuum of agents of mass $N_t$, divided amongst 3 age groups: young dependants ($N^y_t$), workers ($N^w_t$), and retirees ($N^r_t$). $\tilde{n}_{t,t+1}N^y_t$ individuals are born every period and become workers with probability $1 - \omega^y_y$. Workers retire with a probability $1 - \omega^w_w$, and retirees die and leave the economy with a probability $1 - \omega^r_r$. As a result, the population dynamics are

\begin{align}
N^y_{t+1} &= \tilde{n}_{t,t+1}N^y_t + \omega^y N^y_t = (\hat{n}_{t,t+1} + \omega^y)N^y_t = n_{t,t+1}N^y_t, \\
N^w_{t+1} &= (1 - \omega^y)N^y_t + \omega^w N^w_t, \\
N^r_{t+1} &= (1 - \omega^w)N^w_t + \omega^r_{t+1}N^r_t.
\end{align}

The young dependency ratio is $\zeta^y_t \equiv N^y_t / N^w_t$ and the old dependency ratio is $\zeta^r_t \equiv N^r_t / N^w_t$. The stock of workers ($\Gamma^w_t$) who influence the innovation process is

\begin{equation}
\Gamma^w_t \equiv \kappa_y (1 - \omega^y)N^y_t + (1 - \lambda^w \omega^w)\Gamma^w_{t-1},
\end{equation}

where $\kappa_y$ denotes the share of new workers at period $t$ who participate in innovation, and $0 < \lambda^w \omega^w \leq 1$ denotes the fraction of workers who retire or are no longer active in the innovation sector. As such the average working life of a worker who joins the innovation sector is $1 / (\lambda^w \omega^w)$. We denote $\gamma^w_t = \Gamma^w_t / N_t$ the share of innovation workers.

The dependent young passively acquire human capital, while workers and retirees decide their consumption to maximise welfare. As in Gertler (1999), we make two key assumptions to simplify the model such that household decision rules aggregate linearly. An individual faces two idiosyncratic risks: loss of wage income at retirement and time of death. There is a perfect annuity market allowing retirees to insure against time of death. They turn their wealth over to perfectly competitive financial intermediaries which invest the proceeds and pay back a return of $R_t / \omega^r_{t-1} \omega^r_t$ for surviving retirees.

The uncertainty about the employment tenure does not affect workers since they are assumed to be risk-neutral. To provide a motive for consumption smoothing, we assume that individual preferences belong to the recursive non-expected utility family. Thus, for $z = \{w, r\}$ we assume that the agent $j$ selects consumption and asset holdings to maximise

\begin{equation}
V^j_t = \left\{ (C^{jz})^{\rho_U} + \beta_t^{zj} (E_t[V^j_{t+1} | z])^{\rho_U} \right\}^{1/\rho_U}
\end{equation}
subject to

\[
C_{t}^{jz} + FA_{t+1}^{jz} = R^{z}_{t} FA_{t}^{jz} + W_{t} \xi_{t}^{jz} I^{z} + d_{t}^{z} - \tau_{t}^{jz} I^{z}
\]

where \( \beta^{z}_{t,t+1} \) is the discount factor, which is equal to \( \beta \) for workers and \( \beta \omega^{r}_{t,t+1} \) for retirees, \( R^{z}_{t} \) is the return on assets, which is equal to \( R_{t} \) for workers and \( R_{t} / \omega^{r}_{t-1,t} \) for retirees. \( W_{t} \) is the wage, \( \xi_{t}^{jz} \) is the effective unit of labour supplied by worker \( j \), and \( I^{z} \) is an indicator function that takes the value of one when \( z = w \) and zero otherwise; thus we assume retirees do not work and workers’ labour supply is fixed. \( FA_{t}^{jz} \) are the assets acquired from the financial intermediary and \( d_{t}^{z} \) is the dividend from the financial intermediary. Finally, \( \tau_{t}^{jz} \) is the tax a worker \( j \) pays to support the young with the total transfer at time \( t \) given by \( \tau_{t} = \int_{0}^{N^{w}_{t}} \tau_{t}^{jz} \).

The consumption functions of workers and retirees, are

\[
C_{t}^{w} = \varsigma_{t} [R_{t} FA_{t}^{w} + H_{t}^{w} + D_{t}^{w} - T_{t}^{w}] \quad \text{and} \quad C_{t}^{r} = \varepsilon_{t} \varsigma_{t} [R_{t} FA_{t}^{r} + D_{t}^{r}],
\]

where, \( H_{t}^{w} \) is the present value of gains from human capital, \( T_{t}^{w} \) is the present value of transfers, \( D_{t}^{z} \) is the present value of dividends for \( z = \{w, r\} \). \( \varsigma_{t} \) denotes the marginal propensity of consumption of workers and \( \varepsilon_{t} \varsigma_{t} \) for retirees (where \( \varepsilon_{t} > 1 \)). Given different marginal propensities the distribution of asset holdings affect aggregate demand and the marginal propensities are functions of fertility \((\tilde{n}_{t,t+1})\) and longevity \((\omega_{t,t+1}^{r})\) and are directly linked to the expected path of interest rates. As such, variations in population dynamics and interest rates affect aggregate demand and this effect may depend on the demographic composition in the economy.

Society taxes workers to provide for the young and their education, which increases the effective labour units they supply when they become workers. The amount of investment in education \( (I_{t}^{y}) \) at each period is determined by equating the marginal cost of obtaining resources from current workers (depressing their consumption at \( t \)) and the marginal benefits of higher effective labour supply, which leads to higher life cycle consumption. The greater the young dependency ratio, \( \zeta_{t}^{y} \), the greater the burden on the current generation of workers to finance education, depressing investment in it. In contrast, lower fertility boosts education. Ludwig et al. (2012) show this adjustment mechanism is important to account for the effects of the demographic transitions on the macroeconomy.

Let \( \xi_{t} \) be the average effective units, or productivity, at period \( t \). Each young person who becomes a worker at the end of period \( t \) provides \( \xi_{t+1}^{y} \) effective units. We assume

\[
\xi_{t+1}^{y} = \rho_{E} \xi_{t} + \frac{\chi_{E}}{2} \left( \frac{I_{t}^{y}}{\xi_{t}} \right)^{2} \xi_{t},
\]

where \( \rho_{E} \) and \( \chi_{E} \) are parameters.
where $\rho_E < 1$ and denotes the obsolescence of labour skills and $I_t^y = \frac{\tau_t}{W_t N_t^y}$ is the effective expenditure on the young, the ratio between total funds and their labour cost. The evolution of workers effective labour units is then given by

$$\xi_{t+1} = \omega w N_{t+1}^w \xi_t + (1 - \omega y) \frac{N_{t+1}^y}{N_{t+1}^w} \xi_{t+1}.$$ 

(24)

D. Financial Intermediary

The financial intermediary sells assets to the households ($FA^w_t$, $FA^r_t$), holds the capital ($K_t$) and rents it to firms and lends funds ($B_{t+1}$) to innovators and adopters to finance their expenditure (given by $S_t$ and $\Xi_t (Z_t - A_t)$, respectively). Finally, we assume it owns the innovators and adopters enterprises, receiving their dividends at the end of the period.

E. Equilibrium

The symmetric equilibrium is a tuple of endogenous predetermined variables $\{FA^w_{t+1}, K_{t+1}, A_{t+1}, Z_{t+1}, B_{t+1}, \xi_{t+1}\}$ and a tuple of endogenous variables $\{C^z_t, H^w_t, T^w_t, d^z_t, D^z_t, L_t, Y_t, \Xi_t, \mu_t, N^I_t, S_t, V_t, \lambda_t, Y_{c,t}, C_t, U_t, r^k_t, \delta_t, R_t, \Pi^F_t, W_t, P^M_t, \varepsilon_t, \tau_t, t^y_t, \xi_{t+1}^y, \omega_t\}$ for $z = \{w, r\}$ obtained such that: a. Workers and retirees maximize utility subject to their budget constraint and investment in education is such that society’s marginal cost and benefit is equated; b. Input and final firms maximize profits, and firm entry occurs until profits are equal to operating costs; c. Innovators and adopters maximise their gains; d. The financial intermediary selects assets to maximize profits, and its profits are shared amongst retirees and workers according to their share of assets; and e. Consumption goods, capital, labour and asset markets clear.

All agents take as given the initial values of all the predetermined variables $\{FA^z_t, K_t, A_t, Z_t, \xi_t, FA^r_t, B_t\}$ and the exogenous predetermined variables $\{N^V_t, N^w_t, N^r_t, N_t\}$ specified by the population dynamics. Details of all the equilibrium conditions are shown in the online appendix. The market clearing equilibrium conditions determine the labour used in production, the dynamics of the capital stock, aggregate consumption, added value output from supply and demand sides and finally the asset market flows for retirees and workers. Under these conditions, the added value output is given by

$$Y_t = Y_{c,t} - A_t^{1-\theta} M_t - \Omega \bar{\Psi}_t = C_t + I_t + S_t + \Xi_t (Z_t - A_t) + \tau_t$$

(25)

We must then define $\bar{\Psi}_t$ such that a balanced growth path exists. Comin and Gertler (2006) select the current value of capital stock. Given that in their model the price of capital is determined at time $t$, $\bar{\Psi}_t$ fluctuates accordingly ensuring stability. We simplify our model to consider only one sector and thus the price
of capital and the value of the capital stock are constant at \( t \), invalidating this choice of scaling factor. We therefore select the current value of adopted goods as our scaling factor. Thus,

\[
\tilde{\Psi}_t \equiv V_t A_t.
\]

**F. Calibration and Steady State**

The quantity variables of our model are driven by: the exogenously given fertility rate \((\bar{n})\), the endogenous growth of effective labour \((\xi_t)\) and the endogenous process of invention and adoption of new intermediate goods \((A_t)\), which increases the productivity of capital and labour. We normalize certain variables relative to final goods output to obtain a system of equations for the stationary steady state. Throughout the calibration, we set one period of the model to correspond to one year.

Individuals are young for 20 years on average, with the probability of becoming a worker \((1 - \omega^y) = 0.05\). They work on average from 21 to 60, with the probability of retirement \((1 - \omega^r) = 0.025\), and are retired on average from 61 until 70, and thus \( \omega^r = 0.9\). At steady state, the ratio of young to workers is 70 percent, the ratio of retirees to workers, 22 percent and retirees hold around 17 percent of the assets. The measure of workers in innovation \((\gamma^{yw})\) depends on \( \kappa_y \), which is set such that \( \gamma^{yw} \) matches the share of R&D workers in US population, and \( \lambda_y \), set to make the average age of innovation workers to be 40 (slightly lower than the average age of employed scientists reported in the Survey of Doctorate Recipients (SDR) of the National Science Foundation - 2013).

For the parameters that govern the innovation process, we follow Comin and Gertler (2006) closely. We set obsolescence \((\phi)\) and productivity in innovation \((\chi)\) so growth per working age person is 0.024 and share of research expenditures in total GDP is 0.012. The mark-up for intermediate goods is 1.6. The elasticity of intermediate goods with respect to R&D \((\rho)\) is 0.9. Average adoption time is 10 years thus \( \lambda = 0.1\). The elasticity of this rate to increasing intensity \((\epsilon_\lambda)\) is set to 1. The price mark-up elasticity to entry \((\epsilon_\mu)\) is 1.

The link between demographics and innovation depends on the total share of workers in innovation, parameter \( \gamma^{yw} \), and the elasticity of invention to the share of workers, parameter \( \rho^{yw} \). Denote the share of innovation workers of each age \( j \geq 20 \) as \( S_{\gamma,j} \), and thus \( \sum_j S_{\gamma,j} = \gamma^{yw} \). We use the age distribution of inventions from Jones (2010) to set the productivity of a worker of each age relative to a 20 year old worker, denoted \( \Phi_j \). In Jones (2010), the frequency of great inventions of 38 years old workers is 0.042 while the frequency for 20 year old workers is 0.002 and hence 38 year old workers are roughly 21 times more productive than their 20 year old counterparts. Both of these variables are depicted in Figure 4. The reduced-form innovation framework in the model relates a single population measure, \( \gamma^{yw} \), and its elasticity, to the production of ideas. In order to reflect
the information on the entire distribution of invention across ages we use each age share \((S_{\gamma,j})\) and its respective productivity measure \(\Phi_j\) to obtain an economy wide productivity measure \(\rho_{yw}\) (which refers to our population measure, \(\gamma_{yw}\)). Thus, we set \(\rho_{yw}\) such that \(\sum_j S_{\gamma,j} \Phi_j = (\gamma_{yw})^{\rho_{yw}+1}\). We use \(\rho_{yw} + 1\) since \(\Phi_j\) denotes a relative productivity. As a result, were the probability that a worker invents a new prototype independent of age and \(\Phi_j = 1\) for all \(j\), then \(\rho_{yw} = 0\) and innovation is not influenced by the economy’s demographic composition or age profile. Our calibration reflects the fact that demographic composition matters, the middle aged are more productive, and \(\rho_{yw} = 0.51\).

Finally we set the standard macro parameters, in line with Comin and Gertler (2006)). The discount factor \(\beta = 0.96\); the capital share \(\alpha = 0.33\); steady state capital utilisation \(\bar{U} = 80\) percent; the yearly depreciation rate \(\delta(\bar{U}) = 0.08\); the elasticity of the change in the depreciation rate with respect to utilisation is 0.33; the share of intermediate goods \(\gamma_l = 0.5\); and the mark-up in the consumption sector \(\mu = 1.1\). Finally, following Gertler (1999) we set the intertemporal elasticity of substitution \((1/(1 - \rho_U)) = 0.25\).

IV. Simulation Results

We first examine how well our theoretical model can explain the consequences of demographic changes predicted in our empirical exercises. We then analyse the effect of an increase in longevity, holding population constant, using different specifications of the model to highlight the key transmission mechanisms.

A. Simulation: Projection

To incorporate the projections into the macro model, we set a path for the fertility rate \(\tilde{n}\), the longevity parameter \(\omega^r\) and the probability a young dependant
becomes a worker \((1 - \omega_y)\) such that we match the change in the UN projected population growth, the change in the share of workers and retirees in each country. Therefore, as in the estimation, the changes in age structure describe the demographic shock. We use actual population data 2005-2015 and UN projections 2016-2035. For instance, U.S. population growth is expected to fall from 0.9 to 0.55 percent, the share of workers (ages 20 to 60) to fall from 55 to 50 percent and the share of retirees (over 60) to rise from 19 to 27 percent (see Figure 5). The simulation starts at \(t= 2005\), from the steady state of the model, and all agents are able to predict the population dynamics for the next 30 years. As noted above, our focus is on how the projected demographic changes affect the macroeconomic variables.

Figure 5 shows the results for U.S., Japan, Italy and France from 2010 till 2030 together with the empirical predictions. Our model does a fairly good job in matching the predicted path of real rates and output growth for the countries in our sample (in the online appendix we show the estimation and theoretical simulations based on the UN predictions for four additional countries). For countries that are expected to suffer significant falls in fertility (e.g. Greece, Spain and Japan) the model projections slightly overpredict the fall in growth, while for countries where fertility is expected to fall less (e.g. Australia, Canada and U.S.) our growth projections match well with the empirical results. In general the model tends to underpredict the fall in interest rates but as we discussed in the empirical section in some specifications we find a smaller impact and as such the benchmark results should be interpreted as a lower bound. Gagnon et al. (2016) build a life-cycle model fully accounting for age heterogeneity but

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14 As we do not alter the steady state to match the characteristics of each country, focusing on the changes in macroeconomic variables after a demographic change, we discard the first five years of the simulation to decrease the influence of the initial steady state on the results.

15 Given that at each year population features are changing, the theoretical model can sometimes display sharp movements, particularly in growth rates. In most cases this occurs in 2015 when the shift from historical to projected data occurs. For instance, see the population growth path used for the U.S. in Figure 5.
as opposed to our framework, assumes an exogenous path for productivity. They find that the expected drop in interest rates from the Great Recession until 2030 is around 75 basis points, while our projection shows a bigger fall, of around 180 basis points. The link between demographics and productivity in the future is therefore a relevant factor in assessing and comparing the predictions.

The effect of demography on innovation is important in the match between our theory and our estimates. That is driven by the elasticity of technological creation to young workers ($\rho_{yw}$) and the changes in the share of young workers in total population ($\gamma_{yw} = \Gamma_{yw}/N_t$). As longevity increases, this share falls. We modify the creation of ideas such that innovation is linked to the ratio of young worker to the total number of workers ($\hat{\gamma}_{yw} = \Gamma_{yw}/N_{w,t}$), reducing the effect of changes in longevity on the results. Nonetheless, we find that the theoretical projections are very similar (results are shown in the online appendix). The two main reasons for this are: (i) the expected drop in fertility is important in depressing $\gamma_{yw}$ and subsequently innovation and (ii) as we re-calibrate the relationship between demographics and innovation following the same procedure but now with the modified shares $\hat{S}_{\gamma,j}$, $\rho_{yw}$ increases to 0.6, thus the elasticity of innovation to age structure is higher in this specification.

![Figure 6. Simulation: Projection](image-url)
B. Simulation: Ageing

Most economies during our estimation period experienced increasing life expectancy and falling fertility resulting in an increasing share of retirees. We reflect these changes in the model by smoothly increasing $\omega$ such that the average retiree lives an additional 10 years, increasing societies’ average age and doubling the share of retirees. We hold population growth ($g^n = N_{t+1}/N_t$) constant and as such, fertility must decrease during the first part of the adjustment process. As a result we are able to separate the effects of a drop in fertility and ageing while controlling for population growth as done in our estimation. Once again, fertility and mortality translate into changes in dependency ratios (young - $\zeta_y$, and old - $\zeta_r$), which concisely describe the age structure with 3 generations.\(^{16}\)

We do this exercise for different specifications of our model. All results are presented in Figure 7. In the benchmark specification (depicted as Benchmark) we observe that the effects of increased longevity and decreased fertility in our theoretical model matches our empirical estimates. In the medium run we observe lower growth ($g$), lower investment and lower interest rates ($R$). These effects are a product of three key mechanisms through which demography impacts the economy.

First, increased longevity means workers expect to live longer, so they increase savings and asset accumulation and reduce consumption causing real interest rates to fall. The additional savings are allocated to investment in capital and innovation, pushing the growth rate of output up. Therefore, life-cycle consumption adjustment, our first mechanism, which is predominantly a demand or expenditure effect, leads to an increase in growth rates.\(^{17}\) This mechanism matches our estimates where ageing has a negative effect on interest rates. Our model indicates that this is a result of workers savings more and occurs despite the higher proportion of retirees ($\zeta_r$ increases), who are dissaving.

The second mechanism works through the adjustment of education to the decrease in fertility that leads to a drop in the ratio of young dependants ($\zeta_y$). As workers increase their savings for retirement, investment in education falls. However, as $\zeta_y$ falls, the per capita investment in education increases, leading to a growth in human capital ($g^\xi$). This human capital effect pushes the growth rate up, but it has a small contribution when technology is endogenous.\(^{18}\)

Finally, the third mechanism works through the proportion of young workers having a positive effect on the productivity of the innovation process. Ageing

\(^{16}\)Note that as population growth is kept constant the inclusion of a population measure on the congestion factor $[(\Psi_t)^\rho (S_t)^{1-\rho} N_t^{\rho\psi}]$ does not affect the results of this simulation exercise. Effectively, in this case the share of workers in innovation is equal to scale of workers in innovation.

\(^{17}\)Our model cannot generate a paradox of thrift such that greater desire to save decreases aggregate demand sufficiently to mean that no additional savings is done. As a result, additional resources always flow to the innovation sector increasing growth. Altering the aggregate demand features of the model may generate stronger negative effects on growth due to lower consumption.

\(^{18}\)Allowing for the accumulation of human capital during the working life as well as during youth might increase its importance (see Ludwig et al. (2012) for a framework where accumulation also occurs during the working life).
causes ($\gamma_{yw}$) to fall, reducing invention. This process is particularly strong when fertility falls. The lags between innovation and adoption mean that this negative effect takes time but eventually it offsets the positive effect due to increased savings so after 40 years growth reaches its lowest point. As the old dependency ratio ($\zeta_r$) reaches its new stationary level, fertility increases, returning back to its steady state value. As a result, only the change in longevity influences the long-term equilibrium. As the average age of the population has increased, $\gamma_{yw}$ is lower at the new steady state and the final effect of the growth rate is mildly negative. This direct link between innovation and demographic structure, which we refer as our main aggregate supply effect, is supported by our estimates. Ageing leads to lower patent applications and thus to potentially lower contribution of innovation to growth.

![Figure 7. Simulation: Ageing](image)

We run three additional simulations next to the benchmark model to assess how each mechanism contributes to lower investment, growth and real rates.$^{19}$

**Absence of Aggregate Supply Channel:** Here we eliminate the link between innovation and demographic structure, assuming workers of all ages contribute equally to innovation, setting $\rho_{yw} = 0$. We denote this simulation in Figure 7.

$^{19}$We also consider an extension where the second mechanism is reinforced by assuming that not only the share of young workers affects innovation, but the new workers’ higher level of human capital also boosts the productivity of innovation. We find that in this case the negative effect of fertility on innovation is only mildly offset.
as No AS Channel ($\rho_{yw} = 0$). We observe that the long-run effect on growth, investment and interest rate are now positive. Both human capital accumulation (second mechanism) and increase in savings due to the life-cycle consumption channels (first mechanism) in general accelerate growth and lead to an increase in investment. The life-cycle mechanism still leads to a decrease in the marginal propensity to consume (MPC) of workers, however, the additional savings have a productive use in the innovation sector and hence real rates increase. Note that in a model without endogenous growth, the increase in longevity and subsequent fall in the MPC of workers, leads to capital deepening and lower real rates, as shown in Carvalho et al. (2016). However, introducing an innovation sector implies that the additional savings boost innovation and productivity such that the marginal product of capital does not decrease.

**Shifting Age Distribution:** Jones (2010) argues that through time the age distribution of inventors at the moment of a great invention is shifting to the right. Great inventions are happening later in the life of inventors. We therefore consider the impact of the ageing process together with a shift in age productivity curve such that the peak productivity occurs at 43 years old and not at the age of 38, as in our benchmark calibration (see the online appendix for details). Such a shift, according to our calibration, would lead to a change in the elasticity of technological creation to population age structure (parameter $\rho_{yw}$). We denote this simulation in Figure 7 as Shifting Age Distribution. We find that as the population ages in tandem with an increase in the productivity of innovation of older workers, the negative effects of longevity are offset. Thus, although the lower fertility during the transition still produces a decrease in output growth, population ageing leads to a smaller effect on growth, investment and interest rates.

**Pay-as-you-go Pensions and Health Expenditures:** The initial increase in savings and reduced aggregate demand that results from the changes in demographic structure are not sufficient to create a decrease in the demand for new varieties $A$ such that the incentives for innovation are diminished. As a result, this expenditure effect does not generate lower growth rates. However, most OECD economies employ a pay-as-you-go pension scheme in which workers are taxed to fund payments to an increasing share of retirees. Moreover, governments are expected to continue financing ever increasing health expenditures for the ageing population (see the ageing report - European Commission (2015)). We account for these by augmenting our benchmark model with a pay-as-you pension scheme and fully funded expenditures in health care. The details of this model extension are shown in the online appendix. We set the replacement ratio (ratio of pension payment to labour income) to 40 percent matching most European pension schemes and the ratio of health expenditure on old agents per capita to GDP of 12 percent, close to the numbers reported in European Commission (2015) and in the US Medicare program (see AARP Public Policy Institute (2009) and Curto et al. (2017)).
In this extension, denoted *Pension and Health*, ageing leads to a contraction of workers’ aggregate demand as they are forced to transfer a higher share of their earnings to fund health and pension expenditures, but in contrast to what we observe in the benchmark model, total savings are not directly affected. These additional aggregate demand effects are sufficiently strong to generate a decrease in the output growth rate in the long-run. However, this decrease is relatively small compared to the effect generated by the supply channel, i.e. when $\rho_{yw} > 0$. We therefore conclude that the link between demographics and innovation is crucial in aligning the theoretical and empirical results of lower growth, investment and real rates due to increased longevity and low fertility.

V. Conclusions

Gordon (2012) asks how much further could the frontier growth rate decline? We provide an analysis that measures this decline focusing on the impact of demographic structure on the macroeconomy. Our results indicate that the age profile of the population has both economically and statistically significant impacts on output growth, investment, savings, hours worked per capita, real interest rates and inflation. The magnitude of the long-term impact is large. Demographic factors are predicted to depress average annual long-term GDP growth over the current decade, 2015-2025, by 0.64 percent in our sample of OECD countries. We also provide evidence of the link between demographic structure and innovation: patent applications are positively affected by the share of the middle-aged and negatively by retirees. Our empirical results are robust to a variety of factors including time effects, controlling for mortality and fertility trends and open economy dynamics.

We develop a theoretical model that incorporates age heterogeneity and endogenous productivity. This model highlights three main channels by which demographics affects the macroeconomy: i) through life-cycle consumption decisions, ii) through incentives to alter human capital accumulation and iii) through the influence of young workers on the innovation process. The calibrated theoretical model is able to replicate most of our empirical findings, and indicates that the third channel, the demographic effect of ageing on innovation, is a particularly important cause of reduced long-run growth. Our results indicate that the current trend of population ageing and reduced fertility, expected to continue in the next decades, may contribute to reduced output growth and real interest rates across OECD economies.

References


