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Ex Ante Versus Ex Post Regulation of Bank Capital

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Abstract

The current debate on the new Basel Accord gives rise to a natural question about the appropriate form of capital regulation. We construct a simple framework to analyze this issue. In our model the risk carried by a bank as well as managerial risk preference are a bank's private information. We show that ex ante constraints waste the superior risk information of a bank, while an ex post regime makes full use of it. However, the latter is more vulnerable to the problem of unknown managerial risk-aversion. The results imply that the two regimes are complements, rather than substitutes. Further, under plausible conditions, an ex post regime emerges as the dominant element of the optimal combination. We use the results to shed light on current policy concerns. In particular, our results provides theoretical underpinning for the inclusion of pillar 2 alongside pillar 1 in Basel II.

KEYWORDS: *Ex Ante Regulation, Ex Post Regulation, Asymmetric Information, Safety Loss, Overprotection Loss, Safety Bias, Basel II.*
JEL CLASSIFICATION: G28, D82, L51.

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1 Introduction

The Basel Accord introduced the first international standard for risk based bank capital in 1988. In the ensuing fifteen years, a large literature has analyzed its impact. The original rules have been severely criticized and new forms of capital regulation have been suggested. Some of these ideas are embodied in Basel II, the new capital adequacy framework which is set to replace the Accord by 2006. In this paper, we study different forms of capital regulation, and apply our results to clarify the theoretical foundation for the Basel II architecture.

The principal justification of capital regulation rests on two factors. First, it has a role in ameliorating moral hazard induced by deposit insurance(1). The problem would not arise if deposit insurance premia reflect true portfolio risk. However, typically the regulator does not have full information on the portfolio risk of a particular bank, which makes setting actuarially fair risk-based premiums difficult(2). This implies a scope for additional solvency regulation. Indeed, as several authors have shown, risk-based capital regulation plays a role in designing incentive-compatible risk-sensitive deposit insurance under asymmetric information(3). Second, capital regulation also serves to mitigate negative externalities arising from the failure of a bank. The importance of this "systemic risk" problem has been underlined by the recent financial turmoil in South East Asia, Russia and South America(4).

While the above discusses reasons behind capital regulation(5), this paper focuses on a related but different question. Given that capital regulation is an integral part of bank regulation,

(1)Kahane (1977), Koehn and Santomero (1980), Kim and Santomero (1988) and Rochet (1992) show that a flat-rate capital requirement might increase risk in bank portfolio. Relaxing the assumption of zero net present value assets, Gennotte and Pyle (1991) show that capital standards based on accounting value could lead to an increase in risk taking. Kim and Santomero (1988) and Rochet (1992) show how a move to risk-adjusted capital requirements might solve the problem and calculate optimal risk weights in a setting with no asymmetric information. Besanko and Kanatas (1996) consider a model with moral hazard, and show that capital regulation can cause risk of bank assets to increase because regulation leads to effort aversion by bank management. See Calomiris (1999) for a succinct overview of the reasons behind and the problems arising from deposit insurance.

(2)See Chan, Greenbaum and Thakor (1992).

(3)For example, papers by Bond and Crocker (1993), Giammarino, Lewis and Sappington (1993) and Freixas and Gabillon (1998) adopt a mechanism design approach to derive risk based deposit insurance. Further, Flannery (1991) shows that if the regulator observes bank risk with an error, the negative impact of the error can be minimized by combining capital requirements with risk based deposit insurance premia.

(4)Since the mid 90s there has been a growing literature on systemic risk. See de Bandt and Hartmann (2000) for a detailed survey.

(5)See Santos (2000) for an extensive survey of the literature on capital regulation.

what form should it take?

Broadly speaking, there are two distinct forms of capital regulation. The 1988 Basel Accord imposes minimum capital requirements as a fixed proportion of a bank's risk weighted assets. This imposes an *ex ante constraint* on risk taking by a bank, and forces an exogenous link between risk and capital. An alternative design gives a bank freedom to choose capital and portfolio risk. Regulatory intervention is triggered if losses exceed a certain threshold. The threat of intervention *endogenously induces* a link between risk and capital. Since such a scheme conditions on the *outcome* of bank actions, this is referred to as *ex post regulation*.

While Basel I imposes pure ex ante constraints, the new capital adequacy framework Basel II combines ex post features with ex ante constraints. Pillar 1 of Basel II is dominated by ex ante rules, while pillar 2 introduces explicit ex post elements. Therefore natural questions arise about the efficiency of different forms of capital regulation, and how they fit together. In spite of the considerable practical importance of this issue, the theoretical literature offers little consensus or clarity on the comparison of different forms of capital regulation. This paper is an attempt to fill this gap. In doing so, we clarify the theoretical reasons for inclusion of pillar 2 alongside pillar 1. To the best of our knowledge this is the first paper to address this issue.

Under full information, all regimes are equally efficient. However, as Fama (1985) and others have noted, information asymmetries are endemic to the problem of regulating banks. Once this is taken into account, we show that ex ante and ex post regimes have very different properties. Therefore, questions about when a particular form should be used, and how different forms fit together, are not only of practical relevance, but also theoretically interesting.

As Fama (1985) points out, banks typically have superior information on their clients, and this is fundamental to explaining the special nature of banks[(6)]. Therefore, a bank can usually make a more precise estimate of its own portfolio risk compared to the regulator.

A further source of information asymmetry is the risk preference of a bank, which is typically the bank's private information. In the event of bankruptcy, shareholders usually lose only the capital they contribute. Such limited liability makes their payoff a convex function of bank returns. However, managers often stand to lose much more. First, as Benston etal (1986) note, managers sink non-diversifiable human capital in the firm, which is lost in the event of bankruptcy. The risk taking incentives of bank managers decrease in the degree to which their

[(6)]See James (1987) for empirical evidence of this. See also Freixas and Rochet (1997) (chapter 2) for a discussion on the role of asymmetric information in justifying the existence of banks.

non-diversifiable human capital is bank specific. Second, unlike shareholders, managers often face sanctions after failure. Gilson (1989) and Gilson and Vetsuypens (1994) report that after filing for bankruptcy, managers in the US suffer large personal costs. After bankruptcy, half of the managers are fired and those that are not, on average receive only 35% of their previous compensation. Third, a point related to personal costs is concern for reputation. As noted by Hirshleifer and Thakor (1992) and Diamond (1989), managers tend to be conservative in their risk taking to build reputation. Fourth, as Armour (2005) notes, large creditors often demand personal guarantees from owner-managers, sidestepping the legal shield of limited liability. Finally, creditors may initiate legal action against the manager which, apart from causing personal distress, may lead to fines not covered by liability insurance.

These factors are likely to make the manager's payoff function concave. However, contractual elements such as a golden parachute and liberal pension benefits that survive the failure of the bank can mitigate such managerial conservatism. The degree of risk aversion ultimately depends on the extent to which the bank is controlled by the management, the human capital investment by the manager, reputational concerns, extent of legal shield, as well as the form of management compensation. The extent of bank capital can also influence the degree of risk aversion. Empirical studies find that management controlled institutions tend to be more conservative than stockholder controlled organizations. For example, Saunders, Strock and Travlos (1990) study the relationship between bank ownership structure and risk taking. They find support for the hypotheses that stockholder controlled banks take more risk compared to manager controlled banks, and that deregulation makes this difference more pronounced.

The discussion above motivates our modelling of informational structure. True portfolio risk as well as the degree of managerial risk aversion are private information of a bank. We judge the success of any regulatory regime against this backdrop of multi-dimensional information asymmetries.

Our main results are as follows. We show that an ex ante regime makes poor use of the expertise of a bank in measuring risk. An ex post regime, on the other hand, fully incorporates the superior private information of a bank on underlying risk; but in doing so, becomes more sensitive to the problem of unknown managerial risk aversion compared to an ex ante regime. This implies that the two regimes are vulnerable to different dimensions of information asymmetry, and their comparison depends on the relative importance of the sources of informational asymmetries. This also suggests that a combination of the two regimes is potentially welfare improving. Even though Basel II combines the two forms of regulation, the theoretical literature largely treats

them as substitutes and advocates one or the other. In contrast, we show that a combined regime outperforms either regime. Moreover, if ex post penalties are sufficiently regressive[7], (in our model, a linear penalty function satisfies this), we get a striking characterization: when solvency is the main regulatory concern, and the bank is well capitalized, the optimal mixture involves a strong form of ex post regulation, combined with a much weaker version of ex ante regulation.

The results above are derived assuming that a bank has superior information on underlying risk. If, however, the bank's ability to assess risk is poor, we show that it is optimal to use only ex ante regulation. The result underlines the importance of tailoring regulation according to the regulator's assessment of the risk management ability of a bank (through, for example, backtesting of a bank's internal VaR model). Such customization is indeed a feature of Basel II.

We relate our results to current trends in regulation and draw policy conclusions. The rest of the paper is organized as follows. Sections 2 and 3 introduce the model and specify the regulatory benchmark. Sections 4 and 5 analyze ex ante and ex post regulation. Section 6 compares the two regimes, and section 7 characterizes the optimal combination. Section 8.1 considers extensions of the model including the case of a bank with no private information on underlying risk. Section 9 relates our results to Basel II, draws policy prescriptions, and discusses extensions, and section 10 concludes. Proofs are collected in the appendix.

[7] A penalty structure is regressive if a higher penalty has a greater marginal impact on the optimal choice of types with lower risk aversion.

2 The model

Let us briefly summarize our model to start with. There are two players: a bank manager and a regulator. The manager can invest in a riskless asset and a risky asset, and chooses a portfolio to maximize expected utility. The regulator does not know either the exact risk of the risky asset, or the degree of risk aversion of the manager. The regulator can observe the fraction invested in the risky asset as well as the bank's capital, and wants to limit the probability of bankruptcy. However, given the information asymmetries, the regulator cannot ensure first best. Regulation might lead to a deviation from the optimal probability of failure. A safety loss occurs if the probability of failure is too high, and overprotection loss occurs if it is too low. The regulator's objective is to minimize a weighted average of the two kinds of losses.

Let us start describing the model by specifying the investment opportunities facing the bank and its objective function.

Investment opportunities of the bank The total investment is normalized to 1. The bank chooses efficient portfolios of assets. Portfolios are constructed by investing a fraction $(1-\alpha)$ on a riskless asset, and a fraction $\alpha \in \Re$ on a risky asset[8].

The return from the risky asset is normally distributed with mean m and standard deviation s. The return from the riskless asset is normalized to 0, and the efficient frontier is a ray from the origin given by $m = \beta s$. Let $\widetilde{V}$ denote the return from the bank's portfolio. $\widetilde{V} \sim N(\alpha m, \alpha s)$. Let σ denote the portfolio risk. Then $\sigma = \alpha s$ and $\widetilde{V} \sim N(\beta\sigma, \sigma)$.

The bank's objective function The manager decides the level of bank capital K and the portfolio fraction α. We ignore the details of determination of K[9], and simply assume that once decided, it is fixed in the short run at a finite level, and the manager determines risk given this level of capital. This is the natural approach here, because the regulator can observe K, but not all components of portfolio risk. Thus for *any* given K, the regulator must decide how much risk is optimal and needs to regulate banks (i.e. influence the risk adopted given any capital level) to bring them in line with the social optimum. This also reflects the

[8] This can also be thought of as the tangency portfolio between the efficient frontier with all assets and that with only risky assets.

[9] The literature offers various theories for determining K. For example, the pecking order hypothesis stresses the availability of internal funds relative to external debt and equity, while an alternative trade-off theory relies on weighing the balance between interest tax shields and bankruptcy costs.

standard practice in banks to treat capital as given for risk management purposes. Current capital is first allocated to the various business units down to individual portfolio managers. Then, portfolios are chosen given the allocated capital.

The level of K matters for the manager's decision about α because, as discussed in the introduction, capital is one of the factors influencing the degree of managerial risk aversion[10]. While observing capital can give the regulator important information on the nature of the distribution of risk aversion, our analysis rests on the general assumption that the degree of risk aversion is not a deterministic function of K, so that for any given K the regulator faces uncertainty about ρ. This is plausible because, as discussed in the introduction, other factors such as non-diversifiable human capital investment, erosion of reputation and loss of future income in the wake of bankruptcy, as well as personal liability issues also influence the degree of a manager's risk aversion.

We now formalize the managerial objective function. Once this is done, we specify the regulator's objective function, and finally, formalize our main assumption about the regulator facing a non-trivial problem of uncertainty for any given level of capital.

Following the base model used by Freixas and Rochet (1997), we assume that the bank manager's von Neumann-Morgenstern utility function is given by

$$u(\widetilde{W}) = -\exp(-\rho\,\widetilde{W})$$

where $\widetilde{W}$ denotes the total return from investment and ρ is the manager's risk aversion parameter, which depends on bank capital and other factors as discussed above. In the following, we refer to the parameter ρ as the manager's "type."

The total return $\widetilde{W}$ is given by $1+\widetilde{V}$. Thus expected utility is given by

$$Eu(\widetilde{W}) = E\left(-\exp\left(-\rho\left(1+\widetilde{V}\right)\right)\right) = -\exp\left(-\rho\left(1+\alpha\beta s-\frac{\rho}{2}\alpha^2 s^2\right)\right).$$

It follows that the manager can simply maximize $\alpha\beta s-\frac{\rho}{2}\alpha^2 s^2$ with respect to α. Therefore, the optimal portfolio for a manager of type ρ given underlying risk s is

$$\alpha^* = \frac{\beta}{\rho s}. \tag{2.1}$$

The optimal risk of type ρ is $\sigma^* = \alpha^* s$, implying that $\sigma^* = \frac{\beta}{\rho}$.

[10] For example, under decreasing absolute risk aversion, higher capital brings about lower risk aversion.

Information Next, we specify the informational structure. The regulator faces two dimensions of asymmetric information. First, the type ρ is the bank manager's private information. From the perspective of the regulator, ρ is a random variable which is uniformly distributed on the interval $[\rho_L, \rho_H]$.

Second, the standard deviation s of the risky asset is a random variable uniformly distributed on the interval $[s_L, s_H]$. The realization of s is the private information of the bank manager. The regulator knows only the distribution of s.

The regulator's objective function As noted by Kim and Santomero (1988), Dimson and Marsh (1995), Hellmann, Murdock and Stiglitz (2000) and other authors, regulators face a trade-off when determining the amount of regulatory capital to be set. On the one hand, very high capital requirements impose inefficiently high costs on banks[(11)]. Further, as noted by Dimson and Marsh (1995), a high capital requirement might also inhibit competition by acting as an entry barrier. On the other hand, too little capital impairs solvency. This trade-off between the regulators' safety goal and preservation of efficiency implies a socially optimal probability of default. The regulator attempts to ensure that portfolio risk and capital in a bank are consistent with this probability.

Therefore, in the social optimum, capital (K) and portfolio risk ($\sigma = \alpha s$) should be such that the capital offsets portfolio losses with a specified probability p (e.g. 0.05% when using annual returns). Given capital K, portfolio choice should be such that $Prob(\widetilde{V} < -K) = p$. Let Φ denote the standard Normal cumulative distribution function. Then in the social optimum, $\Phi(-K/\sigma - \beta) = p$. From this, for any given capital level K, the **socially optimal portfolio risk** for any $s \in [s_L, s_H]$ is given implicitly by

$$\frac{K}{\sigma} \;=\; \Phi^{-1}(1-p) - \beta \;\equiv\; C$$

Therefore, the socially optimal risk (denoted by $\sigma_{\mathbf{0}}$) is

$$\sigma_{\mathbf{0}} = \frac{K}{C}. \tag{2.2}$$

(11) Seminal work by Leland and Pyle (1977) shows how a cost of capital can arise due to information asymmetry that distorts investment decisions when the bank must raise funds from uninformed outsiders. Further, Besanko and Kanatas (1996) show that recapitalization might dilute the effort-provision incentives of the management and lower share price. Hellmann et al. (2000) note that if capital were not costly, the problem of excessive risk taking in the pursuit of private gains by the bank ownership would hardly be as severe as empirical evidence suggests, as regulators would simply set very high capital requirements and banks would comply willingly.

We assume that $C > 0$[12].

For any given K, whenever actual risk is different from the socially optimal risk, there is an efficiency loss. If the actual risk ratio exceeds the optimal risk, we say there is a **"loss of safety"** because the probability that the bank defaults is higher than the regulatory optimum. On the other hand, when the actual risk falls below the optimal risk, we say there is an **"overprotection loss."**

Given the capital K held by the bank, the regulator attempts to implement the socially optimal portfolio risk $\sigma_{\mathbf{0}}$. This is a non-trivial problem under asymmetric information. The usual optimization problem is simply to minimize the deviation of the actual portfolio risk from the optimal risk. However, this is not necessarily appropriate for bank regulation. Given a level of capital, if the actual portfolio risk is lower than the optimal risk (i.e. overprotection loss occurs), it principally penalizes the shareholders of the bank. The reason is that under limited liability the payoff of shareholders is a convex function of the return from investment, implying that they prefer higher risk. If, on the other hand, actual risk is higher than the optimal level (i.e. safety loss occurs), it is primarily a problem for depositors, whose payoff is a concave function of the return from investment, implying a preference for lower risk. In general, the regulator might attach different weights to the interests of shareholders and depositors, i.e. different weights to overprotection and safety loss. Dewatripont and Tirole (1994) argue that solvency (i.e. protection against safety loss) is the main goal of bank regulation because banks are not like other financial institutions. The important difference is that banks have a large number of small depositors who face a free riding problem resulting in poor provision of monitoring of the bank. Therefore, the regulator, who represents the interests of the depositors, has a monitoring role.

A further argument in favor of relatively greater regulatory concern about solvency is the presence of externalities from bank failure. Given significant systemic risk, the regulator needs to ensure that banks internalize the externality. In such cases, the regulator would attach a greater weight to safety loss. On the other hand, in economies that are particularly subject to credit crunches, the regulator might be more concerned with overprotection loss (while still imposing regulation to curb extreme risk taking).

We propose a formulation that encompasses such concerns through a regulatory objective function that allows for safety loss and overprotection loss to be weighted differently. The loss is

[12] This is not unreasonable, since for p not too high (which should be the case), $\Phi^{-1}(1-p)$ is large, and the slope of the efficient frontier β is typically less than 1.

given by:

$$L = \omega \ \max\left(1 - \frac{\sigma_{\mathbf{0}}}{\sigma}, 0\right) + (1-\omega) \ \max\left(\frac{\sigma_{\mathbf{0}}}{\sigma} - 1, 0\right), \tag{2.3}$$

where $\omega \in [0,1]$. The loss function is the weighted sum of the two types of losses described above. The first element of the sum is the weighted safety loss and the second element is the weighted overprotection loss. We should emphasize that allowing ω to take any value in the unit interval makes this is a more general regulatory objective function compared to simple loss minimization. The weight ω can be thought of as the regulator's **"safety bias."** $\omega > .5$ represents a positive safety bias. Clearly, if $\omega = 0$, no regulation is trivially the best solution. We ignore this trivial case and assume $\omega > 0$.

Ensuring Non-Trivial Regulation From (2.2), the socially optimal portfolio risk is given by K/C. From equation (2.1), the optimal portfolio risk of a bank with risk aversion ρ is β/ρ. If $K/C < \beta/\rho$, there is a safety loss. Of course, for regulation to have any bite at all, there must be some types of bank managers such that $K/C < \beta/\rho$. Let ρ_0 be the type whose optimal portfolio coincides with the socially optimal portfolio. Thus,

$$K/C = \beta/\rho_0. \tag{2.4}$$

We proceed with the following general specification. We assume that the managerial risk aversion is not a deterministic function of bank capital. Specifically, we assume that for any finite K chosen by the manager, the support of the distribution of ρ is such that

$$\rho_H \geqslant \rho_0 > \rho_L. \tag{2.5}$$

If K is infinite, the regulatory problem is obviously trivial, as the risk chosen by the manager is never greater than the socially optimal risk, which itself is infinite. However, for any K that is not unboundedly large, so long as the above inequality holds[13], there are types $\rho < \rho_0$ whose optimal risk is higher than the socially optimal risk in the absence of regulation. It follows that, if the regulator observes a finite K in a bank, he faces non-trivial uncertainty about the bank manager's type[14], and the regulatory problem is non-trivial.

[13] For example, a sufficient condition for this is that the distribution of ρ shifts down (in the sense of being first order stochastically dominated) at a high enough rate as K increases, so that at any level of K there are types who want to take more risk than the socially optimal risk.

[14] If $\rho_0 \leq \rho_L$, regulation is unnecessary. On the other hand, if we make the stronger assumption that $\rho_0 > \rho_H$, our results remain qualitatively unaffected.

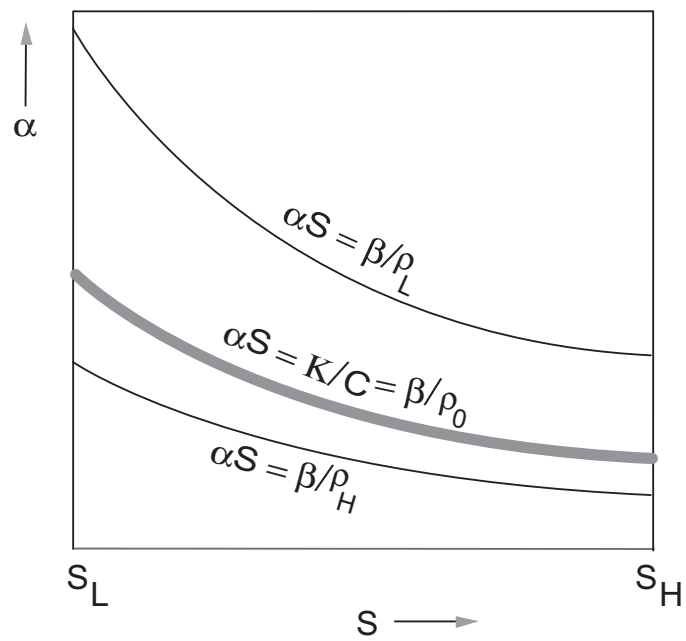

Figure 1: The unregulated optimum. For any type ρ, the optimal portfolio function is the rectangular hyperbola $\alpha s = \beta/\rho$. The thicker hyperbola is the socially optimum portfolio risk function $\alpha s = K/C$. This is also, by definition, the optimal hyperbola of type ρ_0. Types $\rho < \rho_0$ take more than socially optimal risk (optimal hyperbolas above the thicker hyperbola), and types $\rho > \rho_0$ take less than socially optimal risk (optimal hyperbolas below the thicker hyperbola).

A Picture of the Model Our model is summarized in Figure 1. For any given type ρ, αs is a constant given by β/ρ. Thus in $\alpha - s$ space, the optimal choice of ρ is a rectangular hyperbola. The socially optimal portfolio risk is K/C, another constant. Thus this is also a rectangular hyperbola - and coincides with the optimal choice of type ρ_0. The figure shows the hyperbolas associated with types ρ_H, ρ_L and ρ_0. The positions of the three hyperbolas reflect assumption (2.5).

For each type $\rho < \rho_0$, the optimal choice hyperbola is entirely above the hyperbola of type ρ_0. Each such type ρ generates a safety loss (but no overprotection loss). For example, the area between the topmost and the middle hyperbolas is the safety loss generated by type ρ_L. Similarly, each type $\rho > \rho_0$ generates an overprotection loss (but no safety loss). For example, the area between the middle and lowest hyperbolas is the overprotection loss generated by type ρ_H. The regulator's objective is to minimize a weighted average of these losses, where ω denotes the weight attached to safety loss. We show next how ex ante and ex post regimes attempt to achieve this objective. However, we need to specify the regulatory benchmark first.

3 The Regulatory Benchmark

Regulators typically only restrain risk taking. We know of no instance when a regulator sets out to encourage conservative banks to take on further risk. To reflect this, we impose the restriction that under full information, for any $\rho > \rho_0$, no regulation is imposed.

We should emphasize that realism is the main reason for this modelling choice, and removing the restriction would leave our results qualitatively unchanged[15]. However, imposing the restriction makes it clear that the results comparing the two regimes are not influenced by any relative advantage/disadvantage of any regime in encouraging risk taking. We feel this is important to know for policy applications.

For any $\rho < \rho_0$, and for any s, the optimal regulation forces a choice of α (either through an ex ante constraint or through ex post penalties) such that $\alpha s = \beta/\rho_0$. The only loss then is the overprotection loss arising from types $\rho > \rho_0$. Therefore, the **regulatory benchmark loss** (denoted by $EL_{\mathbf{0}}$) is strictly positive and given by $(1-w)\int_{\rho_0}^{\rho_H}(\sigma_{\mathbf{0}}/\sigma - 1)\frac{d\rho}{(\rho_H-\rho_L)}$. Using $\sigma = \alpha^* s$ and equations (2.1), (2.2) and (2.4), $\sigma_{\mathbf{0}}/\sigma = \rho/\rho_0$. Therefore,

$$EL_{\mathbf{0}} = (1-w)\int_{\rho_0}^{\rho_H}\left(\frac{\rho}{\rho_0} - 1\right)\frac{d\rho}{(\rho_H-\rho_L)}. \tag{3.1}$$

The fact that under full information, regulation cannot attain zero loss is of course due to our assumption that regulation does not encourage risk taking, and therefore cannot eliminate the overprotection loss arising from types that adopt lower risk than is socially optimal.

For any regulation, $(EL - EL_{\mathbf{0}})$ is the **distortion** caused by the presence of asymmetric information. It is useful to note that the **unregulated distortion** provides an upper bound to regulatory distortion. If the distortion under any regulation exceeds this level, that regulation is clearly useless and should not be applied. The unregulated distortion is given by $EL_{\mathrm{U}} - EL_{\mathbf{0}} = \omega \int_{\rho_L}^{\rho_0}\int_{s_L}^{s_H}\left(1-\frac{\sigma_{\mathbf{0}}}{\sigma}\right)\frac{ds}{(s_H-s_L)}\frac{d\rho}{(\rho_H-\rho_L)}$, which can be rewritten as:

$$EL_{\mathrm{U}} - EL_{\mathbf{0}} = \omega \int_{\rho_L}^{\rho_0}\left(1-\frac{\rho}{\rho_0}\right)\frac{d\rho}{(\rho_H-\rho_L)}. \tag{3.2}$$

[15] Removing the restriction would imply that for low values of ω, optimal regulation involves a lower limit on risk (ex ante) and a reward for higher risk (ex post). This does not change either the comparison with benchmark (which itself changes down to zero loss), or the comparison across regimes, but this does have the problem of implying somewhat unrealistic regulatory forms.

4 Ex Ante Regulation

Ex ante regulation in the current and future bank capital adequacy framework (Basel I and II) is characterized by a rule that establishes a direct link between capital and portfolio risk. If the regulator could observe the underlying risk s, the optimal regulatory regime would simply set a capital requirement $K = C\alpha s$ so that the portfolio risk would coincide with the socially optimal risk (given by (2.2)). This would achieve a zero loss.

However, the actual realization of s is the bank's private information, and regulation is based on the risk as measured by the regulator. For any given level of K, the regulator, by minimizing regulatory loss, specifies a **"regulatory risk estimate"** $\widehat{s} \in [s_L, s_H]$[16]. This implies an estimated portfolio risk $\alpha\widehat{s}$. Given this estimate, the regulator asks the bank to adopt portfolios such that estimated portfolio risk does not exceed the optimal risk K/C.

For any given capital level K, and for any choice of regulatory estimate $\widehat{s} \in [s_L, s_H]$, such regulation leads to a maximum permissible α, denoted by $\overline{\alpha}$, and given by $\overline{\alpha}\,\widehat{s} = K/C$, i.e.

$$\overline{\alpha} = \frac{K}{C\,\widehat{s}} \tag{4.1}$$

This implies that choosing a $\widehat{s}$ is equivalent to choosing a portfolio constraint $\overline{\alpha}$, shown as the horizontal line in figure 2. A higher regulatory risk estimate leads to a lower $\overline{\alpha}$, that is, a more stringent portfolio constraint. In what follows, we use $\overline{\alpha}$ and $\widehat{s}$ interchangeably.

Next, we need to characterize the optimal choice of $\widehat{s}$, obtained by minimizing the expected loss. The next section shows that in minimizing expected loss, the regulator faces a trade-off.

4.1 Trade-off Between Safety Loss and Overprotection

For types $\rho < \rho_0$, both safety loss and overprotection loss occur. Figure 2(a) shows that for a type $\widetilde{\rho} < \rho_0$, an overprotection loss occurs for $s < \widehat{s}$ (the constrained portfolio is below the socially optimum portfolio). This is shown as the shaded area to the left of $\widehat{s}$. Further, a safety loss occurs for $s > \widehat{s}$ (the shaded area to the right of $\widehat{s}$). For types $\rho > \rho_0$, only an overprotection loss occurs. Figure (b) shows the overprotection loss for a type $\widetilde{\rho}' > \rho_0$ for which the constraint binds for some values of s. The overprotection loss is given by the area between

[16] Risk can also be measured by banks under restrictive guidelines set by the regulator. In this case we can think of the regulator forcing a particular estimate of s through the restrictions.

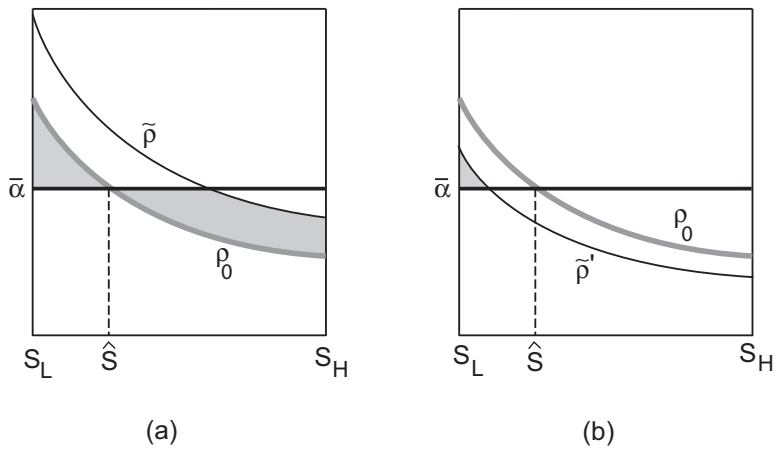

Figure 2: The thick hyperbola is the regulatory optimum (i.e. the optimal hyperbola of type ρ_0), while the horizontal line is the portfolio constraint. Figure (a) shows the safety loss and overprotection loss from an ex ante constraint for any type $\widetilde{\rho} < \rho_0$. In figure (b), the area between the two curves plus the shaded area is the overprotection loss from any type $\widetilde{\rho}' < \rho_0$.

the two hyperbolas and the shaded area. The area between the hyperbolas is part of benchmark loss. The shaded area is the extra overprotection loss over and above benchmark loss.

Finally, if the choice of $\widehat{s}$ is moved to the left, this reduces overprotection loss from any type constrained by regulation, but increases safety loss. The opposite happens if $\widehat{s}$ is moved to the right. Thus, a trade-off arises between safety and overprotection.

4.2 Minimizing Regulatory Loss

The optimal regulatory estimate $\widehat{s}^*$ finds the optimal balance between the twin losses and minimizes the expected regulatory loss. The following result establishes that as the safety bias ω increases (i.e. the regulator becomes more conservative), the regulatory risk estimate becomes upwardly biased.

Lemma 1 (Optimal regulatory estimate)

(a) *For any $\omega \in (0,1)$ there exists $\widehat{s}^* \in (s_L, s_H)$ such that the expected regulatory loss is minimized at $\widehat{s}^*$.*

(b) *$\widehat{s}^*$ is increasing in ω. Further, as $\omega \to 1$, $\widehat{s}^* \to s_H$.*

4.3 The Role of Information on Underlying Risk and Managerial Risk Aversion

An ex ante regime makes poor use of the expertise of a bank in measuring its risk exposure. The regulator imposes a portfolio upper bound, and for all values of s at which this binds, the bank is forced to adopt the same portfolio. Therefore, the information of the bank on those values of s is lost, creating a distortion. Further, the distortion (given by $(EL^* - EL_{\mathbf{0}})$, where EL^* denotes minimized loss) rises as the uncertainty about s increases. It follows that the extent of uncertainty about s is crucial. The following result confirms this intuition.

Proposition 1 (Importance of information on underlying risk)

(a) *As the uncertainty about s increases, the distortion under an ex ante regime increases so that beyond a certain level of uncertainty, no regulation is optimal.*

(b) *As the uncertainty about s vanishes, an ex ante regime attains the regulatory benchmark.*

The next result shows that the precision of information about ρ is not as important. It is the uncertainty about s that is crucial in evaluating the performance of an ex ante regime.

Proposition 2 (Relative unimportance of information on managerial risk aversion)

(a) *As the uncertainty on managerial risk aversion parameter ρ increases, $(EL^* - EL_{\mathbf{0}})$ is bounded above.*

(b) *As the uncertainty about ρ vanishes, $(EL^* - EL_{\mathbf{0}})$ goes to a strictly positive lower bound for any type on which regulation binds.*

(c) *Finally, the extent of both bounds depend on the extent of uncertainty about s. As the uncertainty about s rises, the lower bound tends to unregulated distortion, making regulation useless. As the uncertainty about s vanishes, the upper bound tends to zero.*

The intuition for part (a) is that all types for which the constraint binds are pooled at the portfolio upper bound. Thus, managers are not free to choose high risk portfolios. This

reduces the sensitivity of ex ante regulation to the uncertainty about risk preference of the bank manager. The intuition for part (b) is that whenever the regulatory upper bound binds, no use is made of the bank's private information about s, which generates a distortion. Finally, part (c) results from the fact that as the uncertainty about s increases, either $(\widehat{s} - s_L)$ (and therefore overprotection loss) or $(s_H - \widehat{s})$ (and therefore safety loss) must keep rising. The result shows that the scope for sensitivity to the uncertainty about ρ depends crucially on the uncertainty about s. If the latter is low, the extent of uncertainty about ρ makes little difference. On the other hand, if the uncertainty about s is high, even very precise information about ρ cannot stop the distortion rising to the unregulated level, making regulation useless.

5 Ex Post Regulation

Under ex post regulation, the bank is allowed to choose portfolio risk without any ex ante constraint. If the losses exceed a specified tripwire, corrective action is imposed. This induces a link between capital and portfolio risk. Such a regime can be thought of as generating an endogenous "soft-link" between capital and portfolio risk that contrasts with the exogenous "hard-link" under ex ante regulation.

Kupiec and O'Brien (1997) proposed an ex post scheme for capital regulation of market risk. Market risk refers to risk in the trading book of the bank, which is a fraction of the total investment by a bank. Under this proposal, known as the "precommitment" approach, a bank announces a certain level of capital for market risk, and chooses portfolio risk without any ex ante constraint. However, if the bank suffers losses that exceed the pre-announced level of capital, it is fined. A large fine in the event of a loss might be credible if the trading book capital is a small part of total capital.

Here, we investigate ex post regulation for the whole bank. The above approach may not work as fines are difficult to enforce when losses are large relative to the bank capital. Therefore, we restrict attention here to well capitalized banks. We should note at the outset that our aim is not to prescribe pure ex post regulation for all banks. Indeed, as the next section shows, the optimal regime combines the two forms of regulation. But as a first step in our analysis we need to study a pure ex post approach in order to understand its properties.

Let us now discuss our approach to ex post incentives. To obviate the problem of fines that may be difficult or even undesirable to impose on a bank that has already suffered losses,

one can resort to more general penalties available in the form of gradual interventions by the regulator. For instance, imposing constraints (e.g. stricter ex ante control in future) on bank risk taking reduces freedom to choose portfolio in future, and therefore reduces the market value of bank capital. A well known example is the structured early intervention and resolution proposed originally by Benston and Kaufman (1988) (clarified further by SFRC (1989)), and subsequently revised and incorporated as "prompt corrective actions" in the Federal Deposit Insurance Corporation Improvement Act (FDICIA) of 1991[17]. As SFRC (2000) notes, "it is conceivable that the gradual penalties embodied in the structured early intervention and resolution system ... could help to make the precommitment approach, as applied to the entire bank, credible"[18]. So long as capital is not too low, this approach is feasible.

We model regulatory intervention following this idea. We assume that corrective actions impose a cost on the shareholders as well as the manager of the bank, and the gradual nature of intervention implies that this cost is an increasing function of the extent of loss. While the cost on the shareholders is limited by the level of capital, the regulator might be able to impose additional private costs on the manager. For example, the regulator might initiate action to prevent a manager of a failed bank from being appointed to a similar job for a few years[19]. Further, the fact that the regulator starts penalizing actions could be detrimental to the manager's (and the bank's) reputation. In what follows, the word "penalty" refers to such regulation-imposed costs. The penalty is finite, and is limited by the extent of capital as well as the costs that can be imposed on the manager. We assume that the expected managerial cost through penalties increases in the total losses of the bank.

We assume that a penalty applies when $\widetilde{V} < -\theta K$, $0 \leq \theta < 1$. In general, the penalty function is an increasing function of the extent of the breach (given by $(-\widetilde{V} - \theta K)$). A general form of the penalty function is given by $f(-\widetilde{V} - \theta K)$, where $f' > 0$.

We analyze ex post regulation by assuming (stated formally in section 5.2) that the regulator can control all types of managers through such penalties. However, for banks with very low capital,

[17] These include compulsory capital restoration plans, asset growth limits, restriction on transactions with affiliates, restrictions on deposit interest rates and liquidation. See Benston and Kaufman (1997) for a review of the effectiveness of the act.

[18] Taylor (2002) discusses a similar idea.

[19] Such measures are not uncommon. For example in the UK, a director of an insolvent company can be legally disqualified even in the absence of any dishonest behavior, if the director was negligent or incompetent. Under the 1986 Company Directors Disqualification Act, individuals can be forbidden from acting as a director for up to 15 years.

there might be little scope for early intervention. Moreover, the manager might have very low risk aversion (could bet the bank). In such cases, a pure ex post approach is clearly not effective. Therefore, we implicitly restrict attention here to well capitalized banks, and consider other cases in section 8.2, which relaxes the assumption of full control through penalties. Basically, if the scope for penalties is low, regulation must include some degree of ex ante constraints, but, as we argue, this does not alter the qualitative nature of our results.

Indeed, while we analyze pure ex post regulation in this section, which is important in order to understand the properties of this regime, we show later that some level of ex ante control should always be present in optimal regulation (see sections 7 and 8.2). Therefore, the problem of controlling managerial types is less severe in optimal regulation than the discussion above might indicate.

5.1 Trade-off Between Safety Loss and Overprotection

Under an ex post regime, since the same penalties apply to all types, two different types cannot be induced to choose the same portfolio[(20)]. If some type adopts the socially optimal portfolio risk, more risk averse types adopt portfolios with lower risk, causing an overprotection loss to arise, while less risk averse types adopt higher-risk portfolios, and generate a safety loss. A greater penalty leads to lower risk taking by all types. This reduces safety loss (i.e. reduces the proportion of types adopting greater than socially optimal risk) - but at the same time increases overprotection loss. Thus a trade-off arises between safety and overprotection.

Clearly, the reason behind the trade-off is fundamentally different from that behind the trade-off in ex ante regulation, which arises because an ex ante regime makes poor use of the bank's private information about risk. Here, the trade-off arises only due to the uncertainty about the bank manager's risk preference.

[(20)] To see this, suppose two different types adopt the same portfolio for some value of s. Then, they face exactly the same penalty in each event of breach, and the same probability of a breach. Therefore, they face the same distribution of outcomes. But if two different types face exactly the same distribution of returns, their optimal choice of portfolio cannot be the same, which is a contradiction.

5.2 The Regulator's Problem: Regulatory Target Type

The expected utility of the manager is $Eu(W) = E\left[-\exp\left(-\rho(1+\widetilde{X})\right)\right]$ where

$$\widetilde{X} = \begin{cases} \widetilde{V} & \text{if } \widetilde{V} \geqslant -\theta K \\ \widetilde{V} - f(-\widetilde{V} - \theta K) & \text{if } \widetilde{V} < -\theta K \end{cases} \tag{5.1}$$

The optimal portfolio choice maximizes expected utility. Under any penalty $f(\cdot)$, let $\widehat{\rho}$ be the type such that its optimal risk coincides with the regulatory optimum.

We assume that the regulator can control all types through penalties, so that, in particular it is possible to have $\widehat{\rho} = \rho_L$, i.e. the domain of $\widehat{\rho}$ includes all types less risk averse than ρ_0:

$$\widehat{\rho} \in [\rho_L, \rho_0] \tag{5.2}$$

We discuss the consequences of relaxing this in section 8.2. The result below clarifies that $\widehat{\rho}$ serves to index the level of penalties.

> **Lemma 2** *In equilibrium, for any punishment scheme $f(\cdot)$, there is a type $\widehat{\rho} \in [\rho_L, \rho_0]$ such that the optimal risk of type $\widehat{\rho}$ coincides with the socially optimal risk. $\widehat{\rho}$ decreases as the level of penalties rises, and serves as an index of the level of penalties.*

If no penalties are imposed, the regulatory optimum risk coincides with the risk adopted by type $\widehat{\rho} = \rho_0$, where ρ_0 is defined in section 3. With positive penalties, each type takes a lower risk, and the socially optimal risk coincides with the risk adopted by type $\widehat{\rho} < \rho_0$. As penalties rise, $\widehat{\rho}$ decreases, and $\widehat{\rho} = \rho_L$ denotes the maximum level of penalties.

Since $\widehat{\rho}$ indexes the level of penalties, the regulatory problem of choosing the optimal penalty is equivalent to choosing $\widehat{\rho}$ to minimize loss. This can therefore be interpreted as the **"regulatory target"** type. Before minimizing loss, we note an important property of ex post regulation.

5.3 The Role of Information on Underlying Risk

> **Proposition 3 (Use of bank's expertise)** *Under ex post regulation, all information on s is used so that the expected regulatory loss is independent of the distribution of the underlying risk.*

The result implies that even if the regulator's information on s is very poor, this is not a matter for concern, since ex post regulation makes full use of the information of the bank. Ex ante regulation imposes a cap on the share of the risky component in the bank's portfolio, preventing full use of the bank's superior information on risk. Under an ex post regime, the prospect of punishment makes a bank choose a lower portfolio risk for each level of s compared to the unrestricted optimum. However, a bank can still choose different portfolios for different realizations of s, making full use of their superior information on risk.

5.4 Minimizing Loss

The regulator's problem under ex post regulation is to determine the optimal regulatory target $\widehat{\rho}^{\,*}$, which minimizes expected loss. This is derived below.

Lemma 3 (Optimal regulatory target) *There exists $\omega_* < 1$ such that*

(a) *for $\omega \leq \omega_*$, the expected loss is minimized by imposing no penalties, and thus $\widehat{\rho}^{\,*} = \rho_0$.*

(b) *For $\omega > \omega_*$ there exists $\widehat{\rho}^{\,*} \in (\rho_L, \rho_0)$ which minimizes the expected regulatory loss.*

(c) *Finally, $\widehat{\rho}^{\,*}$ is decreasing in ω, and as $\omega \to 1$, $\widehat{\rho}^{\,*} \to \rho_L$.*

5.5 The Role of Information on Managerial Risk Aversion

Proposition 3 shows that in sharp contrast with ex ante regulation, under an ex post regime the uncertainty about s is irrelevant, as such a regime makes full use of the bank's private information on s. However, the opposite is true of the regulator's information on the extent of risk aversion of the bank manager. The following result shows that unlike in an ex ante regime, uncertainty about ρ is very important under ex post regulation.

Proposition 4 (Importance of information on the manager's type)

(a) *The expected loss is increasing in the uncertainty about ρ.*

(b) *Further, as uncertainty about ρ vanishes, an ex post regime attains the regulatory benchmark.*

6 Comparing Ex Ante and Ex Post Regulation

From propositions 1 and 2 the extent of information on s - rather than that on ρ - is the critical factor in evaluating the performance of an ex ante regime. As the uncertainty about s increases starting from zero, the distortion goes from zero to the maximum (the unregulated level). Further, proposition 3 shows that information on s is irrelevant for the performance of ex post regulation, and proposition 4 shows that the extent of information on ρ is critical for evaluating the performance of an ex post regime. Thus, clearly, the comparison of the two regimes depends on the relative spreads of the distributions of ρ and s. If the spread of the distribution of s is high relative to that of ρ, ex post regulation produces a lower regulatory loss, and vice versa.

7 Combining Ex Ante and Ex Post Regulation

The complementary nature of the two regimes implies that a combination might be useful. The following result characterizes the optimal combination.

Let γ denote the fraction of types for which the optimal portfolio risk is above the social optimum in the absence of regulation. These are the types on which regulation is potentially binding. Formally,

$$\gamma = \frac{\rho_0 - \rho_L}{\rho_H - \rho_L}. \tag{7.1}$$

Let $\widehat{\rho}^{**}$ and $\widehat{s}^{**}$ denote the optimal regulatory target type and the optimal regulatory risk estimate (respectively) in the combined regime.

Proposition 5 (Characterizing the optimal combination)

(a) $\widehat{s}^{**} > s_L$ - *i.e. the optimal combination always includes a non-trivial version of ex ante constraints.*

(b) *If* $\gamma = 1$, $\widehat{\rho}^{**} < \rho_0$.

(c) *If* $\gamma < 1$, $\exists\ \omega_{**} < 1$ *such that* $\widehat{\rho}^{**} < \rho_0$ *for all* $\omega > \omega_{**}$.

The result says that the optimal combined regime always includes a non-trivial ex ante constraint (i.e. relaxing the constraint all the way so that it never binds is not optimal). Further,

the extent of ex post incentives depends on the unregulated optimum. If the unregulated optimum risk of all types is above the socially optimal risk, the optimal combination always includes ex post incentives. However, if some types adopt a risk below the social optimum even without regulation, it is as if the effect of a certain level of ex post incentives is already in place, and thus ex post penalties have no further role for low values of ω. In such cases, ex post incentives play a role in the optimal combination only when the safety bias ω is high enough.

Finally, if the regulator is mostly concerned about depositor protection and/or faces significant systemic risk, so that ω is close to 1, we get a striking characterization. In this case, the optimal mixture is asymmetric. One regime is applied in a strong form, and the other in a weak form.

> **Proposition 6 (Asymmetry under high safety bias)** *For ω close to 1, the optimal combination is asymmetric and involves either* **(a)** *strong ex post penalties ($\widehat{\rho}$ close to ρ_L) and weak ex ante constraints ($\widehat{s}$ close to s_L), or* **(b)** *strong ex ante constraints ($\widehat{s}$ close to s_H) and weak ex post penalties ($\widehat{\rho}$ close to ρ_L).*
>
> *Further, if an ex post regime with $\widehat{\rho} = \rho_L$ generates a lower (greater) overprotection loss compared to an ex ante regime with $\widehat{s} = s_H$, case* **(a)** *(case* **(b)***) obtains.*

The intuition is as follows. Given strong ex post penalties (say), safety loss is very low. At this point, a marginal increase in ex ante constraints does not reduce safety loss very much further, but it does increase overprotection loss. Therefore, if strong ex post penalties are already in place, ex ante constraints should be weakened, and vice versa. In addition, either regime, applied in a strong enough form, is capable of eliminating safety loss. It follows that the regime chosen to be applied in a strong form is the one that generates a lower overprotection loss when applied in a strong form.

Which regime is likely to have a lower overprotection loss when applied in a strong form? Clearly, the answer is ex post regulation if the uncertainty about the manager's type ρ is low relative to that about the underlying risk s. However, the same answer might apply even if the latter condition is not satisfied. A desirable property of penalties is that the penalty is **"regressive"** in the sense that the marginal impact of a rise in penalties is higher on less risk averse types. Since the marginal reduction in risk is lower for more risk averse types, the optimal risks adopted by the different types get "bunched" together as the penalty level rises. This **bunching effect** reduces the effective spread of the distribution of ρ. Thus, if a penalty structure is sufficiently regressive, the overprotection loss generated by a strong form of ex post regulation is lower than that under a strong form of ex ante regulation. Therefore, an ex post

regime is the dominant element of the optimal combination (case (a) in the proposition above). The following example shows that linear penalties satisfy this property.

7.1 Regressive Property and Bunching Effect: Linear Penalties

As an example of ex post penalty functions, we look at the case of linear penalties. We show below that linear penalties are regressive in the sense described above. Further, the regressive property and consequent bunching effect is strong enough so that ex post regulation emerges as the dominant regime under high safety bias.

A linear penalty function is given by $f(-\widetilde{V}-K) = \delta(-\widetilde{V}-K)$, where δ is a positive constant[21]

For general penalties, we used $\widehat{\rho}$ as the index of the level of penalties. We can do the same here, but now δ is an equivalent but more natural index of penalties. In what follows, we will use δ to index penalties for ease of calculation. Since $\widehat{\rho}$ maps one-for-one into δ, this is without loss of generality.

Under this penalty,

$$\begin{aligned} Eu(W) &= -\exp\left(-\rho\left[1+\beta\sigma+\delta\sigma Z-\frac{\rho}{2}\sigma^2\,(1+\delta)^2\right]\right)\Phi\left(-Z+\sigma\,\rho\,(1+\delta)\right) \\ &\quad -\exp\left(-\rho\left[1+\beta\sigma-\frac{\rho}{2}\sigma^2\right]\right)\Phi\left(Z-\sigma\rho\right), \end{aligned}$$

where $Z = K/\sigma + \beta$, and $\sigma = \alpha s$. Let σ^* denote the optimal risk. The marginal impact of penalties on risk is given by $\partial\sigma^*/\partial\delta$, which is negative. As described before, a penalty structure is regressive if the absolute value of this derivative is decreasing in ρ at each δ. Formally, a penalty is regressive if

$$\frac{\partial}{\partial\rho}\left|\frac{\partial\sigma^*}{\partial\delta}\right| < 0 \;\; \forall\, \delta > 0. \qquad \text{(Regressive Penalties)}$$

As mentioned above, if this is the case, as δ increases, the optimal risk of lower types fall faster and thus the optimal risks across types get "bunched."

As the algebra is too complex to be of help, we show the behavior through simulations. Figure 3 plots the optimal choice of σ for different types ρ as the penalty rises. The figure shows that

[21] Note that our assumption of normality of returns implicitly implies log returns. When log return equals minus infinity the actual loss is finite and equal to the initial value of the bank's assets. Since penalties are also therefore defined in the log return domain, they are finite.

linear penalties are regressive and therefore give rise to a bunching effect. Next, figure 4 shows the optimal combination of the two regimes for different values of safety bias. Ex post penalties kick in once ω exceeds 0.25, and then steadily get stronger. The ex ante constraint $\overline{\alpha}$ displays a "U" shape as a function of ω. Initially, the constraint gets stronger ($\overline{\alpha}^*$ falls). As ω rises, the optimal regulation relaxes ex ante constraints and employs more stringent ex post measures. Accordingly, for high safety bias, ex post regime is dominant. Note that the spread of s is relatively small, yet a strong bunching effect makes ex post the dominant regime for high safety bias.

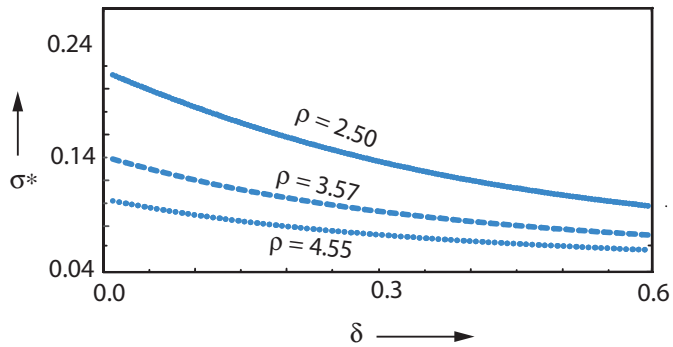

Figure 3: Bunching of optimal risk under regressive penalties.

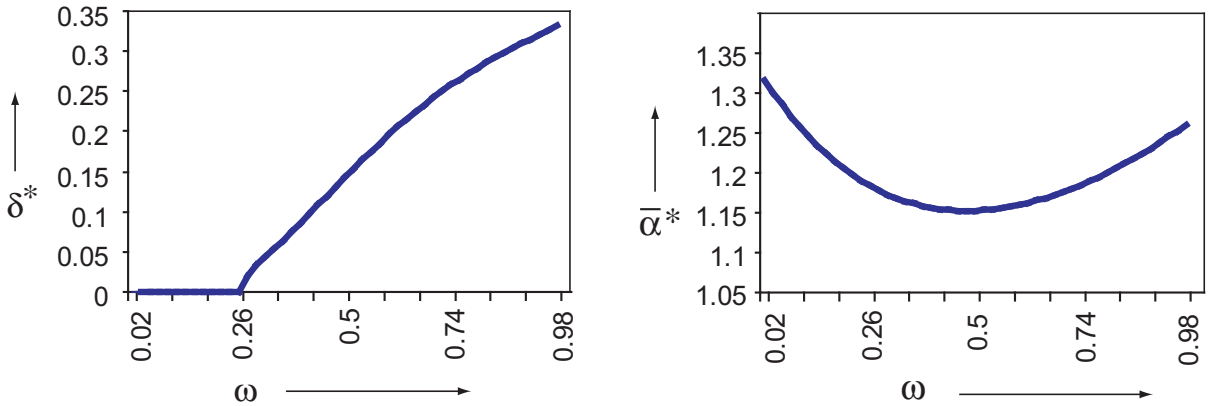

Figure 4: The ex post penalty level (δ^*) and the ex ante constraint ($\overline{\alpha}^*$) in the optimal combination for different values of the safety bias ω.

We use the following parameter values[22]: $\rho_L = 2.5, \rho_H = 4.6$, $s_L = 0.1$, $s_H = 0.2$, $\beta = 0.5$,

[22] While we report simulations for only one set of parameter values, we have carried out simulations for a wide range of parameter values, and obtained similar results in all cases.

$K = 0.15$ and $p = 0.05$. $C \equiv \Phi^{-1}(1-p) - \beta = 1.644 - .5 = 1.144$. The socially optimal risk is $K/C = 0.13$. Further, the type such that the optimal risk coincides with the socially optimal risk is given by $\rho_0 = \beta C/K = 3.8$. Therefore, types $\rho \in (3.8, 4.6)$ initially adopt risk below the socially optimal risk, and types below 3.8 initially adopt risk exceeding the socially optimal risk.

8 Extensions

8.1 Poor Bank Information on Risk

We have assumed so far that the bank knows the true portfolio risk. What if we relax this assumption? Recall that the principal virtue of ex post regulation is that it makes full use of the bank's specialized information on underlying risk. If the bank has no such specialized information, the advantage vanishes. On the other hand, the relative advantage of ex ante regulation, arising from the fact that it is less sensitive to regulatory uncertainty about the risk preference of the bank, still holds. Therefore, for a bank with poor risk management, pure ex ante regulation optimal. Only when regulators are satisfied that a bank has developed sufficient expertise in measuring risk should they move towards including ex post incentives. This is the intuition behind the following result.

Proposition 7 *If the bank has no private information on s, regulating through only ex ante constraints is optimal.*

8.2 Relaxing assumption 5.2

In section 5, we assume that the regulator can control even the least risk averse type with penalties, so that $\widehat{\rho} \in [\rho_L, \rho_0]$. However, if a bank has accumulating losses and low capital, the prospect of penalties might induce it to bet the bank. In such cases, penalties are unable to control the bank. To extend our analysis to such cases, assume that ρ_L is very low (or even zero, or negative), and there is a critical type $\rho_0 < \rho_C < \rho_L$ such that penalties fail to control types $\rho \in [\rho_L, \rho_C]$.

In this case, we need to impose ex ante rules along with ex post penalties to control these types. In other words, while a pure ex ante approach does not work, a combined regime is optimal.

The analysis in section 7 applies. However, proposition 6 no longer holds in the stated form. A moderately strong version of ex ante regime must always be present, and once we redefine this level as the weakest feasible level of ex ante regulation, the proposition applies.

9 Ex ante and Ex post Regulation In Basel II

The first international standard for risk based bank capital requirements was proposed in 1988 by the Basel Committee on Banking Supervision. A new capital adequacy framework (Basel II) is set to replace the existing agreement by 2006. The 1988 Accord imposes a fairly rigid ex ante regime. In contrast, the three pillars that constitute Basel II: (1) minimum capital requirements, (2) supervisory review of capital adequacy, and (3) market discipline, include both ex ante and ex post forms of regulation[23].

Our results provide a theoretical explanation for the approach taken in pillar 1 and the inclusion of pillar 2. First, let us consider how our results relate to pillar 1. For banks with poor expertise in risk measurement, we show that only ex ante regulation should be used. On the other hand, sophisticated banks should be regulated through a mixture of ex ante and ex post regulation. This is consistent with pillar 1, in which banks are subject to different regimes according to their degree of sophistication. For example, as stated in pillar 1, banks can opt for a pure ex ante credit risk regime known as the Standardized Approach, which is very similar to the 1988 Basel Accord[24]. More sophisticated banks are given the option of choosing an internal rating based (IRB) approach[25], in which some risk factors are specified by banks themselves, thus combining ex ante and ex post features.

The fact that some aspects of risk are specified by the banks introduces an element of ex post regulation in an otherwise ex ante environment. As Rochet (1999) points out, there is little theoretical difference between a system in which banks decide their own capital and risk and

[23] The same is true for the 1996 Amendment that is concerned with capital requirements for the bank's trading book. The Amendment is essentially left unchanged under Basel II.

[24] However, unlike Basel 1988 where each debt attracts a capital charge that only depends on the type of debtor (sovereign, bank, corporate, householder), the Standardized Approach is also based on "external" credit ratings.

[25] The IRB approach itself includes two variants: a "foundation" version which is close to the standardized approach, and an "advanced" version, which allows greater flexibility. Similarly, for operational risk an ex ante regime called the Basic Indicator Approach is offered as an alternative to a hybrid regime, the Advanced Measurement Approach.

are penalized ex post (our model), and a system in which banks use their internal models to measure and report risk, and the regulator checks whether the model is accurate through backtesting.

Even though pillar 1 makes some room for ex post measures, it is still dominated by ex ante requirements. The hybrid nature of Basel II is more importantly highlighted by pillar 2, which introduces explicit ex post features. Pillar 2 requires banks to hold additional capital for the risk not taken into account in pillar 1[26]. The banks are directly responsible for measuring these additional risks and hold capital against them. If additional risks are deemed not to be assessed properly, the regulator intervenes with measures similar to the corrective actions in the FDICIA (1991)[27]. Here losses could well be an indicator for poor risk management, which would make our modelling of ex post regulation fit well with actual pillar 2 interventions[28].

Finally, pillar 3 requires greater disclosure from banks, and can potentially introduce ex post penalties in the form of an adverse market reaction to bad news (for example, when banks disclose higher risk exposure). However, market discipline is outside the scope of the current paper.

9.1 Policy Recommendation

The principal policy recommendation arising from our results is that compared to current and proposed regulation, we should go much further in relying on ex post incentives for sophisticated and well capitalized banks. This is especially true for prudential regulation largely concerned with depositor safety. The following observations lend support to this claim. Given the increasing sophistication of financial products and banks' specialized information on the risk profile of their portfolios, it is unlikely that the regulator is able to detect underlying risk very

[26] These are: (i) risks considered under pillar 1 that are not fully captured by the pillar 1 process (e.g. credit concentration risk), (ii) factors not taken into account by the Pillar 1 process (e.g. interest rate risk in the banking book, business and strategic risk) as well as (iii) factors external to the bank (e.g. business cycle effects).

[27] Principle 4 of pillar 2 explicitly incorporates early intervention "to prevent capital from falling below the minimum levels required to support the risk characteristics of a particular bank and should require rapid remedial action if capital is not maintained or restored."

[28] The Accord has not clearly specified what triggers intervention. The Basel Committee on Banking Supervision (2004) states "Supervisors are expected to evaluate how well banks are assessing their capital needs relative to their risks and to intervene, when appropriate. ... when deficiencies are identified, prompt and decisive action can be taken to reduce risk or restore capital."

precisely. In this case, ex ante regulation wastes valuable information while ex post incentives make full use of such information. Further, making use of a process such as the supervisory review provision in Basel II, the regulator can establish a close relation with a bank, which reduces uncertainty about managerial risk preferences. This reinforces the suitability of ex post elements.

Although the informational disadvantage of the regulator is acknowledged by the Basel Committee[29], more work needs to be done to slim down the New Accord, with greater reliance for sophisticated and well capitalized banks on supervision (pillar 2) and market discipline (pillar 3), and less on detailed ex ante rules (pillar 1).

9.2 Two Issues for Future Research

While our analysis generates robust insights, there are certain practical problems with ex ante and ex post regulation that are beyond the scope of our simple model. As Jones (2000) points out, banks might make cosmetic adjustments that reduce the regulatory measure of risk, and therefore capital requirement, even though actual risk is unchanged. Such "regulatory capital arbitrage" reduces the effectiveness of ex ante regulation.

A second practical problem faced by both regimes is regulatory forbearance, which undermines enforcement. As Calomiris (1999) as well as SFRC (2000) discuss, a potential solution to the problem of forbearance is to make use of market discipline induced by market price signals of mandatory subordinated debt. Clearly, to the extent that such measures can reduce the scope of forbearance, they enhance the relative efficacy of ex post regimes.

Finally, it should be added that capital arbitrage and forbearance might in some cases strengthen regulation. As Alan Greenspan has noted, the former might act as a safety valve[30]. Forbearance, on the other hand, could optimally lead to a relaxation of ex post regulation under aggregate shocks. A formal analysis of these problems is a matter for future research.

[29] The Basel Committee (2003) states "it is inevitable that a capital adequacy framework, even the more forward looking New Accord, will lag to some extent behind the changing risk profiles of complex banking organizations, particularly as they take advantage of newly available business opportunities."

[30] Remarks before the Conference on Capital Regulation in the 21st Century, Federal Reserve Bank of New York, New York, NY1 February 26, 1998.

10 Conclusion

The 1988 Basel Accord imposed ex ante capital regulation. In contrast, Basel II has moved towards more hybrid regimes. This motivates our question about the role of different approaches to capital regulation. Despite its practical importance, the theoretical literature offers little clarity on this issue. Our contribution is to specify a simple framework to analyze the properties of different forms of capital regulation, helping to determine the optimal regime under a variety of circumstances. We show that in both ex ante and ex post regimes, the regulator must resolve a trade-off between safety loss and overprotection. Interestingly, the nature of the trade-off is very different under the two regimes. Ex ante regulation is not very sensitive to the information asymmetry about managerial risk aversion, but makes poor use of the private information of the bank on underlying risk. Ex post regulation, on the other hand, makes full use of the bank's superior information on risk, but is more vulnerable to the problem of unknown managerial risk aversion.

It follows that the relative importance of the two sources of regulatory uncertainty determines which regime is preferable. Further, the fact that the two regimes are sensitive to different dimensions of information asymmetry suggests that a combination is useful. Indeed, we show that a combined regime outperforms either regime. Moreover, under penalties with a plausible regressive property, we get a striking characterization: if the regulator is mostly interested in protecting depositors and/or faces significant systemic risk, the optimal regime combines a strong version of ex post regulation with a weak version of ex ante regulation.

The discussion above assumes that the bank has superior information on underlying risk. If, however, the bank's information on risk is poor, it is optimal to use only ex ante regulation. This explains why it is important for the regulator to assess the risk management abilities of a bank before choosing the appropriate form of regulation.

Our results support the move of the Basel Committee towards schemes offering different combinations of ex ante and ex post rules, and in particular the introduction of pillar 2. However, our policy prescription disagrees with the Basel II framework in that we find stronger emphasis should be given to ex post rules in regulating sophisticated and well capitalized banks. We envisage that, over time, as regulators have the opportunity to observe the risk-taking behavior of individual banks and become confident about their risk management skills, increasingly flexible forms of ex post regulation would be allowed. At the same time, ex ante constraints would continue to play a role as safeguards against extreme risk taking.

Appendix: Proofs

A.1 Proof of Lemma 1

The proof proceeds through the following steps.

A.1.1 Step 1: Deriving EL

We first define three functions $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$, $\mathbf{s}_c(\rho)$

For any given ex ante constraint, let ρ_1 be such that for types $\rho > \rho_1$, the constraint does not bind for any value of s. Further, let $\rho_2 < \rho_1$ be such that for types $\rho < \rho_2$, the constraint binds for all s. Thus for types $\rho \in [\rho_2, \rho_1]$, the constraint binds for some values of s. Figure 5 shows the optimal hyperbolas for types ρ_1 and ρ_2.

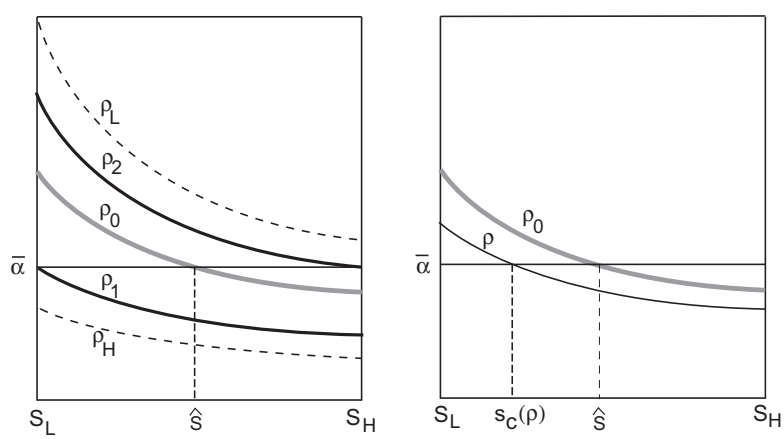

Figure 5: The figure on the left shows the optimal hyperbolas of ρ_1 and ρ_2. The figure on the right shows $s_c(\rho)$ for type $\rho \in (\rho_0, \rho_1)$.

ρ_1 and ρ_2 are formally defined as follows.

ρ_1 is implicitly given by $\dfrac{\beta}{\rho_1} = \overline{\alpha} s_L$, where $\overline{\alpha} = K/(C\widehat{s})$. Using this, and using equation (2.4),

$$\rho_1 = \rho_0 \frac{\widehat{s}}{s_L}. \tag{A.1}$$

Further, if $\beta/\rho > \overline{\alpha} s_L$ for all ρ, then $\rho_1 = \rho_H$.

Similarly, ρ_2 is implicitly given by $\dfrac{\beta}{\rho_2} = \overline{\alpha} s_H$. Again, using equation (2.4),

$$\rho_2 = \rho_0 \frac{\widehat{s}}{s_H}. \tag{A.2}$$

Further, if $\beta/\rho < \overline{\alpha} s_H$ for all ρ, then $\rho_2 = \rho_L$.

For any $\rho \in (\rho_2, \rho_1)$, let $s_c(\rho)$ be the value of s at which the portfolio constraint cuts the optimal hyperbola for type ρ. Therefore, for $s \leqslant s_c(\rho)$, the optimal α for the bank is above $\overline{\alpha}$, and for $s > s_c(\rho)$, the optimal α for the bank is below $\overline{\alpha}$. Thus $s_c(\rho)\ \overline{\alpha} = \beta/\rho$. From equation (4.1), we know that $\overline{\alpha} = K/(C\widehat{s})$. Using this, and the expression for ρ_0 from (2.4), we have $s_c(\rho) = (\rho_0/\rho)\widehat{s}$.

For any $\rho \leqslant \rho_2$, we define $s_c(\rho) = s_H$, and for any $\rho \geqslant \rho_1$, we define $s_c(\rho) = s_L$.

Thus we have

$$s_c(\rho) = \begin{cases} \dfrac{\rho_0}{\rho}\,\widehat{s} & \text{for } \rho_2 < \rho < \rho_1 \\ s_L & \text{for } \rho \geqslant \rho_1 \\ s_H & \text{for } \rho \leqslant \rho_2 \end{cases} \tag{A.3}$$

Next we derive EL

For notational convenience, let

$$\Delta_\rho \equiv (\rho_H - \rho_L), \quad \text{and} \quad \Delta_s \equiv (s_H - s_L). \tag{A.4}$$

For any $\rho > \rho_0$, safety loss is zero, but an overprotection loss occurs. The overprotection loss for any such ρ is given by

$$\begin{aligned} OL(\rho, \widehat{s}) &= \int_{s_L}^{s_c(\rho)} \left(\frac{\sigma_{\mathbf{0}}}{\overline{\alpha} s} - 1\right) \frac{ds}{\Delta_s} + \int_{s_c(\rho)}^{s_H} \left(\frac{\sigma_{\mathbf{0}}}{\alpha^* s} - 1\right) \frac{ds}{\Delta_s} \\ &= \int_{s_L}^{s_c(\rho)} \left(\frac{\widehat{s}}{s} - 1\right) \frac{ds}{\Delta_s} + \int_{s_c(\rho)}^{s_H} \left(\frac{\rho}{\rho_0} - 1\right) \frac{ds}{\Delta_s}, \end{aligned} \tag{A.5}$$

where the first term in the second step follows from $\sigma_{\mathbf{0}} = K/C$ (equation (2.2)) and $\overline{\alpha} s = (K/C)(s/\widehat{s})$ (equation (4.1)). Further, the second term in the second step is obtained using $\alpha^* s = \beta/\rho$ (equation (2.1)) and $K/C = \beta/\rho_0$ (equation (2.4)).

For $\rho \geqslant \rho_1$, $s_c(\rho) = s_L$, so the first term vanishes.

For any $\rho < \rho_0$, overprotection loss would be zero if there were no ex ante constraint. But given an ex ante constraint, an overprotection loss occurs also for $\rho < \rho_0$. This is given by

$$\int_{s_L}^{\widehat{s}} \left(\frac{\widehat{s}}{s} - 1\right) \frac{ds}{\Delta_s}.$$

Further, for any $\rho < \rho_0$, the safety loss is given by

$$SL(\rho, \widehat{s}) = \int_{\widehat{s}}^{s_c(\rho)} \left(1 - \frac{\widehat{s}}{s}\right) \frac{ds}{\Delta_s} \; + \; \int_{s_c(\rho)}^{s_H} \left(1 - \frac{\rho}{\rho_0}\right) \frac{ds}{\Delta_s}. \tag{A.6}$$

For $\rho \leqslant \rho_2$, $s_c(\rho) = s_H$, so the second term vanishes.

Thus expected loss is given by

$$\begin{aligned} EL \;=\; & (1-\omega) \int_{\rho_0}^{\rho_1} OL(\rho, \widehat{s}) \frac{d\rho}{\Delta_\rho} \\ & + (1-\omega) \int_{\rho_1}^{\rho_H} \left(\frac{\rho}{\rho_0} - 1\right) \frac{d\rho}{\Delta_\rho} \\ & + (1-\omega) \int_{\rho_L}^{\rho_0} \int_{s_L}^{\widehat{s}} \left(\frac{\widehat{s}}{s} - 1\right) \frac{ds}{\Delta_s} \frac{d\rho}{\Delta_\rho} \\ & + \omega \int_{\rho_2}^{\rho_0} SL(\rho, \widehat{s}) \frac{d\rho}{\Delta_\rho} \\ & + \omega \int_{\rho_L}^{\rho_2} \int_{\widehat{s}}^{s_H} \left(1 - \frac{\widehat{s}}{s}\right) \frac{ds}{\Delta_s} \frac{d\rho}{\Delta_\rho}, \end{aligned} \tag{A.7}$$

where $OL(\rho, \widehat{s})$ and $SL(\rho, \widehat{s})$ are given by equations (A.5) and (A.6) respectively.

A.1.2 Step 2

The proof proceeds through the following lemma.

Lemma 4 $\dfrac{\partial EL}{\partial \rho_1} = \dfrac{\partial EL}{\partial \rho_2} = \dfrac{\partial EL}{\partial s_c(\rho)} = 0.$

Proof: From (A.3), $s_c(\rho_1) = s_L$. Using this,

$$\frac{\partial EL}{\partial \rho_1} \;=\; (1-\omega) \left(\frac{\rho_1}{\rho_0} - 1\right) \frac{1}{\Delta_\rho} - (1-\omega) \left(\frac{\rho_1}{\rho_0} - 1\right) \frac{1}{\Delta_\rho} \;=\; 0.$$

Next,

$$\frac{\partial EL}{\partial \rho_2} \;=\; -\omega \left(\int_{\widehat{s}}^{s_H} \left(1 - \frac{\widehat{s}}{s}\right) \frac{ds}{\Delta_s}\right) \frac{1}{\Delta_\rho} + \omega \left(\int_{\widehat{s}}^{s_H} \left(1 - \frac{\widehat{s}}{s}\right) \frac{ds}{\Delta_s}\right) \frac{1}{\Delta_\rho} \;=\; 0.$$

Finally,

$$\begin{aligned}\frac{\partial EL}{\partial s_c(\rho)} &= (1-\omega)\int_{\rho_0}^{\rho_1}\left(\left(\frac{\widehat{s}}{s_c(\rho)}-1\right)-\left(\frac{\rho}{\rho_0}-1\right)\right)\frac{1}{\Delta_s}\frac{d\rho}{\Delta_\rho} \\ &\quad +\omega\int_{\rho_2}^{\rho_0}\left(\left(1-\frac{\widehat{s}}{s_c(\rho)}\right)-\left(1-\frac{\rho}{\rho_0}\right)\right)\frac{1}{\Delta_s}\frac{d\rho}{\Delta_\rho}\end{aligned}$$

Now, from (A.3), $s_c(\rho) = (\rho_0/\rho)\widehat{s}$ for $\rho \in (\rho_2, \rho_1)$. Using this, we see that $\dfrac{\partial EL}{\partial s_c(\rho)} = 0$. This completes the proof of the lemma.‖

We are now ready to prove parts (a) and (b) of the proposition.

Part (a)

The derivative of expected loss with respect to $\widehat{s}$ is given by $\dfrac{\partial EL}{\partial \rho_1}\dfrac{\partial \rho_1}{\partial \widehat{s}}+\dfrac{\partial EL}{\partial \rho_2}\dfrac{\partial \rho_2}{\partial \widehat{s}}+\dfrac{\partial EL}{\partial s_c(\rho)}\dfrac{\partial s_c(\rho)}{\partial \widehat{s}}+\dfrac{\partial EL}{\partial \widehat{s}}$. From lemma 4, the first three terms are zero. Thus we only need to differentiate the terms directly involving $\widehat{s}$. Thus the derivative of expected loss with respect to $\widehat{s}$ is given by:

$$\begin{aligned}\frac{dEL}{d\widehat{s}} &= (1-\omega)\int_{\rho_0}^{\rho_1}\left(\int_{s_L}^{s_c(\rho)}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}+(1-\omega)\int_{\rho_L}^{\rho_0}\left(\int_{s_L}^{\widehat{s}}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho} \\ &\quad -\omega\int_{\rho_2}^{\rho_0}\left(\int_{\widehat{s}}^{s_c(\rho)}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}-\omega\int_{\rho_L}^{\rho_2}\left(\int_{\widehat{s}}^{s_H}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}.\end{aligned}$$

Let A and B denote the coefficients of $(1-\omega)$ in the first and second terms (respectively) on the right hand side, and let D and F denote the coefficients of ω in third and fourth terms (respectively) on the right hand side.

At $\widehat{s} = s_L$, $\rho_1 = \rho_0$, and $\frac{dEL}{d\widehat{s}} = -\omega(D+F) < 0$, and at $\widehat{s} = s_H$, $\rho_2 = \rho_0$, and $\frac{dEL}{d\widehat{s}} = (1-\omega)(A+B) > 0$. Thus there is an interior minimum and the first order condition for loss minimization is given by $\frac{dEL}{d\widehat{s}} = 0$. This implicitly defines the optimal regulatory estimate $\widehat{s}^{\,*}$. Further, the second order condition for minimization holds, so that $\dfrac{d^2EL}{d\widehat{s}^{\,2}} > 0$.

Part(b)

From the first order condition, differentiating with respect to ω, we obtain

$$\frac{d\widehat{s}^{\,*}}{d\omega} = \frac{A+B+D+F}{(1-\omega)\rho_1+\omega\rho_2-\rho_L}\ \widehat{s} > \ 0.$$

Finally, as $\omega \to 1$, $\dfrac{dEL}{d\widehat{s}} \to -\displaystyle\int_{\rho_2}^{\rho_0}\left(\int_{\widehat{s}}^{s_c(\rho)} \frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho} - \int_{\rho_L}^{\rho_2}\left(\int_{\widehat{s}}^{s_H} \frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}$. For any $\widehat{s} < s_H$, this is strictly negative. Further, the expression is zero at $\widehat{s} = s_H$ (the second term is clearly zero, and at $\widehat{s} = s_H$, $\rho_2 = \rho_0$ - thus the first term is also zero). Thus at $w = 1$, the expected loss is minimized at $\widehat{s} = s_H$. This completes the proof of lemma 1.‖

A.2 Proof of Proposition 1

Part (a)

Information about s is captured by the interval $[s_L, s_H]$. A smaller interval implies better information about s. With any change in s_L or s_H, $\widehat{s}$ adjusts optimally. For the proof, we will adjust $\widehat{s}$ so that the relative probability mass attached to the support of safety loss and overprotection loss remains constant. Under this adjustment, we show that as the interval shrinks, EL falls. Since the EL under optimal adjustment cannot be higher, the optimized loss, denoted EL^*, falls as well.

Let M_0 denote the ratio $\dfrac{\widehat{s} - s_L}{s_H - s_L}$. Whenever s_H and/or s_L change, we change $\widehat{s}$ so that M_0 remains constant. Since M_0 is constant, so is $1 - M_0 = (s_H - \widehat{s})/(s_H - s_L)$.

The benchmark loss is $EL_{\mathbf{0}}$ given by (3.1). The expression for $EL^* - EL_{\mathbf{0}}$ can be rewritten as

$$\begin{aligned} EL^* - EL_{\mathbf{0}} &= (1-\omega)\, M_0 \left[\int_{\rho_0}^{\rho_1} L_1(s_L)\frac{d\rho}{\Delta_\rho} + \int_{\rho_L}^{\rho_0} L_2(s_L)\frac{d\rho}{\Delta_\rho}\right] \\ &\quad + \omega\,(1-M_0)\left[\int_{\rho_2}^{\rho_0} L_3(s_H)\frac{d\rho}{\Delta_\rho} + \int_{\rho_L}^{\rho_2} L_4(s_H)\frac{d\rho}{\Delta_\rho}\right] \end{aligned} \tag{A.8}$$

where

$$L_1(s_L) = \int_{s_L}^{s_c(\rho)} \left(\frac{\widehat{s}}{s} - \frac{\rho}{\rho_0}\right)\frac{ds}{\widehat{s} - s_L} \tag{A.9}$$

$$L_2(s_L) = \int_{s_L}^{\widehat{s}} \left(\frac{\widehat{s}}{s} - 1\right)\frac{ds}{\widehat{s} - s_L} \tag{A.10}$$

$$\begin{aligned} L_3(s_H) &= \int_{\widehat{s}}^{s_c(\rho)} \left(1 - \frac{\widehat{s}}{s}\right)\frac{ds}{(s_H - \widehat{s})} + \int_{s_c(\rho)}^{s_H} \left(1 - \frac{\rho}{\rho_0}\right)\frac{ds}{(s_H - \widehat{s})} \\ &= \int_{\widehat{s}}^{s_c(\rho)} \left(\frac{\rho}{\rho_0} - \frac{\widehat{s}}{s}\right)\frac{ds}{(s_H - \widehat{s})} + \left(1 - \frac{\rho}{\rho_0}\right) \end{aligned} \tag{A.11}$$

$$L_4(s_H) = \int_{\widehat{s}}^{s_H} \left(1 - \frac{\widehat{s}}{s}\right)\frac{ds}{(s_H - \widehat{s})} \tag{A.12}$$

Now,

$$\frac{dEL^*}{ds_L} = \frac{\partial EL^*}{\partial s_L} + \frac{\partial EL^*}{\partial \widehat{s}} \frac{d\widehat{s}^*}{ds_L}$$

But $\frac{\partial EL^*}{\partial \widehat{s}} = 0$ from the first order condition for minimum loss. Thus

$$\begin{aligned} \frac{dEL^*}{ds_L} &= \frac{\partial EL^*}{\partial s_L} = \frac{\partial (EL^* - EL_{\mathbf{0}})}{\partial s_L} \\ &= (1-\omega)\, M_0 \left[\int_{\rho_0}^{\rho_1} \frac{\partial L_1(s_L)}{\partial s_L} \frac{d\rho}{\Delta_\rho} + \int_{\rho_L}^{\rho_0} \frac{\partial L_2(s_L)}{\partial s_L} \frac{d\rho}{\Delta_\rho} \right] \end{aligned}$$

From (A.3), $s_c(\rho) = (\rho_0/\rho)\widehat{s}$. Using this, for $\rho \in [\rho_0, \rho_1]$,

$$\begin{aligned} \frac{\partial L_1(s_L)}{\partial s_L} &= -\frac{\widehat{s}}{(\widehat{s} - s_L)} \left[\left(\frac{1}{s_L} - \frac{1}{s_c(\rho)} \right) - \int_{s_L}^{s_c(\rho)} \left(\frac{1}{s} - \frac{1}{s_c(\rho)} \right) \frac{ds}{(\widehat{s} - s_L)} \right] \\ &< -\frac{\widehat{s}}{(\widehat{s} - s_L)} \left[\left(\frac{1}{s_L} - \frac{1}{s_c(\rho)} \right) - \int_{s_L}^{\widehat{s}} \left(\frac{1}{s} - \frac{1}{s_c(\rho)} \right) \frac{ds}{(\widehat{s} - s_L)} \right] \\ &= -\frac{\widehat{s}}{(\widehat{s} - s_L)} \int_{s_L}^{\widehat{s}} \left(\frac{1}{s_L} - \frac{1}{s} \right) \frac{ds}{(\widehat{s} - s_L)} < 0, \end{aligned}$$

where the second step follows from the fact that for $\rho \in [\rho_0, \rho_1]$, $\widehat{s} > s_c(\rho)$. Next,

$$\begin{aligned} \frac{\partial L_2(s_L)}{\partial s_L} &= -\frac{1}{(\widehat{s} - s_L)} \left[\left(\frac{\widehat{s}}{s_L} - 1 \right) - \int_{s_L}^{\widehat{s}} \left(\frac{\widehat{s}}{s} - 1 \right) \frac{ds}{(\widehat{s} - s_L)} \right] \\ &= -\frac{1}{(\widehat{s} - s_L)} \int_{s_L}^{\widehat{s}} \left(\frac{\widehat{s}}{s_L} - \frac{\widehat{s}}{s} \right) \frac{ds}{(\widehat{s} - s_L)} < 0. \end{aligned}$$

Using the above two derivatives, we see that

$$\frac{dEL^*}{ds_L} < 0. \tag{A.13}$$

Similarly,

$$\frac{dEL^*}{ds_H} = \omega\, (1 - M_0) \left[\int_{\rho_2}^{\rho_0} \frac{\partial L_3(s_H)}{\partial s_H} \frac{d\rho}{\Delta_\rho} + \int_{\rho_L}^{\rho_2} \frac{\partial L_4(s_H)}{\partial s_H} \frac{d\rho}{\Delta_\rho} \right]$$

Now,

$$\frac{\partial L_3(s_H)}{\partial s_H} = -\widehat{s} \int_{\widehat{s}}^{s_c(\rho)} \left(\frac{1}{s_c(\rho)} - \frac{1}{s} \right) \frac{ds}{(s_H - \widehat{s})^2} > 0,$$

where the last step follows from the fact that for $\rho \in [\rho_2, \rho_0]$, $\widehat{s} < s_c(\rho)$. Finally,

$$\begin{aligned} \frac{\partial L_4(s_H)}{\partial s_H} &= \frac{1}{(s_H - \widehat{s})} \left[\left(1 - \frac{\widehat{s}}{s_H} \right) - \int_{\widehat{s}}^{s_H} \left(1 - \frac{\widehat{s}}{s} \right) \frac{ds}{(s_H - \widehat{s})} \right] \\ &= \frac{1}{(s_H - \widehat{s})} \int_{\widehat{s}}^{s_H} \left(\frac{\widehat{s}}{s} - \frac{\widehat{s}}{s_H} \right) \frac{ds}{(s_H - \widehat{s})} > 0. \end{aligned}$$

Using the above two derivatives, we see that $\dfrac{dEL^*}{ds_H} > 0$. From this and A.13, we can conclude that as s_H falls and s_L increases (i.e. uncertainty about s decreases), the optimal value of expected loss falls.

This proves that the expected loss (and therefore distortion) is increasing in the uncertainty about s. Next, we need to show that the distortion increases beyond the unregulated distortion, so that no regulation becomes optimal.

As $s_H \to +\infty$, either (a) $\widehat{s}$ increases with s_H so that $\widehat{s}/s_H$ remains finite, or (b) $\widehat{s}/s_H \to 0$. It can be easily checked (by integrating and taking limits) that in case (a), the third term in the expression for EL given by (A.7) goes to $+\infty$. Thus the distortion exceeds unregulated distortion. Clearly, therefore, there is a critical value of s_H such that beyond that critical value, no regulation is optimal. In case (b), the fourth term in the same expression goes to $\omega \int_{\rho_2}^{\rho_0} (1 - \frac{\rho}{\rho_0}) \frac{d\rho}{\Delta_\rho}$, while the fifth term goes to $\omega \int_{\rho_L}^{\rho_2} \frac{d\rho}{\Delta_\rho}$. Using these, $(EL^* - EL_{\mathbf{0}}) \to \overline{L}$ where

$$\overline{L} \;\geq\; \omega \left[\int_{\rho_2}^{\rho_0} \left(1 - \frac{\rho}{\rho_0}\right) \frac{d\rho}{\Delta_\rho} + \int_{\rho_L}^{\rho_2} \frac{d\rho}{\Delta_\rho} \right] \;>\; \omega \int_{\rho_L}^{\rho_0} \left(1 - \frac{\rho}{\rho_0}\right) \frac{d\rho}{\Delta_\rho}$$

But the last expression is the unregulated distortion, given by (3.2). Thus as the uncertainty about s increases, at some point the distortion exceeds the unregulated distortion, making no regulation optimal.

Part (b)

Let $\widetilde{s}$ denote the true value of s. As uncertainty about s vanishes, $s_H \longrightarrow \widetilde{s}$, and $s_L \longrightarrow \widetilde{s}$. Of course, $\widehat{s} \longrightarrow \widetilde{s}$. From equations (A.1) and (A.2), ρ_1 and ρ_2 both go to ρ_0. From the expression for expected loss given by (A.7), we see that $EL \longrightarrow (1-w) \int_{\rho_0}^{\rho_H} \left(\frac{\rho}{\rho_0} - 1\right) \frac{d\rho}{\Delta_\rho}$, which is the regulatory benchmark loss $EL_{\mathbf{0}}$ (given by (3.1)). $\|$

A.3 Proof of Proposition 2

Part (a)

$(EL^* - EL_{\mathbf{0}})$ is given by (A.8). From (A.9) and (A.10), for $\rho \in [\rho_0, \rho_1]$, $L_1(s_L) < L_2(s_L)$ where the inequality follows from the fact that for $\rho \in [\rho_0, \rho_1]$, both $s_c(\rho) < \widehat{s}$ and $\frac{\rho_0}{\rho} < 1$. Next, from (A.3), $\rho/\rho_0 = \widehat{s}/s_c(\rho)$. Now, $\widehat{s}/s_c(\rho) > \widehat{s}/s$ for $s > s_c(\rho)$. Therefore,

$$\int_{s_c(\rho)}^{s_H} \left(1 - \frac{\rho}{\rho_0}\right) \frac{ds}{\Delta_s} < \int_{s_c(\rho)}^{s_H} \left(1 - \frac{\widehat{s}}{s}\right) \frac{ds}{\Delta_s} \tag{A.14}$$

Using this, from (A.11) and (A.12), $L_3(s_H) < L_4(s_H)$.

Using $L_1(\cdot) < L_2(\cdot)$ and $L_3(\cdot) < L_4(\cdot)$ in (A.8),

$$\begin{aligned}(EL^* - EL_{\mathbf{0}}) &< (1-\omega)\, M_0 \left(\frac{\rho_1 - \rho_L}{\rho_H - \rho_L}\right) L_2(s_L) + \omega\,(1 - M_0) \left(\frac{\rho_0 - \rho_L}{\rho_H - \rho_L}\right) L_4(s_H) \\ &\leq (1-\omega) \int_{s_L}^{\widehat{s}} \left(\frac{\widehat{s}}{s} - 1\right) \frac{ds}{\Delta_s} + \omega \int_{\widehat{s}}^{s_H} \left(1 - \frac{\widehat{s}}{s}\right) \frac{ds}{\Delta_s},\end{aligned}$$

where the last step follows by substituting the values of $L_2(\cdot)$ and $L_4(\cdot)$ and the fact that the ratios appearing before $L_2(\cdot)$ and $L_4(\cdot)$ in the first line are both less than 1. The final expression on the right hand side of the equation above is an upper bound to the distortion under ex ante regulation, and is clearly independent of the distribution of ρ.

Part (b)

Next, we calculate the expected regulatory loss when the regulator has full information on ρ. For any $\rho > \rho_0$, it is optimal to not regulate, and the loss coincides with the benchmark loss. For any given $\rho \in [\rho_L, \rho_0]$, the expected regulatory loss is given by:

$$EL(\rho) = (1-\omega) \int_{s_L}^{\widehat{s}} \left(\frac{\widehat{s}}{s} - 1\right) \frac{ds}{\Delta_s} + \omega \left[\int_{\widehat{s}}^{s_c(\rho)} \left(1 - \frac{\widehat{s}}{s}\right) \frac{ds}{\Delta_s} + \int_{s_c(\rho)}^{s_H} \left(1 - \frac{\rho}{\rho_0}\right) \frac{ds}{\Delta_s}\right]$$

Clearly, this is strictly positive for any $\widehat{s} \in [s_L, s_H]$. Now, for any given $\rho \in [\rho_L, \rho_0]$, the benchmark loss is 0. Thus for any such ρ, $EL(\rho) - EL_{\mathbf{0}}(\rho) > 0$. Finally, note that the expected loss if ρ is known cannot be greater than the expected loss if ρ is not known. Thus the expression on the right hand side of (A.15) is the lower bound to distortion, which is strictly positive for all types for which regulation is binding.

To check that the lower bound is not higher than the upper bound, note that the for any $\rho \in [\rho_L, \rho_1]$, $s_c(\rho) = s_H$ and the lower bound coincides with the upper bound. For any $\rho \in (\rho_1, \rho_0)$, $\widehat{s} < s_c(\rho) < s_H$, and from (A.14), the lower bound is strictly lower than the upper bound.

Part (c)

As the uncertainty about s vanishes, so that s_H, s_L and $\widehat{s}$ coincide at the realized value of s, the upper bound clearly goes to zero. Finally, we need to show that as uncertainty about s increases, the lower bound becomes the unregulated distortion, making regulation useless. Let B_1 and B_2 denote the coefficients of $(1-\omega)$ and ω (respectively) in the expression for $EL(\rho)$ given by equation (A.15). B_1 can be written as $B_1 = \dfrac{\ln \widehat{s} - \ln s_L}{s_H/\widehat{s} - s_L/\widehat{s}} - \dfrac{\widehat{s}/s_H - s_L/s_H}{1 - s_L/s_H}$. Next, B_2

can be written as

$$B_2 = \frac{\widehat{s}}{s_H - s_L}\left(\frac{\rho_0}{\rho} - 1 - \ln\left(\frac{\rho_0}{\rho}\right)\right) + \left(1 - \frac{\rho}{\rho_0}\right)\left(\frac{s_H - (\rho_0/\rho)\widehat{s}}{s_H - s_L}\right)$$

As $s_H \to +\infty$, either (a) $\widehat{s}$ increases with s_H so that $\widehat{s}/s_H$ remains finite, or (b) $\widehat{s}/s_H \to 0$. In case (a), B_2 is finite, but $B_1 \to \infty$. Thus the expected loss exceeds unregulated loss (and therefore the distortion exceeds unregulated distortion), making regulation is useless. In case (b), $B_1 \to 0$, $B_2 \to (1 - \rho/\rho_0)$, thus $EL(\rho) \to \omega(1 - \rho/\rho_0)$. But for any $\rho \in [\rho_L, \rho_0]$, this is exactly the loss when no regulation is applied. Now, for any given $\rho \in [\rho_L, \rho_0]$, the benchmark loss is 0, thus this is also the unregulated distortion. Thus as the uncertainty about s increases, the lower bound becomes the unregulated distortion, making regulation useless.

A.4 Proof of Lemma 2

First, in the absence of penalties, we know from definition (2.4) that the socially optimal risk coincides with the optimal risk of type ρ_0. With positive penalties, each type takes a lower risk compared to the case of no penalties, and thus the socially optimal risk now coincides with the optimal risk of some type less risk averse than type ρ_0. Thus $\widehat{\rho} \leqslant \rho_0$.

Second, suppose the equilibrium penalties are such that the optimal risk of type ρ_L is lower than the socially optimal risk $\frac{K}{C}$. Then expected overprotection loss can be reduced by imposing a lesser punishment (which raises optimal risk for all ρ) without incurring any safety loss. Contradiction. Thus $\widehat{\rho} \geqslant \rho_L$.

Finally, as penalties rise, optimal risk of each type falls, and therefore $\widehat{\rho}$ falls. In equilibrium, each level of penalty is associated with a unique $\widehat{\rho} \in [\rho_L, \rho_0]$, and thus $\widehat{\rho}$ can be used to index the level of penalties.‖

A.5 Proof of Proposition 3

The proof proceeds through the following lemma.

Lemma 5 *The optimal choice of portfolio under ex post penalties (denoted by α_P^* is given by the general form*

$$\alpha_P^* s = \lambda(\rho, \widehat{\rho})\frac{\beta}{\rho}, \tag{A.15}$$

where $0 < \lambda(\rho, \widehat{\rho}) < 1$.

Proof: From section 5.2, the expected utility of the manager under an ex post regime is $Eu(W) = E\left[-\exp\left(-\rho(1+\widetilde{X})\right)\right]$ where $\widetilde{X}$ is given by equation (5.1). From this, we obtain:

$$\begin{aligned} Eu(W) &= \int_{-\infty}^{-\theta K} \left[-\exp\left(-\rho\left[1+\widetilde{V}-f(-\widetilde{V}-\theta K)\right]\right)\right]\psi(\widetilde{V})\,d\widetilde{V} \\ &\quad + \int_{-\theta K}^{\infty} \left[-\exp\left(-\rho\left[1+\widetilde{V}\right]\right)\right]\psi(\widetilde{V})\,d\widetilde{V} \end{aligned}$$

Now, the distribution of $\widetilde{V}$, and therefore, from above, the expected utility, depends on α or s only through the standard deviation of the distribution given by $\sigma = \alpha s$. Thus, as in the unregulated case, it is still true that the optimal portfolio satisfies $\alpha s =$ constant. Of course, the optimal portfolio under a penalty is lower than the unregulated optimum. In general, under ex post penalties, the chosen optimal risk exposure is a fraction λ of the unregulated optimum, where λ depends only upon the type ρ of the bank and the penalty function $f(\cdot)$ (and not on s). We argued in section 5.2 that $\widehat{\rho}$ indexes the penalties. Thus λ is a function of ρ and $\widehat{\rho}$.‖

Further, since by definition, the socially optimal risk $\sigma_{\mathbf{0}} = K/C$ coincides with the optimal risk of type $\widehat{\rho}$, we have

$$\lambda(\widehat{\rho},\widehat{\rho})\frac{\beta}{\widehat{\rho}} = \frac{K}{C}. \tag{A.16}$$

For any $s \in [s_L, s_H]$, the expected loss is given by

$$EL(s) = (1-\omega)\int_{\widehat{\rho}}^{\rho_H}\left(\frac{K/C}{\alpha_{\mathrm{P}}^{*}s}-1\right)\frac{d\rho}{\Delta_\rho} + \omega\int_{\rho_L}^{\widehat{\rho}}\left(1-\frac{K/C}{\alpha_{\mathrm{P}}^{*}s}\right)\frac{d\rho}{\Delta_\rho}$$

(2.4) defines ρ_0 such that $\beta/\rho_0 = K/C$. Using this, and (A.16), for any s, the expected loss can be rewritten as:

$$EL(s) = (1-\omega)\int_{\widehat{\rho}}^{\rho_H}\left(\frac{\rho}{\lambda(\rho,\widehat{\rho})\rho_0}-1\right)\frac{d\rho}{\Delta_\rho} + \omega\int_{\rho_L}^{\widehat{\rho}}\left(1-\frac{\rho}{\lambda(\rho,\widehat{\rho})\rho_0}\right)\frac{d\rho}{\Delta_\rho} \tag{A.17}$$

The right hand side is independent of s - thus $EL(s)$ does not depend on s, i.e. regulatory loss is a constant function of s. Therefore, the expected loss is

$$EL = EL(s). \tag{A.18}$$

This completes the proof.‖

A.6 Proof of Lemma 3

Define a function $H(\rho, \widehat{\rho})$ as follows:

$$H(\rho, \widehat{\rho}) \equiv \left(\frac{\rho}{\lambda(\rho, \widehat{\rho})\rho_0} - 1 \right). \tag{A.19}$$

The following lemma is useful. Let H_1 and H_2 denote the partial derivative of H with respect to the first and second arguments respectively.

Lemma 6 (a) $H(\widehat{\rho}, \widehat{\rho}) = 0$, (b) $H_1(\rho, \widehat{\rho}) > 0$, (c) $H_2(\rho, \widehat{\rho}) < 0$.

Proof: (a) From equations (A.16) and (2.4), $\lambda(\widehat{\rho}, \widehat{\rho})(\beta/\widehat{\rho}) = K/C = \beta/\rho_0$. The result is immediate. **(b)** Given that the same penalty function applies to all types, it must be that the risk adopted by types less risk averse is higher. Thus it must be that the optimal risk is decreasing in ρ. Therefore, $\lambda(\rho, \widehat{\rho})/\rho$ is decreasing in ρ, which implies that $H_1 > 0$. **(c)** A higher $\widehat{\rho}$ implies a lower penalty. Thus $\lambda(\rho, \widehat{\rho})$ is increasing in $\widehat{\rho}$, which implies $H_2 < 0$.‖

From equation A.18, the expected loss is given by equation A.17. Using (A.19), the expected loss can be rewritten as:

$$EL = (1-\omega) \int_{\widehat{\rho}}^{\rho_H} H(\rho, \widehat{\rho}) \frac{d\rho}{\Delta_\rho} - \omega \int_{\rho_L}^{\widehat{\rho}} H(\rho, \widehat{\rho}) \frac{d\rho}{\Delta_\rho} \tag{A.20}$$

From this, and using $H(\widehat{\rho}, \widehat{\rho}) = 0$ from lemma 6,

$$\frac{\partial EL}{\partial \widehat{\rho}} = (1-\omega) \int_{\widehat{\rho}}^{\rho_H} H_2(\rho, \widehat{\rho}) \frac{d\rho}{\Delta_\rho} - \omega \int_{\rho_L}^{\widehat{\rho}} H_2(\rho, \widehat{\rho}) \frac{d\rho}{\Delta_\rho} \tag{A.21}$$

The proof proceeds as follows. In **step 1** we work out the optimal $\widehat{\rho}$ under a specific assumption, then in **step 2**, we relax the assumption and derive the optimal $\widehat{\rho}$.

Step 1 Let us assume that $\rho_0 = \rho_H$ - i.e. all managerial types $\rho \in [\rho_L, \rho_H]$ are less risk averse than the socially optimal risk aversion ρ_0.

Let $\widehat{\widehat{\rho}}^*$ be the optimal value of $\widehat{\rho}$ in this case.

From equation (A.21), for any $\omega \in (0, 1)$, at $\widehat{\rho} = \rho_L$,

$$\left. \frac{\partial EL}{\partial \widehat{\rho}} \right|_{\widehat{\rho}=\rho_L} = (1-\omega) \int_{\rho_L}^{\rho_H} H_2(\rho, \rho_L) \frac{d\rho}{\Delta_\rho} < 0$$

and at $\widehat{\rho} = \rho_H$,

$$\left.\frac{\partial EL}{\partial \widehat{\rho}}\right|_{\widehat{\rho}=\rho_H} = -\omega \int_{\rho_L}^{\rho_H} H_2(\rho, \rho_H) \frac{d\rho}{\Delta_\rho} > 0.$$

Thus for all $\omega \in (0,1)$ there is an interior minimum. At this interior minimum, $\frac{\partial EL}{\partial \widehat{\rho}} = 0$, and $\frac{\partial^2 EL}{\partial \widehat{\rho}^2} > 0$. From the first order condition, differentiating with respect to ω, at $\widehat{\rho} = \widehat{\widehat{\rho}}^*$, $\left(\frac{\partial^2 EL}{\partial \widehat{\rho}^2}\right)\left(\frac{\partial \widehat{\widehat{\rho}}^*}{\partial \omega}\right) = \int_{\rho_L}^{\rho_H} H_2(\rho, \widehat{\rho}) \frac{d\rho}{\Delta_\rho}$ From the second order condition, the first term on the left hand side is positive. From lemma 6, the right hand side is negative. Thus $\frac{\partial \widehat{\widehat{\rho}}^*}{\partial \omega} < 0$.

Step 2 Under the assumption in step 1 that $\rho_0 = \rho_H$, $\widehat{\rho}$ can take any value between ρ_L and ρ_H. Now we relax the assumption in step 1 and assume, as usual, $\rho_L < \rho_0 < \rho_H$. Thus the optimal value of $\widehat{\rho}$ is now given by $\widehat{\rho}^* = \min(\widehat{\widehat{\rho}}^*, \rho_0)$.

We showed above that $\widehat{\widehat{\rho}}^*$ is decreasing in ω. Further, from (A.21), at $\omega = 0$, the first order condition is satisfied at $\widehat{\rho} = \rho_H$. Thus, clearly, there is $0 < \omega_* < 1$ such that at $\omega = \omega_*$, $\widehat{\widehat{\rho}}^* = \rho_0$. Thus $\widehat{\rho}^* = \rho_0$ for $\omega \leq \omega_*$ and for $\omega > \omega_*$, $\rho_L \leq \widehat{\rho}^* < \rho_0$.

Finally, from (A.21), in the limit as $\omega \to 1$, the first order condition is satisfied at $\widehat{\rho} = \rho_L$. This completes the proof.‖

A.7 Proof of Proposition 4

Part (a)

Information about ρ is captured by the interval $[\rho_L, \rho_H]$. A smaller interval implies better information about ρ. With any change in ρ_L or ρ_H, $\widehat{\rho}$ is adjusted optimally. For the proof, we will adjust $\widehat{\rho}$ so that the relative probability mass attached to the support of safety loss and overprotection loss remains constant. Under this adjustment, we show that as the interval shrinks, EL falls. Since the EL under optimal adjustment cannot be higher, the optimized loss, denoted EL^*, falls as well.

Let M_1 denote the ratio $\dfrac{\rho_H - \widehat{\rho}}{\rho_H - \rho_L}$. As ρ_L and/or ρ_H changes, we change $\widehat{\rho}$ to keep M_1 constant. Note that since M_1 is constant, so is $1 - M_1 = (\widehat{\rho} - \rho_L)/(\rho_H - \rho_L)$.

The expected regulatory loss under ex post regulation is given by equation (A.20). This can be rewritten as follows:

$$EL = (1-\omega)\, M_1 \int_{\widehat{\rho}}^{\rho_H} H(\rho, \widehat{\rho}\,) \frac{d\rho}{(\rho_H - \widehat{\rho}\,)} \;-\; \omega\,(1 - M_1) \int_{\rho_L}^{\widehat{\rho}} H(\rho, \widehat{\rho}\,) \frac{d\rho}{(\widehat{\rho} - \rho_L)} \qquad \text{(A.22)}$$

Recall that EL^* denotes the minimized value of EL.

$$\frac{dEL^*}{d\rho_L} = \frac{\partial EL^*}{\partial \rho_L} + \frac{\partial EL^*}{\partial \widehat{\rho}} \frac{d\widehat{\rho}}{d\rho_L} \tag{A.23}$$

But $\frac{\partial EL^*}{\partial \widehat{\rho}} = 0$ by definition of EL^*. Thus, with M_1 held constant,

$$\frac{dEL^*}{d\rho_L} = \omega \frac{(1-M_1)}{\widehat{\rho}-\rho_L} \left[H(\rho_L, \widehat{\rho}) - \int_{\rho_L}^{\widehat{\rho}} H(\rho, \widehat{\rho}) \frac{d\rho}{(\widehat{\rho}-\rho_L)} \right] < 0,$$

where the last step follows from the fact that $H(\cdot,\cdot)$ is increasing in ρ (from lemma 6 in section **A**.6).

Next, with M_1 held constant, $\frac{dEL^*}{d\rho_H} = \frac{\partial EL^*}{\partial \rho_H} + \frac{\partial EL^*}{\partial \widehat{\rho}} \frac{d\widehat{\rho}}{d\rho_H}$. But $\frac{\partial EL^*}{\partial \widehat{\rho}} = 0$ by definition of EL^*. Thus, with M_1 held constant,

$$\frac{dEL^*}{d\rho_H} = (1-\omega) \frac{M_1}{(\rho_H - \widehat{\rho})} \left[H(\rho_H, \widehat{\rho}) - \int_{\widehat{\rho}}^{\rho_H} H(\rho, \widehat{\rho}) \frac{d\rho}{(\rho_H - \widehat{\rho})} \right] > 0,$$

where the last step again follows from the fact that $H(\cdot,\cdot)$ is increasing in ρ.

Thus $\frac{dEL}{d\rho_L} < 0$, and $\frac{dEL}{d\rho_H} > 0$. We can conclude that as ρ_H falls and ρ_L increases (i.e. uncertainty about ρ decreases), the optimal value of the expected loss falls. Thus the expected loss is increasing in uncertainty about ρ.

Part (b)

Let $\widetilde{\rho}$ denote the true value of ρ. As uncertainty about ρ vanishes, $\rho_H \longrightarrow \widetilde{\rho}$, and $\rho_L \longrightarrow \widetilde{\rho}$. Now, if $\widetilde{\rho} > \rho_0$, it is optimal to not impose regulation. Thus for $\widetilde{\rho} \in (\rho_0, \rho_H]$, $\widehat{\rho} = \rho_0$. On the other hand, for $\widetilde{\rho} < \rho_0$, as uncertainty about ρ vanishes, penalties would be adjusted optimally so that the optimal risk of type $\widetilde{\rho}$ coincides with socially optimal risk. Thus the loss from any $\rho < \rho_0$ vanishes, and the only loss remaining is the overprotection loss from types $\rho > \rho_0$. But this is exactly the regulatory benchmark loss.‖

A.8 Proof of proposition 5

Before we can prove the statements in parts (a)-(c), we need to derive the expected loss and its derivatives with respect to $\widehat{s}$ and $\widehat{\rho}$.

the expression for expected loss is very similar to the loss derived under ex ante regulation (the loss is given by (A.7)). The only change is that the optimal risk for type ρ has changed from β/ρ to $\lambda(\rho, \widehat{\rho})\beta/\rho$ (equation A.15).

Define ρ_* and ρ_{**} analogously to ρ_1 and ρ_2 defined by equations (A.1) and (A.2) respectively. ρ_* is implicitly given by

$$\lambda(\rho_*, \widehat{\rho})\frac{\beta}{\rho_*} = \overline{\alpha} s_L, \tag{A.24}$$

where $\overline{\alpha} = K/(C\widehat{s})$. Further, if the left hand side is greater than the right hand side for all ρ (so that $\lambda(\rho_*, \widehat{\rho})\beta/\rho_* > \overline{\alpha} s_L$), then $\rho_* = \rho_H$.

Similarly, ρ_{**} is implicitly given by $\lambda(\rho_{**}, \widehat{\rho})\dfrac{\beta}{\rho_{**}} = \overline{\alpha} s_H$. Further, if the left hand side is less than the right hand side for all ρ (so that $\lambda(\rho_L, \widehat{\rho})\beta/\rho_L < \overline{\alpha} s_H$), then $\rho_{**} = \rho_L$.

Finally, for any $\rho \in [\rho_{**}, \rho_*]$, define $s_*(\rho)$ analogously to $s_c(\rho)$ defined by equation A.3. Let $s_*(\rho)$ be the value of s at which the optimal portfolio hyperbola of type ρ under punishment cuts the ex ante portfolio constraint. Thus $\overline{\alpha} s_*(\rho) = \lambda(\rho, \widehat{\rho})(\beta/\rho)$. From equation 4.1, we know that $\overline{\alpha} = K/(C\widehat{s})$. Using this, and the expression for ρ_0 from (2.4), we can rewrite the above as:

$$s_*(\rho) = \lambda(\rho, \widehat{\rho})\frac{\rho_0}{\rho}\widehat{s}. \tag{A.25}$$

For any $\rho < \rho_{**}$, we define $s_*(\rho) = s_H$, and for any $\rho > \rho_*$, we define $s_*(\rho) = s_L$.

The expression for expected loss is obtained from equation (A.7) by substituting ρ_* for ρ_1, ρ_{**} for ρ_2, $s_*(\rho)$ for $s_c(\rho)$, and $\lambda(\rho, \widehat{\rho})\beta/\rho$ for β/ρ.

Carrying out the above substitutions, and using definition (A.19), the expected loss is given by

$$\begin{aligned} EL \;=\; & (1-\omega)\int_{\widehat{\rho}}^{\rho_*} OL(\rho, \widehat{s}, \widehat{\rho})\frac{d\rho}{\Delta_\rho} + (1-\omega)\int_{\rho_*}^{\rho_H} H(\rho, \widehat{\rho})\frac{d\rho}{\Delta_\rho} \\ & + (1-\omega)\int_{\rho_L}^{\widehat{\rho}}\int_{s_L}^{\widehat{s}}\left(\frac{\widehat{s}}{s} - 1\right)\frac{ds}{\Delta_s}\frac{d\rho}{\Delta_\rho} + \omega\int_{\rho_{**}}^{\widehat{\rho}} SL(\rho, \widehat{s}, \widehat{\rho})\frac{d\rho}{\Delta_\rho} \\ & + \omega\int_{\rho_L}^{\rho_{**}}\int_{\widehat{s}}^{s_H}\left(1 - \frac{\widehat{s}}{s}\right)\frac{ds}{\Delta_s}\frac{d\rho}{\Delta_\rho}, \end{aligned}$$

where

$$\begin{aligned} OL(\rho, \widehat{s}, \widehat{\rho}) \;&=\; \int_{s_L}^{s_*(\rho)}\left(\frac{\widehat{s}}{s} - 1\right)\frac{ds}{\Delta_s} \;+\; \int_{s_*(\rho)}^{s_H} H(\rho, \widehat{\rho})\frac{ds}{\Delta_s}. \\ SL(\rho, \widehat{s}, \widehat{\rho}) \;&=\; \int_{\widehat{s}}^{s_*(\rho)}\left(1 - \frac{\widehat{s}}{s}\right)\frac{ds}{\Delta_s} \;-\; \int_{s_*(\rho)}^{s_H} H(\rho, \widehat{\rho})\frac{ds}{\Delta_s}. \end{aligned}$$

The derivative of expected loss with respect to $\widehat{s}$ is given by $\dfrac{\partial EL}{\partial \rho_*}\dfrac{\partial \rho_*}{\partial \widehat{s}} + \dfrac{\partial EL}{\partial \rho_{**}}\dfrac{\partial \rho_{**}}{\partial \widehat{s}} + \dfrac{\partial EL}{\partial s_*(\rho)}\dfrac{\partial s_*(\rho)}{\partial \widehat{s}} + \dfrac{\partial EL}{\partial \widehat{s}}$. It is straightforward to verify that $\dfrac{\partial EL}{\partial \rho_*} = \dfrac{\partial EL}{\partial \rho_{**}} = \dfrac{\partial EL}{\partial s_*(\rho)} = 0$. (The proof of this is

exactly analogous to that of lemma 4, so we omit the details here.) Thus the derivative of expected loss with respect to $\widehat{s}$ is given by:

$$\begin{aligned}\frac{\partial EL}{\partial \widehat{s}} &= (1-\omega)\int_{\widehat{\rho}}^{\rho_*}\left(\int_{s_L}^{s_*(\rho)}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}+(1-\omega)\int_{\rho_L}^{\widehat{\rho}}\left(\int_{s_L}^{\widehat{s}}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}\\ &\quad -\omega\int_{\rho_{**}}^{\widehat{\rho}}\left(\int_{\widehat{s}}^{s_*(\rho)}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}-\omega\int_{\rho_L}^{\rho_{**}}\left(\int_{\widehat{s}}^{s_H}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}. \qquad \text{(A.26)}\end{aligned}$$

Next, let us obtain the derivative of expected loss with respect to $\widehat{\rho}$.

From (A.16) and (2.4), $\lambda(\widehat{\rho},\widehat{\rho})(\beta/\widehat{\rho}) = \beta/\rho_0$. Thus, from equation A.25, $s_*(\widehat{\rho}) = \widehat{s}$ for $\rho \in (\rho_{**},\rho_*)$. Using this, and $H(\widehat{\rho},\widehat{\rho}) = 0$ from lemma 6 in section **A**.6,

$$\begin{aligned}\frac{\partial EL}{\partial \widehat{\rho}} &= (1-\omega)\int_{\widehat{\rho}}^{\rho_*}\left(\frac{s_H-s_*(\rho)}{\Delta_s}\right)H_2(\rho,\widehat{\rho})\ \frac{d\rho}{\Delta_\rho}+(1-\omega)\int_{\rho_*}^{\rho_H}H_2(\rho,\widehat{\rho})\ \frac{d\rho}{\Delta_\rho}\\ &\quad -\omega\int_{\rho_{**}}^{\widehat{\rho}}\left(\frac{s_H-s_*(\rho)}{\Delta_s}\right)H_2(\rho,\widehat{\rho})\ \frac{d\rho}{\Delta_\rho}, \qquad \text{(A.27)}\end{aligned}$$

where $H_2(\cdot,\cdot)$ denotes the derivative of $H(\cdot,\cdot)$ with respect to the second argument. From lemma 6 in section **A**.6, $H_2(\cdot,\cdot) < 0$. We are now ready to prove parts (a)-(c).

Part (a)

The weakest possible ex ante constraint is given by $\widehat{s} = s_L$. Let us evaluate the derivative given by (A.26) at $\widehat{s} = s_L$. From equation (A.16), for type $\widehat{\rho}$, $\lambda(\widehat{\rho},\widehat{\rho})\dfrac{\beta}{\widehat{\rho}} = \dfrac{K}{C}$. Using $\overline{\alpha} = K/(C\widehat{s})$ and $\widehat{s} = s_L$ in equation (A.24), $\lambda(\rho_*,\widehat{\rho})\dfrac{\beta}{\rho_*} = \dfrac{K}{C}$. From the previous two equations, $\frac{\lambda(\widehat{\rho},\widehat{\rho})}{\widehat{\rho}} = \frac{\lambda(\rho_*,\widehat{\rho})}{\rho_*}$, which implies that at $\widehat{s} = s_L$, $\rho_* = \widehat{\rho}$. Using this, and given any $\widehat{\rho} \in (\rho_L,\rho_0]$,

$$\left.\frac{\partial EL}{\partial \widehat{s}}\right|_{\widehat{s}=s_L} = -\omega\int_{\rho_{**}}^{\widehat{\rho}}\left(\int_{s_L}^{s_*(\rho)}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho} - \omega\int_{\rho_L}^{\rho_{**}}\left(\int_{s_L}^{s_H}\frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho}$$

Now, $\rho_L \leqslant \rho_{**} < \rho_H$, and for all $\rho < \rho_H$, $s_*(\rho) > s_L$. Thus the integrals in both terms are non-negative and the integral in the first term is strictly positive. Therefore, $\left.\dfrac{\partial EL}{\partial \widehat{s}}\right|_{\widehat{s}=s_L} < 0$. Thus any optimal regime must involve $\widehat{s} > s_L$.

Part (b)

Given $\gamma = 1$, the weakest possible ex post regime sets $\widehat{\rho} = \rho_H$. Now, any optimal regulatory regime must involve setting at least the weakest possible ex post incentive regime - i.e. an ex

post regime that sets $\widehat{\rho} = \rho_H$. Let us evaluate the derivative given by (A.27) at $\widehat{\rho} = \rho_H$. If $\gamma = 1$ and $\widehat{\rho} = \rho_H$, then for all ρ, $\lambda(\rho_*, \widehat{\rho})\beta/\rho_* > \overline{\alpha} s_L$. Thus $\rho_* = \rho_H$. Using this,

$$\left.\frac{\partial EL}{\partial \widehat{\rho}}\right|_{\widehat{\rho}=\rho_H} = -\,\omega \int_{\rho_{**}}^{\rho_H} \left(\frac{s_H - s_*(\rho)}{\Delta_s}\right) H_2(\rho, \rho_H) \; \frac{d\rho}{\Delta_\rho} \;\; > \;\; 0,$$

where the last step follows from the fact that from lemma 6 (in section **A**.6), $H_2(\cdot, \cdot) < 0$. Thus any optimal regime must involve $\widehat{\rho} < \rho_H$.

Part (c)

Finally, suppose $\gamma < 1$. In this case, the weakest possible ex post regime involves imposing no penalties - so that $\widehat{\rho} = \rho_0$.

The procedure used in part (b) does not work any more, but note that as ω goes to 1, the first and third terms in the expression for $\frac{\partial EL}{\partial \widehat{\rho}}$ (given by equation (A.27)) go to 0, while the second term is positive and bounded away from 0. Thus for ω high enough, $\frac{\partial EL}{\partial \widehat{\rho}} > 0$. Further, as ω goes to 0, the second term in the expression for $\frac{\partial EL}{\partial \widehat{\rho}}$ goes to 0, while the first and third terms are negative and bounded away from 0. Thus for ω low enough, $\frac{\partial EL}{\partial \widehat{\rho}} < 0$. Thus there is $\omega_{**} \in (0, 1)$ such that for $\omega = \omega_{**}$, $\frac{\partial EL}{\partial \widehat{\rho}} = 0$.

Thus for $\omega \leq \omega_{**}$, it is optimal to not impose a penalty (i.e. $\widehat{\rho}^{\,*} = \rho_0$), and for $\omega > \omega_*$, $\widehat{\rho}^{\,*} < \rho_0$. This completes the proof.‖

A.9 Proof of proposition 6

First, we show that both regimes are effective in reducing safety loss. As ω approaches 1, only safety loss matters to the regulator (safety-first regulation) - and by applying either stringent ex ante constraints or harsh penalties, safety loss can be eliminated. This is shown below.

Lemma 7 *Under both regimes,* $\lim_{\omega \to 1} EL = 0$.

Proof: The optimized value of the loss under ex ante regulation (denoted by EL^*_{EA}) is given by substituting $\widehat{s}^{\,*}$ for $\widehat{s}$ in (A.7). The term $(1 - \omega)$ appears in the first three terms. Thus as $\omega \to 1$, the first three terms go to 0. Next, from lemma 1, we know that $\widehat{s}^{\,*}$ is increasing in ω, and $\lim_{\omega \to 1} \widehat{s}^{\,*} = s_H$. Further, from (A.2), as $\widehat{s} \to s_H$, $\rho_2 \to \rho_0$. It can then be easily seen from (A.7), that as $\omega \to 1$, the coefficients of ω in the last two terms go to 0. Thus $\lim_{\omega \to 1} EL^*_{\text{EA}} = 0$.

Second, The optimized value of loss under ex post regulation (denoted by EL^*_{EP}) is given by substituting $\widehat{\rho}^{\,*}$ for $\widehat{\rho}$ in (A.20). As $\omega \to 1$, $\widehat{\rho}^{\,*} \to \rho_L$ and thus the second term in the expression for expected loss goes to 0, and since $(1-\omega)$ goes to 0, the first term goes to 0 as well. Thus $\lim_{\omega\to 1} EL^*_{\text{EP}} = 0.\|$

Now, consider ω close to 1. From the lemma above, we know that both regimes are good at reducing safety loss, and thus optimal regulation must involve either $\widehat{\rho}$ close to ρ_L or $\widehat{s}$ close to s_H or both.

For $\widehat{\rho}$ close to ρ_L, $\rho_{**} = \rho_L$. For any $\omega < 1$, and any $\widehat{s} < s_H$, from equation (A.26),

$$\lim_{\widehat{\rho}\to\rho_L} \frac{\partial EL}{\partial \widehat{s}} = (1-\omega)\int_{\rho_L}^{\rho_*}\left(\int_{s_L}^{s_*(\rho)} \frac{1}{s}\frac{ds}{\Delta_s}\right)\frac{d\rho}{\Delta_\rho} \quad > \quad 0.$$

Thus if $\widehat{\rho} \to \rho_L$, the optimal $\widehat{s} \to s_L$.

Similarly, as $\widehat{s} \to s_H$, $\rho_{**} \to \rho_0$ and $s_*(\rho) \to s_H$ for $\rho \in [\rho_L, \widehat{\rho}]$. For any $\omega < 1$, and any $\widehat{\rho} > \rho_L$, from equation (A.27),

$$\lim_{\widehat{s}\to s_H} \frac{\partial EL}{\partial \widehat{\rho}} \quad = \quad (1-\omega)\left(\int_{\widehat{\rho}}^{\rho_*}\left(\frac{s_H - s_*(\rho)}{\Delta_s}\right) H_2(\rho,\widehat{\rho})\,\frac{d\rho}{\Delta_\rho} + \int_{\rho_*}^{\rho_H} H_2(\rho,\widehat{\rho})\,\frac{d\rho}{\Delta_\rho}\right) \;<\; 0,$$

where the last inequality follows from the fact that, from lemma 6 (in section **A**.6), $H_2 < 0$. Thus if $\widehat{s} \to s_H$, the optimal $\widehat{\rho} \to \rho_0$.

The above shows that we must either have (a) $\widehat{\rho}$ close to ρ_L (strong ex post regulation) and $\widehat{s}$ close to s_L (weak ex ante regulation), or (b) $\widehat{\rho}$ close to ρ_0 (weak ex post regulation) and $\widehat{s}$ close to s_H (strong ex ante regulation).

Finally, since both regimes can reduce safety loss to any arbitrarily low level, the choice between scenarios (a) and (b) depends on how much overprotection loss each regime generates to reduce safety loss to any given level. The regime that is applied in a strong form must be the one that generates less overprotection loss when applied in a strong form.$\|$

A.10 Proof of proposition 7

If the bank has no private information on s, its optimal portfolio decision is a constant function of s. In other words, a bank of type ρ simply decides an optimal α, denoted by $\alpha^*(\rho)$. Note that now it is no longer possible to implement a portfolio choice of $\alpha(s)$ such that $\alpha(s)s = K/C$. The best now achievable (regulatory second best) is $\alpha(\rho)$ such that $\alpha(\rho)E(s) = K/C$.

Let ρ_{SB} be the type such that $\alpha^*(\rho_{\text{SB}})\ E(s) = K/C$. Given any K, the expected regulatory loss is then given by

$$(1-\omega)\int_{\rho_{\text{SB}}}^{\rho_H}\left(\frac{K/C}{\alpha^*(\rho)\ E(s)}-1\right)\frac{d\rho}{\Delta_\rho}+\omega\int_{\rho_L}^{\rho_{\text{SB}}}\left(1-\frac{K/C}{\alpha^*(\rho)\ E(s)}\right)\frac{d\rho}{\Delta_\rho}$$

The new regulatory benchmark is given by the loss when $\rho \in [\rho_L, \rho_{\text{SB}}]$ adopt $\alpha^*(\rho_{\text{SB}})$ and types $\rho > \rho_{\text{SB}}$ adopt their optimal portfolio. Thus the benchmark loss is given by the first term in the expression for expected loss hand above.

By imposing $\overline{\alpha} = \alpha^*(\rho_{\text{SB}})$, ex ante regulation can attain the regulatory benchmark. Thus ex ante regulation is optimal.

It remains to show that ex post regulation is not optimal. Under ex post regulation, given a particular penalty scheme, the optimal portfolio of some type $\widehat{\rho}$ coincides with $\alpha^*(\rho_{\text{SB}})$. Given positive penalties, $\rho_L \leqslant \widehat{\rho} < \rho_{\text{SB}}$. Thus all types $\rho \in [\rho_L, \widehat{\rho})$ generate greater loss than type ρ_{SB}. Under benchmark loss, all these types generate the same loss as ρ_{SB}. Further, types $\rho \in (\widehat{\rho}, \rho_H]$ take lower risks than in the case of no penalty, and generate a greater overprotection loss compared to the case of no penalty, and the case of no penalty corresponds with the benchmark loss for these types. Thus the expected regulatory loss is above the benchmark loss, and ex post regulation is suboptimal.‖

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