Tobin's Q, Asymmetric Information and Aggregate Stock Market Valuations

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Abstract

Estimates of Tobin's Q for the United States using publicly available data present an apparent puzzle: it is systematically less than unity. This paper sets out a simple model consistent with rational stock market valuation under conditions of asymmetric information that provides a possible explanation of this puzzle

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Estimates of Tobin's Q for the United States using publicly available data present an apparent puzzle. It is usually assumed that the ratio of the stock market's valuation of US corporations to their underlying recorded assets at replacement cost should be close to unity, at least on average. Indeed, if there are significant monopoly rents, or if (as has been claimed by a number of authors in recent years - eg, Hall, 2000; Laitner & Stolyarov, 2003) statisticians typically under-record intangible assets, then it would be expected that Tobin's Q would typically be above unity. In fact the data show that it has typically been well *below* unity.

One possible response to this apparent puzzle is to conclude that the data must be wrong. This would require that statisticians habitually overestimated the replacement cost of capital for most of period over which Q can be calculated, but (at least according to the authors cited above) switched to *under*estimating it in the immediate past. Understandably, statisticians, who spend a lot more time and effort on data construction than economists do, would dispute both these claims. It therefore seems worth pursuing the possibility that the statisticians, rather than the economists, are getting it right.

There may indeed be an alternative explanation. Since Grossman and Stiglitz (1980) it has been established that a perfectly efficient stock market, in which price always equals underlying value, must be a logical impossibility, due to the costs of information gathering. This paper explores the link between rational stock market valuations and the corporate sector's underlying aggregate value, in the presence of asymmetric information. It asks whether systematic differences between price and underlying value are a possible alternative explanation of the properties of recorded q.

1. The Puzzle of Tobin's Q

Table 1, using data from Wright (2004,2005) shows that mean values of Tobin's Q, constructed from a range of publicly available data, have been well below unity in the postwar era both for the US business sector as a whole,[1] and for the better documented non-farm non-financial corporate sector. The same feature holds in data for the nonfinancial corporate sector over the course of the entire twentieth

[1] Note that Laitner & Stolyarov (*op cit*) derive Tobin's Q estimates for the business sector that have an average value *greater* than unity in the period 1953-2000, and use this as supporting evidence for claims that there are significant quantities of intangible assets. Wright (2005) shows however that this feature of their data is due to major errors in data construction.

century; and for quoted companies (defining Q on an equity basis) on data since 1871.

Table 1. Alternative Estimates of Tobin's Q			
	Geometric Averages		
	1953-2000	1900-2002	1871-2002
All business	0.859*	n/a	n/a
Nonfarm, Nonfinancial Corporations	0.670*	0.648**	n/a
S&P 500 Companies[2]	0.590-0.695**	0.525-0.707**	0.545-0.862**

Sources: * Wright (2004); **Wright (2005)

2. Stock Market Valuation with Asymmetric Information

2.1. Firms

Consider the value maximisation problem of a representative unleveraged firm. Our analysis is standard, and closely related to that of Hayashi (1982); Abel and Blanchard (1986) and many others. Let Ω_t represent information available to the firm at the end of period t. Assume the firm maximises the value of the firm:

$$V_t = E_t \left[\Theta_{t+1} \left(D_{t+1} + V_{t+1} \right) | \Omega_t \right] \tag{2.1}$$

where D_t is the total net cashflow to equity holders.[3] The firm makes investment decisions, and chooses factors of production, to maximise its current value, and simultaneously prices its resulting maximised value with reference to Θ_{t+1}, the one-period ahead stochastic discount factor. Because the firm's information set is greater than the public information set, due to the costs of information-gathering, the firm is in a unique position to assess its own underlying value. It does so, however, using a market-based measure of (stochastic) opportunity cost.

2.2. Stock Markets

Assume that stock markets price the same income flow, using the same stochastic discount factor, but on the basis of public information, S_t:

$$P_t = E_t \left[\Theta_{t+1} \left(D_{t+1} + P_{t+1} \right) | S_t \right] \tag{2.2}$$

[2] Ratio of stock price to net worth per share, derived by cumulating retentions. Estimates require starting values for Q: range shown based on initial values of 0.5 and 2 in 1871. For further details of calculations see Wright (2004).

[3] (For referees: See Endnote A)

where P_t = stock market value. Rearranging (2.1) and (2.2), we have the no arbitrage pricing condition

$$E_t\left[\Theta_{t+1}\left(1+I_{t+1}\right)|\Omega_t\right] = E_t\left[\Theta_{t+1}\left(1+R_{t+1}\right)|S_t\right] = 1 \tag{2.3}$$

where $I_{t+1} = \frac{D_{t+1}+\Delta V_{t+1}}{V_t}$ is the internal return, and $R_{t+1} = \frac{D_{t+1}+\Delta P_{t+1}}{P_t}$ is the stock return. With symmetric information, this condition would be automatically satisfied by the equality of P_t with V_t for all t and hence of the stock return with the firm's internal return. More generally, however, with costly information (which here can be viewed as providing the rationale for the existence of firms), the information sets, and hence the two returns must differ. Since the underlying income flow being valued is identical, this must in turn imply that P_t must differ from V_t.[4] The pricing condition will nonetheless provide a crucial link between the stock market's valuation and underlying value.

2.3. Tobin's q and "Noise"

For simplicity we assume constant returns to scale and no investment adjustment costs,[5] and hence that firms' investment ensures aggregate V_t is always equal to the replacement cost of capital, which we shall assume that statisticians measure perfectly in aggregate. These are very strong assumptions, but we make them solely for the sake of isolating the potential role of asymmetric information. There is no necessary conflict between these assumptions and that of asymmetric information, so long as market participants cannot infer value at the individual firm level from the aggregate.[6] Tobin's Q will then be given by

$$Q_t = \frac{P_t}{V_t} \tag{2.4}$$

In contrast to the usual interpretation of Q, as reflecting investment adjustment costs, in this framework it simply captures the deviation of market prices from their underlying value if there were symmetric information. We are interested in whether in this restrictive framework Q may differ systematically from unity.

[4] While this feature is a logical implication of rational pricing under asymmetric information it may of course also arise under other conditions.

[5] (For referees: See Endnote B)

[6] (For referees: see Endnote C)

We first make the pricing condition in (2.3) more transparent by log-linearising. Letting $r_t = \ln(1 + R_t)$, $q_t = \ln Q_t$, etc, we can use the Campbell-Shiller (1988) approximation to write

$$r_{t+1} \simeq \varphi + \Delta p_{t+1} + (1 - \rho)(d_{t+1} - p_{t+1}) \tag{2.5}$$

where $\rho = \frac{1}{1+\exp(\overline{d-p})}$; $\varphi = \ln\left(1 + \exp(\overline{d-p})\right) - (1-\rho)(\overline{d-p})$ and $\overline{d-p} = E(d-p)$. For values of $\overline{q}$ sufficiently close to zero, we can apply the same log-linearisation coefficients to $i_t = \ln(1 + I_t)$ (replacing p with v) and relate the two log returns by writing:

$$r_{t+1} \simeq i_{t+1} + \varepsilon_{t+1} \tag{2.6}$$

where, using (2.4) and (2.5)

$$\varepsilon_{t+1} = \rho q_{t+1} - q_t \tag{2.7}$$

Following standard practice, we refer to ε_{t+1} as "noise" even though in our framework we assume it does not arise from any irrationality.

Assuming log normality, taking unconditional expectations of the pricing condition (2.3), using (2.6) and ruling out a deterministic trend in q implies

$$\overline{q} \simeq \frac{1}{1-\rho} E\left[cov_t(\varepsilon_{t+1}, \theta_{t+1}) + cov_t(\varepsilon_{t+1}, i_{t+1}) + \frac{var_t(\varepsilon_{t+1})}{2}\right] \tag{2.8}$$

Covariances and variances may in principle be time-varying. But, so long as they are themselves stationary, q_t will also be stationary (hence p_t and v_t must be cointegrated). This general result (which places no other restriction on the process for q_t than that it be stationary) arises simply from the minimal assumption that the market and the representative firm use the same stochastic discount factor.[7]

3. Can "Noise" Explain Apparent Under-Pricing?

From (2.8) it is evident that $\overline{q}$ may in principle be less than zero. This then offers at least a possible explanation for the observed less-than-unit geometric mean of Tobin's Q. A relatively simple calibration exercise, exploiting some further identifying assumptions, can be used to assess its plausibility.

To simplify, we assume, first that conditional covariances are time-invariant;[8] and second that $cov_t(\varepsilon, i) = 0$. The second assumption is a stronger one, but is

[7] (For referees: see Endnote D)

[8] This is an innocuous assumption as long as they are themselves stationary.

consistent with efficient information-gathering by market participants.[9] We can then re-write (2.8) as:

$$\overline{q} \simeq \frac{var_t(\varepsilon_{t+1})}{1-\rho}\left[\beta_{\theta\varepsilon} + \frac{1}{2}\right] \tag{3.1}$$

where $\beta_{\theta\varepsilon} = cov_t(\theta_{t+1}, \varepsilon_{t+1})/var_t\,(\varepsilon_{t+1})$ is the response of the stochastic discount factor to a unit innovation in ε_{t+1}. Since ε_{t+1} will by assumption be observationally indistinguishable from i_{t+1} using the market information set, S_{t+1} we can on *a priori* grounds set $\beta_{\theta\varepsilon} = \beta_{\theta i} = \beta_{\theta r}$. This latter coefficient can be treated as known in our framework, since, using standard assumptions[10] the log of the mean equity premium must equal, with opposite sign, the mean covariance of the stock return with the unobservable stochastic discount factor. That is, if R^s_{t+1} is the safe return, and we define

$$\eta \equiv \log E\left(\frac{1+R_{t+1}}{1+R^s_{t+1}}\right) = -cov_t(\theta_{t+1}, r_{t+1}) = \beta_{\theta r} var_t\,(r_{t+1}) \tag{3.2}$$

then, given that we can observe both η and $var_t\,(r_{t+1})$, and hence $\beta_{\theta r}$, the only unknown element in (3.1) is the conditional variance of ε_{t+1}. So we can can infer the required degree of volatility of "noise" if this is to be the sole explanatory factor of the observed negative mean of q.

It turns out that, given available emprical estimates, the above expression simplifies yet further. A mid-range estimate of η (see, eg, Campbell *et al, op cit*) is of the order of 0.04. Conveniently, this is also a reasonable estimate of both $var(r_{t+1})$[11] and the linearisation parameter $1-\rho$, so the expression in (3.1) simplifies further to

$$\overline{q} \simeq -\frac{1}{2}\frac{var_t(\varepsilon_{t+1})}{var_t(r_{t+1})}$$

The figures for Tobin's Q for the nonfinancial corporate sector shown in Table 1 imply $\overline{q} \simeq -\frac{1}{3}$. Thus if we assumed that "noise" was the sole explanatory factor, we would need the conditional variance of the noise process to be around two thirds of the conditional variance of the log stock return.

[9] (For referees: see Endnote E)
[10] For a standard treatment,see eg, Campbell, Lo and Mackinlay, 1997, Ch 8.
[11] (For referees: See Endnote F)

4. Conclusions

This paper has set out a simple model consistent with rational stock market valuation under asymmetric information that appears to provide a possible explanation for the empirical puzzle that Tobin's Q is systematically less than unity. The intuition for this result is that, viewed as an investment asset, corporate capital is relatively safe compared to the stocks and shares that provide notional title to that capital. It is relatively safe because it is relatively uncorrelated with the stochastic discount factor, for the simple reason that its return is only imperfectly observable.

We make no claim that this is actually the sole explanation of Tobin's Q being predominantly below unity. Indeed, if it were, this would require "noise" to be the dominant element in the variability of aggregate stock returns. But our analysis does suggest that the empirical implications of less than perfect market efficiency for aggregate stock market valuations are potentially significant.

References

[1] Abel, A and Blanchard, O (1986), "The Present Value of Profits and Cyclical Movements in Investment" *Econometrica* 54(2) pp 249-273

[2] Campbell, J and Shiller, R (1988) "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies*, 1. 195-227

[3] Campbell, J, Lo, A J and MacKinlay, A C (1997), *The Econometrics of Financial Markets*, Princeton University Press, Princeton, NJ

[4] Grossman, S, Stiglitz, J (1980), "On the Impossibility of Informationally Efficient Markets", *American Economic Review*, 70(3), June 1980, pages 393-408.

[5] Hall, R (2001) "The Stock Market and Capital Accumulation," *American Economic Review* 91:1185-1202

[6] Hayashi, F(1982), "Tobin's Marginal q and Average q: A Neoclassical Interpretation", *Econometrica*, 50(1), Jan. 1982, pages 213-24.

[7] Laitner, J and Stolyarov, D (2003), “Technological Change and the Stock Market”, *American Economic Review,* vol. 93, no. 4 1240-67

[8] Wright, S (2004) “Measures of Stock Market Value and Returns for the US Nonfinancial Corporate Sector, 1900-2002”, *Review of Income and Wealth* 50 (4) pp 561-584

[9] Wright, S (2005) “Technological Change and the Stock Market, A Comment", Under revision for *American Economic Review*

Workings

Derivation of (2.6) and (2.7)

If we apply the same linearisation as in (2.5) to the log internal return (which is acceptable as long as $\overline{q} \simeq 0 \rightarrow \overline{d-v} \simeq \overline{d-p}$)

$$i_{t+1} \equiv \ln\left(1+I_{t+1}\right) \simeq \varphi + \Delta v_{t+1} + (1-\rho)(d_{t+1} - v_{t+1}) \tag{4.1}$$

and subtract from (2.5) we have, using (2.4) and (2.7)

$$\begin{aligned} r_{t+1} - i_{t+1} &\simeq \Delta p_{t+1} - \Delta v_{t+1} + (1-\rho)\left[d_{t+1} - p_{t+1} - (d_{t+1} - v_{t+1})\right] \\ &\simeq \Delta q_{t+1} - (1-\rho) q_{t+1} \\ &\simeq \rho q_{t+1} - q_t \\ &\simeq \varepsilon_{t+1} \end{aligned}$$

Derivation of (2.8)

Substituting from (2.6) into (2.3), and using $\theta_t = \log \Theta_t$ gives

$$E_t \exp(\theta_{t+1} + i_{t+1}) | \Omega_t = E_t \exp(\theta_{t+1} + i_{t+1} + \varepsilon_{t+1}) | S_t$$

taking logs, exploiting lognormality, and letting $\sigma_x^2 = var_t(x_{t+1})$; $\sigma_{xy} = cov_t(x_{t+1}, y_{t+1})$,

$$\begin{aligned} & E_t \left[(\theta_{t+1} + i_{t+1}) + \frac{1}{2}\left(\sigma_\theta^2 + \sigma_i^2 + 2\sigma_{i\theta}\right)\right] | \Omega_t \\ = \; & E_t \left[(\theta_{t+1} + i_{t+1} + \varepsilon_{t+1}) + \frac{1}{2}\left(\sigma_\theta^2 + \sigma_i^2 + \sigma_\varepsilon^2 + 2\sigma_{i\theta} + 2\sigma_{\varepsilon\theta} + 2\sigma_{\varepsilon i}\right)\right] | S_t \end{aligned}$$

If we apply the law of iterated expectations repeatedly to derive unconditional expectations, and assume homoscedasticity, the differences in information sets disappear, and we get

$$\begin{aligned} & E \left[(\theta_{t+1} + i_{t+1}) + \frac{1}{2}\left(\sigma_\theta^2 + \sigma_i^2 + 2\sigma_{i\theta}\right)\right] \\ = \; & E \left[(\theta_{t+1} + i_{t+1} + \varepsilon_{t+1}) + \frac{1}{2}\left(\sigma_\theta^2 + \sigma_i^2 + \sigma_\varepsilon^2 + 2\sigma_{i\theta} + 2\sigma_{\varepsilon\theta} + 2\sigma_{\varepsilon i}\right)\right] \end{aligned}$$

which allows us to cancel terms,

$$0 = E\left(\varepsilon_{t+1}\right) + E\left[\frac{\sigma_\varepsilon^2}{2} + \sigma_{\varepsilon\theta} + \sigma_{\varepsilon i}\right]$$

Hence, using (2.7),

$$-E\left(\varepsilon_{t+1}\right) \simeq -\overline{\Delta q} + (1-\rho)\overline{q} \simeq E\left[\frac{\sigma_\varepsilon^2}{2} + \sigma_{\varepsilon\theta} + \sigma_{\varepsilon i}\right]$$

ruling out a deterministic trend in q implies $\overline{\Delta q} = 0$, hence we can solve to give (2.8).

Derivation of (3.1)

To get (3.1) we set $\sigma_{\varepsilon i} = 0$ and apply the homoscedasticity assumption (which, it should be noted, simply implies that conditional variances and covariances are constant over time - it does not imply that they equal their unconditional values).

Endnotes for Referees

A.We could equivalently consider the problem of a leveraged firm in a Miller-Modigliani setting, where leverage does not affect firm value. In this case D_t would be the net cashflow to equity and bondholders. This would be more consistent with the data on q, which include the market value of debt in the numerator; but we simplify here for the purposes of exposition.

B. Since we are only concerned with long-run features we could equally well assume that adjustment costs have mean zero, which is explicitly or implicitly assumed in most q theories of investment; and that any variance and covariance terms associated with adjustment costs have a minimal impact on mean q.

C. Here we respond to a number of points raised by past referees relating to the issue of measurement of aggregate vs firm level capital. 1) Aggregate capital could in principle be measured perfectly if aggregate investment figures and depreciation factors were measured perfectly, which does not require firm level information. 2) Typically there will be considerable time lags in data collection, and even if we accept that statisticians get it right in the end they typically do so through a process of regular revisions. Thus it is highly unlikely that Q_t will be part of S_t, the market information set. This in itself precludes any possible arbitrage based on Q_t. But even if aggregate Q_t *were* part of S_t, and thus investors knew that the market was over- or under-valued in aggregate, their informational disadvantage at the firm level would not allow them to exploit this information. 3) Some firm level data on assets are of course available from balance sheets, but only at book value; and in the era of Enron it seems reasonable to assume that this gives at best an imperfect picture of firm-level replacement cost assets.

D. If $\overline{q} \neq 0$, then, to the extent that the linearisation coefficents are sufficiently different, there will be an additional term in $\overline{d-p}$, the mean "cashflow" yield, so the result will still hold as long as the cashflow yield is stationary for which there is strong evidence. See Robertson, D and Wright, S (2004a), "Dividends, Total Cashflows to Shareholders and Predictive Return Regressions (under revision for *Review of Economics and Statistics*)

E. It is perhaps worth stressing that this does *not* require ε_t itself to be white noise. So far all that we have established is the stationarity of q. Hence we can write $q = \overline{q} + a(L)\omega_t$ where ω_t is an innovation, hence, from (2.7) $\varepsilon_t \simeq \overline{\varepsilon} + (\rho - L)a(L)\omega_t$. Similarly we can assume some process for $i_t = b(L)u_t$ where u_t may be some structural innovation like a technology shock. Efficient use of S_t would imply $E(\omega_t u_t) = 0$, and hence $cov(\varepsilon_t, i_t) = 0$, but this makes no restriction on the time series properties of ε_t.

F. Assuming an annual volatility of 20%: ie, we use the unconditional variance; but the answer would barely differ if there were some predictability of r_{t+1}. (The approximate equality of η and $var_t(r_{t+1})$ implies $\beta_{\theta r} \simeq -1$, which we would expect)