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Delegation of Monetary Policy: More than a Relocation of the Time-Inconsistency Problem*

John Driffill
Birkbeck College, University of London, and CEPR, and

Zeno Rotondi
Capitalia and University of “Tor Vergata”, Rome

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Abstract

It has been argued that delegation of monetary policy to an independent central bank, which acts as an agent for the government, does not mitigate the problem of time-inconsistency, but merely relocates it. We argue here that this is not so, and that delegation enables a wider class of economies to sustain zero inflation than would be able to do so in its absence. We consider an economy in which the government faces re-appointment costs, that is, costs associated with sacking one central banker and replacing them with another, costs which are intended to protect central bank independence. We show that, by means of suitable announcements of incentive schemes for the central bank, combined with appropriate actually implemented schemes, delegated policy enables zero inflation to prevail in economies in which

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it could not do so without delegated policy. These economies are ones that have relatively high discount rates (and so low discount factors).

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1 Introduction

The monetary delegation approach to solving potential time-inconsistency problems has recently been questioned by McCallum (1995, 1997). He argues that, as there is always the temptation for the government to renege on the chosen monetary institution, the institutional remedies proposed in this approach do not fix the problem of time inconsistency. They merely relocate it.1,2

Persson and Tabellini (1999), amongst others, have replied to this criticism by observing that: “in the model that dominates the literature, what is needed is a high cost for changing the institution within the time horizon of existing nominal contracts” and “the cost of suddenly changing the institution could also be a loss of reputation”.

1Posen (1995) also criticises the delegation approach and provides some empirical findings that, in contrast to the previous empirical evidence, suggest that the relationship between central bank independence and disinflationary credibility is not supported. He concludes that central bank independence alone is not sufficient. The presence of a coalition in society committed to protecting the central bank independence is necessary for achieving low inflation. In particular he concludes that, as the financial community is the critical constituency that influences the central bank, the outcomes of monetary policy will predominantly reflect that of the monetary policy desired by the financial community. However, no formal analysis is presented and as shown by Alesina (1995) his empirical findings are controversial.

2McCallum refers his criticism only to the Walsh contracting approach. Also Obstfeld and Rogoff (1996) make the same criticism referring to the Walsh solution. There is the view that the Rogoff’s approach is immune to McCallum’s criticism. In that regard Alesina (1995, p.289) wrote: ‘it is institutionally harder to dismiss a “conservative” central banker than it is for the policymaker simply to renege on a policy announcement made without the independent conservative agent’. However in his argument there is the implicit assumption that the presence of reappointment costs will deter the government from over-ruling the banker or sacking him and appointing a less conservative one. If, following Jensen (1997), we interpret the principal-agent approach as a complex institutional arrangement based on a structure of incentives that is costly to change, then Alesina’s argument should hold also for the Walsh approach.
This premise, which has been implicitly assumed in the standard literature on delegation, has been recently challenged by Jensen (1997). He explicitly introduces the delegation stage in the government policy choice and adds a quadratic cost for reappointments. In the static one-shot game version higher costs of reappointment reduce the inflationary bias but never remove it. An exception is represented only by the extreme and unrealistic case where the weight on reappointment costs in the government’s loss function is infinite. In this situation optimal monetary delegation is not subject to a credibility problem, but all that matters in the loss function are reappointment costs. Moreover if the game is repeated over an infinite horizon, along the lines of Barro and Gordon (1983), the presence of reappointment costs worsens the credibility of optimal monetary policy under delegation compared to the case when monetary policy is conducted directly by the government.

These results imply a negative view of the contracting solution and in general of the monetary delegation approach. Jensen concludes by suggesting that too much emphasis has been given to the approach of monetary delegation and that research should focus on other directions, in particular on the relationships between time inconsistency and structural policies.

In the present analysis we show that the paradoxical result, that reappointment costs worsen the credibility of optimally delegated monetary policy, results from an implicit limitation on the range of policies that are considered. The analysis has restricted the set of announced incentive schemes (or contracts) that the government might present to the central bank to maintain zero inflation. When this restriction is removed, the opposite, less paradoxical result is restored, namely that costs of reappointing the central banker do in fact enable zero inflation to emerge from a credible policy in circumstances (i.e., when the government is sufficiently impatient) in which it would not do so in the absence of such costs. On this basis we are able to assert that delegation does not merely relocate the time inconsistency problem. Rather it does more than this. In some circumstances it does indeed provide a solution to it.

2 The model

We use the same notation as Jensen (1997). The supply function is given by the standard expectations-augmented Phillips curve
\[ y_t = \alpha (\pi_t - \pi_t^e), \quad (1) \]

where for simplicity the natural level of output is normalised to zero; \( \pi_t, \pi_t^e \) are the actual and expected inflation rate respectively.

The government’s loss function is expressed by

\[ \bar{L}_t = \left[ \pi_t^2 + \lambda (y_t - \bar{y})^2 + \varphi (f_t - f_t^a)^2 \right], \quad (2) \]

where deviations of output and inflation from the targets are weighted by 1 and \( \lambda \) respectively, with \( \lambda > 0 \). As usual in the time inconsistency literature the output target is assumed to be greater than the natural level, \( \bar{y} > 0 \). But in contrast with the previous literature on monetary delegation, there is now an additional cost on the reappointment of the central banker, expressed by the difference between the announced incentive scheme with the penalty \( f_t^a \) and the realised scheme with incentive \( f_t \). In particular if \( f_t \neq f_t^a \) we will say that the central banker has been reappointed, which in the present framework will happen at some cost to the government. The parameter \( \varphi \) reflects the distaste for reappointment costs relative to the other costs in the loss function. When monetary policy is delegated we have \( \varphi > 0 \). When there is no delegation \( \varphi \) is set equal to zero. As observed by Jensen, if \( f_t \) and \( f_t^a \) are understood not simply as the contract offered to the central banker, but as referring to a complex system of monetary regulations, it seems natural to assume that a small change is less costly than a bigger one. This may justify the use of a quadratic cost of reappointments with an assigned weight \( \varphi \).

Monetary policy is delegated by the government to a central banker whose loss function is the following

\[ L_t^b = \left[ \pi_t^2 + \lambda (y_t - \bar{y})^2 + 2 f_t \pi_t \right]. \quad (3) \]

Here the central banker is fined with the penalty \( 2 f_t \) for inflation rates greater than zero. As we will see later on, the optimal incentive scheme that allows the government to eliminate the inflation bias is \( f_t = \lambda \alpha \bar{y} \).

\(^3\) An alternative way of modelling reappointment costs would be to assume that if the government reneges on his announcement it will also incur a fixed cost and that this fixed component is relatively more important than that dependent on the size of the modifications of the given institutional arrangement. This idea is captured, for example, in the work of Lohmann (1992).
In each period the timing of moves is the following. In stage zero, the government delegates monetary policy to a central banker and announces an incentive scheme $f_t^a$. In stage 1, the private sector forms expectations about inflation and sets wages. In stage 2, the government sets actual conditions $f_t$ for monetary policy. Finally, in stage 3, the CB sets actual inflation.

In the discretionary regime the central banker minimises the discounted value of his loss function, $\sum_{t=-\infty}^{\infty} \beta^{-t} L_b^t$, subject to (1) by taking inflation expectations and actual conditions for monetary policy $f_t$ as given. The parameter $0 < \beta < 1$ is the discount factor of the central banker. It is assumed that the central banker and the government have the same discount factor.

From the first order condition we obtain the central banker’s reaction function

$$\pi_t = \frac{\lambda \alpha^2 \pi_t^c + \lambda \alpha \overline{\pi} - f_t}{\Lambda},$$

where $\Lambda = (1 + \lambda \alpha^2)$. When choosing actual monetary conditions for monetary policy, the government must take its prior announcements and inflation expectations as given but incorporates the behaviour of the central banker in its decision problem. Thus it minimises the discounted value of its loss function, $\sum_{t=-\infty}^{\infty} \beta^{-t} L_s^t$, with respect to $f_t$ subject to (1) and (4). The minimisation yields the following optimal incentive scheme

$$f_t = \frac{\varphi \Lambda}{1 + \varphi \Lambda} f_t^a,$$

where $f_t^a$ is the announcement chosen by government. As observed by Jensen the assumption of prohibitive costs of reappointment when monetary policy is delegated by the government to a central banker eliminates by definition the issue of the credibility of optimal monetary delegation. From (5) one can see that announcements will always be fulfilled if the government’s only concern is reappointment costs, i.e. when $\varphi \to +\infty$.

The private sector’s inflation expectations are obtained by substituting (5) into (4). After taking expectations we get

$$\pi_t^{e,NCD} = \lambda \alpha \overline{\pi} - \frac{\varphi \Lambda}{1 + \varphi \Lambda} f_t^a.$$

Finally the government chooses the optimal announcement. When making this choice the government internalises the effects of its decision on the
central banker’s behaviour, on its own behaviour when choosing actual monetary conditions, and on the private sector’s expectations, Minimising the government’s loss function with respect to \( f_t^a \) subject to (4), (5) and (6) yields

\[
f_t^{a,NCD} = \frac{\Lambda (1 + \varphi \Lambda) \lambda \alpha \bar{y}}{1 + \varphi \Lambda^2}.
\]  

Expression (7) implies that

\[
f_t^{NCD} = \frac{\varphi \Lambda^2 \lambda \alpha \bar{y}}{1 + \varphi \Lambda^2}.
\] 

Here we can observe that if \( \varphi \) tends to infinite we have \( f_t^{a,NCD} = \lambda \alpha \bar{y} \) and \( f_t^{NCD} = f_t^{a,NCD} \). Thus, if reappointment costs are prohibitive, in the static one-shot game version of Jensen’s model optimal monetary delegation is not subject to a credibility problem. However, the more realistic case is when these costs are not all that matters in the government’s loss function, or in other words when in expression (2) the weight \( \varphi \) is not infinite.

The equilibrium inflation rate will be under the discretionary regime with delegation

\[
p_t^{NCD} = \frac{\lambda \alpha \bar{y}}{1 + \varphi \Lambda^2}.
\] 

If the government does not delegate monetary policy, i.e. \( \varphi = 0 \), it is straightforward to show that if the government behaves in a discretionary manner the equilibrium inflation rate would be \( p_t^{NC} = \lambda \alpha \bar{y} \). From expression (9) we can see that delegation reduces the inflation bias but does not remove it. On the contrary if the government could idealistically precommit to an announced policy rule before expectations are formed then the government would not need to delegate monetary policy in order to eliminate the inflation bias and the optimal policy rule, or the precommitment policy rule, would be in this deterministic case to set \( p_t^{PR} = 0 \). Comparing the government’s losses under the equilibrium with precommitment and the equilibrium with discretion it is possible to see that in the case of delegation the loss is lower than in the case when the government conducts monetary policy directly and behaves in a discretionary manner, but is greater than in the precommitment equilibrium.

In the subsequent sections we will consider the situation when the policy game is repeated for an infinite number of periods in order to study the
precommitment technology where the private sector punishes deviations by a one-period reversion to expectations given by the discretionary solution.

3 Jensen’s paradox

Let us examine the situation when repeated interactions among the players take place. In particular assume that the game is repeated for an infinite number of periods. In this case Barro and Gordon (1983) have shown that, if the private sector adopts a punishment strategy triggered by any observed deviation from optimal policy and if the government does not discount the future too heavily, it is possible that the future cost to the government of loss of reputation may more than outweigh the current gain from deviation.4

By assuming that the private sector reverts for one period to the discretionary solution whenever a deviation from optimal policy is observed, Jensen has found that the minimal requirement for the patience of the government is given by $\beta \geq \hat{\beta} \equiv 1/\Lambda$. If the discount factor, $\beta$, is sufficiently high optimal monetary policy is a perfect Nash equilibrium and therefore it is also credible. Alternatively if the discount factor is not sufficiently high, optimal monetary policy is not credible. In order to achieve the precommitment solution the government might consider delegating monetary policy to a central banker with the optimal incentive scheme $f_t = \lambda \alpha \gamma$ and try again to maintain a reputation for low inflation. Also in this case the credibility of optimal monetary delegation, where credibility is understood as the ability to carry out optimal monetary policy, can be studied by examining simple punishment strategies based on a one-period reversion to the discretionary solution.5 Consider the following strategy combinations:

4 This framework – a reputational model with trigger strategies – suffers from various well-known weaknesses, prominent among them the multiplicity of solutions. Backus and Drifill (1985), among many others, have commented on the problems inherent in the game-theoretic framework used by Barro and Gordon (1983), on which Jensen’s analysis is based. Other formulations of the model, such as those of Herrendorf (1998) and al-Nowaihi and Levine (1996), who also examine the credibility of optimal monetary delegation, avoid these problems and in some ways offer a more satisfactory game-theoretic framework for modelling reputation. Note also that in the present analysis, the central bank is entirely passive. It merely plays the time-consistent discretionary policy – the Nash equilibrium of the one-shot game. Rotondi (2000) introduces the possibility that the central banker, as well as the government, establishes a reputation for low inflation, in a framework similar to that of Jensen.

5 As observed by Jensen it is not necessary to analyse explicitly the cases when the
Government plays:

\[ f_t = \begin{cases} f_t^a = \lambda \alpha \bar{y} & \text{if } \pi_{t-1}^e = \pi_{t-1}^e; \\ f_t^a = f_t^{a, NCD} & \text{and } f_t = f_t^{NCD} & \text{if } \pi_{t-1}^e \neq \pi_{t-1}^e. \end{cases} \]  

(10)

Private sector plays:

\[ \begin{align*} 
\pi_t^e &= 0 \text{ if } \pi_{t-1}^e = \pi_{t-1}^e; \\
\pi_t^e &= \pi_t^{e, NCD} \text{ if } \pi_{t-1}^e \neq \pi_{t-1}^e. 
\end{align*} \]  

(11)

The expressions of \( \pi_t^{e, NCD} \) and \( f_t^{NCD} \) are found by substituting \( f_t^{a, NCD} \) in expressions (5) and (6) respectively. If there is a deviation from the announced optimal delegation the government minimises the loss function with respect to \( f_t \) subject to \( \pi_t^e = 0 \) and \( f_t^{a, DD} = \lambda \alpha \bar{y} \). This yields the following values:

\[ \begin{align*} 
\pi_t^{DD} &= \frac{\lambda \alpha \bar{y}}{(1 + \varphi \Lambda) \Lambda}; \\
f_t^{DD} &= \frac{\varphi \Lambda \alpha \bar{y}}{1 + \varphi \Lambda}. 
\end{align*} \]  

(12) \hspace{1cm} (13)

According to the above strategies the condition of no deviation for the government will be

\[ \tilde{L}_t^{PR} - \tilde{L}_t^{DD} \leq \beta \left( \tilde{L}_{t+1}^{NCD} - \tilde{L}_{t+1}^{PR} \right), \]  

(14)

which implies that

\[ \beta \geq \beta^D (\varphi) = \frac{1 + \varphi \Lambda^2}{\Lambda (1 + \varphi \Lambda)}. \]  

(15)

Now we can compare the condition for the credibility of optimal monetary delegation with the condition for the credibility of optimal monetary policy when monetary policy is conducted directly by the government. With this aim Jensen has proved the following proposition:

announcement of the government is \( f_t^e \neq \lambda \alpha \bar{y} \) as we can rule them out through a reversion to the discretionary solution for any value of the discount factor.
Proposition 1 For all $\varphi > 0$, $\hat{\beta}^D (\varphi) > \hat{\beta}$.

PROOF: 
As $\lim_{\varphi \to 0} \hat{\beta}^D (\varphi) = \hat{\beta}$ and given that $\partial \hat{\beta}^D (\varphi) / \partial \varphi > 0$ it follows that $\hat{\beta}^D (\varphi) > \hat{\beta}$ for all $\varphi > 0$.

Thus the premise made by the standard literature on delegation that it is the presence of reappointment costs that makes delegation to an independent central banker more credible than the conduction of monetary policy itself must be considered false according to Jensen’s analysis. The intuition for this result is the following. The punishment subsequent to a deviation becomes weaker the higher is the weight on reappointment costs. Also the gain from deviating decreases with $\varphi$ but less than the reduction in the cost deriving from the loss of reputation. The reason is that the reduction of the gain from deviating results from several opposing forces which mitigate the effect of an increase in $\varphi$.

4 The case when all announcements are feasible

Here we question Jensen’s paradoxical result that reappointment costs worsen the credibility of optimal monetary delegation. We will show that this surprising finding is based on his not considering what happens when all announcements are feasible for maintaining a reputation for zero inflation. Actually, by assuming a reversion to the no-commitment solution when the announcement $f^a_t$ differs from the optimal one (equal to $\lambda \alpha \bar{y}$), he restricts the attention of the analysis to the case in which $f^a_t = \lambda \alpha \bar{y}$ when the government plays its reputational strategy and $f^a_t \neq \lambda \alpha \bar{y}$ when it does not. We consider a game in which the government makes some announcement to the central bank of the incentive scheme, and this announced scheme need not necessarily be the optimal one. Nevertheless the actual scheme implemented will be the optimal one, when the government plays its reputational strategy.

Consider the following strategies. In each period when it plays the reputational strategy (that is, when it is following its commitment policy), the government announces an incentive scheme $f^a_t = \omega$, with $\omega > \lambda \alpha \bar{y}$. People expect inflation of zero ($\pi^e_t = 0$). The government then actually implements an incentive scheme with $f_t = \lambda \alpha \bar{y}$, i.e., the actual penalty on the central
bank for creating inflation is less than the pre-announced one. It is in fact equal to the penalty that induces the central bank to deliver zero inflation. With this scheme in place, the central bank duly delivers inflation of the expected rate $\pi_t = 0$.

When the government cheats (deviates from the commitment policy), it announces the incentive scheme as in the commitment policy, that is $f_t^a = \omega$, and people respond by expecting zero inflation ($\pi_t^e = 0$), but then the government implements a different incentive scheme than the one above. In fact it follows the discretionary policy, and implements the scheme that minimizes its expected loss for this period, given the announced scheme and the public’s expectations of inflation. The best scheme to implement satisfies (5) above. The government therefore implements $f_t = \omega \frac{\varphi \Lambda}{1 + \varphi \Lambda}$ and the central bank delivers inflation $\pi_t = \frac{\lambda \alpha \bar{y}}{\Lambda} - \frac{\varphi \omega}{1 + \varphi \Lambda}$.

Following a period in which the government has deviated, it is punished. In this play of the game, the government plays the discretionary policy. The government announces an incentive scheme $f_t^{\text{a, NCD}} = \frac{\lambda \alpha \bar{y} \Lambda (\varphi \Lambda + 1)}{\varphi \Lambda^2 + 1}$, people expect inflation $\pi_t^e = \lambda \alpha \bar{y} - f_t^{\text{a, NCD}} \frac{\varphi \Lambda}{\varphi \Lambda + 1}$, the actual incentive scheme is $f_t = f_t^{\text{a, NCD}} \frac{\varphi \Lambda}{\varphi \Lambda + 1}$, and actual inflation turns out as expected.

The payoffs to the government of two of the scenarios set out above – commitment and deviation – depend on the value of the announced incentive $\omega$.

Looking at the payoffs, we have for commitment,

\[ L_t^{PR} = \lambda \left[ \alpha (\pi_t - \pi_t^e) - \bar{y} \right]^2 + \pi_t^2 + \varphi (f_t - \omega)^2 \]

\[ = \lambda \bar{y}^2 + \varphi \lambda^2 \alpha^2 \bar{y}^2 - 2 \varphi \lambda \alpha \bar{y} \omega + \varphi \omega^2. \]

This can be written as

\[ L_t^{PR} = \lambda \bar{y}^2 + \varphi (\lambda \alpha \bar{y} - \omega)^2. \]  

For deviation, we have

\[ L_t^{DD} = \lambda \left[ \alpha (\pi_t - \pi_t^e) - \bar{y} \right]^2 + \pi_t^2 + \varphi (f_t - \omega)^2 \]

\[ = \lambda^2 \alpha^2 \bar{y}^2 \varphi + \varphi \omega^2 \lambda \alpha^2 + \lambda \bar{y}^2 \varphi + \lambda \bar{y}^2 + \varphi \omega^2 \]

\[ = \frac{\varphi \lambda \alpha^2 + \varphi + 1}{(\varphi \lambda \alpha^2 + \varphi + 1)} \] (16)
This can be written as

\[ L_t^{DD} = \frac{\lambda \bar{y}^2}{\Lambda} + \frac{\omega^2\varphi}{1 + \varphi\Lambda}. \] (17)

For discretion, we have:

\[
L_{t+1}^{NCD} = \lambda \left[ \alpha (\pi_{t+1} - \pi_{t+1}^e) - \bar{y}_{t+1} \right]^2 + \varphi \left( f_{t+1} - f_{t+1}^{a,NCD} \right)^2 \\
= \frac{(\lambda \alpha^2 + 1) \bar{y}^2 \lambda (\varphi \alpha^2 + \varphi + 1)}{\varphi \lambda^2 \alpha^4 + 2\varphi \lambda \alpha^2 + \varphi + 1};
\]

and this can be written as

\[ L_{t+1}^{NCD} = \frac{\bar{y}^2 \lambda (\varphi \Lambda + 1) \Lambda}{\varphi \Lambda^2 + 1}. \] (18)

How do these payoffs change as \( \omega \) is varied? Jensen’s model of delegation with commitment uses \( \omega = \lambda \alpha \bar{y} \). In order for the proposed solution to be sustainable with one-period punishment by reversion to discretion, we need to have

\[
(L_t^{PR} - L_t^{DD}) + \beta (L_{t+1}^{PR} - L_{t+1}^{NCD}) \leq 0.
\] (19)

At \( \omega = \lambda \alpha \bar{y} \), the losses for commitment (\( L_t^{PR} \) or \( L_{t+1}^{PR} \)) are at a minimum and they equal \( \lambda \bar{y}^2 \). They are quadratic in \( \omega \), and increase for smaller or larger \( \omega \). The loss for deviation (when \( \omega = \lambda \alpha \bar{y} \)) are

\[ L_t^{DD} = \frac{\lambda \bar{y}^2}{\Lambda} + \frac{(\lambda \alpha \bar{y})^2 \varphi}{1 + \varphi \Lambda} = \frac{\lambda \bar{y}^2 (1 + \varphi \Lambda^2)}{\Lambda (1 + \varphi \Lambda)}, \]

and this is less than the loss for commitment. But it important to note is that the loss under deviation is increasing in \( \omega \) at this value of \( \omega \), while the loss under discretion is locally constant. It starts to rise as \( \omega \) increases, but only very slowly at first. Thus as \( \omega \) is increased above Jensen’s value of \( \lambda \alpha \bar{y} \), the temptation to cheat, which is the gap between the two \( (L_t^{PR} - L_t^{DD}) \), gets less. The punishment for cheating, \( (L_{t+1}^{NCD} - L_{t+1}^{PR}) \), also gets less as the value of \( \omega \) is increased, but it does so slowly at first because \( L_{t+1}^{NCD} \) is independent of \( \omega \) and \( L_{t+1}^{PR} \) is locally constant. So increasing the value of \( \omega \) above \( \lambda \alpha \bar{y} \) by a little bit is likely to make the commitment outcome easier to sustain.
In other words, an increase in the strength of $\omega$, the announced anti-inflation incentive given to the central bank, has a first order effect on the loss associated with the cheating scenario, but only a second order effect on the losses of the reputation solution, and no effect on the loss that is got in the punishment phase. There is a first order effect on the temptation to cheat, but only a second order effect on the punishment for cheating. Figure 1 plots the losses associated with the commitment outcome, deviation, and punishment, for different values of $\omega$.

Interestingly, the losses under reputation and under cheating actually become equal at one value of $\omega$. The curves representing the two loss functions become tangent at that point. At all other points the losses under cheating are less than the losses under reputation. If the loss under deviation (equation 17) is set equal to the loss under reputation (equation 18) then one obtains

$$\frac{\lambda \bar{y}^2}{\Lambda} + \frac{\omega^2 \varphi}{1 + \varphi \Lambda} = \lambda \bar{y}^2 + \varphi (\lambda \alpha \bar{y} - \omega)^2.$$  

The only value of $\omega$ that solves this equation is

$$\omega = \frac{(1 + \Lambda \varphi) \lambda \alpha \bar{y}}{\Lambda \varphi},$$

and thus the two functions are equal for this value of $\omega$. The slopes of the functions $L_{t}^{DD}$ and $L_{t}^{PR}$ are $\frac{2 \omega \varphi}{1 + \varphi \Lambda}$ and $-2 \varphi (\lambda \alpha \bar{y} - \omega)$ respectively. Equating them gives the same value of $\omega$ as above. So the two functions are tangents to each other at this point. At this point, the losses are

$$L_{t}^{DD} = L_{t}^{PR} = \lambda \bar{y}^2 + \varphi (\lambda \alpha \bar{y} - \frac{(1 + \Lambda \varphi) \lambda \alpha \bar{y}}{\Lambda \varphi})^2$$

$$= \lambda \bar{y}^2 - \varphi (\frac{\lambda \alpha \bar{y}}{\Lambda \varphi})^2$$

$$= \lambda \bar{y}^2 \left(\frac{\Lambda^2 \varphi + \Lambda - 1}{\Lambda^2 \varphi}\right).$$

Compared with the loss under discretion (which is $L_{t+1}^{NCD} = \frac{\bar{y}^2 \lambda (\varphi (\Lambda + 1) \Lambda)}{\varphi (\Lambda + 1) \Lambda}$) it is evident that the loss under discretion is lower than the loss under either reputation or deviation, for this value of $\omega$. So evidently the value of $\omega$ for which the critical value of the discount factor is minimized is lower than this one.
It is possible to find the value of the announcement $\omega$ for which the critical value of $\beta$ needed to sustain the reputational solution is at a minimum. This is done in the following paragraphs.

When the government is considering whether or not to deviate from the reputation solution, the criterion for not deviating is given by equation (19) above, in which $L_{t}^{PR}$ the reputational loss is given by (16), the loss under deviation $L_{t}^{DD}$ is given by (17) and the loss under the discretionary policy $L_{t+1}^{NCD}$ is given by (18). The parameter $\Lambda$ is defined as $\Lambda \equiv 1 + \lambda \alpha^2$. Thus the critical value of $\beta$ satisfies

$$\beta = \frac{L_{t}^{PR} - L_{t}^{DD}}{L_{t+1}^{NCD} - L_{t+1}^{PR}}.$$  

The numerator of this expression can be written as

$$L_{t}^{PR} - L_{t}^{DD} = \lambda \bar{y}^2 + \phi(\lambda \alpha \bar{y} - \omega)^2 - \frac{\lambda \bar{y}^2}{\Lambda} - \frac{\omega^2 \phi}{1 + \varphi \Lambda};$$

which with a little manipulation becomes

$$L_{t}^{PR} - L_{t}^{DD} = \frac{1}{\Lambda (1 + \varphi \Lambda)} [\lambda \alpha \bar{y} (1 + \varphi \Lambda) - \omega \varphi \Lambda]^2$$

$$= \frac{\varphi^2 \Lambda}{(1 + \varphi \Lambda)} \left[ \lambda \alpha \bar{y} \left( \frac{1 + \varphi \Lambda}{\varphi \Lambda} \right) - \omega \right]^2$$

$$= \frac{\varphi^2 \Lambda}{(1 + \varphi \Lambda)} \left[ \frac{\lambda \alpha \bar{y}}{\varphi \Lambda} + \lambda \alpha \bar{y} - \omega \right]^2,$$

while the denominator gives

$$L_{t+1}^{NCD} - L_{t+1}^{PR} = \bar{y}^2 \lambda \left( \frac{\Lambda + \varphi \Lambda^2}{1 + \varphi \Lambda^2} \right) - \lambda \bar{y}^2 - \phi(\lambda \alpha \bar{y} - \omega)^2;$$

which with some manipulation becomes

$$L_{t+1}^{NCD} - L_{t+1}^{PR} = \frac{\bar{y}^2 \lambda^2 \alpha^2}{1 + \varphi \Lambda^2} - \phi(\lambda \alpha \bar{y} - \omega)^2$$

$$= \phi \left[ \frac{\bar{y}^2 \lambda^2 \alpha^2}{\varphi (1 + \varphi \Lambda^2)} - (\lambda \alpha \bar{y} - \omega)^2 \right]$$

$$= \phi \left[ \frac{\bar{y} \lambda \alpha}{\sqrt{\varphi (1 + \varphi \Lambda^2)}} + (\lambda \alpha \bar{y} - \omega) \right] \left[ \frac{\bar{y} \lambda \alpha}{\sqrt{\varphi (1 + \varphi \Lambda^2)}} - (\lambda \alpha \bar{y} - \omega) \right].$$
Now all this can be put back together. The expression for the critical $\bar{\beta}$ can be written as

$$\beta = \frac{L_t^{PR} - L_t^{DD}}{L_t^{NC} - L_t^{PR}}$$

$$= \frac{\varphi \Lambda}{(1 + \varphi \Lambda)} \frac{(A - \omega')^2}{[B - \omega'][B + \omega']}$$

in which $\omega' \equiv \omega - \lambda \alpha \bar{y}, A \equiv \frac{\lambda \alpha \bar{y}}{\varphi \Lambda}, B \equiv \frac{\lambda \alpha \bar{y}}{\sqrt{\varphi(1 + \varphi \Lambda^2)}}$.

We want to choose $\omega$ to minimize the critical value. We are looking at values of $\omega'$ that lie in the range $(0, B)$. That is equivalent to looking at values of $\omega$ that are at least as great as in the Jensen solution and which go up to the value at which the punishment for cheating becomes zero, i.e., where the loss due to discretion equals the loss under reputation. At the minimum critical value,

$$-\frac{2}{A - \omega'} + \frac{1}{B - \omega'} - \frac{1}{B + \omega'} = 0.$$ 

Multiplying through by $(A - \omega')(B - \omega')(B + \omega')$ and tidying up gives

$$\omega' = B^2/A,$$

and the value of the function at the minimum point is

$$\bar{\beta}^{D^*} = \min_{\omega' \in (0, B)} \left[ \frac{L_t^{PR} - L_t^{DD}}{L_t^{NC} - L_t^{PR}} \right]$$

$$= \frac{\varphi \Lambda}{(1 + \varphi \Lambda)} \frac{(A - \frac{B^2}{A})^2}{[B - \frac{B^2}{A}][B + \frac{B^2}{A}]}$$

$$= \frac{\varphi \Lambda}{(1 + \varphi \Lambda)} \frac{A^2 - B^2}{B^2}$$

$$= \frac{\varphi \Lambda}{(1 + \varphi \Lambda)} \left( \frac{\lambda^2 \alpha^2 \bar{y}^2}{\varphi^2 \Lambda^2} - \frac{\lambda^2 \alpha^2 \bar{y}^2}{\varphi (1 + \varphi \Lambda^2)} \right) \frac{\varphi (1 + \varphi \Lambda^2)}{\lambda^2 \alpha^2 \bar{y}^2}$$

$$= \frac{1}{\Lambda(1 + \varphi \Lambda)}.$$ 

Since this value is less than $1/\Lambda$ this proves that the critical $\beta$ under delegation with any announcement is less than under simple discretion.
The result is illustrated in Figure 2, in which the function (20) is plotted against $\omega'$. It shows that at $\omega' = 0$, the critical value of the discount factor equals $\beta^D (> 1/\Lambda)$ the value obtained by Jensen. As $\omega'$ increases, the critical value of $\beta$ falls until it reaches a minimum, at $\omega' = B^2/A$, when the critical value is $\beta^{Ds}$ as defined above. For values of the incentive $\omega'$ greater than $B^2/A$ the value of the critical discount factor rises again, and approaches infinity as $\omega'$ approaches $B$. A government with any given value of the discount factor would wish to employ the lowest value of the announced incentive scheme $\omega'$ consistent with sustaining zero inflation in the reputational equilibrium. The left hand branch of the relationship between $\omega'$ ($\in (0, B)$) and $\beta$ is therefore the relevant one. Delegation with an announced incentive scheme and reappointment costs therefore permits some types of government, those types whose discount factor $\beta$ lies between $1/\Lambda$ and $\beta^{Ds}$ ($= \frac{1}{\Lambda(1+\psi_M)} < 1/\Lambda$), who would not have been able to sustain zero inflation in the absence of delegation with reappointment costs, to sustain zero inflation in the reputational equilibrium.

Therefore this argument has shown that if the government is allowed to make any announcement of $\omega$ then it is possible to have zero inflation using delegation with a lower discount factor than is needed to sustain zero inflation with no delegation of policy.

It may be argued that this result is merely a curiosity, because it involves the government in making an announcement about $\omega$ that is not honoured. The actual value of the incentive that is used is always less than the announced one. This behaviour is expected, and the private sector expects and gets zero inflation – so they are happy. Arguably this scenario does not correspond with the behaviour any government and central bank in practice.

A counter-argument to such a position is that the scenario presented above is just as realistic as Svensson’s (1997) idea that optimal inflation and output stabilization can be achieved by giving a central bank a negative inflation target – knowing all the while that discretionary behaviour by the central bank will lead to its generating zero inflation on average.

The key point that we would stress here is the logical one that Jensen’s result is based on an implicit assumption that may be too strong. If this assumption is relaxed, then his result disappears, and delegation with reappointment costs does indeed solve the time-inconsistency problem. It does

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But notice that the equilibrium the static one-shot game shares this feature. The actual value of the incentive that is used there is always less than the announced one.
not merely relocate it.

5 Conclusion

In this paper we have argued that Jensen’s result, to the effect that introducing costs of re-appointing a central banker does not make it any easier to sustain a zero-inflation outcome, is based on his making the assumption that the government must announce the same incentive scheme as the one that it actually uses in practice in the reputational play of the game. If this assumption is relaxed, then the result disappears also: re-appointment costs may make it easier to sustain the zero-inflation outcome. The reason is that the announcement of the incentive scheme at the start of the period affects the incentives of the government later in the period when it puts in place the actual incentive scheme. By announcing a more vigorous scheme at the start of the period, the government has less incentive to deviate from the reputational zero-inflation outcome. However, the proposed solution does have the unattractive feature that the announced incentive differs from the actually implemented one. From certain perspectives this may seem an odd model of reputational behaviour. However is perhaps a widespread observation in politics, that politicians make promises, generally in a effort to get votes, by which they bind themselves to carrying though certain policies. There is likely to be slippage from these commitments when the time comes to deliver on them. Voters know and expect this, but nevertheless there are political costs to be paid by politicians who appear to have made no effort to honour their pledges, and whose actual policies deviate grossly from the announced ones. Therefore we would argue that our solution in this paper contains important elements of descriptive realism.

The key point that has been made here is that delegating monetary policy to an independent central banker, with an announced or promised incentive scheme (or equivalently, contract for the central banker) that is costly to change, can, under circumstances which are not extreme, be more credible than the conduct of monetary policy without delegation. This results from reappointment costs.

But there are important distinctions to be made between the analysis undertaken here and the standard theory of monetary delegation, based on the static one-shot game. First we have shown that the delegation solution
for time-inconsistency can be conducive to reputation-building for the government and hence is not merely an alternative to the reputational solution, as is usually claimed in the standard theory, but adds something to it.

Moreover, in much of the literature on delegation of monetary policy, incentive schemes or policy targets are introduced in order to constrain the behaviour of the central banker according to the objectives of the government. The view of the delegation process that emerges from the analysis in this paper is rather different, as one of the main effects of the reappointment costs is, in certain circumstances, to enhance a government’s reputation for low inflation. Because delegation is done long in advance (in advance, that is, of inflation expectations being formed, or shocks – not modelled here – hitting the system) the government is led to announce an incentive scheme for the central bank that has good long-term properties. As subsequent changes to the incentive scheme are costly, the announcements constrain the behaviour of the government, and limit its interference, as it were, in the central bank’s policy making. Hence we agree with McCallum’s (1997, p.109) observation that “...the main effect of such arrangements [as those of New Zealand’s] is not principally to constrain the central bank to act in accordance with the government’s objectives, but rather to constrain the government by increasing the difficulty of its bringing pressure to inflate upon the central bank.... Arrangements such as those of New Zealand’s, therefore, give the central banks an increased opportunity to behave in a rule-like, committed fashion”.

REFERENCES


Figure 1 – Losses associated with the pre-commitment, deviation and punishment outcomes as a function of the announcement of the government.

\[ L_{\text{PR}}(\omega), \quad L_{\text{DD}}(\omega), \quad L_{\text{NCD}}(\omega) \]

\[ \omega' = \omega - \lambda \alpha \bar{y} \]
Figure 2 – The set of feasible announcements and the optimal announcement of the government

\[ \hat{\beta}^{D}(\omega' = 0) \]

\[ \hat{\beta} = 1 / \Lambda \]

\[ \beta \]

\[ \hat{\beta}^{D} \]

\[ \omega' = \omega - \lambda \bar{\alpha} \bar{y} \]