Credit Booms and Freezes

Arup Daripa
Birkbeck, University of London

October, 2018

Abstract

We show that short-term borrowing by intermediaries creates a rollover-coordination problem endogenously. We then introduce ambiguity and show that the coordination outcome is discontinuous in ambiguity attitude: starting from neutrality, any aversion implies a complete collapse for every value of fundamentals for which coordination matters, whereas any ambiguity loving causes successful coordination for all such values of fundamentals - a "lending euphoria." These help clarify credit booms even when backed by sub-prime assets as well as sudden credit freezes at the advent of bad news even for borrowers investing in non-sub-prime assets.

JEL CLASSIFICATION: G2, C7, E5.

KEYWORDS: Rollover coordination, global games, ambiguity, ambiguity-sensitivity, α-maxmin expected utility, multiple equilibria, credit booms, liquidity freezes

1 Introduction

In the run up to the financial crisis of 2007-08, much of the funding raised by financial intermediaries was supplied through arrangements that were inherently fragile. Intermediaries made widespread use of short-term credit sources such as repo markets, auction-rate securities markets and asset-backed commercial paper markets. Since short-term loans must be rolled over frequently, and funding liquidity is maintained only if enough lenders roll over loans, such markets depend critically on coordination among lenders. The consequent vulnerability to runs due to coordination failure makes such markets fragile.[1] A further crucial aspect of such markets was that the underlying assets had, in many cases, weak fundamentals. For example, a significant proportion of asset-backed securities issued in the years preceding the crisis were backed by subprime mortgages. A natural question, then, is why do such fragile arrangements proliferate successfully and further, why do they succeed even when backed by assets with relatively weak fundamentals?

Related questions arise from the subsequent collapse of coordination among lenders across markets. Once the crisis arrived in 2007 following bad news from housing markets, financial markets experienced multiple rounds of systemic runs in short-term credit markets. As Gorton and Metrick (2012) document, there was a run on repo markets which spread from subprime housing assets to non-subprime assets with no direct connection to the housing market. A similar run took place in the asset-backed commercial paper market, even for paper not exposed to subprime mortgages.[2] Further, a widespread freeze of liquidity markets followed the failure of Lehman Brothers in September 2008. A run ensued on money-market mutual funds and the interbank market was severely disrupted. Further questions, then, are why might liquidity mar-

[1] In a typical repo arrangement, a financial intermediary purchases an asset (e.g. a mortgage-backed security) with face-value 100 (say) and finances the purchase by borrowing. A lender is willing to lend with a certain haircut (say 10% - i.e. the lender willing to lend 90), with the asset held by the lender as the collateral. The arrangement is short-term and must be rolled-over frequently to maintain funding. As Gorton and Metrick (2012), Gorton (2012) explain, a repo loan is similar to a bank deposit, and if lenders, concerned by some bad news about the collateral, raise the haircut, the intermediary faces a sudden withdrawal of funds - a scenario similar to a bank run. Similar coordination-dependence and consequent fragility characterized markets such as those for asset-backed commercial paper and auction-rate securities.

[2] See Covitz, Liang and Suarez (2009).

kets freeze suddenly, why runs spread to non-subprime assets, and why the failure of Lehman Brothers had a such a large negative impact across liquidity markets.

In this paper, we first show that a dependence on lender coordination for funding – and consequent fragility – arises endogenously. Given such dependence on coordination, we then show that a common thread of explanation ties the questions posed above. If the environment becomes characterized by ambiguity, the coordination outcome changes discontinuously when lender outlook changes and the equilibrium becomes "detached from fundamentals" in the following sense. Under an (even slightly) optimistic outlook, a "lending euphoria" ensues, so that lending coordination succeeds for every value of fundamentals for which coordination matters. Thus even borrowers investing in projects with relatively weak fundamentals find it easy to raise and rollover funds. However, once some bad news arrives so that the outlook becomes (even slightly) pessimistic, lenders act in a panic-stricken manner so that lending collapses across all values of fundamentals, implying even borrowing based on prime assets is not immune to runs. Thus for every value of fundamentals, there are two extreme equilibria, and equilibrium selection is tied to lender outlook. This is a complete departure from the usual unique equilibrium under global games without ambiguity. We identify the precise mechanism accounting for such dramatically different outcomes under ambiguity. Further, we show that the a consequence of the discontinuous change in coordination is that policy itself might act as a trigger for liquidity crises, and clarify why this accounts for the Lehman Brothers failure having a large negative impact on liquidity.

We clarify the results and their implications below.

First, we establish the emergence of a coordination problem among lenders as an endogenous outcome. In the literature on panic-driven bank runs following the seminal work of Diamond and Dybvig (1983), a coordination problem is typically assumed to arise. In a three-period model, funding is raised and invested initially, with investments maturing in the last period. In the middle period, success or failure of coordination among lenders decides whether funding continues or a run arises requiring liquidation of investments. Here we adopt this model, but allow intermediaries to invest in liquid assets alongside illiquid ones in the initial period. Enough aggregate liquidity would allow liquidation of assets in the middle period without a fire-sale, rendering rollover coordination irrelevant and removing the possibility of runs. We then show that, nevertheless, optimal liquidity choice of intermediaries is such that a coordination problem *necessarily* arises in equilibrium. Fragility of funding is therefore an endogenous outcome.

Second, we investigate how the coordination outcome responds to the emergence of ambiguity when agents are ambiguity-sensitive. As is well known, in a complete information setting, multiple coordination equilibria arise: for all values of fundamentals for which coordination matters, there is an equilibrium characterized by coordination success and another by failure and runs.[3] These are sunspot equilibria detached from fundamentals and the theory cannot say what guides the choice of equilibrium. The literature on global games[4] shows, in contrast, that introducing a small amount of incomplete information can remove the indeterminacy and a unique equilibrium obtains for every value of fundamentals. Here, we model coordination using a standard global game: there is a unit measure of lenders to each intermediary, and each such lender receives a signal of the state of fundamentals θ for that intermediary with some noise. There is an interval $[\underline{\theta}, \overline{\theta}]$ over which coordination matters, and (as standard in the global games literature) in the case without ambiguity, there is an equilibrium threshold θ^* in the interval such that coordination succeeds for and only for values of θ above the threshold. We then introduce ambiguity in signals and show that the nature of the coordination outcome in global games changes completely. Specifically, we model preference towards ambiguity using the well-known α-maxmin representation[5] which captures the degree of ambiguity aversion or loving parsimoniously through the parameter α: an agent is ambiguity averse, neutral or loving according as α exceeds, is equal to or lower than $1/2$. We introduce ambiguity in the global games setting by assuming that each agent perceives a possible bias (positive or negative) in their own signal relative to that of others. As the noise goes to zero, the bias vanishes as well, but at a slower rate. Then, for small values of noise,[6] we show that if lenders are even slightly ambiguity-loving (i.e. for *any* $\alpha < 1/2$), coordination succeeds for *all* values of θ in the interval $[\underline{\theta}, \overline{\theta}]$. We call this case "lending euphoria." Similarly, for *any* $\alpha > 1/2$,

[3]See, for example, Diamond and Dybvig (1983).

[4]See, for example, Morris and Shin (1998, 2003).

[5]See **?**, **?**, **?**. In the α-maxmin representation, an agent puts a weight α on the most pessimistic outcome and the remaining weight on the most optimistic outcome. The maxmin expected utility representation (Gilboa and Schmeidler, 1989) obtains as a special case with extreme ambiguity aversion ($\alpha = 1$).

[6]in other words, the results hold not just in the limit as noise goes to zero, but away from the limit as well.

coordination collapses for all such values of θ. Figure 1 shows how the coordination outcome varies discontinuously as a function of the ambiguity-sensitivity parameter α.

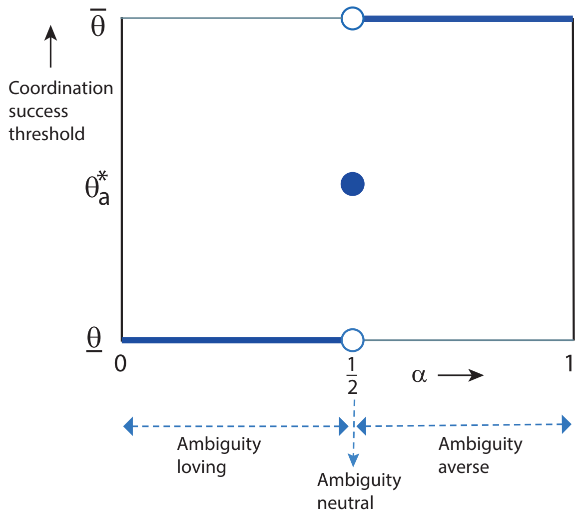

Figure 1: The diagram shows how the coordination threshold (the value such that coordination succeeds for and only for higher values of fundamentals) changes discontinuously with ambiguity attitude. If lenders are insensitive to ambiguity ($\alpha = 1/2$), there is a unique equilibrium for all θ as in the case without ambiguity: rollover succeeds above a threshold $\theta^*_a \in [\underline{\theta}, \overline{\theta}]$. Any degree of ambiguity aversion (any $\alpha > 1/2$) then leads to a complete collapse of coordination for all values of θ for which coordination matters, i.e. for all $\theta \in [\underline{\theta}, \overline{\theta}]$. In this case the threshold jumps up to $\overline{\theta}$. Similarly, any degree of ambiguity loving (any $\alpha < 1/2$) leads to coordination success for all values of θ in the interval so that the threshold jumps down to $\underline{\theta}$.

Note that in the global games model without ambiguity, the coordination outcome depends on fundamentals: success is achieved only if the fundamentals are high enough. The results here show that, once we introduce ambiguity in a global game setting, the coordination outcome becomes detached from fundamentals: for all values of fundamentals for which coordination matters we again obtain the same multiple equilibria as under complete information, but with one crucial difference: now the equilibria are not sunspot driven – they are tied fully to a preference parameter related to ambiguity-attitude (or degree of pessimism/optimism). This allows us to predict the choice of equilibrium under different circumstances and show when a lending euphoria or collapse might occur. These results also contribute to the literature on global games.

The mechanism that gives rise to both euphoria and collapse works as follows. Under ambiguity, if agents are even slightly pessimistic (or ambiguity averse), each agent tries to take the coordinating action "after others" - i.e. if an agent perceives that other agents will coordinate for values of fundamentals above $\widehat{\theta}$, their optimal strategy would be to take the coordinating action above some $\widehat{\widehat{\theta}} > \widehat{\theta}$. Since all agents try to get above the coordinating threshold of others in this way, coordination naturally unravels for all values of fundamentals, giving rise to an indiscriminate panic. Since this is true for any degree of ambiguity aversion, this is a discontinuous break in coordination. Similarly, with even the slightest degree of optimism (or ambiguity loving), each agent tries to take the coordinating action "before others" - i.e. in this case $\widehat{\widehat{\theta}} < \widehat{\theta}$. All agents trying to get below the coordinating threshold of others then leads to coordination success for all values of fundamentals - an indiscriminate "euphoria" in lending.

Finally, we discuss implications of our results for policy. We show that the results imply that policy itself can precipitate a sudden liquidity freeze when it does not conform to market expectations, which sheds light on the fact that the policy decision not to rescue Lehman Brothers had a large negative impact on liquidity markets. The results further suggest that breaking up large banks (a course of action that has been suggested by some policymakers) might worsen systemic stability.

1.1 Related literature

Rochet and Vives (2004) and Vives (2014) study global game coordination models of loan rollover in the standard three-period framework. Here we study a global game model and show that a coordination problem among lenders arises endogenously through insufficient equilibrium liquidity choice in the intermediation sector. Further, the focus of our work is different: we study the interaction between the coordination outcome and ambiguity in signals.

More direct antecedents in studying coordination under ambiguity are papers by Ui (2009) and **?**, who study the effect of ambiguity in a two-player global game with players' preferences represented by maxmin expected utility. In these models, a small degree of ambiguity has a small effect in reducing coordination. Our model assumes a continuum of agents, and our results on the relation between ambiguity attitude and the nature of coordination outcome (collapse or euphoria in lending) and the fact that the coordination outcome gets detached from fundamentals are new and very different from results in these papers. We also clarify the precise mechanism behind the collapse and euphoria: when agents think their own signal might be biased (upwards or downwards) relative to signals of others, even an arbitrarily small degree of ambiguity aversion (loving) results in each agent adopting the coordinating action above a threshold that is above (below) the coordination threshold of others, destroying (making a success of) the possibility of coordination. This is again very different from the papers mentioned. However, in terms of modeling ambiguity and results for the case of ambiguity aversion, there are some similarities and differences with **?** worth mentioning. **?** assumes a perceived bias in signals with a fixed interval of values for the bias and shows that ambiguity reduces coordination, and when signal noise goes to zero, so that ambiguity-to-noise ratio goes to infinity, coordination breaks down. Our modelling of ambiguity is similar: we assume a perceived bias in signals, but also allow the bias to vanish with signal noise, albeit more slowly. Our results are then derived for small noise - so our collapse and euphoria results are not limiting results, but also hold for a finite ambiguity-to-noise ratio.

Papers by Angeletos, Hellwig and Pavan (2006), Angeletos and Werning (2006), Angeletos, Hellwig and Pavan (2007) have shown that multiple equilibria can arise in a global games setting. This paper shows another reason for multiplicity to arise: under signal ambiguity, two extreme equilibria arise for each value of fundamentals - one with successful coordination and the other with runs. This multiplicity is similar to that under complete information with the crucial difference that these are not sunspot equilibria - equilibrium choice is tied down by the preference parameter capturing ambiguity attitude.

The question of market freezes has been studied by Acharya, Gale and Yorulmazer (2011). In their work, a freeze arises from the evolution of debt-capacity of assets in repeated rounds of rollovers. Here, the model is of a traditional "panic." Since the seminal work of Diamond and Dybvig (1983), panics have been modeled as selecting the "bad" equilibrium out of multiple coordination equilibria. The global games approach[7] allows linking equilibrium choice to fundamentals, and derivation of a unique equilibrium for every value of fundamentals. The contribution of this paper is to show that once ambiguity is introduced in a global games setting, the results change dra-

[7]See the important contribution of Goldstein and Pauzner (2005).

matically. For every value of fundamentals for which coordination matters, the unique outcome is a collapse of coordination under any degree of pessimism, and successful coordination under any degree of optimism. Thus, in sharp contrast with the case without ambiguity, the coordination outcome becomes independent of fundamentals and depends entirely on ambiguity attitude.

Finally, it worth mentioning the important literature – largely developed since the last crisis – that focuses on amplification of crises through spirals of asset price movements arising through changes in the balance sheets of intermediaries (see Brunnermeier and Pedersen, 2009, Adrian and Shin, 2010).[8] Our work complements this approach in the sense that euphoric coordination under optimism sets-up the scenario for a subsequent fall, and once some bad news turns optimism to even slightest pessimism, coordination collapses completely, which would lead to fire sales, and subsequent spirals of loss could then further exacerbate the problem.

2 The model

Let us first summarize the model and results. We study a three period model in which large numbers of lenders extend short term loans initially (period 0) to a unit measure of intermediaries. Intermediaries invest 1 unit in a long term project and also decide how much liquidity to hold. In period 1, each lender to an intermediary must decide whether to rollover their loan. If enough lenders decide not to rollover loans to an intermediary, it faces liquidation in period 1. Assets of a liquidated intermediary are sold in the secondary market to other intermediaries at a "cash-in-the-market" price. If intermediaries carry enough aggregate liquidity, the secondary market price is high, so that those that are liquidated do not face a fire sale of assets. In this case lenders to an intermediary face no coordination problem (rolling over becomes a dominant strategy). We show that this cannot happen in equilibrium. In equilibrium, markets are necessarily illiquid to some extent and therefore a coordination problem arises endogenously. Later we introduce the possibility that in period 1, the environment becomes characterized by ambiguity and show that the coordination outcome varies discontinuously with respect to ambiguity attitude.

[8]See also Brunnermeier (2009) for an overview of the approach, and Shleifer and Vishny (2011) for a discussion of fire sales in general.

Our modelling of the secondary market for assets (intermediaries buying assets of liquidated intermediaries at cash-in-the-market prices) follows Acharya and Yorulmazer (2008). The difference is that in our model the choice of liquidity held by an intermediary is endogenous.[9]

Let us now specify the model in detail.

The banking system comprises a unit measure of identical financial intermediaries. Each intermediary has a project that is operated over three periods, 0, 1 and 2. Each intermediary's project requires 1 unit of funding initially, which must be maintained in period 1. In period 2 the returns are realized. If in period 1 investment by the intermediary falls below 1, the project must be liquidated. Instead of this risky project, the intermediary can also invest in a safe asset at the initial period. The risk-free net return is normalized to 0.

There is a large mass of potential lenders. Lenders are risk neutral. Each lender to an intermediary lends 1 unit of funds initially, and must decide in period 1 whether to rollover the unit to the intermediary or to withdraw and invest in the safe asset.

For simplicity, this type of short term funding is assumed to be the only source of funds for an intermediary. Initially intermediary i borrows funds of $1 + \psi_i$, where $\psi_i \geqslant 0$. Out of this, 1 unit is invested into the long-term project and ψ_i is held in liquid assets, which, in this case, is the risk-free asset.[10] We refer to ψ_i as the "liquidity carried by intermediary i." We assume that ψ_i can be chosen from the interval $[0, \overline{\psi}]$, where $\overline{\psi}$ is large in the sense that if all intermediaries held a liquidity of $\overline{\psi}$, the market in period 1 would have enough market liquidity so that there would be no rollover problem.[11]

If in period 1 sufficient funds are rolled over so that the project of intermediary i does not have to be liquidated (i.e. the 1 unit of investment can be maintained in period 1), the return of intermediary i in period 2 depends on the fundamentals of its project, captured by project "type" θ_i. Each intermediary i draws a project type θ_i independently from a uniform distribution on $[0, 1]$.

---

[9]Indeed, in concluding their paper, Acharya and Yorulmazer remark that it would be interesting to endogenize banks' ex ante choice of a portfolio of liquid and illiquid assets. This is precisely what we do here.

[10]We show later that in equilibrium there is reason to choose $\psi_i > 0$.

[11]We state this assumption more formally later (as assumption 1 in section 3.2) once the model is fleshed out and all relevant notation has been introduced.

Specifically, if the project of intermediary i is not liquidated in period 1, in period 2 it returns $R > 1$ (success state) with probability θ_i , and 0 (failure state) with the residual probability.

In the event the project of an intermediary is liquidated, its assets are sold off to other intermediaries (those not facing liquidation). The price is determined through "cash-in-the-market" pricing[12], which is the total available liquidity held by non-liquidated intermediaries divided by the total assets in liquidation. This is determined endogenously.

Notation In the following analysis, we consider the problem of a typical intermediary i. Some of the variables are choice variables of i (e.g. i chooses to hold liquid assets of ψ_i), while other relevant variables are determined by aggregate actions or actions of intermediaries other than i. To distinguish these from variables that i can control, such variables are denoted by adding a hat. For example, the liquidity held by other intermediaries is denoted by $\widehat{\psi}$ and the cash-in-the-market price of liquidated assets, which depends on the entire profile of liquidity across the intermediation sector, is denoted by $\widehat{m}$.

The cash-in-the-market price and liquidation payoff of lenders and intermediary The asset does not grow between the initial and the first period so that, for any intermediary, 1 unit of asset is liquidated in the event sufficient funds are not successfully rolled over. Thus total assets on sale in the secondary market is simply the measure of liquidating intermediaries (times 1). The cash-in-the-market price of assets in the secondary market is given by

$$\begin{aligned}\widehat{m} &= \frac{\text{Available liquidity in equilibrium}}{\text{Total assets on sale}} \\ &= \frac{\text{Total liquidity carried by non-liquidating intermediaries}}{\text{Measure of liquidating intermediaries}}.\end{aligned}$$

We assume that an intermediary operates under a sequential service constraint. Lenders who do not rollover their loan at the start of period 1 are placed in a queue. Initially, lenders are returned 1 using the liquid assets ψ_i. If this runs out and there are still more

[12]See Allen and Gale (1994) and Allen and Gale (1998).

lenders who did not rollover, the intermediary must liquidate the project which yields $\widehat{m}$. All remaining lenders (including those who did rollover) get paid an equal share of $\widehat{m}$. Note that once ψ_i is used to pay off a measure ψ_i of lenders, remaining lenders are measure 1. Therefore they each get paid $\widehat{m}$.

Note that if all lenders (measure $1 + \psi_i$) to intermediary i withdraw at the start of period 1, a fraction $\psi_i/(1 + \psi_i)$ get paid 1 and the rest get paid $\widehat{m}$. In this case the liquidation payoff of a lender to intermediary i is $\frac{\psi_i}{1+\psi_i} + \frac{1}{1+\psi_i}\widehat{m}$.

The liquidation payoff of intermediary i is zero.

The payoff of intermediary and lenders if rollover succeeds Now consider the payoff of an intermediary that does not need to liquidate in period 1. Such an intermediary earns a return in period 2 from two sources: its original investment project as well any assets purchased in period 1.

We assume that each unit of assets purchased by an intermediary is used to augment the intermediary's own investment project and adds to its returns by δR, where $\delta < 1$ represents some loss of returns due to change in hands (or due to the fact that assets from one project does not fit exactly with another project).

Therefore, in the event its own project is not liquidated, intermediary i with liquidity ψ_i buys $\psi_i/\widehat{m}$ units in the secondary market, and gets a return of $R + \frac{\psi_i}{\widehat{m}}\delta R$ with probability θ (and 0 with the residual probability).[13]

Note that the lenders to intermediary i get paid only if i succeeds (probability θ_i). It follows that if a lender rolls over his loan to intermediary i, and the project of the intermediary continues to period 2, the payoff of the lender is $\theta_i r - 1$.

[13] Here we assume that the purchased assets are subsumed in the own project of the intermediary - and a positive return arises only when the intermediary's own project succeeds. The interpretation is that the success probability is determined by factors such as management efficiency which then applies to the purchased assets as well. This makes for the simplest model. However, we could alternatively assume that the purchased assets generate a return independently of the original project, so that the intermediary gets R with θ, and $\frac{\psi_i}{\widehat{m}}\delta R$ with an independent probability θ'. This would complicate the algebra but the qualitative conclusions of the model would remain unchanged.

2.1 The timeline

The timeline of the model is as follows.

Period 0: Intermediaries simultaneously choose how much liquidity to carry, i.e. each intermediary i chooses ψ_i, and borrows $(1+\psi_i)$. Of the funds raised, 1 is invested in the long-term project and ψ_i is carried as liquid asset. Each intermediary promises to pay a rate of return of $(1+r)$ in period 2 if the project survives (i.e. sufficient funds are rolled over in period 1) and succeeds.

Period 1: Each intermediary needs total funds of 1 to continue with the project. If lenders renew funds of at least 1, investment proceeds as normal. Otherwise, the investment is liquidated at the cash-in-the-market price $\widehat{m}$ (determined endogenously in equilibrium).

Period 2: If the investment survived in period 1, the intermediary augments own project by buying assets from the liquidated intermediaries. The project of intermediary i succeeds with probability θ_i and in the event of success, the lenders are paid the promised returns.

2.2 The structure of the argument

The structure of the argument in section 3 is as follows.

1. Note that in equilibrium, the aggregate choice of liquidity holding by intermediaries in period 0 drives the value of $\widehat{m}$, which is the secondary market value of assets that an intermediary receives in period 1 in the event of liquidation. Any given intermediary i is atomistic and takes $\widehat{m}$ is given: its own actions cannot change $\widehat{m}$. To solve for the equilibrium, initially we take $\widehat{m}$ as given and solve the rollover coordination game in period 1 for intermediary i.

   We model the period 1 rollover problem using a global game coordination model. Each lender receives a signal of θ_i with a small noise. We consider monotone equilibria, and show that a unique equilibrium exists. We derive the equilibrium as noise vanishes. This gives us a threshold function $\theta_i^*(\widehat{m})$: for any given $\widehat{m}$, rollover succeeds if and only if $\theta_i \geqslant \theta_i^*(\widehat{m})$.

2. Once we have solved the period 1 rollover problem, we fold back to analyze the problem of choosing liquidity holding ψ_i in period 0. Once ψ_i^* is known for all intermediaries, we can calculate aggregate liquidity which gives us the equilibrium value $\widehat{m}$. Substituting this in $\theta_i^*(\widehat{m})$, we get the equilibrium value θ_i^* of the coordination threshold. Our main result in this part of the paper is that the choice of liquidity and therefore the value of $\widehat{m}$ is such that a non-trivial coordination problem necessarily arises for every intermediary in equilibrium.

Once we establish the endogenous coordination problem and derive the equilibrium coordination threshold (section 3), we introduce ambiguity (section 4) and show that the nature of the coordination outcome changes dramatically. Section 5 then discusses implications for policy, and section 6 concludes.

3 Endogenous Coordination Problem

In this section we characterize the equilibrium behavior of lenders to any intermediary. Using a global games approach, we first analyze (in section 3.1) the rollover problem in period 1 for any given value of secondary market price $\widehat{m}$. We then fold back and analyze the liquidity choice problem in period 0 (in section 3.2), and establish the equilibrium value of $\widehat{m}$, which then allows us to characterize the equilibrium rollover problem in period 1. The main result is that the optimal liquidity choice in period 0 leads to a value of $\widehat{m}$ such that a rollover-coordination problem necessarily arises in equilibrium.

3.1 Solving the rollover problem in period 1

First, we specify a global game model of rollover coordination among lenders to any intermediary i. We then solve for the equilibrium coordination threshold for any given $\widehat{m}$.

3.1.1 Incomplete Information and Signals

As noted in section 2 above, the state of fundamentals θ_i for intermediary i is drawn from a uniform distribution on the interval $[0, 1]$.

For intermediary i, when rollover succeeds for a project of type θ_i, a lender to i receives an expected payoff of $\theta_i r$. It follows that not rolling over is a dominant strategy for $\theta_i < \underline{\theta}$ where $\underline{\theta} r = 1$, i.e.

$$\underline{\theta} = \frac{1}{r} \tag{1}$$

For any $\theta_i < \underline{\theta}$, the net expected return from rolling over is negative even if all others roll over. Therefore not rolling over is the dominant strategy in this case.

Next, we assume that for any i, when θ_i is close enough to 1, outside investors with plenty of liquidity enter the market in period 1 to buy any assets from liquidation. In this case the asset market is liquid, so that $\widehat{m} \geqslant 1$. This assumption provides an upper bound to the coordination problem region. Specifically, we assume that there is a (small) $\Delta > 0$ such that for $\theta_i > 1 - \Delta$, the asset market is liquid at date 1 so that $\widehat{m} \geqslant 1$. We have

$$\overline{\theta} = 1 - \Delta. \tag{2}$$

For any $\theta_i > \overline{\theta}$, the net expected return from rolling over is non-negative even if the project is liquidated. Therefore rolling over is the dominant strategy in this case.

For $\theta_i \in [\underline{\theta}, \overline{\theta}]$, whether investment proceeds depends on the size of the total funds raised. In this interval a coordination problem arises, and would give rise to multiple equilibria if θ_i were common knowledge. Here, we model the coordination problem as a global game: the type θ_i of an intermediary is not common knowledge - lenders to any intermediary receive signals as follows.

Each lender to any intermediary i receives a signal x of θ_i in period 1, where x is drawn from a uniform distribution on $[\theta_i - \varepsilon_i, \theta_i + \varepsilon_i]$, $\varepsilon_i > 0$. Conditional on the true state θ_i, the signals are independent across agents. After receiving the signal, lenders simultaneously decide whether to roll over their loan or withdraw. The decisions made result in aggregate loan rollover of $L_i(\theta_i)$ to each intermediary i.

Finally, a technical requirement. To ensure that the signal intervals are well defined at the boundaries of the relevant range of fundamentals, we assume that $1/r > 2\varepsilon_i$ and $\overline{\theta} + 2\varepsilon_i < 1$, which can be rewritten as $2\varepsilon_i < \Delta$.

3.1.2 Equilibrium

We consider monotone equilibria. In a monotone equilibrium, there is a threshold x^* such that agents rollover their loan if and only if $x \geqslant x^*$, and there is a threshold value of fundamentals θ^* such that rollover is successful if and only if $\theta \geqslant \theta^*$.[14]

There are two possibilities. First, the ψ_i values chosen in period 0 could be high so $\widehat{m}$ is at least 1. Second, ψ_i values could be such that $\widehat{m} < 1$.

Consider the first case. It is easy to see that in this case no coordination problem would arise in period 1.

Lemma 1. *If $\widehat{m} \geqslant 1$, intermediary i faces no coordination problem in period 1, so that $\theta_i^* = \underline{\theta}$.*

Proof: The expected payoff of a lender to intermediary i who rolls over the loan is either $\theta r > 1$ (in the case loans are rolled over in period 1 so the investment proceeds to the final stage), or $\widehat{m}$ (if not enough loans are rolled over so investment is liquidated in period 1). If $\widehat{m} \geqslant 1$, it is obvious that for a lender, rolling over the initial loan of 1 is a dominant strategy (the payoff is 1 or more irrespective of whether rollover succeeds or fails, i.e. irrespective of the actions of other lenders). It follows that intermediary i faces no coordination problem and $\theta_i^* = \underline{\theta}$.‖

Next, we derive the coordination threshold θ_i^* in the second case where $\widehat{m} < 1$. Later when we analyse the first stage at which ψ_i is chosen, we show that the in equilibrium we must have $\widehat{m} < 1$ so that this case is the relevant one, also implying that a non-trivial coordination problem necessarily arises in equilibrium.

To reduce notation, **we suppress the subscript i in the following calculation**. Later we reintroduce the subscript.

The aggregate size of the loan to an intermediary is the mass of agents who receive

[14] The restriction to monotone equilibria is without loss of generality. The unique monotone equilibrium in the model is also the only equilibrium irrespective of the strategies considered. From the uniqueness proofs of Morris and Shin (2003, 2004), it follows directly that investing for and only for $x \geqslant x^*$ is the only strategy that survives iterative elimination of strictly dominated strategies.

$x \geqslant x^*$. Thus

$$L(\theta) = \begin{cases} 0 & \text{if } \theta < x^* - \varepsilon, \\ \dfrac{\theta + \varepsilon - x^*}{2\varepsilon} & \text{if } x^* - \varepsilon \leqslant \theta < x^* + \varepsilon, \\ 1 & \text{if } \theta \geqslant x^* + \varepsilon. \end{cases}$$

Clearly, total rollover increases in θ.

First, given any signal cutoff x^*, let us calculate the threshold θ^* such that successful rollover occurs if and only if $\theta \geqslant \theta^*$. This is given by

$$(1 + \psi)L(\theta^*) = 1.$$

Solving,

$$x^* = \theta^* - \varepsilon \left(\frac{1 - \psi}{1 + \psi} \right). \tag{3}$$

Next, given that rollover succeeds if and only if $\theta \geqslant \theta^*$, let us calculate the signal cutoff x^*. The expected payoff of an agent with signal x who rolls over his loan (advanced in period 0) at the start of period 1 is given by

$$\Pr(\theta \geqslant \theta^* | x) E\Big(\theta r | \{\theta \geqslant \theta^*, x\} \Big) + \Pr(\theta < \theta^* | x) E\Big(\widehat{m} | \{\theta < \theta^*, x\} \Big) - 1$$

Recall that if all lenders (measure $1 + \psi$) withdraw at the start of period 1, a fraction $\psi/(1 + \psi)$ get paid 1 and the rest get paid $\widehat{m}$. Therefore, if an agent does not rollover at the start of period 1, their payoff is

$$\Pr(\theta \geqslant \theta^* | x) + \Pr(\theta < \theta^* | x) \left(E\Big(\frac{\psi}{1 + \psi} + \frac{1}{1 + \psi} \widehat{m} | \{\theta < \theta^*, x\} \Big) \right) - 1$$

Comparing the two expressions above, the net benefit from rollover is

$$\begin{aligned} V(\theta^*, x) = \Pr(\theta \geqslant \theta^* | x) E\Big(\theta r - 1 | \{\theta \geqslant \theta^*, x\} \Big) \\ - \Pr(\theta < \theta^* | x) E\Big((1 - \widehat{m}) \frac{\psi}{1 + \psi} | \{\theta < \theta^*, x\} \Big) \end{aligned} \tag{4}$$

For any given θ^*, x^* solves the indifference condition

$$V(\theta^*, x^*) = 0. \tag{5}$$

Using equation (3) to write x^* in terms of θ^*, and substituting in the expression for $V(\theta^*, x^*)$ from above, then using (5), we get a single equation in θ^*.

Let us now reintroduce the subscript *i*.

Solving the single equation in θ_i^*, we get following (unique) rollover threshold.

$$\theta_i^* = \frac{1}{r} + \frac{(1-\widehat{m})}{(1+\psi_i)r} - \varepsilon_i \left(\frac{\psi_i}{1+\psi_i}\right). \tag{6}$$

The details of the derivation is in the appendix.

Taking the limit of the right hand side of equation (6) as $\varepsilon_i \to 0$, we get the following result.

Lemma 2. *As noise vanishes, the unique rollover threshold is given by*

$$\theta_i^* = \frac{1}{r} + \frac{(1-\widehat{m})}{(1+\psi_i)r} \tag{7}$$

In other words, lenders successfully coordinate to roll over funds for any intermediary i for whom $\theta \geqslant \theta_i^*$, and investment is liquidated otherwise.

Note that if $\widehat{m} = 1$, $\theta_i^* = 1/r = \underline{\theta}$. In this case there is no coordination problem - it is dominant strategy to rollover for any value of $\theta \geqslant 1/r$. We now solve the problem of liquidity choice in period 0, and show that in equilibrium we must have $\widehat{m} < 1$ so that a non-trivial coordination problem necessarily arises in period 1.

3.2 Equilibrium liquidity choice in period 0

Having solved the equilibrium in the rollover coordination game in period 1, we now solve the problem of choosing liquidity in period 0. Recall that ψ is chosen from $[0, \overline{\psi}]$. We want to show that in equilibrium liquidity holding in period 0 is such that a coordination problem necessarily arises in period 1. For this result to be non-trivial, there must be scope for liquidity to be high enough so that this does not hold.

Recall from Lemma 1 that if $\widehat{m} \geqslant 1$, no coordination problem arises, so that $\theta_i^* = \underline{\theta}$ for all intermediaries i. Accordingly we assume the following.

Assumption 1. *The upper limit $\overline{\psi}$ is high enough so that if all intermediaries hold liquidity of $\overline{\psi}$, the cash-in-the-market price is $\widehat{m} \geqslant 1$.*

Next, holding liquidity ψ_i benefits intermediary i in two ways. First, a higher ψ_i lowers θ_i^*, allowing rollover coordination to succeed for more values of θ_i. Second, in the event rollover does succeed, intermediary i can buy, in period 1, assets of intermediaries who liquidate in that period. These assets then augment i's project. If i's project succeeds in period 2, the net secondary market benefit from holding ψ_i is $\delta R \psi_i / \widehat{m} - r\psi_i$.

Now, if the net benefit is always strictly positive, the model would only have uninteresting equilibria in which all intermediaries hold a lot of liquidity in period 0 (in our model, each intermediary would choose $\psi_i = \overline{\psi}$), the market price $\widehat{m}$ is at least 1, and there is no rollover coordination problem in period 1. For there to be even the possibility of a coordination problem in period 1, it must be that r exceeds $R\delta$.[15] Accordingly, we make the following assumption.

Assumption 2. $\delta R < r$.

Now, given the threshold $\widehat{\theta}_j^*$ for intermediary $j \neq i$, the available liquidity from intermediary j in the market is $(1 - \widehat{\theta}_j^*)\widehat{\psi}_j$. Therefore total available liquidity is $\int_{j\in[0,1]}(1 - \widehat{\theta}_j^*)\widehat{\psi}_j dj$. Total assets on sale is the measure of liquidating intermediaries given by $\int_{j\in[0,1]} \widehat{\theta}_j^* dj$. It follows that the cash-in-the-market price of assets in the event of liquidation at date 1 is given by

$$\widehat{m} = \frac{\int_{j\in[0,1]}(1 - \widehat{\theta}_j^*)\widehat{\psi}_j dj}{\int_{j\in[0,1]} \widehat{\theta}_j^* dj}. \tag{8}$$

We show next that there is a unique symmetric equilibrium in the liquidity choice problem in period 0 which implies that a coordination problem must arise in period 1.

Proposition 1. (Endogenous coordination problem) *Under assumptions 1 and 2, there is a unique symmetric equilibrium in which each intermediary holds liquidity $\psi^* \in (0, \overline{\psi})$. Further, in equilibrium, the cash-in-the-market price for liquidated assets in period 1 satisfies $\widehat{m} < 1$, so that a non-trivial rollover coordination problem necessarily arises among lenders to any intermediary.*

[15] To see this is necessary, suppose $\delta R \geqslant r$. Then for any $\widehat{m} < 1$, $\frac{\delta R}{\widehat{m}} > r$. In this case, payoff of intermediary i increases in ψ_i, so that each intermediary would optimally choose $\overline{\psi}$, which implies $\widehat{m} \geqslant 1$, which makes it a dominant strategy to rollover all loans so that there is no rollover coordination problem. Another way to say this is that if r were determined within a larger model in which we did not impose an upper limit $\overline{\psi}$ exogenously, any equilibrium would involve an inequality such as the one stated in the assumption - as aggregate demand for liquidity would be unboundedly high otherwise, pushing up r.

From the result above we know that the unique equilibrium is a symmetric equilibrium in which all intermediaries hold liquidity $\psi_i^* = \psi^*$ in period 1. From equation (7), it follows that each intermediary has the same success threshold $\theta_i^* = \theta^*$. As noise vanishes, this is given by

$$\theta^* = \frac{1}{r} + \frac{1 - \widehat{m}}{(1 + \psi^*) r}.$$

Further, from equation (8), the value of $\widehat{m}$ becomes

$$\widehat{m} = \frac{(1 - \theta^*)\, \psi^*}{\theta^*}. \tag{9}$$

Using the value of $\widehat{m}$ in the equation for θ^* above, we can solve for the value of θ^*. This is shown in the result below.

Proposition 2. *The unique symmetric rollover threshold across intermediaries is given by θ^* where $\underline{\theta} < \theta^* \leqslant \overline{\theta}$. As noise vanishes, this is given by*

$$\theta^* = \min \left[1 - \Delta, \frac{1}{r} \left(1 + \sqrt{1 - \frac{r\, \psi^*}{1 + \psi^*}} \right) \right]$$

Further, for Δ small, $\theta^* < \overline{\theta}$.

4 Lender Coordination under Ambiguity

We now introduce ambiguity in the model. Suppose, after initial liquidity decisions are made, the environment becomes characterized by ambiguity in period 1, and lenders are ambiguity sensitive. We study the effect of such ambiguity on the coordination problem in period 1 and show that coordination either succeeds for all values of fundamentals for which coordination matters (optimistic agents), or breaks down completely for all such values of fundamentals (pessimistic agents). For each value of fundamentals there are, therefore, multiple equilibria. However, these are not sunspot equilibria: equilibrium is determined completely by ambiguity attitude.

Let θ_a^* denote the coordination-success-threshold value of fundamentals under ambiguity. We show that starting from any coordination threshold $\theta_a^* \in [\underline{\theta}, \overline{\theta})$ under ambiguity-neutrality (in which case there is a unique equilibrium for every value of fundamentals as in the case without ambiguity), the threshold jumps discontinuously to $\overline{\theta}$ under ambiguity-aversion (or pessimism) and jumps down discontinuously to $\underline{\theta}$

under ambiguity-loving (optimism). This also implies that under ambiguity-sensitivity, multiple extreme equilibria obtain for every value of fundamentals for which coordination matters.

4.1 Preference under ambiguity

Suppose in period 1 lenders face ambiguity about the quality of their own signal. Specifically, suppose lenders think that their signal might have some bias relative to the signals received by others.

Suppose lender k to intermediary i believes that their signal is drawn from some distribution from a *set* of distributions with means $\theta_i + \eta_k$, where the bias $\eta_k \in [-\varepsilon_i^\beta, \varepsilon_i^\beta]$, where $0 < \beta < 1$.[16] Note that as signal noise $\varepsilon_i \to 0$, ε_i^β also vanishes, but at a slower rate.

The preferences of the buyers follow the well-known α-maxmin expected utility representation.[17] The α-maxmin expected utility of agent k lending to intermediary i is given by

$$U_k(\theta_a^*, x|\alpha) = \alpha \min_{\eta_k \in \in [-\varepsilon_i^\beta, \varepsilon_i^\beta]} V_k(\theta_a^*, x|\eta_k) + (1-\alpha) \max_{\eta_k \in [-\varepsilon_i^\beta, \varepsilon_i^\beta]} V_k(\theta_a^*, x|\eta_k) \tag{10}$$

We suppress the subscript i in the following for clarity of notation. Since we are simply analyzing the problem faced by lenders to intermediary i, this causes no interpretation problems.

Consider a bias $\eta_k > 0$ for agent k. Perceiving own signal bias η_k, the agent with signal x thinks θ is distributed uniformly on $[x - \eta_k - \varepsilon, x - \eta_k + \varepsilon]$. The net payoff $V_k(\theta_a^*, x|\eta_k)$ from rolling over under such a signal is then given by an expression analogous to

[16] Here we are assuming that i perceives own signal to have a bias relative to others, who get an unbiased signal. This is the simplest way to introduce a perception of own-signal-bias. However, we could assume that i perceives everyone's signal to have some common bias, but own signal to have a further bias in either direction - this would not change anything other than introducing an extra term denoting perceived common bias. All that matters is that there is a perceived relative bias.

[17] See **?**, **?**, **?**. In the α-maxmin representation, an agent puts a weight α on the most pessimistic outcome and the remaining weight on the most optimistic outcome. The maxmin expected utility representation (Gilboa and Schmeidler, 1989) obtains as a special case with extreme ambiguity aversion ($\alpha = 1$).

equation (4).

$$
\begin{aligned}
V_k(\theta_a^*, x|\eta_k) &= \Pr(\theta \geqslant \theta_a^*|x)E\Big(\theta r - 1|\{\theta \geqslant \theta_a^*, x\}\Big) \\
&\qquad - \Pr(\theta < \theta_a^*|x)E\Big(G|\{\theta < \theta_a^*, x\}\Big) \\
&= \frac{1}{2\varepsilon}\int_{\theta_a^*}^{x-\eta_k+\varepsilon} (\theta\, r - 1)\, d\theta - \frac{1}{2\varepsilon}\int_{x-\eta_k-\varepsilon}^{\theta_a^*} G\, d\theta \qquad (11)
\end{aligned}
$$

where

$$
G \equiv (1 - \widehat{m})\frac{\psi}{1+\psi}. \qquad (12)
$$

Note that $G > 0$.

It is straightforward to see that $V_k(\theta_a^*, x|\eta_k)$ is decreasing in η_k. Therefore the minimizing case obtains when $\eta_k = \varepsilon_i^\beta$ and the maximizing case obtains when $\eta_k = -\varepsilon_i^\beta$.

The intuition for this is clear: for a positive bias η_k, the agent with signal x thinks θ is distributed uniformly on $[x - \eta_k - \varepsilon, x - \eta_k + \varepsilon]$. In other words, the perceived interval over which θ is distributed is shifted down, making for a lower perceived probability that θ exceeds θ_a^* (i.e. a lower probability that coordination succeeds), leading to a lower payoff. Similarly, a negative bias shifts the perceived distribution up and raises the perceived probability that coordination succeeds. Therefore the minimizing case obtains under the highest positive bias, and the maximizing case obtains under the most negative bias.

Using this in equation (10), The α-maxmin net expected utility of agent k from rolling over is therefore given by

$$
U_k(\theta_a^*, x) = \alpha V_k(\theta_a^*, x|\eta_k = \varepsilon^\beta) + (1-\alpha)V_k(\theta_a^*, x|\eta_k = \ -\varepsilon^\beta) \qquad (13)
$$

where the expressions for V_k are given by substituting the appropriate value of η_k in equation (11).

4.2 Lending euphoria and sudden liquidity freeze

Proposition 3. (Coordination under ambiguity) *Suppose each lender k to intermediary i maximizes α-maxmin expected utility assuming a bias in own signal of ε_i^{β} relative to other lenders to i, where $0 \leqslant \beta < 1$.*

1. *(Sudden liquidity freeze) For any $\alpha < 1/2$, there is a $\varepsilon_1(\alpha) > 0$ such that for $\varepsilon_i < \varepsilon_1(\alpha)$, coordination breaks down for all values of $\theta \in [\underline{\theta}, \overline{\theta})$.*
2. *(Lending euphoria) For any $\alpha > 1/2$, there is a $\varepsilon_2(\alpha) > 0$ such that for $\varepsilon_i < \varepsilon_2(\alpha)$, coordination succeeds for all values of $\theta \in [\underline{\theta}, \overline{\theta})$.*
3. *For $\alpha = 1/2$, equilibrium is the same as without ambiguity: a unique equilibrium obtains for all θ, with coordination success for and only for $\theta \geqslant \theta_a^*$, for some $\theta_a^* \in [\underline{\theta}, \overline{\theta}]$.*

Figure 1 (in the introduction) shows the equilibrium coordination outcome as a function of α.

The result shows that under ambiguity, for small noise, coordination collapses for all values of θ for which coordination matters if investors are even slightly ambiguity averse or pessimistic (i.e. for any $\alpha > 1/2$). Similarly, for all such values of θ, lenders successfully coordinate to rollover loans if they are ambiguity loving or optimistic (i.e. for any $\alpha < 1/2$).

Under ambiguity neutrality, the result is qualitatively the same as in the case without ambiguity. There is a unique equilibrium for every value of fundamentals: coordination succeeds if and only if θ exceeds a threshold. Clearly the coordination outcome in this case depends directly on the value of fundamentals. The result above shows that with ambiguity-sensitive agents the coordination outcome no longer depends on fundamentals. For every value of the fundamentals in the coordinating interval, coordination succeeds or fails in equilibrium according as α is lower or higher than $1/2$. Thus there are multiple extreme equilibria for every value of fundamentals. This is just as under complete information, but with one crucial difference: under complete information we get sunspot equilibria whereas here the equilibrium is not indeterminate - equilibrium selection depends entirely on the ambiguity attitude encapsulated by the parameter α.

Intuition: Suppose an agent calculates that other agents would rollover loans above a signal threshold y^*. In the case of ambiguity aversion or pessimism, an agent puts more weight on own signal to be biased upwards relative to others. We show that for even the slightest degree of ambiguity aversion, the agent optimally wants to rollover using a higher signal threshold compared to others. In other words, the agent rolls over own funds only when others have established a coordination threshold y^*, and then sets own threshold a little above y^*. Note that this agent is not contributing to coordination at all. If everyone behaves this way, it is impossible to have an interior solution for y^*, implying that coordination necessarily fails for any value of fundamentals for which it matters. Here rollover happens only when it is a dominant strategy for each agent to roll over, i.e. when $\theta \geqslant \overline{\theta}$. Similarly, even for the slightest degree of ambiguity loving or optimism, an agent wants to take the coordinating action using a threshold that is below y^*. If everyone behaves this way – taking the coordinating action before others – the only possibility in equilibrium is that coordination succeeds for all values of fundamentals for which it matters, i.e. for all $\theta \geqslant \underline{\theta}$. In this case coordination fails only when it is a dominant strategy for each agent not to rollover.

The proof outline: The proof steps reflect the intuition discussed above. Consider the problem of lender k to intermediary i who receives signal x and considers y^* to be the rollover signal threshold for all other lenders to intermediary i. To get an interior threshold in equilibrium, we need all lenders to rollover above (and only above) a common threshold. For this to be true, we therefore need k to be indifferent between rolling over and not rolling over at signal $x = y^*$. The proof calculates the net α-maxmin expected utility from rolling over at $x = y^*$ and shows that for $\alpha \neq 1/2$, i.e. in the case where agents are sensitive to ambiguity, the indifference condition is not met. In particular, for small values of ε, the net utility from rolling over is strictly negative if $\alpha > 1/2$ and strictly positive if $\alpha < 1/2$. It follows that, for $\alpha > 1/2$, the optimal signal threshold for agent k is $x_k^* > y^*$. Since this is true for every agent k, coordination collapses for all values of fundamentals for which coordination matters - i.e. the threshold value of fundamentals above which coordination succeeds becomes $\overline{\theta}$. Similarly, for $\alpha < 1/2$, we have $x_k^* < y^*$. Since this is true for all agents, the only possible equilibrium outcome is that lender rollover always succeeds (the threshold value of fundamentals above which coordination succeeds becomes $\underline{\theta}$).

4.3 Proof of Proposition 3

Consider the problem of lender k to intermediary i who receives signal x. Henceforth, **we suppress the subscript i to reduce notational clutter.** Since all variables relating to the intermediary refers to intermediary i, this causes no interpretation problems.

Recall that $U_k(\theta_a^*, x)$, given by equation (13), denotes the α-maxmin net expected utility from rolling over for agent k. We now derive the value of θ_a^* in terms of y^* and substitute in the expression for $U_k(\theta_a^*, x)$. Clearly, y^* is such that $(1+\psi)L(\theta_a^*|y^*) = 1$, which implies (using equation (3))

$$(1+\psi)\frac{\theta_a^* + \varepsilon - y^*}{2\varepsilon} = 1. \tag{14}$$

Substituting the value of θ_a^* from above in the expression for $U_k(\theta_a^*, x)$, we get an expression for α-maxmin net expected utility from rolling over as a function of y^* and x, denoted by $U_k(y^*, x)$.

The result below considers $U_k(y^*, x)$ at $x = y^*$.

Lemma 3. *There is $\varepsilon_* > 0$ such that for any $\varepsilon < \varepsilon_*$,*

$$U_k(y^*, y^*) < 0 \quad \textit{if } \alpha > 1/2 \textit{ and}$$
$$U_k(y^*, y^*) > 0 \quad \textit{if } \alpha < 1/2$$

Further, for $\alpha = 1/2$, the statement in item 3 in proposition 3 holds.

The proof, which requires going through several steps of algebra to derive the expression for $U_k(y^*, y^*)$, is relegated to the appendix. The result proves the third part of proposition 3. To prove the first two, we proceeds as follows.

Let x_k^* denote the signal threshold for agent k. This is given implicitly by $U_k(y^*, x) = 0$.

Let us now show that the lemma above implies that the optimal choice of x_k^* cannot be equal to y^* for an interior y^*. Consider first the case of ambiguity aversion.

Case 1: $\alpha > 1/2$. In this case $U_k(y^*, y^*) < 0$ for small ε. Since payoff of agent k is increasing in own signal, we can only have $U_k(y^*, x_k^*) = 0$ where $x_k^* > y^*$. Given any signal cutoff y^* above which others lend, each agent k wants to lend above a cutoff $x_k^* > y^*$. It follows that the only possible equilibrium involves agents rolling over only when it is dominant strategy to do so, implying that $\theta^* = \overline{\theta}$.

Next, consider the case of ambiguity loving.

Case 2: $\alpha < 1/2$. In this case $V_k(y^*, y^*) > 0$ for small ε. Since payoff of agent k is increasing in own signal, we can only have $V_k(y^*, x_k^*) = 0$ where $x_k^* < y^*$. Then the only possibility in equilibrium is that agents roll over loans unless it is a dominated strategy. This implies that rollover succeeds for all values of θ for which coordination matters (i.e. for all $\theta \in [\underline{\theta}, \overline{\theta})$), implying $\theta^* = \underline{\theta}$.

Finally, the lemma above proves the claim for the case of $\alpha = 1/2$. This completes the proof.‖

5 Policy implications

5.1 Policy as trigger for a systemic liquidity freeze

So far we have assumed intermediaries are atomistic. Suppose now that the intermediation sector comprises of a total measure $\gamma \in (0, 1)$ of a few large intermediaries, and measure $(1 - \gamma)$ of atomistic ones. Liquidation of an atomistic intermediary has no impact on the secondary market. However, a large intermediary is systemic in the following sense. If such an intermediary is liquidated in period 1, the secondary market supply of assets would increase, pushing down the cash-in-the-market price of assets. Since this could happen with positive probability, this would reduce liquidation payoff for all intermediaries ex ante, causing an upward jump up in their coordination threshold θ_i^*. In other words, the possibility of liquidation of a large intermediary leads to a reduction in coordination success for all.

In view of such systemic importance, suppose all agents expect large intermediaries to be given liquidity support with probability 1 in the event they face liquidation. We refer to such intermediaries as too unexpected to fail (TUTF). Clearly, if policy were to follow market expectation, such intermediaries would never need to liquidate. Irrespective of ambiguity in signals about fundamentals, the net payoff from rolling over a loan to a TUTF intermediary is then simply $\theta r - 1$, and it is a dominant strategy to lend for any $\theta > 1/r = \underline{\theta}$. In other words, for any TUTF intermediary, coordination succeeds for every value of fundamentals for which coordination matters, irrespective of ambiguity and ambiguity attitude.

Suppose now that a shock hits a TUTF intermediary reducing its fundamentals to $\theta < 1/r$. In this case not rolling over the initial loan is a dominant strategy for each lender and therefore the intermediary faces liquidation. As noted above, the market expectation is that liquidity support would be forthcoming.[18] The regulator can either provide liquidity support, or act against market expectation and choose not to do so (perhaps because the regulator feels θ is so low that it is optimal to let the intermediary go into liquidation). In the latter case, what is the impact on the market?

A policy that deviates from market expectation is likely to reduce market expectation of the probability that other large intermediaries in a similar position would be rescued from 1 to some $p < 1$. Lenders now expect that with positive probability large intermediaries will be exposed to lender coordination. As we show in the result below, under signal ambiguity, this is enough to give rise to the same cases as discussed in Proposition 3. Since an environment where a large intermediary receives a shock to returns is likely to be a downturn, lenders are likely to be ambiguity averse. In such a situation, small intermediaries already face a collapse of lender coordination. A change in market expectation about rescue of large intermediaries causes lenders to run on all large intermediaries as well. Thus a systemic run on liquidity is triggered by the policy decision. This helps explain the sudden sector-wide collapse of liquidity in the wake of the Lehman Brothers failure.

Corollary 1. (Sudden liquidity freeze triggered by policy) *Suppose an unexpected regulatory action (or inaction) reduces market expectation about the probability of rescue of large intermediaries from 1 to $p < 1$. In the presence of signal ambiguity and ambiguity-averse lenders, lender coordination collapses for every value of fundamentals for which coordination matters across large intermediaries as well.*

5.2 Too-big-to-fail and too-unexpected-to-fail

While our work shows the importance of considering market expectations, a different issue, much discussed in the literature, is the consequence of a financial institution being too big to fail (TBTF). What does our work imply about the notion of TBTF?

[18]Indeed, so long as $\theta > 1/R$, the intermediary is solvent but illiquid, and liquidity support is the efficient policy. Such a region of fundamentals where intermediaries are solvent but illiquid is a standard feature of coordination models. See, for example, Rochet and Vives (2004), Vives (2014).

In particular, does a large institution regarded as TBTF help with or exacerbate the crisis? It is quite likely that TBTF intermediaries are also TUTF. In this case having a TBTF institution is helpful in limiting the systemic spread of a liquidity crisis, assuming policy conforms to market expectations. The same reasoning shows why breaking up a TBTF institution exacerbates the systemic crisis. If broken up, none of the parts might be TUTF, in which case, assuming lenders are pessimistic (or ambiguity-averse) in a downturn, all parts experience liquidity runs. This increases systemic vulnerability.

Corollary 2. (Breaking up TUTF intermediaries increases systemic risk) *So long as policy conforms to market expectations, having TUTF intermediaries is insurance against systemic run. A policy of breaking up too-big-to-fail intermediaries might reduce systemic stability if the large intermediaries in question are also TUTF.*

6 Conclusion

We analyse a model of short-term lending and show that a coordination problem among lenders arises endogenously. We show that the introduction of ambiguity has a dramatic effect on the nature of the coordination outcome. Without ambiguity, we have the standard global games result that there is a unique equilibrium for every value of fundamentals: coordination succeeds if and only if the value of fundamentals exceeds a threshold. The introduction of ambiguity changes this completely. First, equilibria become detached from fundamentals: for every value of fundamentals for which coordination matters, there are two extreme equilibria - one in which coordination succeeds and another with complete unravelling of coordination. Indeed, this multiplicity is reminiscent of the indeterminacy under complete information, in which case there are exactly these two equilibria for every value of fundamentals. However, the crucial difference is that, unlike the case of complete information, equilibria under ambiguity are not sunspot equilibria - equilibrium selection is completely determined by ambiguity preference. Second, the coordination outcome varies discontinuously with respect to ambiguity attitude (as shown in figure 1) around ambiguity neutrality. When agents are ambiguity neutral, there is no qualitative difference in the nature of equilibrium under ambiguity compared to the case without ambiguity. However, for even the slightest ambiguity aversion, coordination unravels completely (irrespective of fundamentals) and even the slightest ambiguity loving preference makes every lender rollover loans, making for successful rollover coordination (again, irrespective of fundamentals). We identify the precise mechanism that causes such extreme outcomes to arise under even slight ambiguity-sensitivity.

The results explain why lending arrangements are typically fragile, why a lending euphoria can emerge under an optimistic outlook so that rollover succeeds even when an intermediary might be investing in sub-prime assets, why such arrangements can break down suddenly if the lender outlook changes and why a run can arise for loans funding not just subprime assets, but also prime ones.

Our results also show that policy itself can act as a trigger for a systemic liquidity freeze. If lenders expect some intermediaries to be rescued by the central bank (or some other liquidity provider of last resort), they would be immune to the problem of a lender-coordination-collapse. However, if policy does not follow market expectations and thereby alters the latter, there is again a breakdown of coordination implying a liquidity freeze. Since the Lehman Brothers failure was a shock to market expectation about Fed rescue policy, our results explain the sudden widespread liquidity freeze triggered by this event.

The results imply that there are two ways for regulatory policy to reduce the systemic vulnerability to ambiguity. Any policy that reduces the coordination problem helps improve the system's defence against ambiguity. Further, any policy that creates an expectation that an intermediary would be rescued in a crisis also has the same effect. Examples of policies that belong to the first category include requiring banks to hold more liquid assets - for example the liquidity coverage ratio requirement of Basel III; requiring banks to fund projects fully or partially with stable funds (long term debt, equity, insured deposits) - for example the net stable funding ratio requirement of Basel III. A policy of explicit or implicit promise of liquidity support belongs to the second category.

Gorton (2012) defines systemic risk as loss of investor confidence in financial intermediary debt. The theory presented here articulates this idea. The paper shows how the interaction of a coordination problem and ambiguity leads to a lending euphoria, followed by a sudden loss of confidence and collapse of liquidity at the advent of some bad news. In future research, we hope to pursue richer models using this core idea.

Appendix: Proofs

A.1 Derivation of equation (6)

To reduce notational clutter, throughout the following calculations, we suppress the subscript i.

From equation (4), the net benefit for intermediary i from rollover is

$$V(\theta^*, x) = \frac{1}{2\varepsilon}\int_{\theta^*}^{x+\varepsilon} (\theta\, r - 1)\, d\theta - \frac{1}{2\varepsilon}\int_{x-\varepsilon}^{\theta^*} G\, d\theta$$

where $G = (1-\widehat{m})\frac{\psi}{1+\psi}$. Integrating and simplifying, we get

$$V(\theta^*, x) = \frac{1}{4\varepsilon}\Big(r\left((x+\varepsilon)^2 - (\theta^*)^2\right) + 2(\theta^* - x)(1-G) - 2(1+G)\varepsilon\Big).$$

For any given θ^*, x^* solves the indifference condition $V(\theta^*, x^*) = 0$. Using equation (3) to write x^* in terms of θ^*, and substituting in the expression for $V(\theta^*, x^*)$ from above and equating that expression to 0, we get the following single equation in θ^*:

$$\frac{\psi}{(1+\psi)^2}\Big((r\,\theta^* - 1)(1+\psi) + \varepsilon r\psi\Big) - \frac{G}{1+\psi} = 0.$$

Using the value of G, this becomes

$$\frac{\psi}{(1+\psi)^2}\Big((r\,\theta^* - 1)(1+\psi) - (1-\widehat{m}) + \varepsilon r\psi\Big) = 0.$$

Solving this gives us the expression for θ_i^* in equation (6).

A.2 Proof of Proposition 1

We prove that each intermediary i has a well-defined best-response *function* mapping the liquidity choice of others to choice of ψ_i. We show that the best response function satisfies all the requirements for Brouwer's fixed-point theorem, from which the proof follows. But first, we need to establish the signs of some terms. This is done in section A.2.1. Then in section A.2.2 we establish the best response function. Section A.2.3 then completes the proof.

Let $\widehat{\psi}$ denote the aggregate liquidity others decide to hold in stage 1. This is given by

$$\widehat{\psi} = \int_{j \in [0,1], j \neq i} \psi_j dj.$$

Note that if each $j \neq i$ chooses $\psi_j = 0$, $\widehat{\psi} = 0$ and if $\psi_j = \overline{\psi}$ for all $j \neq i$, we have $\widehat{\psi} = \overline{\psi}$. Therefore $\widehat{\psi} \in [0, \overline{\psi}]$.

A.2.1 Preliminaries

The proof proceeds through the following lemma.

Lemma 4. *The following properties hold:*

1. *θ_i^* is decreasing in ψ_i and $\widehat{\psi}$.*
2. *$\dfrac{\partial \theta_i^*}{\partial \psi_i}$ is increasing in ψ_i and $\widehat{\psi}$.*

Proof:

1. First, consider the derivative of θ_i^* with respect to ψ_i:

$$\frac{\partial \theta_i^*}{\partial \psi_i} = -\frac{(1-\widehat{m})}{r(1+\psi_i)^2} < 0. \tag{A.1}$$

Next, note that $\widehat{\psi}$ enters θ_i^* only through $\widehat{m}$. It follows that

$$\frac{\partial \theta_i^*}{\partial \widehat{\psi}} = \frac{\partial \theta_i^*}{\partial \widehat{m}} \frac{\partial \widehat{m}}{\partial \widehat{\psi}}.$$

Now, from the expression for θ_i^* (equation (7)) it is clear that the first term on the right hand side is strictly negative, Further, it is clear from the expression for $\widehat{m}$ (equation (8)) that an increase in $\widehat{\psi}$ raises $\widehat{m}$, so that the second term is strictly positive. It follows that $\frac{\partial \theta_i^*}{\partial \widehat{\psi}} < 0$.

2. From equation (A.1), $\dfrac{\partial^2 \theta_i^*}{\partial \psi_i^2} = \dfrac{2(1-\widehat{m})}{r(1+\psi_i)^3} > 0$. Further, it is clear that $\dfrac{\partial \theta_i^*}{\partial \psi_i}$ is strictly increasing in $\widehat{m}$, which is strictly increasing in $\widehat{\psi}$. It follows that $\dfrac{\partial^2 \theta_i^*}{\partial \widehat{\psi} \partial \psi_i} > 0$. This completes the proof of lemma 4.‖

The payoff of intermediary i from holding liquidity $\psi_i > 0$ is

$$\begin{aligned}\Pi_i &= \Pr(\theta \geqslant \theta_i^*)E(\theta_i|\theta_i \geqslant \theta_i^*)\left(R + \frac{\psi_i}{\widehat{m}} R\,\alpha - (1+\psi_i)r\right) \\ &= \left(\int_{\theta_i^*}^{1} \theta d\theta\right)\left(R - r + \psi_i \widehat{Z}\right)\end{aligned}$$

where

$$\widehat{Z} \equiv \frac{1}{\widehat{m}} R\,\alpha - r. \tag{A.2}$$

Note that $\widehat{Z}$ is independent of ψ_i.

We prove the following result next.

Lemma 5. *In equilibrium $R - r + \psi_i \widehat{Z} > 0$.*

Proof: Note first that profit must be strictly positive in equilibrium. This can be seen as follows. If $\psi_i = 0$, $\theta_i^* = \overline{\theta} = 1 - \Delta$. The first component of the expression for profit above is still strictly positive. The second component is $R - r > 0$. Since profit is strictly positive at $\psi_i = 0$, it cannot be zero at any $\psi_i > 0$ (or i could deviate profitably by reducing ψ_i). Since $\Pi_i > 0$, and since the first component in the expression for Π_i is always strictly positive, it follows that the second component is strictly positive.$\|$

The proof now proceeds through the following lemma.

Lemma 6. *In equilibrium $\widehat{Z} \leqslant 0$.*

Proof: Suppose not. Then $\widehat{Z} > 0$. Consider the problem of choosing ψ_i at stage 0. By raising ψ_i, θ_i^* falls which improves the expected payoff. Further, the marginal cost per unit of liquidity is r, while the secondary market generates a benefit of $\frac{1}{\widehat{m}} R\,\alpha$ per unit of liquidity. If $Z > 0$, the benefit from a unit of liquidity therefore exceeds the cost. Therefore each i would like to hold $\psi_i = \overline{\psi}$. From assumption 1, this implies $\widehat{m} \geqslant 1$. Therefore $\frac{1}{\widehat{m}} R\,\alpha \leqslant R\delta < r$, where the second inequality follow from assumption 2, which in turn implies $\widehat{Z} \not> 0$. Thus we have a contradiction.$\|$

The next step is to show that there is a well-defined, continuous best response function for intermediary i mapping $\widehat{\psi}$ values to ψ_i values. Once we establish this, and given that this is a mapping from a compact convex set onto itself, we can invoke a fixed-point theorem to establish the existence of an equilibrium.

First, we need to sign the second derivative of Π_i with respect to ψ_i and the cross partial with respect to ψ_i and $\widehat{\psi}$.

We have

$$\frac{\partial \Pi_i}{\partial \psi_i} = -\theta_i^* \frac{\partial \theta_i^*}{\partial \psi_i}\left(R - r + \psi_i \widehat{Z}\right) + \left(\int_{\theta_i^*}^{1} \theta d\theta\right) \widehat{Z} \tag{A.3}$$

It follows that

$$\begin{aligned}
\frac{\partial^2 \Pi_i}{\partial \psi_i^2} &= \left(R - r + \psi_i \widehat{Z}\right)\left(-\left(\frac{\partial \theta_i^*}{\partial \psi_i}\right)^2 - \theta_i^* \frac{\partial^2 \theta_i^*}{\partial \psi_i^2}\right) + \widehat{Z}\left(-\theta_i^* \frac{\partial \theta_i^*}{\partial \psi_i}\right), \\
\frac{\partial^2 \Pi_i}{\partial \widehat{\psi} \partial \psi_i} &= \left(R - r + \psi_i \widehat{Z}\right)\left(-\left(\frac{\partial \theta_i^*}{\partial \widehat{\psi}}\right)\left(\frac{\partial \theta_i^*}{\partial \psi_i}\right) - \theta_i^* \frac{\partial^2 \theta_i^*}{\partial \widehat{\psi} \partial \psi_i}\right) \\
&\quad - \theta_i^* \frac{\partial \theta_i^*}{\partial \psi_i}\left(\psi_i \frac{\partial \widehat{Z}}{\partial \widehat{\psi}}\right) + \frac{\partial \widehat{Z}}{\partial \widehat{\psi}}\left(\int_{\theta_i^*}^{1} \theta d\theta\right) + \widehat{Z}\left(-\theta_i^* \frac{\partial \theta_i^*}{\partial \widehat{\psi}}\right).
\end{aligned}$$

Using the signs of terms from lemmas 4, 5 and 6, it follows that all right hand side terms in both equations are non-positive, with some being strictly negative in each equation (e.g. the first term in each equation). Therefore

$$\frac{\partial^2 \Pi_i}{\partial \psi_i^2} < 0, \tag{A.4}$$

$$\frac{\partial^2 \Pi_i}{\partial \widehat{\psi} \partial \psi_i} < 0. \tag{A.5}$$

A.2.2 Best response function

The previous section established signs of certain terms. Using these, the next lemma now establishes crucial properties of the best response of intermediary i given actions by other intermediaries.

Lemma 7. 1. *Suppose other intermediaries set $\widehat{\psi} = \overline{\psi}$. The best response of intermediary i is given by $B_i(\overline{\psi}) = 0$. Further, the same best response of 0 applies to values of $\widehat{\psi}$ close to $\overline{\psi}$.*

2. *Suppose other intermediaries set $\widehat{\psi} = 0$. The best response of i is $B_i(0) = \overline{\psi}$. Further, the same best response of $\overline{\psi}$ applies to values of $\widehat{\psi}$ close to 0.*

Proof:

1. If intermediaries other than i choose $\widehat{\psi} = \overline{\psi}$, from assumption 1, $m \geqslant 1$. In this case, we have $\theta^* = \underline{\theta}$ for all intermediaries, and the payoff of intermediary i becomes

$$\Pi_i = \left(\int_{\underline{\theta}}^{1} \theta d\theta\right)\left(R - r + \psi_i\left(\frac{R\delta}{\widehat{m}} - r\right)\right).$$

Here, $\underline{\theta}$ does not change when ψ_i changes. Therefore the first term does not depend on ψ_i. Since $R\delta < r$ (from assumption 2) and $\widehat{m} \geqslant 1$, the coefficient of ψ_i in the second term is strictly negative. It follows that $\dfrac{\partial \Pi_i}{\partial \psi_i} < 0$. Therefore, if others choose high liquidity $\overline{\psi}$, the optimal choice for i is $B_i(\overline{\psi}) = 0$. By continuity, $\dfrac{\partial \Pi_i}{\partial \psi_i} < 0$ also for values of $\widehat{\psi}$ close to $\overline{\psi}$. Therefore $B_i(\widehat{\psi}) = 0$ for $\widehat{\psi}$ at or close to $\overline{\psi}$.

2. Next, as $\widehat{\psi}$ goes to zero and $\widehat{\theta}^*$ rises to $\overline{\theta} = 1 - \Delta$, the payoff of intermediary i is

$$\Pi_i = \left(\int_{\theta_i^*}^{1} \theta d\theta\right)\left(R - r + \psi_i\left(\frac{R\delta}{\widehat{m}} - r\right)\right)$$

In this case only a small measure Δ survive in period 1. The market price $\widehat{m}$ for the assets of liquidated intermediaries goes to $\widehat{\psi}\Delta/(1-\Delta)$ which goes to zero with $\widehat{\psi}$. In this case, clearly, the coefficient of ψ_i in the second term is strictly positive. It then follows that both terms on the right hand side are increasing in ψ_i, implying that $\dfrac{\partial \Pi_i}{\partial \psi_i} > 0$. Therefore, $B_i(\widehat{\psi}) = \overline{\psi}$ for $\widehat{\psi}$ close to and equal to 0.$\|$

Constructing the best response Let us now construct the best response of intermediary i as $\widehat{\psi}$ varies over $[0, \overline{\psi}]$. The rest of this section shows that the best response of intermediary i is a function, and is as depicted in figure 2 below, which satisfies all conditions for applying Brouwer's fixed point theorem. The next section then completes the proof by invoking the fixed point theorem to establish existence.

Step 1: The point L: Fix $\psi_i = \overline{\psi}$ and consider how $\frac{\partial \Pi_i}{\partial \psi_i}$ varies as $\widehat{\psi}$ varies over $[0, \overline{\psi}]$. We know from Lemma 7 above that for $\widehat{\psi} = 0$, $\frac{\partial \Pi_i}{\partial \psi_i} > 0$ for all values of ψ_i, including $\psi_i = \overline{\psi}$. Further, we also know that for values of $\widehat{\psi}$ close to $\overline{\psi}$, $\frac{\partial \Pi_i}{\partial \psi_i} < 0$ for all ψ_i, including $\psi_i = \overline{\psi}$. Further, $\frac{\partial \Pi_i}{\partial \psi_i}$ is continuous in $\widehat{\psi}$. Therefore, fixing $\psi_i = \overline{\psi}$, as $\widehat{\psi}$

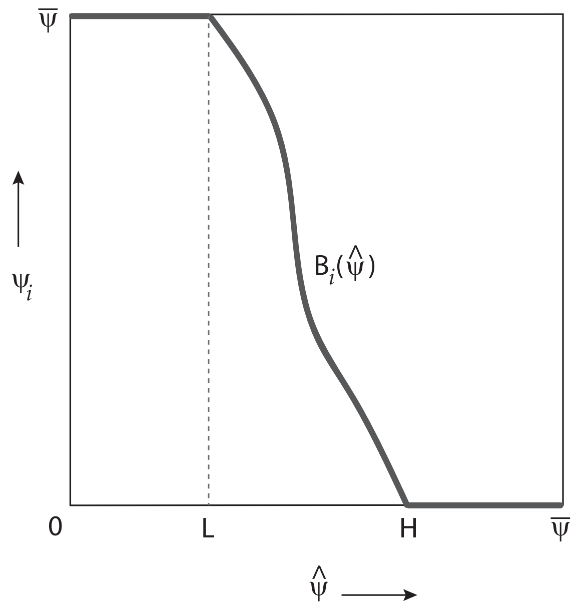

Figure 2: Best response function of intermediary i

increases from 0, there is some value of $\widehat{\psi}$ for which $\frac{\partial \Pi_i}{\partial \psi_i} = 0$. Next, since $\frac{\partial \Pi_i}{\partial \psi_i}$ is strictly decreasing in $\widehat{\psi}$, (inequality (A.5)), this value is also unique.

Let this unique point be $\widehat{\psi} = L$ (shown in figure 2).

Step 2: The point H: Now fix $\psi_i = 0$ and again consider how $\frac{\partial \Pi_i}{\partial \psi_i}$ varies as $\widehat{\psi}$ varies over $[0, \overline{\psi}]$. Using the same arguments as in step 1 above, it follows that there is a unique value H (as shown in figure 2) such that, fixing $\psi_i = 0$, $\frac{\partial \Pi_i}{\partial \psi_i} = 0$ at $\widehat{\psi} = H$.

Step 3: Constructing the best response function: From step 1, we know that for $\widehat{\psi} \leqslant L$, $\frac{\partial \Pi_i}{\partial \psi_i} > 0$ for all values of ψ_i and therefore, for $\widehat{\psi} \leqslant L$, the unique response is $B_i(\widehat{\psi}) = \overline{\psi}$. Similarly, from step 2, it follows that for $\widehat{\psi} \geqslant H$, the unique best response is $B_i(\widehat{\psi}) = 0$.

Next, we need to derive the best response of i for $\widehat{\psi} \in (L, H)$.

We know from above that, with $\psi_i = \overline{\psi}$, $\frac{\partial \Pi_i}{\partial \psi_i} = 0$ for $\widehat{\psi} = L$. Next, with $\psi_i = 0$, $\frac{\partial \Pi_i}{\partial \psi_i} = 0$ for $\widehat{\psi} = H$.

From inequality (A.5), $\frac{\partial \Pi_i}{\partial \psi_i}$ decreases strictly in $\widehat{\psi}$.

It then follows that, with $\psi_i = \overline{\psi}$, the derivative $\frac{\partial \Pi_i}{\partial \psi_i} < 0$ for any value of $\widehat{\psi} \in (L, H)$. Similarly, with $\psi_i = 0$, the derivative $\frac{\partial \Pi_i}{\partial \psi_i} > 0$ for any value of $\widehat{\psi} \in (L, H)$.

Further, from inequality (A.4), $\frac{\partial \Pi_i}{\partial \psi_i}$ is strictly decreasing in ψ_i. Therefore for any $\widehat{\psi} \in (L, H)$, there is a unique ψ_i at which $\frac{\partial \Pi_i}{\partial \psi_i} = 0$.

This proves that for any $\widehat{\psi} \in (L, H)$, there is a unique best response $B_i(\widehat{\psi}) \in (0, \overline{\psi})$ given by

$$\frac{\partial \Pi_i}{\partial \psi_i} = 0. \tag{A.6}$$

We have established that there is a well-defined best-response function

$$B_i : [0, \overline{\psi}] \mapsto [0, \overline{\psi}].$$

Step 4: $B_i(\cdot)$ is continuous: Π_i is continuous in ψ_i and $\widehat{\psi}$, and ψ_i is chosen from a non-empty compact set $[0, \overline{\psi}]$. It then follows from Berge's Maximum Theorem that the best response function is continuous.

It remains to be shown that the best response function is downward sloping over (L, H).

Step 5: $B_i(\cdot)$ decreases over (L, H): How does the best response of i change as $\widehat{\psi}$ varies over (L, H)? To see this, recall that the equation (A.6) above implicitly defines $B_i(\widehat{\psi})$. Totally differentiating equation (A.6), we get

$$\frac{\partial^2 \Pi_i}{\partial \psi_i^2} \frac{\partial B_i}{\partial \widehat{\psi}} + \frac{\partial^2 \Pi_i}{\partial \widehat{\psi} \partial \psi_i} = 0$$

implying that

$$\frac{\partial B_i}{\partial \widehat{\psi}} = -\left(\frac{\partial^2 \Pi_i}{\partial \widehat{\psi} \partial \psi_i} \right) / \left(\frac{\partial^2 \Pi_i}{\partial \psi_i^2} \right) < 0,$$

where the last inequality follows from inequalities (A.4) and (A.5).

Therefore, the best response function is a strictly decreasing function over (L, H). This completes the derivation of the best response function, which is depicted in figure 2.

We are now ready to complete the proof of the result.

A.2.3 Proof of Proposition 1 (Completed)

We know from above that the best response function $B_i(\widehat{\psi})$ is continuous and maps the compact and convex set $[0, \overline{\psi}]$ to itself. From Brouwer's fixed point theorem, we know that there exists ψ^* such $B_i(\psi^*) = \psi^*$. Any such ψ^* in a symmetric equilibrium.

Further, we know from lemma 7 that if $\widehat{\psi}$ is high so that $\widehat{m} \geqslant 1$, the best response is $B_i(\widehat{\psi}) = 0$. It follows that any fixed point must occur at some value of $\widehat{\psi}$ for which $\widehat{m} < 1$. In other words, if each intermediary choosing ψ^* is an equilibrium then it must be that ψ^* is such that $\widehat{m} < 1$, implying that a non-trivial coordination problem necessarily arises in equilibrium.

Finally, from the best-response function in figure 2 it is clear that there is a unique fixed point, so that the equilibrium is unique. Formally, this can be shown by making use of the contraction mapping theorem as follows.

Define $B_i^{-1}(0) = L$ and $B_i^{-1}(\overline{\psi}) = H$. Let $f \equiv B^{-1}$. Then f maps the closed interval $[0, \overline{\psi}]$ into itself. Let $K = \frac{H-L}{\overline{\psi}}$. Note that $K < 1$.

Now,

$$f(\overline{\psi}) - f(0) = H - L = K(\overline{\psi} - 0).$$

Next, for any $a, b \in (0, \overline{\psi})$ where $b > a$, we have

$$f(b) - f(a) < H - L = K(\overline{\psi} - 0) < K(b - a)$$

Therefore, the function f satisfies the Lipschitz condition

$$|f(b) - f(a)| \leqslant K|b - a|$$

where $K < 1$. Then f is a contraction mapping, and therefore, from the contraction mapping theorem, has a unique fixed point $f(\psi^*) = \psi^*$.

Since no fixed points can be located if $\psi_i = 0$ or $\psi_i = \overline{\psi}$, it follows that there is a unique fixed point $\psi^* = B_i(\psi^*)$, where $\psi^* \in (0, \overline{\psi})$. This completes the proof.$\|$

A.3 Proof of Proposition 2

Substituting the value of $\widehat{m}$ (from equation (9)) on the right hand side of equation (7), we get

$$\theta^* = \frac{1}{r}\left(1 + (1 - \frac{\psi^*}{\theta^*(1+\psi^*)}\right).$$

The solutions are

$$\theta^* = \frac{1}{r}(1 \pm \sqrt{D})$$

where

$$D = 1 - \frac{r\psi^*}{1+\psi^*}. \tag{A.7}$$

Recall that in equilibrium we must have $\widehat{m} < 1$. We know from equation (9) that in equilibrium, $\widehat{m} = (1-\theta^*)\psi^*/\theta^*$. Since this is true for any value of $\theta^* \in [\underline{\theta}, \overline{\theta}]$, this must hold at $\underline{\theta} = 1/r$. Using this, we have $\frac{r\psi^*}{1+\psi^*} < 1$. Using this inequality in the expression for D above (equation (A.7)), it follows that $D > 0$.

Now consider the solution $\theta_1^* = \frac{1}{r}(2 - \sqrt{D})$. Since $D > 0$, $\theta_1^* < \frac{1}{r}$. Therefore $\theta_1^* < \underline{\theta}$ and therefore this solution is rejected. The right solution is therefore given by

$$\theta^* = \frac{1}{r}(1 + \sqrt{D})$$

where D is given by equation (A.7). It is clear that $\theta^* > \frac{1}{r} = \underline{\theta}$. Also, by construction, $\theta^* \leqslant \overline{\theta}$.

Next, consider the second part of the result.

Suppose in equilibrium $\theta^* = \overline{\theta}$. Let us show this leads to a contradiction. We know, from Proposition 1 that in equilibrium $\psi^* \in (0, \overline{\psi})$.

Now, suppose intermediary i holds liquidity ψ_i. Recall that any intermediary gets a positive payoff only if it successfully rolls over loans in period 1 and succeeds in period 2. In the success state, the payoff of i is $R - (1+\psi_i)r + R\delta\frac{\psi_i}{\widehat{m}}$. Note that only types at or above $\overline{\theta} = 1 - \Delta$ successfully rollover loans in period 1. Therefore a measure $1 - \Delta$ of intermediaries are liquidated in period 1. It follows that the secondary market price of assets is

$$\widehat{m} = \frac{\Delta\psi^*}{1-\Delta}.$$

Using this, the payoff of an intermediary i in the success state is $R - (1+\psi_i)r + R\delta\frac{1-\Delta}{\Delta}\frac{\psi_i}{\psi^*}$, which can be rewritten as

$$R - r + \psi_i\left(R\delta\frac{1-\Delta}{\Delta}\frac{1}{\psi^*} - r\right).$$

Since $\psi^* < \overline{\psi}$, ψ^* is bounded above. It follows that the coefficient of ψ_i is strictly positive for Δ small. This implies that the optimal ψ_i is $\overline{\psi} \neq \psi^*$, which is a contradiction.

This implies that if Δ is small, in equilibrium we must have $\theta^* < \overline{\theta}$. This completes the proof.‖

A.4 Proof of Lemma 3

Step 1. First, we calculate the value of $U_k(y^*, y^*)$.

From equation (11), we know that

$$V_k(\theta_a^*, x|\eta_k) = \frac{1}{2\varepsilon}\int_{\theta_a^*}^{x-\eta_k+\varepsilon}(\theta\, r - 1)\, d\theta - \frac{1}{2\varepsilon}\int_{x-\eta_k-\varepsilon}^{\theta_a^*} G\, d\theta$$

Integrating, we get

$$V_k(\theta_a^*, x|\eta_k) = \frac{r}{4\varepsilon}\left((x-\eta_k+\varepsilon)^2 - (\theta_a^*)^2\right) - \frac{1}{2\varepsilon}\left((x-\eta_k+\varepsilon) - \theta_a^*\right) - \frac{G}{2\varepsilon}\left(\theta_a^* - (x-\eta_k-\varepsilon)\right)$$

Let

$$\gamma \equiv \frac{1-\psi}{1+\psi} \tag{A.8}$$

From equation (14),

$$\theta_a^* = y^* + \gamma\varepsilon \tag{A.9}$$

Using this value of θ_a^* in the expression for $V_k(\theta_a^*, x)$ above, we get

$$V_k(y^*, x|\eta_k) = \frac{r}{4\varepsilon}\left((x-\eta_k+\varepsilon)^2 - (y^*+\gamma\varepsilon)^2\right) - \frac{1}{2\varepsilon}\left((x-\eta_k+\varepsilon) - (y^*+\gamma\varepsilon)\right) - \frac{G}{2\varepsilon}\left((y^*+\gamma\varepsilon) - (x-\eta_k-\varepsilon)\right)$$

Putting $x = y^*$, and simplifying, we get

$$\begin{aligned}
V_k(y^*, y^*|\eta_k) &= \frac{r}{4\varepsilon}\Big(\varepsilon(1-\gamma) - \eta_k\Big)\Big(2y^* + \varepsilon(1+\gamma) - \eta_k\Big) \\
&\qquad - \frac{1}{2\varepsilon}\Big(\varepsilon(1-\gamma) - \eta_k\Big) - \frac{G}{2\varepsilon}\Big(\varepsilon(1+\gamma) + \eta_k\Big) \\
&= \frac{1}{2\varepsilon}\Bigg(\Big(\varepsilon(1-\gamma) - \eta_k\Big)\Big((2y^* + \varepsilon(1+\gamma) - \eta_k)\frac{r}{2} - 1\Big)\Bigg) - \frac{G}{2\varepsilon}\Big(\varepsilon(1+\gamma) + \eta_k\Big) \\
&= \frac{1}{2\varepsilon}\Bigg(\Big(\varepsilon(1-\gamma) - \eta_k\Big)\Big((y^* r - 1) + \frac{r}{2}(\varepsilon(1+\gamma) - \eta_k)\Big)\Bigg) - \frac{G}{2\varepsilon}\Big(\varepsilon(1+\gamma) + \eta_k\Big)
\end{aligned}$$

Let

$$A(\eta_k) \equiv (y^* r - 1) + \frac{r}{2}(\varepsilon(1+\gamma) - \eta_k) \tag{A.10}$$

Using this, rearranging terms, and simplifying

$$V_k(y^*, y^*|\eta_k) = \frac{1}{2}\Big((1-\gamma)A(\eta_k) - (1+\gamma)G\Big) - \frac{\eta_k}{2\varepsilon}\Big(A(\eta_k) + G\Big) \tag{A.11}$$

We know from section 4.1 that the minimizing case obtains when $\eta_k = \varepsilon_i^\beta$ and the maximizing case obtains when $\eta_k = -\varepsilon_i^\beta$. It follows that the α-maxmin net expected utility from rolling over at signal $x = y^*$ is given by

$$U_k(y^*, y^*) = \alpha V_k(y^*, y^*|\eta_k = \varepsilon^\beta) + (1-\alpha)V_k(y^*, y^*|\eta_k = -\varepsilon^\beta)$$

Substituting the appropriate value of η_k in equation (A.11),

$$\begin{aligned}
U_k(y^*, y^*) &= \frac{1}{2}\Bigg((1-\gamma)\Big(\alpha A(\varepsilon^\beta) + (1-\alpha)A(-\varepsilon^\beta)\Big) - (1+\gamma)G\Bigg) \\
&\qquad - \frac{\alpha}{2}\varepsilon^{\beta-1}(A(\varepsilon^\beta) + G) - \frac{(1-\alpha)}{2}\Big(-\varepsilon^{\beta-1}\Big)\Big(A(-\varepsilon^\beta) + G\Big)
\end{aligned}$$

Let $A_0(\varepsilon)$ denote the first term in the expression on the right hand side above. Using this and simplifying,

$$U_k(y^*, y^*) = A_0(\varepsilon) - \frac{\varepsilon^{\beta-1}}{2}\Bigg(\Big(\alpha A(\varepsilon^\beta) - (1-\alpha)A(-\varepsilon^\beta)\Big) + (2\alpha - 1)G\Bigg)$$

Substituting the value of the A terms from equation (A.10) and simplifying,

$$U_k(y^*, y^*) = A_0(\varepsilon) + \varepsilon^{\beta-1}\Bigg(\Big(\frac{1}{2} - \alpha\Big)\Big((y^* r - 1) + G + \frac{r}{2}\varepsilon(1+\gamma)\Big) + \frac{r}{2}\varepsilon^\beta\Bigg) \tag{A.12}$$

Step 2. Let us now show that for ε small, this expression is negative or positive according as $\alpha \gtrless 1/2$.

To see this, note that $\lim_{\varepsilon\to 0} A(\varepsilon^{\beta}) = \lim_{\varepsilon\to 0} A(-\varepsilon^{\beta}) = y^* r - 1$. From the value of θ^* derived in equation (A.9), we know that $y^* \to \theta^*$ as $\varepsilon \to 0$. Further, since $\theta^* \geqslant \underline{\theta} = 1/r$, it follows that

$$\lim_{\varepsilon\to 0} A(\varepsilon^{\beta}) = \lim_{\varepsilon\to 0} A(-\varepsilon^{\beta}) = \theta^* r - 1 \geqslant 0.$$

It follows that the term $A_0(\varepsilon)$ in equation (A.12) is finite and goes to a finite limit as ε goes to zero.

Next, consider the coefficient of $\varepsilon^{\beta-1}$ in equation (A.12). We know from above that $(y^* r - 1) \to (\theta^* r - 1) \geqslant 0$. This, combined with the fact that $G > 0$ (equation (12)) implies that as $\varepsilon \to 0$, this coefficient term goes to $\left(\frac{1}{2} - \alpha\right)(\theta^* r - 1 + G)$. It is then clear that for small ε, this term is finite and strictly positive for $\alpha < 1/2$ and strictly negative for $\alpha > 1/2$.

Now, $\varepsilon^{\beta-1}$ increases without bound as ε vanishes. Therefore, for small values of ε, the sign of $U_k(y^*, y^*)$ depends on the sign of the second term, which in turn depends on the sign of the coefficient of $\varepsilon^{\beta-1}$.

It follows that for ε small enough, $U_k(y^*, y^*) < 0$ if $\alpha > 1/2$, and $U_k(y^*, y^*) > 0$ if $\alpha < 1/2$.

Step 3. Finally, we consider the case of $\alpha = 1/2$. Putting $\alpha = 1/2$ in equation (A.12),

$$\begin{aligned} U_k(y^*, y^* | \alpha = 1/2) &= \frac{1}{2}\left((1-\gamma)\left((1/2)A(\varepsilon^{\beta}) + (1/2)A(-\varepsilon^{\beta})\right) - (1+\gamma)G\right) + \varepsilon^{2\beta-1}\frac{r}{2} \\ &= \frac{1}{2}\left((1-\gamma)\left((y^* r - 1) + \frac{r}{2}\varepsilon(1+\gamma)\right) - (1+\gamma)G\right) + \varepsilon^{2\beta-1}\frac{r}{2} \end{aligned}$$

Equating the above to 0 and solving,

$$y^* = \frac{1}{r} + \frac{1}{r}\frac{1+\gamma}{1-\gamma}G - \frac{\varepsilon^{2\beta-1}}{2(1-\gamma)} - \frac{1+\gamma}{2}\varepsilon$$

Using equations (A.9), (12) and (A.8) and simplifying,

$$\theta_a^* = \theta^* - \frac{\psi}{1+\psi}\varepsilon - \frac{1+\psi}{4\psi}\varepsilon^{2\beta-1}$$

where θ^* is the coordination-success-threshold in the case without ambiguity, given by equation (7). Note that as $\varepsilon \to 0$, $\theta_a^* \to \theta^*$ if $2\beta - 1 > 0$, i.e. $\beta > 1/2$. If, on the other hand, $\beta < 1/2$, the third term goes to negative infinity as $\varepsilon \to 0$, implying that for $\beta < 1/2$, $\theta_a^* = \underline{\theta}$ for small values of ε.[19] Finally, for $\beta = 1/2$, $\theta_a^* \to \max\left\{\underline{\theta}, \theta^* - \frac{1+\psi}{4\psi}\right\}$. This completes the proof.$\|$

A.5 Proof of Corollary 1

This follows in a straightforward manner from the proof of Proposition 3. We include the details for completeness.

Let θ_a^{**} denote the rollover-success threshold for a large intermediary expected to receive liquidity support with probability p if lenders fail to rollover funds, and let $\widehat{U}_k(\theta_a^{**}, x)$ denote the α-maxmin net expected utility of lender k from rolling over. This is given by

$$\begin{aligned}\widehat{U}_k(\theta_a^{**}, x) = \alpha\Big(pE(\theta r - 1|\eta_k = \varepsilon^\beta) + (1-p)V_k(\theta_a^{**}, x|\eta_k = \varepsilon^\beta)\Big) \\ + (1-\alpha)\Big(pE(\theta r - 1|\eta_k = -\varepsilon^\beta) + (1-p)V_k(\theta_a^{**}, x|\eta_k = -\varepsilon^\beta)\Big)\end{aligned}$$

Now,

$$E(\theta r - 1|\eta_k) = \frac{1}{2\varepsilon}\int_{x-\eta_k-\varepsilon}^{x-\eta_k+\varepsilon}(\theta r - 1)d\theta = 2xr - 1 - 2\eta_k$$

It follows that

$$\widehat{U}_k(\theta_a^{**}, x) = p(2xr - 1 + 2\varepsilon^\beta(1-2\alpha)) + (1-p)U_k(\theta_a^{**}, x)$$

where $U_k(\theta_a^{**}, x)$ is the α-maxmin net expected utility of agent k from rolling over for an intermediary that does not expect to receive liquidity support.

Now, in the case without liquidity support, the α-maxmin net expected utility from rolling over at signal $x = y^*$ is $U_k(y^*, y^*)$ given by equation (A.12). From the above, it is clear that the same for an intermediary with liquidity support is given by

$$\widehat{U}_k(y^*, y^*) = p\big(2y^*r - 1 + 2\varepsilon^\beta(1-2\alpha)\big) + (1-p)U_k(y^*, y^*)$$

[19] In other words, for $\beta > 1/2$ the coordination outcome is as shown in figure 1. For $\beta < 1/2$, θ_a^* is also equal to $\underline{\theta}$ for small values of ε, so that the coordination outcome jumps from full-coordination to complete collapse as α changes from any value less than or equal to $1/2$ to any value above $1/2$.

The first term is finite and goes to a finite limit of $p(2\theta^* r - 1)$ as $\varepsilon \to 0$. However, as the proof of Lemma 3 shows, for $\alpha > 1/2$ ($\alpha < 1/2$), $U_k(y^*, y^*)$ decreases (increases) without bound as ε vanishes. Therefore, if $p < 1$, for small values of ε, $\widehat{U}_k(y^*, y^*) \lesseqgtr 0$ according as $\alpha \gtreqless 1/2$. Then the same proof as Proposition 3 applies, implying that coordination collapses for $\alpha > 1/2$ (ambiguity aversion). This completes the proof.$\|$

References

Acharya, Viral V. and Tanju Yorulmazer, "Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures," *Review of Financial Studies*, 2008, *21* (6), 2705–2742. 8

_ , Douglas Gale, and Tanju Yorulmazer, "Rollover Risk and Market Freezes," *Journal of Finance*, 2011, *66* (4), 1177–1209. 6

Adrian, Tobias and Hyun Song Shin, "Liquidity and Leverage," *Journal of Financial Intermediation*, 2010, *19* (3), 418–437. 7

Allen, Franklin and Douglas Gale, "Liquidity Preference, Market Participation and Asset Price Volatility," *American Economic Review*, 1994, *84*, 933–955. 9

_ and _ , "Optimal Financial Crises," *Journal of Finance*, 1998, *53*, 1245–1284. 9

Angeletos, George-Marios and Iván Werning, "Crises and Prices: Information Aggregation, Multiplicity and Volatility," *American Economic Review*, 2006, *96* (5). 6

_ , Christian Hellwig, and Alessandro Pavan, "Signaling in a Global Game: Coordination and Policy Traps," *Journal of Political Economy*, 2006, *114* (3), 452–484. 6

_ , _ , and _ , "Dynamic Global Games of Regime Change: Learning, Multiplicity and Timing of Attacks," *Econometrica*, 2007, *75* (3), 711–756. 6

Brunnermeier, Markus K., "Deciphering the Liquidity and Credit Crunch 2007-2008," *Journal of Economic Perspectives*, 2009, *23* (1), 77–100. 7

_ and Lasse Heje Pedersen, "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, 2009, *22* (6), 2201–2238. 7

Covitz, Daniel, Nellie Liang, and Gustavo Suarez, "The Anatomy of a Financial Crisis: The Evolution of Panic-Driven Runs in the Asset-Backed Commercial Paper Market," 2009. Working Paper 2009-36, Board of Governors of the Federal Reserve. 1

Diamond, D.W. and P.H. Dybvig, "Bank Runs, Deposit Insurance and Liquidity," *Journal of Political Economy*, 1983, *91*, 401–419. 2, 3, 6

Gilboa, Itzach and David Schmeidler, "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics*, 1989, *18*, 141–153. 3, 19

Goldstein, Itay and Ady Pauzner, "DemandDeposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 2005, *60* (3), 1293–1327. 6

Gorton, Gary B., *Misunderstanding Financial Crises*, Oxford University Press, 2012. 1, 27

_ and Andrew Metrick, "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, 2012, *104* (3), 425–451. 1

Morris, Stephen and Hyun Song Shin, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 1998, *88*, 587–597. 3

_ and _ , "Global Games: Theory and Applications," in L. Hansen M. Dewatripont and S. Turnovsky., eds., *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)*, Cambridge, England: Cambridge University Press, 2003. 3, 14

_ and _ , "Coordination Risk and the Price of Debt," *European Economic Review*, 2004, *48*, 133–153. 14

Rochet, Jean-Charles and Xavier Vives, "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?," *Journal of European Economic Association*, 2004, *2*, 1116–1147. 5, 25

Shleifer, Andrei and Robert Vishny, "Fire Sales in Finance and Macroeconomics," *Journal of Economic Perspectives*, 2011, *25* (1), 29–48. 7

Ui, Takashi, "Ambiguity and Risk in Global Games," 2009. Working paper. 5

Vives, Xavier, "Strategic Complementarity, Fragility, and Regulation," *Review of Financial Studies*, 2014, *27* (12), 3547–3592. 5, 25