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**Monetary Policy in the Presence of  
Imperfect Observability of the  
Objectives of Central Bankers**

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# Monetary Policy in the Presence Of Imperfect Observability Of The Objectives Of Central Bankers

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## **Abstract**

The paper presents a theoretical model for analysis of the imperfect observability of central bank preferences by the private sector on the decisions taken by the monetary authority, and therefore on the inflation rate. It examines in particular the connection which, in the presence of a time inconsistency problem, arises between the observability of the monetary institution's goals and its equilibrium strategies. The model yields innovative results from the technical and economic points of view. From the technical point of view, the study of equilibrium strategies in a simple signalling model allows derivation of the equilibrium outcomes of a monetary policy game already examined by D'Amato and Pistoiesi (1996) and Sibert (2002), without the restrictions that those authors impose on the basis of the types of monetary institution. It is thus possible to identify the conditions on the model's parameters under which a pure separating equilibrium arises, and the conditions under which there instead exists a 'hybrid' equilibrium in which some types of Central Bankers adopt separating strategies (Vickers 1986; D'Amato and Pistoiesi 1996; Sibert 2002) while others adopt pooling strategies similar to those studied by Backus and Driffill (1985). From an economic point of view, the paper shows a number of relations that arise, in equilibrium, between the degree of observability and transparency of the Central Banker's goals and the inflation rate set by the Central Banker.

JEL Classification numbers: E31; E58; E61

Keywords: Monetary Policy, Central Bank.

## 1 Introduction

As shown by the literature on the institutional designs of monetary policy, there are various arguments in favour of delegating monetary policy to agents independent of the government, within an institutional framework which not only guarantees that independence but also imposes precise objectives and constraints on the operations of a Central Banker. The advantages of delegation to an independent banker more inflation-averse than the government, or ‘society’, arise from the desire to prevent the stagflation problems usually associated with time inconsistency and the prevalence of a ‘discretion equilibrium’. However, this approach has been criticised in light of considerations concerning the distribution of the benefits and costs of inflation within a society made up of heterogeneous agents. For example, from the positive point of view, Adam Posen (1995) argues that what matters in the long period in the struggle against inflation is the presence of a strong financial sector with an interest in price stability and willing and able to induce the monetary policy authorities to pursue that objective. From this standpoint, the independence of the Central Bank and its inflation-control targets are unconnected with the interests of society as a whole, and the convenience of a particular institutional arrangement derives not from normative considerations but from the political influence of a particular interest group. This point of view has been analysed further in the literature that connects the design of monetary institutions with the field of ‘political economics’ (see Persson and Tabellini, 2000; Drazen, 2000; Alesina and Gatti, 1995).

Yet the view of institutions as produced by ‘special interests politics’ is not necessarily antithetical to the credibility-based approach (Kydland and Prescott, 1977; Barro and Gordon, 1983; Rogoff, 1985; Lohmann, 1992). It is possible in fact to hypothesise that even in the presence of heterogeneous agents (debtors versus creditors, financial system versus firms’ system, agents whose wages are fixed by long-period contracts versus agents whose remuneration is fixed in flexible markets), the time-inconsistency problem and its solution of strategic delegation (commitment) continues to play a significant role in the design of monetary policy institutions.

An aspect which requires investigation is the problem of the observability of commitment and the possible role assumed by the heterogeneity of agents in this regard. As well known, the results obtained in the literature on commitment and observability (Bonanno, 1992; Bagwell, 1995; Fershtman and Kalai, 1997) show that the benefits deriving to a player from constraining

its actions to a commitment via delegation are essentially connected to the likelihood that the commitment will be observed by the other players.<sup>1</sup>

This paper analyses a simple time inconsistency model in which institutional reforms – i.e. the process of strategic delegation to an independent Banker constrained in its decisions to monetary stability goals – come about in the presence of agents heterogeneous in their ability, or willingness, to invest resources in order to observe and understand those reforms. In the presence of agents not all of which are able to observe and understand the effects of reform of the monetary institutions on the behaviour of the Central Banker in terms of its decisions, monetary policy is implemented in the presence of the Banker’s partial private information, with respect to private agents, about monetary policy objectives.

The aim of the paper is therefore to analyse the effect of the imperfect observability of Central Bank preferences by the private sector on the decisions taken by the monetary authority and therefore on the inflation rate. In the case of a Banker’s multiperiod appointment and imperfect observability of its goals on the part of private agents, the latter, to the extent that they are not directly informed about those goals, may infer them from the decisions taken by the Central Banker.

The paper considers in particular the case of an economy in which a Central Banker is appointed for two periods and information about its objectives is distributed in society as follows: an exogenous fraction  $p$  of agents has incentives and the capacity to invest resources in the observation of the objectives assigned to the independent institution and take account of that information when formulating their expectations about the inflation rate. A fraction  $1 - p$  of agents do not have this information and therefore in the first period formulate expectations about the inflation rate according to their a priori beliefs about the type of Banker, while in the second period their expectations are conditioned by the Banker’s behaviour in the first period.

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<sup>1</sup>This in fact is the result obtained by Fershtman and Kalai (1997) and it contrasts with the one obtained by Bagwell (1995). In this model, there is a small probability of error in the player’s information about his opponent’s actions. This small probability makes the information useless, and the result of the game is identical to that in a game without information. By contrast, in Fershtman and Kalia’s (1997) model there is a small probability that the player is informed about his opponent. In this case, this small probability drastically affects the outcome of the game: that is, unlike in Bagwell’s model, advantages are obtained from the introduction of a commitment via delegation even when the delegation is not perfectly observable.

The monetary policy game describe therefore divides into two periods and has the characteristics typical of signalling games. The temporal sequence of the decisions is as follows: at time  $t = 0$  monetary policy is delegated to an independent Banker with particular preferences regarding the trade-off between inflation and output (unemployment) and which remains in office for two periods. At time  $t = 1$ , given the agents' expectations, the Central Banker fixes an inflation rate considering that future expectations (at time  $t = 2$ ) of fraction  $1 - p$  (those that do not observe the BC's preferences at  $t = 0$ ) will be conditioned by observation of the current decision of the Central Banker. At time  $t = 2$ , the Central Banker decides the money supply, the macroeconomic results are achieved and the game concludes.

Introducing the hypothesis of heterogeneity in the private sector, in a standard monetary policy model (Vickers, 1986), extended to the case of a continuous support of types (Mailath, 1987) enables one to analyse the behaviour of a just-appointed policymaker, or of institutions created ex novo at a given point in time and in a particular place.<sup>2</sup> The presence in the economy of private agents with a different degree of observability with respect to the Central Banker's preferences gives rise to a private learning process which generates more or less strong incentives for the just-appointed policymaker, or the new institutions, to acquire a reputation.

With respect to the previous literature (D'Amato and Pistoiese, 1996; Sibert, 2002), therefore, the contribution of this paper from a technical point of view consists in two extensions: first, it considers the case of agent heterogeneity in terms of the information set; second, it studies and resolves the game without imposing restrictions on the support of the distribution of the agents' beliefs across the possible types of Central Banker. These extensions will enable me to characterize the nature and properties of a 'semi-separating' equilibrium (also called 'partial pooling'), and also to study the relationships between monetary policy strategies in the presence of time inconsistency and the degree of transparency and observability of the objectives of a Central Banker in a simple economy.

The results of the model show that, in the case of an economy characterized by a large number of private agents, about which the Central Banker possesses private information, the hypothesis of a continuum of Central

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<sup>2</sup>An extension of signalling models for monetary policy to the case of a continuous support of types is also present in D'Amato and Pistoiese (1996) and Sibert (2002). Both models substantially confirm Vickers' result (separating equilibrium) that the presence of wet types disciplines the behaviour of tough types.

Bankers signifies that it is not economically convenient for the ‘tougher’ types (strongly inflation-averse), within a given support, to separate themselves from each other, because this gives rise to high signalling costs<sup>3</sup>, whereas it is instead optimal for them join with the type which sets a nil inflation rate. Instead, each ‘wet’ type (those most sensitive to the level of economic activity) in the support, obviously excluding the worst possible type, separates from the one closest to it.<sup>4</sup>

Vice versa, as the private sector’s uncertainty about the true identity of the monetary decreases (economy characterized by a large number of agents informed about the type of Central Banker), the model converges on the results obtained by D’Amato and Pistoiesi (1996) and Sibert (2002): that is, it produces a complete separating equilibrium, so that once again the presence of wet types disciplines the behaviour of tough types.

The introduction of partial observability also has implications for the strategic delegation that would emerge in equilibrium. The presence of signalling costs substantially alters the incentives for commitment and depends on the degree of observability. This study will not concern itself with deriving the government’s optimal strategy. It restricts itself to pointing out that, in the case where the economy is characterized by situations in which observation by the private sector is close to being perfect, introducing a commitment mechanism into monetary policy may be a way to evade signalling costs. Otherwise, there may arise commitment costs sufficiently high to induce the government to reduce the use of delegation and assign inflation-control targets less stringent than in the case of high transparency.

This interpretation may help explain why in the industrialized countries, where institutional conditions are such to guarantee the high observability of the policy-makers’ preferences, monetary policy is often characterized by the presence of legal rules, or by institutions created by constitutional laws (Central Banks independent from political power, pegging of the exchange rate to a strong currency, etc.) which essentially serve the purpose of specifying a partial commitment mechanism.

The rest of the paper is organized as follows. Section 2 presents the monetary policy game, while in Section 3 the model is solved for a separating equilibrium and a pooling equilibrium in the monetary policy game. A number of simulations are performed in Section 4 in order to analyse, other

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<sup>3</sup>In monetary policy models the cost of the signal is represented by the fall of the employment level below the natural rate.

<sup>4</sup>In substance, the equilibrium obtained represents a ‘partial pooling’.

conditions remaining equal, the impact on the inflation rate of an increase in the observability of the Central Banker's preferences. Section 5 sets out the conclusions.

## 2 A monetary policy signalling game

The starting-point is the monetary authority's pay-off functions – which are widely used in the literature and were proposed for the first time by Barro-Gordon (1983, a) – in which a welfare function is assigned for each period  $t$ .

The welfare of the Central Banker (CB) in period  $t$ , depends on actual inflation and unexpected inflation. We thus have the following functions:

$$W_t = w(\pi_t; \pi_t - \pi_t^e), \quad (1)$$

where  $\pi_t$  represents the inflation rate in each period  $t$ , which by hypothesis is completely controlled by the monetary authority, and  $\pi_t^e$  is the inflation expected by the private sector. It is therefore assumed that positive inflation is a cost for the Central Banker, whilst so-called “*surprise inflation*”,  $\pi_t - \pi_t^e$ , gives rise to welfare<sup>5</sup>.

To simplify the analysis, it is also assumed that the welfare function is linear in unemployment, rather than quadratic as in Barro and Gordon (1983, a). This assumption, which has been made by Barro and Gordon (1983, b) and Vickers (1986), is attractive for our model, in that it ensures that the Banker has a dominant strategy during the final period in which it is in office, and therefore enables the expected inflation of the previous period to be considered a constant. Hence  $W_t$  has the following functional form:

$$W_t = -\frac{1}{2}\pi_t^2 + \alpha(\pi_t - \pi_t^e) \quad (2)$$

where  $\alpha \geq 0$  is the parameter of preferences that the Banker assigns to unexpected inflation, and therefore to the trade-off between inflation and unemployment, and  $t = 1; 2$ .

This parameter  $\alpha$  is only in part private information for the Central Banker. In fact, from the point of view of private agents, for the fraction of

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<sup>5</sup> “*Surprise inflation*” is a benefit for the monetary authority because it pushes unemployment below the natural rate, which is assumed to be too high. Alternatively, the benefit can be interpreted in terms of advantages connected with the presence of debt stock issued in nominal terms or short-period rigidity of the tax system. In the rest of the analysis, the model will use mainly the first of these interpretations.

agents equal to  $1 - p$ , it is distributed assuming a priori ‘beliefs’ about the distribution function  $F(\alpha)$  defined in a continuous support  $\alpha \in [a, A^{BC}]$ ;<sup>6</sup> whilst the fraction  $p$  of private agents possesses complete information about the type of Banker because it observes  $\alpha$  at the moment when the Central Banker is appointed and then behaves rationally.

There are various reasons for considering the private sector as divided into two parts. The principal one relates to Posen’s hypothesis that a crucial role is played in society by net nominal creditors (for example, banks and financial companies) which have a greater interest in gathering information about the preferences of a just-appointed policymaker.<sup>7</sup>

The temporal structure of events is the usual sequence of monetary policy games<sup>8</sup>: the private agents form their expectations about the inflation rate of the first period,  $\pi_1^e$  (first stage), the Central Banker observes  $\pi_1^e$  and then chooses the actual inflation of the first period  $\pi_1$  (second stage). In the first stage of the second period, the private agents form their inflation expectation, which will be:  $\pi_2^e = pE(\pi_2|\alpha) + (1 - p)E(\pi_2|\pi_1)$ , that is, the fraction of informed private agents formulates expectations about the inflation rate which are conditioned by the type of Banker  $\alpha$  observed when it takes office, whilst the fraction of private agents uninformed at the moment when the Banker takes office will form its expectations on the basis of what it has observed in the previous period, i.e.  $\pi_1$ , but not  $\alpha$ . Finally, in the second stage of the second period, the Central Banker observes  $\pi_2^e$ , and chooses the actual inflation.

In formulating their expectations conditioned by their information set, the private agents minimize the cost of inflation rate forecast error by means of the following quadratic function of the *pay-offs* for each period  $t$ :

$$u_t = -(\pi_t - \pi_t^e)^2 \quad (3)$$

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<sup>6</sup>Both the distribution function and the support constitute knowledge shared by the players and can be arbitrarily defined. However, it can be shown that the conditions for the existence of a separating equilibrium, under the hypothesis of a continuum of types, can restrict the support (Mailath, 1987).

<sup>7</sup>Alternatively, the division of the private sector into two parts can be seen as the presence in the economy of a fraction of agents  $p$  which sign long-period nominal contracts in anticipation of monetary policy decisions, and which is necessarily interested in knowing the Central Banker’s preferences when it takes office.

<sup>8</sup>See: Backus e Driffill (1985), Vickers (1986), D’Amato e Pistoresi (1996), Sibert (2002).

The solution concept used to determine the optimal strategies is the perfect Bayes-Nash equilibrium. Thus obtained is a pair of inflation rates played by the Central Banker,  $s = \{\pi_1(\alpha), \pi_2(\alpha)\}$ , and a pair of expected inflation rates played by each private agent,  $e = \{\pi_1^e, \pi_2^e\}$ , where  $\pi_1^e = pE(\pi_1|\alpha) + (1-p)E(\pi_1)$  represents the inflation expected in the first period by the entire private sector. It is calculated on the basis of the distribution function of the a priori beliefs of the uninformed private agents and is instead conditioned by the type  $\alpha$  of Central Banker for the informed private agents.  $\pi_2^e = pE(\pi_2|\alpha) + (1-p)E(\pi_2|\pi_1)$  represents the inflation in the second period, which for the informed agents once again depends on the type  $\alpha$ , whilst for the uninformed agents it depends on their inference of the Central Banker's preferences based on the inflation rate observed in the first period.

Finally, for the sake of simplicity, it is assumed that the players do not discount the future, so that the function of the *pay-offs* across the entire time horizon of the game is:  $W = W_1 + W_2$ .<sup>9</sup>

### 3 Equilibrium of the monetary policy signalling game

The BC's optimal strategy can be determined by solving the game backwards. In fact, the equilibrium value of the inflation rate for the second stage of the second period can be easily obtained by solving the following programme

$$\max_{\pi_2} W_2 = -\frac{1}{2}\pi_2^2 + \alpha(\pi_2 - \pi_2^e) \quad (4)$$

the first-order condition of which is:

$$\pi_2 = \alpha \quad (5)$$

In other words, in period  $t = 2$ , there is no future to consider, and the Central Banker's dominant strategy consequently corresponds to (5).

At this point in the first stage of the second period, the informed fraction of private agents  $p$  fixes the expected inflation rate on the basis of the type  $\alpha$  that it has observed at the moment of the appointment:  $E(\pi_2|\alpha) = \alpha$ , while

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<sup>9</sup>Both Vickers (1986) and D'Amato and Pistoiesi (1996) solve the game considering this hypothesis.

the uninformed private agents  $(1 - p)$  anticipate the choice of inflation rate  $\pi_2 = \alpha$  and minimize the forecasting error by formulating their modified expectations on the basis of *Bayes'rule*, that is, on the basis of the inflation rate observed in the first period:  $E(\pi_2|\pi_1) = E(\alpha|\pi_1) = \hat{\alpha}$ .

The  $(1 - p)$  fraction of agents conjecture that in the first period the Central Banker's strategy is  $\pi_1 = \phi(\alpha)$ <sup>10</sup> in the case of a separating equilibrium; instead,  $\pi_1 = 0$  in the case of a *pooling* equilibrium<sup>11</sup>. From this it follows that the  $1 - p$  agents infer the "type" of BC from their observation of  $\pi_1$ .

In general  $\hat{\alpha} = \frac{\alpha^s}{2} \forall \pi_1 = 0$  and  $\hat{\alpha} = \phi^{-1}(\pi_1) \forall 0 < \pi_1 \leq A^{BC}$ , where  $\alpha^s$  is such that  $\phi(\alpha^s) = 0$ : that is, it is the point which divides the *separating* region from the *pooling* region. Hence, given the a priori "beliefs" about the "types" that the BC may assume, the expectations regime for the entire private sector can be summarized as follows::

$$\begin{cases} \pi_2^e = p\alpha + (1 - p)\hat{\alpha} & \text{for } 0 < \pi_1 \leq A^{BC} \\ \pi_2^e = p\alpha + (1 - p)\frac{\alpha^s}{2} & \text{for } \pi_1 = 0 \end{cases} \quad (6)$$

**Lemma 1** *If the Central Banker's strategy in the first period is partial pooling, in the second period the private sector's expectations regime and the BC's strategy are respectively defined by (6) and (5).*

As regards the first period, expected inflation is given by:

$$\pi_1^e = pE(\pi_1|\alpha) + (1 - p)E(\pi_1) \quad (7)$$

In this case, too, it is necessary to consider the difference between the *pooling* region and the *separation* region.

In the *pooling* case  $0 \leq \alpha \leq \alpha^s$  from which it follows that  $\phi(\alpha) = 0$ . Consequently, the expected inflation will be:

$$\pi_1^e = p0 + (1 - p) \int_0^{\alpha^s} 0f(\alpha)d\alpha = 0 \quad (8)$$

Instead, in the *separating* case  $\alpha^s \leq \alpha \leq A^{BC}$ , and therefore  $\pi_1 = \phi(\alpha)$ .Consequently, the expected inflation will be:

$$\pi_1^e = p\phi(\alpha) + (1 - p) \int_{\alpha^s}^{A^{BC}} \phi(\alpha)f(\alpha)d\alpha = p\phi(\alpha) + (1 - p)\bar{\phi} \quad (9)$$

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<sup>10</sup>This is a biunivocal function: that is, there is a one-to-one correspondence on the space of the types and that of the strategies (see Mailath, 1987).

<sup>11</sup>This is a constant function of the space of the types and that of the strategies

and in the entire support one therefore has:

$$\pi_1^e = p\phi(\alpha) + (1-p) \int_0^{A^{BC}} \phi(\alpha)f(\alpha)d\alpha = p\phi(\alpha) + (1-p)\bar{\phi} \quad (10)$$

In the second stage of the first period, the Central Banker regards expectations as given, but takes account of learning by the  $(1-p)$  fraction of private agents when they formulate expectations about inflation in the second period, and may therefore have an incentive to signal its type.<sup>12</sup>

Given the equilibrium result of the second period, the reduced form of the Central Banker's *pay-off* function in the first period is:

$$\max_{\pi_1} W_1 = -\frac{1}{2}\pi_1^2 + \alpha(\pi_1 - \pi_1^e) - \frac{\alpha^2}{2} + \alpha[\alpha - p\alpha - (1-p)\hat{\alpha}] \quad (11)$$

Using the definition of the strategy for separating equilibrium,  $\pi_1 = \phi(\alpha)$ , and the Bayes' rule,  $\hat{\alpha} = \phi^{-1}(\pi_1)$ ,<sup>13</sup> one obtains the first-order condition for the Central Banker:

$$-\phi + \alpha - \alpha(1-p)\frac{d\hat{\alpha}}{d\phi} = 0 \quad (12)$$

which, evaluated in equilibrium, at the point  $\alpha = \hat{\alpha}$  for  $\alpha^s \leq \alpha \leq A^{BC}$ , determines the following first-order homogeneous, non-linear differential equation, the solution of which satisfies a separating equilibrium:

$$\frac{d\phi}{d\alpha} = \frac{\alpha(1-p)}{\alpha - \phi} \quad (13)$$

whilst for  $0 \leq \alpha \leq \alpha^s$

$$\phi(\alpha) = 0 \quad (14)$$

Equation (13) can be solved analytically by separating the variables and integrating to obtain the implicit function that describes the separation strategy. Selection of the relevant branch of the implicit function, as the unique equilibrium of the separation strategy, can be done using Mailath's second condition (1987, p. 1353)<sup>14</sup> and an initial condition. The initial

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<sup>12</sup>The two-period monetary policy game under examination therefore comprises a single-period signalling game.

<sup>13</sup>This expresses the second-period beliefs of the uninformed private agents about the type of Central Banker.

<sup>14</sup>The condition of type monotonicity enables one to establish that the relevant part of the solution of the differential equation has positive slope (for details see the section in the Appendix).

value condition is given by the equality  $\phi(A^{BC}) = \pi_1(A^{BC}) = A^{BC}$ : that is, in a separating equilibrium, the worst possible type of Central Banker has no incentive to signal itself and fixes the inflation rate at the same level as it would do if the game were with complete information.

In particular, given  $\frac{d\phi}{d\alpha} = \frac{1-p}{1-\frac{\phi}{\alpha}}$  and setting  $\frac{\phi}{\alpha} = x$ , it follows that:  $\phi = \alpha x$ , then:  $d\phi = \alpha dx + x d\alpha$ ; Consequently, after some algebraic steps it is possible to rewrite (13) in the following separable form:

$$\frac{1}{\alpha} d\alpha = \frac{1-x}{1-p-x+x^2} dx \quad (15)$$

Integrating both the members of (15) yields:

$$\int \frac{1}{\alpha} d\alpha = \int \frac{1-x}{1-p-x+x^2} dx \quad (16)$$

The integral of the right-hand side of (16) admits three solutions according to whether  $\Delta$  is greater than, less than, or equal to zero.<sup>15</sup>

The first is with  $\Delta < 0$  that is  $\Delta = 1 - 4(1-p) < 0$ ; from which it follows that  $p < \frac{3}{4}$ , and therefore that (16) is equal to:

$$\log \alpha = -\frac{1}{2} \log |x^2 - x + (1-p)| + \frac{1}{2} \frac{2}{\sqrt{3-4p}} \arctan \frac{2x-1}{3-4p} + c_1 \quad (17)$$

The second solution of the integral of (16) is with  $\Delta = 0$  from which it follows that  $p = \frac{3}{4}$ , and therefore that (16) is equal to:

$$\log \alpha = -\frac{1}{2} \left(x - \frac{1}{2}\right)^{-1} - \log \left|x - \frac{1}{2}\right| + c_0 \quad (18)$$

Finally, the third solution of the integral of (16) is with  $\Delta > 0$  from which it follows  $p > \frac{3}{4}$  and that:

$$\log \alpha = -\frac{1}{2} \log |x^2 - x + (1-p)| + \frac{1}{2} \frac{1}{\sqrt{4p-3}} \log \left| \frac{2x-1-\sqrt{4p-3}}{2x-1+\sqrt{4p-3}} \right| + c_2 \quad (19)$$

Consequently by eliminating the auxiliary variable  $x$  it obtain the final form implicit solution, in the original variables of (16) that also admits three solutions.

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<sup>15</sup>The Appendix shows in what intervals the integral equation (16) is verified.

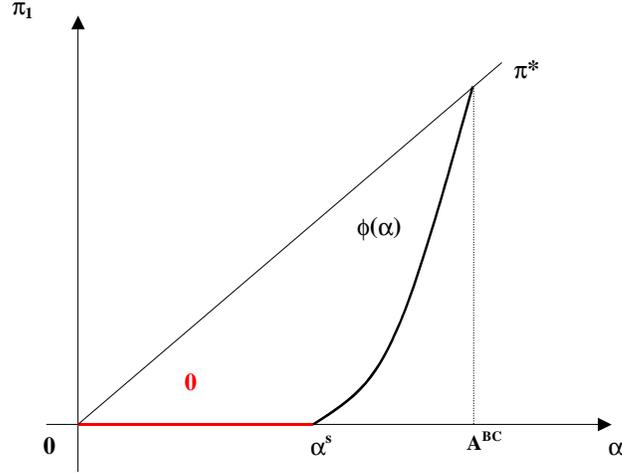


Figure 1: Case with  $p < \frac{3}{4}$ : *Partial Pooling* Equilibrium.  
 $\pi^*$ : inflation level with complete information.  
 $\phi(\alpha)$ : inflation level in separating equilibrium.

The first, with  $p < \frac{3}{4}$ , is

$$\log \alpha = -\frac{1}{2} \log \left| \left( \frac{\phi}{\alpha} \right)^2 - \left( \frac{\phi}{\alpha} \right) + (1-p) \right| + \frac{1}{\sqrt{3-4p}} \arctan \frac{2\frac{\phi}{\alpha} - 1}{3-4p} + c_1 \quad (20)$$

The second with  $p = \frac{3}{4}$  is:

$$\log \alpha = -\frac{1}{2} \left( \frac{\phi}{\alpha} - \frac{1}{2} \right)^{-1} - \log \left| \frac{\phi}{\alpha} - \frac{1}{2} \right| + c_0 \quad (21)$$

The third with  $p > \frac{3}{4}$ , is:

$$\begin{aligned} \log \alpha = & -\frac{1}{2} \log \left| \left( \frac{\phi}{\alpha} \right)^2 - \left( \frac{\phi}{\alpha} \right) + (1-p) \right| + \\ & \frac{1}{2} \frac{1}{\sqrt{4p-3}} \log \left| \frac{2\frac{\phi}{\alpha} - 1 - \sqrt{4p-3}}{2\frac{\phi}{\alpha} - 1 + \sqrt{4p-3}} \right| + c_2 \end{aligned} \quad (22)$$

where  $c_1$ ,  $c_0$  and  $c_2$  respectively represent the integration constants that can be obtained by setting the initial value condition  $\phi(A^{BC}) = A^{BC}$ .

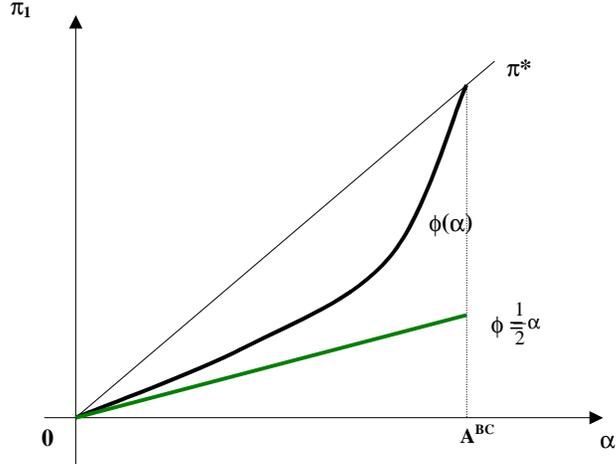


Figure 2: Case with  $p = \frac{3}{4}$ : Complete *Separating Equilibrium*.  
 $\pi^*$ : level with complete information.  
 $\phi(\alpha)$ : inflation level in separating equilibrium.

These solutions are such that the relevant branch of the separating strategy is an increasing convex function whereby  $0 \leq \phi(\alpha) \leq \alpha$ ,  $\phi(A^{BC}) = A^{BC}$ , and  $1 - p \leq \phi' < \infty$ .

Figures 1, 2 and 3 show, respectively for the various regimes of  $p$ , the relevant part of the contourplot of the solution of equation (13) in the signal-types space .

**Proposition 1** *If  $0 \leq p < 3/4$ , for an arbitrary support  $[a, A^{BC}]$  of  $F(\alpha)$ , there exists a partial pooling equilibrium. In the interval  $[\alpha^s, A^{BC}]$ , where  $\alpha^s$  is defined by  $\phi(\alpha^s) = 0$ , there exists a separating equilibrium which satisfies (13). The equilibrium strategies in this interval are therefore the following:  $s^s = \{\pi_1^s(\alpha), \pi_2^*(\alpha)\}$ ,  $e^s = \{\pi_1^e, \pi_2^e\}$ , where  $\pi_1^s(\alpha) = \phi(\alpha)$ ,  $\pi_2^*(\alpha) = \alpha$ ,  $\pi_1^e(\alpha) = p\phi(\alpha) + (1-p)E[\phi(\alpha)]$ ,  $\pi_2^e(\alpha) = pE(\pi_2 | \alpha) + (1-p)E(\pi_2 | \pi_1) = \alpha$ . In the interval  $[0, \alpha^s]$  there exists a pooling equilibrium which satisfied (14). The equilibrium strategies in this interval are therefore  $s^p = \{\pi_1^p(\alpha), \pi_2^*(\alpha)\}$ ,  $e^s = \{\pi_1^e, \pi_2^e\}$ ,  $\pi_1^p(\alpha) = 0$ ,  $\pi_2^*(\alpha) = \alpha$ ,  $\pi_1^e(\alpha) = 0$ ,  $\pi_2^e(\alpha) = p\alpha + (1-p)E(\alpha)$ , where  $E(\alpha)$  is defined as the expected value unconditioned by the distribution function of the a priori beliefs. .*

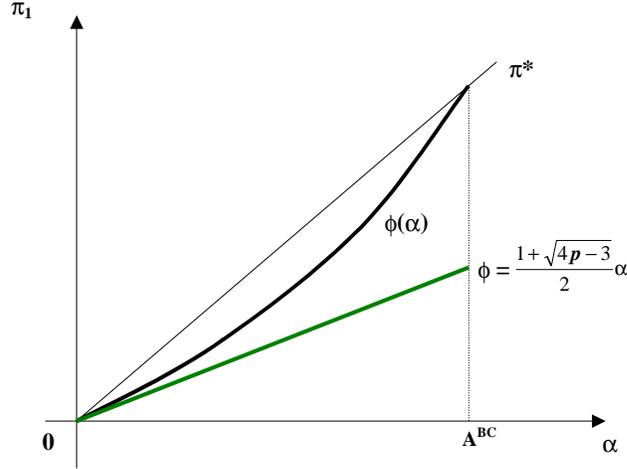


Figure 3: Case with  $p > \frac{3}{4}$  : Complete Separating Equilibrium.

$\pi^*$  : inflation level with complete information..

$\phi(\alpha)$  : inflation level in separating equilibrium.

In order to demonstrate that the separating equilibrium of Proposition 1 exists and is unique, one must consider both Mailath's (1987) regularity conditions and the second-order condition on (11) satisfied for  $\alpha > \alpha^s$  (see the Appendix for the derivation of these conditions). In the *pooling* case, the strategy expressed by (14) will instead be an equilibrium if there is no incentive for the monetary authority to deviate from that equilibrium, given specification of the out-of-equilibrium "beliefs". It must therefore be that:  $w^D(\alpha) < w^P(\alpha)$  for  $0 \leq \alpha \leq \alpha^s$ <sup>16</sup>(see the Appendix for the proof)..

The Central Banker's separating strategy is such that, in the first period, the "types" falling in the interval  $[\alpha^s, A^{BC}]$ , will choose a lower inflation rate than in the case of complete information, thus reducing the inflationary bias. In other words, the risk that the public may revise its beliefs downwards serves as a *commitment* mechanism for the monetary authority. The fraction of private agents  $(1 - p)$  – that is, those uninformed at the moment of the Central Banker's appointment – will anticipate this behaviour and in

<sup>16</sup>In this signalling game there may exist a pooling equilibrium for the entire support  $[a, A^{BC}]$ . This case is analysed in the Appendix.

the first period will set a lower level of expected inflation.

The separating equilibrium breaks down in the part of the interval where  $\phi(\alpha) \leq 0$  i.e. for  $\alpha \in [0, \alpha^s]$ <sup>17</sup>. For all the “types” of Central Banker comprised in this interval to separate themselves, they must play a negative inflation rate in the first period, which implies a result worse than the *pooling* case for  $\pi_1 = 0$ . In other words, the signalling effect, which reduces inflation in the first period, becomes too costly for tough types, that is, for  $0 \leq \alpha \leq \alpha^s$ .

Hence, the difference between the result obtained here and Vickers’ (1981) result is that in Vickers the pooling equilibrium comes about with a positive inflation rate in the period  $t = 1$ , and the player with “small”  $\alpha$  (the toughest type)<sup>18</sup> may separate itself by playing a lower, though still positive, inflation rate. In Vickers, deviating, choosing to separate, is always advantageous with respect to the pooling equilibrium. This does not happen in the model considered here, where a hybrid *pooling/separating* equilibrium is instead obtained.

One implication of the *partial pooling* equilibrium obtained is that there are “intermediate” types which signal themselves more, and which are therefore those with the widest gap between the actual inflation rate and the time-consistent inflation rate à la Barro-Gordon.

**Proposition 2** *If  $3/4 \leq p \leq 1$ , for an arbitrary support  $[a, A^{BC}]$  of  $F(\alpha)$ , in the interval  $[0, A^{BC}]$  there exists a complete separating equilibrium. The complete characterization of the separating equilibrium is given by the following strategies:  $s^s = \{\pi_1^s(\alpha), \pi_2^s(\alpha)\}$ ,  $e^s = \{\pi_1^e, \pi_2^e\}$ , where  $\pi_1^s(\alpha) = \phi(\alpha)$ <sup>19</sup>,  $\pi_2^s(\alpha) = \alpha$ ,  $\pi_1^e(\alpha) = p\phi(\alpha) + (1 - p)E[\phi(\alpha)]$ ,  $\pi_2^e(\alpha) = pE(\pi_2 | \alpha) + (1 - p)E(\pi_2 | \pi_1) = \alpha$ .*

In order to demonstrate that the separating equilibrium of Proposition 2 exists and is unique, it is once again necessary to bear in mind both Mailath’s (1987) regularity conditions and the second-order condition on (11), satisfied for  $\alpha > 0$ , both of which are derived in the Appendix.

The difference of the result presented in Proposition 1 is that, in this case, each type of Central Banker in the interval separates itself from the

<sup>17</sup>See the red segment in Figure 2-1.

<sup>18</sup>In Vickers’ model there are two types of BC: “tough” and “wet”.

<sup>19</sup>More specifically, if  $p = \frac{3}{4}$ ,  $\frac{1}{2}\alpha < \phi(\alpha) < \alpha$ . If  $p > \frac{3}{4}$ ,  $\frac{1+\sqrt{4p-3}}{2}\alpha < \phi(\alpha) < \alpha$ . See the Appendix for the proof.

one closest to it, unlike what happened previously<sup>20</sup>.

A further conclusion to be drawn from the analysis of the different information contexts in which the monetary policy game takes place (i.e. with  $p \leq \frac{3}{4}$ ), is that the closer one approaches the case of the perfect observability of the Central Banker's preferences  $\alpha$  (i.e. with increasingly higher values of  $p$ ), the more the marginal cost of the signal decreases<sup>21</sup>. This is ensured by both the monotonicity of  $\phi$  and the shift of  $\alpha^s$  towards the origin (compare in this regard Figures 1, 2 and 3).

In order to confirm this result, a comparative statics exercise to analyse, as the number increases in the economy of private agents which observe the Central Banker at the moment of its appointment, how the results of the equilibria determined may change. From an analytical point of view, this involves considering the total differential of the implicit solution of the differential equation (13):

$$d\phi F_\phi + dp F_p = 0 \rightarrow \frac{d\phi}{dp} = -\frac{F_p}{F_\phi} \quad (23)$$

Unfortunately, it is analytically not possible to determine either the sign of  $F_p$  or the sign of  $F_\phi$  unambiguously. Consequently, in the next section I shall perform some simulations to determine the impact of a variation  $p$  on the equilibrium determined.

## 4 Some simulations

This section reports some results obtained using simulations<sup>22</sup>, in which enable us to characterize the effects of a variation in the exogenous variable  $p$ : that is, the number of agents informed about the Baker's preferences at the moment of its appointment, in  $\frac{d\phi}{d\alpha} = \frac{\alpha(1-p)}{\alpha-\phi}$ .

Given that our model focuses on the case in which the Central Banker's preferences are only in part private information, we analyse the impact on the equilibrium inflation rate of an increase in the observability of the Banker's preferences for the private sector. In other words, from an analytical point of view, we analyse  $\frac{d\phi}{dp}$ .

Figure 4 illustrates a numerical simulation of equation (13), which shows the relevant part of the contour diagram of the equation, that is, the Central

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<sup>20</sup>In this case, for  $0 \leq \alpha \leq \alpha^s$ , the *pooling* equilibrium break down.

<sup>21</sup>This represents the marginal cost of setting the inflation rate below the optimal level of the complete information case, i.e. below  $\alpha$ .

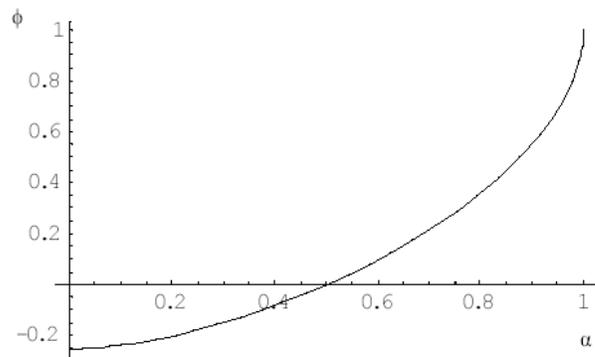


Figure 4: Simulation 1:  $p = 0.1$

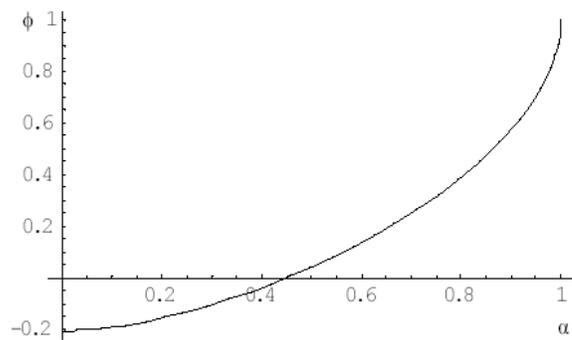


Figure 5: Simulation 2:  $p = 0.2$

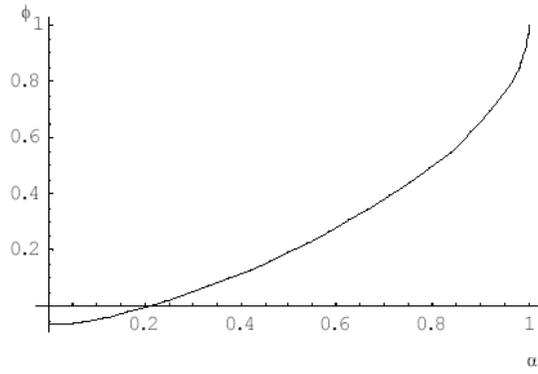


Figure 6: Simulation 3:  $p = 0.5$

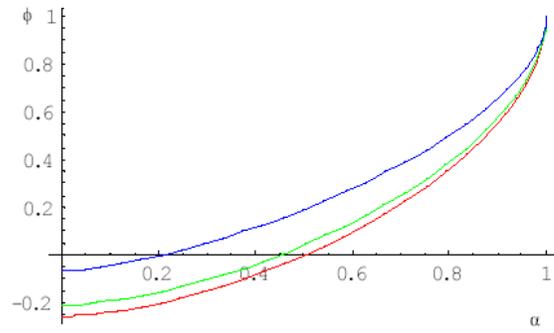


Figure 7: Comparison of simulations 1, 2 and 3

Banker's strategy. It will be noted that if  $p = 0.1$ , the separating strategy  $\phi(\alpha)$  cuts the x-axis at the point  $\alpha^s = 0.5$ .

Figure 5 shows, within the interval of the current parameters, the impact of an increase in observability ( $p = 0.2$ ). In this case,  $\alpha^s = 0.45$ .

Figure 6 shows the case of  $p = 0.5$ , and therefore  $\alpha^s = 0.2$ .

Finally, Figure 7 considers the three simulations simultaneously in a single graph. This suggests that  $\frac{d\phi}{dp} > 0$  and that  $\frac{d\alpha^s}{dp} \leq 0$ . That is to say, if within an economy, the number of agents informed at  $t = 0$  about the Banker's preferences increases, the Central Banker distorts less. Hence the inflation rate grows progressively higher than in the incomplete information case ( $p = 0$ ), so that the results presented in the previous section are confirmed.

## 5 Conclusions

The paper has presented an extension of the monetary policy models presented by D'Amato and Pistoiesi (1996) and by Sibert (2002) to the case in which the private sector is not homogeneous in regard to the observability of the Central Banker's preferences. More precisely, it has assumed that there are two different fractions of private agents: one completely informed about the type of Central Banker when it is appointed; and one about which the Central Banker has private information. The results obtained show that the information context and the degree of transparency and observability of the processes that lead to the strategic delegation of monetary policy significantly influence the nature of the equilibrium.

In the case in which the economy is characterized by high uncertainty among private agents about the identify of the Central Banker (i.e. an economy with a small number of agents that observe the type  $\alpha$  at the moment of the appointment), one obtains a hybrid *pooling/separating* equilibrium in which the wetter types separate from each other, while the tougher types behave as an intermediate type which chooses an inflation rate equal to zero. In equilibrium, therefore, no Central Banker utilizes a strategy with an inflation rate less than zero (as shown by Figure 1). This is in substance is an example in which, even though there are infinite types of Banker and infinite actions, each type does not select a different type in order that it

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<sup>22</sup>The simulations were obtained using the Mathematica 4.0 program.

can be identified. In effect, only the less inflation-averse types reveal their identity unequivocally to the public.

In the case where one moves towards situations of the monetary authority’s perfect observability – in other words, there is a large number of agents informed about type  $\alpha$  in the economy – the pooling equilibrium breaks down and there emerges a complete separating equilibrium where each type selects an inflation rate lower than that selected by the wet type closest to it. In both equilibria (partial *pooling* and complete *separating*), it is the intermediate types that greatly reduce the inflation rate compared to the case in which preferences are fully known (Barro-Gordon).

As pointed out in the introduction, the observability of commitment and the nature of the Banker’s equilibrium strategies influence the design of the government’s strategic delegation.

An interesting problem for future research is modification of the game by introducing an initial stage in which a *continuum* of governments appoint a Central banker whose preferences are not perfectly observable by the private sector. In this case, does a government with certain preferences have the incentive to appoint Bankers with preferences different from its own?

## 6 Appendix

### 6.1 Regularity conditions on the Central Banker’s welfare function

As said in Section 2, in order to demonstrate that the separating equilibrium exists and is unique, one must check Mailath’s (1987) regularity conditions defined on equation (11), which are: belief monotonicity, type monotonicity, and *single crossing*.

Belief monotonicity condition

$$\tilde{W}_{\alpha} = -\alpha(1 - p) < 0$$

This condition represents the Central banker’s incentive to be believed “tough” in combating inflation. In effect,  $\alpha(1 - p)$  represents the marginal cost of the loss of reputation; given  $\alpha$ , the marginal cost is decreasing in  $p$ .

Type monotonicity condition

$$\tilde{W}_{\pi_1\alpha} = 1 > 0$$

This condition represents the marginal benefit due to the inflation surprise, for each given belief, that private agents have about the type of Central Banker. This condition states that the greater the weight that the monetary assigns to unemployment, the greater, at the margin, is the positive effect of inflation for each given belief that  $1 - p$  agents have about the type of Central Banker.

Single crossing condition

$\partial(\tilde{W}_{\pi_1}/\tilde{W}_{\hat{\alpha}})/\partial\alpha = -\pi_1/(-\alpha)^2(1-p) < 0$ , which is satisfied in that it does not change sign for:  $\pi_1 = \phi(\alpha) > 0$ , that is, we restrict the analysis to positive inflation rates. The marginal substitution rate between an increase in inflation in the first period and the consequent loss of reputation in the second period is an increasing monotonic function in  $\alpha$ . In other words, the wetter the Central Banker, the greater the cost that it must bear in terms of future reputation for one additional unit of current inflation.

The second-order conditions

A sufficient condition for (13) to be a local maximum for (11) is the following (see Mailath, 1987, p. 1355):  $\phi'\tilde{W}_{\pi_1\alpha} + \tilde{W}_{\alpha\hat{\alpha}} \geq 0$ . Consequently, given that  $\pi_1 \in \phi$ , and  $\tilde{W}_{\alpha\hat{\alpha}} = -(1-p)$  and  $\tilde{W}_{\pi_1\alpha} = 1$ , we obtain the following restriction in  $\phi$ :  $\phi' > 1 - p$ . Thus from (13), the restriction of the separating equilibrium to the support interval is such that:  $\phi > 0$ .

A sufficient condition for (13) to be a local maximum for (11) is the following (see Mailath, 1987, p. 1355):

$\phi' \left\{ \tilde{W}_{\pi_1\alpha} - \left( \tilde{W}_{\pi_1}/\tilde{W}_{\hat{\alpha}} \right) \tilde{W}_{\alpha\hat{\alpha}} \right\} \geq 0$ , which is satisfied for  $\phi' \left\{ 1 - \frac{(\alpha-\phi)}{\alpha} \right\} \geq 0$ , that is,  $0 \leq \phi \leq \alpha$ .

## 6.2 Separating equilibrium for different information contexts

The integral equation:  $\int_{\alpha}^1 \frac{1}{\alpha} d\alpha = \int_x^1 \frac{1-x}{1-p-x+x^2} dx$  is defined for different intervals according to whether:  $\Delta \begin{matrix} \leq \\ \geq \end{matrix} 0$ .<sup>23</sup>

With  $\Delta < 0$  i.e.  $p < \frac{3}{4}$ , the fraction of the integral in  $dx$  is always verified with,  $0 < x < 1$ , from which follows:

$$\int_{\alpha}^1 \frac{1}{\alpha} d\alpha = \int_x^1 \frac{1-x}{1-p-x+x^2} dx$$

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<sup>23</sup> Where  $0 < x < 1$  and  $0 \leq \alpha \leq 1$ . In effect, it is assumed that the support is equal to one.

the solution of which, as we have seen, is equal to:

$$\log \alpha = -\frac{1}{2} \log \left| \left( \frac{\phi}{\alpha} \right)^2 - \left( \frac{\phi}{\alpha} \right) + (1-p) \right| + \frac{1}{\sqrt{3-4p}} \arctan \frac{2\frac{\phi}{\alpha} - 1}{3-4p} + c_1.$$

Hence if  $\alpha = A^{BC}$  e  $\phi(\alpha) = A^{BC}$  it is possible to calculate  $c_1$  :

$$\log A^{BC} = -\frac{1}{2} \log |1-p| + \frac{1}{\sqrt{3-4p}} \arctan \frac{1}{\sqrt{3-4p}} + c_1$$

$$c_1 = \log A^{BC} + \frac{1}{2} \log |1-p| - \frac{1}{\sqrt{3-4p}} \arctan \frac{1}{\sqrt{3-4p}}.$$

Normalizing for  $A^{BC} = 1$  one obtains:

$$c_1 = \frac{1}{2} \log |1-p| - \frac{1}{\sqrt{3-4p}} \arctan \frac{1}{\sqrt{3-4p}}.$$

Solving in order to determine the critical value of  $\alpha$ , such that  $\phi(\alpha_s) = 0$  in the period  $t = 1$  and  $\pi_2 = \alpha_s$  in the period  $t = 2$ , one finds that for  $p < \frac{3}{4}$ :

$$\log \alpha^s = \frac{1}{\sqrt{3-4p}} \arctan \frac{-1}{\sqrt{3-4p}} - \frac{1}{\sqrt{3-4p}} \arctan \frac{1}{\sqrt{3-4p}}$$

from which one obtains:

$$\alpha^s = e^{-\frac{2}{\sqrt{3-4p}} \arctan \frac{1}{\sqrt{3-4p}}}$$

Hence if the Central Banker has completely private information (i.e.  $p = 0$ ),  $\alpha^s$  the point dividing the separating region from the *pooling* region will be equal to 0.546.<sup>24</sup>

Whith  $p = \frac{3}{4}$ , the fraction of the interval in  $dx$  is verified only if:

$$\alpha > \phi > \frac{1}{2}\alpha$$

because the initial condition is violated for values  $\frac{\phi}{\alpha} < \frac{1}{2}$ . Hence:

$$\int_{\alpha}^1 \frac{1}{\alpha} d\alpha = \int_{x>\frac{1}{2}}^1 \frac{1-x}{1-p-x+x^2} dx$$

This can be easily demonstrated by means of the following limit:

$$\lim_{x \rightarrow \frac{1}{2}^+} \frac{1}{2} \frac{-1}{x - \frac{1}{2}} - \log \left| x - \frac{1}{2} \right| + c_0 = -\infty$$

so that from (21) one has:

$$\log \alpha = -\infty \implies \alpha = 0$$

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<sup>24</sup>This results is confirmed by a numerical simulation of (13) whith  $p = 0$ .

Q.E.D.

Finally, with  $p > \frac{3}{4}$ ,<sup>25</sup> the fraction of the integral in  $dx$  is verified only if:

$$\alpha > \phi > \frac{1 + \sqrt{4p-3}}{2}\alpha,$$

from which follows:

$$\int_{\alpha}^1 \frac{1}{\alpha} d\alpha = \int_{x > \frac{1 + \sqrt{4p-3}}{2}}^1 \frac{1-x}{1-p-x+x^2} dx.$$

The starting-point for proving the above is:

$$\log \alpha = -\frac{1}{2} \log |x^2 - x + (1-p)| + \frac{1}{2} \frac{1}{\sqrt{4p-3}} \log \left| \frac{2x-1-\sqrt{4p-3}}{2x-1+\sqrt{4p-3}} \right| + c_2 \quad (24)$$

where  $c_2$  with  $\alpha = A^{BC}$  and  $\phi(\alpha) = A^{BC}$  is:

$$c_2 = \log A^{BC} + \frac{1}{2} |1-p| - \frac{1}{2} \frac{1}{\sqrt{4p-3}} \log \left| \frac{1-\sqrt{4p-3}}{1+\sqrt{4p-3}} \right|.$$

Considering now:

$$\begin{aligned} \lim_{x \rightarrow \frac{1-\sqrt{4p-3}}{2}} \text{della(24)} &= -\frac{1}{2} \ln(0) + \\ &\frac{1}{2} \frac{1}{\sqrt{4p-3}} \ln \left| \frac{1-\sqrt{4p-3}-1-\sqrt{4p-3}}{1-\sqrt{4p-3}-1+\sqrt{4p-3}} \right| + c_2 = \log \alpha \end{aligned}$$

from which one obtains:

$$\log \alpha = +\infty \implies \alpha = +\infty$$

which is a contradiction.

Consider (24) once again. After short algebraic steps, this can be written in the following form:

$$\begin{aligned} &-\sqrt{4p-3} \log |2x-1+\sqrt{4p-3}| - \sqrt{4p-3} \log |2x-1-\sqrt{4p-3}| + \\ &+\log |2x-1-\sqrt{4p-3}| - \log |2x-1+\sqrt{4p-3}| + 2\sqrt{4p-3} * c_2 = \\ &2\sqrt{4p-3} \log \alpha + \sqrt{4p-3} \log \left( \frac{1}{4} \right), \end{aligned}$$

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<sup>25</sup>In this case, the roots of  $x$  will be:

$$x = \frac{1 \mp \sqrt{4p-3}}{2}$$

$$\text{i.e. } x \in \left[ 0; \frac{1-\sqrt{4p-3}}{2} \right] \cup \left[ \frac{1-\sqrt{4p-3}}{2}; 1 \right].$$

the limit for for which with  $x$  tending to  $\frac{1+\sqrt{4p-3}}{2}$  is:

$$\lim_{x \rightarrow \frac{1+\sqrt{4p-3}}{2}} \log \alpha = -\infty \implies \alpha = 0$$

Q.E.D.

Figures 1, 2 and 3 provide a graphical illustration of the main analytical results for the various information contexts.

### 6.3 Incentives for the Central Banker to deviate from the pooling equilibrium

It was stressed in Section 2 that for the strategy expressed by (14) to be a *pooling* equilibrium, it is necessary that:

$$w^D(\alpha) < w^P(\alpha) \text{ per } 0 \leq \alpha \leq \alpha^s. \quad (25)$$

That is, for (14) to be part of a *pooling* equilibrium, there must be no incentive for the monetary authority to deviate from a zero inflation rate.

It is therefore necessary to calculate the welfare from deviation and the welfare from the equilibrium strategy. The welfare from deviation, in the region  $0 \leq \alpha \leq \alpha^s$ , is:

$$-\frac{1}{2}\pi_1^2 + \alpha [\pi_1 - (1-p)\bar{\phi}] - \frac{1}{2}\pi_2^2 + \alpha [\pi_2 - p\alpha - (1-p)\phi^{-1}(\pi_1)]. \quad (26)$$

The first-order condition on (26) is:

$$-\pi_1 + \alpha - \alpha(1-p) \frac{d\hat{\alpha}}{d\pi_1} = 0. \quad (27)$$

It follows from the first-order condition that  $\pi_1 = \alpha - \alpha(1-p) \frac{\alpha - \phi}{\alpha(1-p)} \rightarrow \pi_1 = \phi$ . Moreover, one intuits that the optimal deviation is never definite.

The welfare from the equilibrium strategy (in other words from *pooling*) is:

$$\alpha [-(1-p)\bar{\phi}] - \frac{1}{2}\alpha^2 + \alpha \left[ \alpha - p\alpha - (1-p) \frac{\alpha^s}{2} \right] \quad (28)$$

Given that the optimal deviation is never definite, we may directly compare (26) (26) with (28), from which it is evident that (25) is always verified. It is thus proved that the strategy expressed by (14) is a *pooling equilibrium*.

## 6.4 Pooling equilibrium for the entire support

As said in Section 2, it is possible that the game has a *pooling* solution for the entire support  $[a, A^{BC}]$ , in correspondence to which all the possible types of Central Banker converge on the choice of a single level of inflation.

The welfare deriving from the pooling strategy is thus:

$$W^P = -\frac{1}{2}(\pi_1^p)^2 + \alpha[\pi_1^p - p\pi_1^p - (1-p)\pi_1^p] - \frac{1}{2}\pi_2^2 + \alpha[\pi^2 - p(\pi_2^e | \alpha) - (1-p)(\pi_2^e | \pi_1^p)] \quad (29)$$

On substituting  $\pi^2 = \alpha$ ,  $(\pi_2^e | \alpha) = \alpha$  and  $(\pi_2^e | \pi_1^p) = \bar{\alpha}$  in (29) one obtains:

$$W^P = -\frac{1}{2}(\pi_1^p)^2 + -\frac{1}{2}\alpha^2 + \alpha(1-p)(\alpha - \bar{\alpha}) \quad (30)$$

The  $\pi_1^p = \bar{\alpha}$  strategy will be part of a pooling equilibrium if there is no incentive for the Central Banker to deviate from that level of inflation, given the specification of the out-of-equilibrium beliefs.

The specification of the out-of-equilibrium beliefs examined here is that of “*passive conjectures*”, according to which the  $1-p$  fraction of private agents infers nothing about the type of Central Banker from observation of a deviation from the pooling strategy.

In this case, the incentive for deviation requires calculation of the *pay-off* function, substituting  $\pi_1 = \pi_1^D$  and  $(\pi_2^e | \pi_1^D) = \bar{\alpha}$  in the Central Banker’s objective function. It is thus possible to obtain the following welfare function deriving from the deviation strategy:

$$W^D = -\frac{1}{2}(\pi_1^D)^2 + \alpha(\pi_1^D - \pi_1^p) - \frac{1}{2}\alpha^2 + \alpha(1-p)(\alpha - \bar{\alpha}) \quad (31)$$

On maximizing (31) with respect to  $\pi_1^D$  one then obtains the optimal deviation, which is:

$$\pi_1^D = \alpha \quad (32)$$

Substituting(32) in (31) we obtain the welfare deriving from the optimal deviation, which will be::

$$W^D = -\frac{1}{2}\alpha^2 + \alpha(\alpha - \pi_1^p) - \frac{1}{2}\alpha^2 + \alpha(1-p)(\alpha - \bar{\alpha}) \quad (33)$$

The deviation will therefore be advantageous if  $W^D > W^P$ , given respectively by (33) and (30), so that after some algebraic steps we obtain:

$$(\alpha - \pi_1^p)^2 > 0 \tag{34}$$

which is always satisfied, so that with passive conjectures, deviation from the *pooling* equilibrium is always advantageous.

## References

- [1] Albanesi S., (2003), “Optimal and Time-Consistent Monetary and Fiscal Policy with Heterogeneous Agents”, *CEPR Discussion Papers* n.3713.
- [2] Albanesi S., Chari V. and Christiano L., (2003), “How Severe Is the Time-Inconsistency Problem in Monetary Policy?”, *Federal Reserve Bank of Minneapolis Quarterly Review Summer*, vol. 27, n. 3, pp. 17-33.
- [3] Alesina A. and Gatti R., (1995), “Independent Central Banks: Low Inflation at Low Costs?”, *American Economic Review Papers and Proceedings*, n. 85, pp. 196-200.
- [4] Alesina A. and Grilli V., (1992), “The European Central Bank: Reshaping Monetary Politics in Europe”, in Persson T. and Tabellini G. (eds.), *Monetary and Fiscal Policy, Vol.1: Credibility*, Cambridge, Mass.: The MIT Press, pp. 247-277.
- [5] Athey S., Atkeson A. and Kehoe P., (2003), “The Optimal Degree of Discretion in Monetary Policy”, *NBER Working Paper Series*, n.10109.
- [6] Backus D. and Driffil J., (1985), “Inflation and Reputation”, *American Economic Review*, vol. 75, pp. 530-538.
- [7] Bagwell K., (1995), “Commitment and Observability in Games”, *Games and Economic Behavior*, n.8, pp.271-280.
- [8] Barro R., (1986), “Reputation in a Model of Monetary Policy with Incomplete Information”, *Journal of Monetary Economics*, vol. 17, pp. 3-20.
- [9] Barro R. and Gordon D. , (1983), “Rules, Discretion, and Reputation in a Model of Monetary Policy”, *Journal of Monetary Economics*, vol. 12, pp. 101-122.
- [10] Blinder A., (1998), *Central Banking in Theory and Practice*, Cambridge: MIT Press.
- [11] Bonanno G., (1992), “Deterrence, Observability and Awareness”, *Economic Notes*, vol.21, n.2 pp. 307-315.

- [12] Canzoneri M., (1985), “*Monetary Policy Games and the Role of Incomplete Information*”, *American Economic Review*, vol. 75, n.5, pp. 1056-1070.
- [13] Chari V., Kehoe P., e Prescott E., (1989), “Time Consistency and Policy”, in Barro R., *Modern Business Cycle Theory*, Cambridge: Harvard University Press.
- [14] Cukierman A., (1993), *Central Bank Strategy, Credibility, and Independence: Theory and Evidence*, Cambridge, Mass.: The MIT Press.
- [15] D’Amato M. and Pistoresi B., (1996), “La Riservatezza del Banchiere Centrale è un Bene o un Male?”, *Politica Economica*, n. 3, pp. 363-389.
- [16] D’Amato M. and Pistoresi B., (1998), “Go and soothe the row. Delegation of monetary policy under private information”, Dipartimento di Scienze Economiche, Università di Salerno, *mimeo*.
- [17] Drazen A., (2000), *Political Economy in Macroeconomics*, Princeton: Princeton University Press.
- [18] Driffill J., (1987), “Macroeconomic policy games with incomplete information: some extensions”, *CEPR Discussion Papers*, n.159.
- [19] Driffill J., (1988), “Macroeconomic policy games with incomplete information: a survey”, *European Economic Review*, n. 32, pp. 533-541.
- [20] Fershtman C. and Kalai E., (1997), “Unobserved Delegation”, *International Economic Review*, vol.38, n.4, pp. 763-774.
- [21] Fisher S., (1995), “Central-Bank Independence Revisited”, *American Economic Review Papers and Proceedings*, n. 85, pp. 201-206
- [22] Franzese R., (1999), “Partially Independent Central Banks, Politically Responsive Governments, and Inflation”, *American Journal of Political Science*, vol. 43, n.3, pp. 681-706.
- [23] Fudenberg, D. and Tirole J., (1991), *Game Theory*, Cambridge, Mass.: The MIT Press.
- [24] Kreps D. and Wilson R., (1982a), “Sequential Equilibria”, *Econometrica*, vol. 52, pp.863-894.
- [25] Kreps D. and Wilson R., (1982b), “Reputation and imperfect Information”, *Journal of Economic Theory*, vol. 27, pp.253-279.

- [26] Kydland F. e E. Prescott, (1977), "Rules Rather than Discretion: the Inconsistency of Optimal Plans", *Journal of Political Economy*, vol. 85, N.3, pp. 473-491.
- [27] Jensen H., (1997), "Credibility of Optimal Monetary Delegation", *American Economic Review*, vol. 87, pp. 911-920.
- [28] Lohmann S., (1992), "Optimal Commitment in Monetary Policy: Credibility versus Flexibility", *American Economic Review*, vol. 82, pp. 150-167.
- [29] Mailath G., (1987a), "Incentive Compatibility in Signaling Games with a Continuum of Types", *Econometrica*, vol.55, pp. 1349-1365.
- [30] Mailath G., (1987b), "*Signaling Games*", in Creedy A. et al. (eds.), *Recent Developments in Game Theory*, Cheltenham: Edward Elgar Publishing Limited, pp. 65-93.
- [31] McCallum B., (1995), "Two Fallacies Concerning Central Bank Independence", *American Economic Review Papers and Proceedings*, n. 85, pp. 207-211.
- [32] McCallum B., (1997), "Crucial Issues Concerning Central bank Independence" *Journal of Monetary Economics*, n.39, pp. 99-112.
- [33] Milgrom P. and Roberts J., (1982a), "Predation e Reputation and Entry Deterrence", *Journal of Economic Theory*, vol. 27, pp.280-312.
- [34] Milgrom P. and Roberts J., (1982b), "Limiting Pricing and Entry under Incomplete information: An Equilibrium Analysis", *Econometrica*, vol. 50, pp. 443-459.,
- [35] Muscatelli A., (1999), "Inflation contracts and Inflation targets under Uncertainty: Why We Might Need Conservative Central Bankers", *Economica*, vol. 66, pp.241-254.
- [36] Persson T. and Tabellini G., (1993), "Designing Institutions for Monetary Stability", *Carnegie-Rochester Conference Series on Public Policy*, n. 39, pp. 53-84.
- [37] Persson T. and Tabellini G. (eds.), (1995), *Monetary and Fiscal Policy. Volume 1: Credibility*, Cambridge, Mass.: The MIT Press.

- [38] Persson T. and Tabellini G., (2000), “*Political Economics. Explaining Economic Policy*”, Cambridge, Mass.: The MIT Press.
- [39] Posen A. S., (1993), “Why Central Bank independence does not cause Low Inflation: There is No Institutional Fix for Politics”, in O’ Brien R. (ed.), *Finance and International Econoy. 7: The Amex Bank Review Prize Essays*, Oxford: Oxford University Press, pp. 41-65.
- [40] Posen A. S., (1995), “Declarations are not enough: Financial Sector Sources of Central Bank Independence”, *Nber Macroeconomics Annals*, pp. 253-274.
- [41] Riboni A., (2003), “Time consistency in dynamic bargaining: The Role of Committees as Substitutes for Commitment”, Department of Economics University of Rochester, *mimeo*.
- [42] Rogoff K., (1985), “The Optimal Degree of Commitment to an Intermediate Monetary target”, *Quarterly Journal of Economics*, vol.100, pp. 1169-1189.
- [43] Rogoff K., (1987), “Reputational Constraints on Monetary Policy”, *Carnegie-Rochester Conference Series on Public Policy*, vol. 26, pp. 141-182.
- [44] Sen A., (2002), *Globalizzazione e Liberta*, Milano: Mondadori.
- [45] Sibert A., (2002), “Monetary Policy with uncertain central bank preferences”, *European Economic Review*, vol. 46, pp. 1099-1110.
- [46] Sibert A., (2003), “Monetary Policy committees: individual and collective reputation”, *Review of Economic Studies*, vol. 70, pp. 649-665.
- [47] Stokey N., (2002), “Rules versus Discretion After Twenty-Five Years”, in Gertler and Rogoff K. (eds.), *NBER Macroeconomics Annual 2002*, vol. 17, Cambridge, Mass.: The MIT Press., pp. 9-45.
- [48] Svensson L., (1995), “Optimal Inflation targets ‘Conservative’ Central Banks, and Linear Inflation Contracts”, *CEPR Discussion Paper*, n. 1249.
- [49] Vickers J., (1986), “Signalling in a Model of Monetary Policy with Incomplete Information”, *Oxford Economic Papers*, vol. 38, pp. 443-455.

- [50] Vincent D., (1998), "Repeated signalling games and dynamic trading relationships", *International Economic Review*, n.39, pp.275-293.
- [51] Walsh C., (1995), "Optimal Contracts for Central Bankers", *American Economic Review*, vol.85, pp. 150-167.