Fiscal policy and asset purchases in a liquidity constrained economy

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Declaration

I declare that the work presented in this thesis is entirely my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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Abstract

This thesis modifies the basic neoclassical DSGE model of Kiyotaki and Moore (2012) by introducing a government which levies distortionary taxes on wages and dividends, consumes general output, issues money, and holds privately-issued equity. The thesis answers two questions – Can discretionary policy relieve the effects of liquidity constraints that limit investment, and thereby stimulate economic activity in normal times? Can discretionary policy ameliorate the effects of an exogenous liquidity shock?

Including distortionary taxes is a unique modification within a branch of literature that extends the work of KM and studies liquidity shocks. The thesis belongs to a branch of this literature which modifies KM's basic model, but none of these papers have distortionary taxes and none examine fiscal policy. The thesis extends the literature with a novel variant of the basic KM model and with a novel set of policies against a liquidity shock.

The results are as follows. Firstly, if money supply is constant and government spending varies to always balance the fiscal budget, then across-the-board tax cuts persistently stimulate the economy and a cut in the rate of tax on dividends ameliorates a liquidity shock without additional distortions. These responses are robust to the model's calibration. Secondly, an increase in government spending, financed by more taxes or selling equity holdings, persistently worsens economic activity and exacerbates a liquidity shock; financing the policies is what brings adverse results. Thirdly, the direct effects of a government equity purchase programme are short-lived – investment rises and new equity is added to the market which partially offsets the government's purchase. Financing the programme with spending cuts do less harm than raising taxes. Adding monetary expansion to the policy mix improves aggregate supply but reduces aggregate demand. When used against a liquidity shock, the programme makes a positive but short-lived difference.

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Dedication

I lovingly dedicate this thesis to my wife – *Vashtie* – who gave up her leisure so that I could have more time to spend on my work. This thesis would not have been possible without her patience, strength, support, and encouragement.

I also dedicate this thesis to my parents – *Roopnarayan* and *Chandra* – who understood and were part of the sacrifices I've made towards this accomplishment.

And finally, I dedicate this thesis to my extended family – *Aunty Jean*, *Marc*, *Dianne*, *Rhys*, and *Kai* – who helped me in many ways, provided many pleasant distractions, and whose company I will miss.

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Chapter 1

Introduction

1.1 Objective and scope

This thesis modifies the basic model of Kiyotaki and Moore (2012) (henceforth, KM) by introducing a government which levies distortionary taxes on wages and dividends, consumes the economy's general output, exclusively issues a non-depreciating perfectly-liquid asset (money), and holds a stock of a depreciable privately-issued partially-liquid asset (equity). The economy is populated by two groups of private agents – workers and entrepreneurs. Asset returns are too low to encourage workers to save, so they consume all their net wages in each period. Entrepreneurs produce and consume, and in each period a fraction of them invest while the rest save. Entrepreneurs sell their stocks of equity and issue new equity to raise liquid funds for investment, but both financing activities are constrained by exogenous frictions (or "liquidity constraints"). The thesis aims to answer two questions about this economy – Can discretionary policy relieve the effects of liquidity constraints that limit investment, and thereby stimulate economic activity in normal times, that is, when there are no other exogenous shocks? Can discretionary policy ameliorate the effects of an exogenous tightening of the liquidity constraints (a "liquidity shock")?

The research questions are answered by three exogenous and temporary policies – cuts in tax rates, an increase in government spending, and a government purchase of equity. There is no debt in the model, and discretionary policies are completely and contemporaneously financed by endogenous variations in one or more of the other policy variables (that is, from among tax rates, government spending, money supply, and government equity holdings). The model is calibrated; discretionary policies are simulated as exogenous stochastic shocks to tax rates, government spending, and government equity holdings; and results are captured and analysed by impulse responses. The thesis therefore theoretically investigates a variety of fiscal policy and fiscal financing scenarios in the KM model.

1.2 Related literature and contribution

The thesis belongs to a set of related works (a "KM-related literature") which all feature KM's liquidity constraints and study liquidity shocks. In particular, the thesis belongs to a strand of the KM-related literature which modifies the basic neoclassical DSGE model of KM. None of the surveyed papers include distortionary taxes, and none of them quantitatively examine fiscal policy. The closest is Driffill and Miller (2013) who algebraically illustrate balanced budget fiscal expansion with lump-sum taxes, and who hypothesise about the inter-temporal complexity of adding distortionary taxes. The thesis is motivated by the gaps in the literature, and it expands existing knowledge on liquidity shocks from a neoclassical perspective.

There is another set of related works within the KM-related literature which introduces KM's liquidity constraints to fairly standard New Keynesian DSGE models. The New Keynesian framework is particularly appealing because it readily accommodates modifications which allow KM's policy of equity purchases to avoid the irrelevance proposition of Wallace (1981). The thesis is related in various ways to members of this group. The closest relative is Kara and Sin (2014), who perform the only other dedicated study of fiscal policy in the KM-related literature. Kara and Sin focus on increasing government spending, both in normal times and against a negative liquidity shock. Such spending is financed by new issues of government bonds, which is an asset that does not exactly resemble money or equity from this model. Moreover, there are structural differences between models which lead to qualitative differences in results.

These structural differences with Kara and Sin (2014) and other related works help identify limitations of the thesis, and help formulate extensions with which to build a programme of future work. The thesis is merely a first step towards understanding fiscal policy in KM's liquidity constrained economy. At the end, a future research programme is proposed which will hopefully develop a robust understanding of fiscal policy and from which advice can be obtained.

1.3 Main results

The intuition which motivates the introduction of distortionary taxes to the KM model is appealing. KM propose that a government should compensate for an unexpected loss of private liquidity with a purchase of partially-liquid assets and a sale of liquid assets. This can be thought of as a direct injection of liquidity. By contrast, tax cuts boost net incomes, and are thus an indirect way of providing liquidity. Adding distortionary taxes not only puts the KM model one step closer to reality, but it opens up the model to exploring an alternative to asset purchases as policy against a liquidity shock.

The experiments of the thesis begin with cutting taxes. The model is uniquely adapted for this work by assumptions of constant money supply and a balanced fiscal budget in which government spending varies according to tax revenue. Across-the-board tax cuts stimulate the economy for a prolonged period of time, and a cut in the rate of tax on dividends is sufficient to ameliorate a liquidity shock without additional distortions, whether positive or negative. A sensitivity analysis concludes that these responses are robust to the model's calibration of structural parameters and the persistence of shocks to tax rates.

The thesis then considers an increase in government spending. Two experiments are successfully simulated, in which the government finances it spending by more taxes and by selling part of its stock of equity. Because government spending is unproductive and does not influence private agents' utility, it merely crowds out private spending and leaves output and employment unchanged. Financing such spending brings adverse and persistent consequences. These discretionary spending policies exacerbate a negative liquidity shock, and the adverse consequences are again the result of their financing.

Finally, the thesis returns to the equity purchase policy of KM that is widely studied in the related literature. Equity purchases are contemporaneously financed by fiscal austerity measures. On its own, the government's purchase raises asset prices and thus improves entrepreneurs' net worth and stimulates investment. However, more investment creates new equity which eventually compensates for the quantity purchased by the government. The direct effects of the equity purchase programme are therefore short-lived. Austerity measures that finance the equity purchase then produce additional effects. Austerity by spending cuts do less harm than by raising taxes. Adding monetary expansion to the policy mix is not a Pareto improvement – the supply-side of the economy benefits but the demand-side suffers.

When used against a liquidity shock, the equity purchase programme makes a positive but short-lived difference. This is a consistent finding in the KM-related literature, but for somewhat different reasons.

1.4 Organisation

The thesis is organised as follows. Chapter 2 defines the novelty of the thesis by surveying the KM-related literature and describing the structure of existing models and their approaches to policy against liquidity shocks. Chapter 3 gives a full description of the model that is used throughout the thesis. The government's behaviour is where the model differs from KM. Chapter 4 describes and justifies the procedures that are involved in calibrating the model, simulating exogenous shocks, and representing the results.

The next three chapters are the substantial chapters of the thesis in which policy variables are shocked, the macroeconomic consequences of such shocks are analysed, and exogenous variations in policy variables are evaluated as measures against a negative liquidity shock. Chapter 5 simulates cuts in tax rates in a variant of the model in which the government maintains a constant money supply and varies its spending to balance the fiscal budget in each period. Tax rates are cut simultaneously and then individually in separate experiments. The two tax rates affect different sides of the economy, and simulating separate shocks to them helps devise an appropriate policy against a liquidity shock. Chapter 6 repeatedly simulates an increase in government spending in variants of the model that differ by the way in which such spending is financed. Three methods of financing are attempted – varying tax rates, selling stocks of entrepreneur-issued equity, and issuing money. Chapter 7 repeatedly simulates a government purchase of equity in variants of the model that differ by the way in which the asset purchase is financed. Five financing arrangements are attempted – issuing money, cutting government spending, issuing money and cutting government spending, raising taxes, and issuing money and raising taxes.

Chapter 8 analyses the sensitivity of the quantitative results of simultaneous tax cuts in Chapter 5 to the calibration of the model's structural parameters and persistence of shocks to tax rates. Chapter 9 summarises the main findings of the thesis, evaluates the experiments, and identifies extensions and opportunities for future research.

Chapter 2

The KM-related literature: a survey

2.1 Introduction

This chapter surveys a set of related works which are directly derived from Kiyotaki and Moore (2012). The thesis calls this set the "KM-related literature". Papers that belong to the group have two common ingredients – they feature KM's liquidity constraints in a DSGE model, and they study liquidity shocks. The chapter describes the current state of the KM-related literature, with particular emphasis on the structure of existing models and the approaches to policy against liquidity shocks. In doing this, the chapter defines the novelty of the thesis.

The KM-related literature can be divided into two groups, based on the structure of the underlying DSGE model – there are papers that modify the KM model, and there are papers that introduce KM's liquidity constraints to other fairly standard models. Papers that modify the KM model all do so modestly, and thereby present neoclassical models which resemble the standard Real Business Cycle framework; Bigio (2010, 2012), Nezafat and Slavík (2012), Shi (2012), and Driffill and Miller (2013) make up this group. Papers that introduce KM's liquidity constraints all do so in fairly standard New Keynesian DSGE models. Del Negro et al. (2011) is the earliest of such work, and from which Kara and Sin (2013, 2014) and Molteni (2014) develop extensions and further modifications. Ajello (2012) is another member of this group, but he develops his own unique framework by translating KM's liquidity constraints into an empirically observable financial friction.

The thesis contributes to the former group with a unique modification of the basic KM model. None of the papers in this group introduce distortionary taxes on wages and income. KM include lump-sum taxes in their "full" model, Shi (2012) also includes lump-sum taxes, and Driffill and Miller (2013) speculate about adding distortionary taxes, while Nezafat and Slavík (2012) do not have a government. Moreover, in the New Keynesian group, Del Negro et al. (2011), Kara and Sin (2013, 2014), and Molteni (2014) have lump-sum taxes; Ajello (2012) has distortionary taxes, but the structure of his model and the objectives of his paper are significantly different from this thesis.

Alternatively, the KM-related literature can be divided by the interest in liquidity shocks. One group – comprising Bigio (2010, 2012), Nezafat and Slavík (2012), Shi (2012), and Ajello (2012) – examines how the shock's transmission and amplification are influenced by the presence of real and financial frictions. The rest of the KM-related literature evaluates policy against the shock. Del Negro et al. (2011), Driffill and Miller (2013), Kara and Sin (2013), and Molteni (2014) follow KM and implement the policy of Holmström and Tirole (1998) (or what is now commonly known as "quantitative easing") in which the government buys a partially liquid privately-issued asset in exchange for a perfectly-liquid government-issued asset. Kara and Sin (2014) apply fiscal policy via government spending. This thesis extends the work of the latter group with a new set of policies, that is, it gives a more comprehensive treatment of fiscal policy, and studies equity purchase programmes that are different from quantitative easing and which are accompanied by fiscal austerity measures.

The rest of the chapter is organised as follows – Section 2.2 briefly describes Kiyotaki and Moore (2012); Section 2.3 surveys papers in the KM-related literature that introduce KM's liquidity constraints to New Keynesian DSGE models; and Section 2.4 surveys the papers that modify the KM model; and Section 2.5 summarises the chapter.

2.2 Kiyotaki and Moore (2012)

KM is a member of a vast literature on DSGE models with financial frictions.[1] This literature examines how real shocks are amplified and more persistent with the presence of financial frictions, and studies the effects of exogenous shocks to these frictions (or "financial

[1]Gertler and Kiyotaki (2010) and Brunnermeier et al. (2012) provide recent and comprehensive surveys of this literature.

shocks"). KM expand knowledge of these issues from a unique and theoretical perspective. They propose a neoclassical DSGE model with a novel feature of two simultaneously binding financial frictions.[2] These frictions (or "liquidity constraints") limit entrepreneurs' abilities to internally and externally raise funds for investment. To self-insure against falling short of liquid funds when an opportunity to invest comes along, entrepreneurs endogenously hold a perfectly liquid government-issued asset (that is, money) alongside another partially liquid privately-issued asset which earns dividends (that is, equity). KM therefore introduce an endogenous demand for money.

KM calibrate their model and qualitatively examine a negative shock to one of the frictions (or a "liquidity shock") which reduces the value of liquid funds that are available to entrepreneurs and thus lowers investment. The liquidity shock resembles a key event of the 2008 crisis, that is, the freezing of second-hand asset markets and widespread emergence of liquidity shortages (Brunnermeier (2009), Moore (2009), Del Negro et al. (2011)). KM demonstrate the effectiveness of a particular policy against a liquidity shock, one that is proposed by Holmström and Tirole (1998) and is now commonly known as "quantitative easing", whereby the government buys large quantities of equity from private agents in exchange for new issues of money. KM's frictions introduce asset heterogeneity, which allows such equity purchases to escape the irrelevance proposition of Wallace (1981). This type of policy is prominently featured in the response to the 2008 crisis. The US Federal Reserve, for instance, purchased privately-held assets and expanded its balance sheet, after which Bernanke (2009) coined the term "credit easing" to refer to such policy.[3]

The KM-related literature is based on an earlier (2008) version of the KM paper which features only the basic model. This thesis is also based on the basic KM model. In the most recent (2012 NBER) version of their paper, KM design a "full" model to address criticism from the KM-related literature. The basic model produces small responses to a liquidity shock, because of a negative co-movement between investment and consumption. The shock

[2] The frictions themselves are not unique. One friction originates from Hart and Moore (1994).

[3] A distinction among the concepts "large-scale asset purchases", "unconventional monetary policy", and "credit easing" is worth mentioning here. Chapter 7 expands this note, and Buiter (2008), Bernanke (2009), Klyuev et al. (2009), and Lyonnet and Werner (2011) give further lessons on the nomenclature. "Unconventional monetary policy " and "large-scale asset purchases" are equivalent but generic names for a set of slightly different policy measures. Table 7.1 lists and compares these measures. KM themselves call their asset purchase programme "open market operations", while Del Negro et al. (2011) call it "non-standard open market operations" and Driffill and Miller (2013) call it "quantitative easing". The meanings of these terms have been reassigned since those papers were written, and now they do not refer to the equity purchase programme that is in KM. Instead, KM's programme describes what policymakers now call "credit easing".

reduces investment, which decreases equity's supply and raises asset prices via a portfolio balance effect. KM's full model includes a storage technology, which introduces endogenous price stickiness for money by being a perfect substitute for the asset. Without storage, asset prices would rise, entrepreneurs' net worth would improve, and investment would increase in a feedback effect which offsets the initial decline after the liquidity shock. With storage, the feedback effect is suppressed and the fall in investment is greater.

2.3 New Keynesian DSGE models with liquidity constraints

2.3.1 Del Negro et al. (2011)

Del Negro et al. (2011) (henceforth, DEFK) is a pioneering work which introduces KM's liquidity constraints to a standard New Keynesian DSGE model along the lines of Christiano et al. (2005) and Smets and Wouters (2007). Their objective is to quantitatively evaluate the effectiveness of the Federal Reserve's credit easing policy in the 2008 crisis. DEFK calibrate their model to match observations of the US economy during the crisis. They simulate the crisis as a negative liquidity shock, to represent the observed freezing of second-hand asset markets and to artificially replicate the Great Recession. Via policy rules, the shock triggers a conventional lowering of the nominal interest rate to its zero lower bound and an unconventional policy response of credit easing.

Kara and Sin (2013) show that if DEFK had not installed KM's liquidity constraints then credit easing in the New Keynesian DSGE model would have had no impact. This is an extension of conclusions by Eggertsson and Woodford (2003) and Cúrdia and Woodford (2011) who show that the irrelevance proposition of Wallace (1981) applies to open market operations in a standard New Keynesian DSGE model and one augmented with nominal frictions. A large-scale asset purchase programme affects the aggregate economy primarily by altering relative asset prices, that is, via a portfolio balance effect of Tobin (1961, 1963, 1969), Brunner and Meltzer (1973), and Friedman (1978).[4] In a frictionless model, the portfolio balance effect is offset by private agents taking advantage of arbitrage opportunities. KM's liquidity constraints make assets sufficiently heterogeneous that the asset purchase programme has its desired impact.[5] This is the motivation behind DEFK's modification of the standard New Keynesian DSGE model. Consequently, DEFK's liquidity shock sig-

[4]See the consensus on this idea from literature surveys by Gagnon et al. (2011) and Joyce et al. (2011b)).
[5]See Andrés et al. (2004) for support on this idea.

nificantly reduces output, investment, and consumption by magnitudes that resemble those actually seen in the US from the third quarter of 2008. By counterfactual experiments, DEFK show that the Great Recession in the US would have been much more severe had it not been for policy intervention by the Fed.

DEFK's main result – that credit easing policy is effective at ameliorating a negative liquidity shock – is shared by Chapter 7 of this thesis, as well as by other papers in the KM-related literature which examine policy. Besides the structural differences and the inclusion of nominal and real frictions, DEFK's model is different from the model in Chapter 7 by featuring only lump-sum taxes.

2.3.2 Kara and Sin (2013, 2014)

Kara and Sin (2013, 2014) add government spending to DEFK. Kara and Sin (2013) closely follow DEFK and examine the effectiveness of a large-scale asset purchase programme against a negative liquidity shock. Their aim is to formulate optimal monetary policy in the tradition of Lucas and Stokey (1983) and Chari et al. (1991). They show that the asset purchase programme makes a significant impact in ameliorating a liquidity shock. Kara and Sin (2013) otherwise provide two useful insights. First, they extend the canonical Wallace-irrelevance result of Eggertsson and Woodford (2003) to large-scale asset purchases in the standard New Keynesian DSGE model. Second, they show that Wallace-irrelevance is overturned in the model by adding KM's liquidity constraints.

Kara and Sin (2014) (henceforth, KS) is the first quantitative and dedicated study of fiscal policy in the KM-related literature. They simulate an exogenous increase in government spending, first on its own and then contemporaneously against a negative liquidity shock. Chapter 6 performs the same set of experiments, but in its own unique framework, and therefore complements KS with a new perspective of government spendings shocks.

Chapter 6 and KS are different in many ways. First, the models are structurally different. Government purchases of equity in Chapter 6 bid up asset prices, which improve private net worth, investment, and consumption. With sticky prices, KS suppress asset price variations, and their responses to government spending shocks are short-lived. One conclusion that can be drawn from this comparison is that asset price fluctuations are essential to the transmission of demand shocks. The presence of nominal rigidities is why New Keynesian models find it difficult to obtain significant aggregate fluctuations, and leaves them to search for additional frictions to enhance shock responses.

Second, DEFK state that their model aims to avoid heterogeneity in private consumption, and so there are no non-Ricardian agents in KS. Workers in KS save by buying bonds and entrepreneur-issued equity. When asset prices rise, workers supply more labour to maintain saving levels, and output rises as a consequence. In this model, workers do not save and are therefore unaffected by asset price changes. Policies which target asset prices, such as the equity purchase programme, have no immediate impact on output in this or any flexible-price model, but they do in KS and other New Keynesian models.

Third, the government in KS collects only lump-sum taxes. Changes in the fiscal balance in KS are much less distorting to private behaviour than they are in Chapter 6 in which tax rates vary. While an increase in government spending improves the economy in KS, it worsens economic activity in all the experiments of Chapter 6 because of higher taxes that are required for financing.

Fourth, the government in KS issues bonds to finance its spending. Bonds are risk-free and perfectly liquid, which are features that are shared by money in this model. The difference between the assets is that the bond yields are determined by a policy rule which depends on output, whereas money's yield (via price appreciations) is determined by the asset's market. One experiment in Chapter 6 assumes an increase in government spending is financed by monetary expansion; this is the closest the thesis comes to KS.

Fifth, liquidity shocks are simulated differently. In Chapter 6, after equity's re-saleability initially falls, it then asymptotically returns to steady state from the next period. In KS, equity's re-saleability falls and stays at that level for three years, after which it immediately and completely returns to steady state.

Finally, KS assume a lower degree of persistence of shocks to government spending, with the persistence parameter set to 0.8, compared to 0.95 in this thesis. The thesis uses the same calibration settings as KS for all except two structural parameters, namely, capital's share of output (0.36 in KS and 0.4 here) and the inverse Frisch elasticity of labour supply (1.92 in KS and 1.0 here). KS calibrate the liquidity constraint parameters according to DEFK, but they calibrate all other structural parameters differently, according to estimates by Smets and Wouters (2007). Chapter 8 shows, albeit for a tax cut experiment in Chapter 5, that the model is sensitive to large changes in the persistence parameter and the calibration of capital's share of output.

2.3.3 Molteni (2014)

Molteni (2014) modifies DEFK by defining equity as a perfectly illiquid asset and adding two types of government bonds – a perfectly liquid short-term type and a partially liquid long-term type. In other words, Molteni proposes a variant of DEFK with three assets of varying liquidity. He examines the economy's response to a negative liquidity shock and the effectiveness of unconventional monetary policy in the form of government issues of short-term bonds. He assumes the most relaxed re-saleability constraint among the KM-related literature (with an agent being able to liquidate 75% of its holdings in any given period), and does this in order to replicate an observed pre-crisis haircut of 25% on 10-year bonds that are issued by the governments of Portugal and Ireland. He disentangles the calibrations of the liquidity constraints and maintains DEFK's value for the borrowing constraint. *Ceteris paribus*, a negative liquidity shock in his model reduces output, consumption, and investment, and causes a delayed decline in employment. He shows that policy intervention significantly ameliorates these responses and gives employment a long-term increase with a hump-shaped trajectory.

2.3.4 Ajello (2012)

Ajello (2012) is the only model in the KM-related literature which has distortionary taxes. His model is otherwise very different from that of this thesis – it includes sticky prices and wages, a stylised banking system, and other structural features of the New Keynesian DSGE paradigm. His focus is on the literature's classic problem of how well financial shocks explain business cycle fluctuations in aggregate variables. Ajello's model is the only one among the KM-related literature that is estimated, and he uses Bayesian methods in the tradition of Smets and Wouters (2007) and An and Schorfheide (2007). Ajello distinguishes his work from DEFK by using a larger sample of observations, one that spans 20 years (1989 – 2010), whereas DEFK concentrate on the period around the 2008 crisis. Ajello's paper is motivated by, and aims to address, two empirical pitfalls of KM.

First, simulating a liquidity shock is not straight-forward because, given their abstract nature, liquidity constraints are difficult to represent by empirically observed variables. DEFK are the first to calibrate the liquidity constraint parameters, and thus initiates quantitative work on the KM model. DEFK follow a theoretical assumption of KM that the tightness of borrowing and re-saleability constraints are the same in steady state. All the papers in the KM-related literature except Shi (2012) and Molteni (2014) also make this assumption. Then the task simplifies to estimating just one parameter. DEFK estimate the re-saleability constraint based on a ratio of US government liabilities to private capital, and their solution is what most of the KM-related literature use for calibration. The only exception is Nezafat and Slavík (2012), who estimate the borrowing constraint as a ratio of external corporate finance to investment. Ajello is wary about exogenously shocking a parameter that is estimated with considerable uncertainty. He proposes a solution that uses an empirically observed variable. He installs banks to intermediate the transfer of equity between saving and investing entrepreneurs. Banks charge an intermediation fee for this service, in the tradition of Chari et al. (1995), Goodfriend and McCallum (2007), and Cúrdia and Woodford (2010). The fee is charged as a fraction of the price that investors (that is, the borrowers in his model) pay for equity. Ajello's intermediation fee is motivated by Kurlat (2013), who shows that adverse selection with heterogenous assets creates a financial market friction that amplifies aggregate shocks, just like KM's liquidity constraints. The intermediation fee is exogenously determined and varies stochastically. Ajello proxies the fee by an interest rate spread, a variable for which data is readily available. He then simulates a "financial shock" as an exogenous increase in the fee.

The second pitfall of KM is a negative co-movement between investment and consumption following a liquidity shock. KM address this problem with endogenous price stability in their full model, and the rest of the New Keynesian KM-related literature, as well as Driffill and Miller (2013), assume nominal rigidities. Ajello shows that sticky wages contribute significantly to the explanatory power of financial shocks – his financial shock with (without, respectively) sticky wages explains 35% (9.5%, respectively) and 60% (49%, respectively) of the variance of output and investment, respectively. The financial shock discourages investment by making it more expensive for investors to borrow. In other words, investors reduce their demand for equity, which depresses the asset's price; this is in contrast to the liquidity shock's rise in equity's price in the basic KM model, a response which also happens in this thesis. Moreover, Ajello's equity price decline discourages savers from selling the asset to banks, and thereby encourages consumption, hence the positive co-movement with investment.

2.4 Modifications of the KM model

2.4.1 Shi (2012)

Shi (2012) points out another flaw in KM – while a liquidity shock raises asset prices and lowers output, such a countercyclical asset price response is hardly observed in recessions. This remark justifies the use of New Keynesian models to simulate recessions from liquidity shocks. Shi rationalises KM's asset price responses to the liquidity shock. He re-designs the basic KM model into a representative household setup and adds government bonds and lump-sum taxes. His model is calibrated slightly differently from most of the KM-related literature. Most notably, he assumes a more relaxed re-saleability constraint (with an agent being able to liquidate 27.3% of its holdings in any given period) than the rest of the KM-related literature except Bigio (2012) and Molteni (2014). Shi finds that asset price increases are robust to the introduction of New Keynesian frictions, namely sticky wages, habit persistence in consumption, and capital adjustment costs. Then he discovers that asset prices are pro-cyclical only to shocks that have an immediate impact on aggregate productivity. He concludes that liquidity shocks alone cannot explain asset price declines in recessions. This thesis somewhat supports Shi's conclusion. Here, exogenous shocks to tax rates, government spending, and government equity holdings are simulated, none of which affect aggregate productivity, and yet asset prices respond counter-cyclically.

2.4.2 Nezafat and Slavík (2012)

Nezafat and Slavík (2012) propose a variant of KM in which the re-saleability constraint is shut down and the borrowing constraint is modelled as a time-varying stochastic variable. Their liquidity shock is an exogenous decline in the borrowing constraint. They proxy the borrowing constraint as a ratio of aggregate corporate funds raised in the markets to aggregate investment. Their paper is in the tradition of Gomes et al. (2003) and Jermann and Quadrini (2012) in a literature that searches for financial frictions that amplify asset price changes following real shocks, and with the aim of matching simulated volatility with empirically observed volatility. Nezafat and Slavík calibrate their model to achieve this objective. The novelty of their paper is that they simulate a productivity shock alongside a liquidity shock.

2.4.3 Bigio (2010, 2012)

Bigio (2010) quantitatively evaluates the liquidity shock in the KM model. He keeps the KM framework intact, calibrates the model, and finds that the shock produces very small output responses. The author cites the explanation of Barro and King (1984) – the shock negatively impacts investment, but this has a minor consequence for the capital stock; the most the capital stock can fall is by the full amount of depreciation (that is, if the shock brings investment down to zero); the shock has no immediate impact on labour; accordingly, with factors of production mostly unchanged, the output response is small. Bigio concludes that business cycle fluctuations cannot be achieved from a liquidity shock in a neoclassical model. He proposes that the shock is amplified if the model has frictions that transmit the effects to the labour market. To support his hypothesis, he augments the KM model with variable capital utilisation, as in Greenwood et al. (1988). He finds that responses to the liquidity shock are magnified tenfold with the friction compared to without.

Bigio (2012) continues his search for frictions with which liquidity shocks are able to generate significant responses. His paper shares the same objective and finds the same result as Ajello (2012). Bigio takes a simpler approach than Ajello by modifying KM's model. In particular, Bigio adds two real frictions to the KM model. First, there is limited enforcement of labour contracts, in the spirit of Hart and Moore (1994), which leaves entrepreneurs to use part of their productive asset (that is, capital) as collateral for loans, as in Kiyotaki and Moore (1997). Second, there is asymmetric information on the quality of capital, as in Kurlat (2013). Bigio simulates an exogenous dispersion of capital's quality, which produces an endogenous drop in liquidity. The frictions imply a decline in employment, and the economy experiences substantial declines in output, consumption, and investment. Bigio therefore solves the negative co-movement problem of the basic KM model. Moreover, all of his shock responses closely match observations from US data in the early stage of the 2008 crisis.

2.4.4 Driffill and Miller (2013)

Driffill and Miller (2013) (henceforth, DM) closely follow DEFK and aim to simulate the Great Recession from a liquidity shock and then evaluate the effectiveness of a policy of asset purchases. But unlike DEFK, DM's core environment is the basic KM model. Bigio (2010) already gives one reason why the KM model cannot produce a deep recession. Another reason is that the liquidity shock reduces investment on impact, but this is partially offset by a contemporaneous feedback effect – equity's supply falls, asset prices rise, entrepreneurs' net worth improves, and investment rises. DM cut off this feedback effect by assuming sticky prices. Compared to the KM model, a liquidity shock in DM's sticky-price variant produces a greater fall in investment and substantial declines in employment and output that closely match observations in the US during the 2008 crisis. DM's results are consistent with Ajello (2012) and Bigio (2012), despite differences in models and calibrations, respectively.

DM then evaluate the effectiveness of KM's asset purchase programme in ameliorating the liquidity shock. DM calibrate their model according to DEFK, which therefore means the asset purchase programme represents the Federal Reserve's credit easing policy during the 2008 crisis. DM find that the policy is highly effective. This is the same result of DEFK, but achieved without a New Keynesian framework. Chapter 7 of this thesis shares the policy evaluation objective with KM, DEFK, and DM. Unlike DM and DEFK, but as in KM, Chapter 7 uses a flexible price model.

Finally, DM theoretically explore the fiscal implications of KM's asset purchase programme. They suggest that, in the aftermath of the programme, if the government must return the economy to its pre-shock state then any dividends earned from holding equity must either be spent or give rise to a reduction in taxes. DM augment their sticky price model with lump-sum taxes on entrepreneurs, and algebraically show that a balanced budget increase in government spending leads to more employment and output but has no change in investment and capital. To limit the scope of their paper, DM avoid an analysis with distortionary taxes, arguing that such an inclusion adds a significant degree of complexity through agents contemporaneously revising their behaviour to changes in expectations of future taxes. That problem is avoided here because the government does not issue debt, and instead balances its flow of funds in each period.

2.5 Chapter summary

Kiyotaki and Moore (2012) contribute to the literature on DSGE models with financial frictions by proposing a neoclassical framework with two simultaneously binding frictions. A set of related works extend from KM, in which either the basic KM model is modified or the pair of frictions are introduced to New Keynesian DSGE models. This thesis joins the former group and adds a government that levies distortionary taxes. This chapter surveys the related literature and shows that such a modification is unique.

Papers in the KM-related literature share a common interest in liquidity shocks. They either look at the shock's transmission or evaluate policy against it. This thesis develops the second line of research in two directions – with a comprehensive treatment of fiscal policy, and with an asset purchase programme that is accompanied by fiscal austerity. Chapter 5 examines tax cuts with a balanced budget rule; Driffill and Miller (2013) is the closest to that chapter because they preserve much of the KM model and theoretically examine expansionary fiscal policy with a balanced budget, albeit with sticky prices and lump-sum taxes. Chapter 6 examines an increase in government spending; Kara and Sin (2014) is the New Keynesian equivalent of that chapter, with both works sharing the same objective and approach, although with structural and other differences between the models. And Chapter 7 examines credit easing policy; KM, Del Negro et al. (2011), Kara and Sin (2013), and Driffill and Miller (2013) are close relatives, with the same objective and approach, but with varying degrees of structural differences among the models.

The survey reveals structural features of the model which help explain differences in results from the rest of the literature and from actual observations. One way that Chapter 6 is different from Kara and Sin (2014) is in the flexibility of asset prices. With perfectly flexible prices, Chapter 6 generates large and persistent responses to shocks to liquidity and government spending; with sticky prices, Kara and Sin (2014) obtain short-lived responses. Variations in asset prices therefore appear essential for the propagation of shocks – they lead to changes in entrepreneurs's worth, and therefore changes in investment and long-term output. Another way that Chapter 6 is different from Kara and Sin (2014) is on the behaviour of workers. Kara and Sin (2014) assume workers save, and therefore any change in asset prices causes immediate changes in labour supply, employment, and output. On the other hand, workers are non-Ricardian in Chapter 6, and are therefore unaffected by asset price changes. Shocks and policy which target asset prices, such as the equity purchase programme, have no immediate impact on output in this model, but they do in New Keynesian models.

Chapter 3

The model

3.1 Introduction

This chapter describes the model that is used throughout the thesis. The model is a modification of the basic framework of Kiyotaki and Moore (2012). In particular, this framework adds a government which levies distortionary taxes on wages and dividends, consumes general output, issues money, and holds privately-issued equity. These variables which define the government's behaviour are known as "policy variables". Each experiment in the thesis makes its own assumptions about which policy variables are exogenously determined and which are endogenously determined. An endogenously determined policy variable evolves according to a policy rule. Such rules are different from the ones found in KM's full model, which has a government.[1] Beyond the differences in government's behaviour among experiments, the rest of the model is the same throughout the thesis.

The chapter is organised as follows. Section 3.2 introduces the model by describing its environment. Sections 3.3 and 3.4 define the behaviour of entrepreneurs and workers, respectively; these sections also derive those agents' dynamic optimising behaviour. Section 3.5 defines the government's behaviour and a menu of assumptions for policy variables; this section is the modification the thesis makes to the basic KM model. Finally, Section 3.6 aggregates the economy and derives dynamic equilibrium conditions for equity, labour, and general output markets. The chapter's appendices contain detailed algebra associated with derivations, simplifications, and proofs.

[1] The full KM model has two policy rules. The first is a fiscal rule that restricts discretionary government spending by limiting it to the deviation of net government asset holdings from steady state. The second is a monetary rule that restricts discretionary open market operations by limiting the ratio of current to steady state government equity holdings to a weighted sum of productivity and liquidity impulse responses.

3.2 The environment

The economy exists over an infinite horizon of discrete time periods. It is populated by a unit-mass continuum of *ex ante* identical entrepreneurs, a unit-mass continuum of identical workers, and a government. The population does not grow or decline, and the economy is closed to the rest of the world. There are no financial intermediaries or formal credit markets. All agents consume a perishable general output, which is produced exclusively by entrepreneurs and is the economy's numeraire. All agents can own and exchange two assets – money and equity. Money is perfectly liquid, does not depreciate, and is exclusively issued by the government; equity is not perfectly liquid, not perfectly re-saleable, depreciates each period, earns dividends, and is exclusively issued by entrepreneurs. Money and equity are traded in competitive markets at perfectly flexible prices, p_t and q_t, respectively, which are both expressed in terms of general output.[2]

3.3 Entrepreneurs

3.3.1 Production

Entrepreneurs are the exclusive owners of capital and a homogeneous Cobb-Douglas technology that produces general output with guaranteed success. At the beginning of period t the representative entrepreneur owns k_t units of capital. The entrepreneur employs l_t hours of labour, and produces y_t units of general output at the end of the period according to

$$y_t = A_t k_t^{\gamma} l_t^{1-\gamma} \tag{3.1}$$

where A_t is a common level of total factor productivity and $\gamma \in (0, 1)$ is the capital elasticity of output. The market for general output is perfectly competitive; then, according to Cobb and Douglas (1928), the production function (3.1) exhibits constant returns to scale and γ is also the share of output accruing to capital. A_t evolves according to a stationary AR(1) process,

$$A_t = (1 - \rho_A)A + \rho_A A_{t-1} + u_t^A \tag{3.2}$$

[2] In other words, p_t and q_t units of general output are exchanged for 1 unit of money and equity, respectively. These are "real" prices. "Nominal" prices are the prices of a unit of general output and equity expressed in terms of money, that is, $1/p_t$ and p_t/q_t, respectively.

where A is the steady state level of productivity, $\rho_A \in (0, 1)$ parameterises the degree of persistency of a stochastic productivity shock, and $u_t^A \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_{uA}^2)$ where σ_{uA}^2 is exogenously determined.

The entrepreneur pays workers w_t units of general output for each labour-hour employed. The rest of the period's general output is the gross profit, or dividends, to the capital used in production,

$$r_t k_t = y_t - w_t l_t \tag{3.3}$$

where r_t is the real rate of return, or dividends, per unit of capital.

3.3.2 Investment

Capital depreciates during the production process, and a fraction, δ, of its stock survives at the end of the period when production is complete. Some time soon after the start of the period, a fraction, π, of entrepreneurs gain access to a homogenous investment technology that converts a unit of general output into a unit of capital with guaranteed success. The production and installation of new capital takes an entire period. Therefore, an entrepreneur who invests i_t units of general output in period t has

$$k_{t+1} = \delta k_t + i_t$$

units of capital at the end of period t.

Entrepreneurs are identical *ex ante* to when investment opportunities are revealed. π is independently and identically distributed across time and entrepreneurs. Who gets an opportunity to invest is exogenously determined. Those without investment opportunities carry on with what they have been doing since the start of the period, that is, producing, consuming, and saving (by purchasing assets); they are the period's "savers". Those with investment opportunities change their behaviour when such opportunities are received; they are the period's "investors". The investment technology has unlimited capacity to produce capital, but any opportunity to use it expires at the end of the period. At the end of the period, investors and savers revert to being *ex ante* identical entrepreneurs until the next period's investment opportunities are revealed.

In an attempt to acquire each unit of general output for the investment technology, an investor publicly issues a new unit of equity. However, the investor simultaneously faces two constraints that make such external financing incomplete. Once an investment project is underway, the entrepreneur's human resource is needed for the entire period to ensure the full amount of new capital is produced. The investor acquires knowledge and skills that are specific to his investment project and cannot be costlessly replicated or replaced. The entrepreneur, however, cannot pre-commit to being involved with the project until its end. Instead, he can guarantee that he will remain with the project for no more than an exogenously determined fraction, θ, of its duration. This implies that he can guarantee a maximum of θ of an investment's new capital will be produced, which further implies that he can guarantee a maximum of θ of new output in the next period when the new capital enters production technologies. Consequently, the investing entrepreneur can credibly raise no more than θ of his investment cost from equity financing. This limitation is called the "borrowing constraint", which is an exogenous feature of the model that has its origins in Hart and Moore (1994) and Kiyotaki and Moore (1997).[3]

The representative entrepreneur holds n_t units of equity at the beginning of the period. The investor cannot sell all of his equity before an investment opportunity expires. Instead, he can liquidate up to a fraction, ϕ_t, of his holdings in period t. This limitation is called the "re-saleability constraint", and is an exogenously determined intrinsic feature of equity.[4] ϕ_t evolves according to a stationary AR(1) process,

$$\phi_t = (1 - \rho_\phi)\phi + \rho_\phi \phi_{t-1} + u_t^\phi \tag{3.4}$$

where ϕ is the steady state value of ϕ_t, $\rho_\phi \in (0, 1)$ parameterises the degree of persistency of a stochastic liquidity shock, and $u_t^\phi \sim^{i.i.d.} \mathcal{N}(0, \sigma_{u\phi}^2)$ where $\sigma_{u\phi}^2$ is exogenously determined.

Borrowing and re-saleability constraints simultaneously bind, and are together called "liquidity constraints". Beyond these limits, the investor completes his investment financ-

[3] An alternative interpretation of θ that is based on moral hazard is proposed by Lorenzoni and Walentin (2007) and repeated by Bigio (2010) – the entrepreneur can run away with a fraction, $(1-\theta)$, of the value of his capital at any time, simply because capital is always under his complete control. In models with formal credit markets, unlike this one, θ appears as a credit market friction – due to a limited ability by lenders to enforce loan contracts, borrowers are required to put up collateral, and they can borrow a fraction, θ, of the value of collateralised assets. If the collateralised assets are risky then $(1-\theta)$ is a haircut. The credit market friction is the most popular representation of θ, owing to Kiyotaki and Moore (1997) who show its macroeconomic significance, and to Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Bernanke et al. (1999) who introduce it into dynamic macroeconomic models. Brunnermeier and Pedersen (2009) gives the constraint an alternative name of "funding liquidity".

[4] The re-saleability constraint arises endogenously out of adverse selection in asset markets in Kiyotaki and Moore (2003), Eisfeldt (2004), Bigio (2012), and Kurlat (2013). With no change in interpretation, Brunnermeier and Pedersen (2009) call the re-saleability constraint "market liquidity".

ing by exchanging money for general output at the market price, p_t. The representative entrepreneur holds m_t units of money at the beginning of the period. The entrepreneur cannot lend money or use it as collateral, and therefore holds

$$m_{t+1} \geq 0 \tag{3.5}$$

units at the end of the period. The entrepreneur's demand for money is motivated by a precaution against falling short of liquidity when financing investment opportunities.[5]

To invest i_t units of general output, the investor issues i_t units of equity. He sells θi_t units of this new equity on the market and receives $\theta i_t q_t$ units of general output in exchange; this amount represents the investment's external finance. The remainder of the investment cost is internally financed – the investor liquidates his money and re-saleable equity to obtain $(i_t - \theta i_t q_t)$ units of general output, and he buys $(1-\theta)i_t$ units of his own new equity. $(i_t - \theta i_t q_t)$ is the value of the investor's equity stake in his own investment, or what KM call the investor's "downpayment" on his investment. Put differently, for every unit of investment, the entrepreneur pays himself $(1-\theta q_t)$ units of general output to acquire $(1-\theta)$ units of his own new equity issue. Or equivalently, for every unit of his own new equity that he retains, the investor effectively pays himself q_t^R units of general output, where

$$q_t^R = \frac{1-\theta q_t}{1-\theta} \tag{3.6}$$

is called the "effective price of inside equity" and varies negatively with the value of θ (see Appendix 3.A).

The non-re-saleable part of his own new equity issue that the investor retains is called his "inside" equity. An entrepreneur's stock of equity that is issued by other entrepreneurs is called his "outside" equity. Inside and outside equity are assumed to be perfect substitutes, that is, they have the same re-saleability constraint and provide the same rate of return. Inside and outside equity are therefore collectively referred to as "equity".

For an investment of i_t units of general output, at the end of the period the investor buys at least $(1-\theta)i_t$ new units of inside equity and remains with at least $(1-\phi_t)\delta n_t$ units

[5]The demand for money is inversely related to the tightness of liquidity constraints, that is, the values of θ and ϕ_t. The tighter the liquidity constraints bind, that is, the smaller the values of θ and ϕ_t, then the greater the need to internally finance investments, and the greater the desire to hold money balances; and conversely.

of non-re-saleable equity. The investor therefore holds

$$n_{t+1}^i \geq (1-\theta)i_t + (1-\phi_t)\delta n_t \tag{3.7}$$

units of equity at the end of the period.

Assumption 1. An entrepreneur with an investment opportunity borrows and liquidates the maximum quantities of equity that liquidity constraints allow.

Assumption 1 is consistent with inter-temporal utility maximisation. More investment in period t means more capital in period $t+1$, and therefore more profits/dividends and more consumption.

With assumption 1, Equation (3.7) becomes binding with equality,

$$n_{t+1}^i = (1-\theta)i_t + (1-\phi_t)\delta n_t \tag{3.8}$$

which, if re-structured, gives the entrepreneur's investment for the period,

$$i_t = \left(\frac{1}{1-\theta}\right) n_{t+1}^i - \left(\frac{1-\phi_t}{1-\theta}\right) \delta n_t \tag{3.9}$$

3.3.3 Consumption and saving

Since capital is created only from investment, then each unit of equity in the economy is backed by a unit of capital. Equity therefore depreciates in tandem with capital. Furthermore, the gross profits that accrue to capital represent dividends to the holders of equity. Then each unit of equity earns r_t units of general output in dividends after period t's production is complete. The entrepreneur pays the government a tax on dividend income at a rate of τ_t^{rn}. His net dividend income is then allocated to consumption and saving, and to investment if the opportunity exists.

An investor in period t consumes c_t^i units of general output, invests i_t units, and saves by accumulating $(n_{t+1}^i - i_t - \delta n_t)$ and $(m_{t+1}^i - m_t)$ units of equity and money, respectively, at their market prices. The investor thus faces a budget constraint for period t,

$$c_t^i + i_t + q_t(n_{t+1}^i - i_t - \delta n_t) + p_t(m_{t+1}^i - m_t) = (1-\tau_t^{rn})r_t n_t \tag{3.10}$$

The investor's budget constraint (3.10) is simplified by substituting Equation (3.9) to obtain (see Appendix 3.B)

$$c_t^i + q_t^R n_{t+1}^i = (1-\tau_t^{rn}) r_t n_t + \left[\phi_t q_t + (1-\phi_t) q_t^R\right] \delta n_t + p_t (m_t - m_{t+1}^i) \tag{3.11}$$

The RHS of Equation (3.11) is the investor's net worth – his net dividends from equity holdings, the value of depreciated equity (where a re-saleable fraction, ϕ_t, is valued at the market price and the non-re-saleable fraction is valued at q_t^R), and net sales of money. The LHS expresses what he does with his net worth.

Alternatively, substituting Equation (3.8) into Equation (3.10) gives the investor's resource constraint (see Appendix 3.B),

$$c_t^i + (1-\theta q_t) i_t = (1-\tau_t^{rn}) r_t n_t + \phi_t q_t \delta n_t + p_t (m_t - m_{t+1}^i) \tag{3.12}$$

The RHS of Equation (3.12) is the total liquid resources available to the investor in period t – net dividends from equity, a re-saleable portion of equity holdings, and net sales of money. The LHS says how he uses these resources – for consumption and financing that portion of his investment for which he cannot borrow.

A saver in period t consumes c_t^s units of general output and saves the rest of his net income by accumulating $(n_{t+1}^s - \delta n_t)$ and $(m_{t+1}^s - m_t)$ units of equity and money, respectively, at their market prices. The saver's budget constraint for period t is

$$c_t^s + q_t(n_{t+1}^s - \delta n_t) + p_t(m_{t+1}^s - m_t) = (1-\tau_t^{rn}) r_t n_t \tag{3.13}$$

3.3.4 Optimising behaviour

When investment opportunities are revealed in period t, the representative investor and saver make optimal choices on $\{c_t^i, i_t, n_{t+1}^i, m_{t+1}^i\}$ and $\{c_t^s, n_{t+1}^s, m_{t+1}^s\}$, respectively. These choices maximise an expected lifetime discounted utility,

$$E_t\left[\sum_{j=t}^{\infty} \beta^{j-t} U_e(c_j)\right] = U_e(c_t) + E_t[\beta U_e(c_{t+1}) + \beta^2 U_e(c_{t+2}) + \dots] \tag{3.14}$$

subject to the respective budget constraints, (3.11) and (3.13), where $E_t[\,\cdot\,]$ is the expectation that is conditional on information available in period t, and $\beta \in (0,1)$ is the subjective discount factor or the inverse rate of time preference. The representative entrepreneur's current utility is assumed to be a natural logarithm of current consumption,

$$U_e(c_t) \equiv \ln c_t$$

Optimal choices are made with uncertainty about investment opportunities in the future. The entrepreneur's first order conditions yield an Euler equation (see Appendix 3.C),

$$\begin{aligned}
&\pi E_t\left[\frac{1}{q_t}([1-\tau^{rn}_{t+1}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q^R_{t+1})U'_e(c^i_{t+1})\right]\\
&\quad+(1-\pi)E_t\left[\frac{1}{q_t}([1-\tau^{rn}_{t+1}]r_{t+1}+\delta q_{t+1})U'_e(c^s_{t+1})\right]\\
&=E_t\left[\frac{p_{t+1}}{p_t}\left(\pi U'_e(c^i_{t+1})+(1-\pi)U'_e(c^s_{t+1})\right)\right] \qquad (3.15)
\end{aligned}$$

Once an entrepreneur makes his optimal choices for period t, the Euler equation (3.15) describes the expectation that $1/q_t$ additional units of equity and $1/p_t$ additional units of money will provide the same marginal utility from consumption in period $t+1$. The expression on the RHS of (3.15) is the expected marginal benefit of holding $1/p_t$ additional units of money in period $t+1$. The expression on the LHS of (3.15) is the expected marginal benefit of holding $1/q_t$ additional units of equity in period $t+1$. Each unit of equity is expected to earn $(1-\tau^{rn}_{t+1})r_{t+1}$ net dividends, and then it depreciates in value. If there is an investment opportunity then a re-saleable portion of depreciated equity, $\delta\phi_{t+1}$, will be valued at the market price, q_{t+1}, while the non-re-saleable portion, $\delta(1-\phi_{t+1})$, will be valued at its replacement cost, q^R_{t+1}. If there is no investment opportunity then the depreciated value of a unit of equity will be δq_{t+1}.

Claim 1. $q_t \neq 1 \iff m^i_{t+1} = 0$

Proof of Claim 1. See Appendix 3.E.

The market price of equity is critical for economic activity. An investor needs at least 1 unit of general output in exchange for every unit of equity issued. If $q_t < 1$ then the investor does not raise enough funds in the market to fulfil his ambition of investing i_t, in which case the investor abandons his opportunity, and then all entrepreneurs become savers. If $q_t > 1$ then the investor materialises his opportunity and sells as much equity as he can within budget and liquidity constraints. To restrict attention to the case where there is investment in the economy, the following assumption is made.

Assumption 2. $q_t > 1$

By Claim 1 and Assumption 2, an investor will not have any money left at the end of a period of investment, that is, $m^i_{t+1} = 0$. He exhausts all of his money in the pursuit of an investment opportunity. In the next period, up to when new investment opportunities are revealed, the current period's investors will be able to replenish their money stocks.

The entrepreneur's logarithmic utility function provides a standard feature that his consumption in each period is a stable fraction, $(1 - \beta)$, of his net worth in that period. From Equations (3.11) and (3.13), Claim 1, and Assumption 2, a representative investor and saver consume, respectively,

$$c^i_t = (1-\beta)([1-\tau^{rn}_t]r_t n_t + \left[\phi_t q_t + (1-\phi_t)q^R_t\right]\delta n_t + p_t m_t) \tag{3.16}$$

$$c^s_t = (1-\beta)([1-\tau^{rn}_t]r_t n_t + q_t \delta n_t + p_t m_t) \tag{3.17}$$

The difference in consumption between the two types of entrepreneurs is given by

$$\begin{aligned} c^s_t - c^i_t &= (q_t - q^R_t)(1-\phi_t)\delta n_t \\ &= (q_t - 1)\left(\frac{1-\phi_t}{1-\theta}\right)\delta n_t \end{aligned} \tag{3.18}$$

Assumption 2 implies $c^s_t > c^i_t$. As an entrepreneur liquidates equity and money for investment financing, he inter-temporally substitutes consumption away from an investing period and towards a saving period. During a period of saving he accumulates equity and money, and does so in an optimal inter-temporal fashion according to the Euler equation (3.15).

Assumption 2 also implies (see Appendix 3.F)

$$\frac{[1-\tau^{rn}_{t+1}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + (1-\phi_{t+1})\delta q^R_{t+1}}{q_t} < \frac{[1-\tau^{rn}_{t+1}]r_{t+1} + \delta q_{t+1}}{q_t} \tag{3.19}$$

that is, an investor's equity portfolio generates a lower rate of return than a saver's equity portfolio. This is because of the limited re-saleability of equity for an investor, which forces him to own inside equity that is valued negatively to the market price of equity. Hence, the return on equity is correlated with consumption. This correlation, along with the re-saleability constraint, is what makes equity risky. Money, on the other hand, is free from these risks. Its return does not depend on having an investment opportunity, and it is perfectly liquid; these are two reasons why entrepreneurs hold money. Additionally, savers accumulate money in preparation for when they receive investment opportunities and expect to face liquidity constraints.

The Euler equation (3.15) simplifies to a portfolio balance equation (see Appendix 3.D),

$$\pi E_t \left[\frac{\left(\frac{p_{t+1}}{p_t}\right) - \left(\frac{[1-\tau^{rn}_{t+1}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q^R_{t+1}}{q_t}\right)}{[1-\tau^{rn}_{t+1}]r_{t+1}n_{t+1} + [\phi_{t+1}q_{t+1} + (1-\phi_{t+1})q^R_{t+1}]\delta n_{t+1} + p_{t+1}m_{t+1}} \right]$$
$$= (1-\pi)E_t \left[\frac{\left(\frac{[1-\tau^{rn}_{t+1}]r_{t+1} + \delta q_{t+1}}{q_t}\right) - \left(\frac{p_{t+1}}{p_t}\right)}{[1-\tau^{rn}_{t+1}]r_{t+1}n_{t+1} + q_{t+1}\delta n_{t+1} + p_{t+1}m_{t+1}} \right] \tag{3.20}$$

Equation (3.20) reflects the portfolio balance theory of Tobin (1958, 1969) and demonstrates substitution between assets when their relative price changes. If q_t rises, for example, then equity's expected return falls, and the entrepreneur substitutes towards money. The substitution represents an increase in demand for money, which in aggregate, *ceteris paribus*, raises p_t. Substitution moves back and forth until expected portfolio returns between having and not having an investment opportunity are equal. The LHS of Equation (3.20) expresses an expected excess return on money over equity if the entrepreneur becomes an investor. The RHS expresses an expected excess return on equity over money if he becomes a saver. The portfolio balance equation says that the *ex ante* identical entrepreneur equates the expected marginal benefits of receiving and not receiving an investment opportunity. He does this by varying how many units of equity and money he holds.

3.4 Workers

Workers are the exclusive owners of labour. They do not own capital or have investment opportunities. In period t the representative worker supplies l^w_t hours of labour to entrepreneurs in exchange for a perfectly flexible gross hourly wage of w_t units of general output. The worker pays the government a tax on wage income at a rate of τ^{wl}_t.

The worker holds n^w_t units of entrepreneur-issued equity and m^w_t units of government-issued fiat money at the beginning of period t. The worker's human resource is non-transferable across time, so he cannot borrow or have negative net worth. His equity and money holdings are therefore always non-negative, that is, for all t,

$$n_t^w \geq 0 \quad \text{and} \quad m_t^w \geq 0 \tag{3.21}$$

The worker pays the government a tax on dividend income at a rate of τ_t^{rn}.

The worker consumes c_t^w units of general output. The rest of his net income is saved by accumulating $(n_{t+1}^w - \delta n_t^w)$ and $(m_{t+1}^w - m_t^w)$ units of equity and money, respectively, at their prevailing market prices. His budget constraint for period t is given by

$$c_t^w + q_t(n_{t+1}^w - \delta n_t^w) + p_t(m_{t+1}^w - m_t^w) = (1 - \tau_t^{wl})w_t l_t^w + (1 - \tau_t^{rn})r_t n_t^w \tag{3.22}$$

Subject to Equations (3.21) and (3.22), the worker maximises an expected lifetime discounted utility,

$$E_t\left[\sum_{j=t}^{\infty} \beta^{j-t} U_w(c_j^w, l_j^w)\right] = U_w(c_t^w, l_t^w) + E_t[\beta U_w(c_{t+1}^w, l_{t+1}^w) + \beta^2 U_w(c_{t+2}^w, l_{t+2}^w) + \ldots] \tag{3.23}$$

The worker's utility is assumed to be additively separable in consumption and leisure,

$$U_w(c^w, l^w) = c^w - \frac{\omega}{1+\nu}(l^w)^{1+\nu}$$

where ω is the relative weight of labour in utility and ν is the inverse Frisch elasticity of labour supply.[6]

3.4.1 Optimising behaviour

At the start of period t the representative worker chooses c_t^w, l_t^w, n_{t+1}^w, and m_{t+1}^w to maximise his expected discounted utility, subject to his budget constraint (3.22). The worker optimally supplies labour until the marginal disutility of work (or equivalently, the marginal utility of leisure) is equal to the real wage rate. First order conditions yield his supply of labour (see Appendix 3.G),

$$l_t^w = \left[\frac{(1 - \tau_t^{wl})w_t}{\omega}\right]^{\frac{1}{\nu}} \tag{3.24}$$

[6]The specification for $U_w(c^w, l^w)$ implies the disutility from work does not directly affect the utility from consumption. Nezafat and Slavík (2012) point out that this utility specification is unusual in the Real Business Cycle literature, but they show that, quantitatively, their model's responses to real and financial shocks are not sensitive to the choice of functional form.

Claim 2. $n^w_{t+j} = 0$ and $m^w_{t+j} = 0$, for $j = 0, 1, 2, \ldots$, that is, the worker will always choose not to hold equity and money.

Proof of Claim 2. From Equations (3.71), (3.73) and (3.74) in Appendix 3.G, if the worker decides to hold equity and money, that is, if $n^w_{t+1} \neq 0$ and $m^w_{t+1} \neq 0$ then

$$\frac{\delta q_{t+1} + (1 - \tau^{rn}_{t+1}) r_{t+1}}{q_t} = \frac{p_{t+1}}{p_t} = \frac{1}{\beta} \tag{3.25}$$

Equation (3.25) says that holding equity and money will not provide any superior (expected) gains above the discounted marginal utility from consumption, $1/\beta$. If the worker has one more unit of general output, he gains as much by consuming it as he expects to gain by saving it. Then there is no reason for the worker to save. The worker saves only if there is a marginal benefit from doing so. □

By Claim 2, the worker's budget constraint (3.22) simplifies to

$$c^w_t = (1 - \tau^{wl}_t) w_t l^w_t \tag{3.26}$$

that is, in each period the worker consumes his entire net wages, thus making him non-Ricardian.[7]

3.5 Government

The government is both the fiscal and monetary authority. As the fiscal authority, the government collects T_t in taxes from entrepreneurs and workers according to

$$T_t = \tau^{rn}_t r_t N_t + \tau^{wl}_t w_t L_t \tag{3.27}$$

[7]Non-Ricardian behaviour in this model arises endogenously. Elsewhere in the literature, where such behaviour is assumed to be exogenous (in Galí et al. (2007), for example), it is justified by such things as lack of access to financial markets, myopia, or fear of saving. Campbell and Mankiw (1989) provide empirical support for the existence of non-Ricardian behaviour, while Mankiw (2000) reviews microeconomic evidence that supports such behaviour. The non-Ricardian feature is how this model departs from the standard Real Business Cycle model and starts to resemble Keynesian IS-LM. Driffill and Miller (2013) algebraically show that the KM model is fundamentally IS-LM by simplifying it to two equations that resemble IS and LM functions. If workers are Ricardian, as in the standard RBC model, then a cut in the income tax rate increases the present value of disposable income, and thus creates a positive wealth effect that induces a rise in saving and a drop in consumption. But here, a cut in the income tax rate increases workers' consumption; this is numerically demonstrated in Chapter 5.

where N_t is the private sector's total equity holdings at the beginning of period t and L_t is the aggregate labour-hours employed in the period's production. The government consumes G_t units of general output, which does not directly affect the utility of workers and entrepreneurs or create any production externalities.[8] $(G_t - T_t)$ is the government's fiscal balance.

As the monetary authority, the government exclusively and costlessly issues/withdraws fiat money at period t's market price, and thereby earns/pays $p_t(M_{t+1} - M_t)$ units of general output as seignorage, where M_t and M_{t+1} are the stocks of money in circulation at the start and end of period t, respectively.

The government owns a non-negative stock of entrepreneur-issued equity. At the start of period t it holds N_t^g units of equity. Over the period the government buys/sells $\left|N_{t+1}^g - \delta N_t^g\right|$ units from/to private agents at period t's market price. These purchases/sales represent changes in the market supply of equity; Section 3.6.1 elaborates more on this activity when describing the equity market.

The government does not issue debt, and therefore balances its overall budget in every period. Its consumption and equity transactions are financed from taxes, dividends, and seignorage according to

$$G_t + q_t(N_{t+1}^g - \delta N_t^g) = T_t + r_t N_t^g + p_t(M_{t+1} - M_t) \tag{3.28}$$

The economy is subject to exogenous stochastic shocks to a non-empty set of policy variables $\Omega_t \subset \{G_t, N_{t+1}^g, M_{t+1}, \tau_t^{rn}, \tau_t^{wl}\}$. Random disturbances to policy variables can be thought of as discretionary policy actions by the government. Ω_t is defined differently in the experiments that follow in Chapters 5 to 7. If they are elements of Ω_t then policy variables evolve according to the same stationary AR(1) process,

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u_t^G \tag{3.29}$$

$$N_{t+1}^g = (1 - \rho_{Ng})N^g + \rho_{Ng} N_t^g + u_t^{Ng} \tag{3.30}$$

$$M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M \tag{3.31}$$

$$\tau_t^{rn} = (1 - \rho_{\tau rn})\tau^{rn} + \rho_{\tau rn}\tau_{t-1}^{rn} + u_t^{\tau rn} \tag{3.32}$$

[8] Canova and Pappa (2011) note that the response by output from a change in government spending is amplified if such spending affects private agents' utility (as in Bouakez and Rebei (2007)) and/or creates production externalities (as in Baxter and King (1993)).

$$\tau_t^{wl} = (1 - \rho_{\tau wl})\tau^{wl} + \rho_{\tau wl}\tau_{t-1}^{wl} + u_t^{\tau wl} \tag{3.33}$$

where, for $X_t \in \{G_t, N_{t+1}^g, M_{t+1}, \tau_t^{rn}, \tau_t^{wl}\}$, X is the steady state value of the variable, ρ_X parameterises the degree of persistency of a stochastic shock to X_t, $u_t^X \sim^{i.i.d.} \mathcal{N}(0, \sigma_{uX}^2)$ where σ_{uX}^2 is exogenously determined, and $E[u_t^Y u_t^X] = 0$ for $X_t \neq Y_t \in \{G_t, N_{t+1}^g, M_{t+1}, \tau_t^{rn}, \tau_t^{wl}\}$. It is assumed that $|\rho_X| < 1$ so that exogenous shocks are temporary events.[9]

If $G_t \in \Omega_t$ and there are exogenous shocks to government spending, then the government follows one or both of the following policy rules:

$$\frac{\tau_t^{wl}}{T_t} = \frac{\tau^{wl}}{T} \tag{3.34}$$

$$\dot{M}_{t+1} = (1 - 2\mathbb{1}_t)\left|\dot{Y}_t\right| \tag{3.35}$$

where τ^{wl} and T are the steady state values of τ_t^{wl} and T_t, respectively, $\dot{M}_{t+1}$ and $\dot{Y}_t$ are the period-t percentage deviations of money and output, respectively, from their steady state levels, and

$$\mathbb{1}_t = \begin{cases} 1 \text{ if the government buys equity in period } t \\ 0 \text{ if the government sells equity in period } t \end{cases} \tag{3.36}$$

Rule (3.34) is used when $\{\tau_t^{rn}, \tau_t^{wl}\} \not\subset \Omega_t$. This rule implies a linear relationship between the two tax rates for given quantities and prices of factors of production; see Appendix 3.J for this linear relationship (that is, Equation (3.75)) and its derivation.

Claim 3. τ_t^{wl} and τ_t^{rn} are positively related if and only if

$$w_t L_t - wL < \left(\frac{\tau^{rn}}{\tau^{wl}}\right) rN$$

for non-zero gross dividends; the relationship is negative otherwise.

Proof of Claim 3. See Appendix 3.J.

Claim 3 says that if gross wages are below their steady state level, regardless of the size of the deviation, then rule (3.34) implies that the linear relationship between tax rates is a positive one. If gross wages are above steady state and if the size of the deviation is

[9]With an estimated DSGE model, Mertens and Ravn (2011) show that responses are different in magnitude, but not in direction, between temporary and permanent fiscal shocks.

smaller than $\tau^{rn}rN/\tau^{wl}$ then tax rates are positively related. However, if gross wages are above steady state and the deviation is larger than $\tau^{rn}rN/\tau^{wl}$ then tax rates are negatively related.

Rule (3.35) is used when $M_t \notin \Omega_t$. If a fiscal deficit (or surplus, respectively) requires the government to sell (or buy, respectively) some of its equity stock then rule (3.35) produces a policy reaction of issuing (or withdrawing, respectively) money to also help reduce the fiscal imbalance. The rule prevents rapid depletion (in a sale) or accumulation (in a purchase) of government equity holdings. It does this by limiting the financing of equity purchases via monetary expansion, by equating the percentage deviations of money and output from steady state.

3.6 The aggregate economy

3.6.1 The equity market

At the beginning of period t, *ex ante* identical entrepreneurs hold a total stock of N_t units of equity. When investment opportunities are revealed at the beginning of the period, those who are savers hold a total of $(1-\pi)N_t$ units of equity. Investors sell their equity over the period in order to finance their investment. Savers are the only agents who buy equity, and they do so from three sources – investors selling ϕ_t of their surviving equity, $\pi\delta N_t$, investors issuing θI_t new (outside) equity, and the government selling (or buying) $(N_{t+1}^g - \delta N_t^g)$. By the end of the period, the aggregate stock of equity held by savers is

$$N_{t+1}^s = (1-\pi)\delta N_t + \phi_t \pi \delta N_t + \theta I_t - (N_{t+1}^g - \delta N_t^g) \tag{3.37}$$

which is re-expressed as an equity market clearing condition,

$$N_{t+1}^s - \delta N_t^s = \phi_t \pi \delta N_t + \theta I_t - (N_{t+1}^g - \delta N_t^g) \tag{3.38}$$

where the RHS (LHS, respectively) is the aggregate supply (demand, respectively) of equity. Savers' accumulation of equity, or aggregate saving, satisfies an aggregate portfolio balance equation, from Equation (3.20),

$$\pi E_t \left[\frac{\left(\frac{p_{t+1}}{p_t}\right) - \left(\frac{[1-\tau^{rn}_{t+1}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q^R_{t+1}}{q_t}\right)}{[1-\tau^{rn}_{t+1}]r_{t+1}N^s_{t+1} + [\phi_{t+1}q_{t+1} + (1-\phi_{t+1})q^R_{t+1}]\delta N^s_{t+1} + p_{t+1}M_{t+1}} \right]$$
$$= (1-\pi)E_t \left[\frac{\left(\frac{[1-\tau^{rn}_{t+1}]r_{t+1}+\delta q_{t+1}}{q_t}\right) - \left(\frac{p_{t+1}}{p_t}\right)}{[1-\tau^{rn}_{t+1}]r_{t+1}N^s_{t+1} + q_{t+1}\delta N^s_{t+1} + p_{t+1}M_{t+1}} \right] \tag{3.39}$$

If the government sells equity to the private sector then

$$N^g_{t+1} - \delta N^g_t < 0$$

Because of the re-saleability constraint, the government can sell at most $\phi_t \delta N^g_t$ units, and therefore

$$\left| N^g_{t+1} - \delta N^g_t \right| \leq \phi_t \delta N^g_t$$

or equivalently, because the expression inside the absolute value brackets is negative,

$$N^g_{t+1} - \delta N^g_t \geq -\phi_t \delta N^g_t$$
$$\implies N^g_{t+1} \geq (1-\phi_t)\delta N^g_t \tag{3.40}$$

Assumption 3. The government buys or sells the maximum quantity of equity within the limits of the re-saleability constraint.

Assumption 3 does not hold in a government sale when the size of the requirement to finance the fiscal account is smaller than the re-saleable value of government's equity holdings. If Assumption 3 does hold then by inequality (3.40), when the government sells equity to private agents, its equity holdings evolve according to

$$N^g_{t+1} = (1-\phi_t)\delta N^g_t \tag{3.41}$$

Entrepreneurs' total equity holdings depreciate each period, and accumulate from investment financing and government sales. Therefore, when the government sells equity to private agents, aggregate private equity holdings evolve according to

$$N_{t+1} = \delta N_t + \phi_t \delta N^g_t + I_t \tag{3.42}$$

If the government buys equity from the private sector then

$$N_{t+1}^g - \delta N_t^g > 0$$

Aside from its budget constraint (3.28), the government has a limit on how many units of equity it can buy – because of the re-saleability constraint, private agents can sell at most $\phi_t \delta N_t$ units to the government, and therefore

$$\left| N_{t+1}^g - \delta N_t^g \right| \leq \phi_t \delta N_t$$

or equivalently, because the expression inside the absolute value brackets is positive,

$$\begin{aligned} N_{t+1}^g - \delta N_t^g &\leq \phi_t \delta N_t \\ \Longrightarrow N_{t+1}^g &\leq \delta N_t^g + \phi_t \delta N_t \end{aligned} \tag{3.43}$$

Assumption 3 holds in a government purchase to prevent the authority from unnecessarily increasing its equity holdings beyond the need to balance its overall budget (3.28). By Assumption 3 and inequality (3.43), when the government buys equity, its equity holdings evolve according to

$$N_{t+1}^g = \delta N_t^g + \phi_t \delta N_t \tag{3.44}$$

Private agents retain their non-re-saleable equity stock, $(1 - \phi_t)\delta N_t$, and create new units from investment financing. Therefore, when the government buys equity, aggregate private equity holdings evolve according to

$$N_{t+1} = (1 - \phi_t)\delta N_t + I_t \tag{3.45}$$

From either Equations (3.41) and (3.42) or Equations (3.44) and (3.45), the aggregate equity stock at the end of a period in which there are government equity purchases or sales is

$$N_{t+1} + N_{t+1}^g = \delta(N_t + N_t^g) + I_t \tag{3.46}$$

Since every unit of equity produces a unit of capital, then the aggregate capital stock, K_t, is matched by the aggregate equity stock,

$$K_t = N_t + N_t^g \tag{3.47}$$

and Equation (3.46) then becomes

$$K_{t+1} = \delta K_t + I_t \tag{3.48}$$

which is the law of motion for the aggregate capital stock.

3.6.2 The labour market

From the production function (3.1), the marginal product of labour is

$$\frac{\partial y_t}{\partial l_t} = (1-\gamma)A_t \left(\frac{k_t}{l_t}\right)^{\gamma}$$

From Equation (3.3), the first order condition for gross profit maximisation with respect to labour is

$$\begin{aligned} &\frac{\partial y_t}{\partial l_t} - w_t = 0 \\ \Longrightarrow\ &(1-\gamma)A_t \left(\frac{k_t}{l_t}\right)^{\gamma} - w_t = 0 \\ \Longrightarrow\ &l_t = k_t \left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}} \end{aligned}$$

which is a typical entrepreneur's demand for labour, given his capital stock.

With an aggregate capital stock, K_t, owned entirely by entrepreneurs, it follows that the aggregate demand for labour is

$$L_t^D = K_t \left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}} \tag{3.49}$$

Appendix 3.H shows that labour demand is wage-elastic. Given the homogeneity and unitary mass of the worker population, the aggregate labour supply labour is, from Equation (3.24),

$$L_t^S = \left[\frac{(1-\tau_t^{wl})w_t}{\omega}\right]^{\frac{1}{\nu}} \tag{3.50}$$

Then the inverse labour demand and supply functions are, respectively,

$$w_t^D = \frac{(1-\gamma)A_t K_t^{\gamma}}{(L_t)^{\gamma}} \tag{3.51}$$

$$w_t^S = \left[\frac{\omega}{1-\tau_t^{wl}}\right](L_t)^{\nu} \tag{3.52}$$

The labour market clears when $L_t^S = L_t^D$, which gives the equilibrium real wage and employment, respectively (see Appendix 3.I),

$$w_t = K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}} (1-\tau_t^{wl})^{-\frac{\gamma}{\gamma+\nu}} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}} \tag{3.53}$$

$$L_t = K_t^{\frac{\gamma}{\gamma+\nu}} \omega^{-\frac{1}{\gamma+\nu}} [(1-\tau_t^{wl})(1-\gamma)A_t]^{\frac{1}{\gamma+\nu}} \tag{3.54}$$

3.6.3 The general output market

Aggregate private consumption is

$$C_t = C_t^i + C_t^s + C_t^w \tag{3.55}$$

where, from Equations (3.16), (3.17) and (3.20), consumption by investors, savers, and workers are, respectively,

$$C_t^i = \pi(1-\beta)([1-\tau_t^{rn}]r_t N_t + \left[\phi_t q_t + (1-\phi_t)q_t^R\right]\delta N_t + p_t M_t) \tag{3.56}$$

$$C_t^s = (1-\pi)(1-\beta)([1-\tau_t^{rn}]r_t N_t + q_t \delta N_t + p_t M_t) \tag{3.57}$$

$$C_t^w = (1-\tau_t^{wl}) w_t L_t \tag{3.58}$$

From Equation (3.12), aggregate investment is given by

$$(1-\theta q_t) I_t = ([1-\tau_t^{rn}]r_t + \phi_t \delta q_t)\pi N_t + \pi p_t M_t - C_t^i \tag{3.59}$$

From Equation (3.1), aggregate production of general output is

$$Y_t = A_t K_t^{\gamma} L_t^{1-\gamma} \tag{3.60}$$

From Equation (3.3), aggregate gross profits are

$$r_t K_t = Y_t - w_t L_t \tag{3.61}$$

And the general output market clears when

$$Y_t = C_t + I_t + G_t \tag{3.62}$$

Appendix 3.A How q_t^R varies with θ

From Equation (3.6),

$$q_t^R = (1 - \theta q_t)(1 - \theta)^{-1}$$

Suppose θ can vary. Then

$$\begin{aligned}
\frac{\partial q_t^R}{\partial \theta} &= (1-\theta)^{-1}\frac{\partial}{\partial \theta}(1-\theta q_t) + (1-\theta q_t)\frac{\partial}{\partial \theta}(1-\theta)^{-1} \\
&= -\frac{q_t}{1-\theta} + \frac{1-\theta q_t}{(1-\theta)^2} \\
&= -\frac{q_t(1-\theta)}{(1-\theta)^2} + \frac{1-\theta q_t}{(1-\theta)^2} \\
&= \frac{-q_t + \theta q_t + 1 - \theta q_t}{(1-\theta)^2} \\
&= \frac{1-q_t}{(1-\theta)^2}
\end{aligned}$$

By Assumption 2, $1 - q_t < 0$ and therefore $\dfrac{\partial q_t^R}{\partial \theta} < 0$ for $\theta \in (0, 1)$.

Appendix 3.B The investor's budget and resource constraints

Substituting Equation (3.9) into Equation (3.10) gives

$$\begin{aligned}
& c_t^i + \frac{1}{1-\theta}n_{t+1}^i - \frac{1-\phi_t}{1-\theta}\delta n_t + q_t\left[n_{t+1}^i - \frac{1}{1-\theta}n_{t+1}^i + \frac{1-\phi_t}{1-\theta}\delta n_t - \delta n_t\right] + p_t(m_{t+1}^i - m_t) \\
&\quad = (1-\tau_t^{rn})r_t n_t \\
\Longrightarrow\ & c_t^i + \left[\frac{1}{1-\theta} + q_t - q_t\frac{1}{1-\theta}\right]n_{t+1}^i = (1-\tau_t^{rn})r_t n_t + \left[\frac{1-\phi_t}{1-\theta} - q_t\frac{1-\phi_t}{1-\theta} + q_t\right]\delta n_t \\
&\quad + p_t(m_t - m_{t+1}^i)
\end{aligned}$$

The coefficients of n_{t+1}^i and δn_t in the above expression are simplified as follows.

$$\begin{aligned}
\frac{1}{1-\theta} + q_t - q_t\frac{1}{1-\theta} &= \frac{1}{1-\theta} + q_t\left[1 - \frac{1}{1-\theta}\right] \\
&= \frac{1}{1-\theta} + q_t\left[\frac{1-\theta-1}{1-\theta}\right] \\
&= \frac{1}{1-\theta} - q_t\frac{\theta}{1-\theta} \\
&= \frac{1-\theta q_t}{1-\theta} \equiv q_t^R
\end{aligned}$$

$$\begin{aligned}
\frac{1-\phi_t}{1-\theta} - q_t\frac{1-\phi_t}{1-\theta} + q_t &= \frac{1-\phi_t}{1-\theta} + q_t\left[1 - \frac{1-\phi_t}{1-\theta}\right] \\
&= \frac{1-\phi_t}{1-\theta} + q_t\left[\frac{1-\theta-1+\phi_t}{1-\theta}\right] \\
&= \frac{1-\phi_t}{1-\theta} + q_t\left[\frac{-\theta+\phi_t}{1-\theta}\right] \\
&= \frac{1-\phi_t-\theta q_t+\phi_t q_t}{1-\theta} \\
&= \frac{1-\phi_t-\theta q_t+\phi_t\theta q_t-\phi_t\theta q_t+\phi_t q_t}{1-\theta} \\
&= \frac{(1-\phi_t)(1-\theta q_t)+\phi_t q_t(1-\theta)}{1-\theta} \\
&= (1-\phi_t)\frac{1-\theta q_t}{1-\theta} + \phi_t q_t \\
&= (1-\phi_t)q_t^R + \phi_t q_t
\end{aligned}$$

These expressions together give the modified budget constraint (3.11),

$$c_t^i + q_t^R n_{t+1}^i = (1-\tau_t^{rn})r_t n_t + \left[\phi_t q_t + (1-\phi_t)q_t^R\right]\delta n_t + p_t(m_t - m_{t+1}^i)$$

Alternatively, substituting Equation (3.8) into Equation (3.10) gives the resource constraint (3.12),

$$\begin{aligned}
& c_t^i + i_t + q_t[(1-\theta)i_t + (1-\phi_t)\delta n_t - i_t - \delta n_t] + p_t(m_{t+1}^i - m_t) = (1-\tau_t^{rn})r_t n_t \\
\Longrightarrow\ & c_t^i + i_t + q_t(1-\theta)i_t + q_t(1-\phi_t)\delta n_t - q_t i_t - q_t\delta n_t = (1-\tau_t^{rn})r_t n_t + p_t(m_t - m_{t+1}^i) \\
\Longrightarrow\ & c_t^i + i_t[1 + q_t(1-\theta) - q_t] + [(1-\phi_t) - 1]q_t\delta n_t = (1-\tau_t^{rn})r_t n_t + p_t(m_t - m_{t+1}^i) \\
\Longrightarrow\ & c_t^i + i_t[1-\theta q_t] + [-\phi_t]q_t\delta n_t = (1-\tau_t^{rn})r_t n_t + p_t(m_t - m_{t+1}^i) \\
\Longrightarrow\ & c_t^i + (1-\theta q_t)i_t = (1-\tau_t^{rn})r_t n_t + \phi_t q_t\delta n_t + p_t(m_t - m_{t+1}^i)
\end{aligned}$$

Appendix 3.C The entrepreneur's first order conditions

From Equations (3.11), (3.13) and (3.14), the investor's Lagrangian is

$$\begin{aligned}
\mathcal{L}_e^i = U_e(c_t^i) - \lambda_t^i\Big(& c_t^i + q_t^R n_{t+1}^i - (1-\tau_t^{rn})r_t n_t - [\phi_t q_t + (1-\phi_t)q_t^R]\delta n_t - p_t(m_t - m_{t+1}^i)\Big) \\
+ \pi E_t\Bigg[\beta\Big\{& U_e(c_{t+1}^i) - \lambda_{t+1}^i\Big(c_{t+1}^i + q_{t+1}^R n_{t+2}^i - (1-\tau_{t+1}^{rn})r_{t+1}n_{t+1}^i - [\phi_{t+1}q_{t+1} \\
& + (1-\phi_{t+1})q_{t+1}^R]\delta n_{t+1}^i - p_{t+1}(m_{t+1}^i - m_{t+2}^i)\Big)\Big\} + \beta^2\Big\{U_e(c_{t+2}^i) - \lambda_{t+2}^i\Big(c_{t+2}^i
\end{aligned}$$

$$
\begin{aligned}
&+ q^R_{t+2} n^i_{t+3} - (1-\tau^{rn}_{t+2}) r_{t+2} n^i_{t+2} - [\phi_{t+2} q_{t+2} + (1-\phi_{t+2}) q^R_{t+2}] \delta n^i_{t+2} \\
&- p_{t+2}(m^i_{t+2} - m^i_{t+3}) \Big) \Big\} + \ldots \Big] \\
+ (1-\pi) E_t \Big[\beta \Big\{ & U_e(c^s_{t+1}) - \lambda^s_{t+1} \Big(c^s_{t+1} + q_{t+1}(n^s_{t+2} - \delta n^s_{t+1}) + p_{t+1}(m^s_{t+2} - m^s_{t+1}) \\
&- (1-\tau^{rn}_{t+1}) r_{t+1} n^s_{t+1} \Big) \Big\} + \beta^2 \Big\{ U_e(c^s_{t+2}) - \lambda^s_{t+2} \Big(c^s_{t+2} + q_{t+2}(n^s_{t+3} - \delta n^s_{t+2}) \\
&+ p_{t+2}(m^s_{t+3} - m^s_{t+2}) - (1-\tau^{rn}_{t+2}) r_{t+2} n^s_{t+2} \Big) \Big\} + \ldots \Big]
\end{aligned}
$$

which gives first order conditions

$$
\begin{aligned}
&\frac{\partial \mathcal{L}^i_e}{\partial c^i_t} = U'_e(c^i_t) - \lambda^i_t = 0 \\
&\implies \lambda^i_t = U'_e(c^i_t) && (3.63)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mathcal{L}^i_e}{\partial c^i_{t+1}} = \pi E_t[\beta\{U'_e(c^i_{t+1}) - \lambda^i_{t+1}\}] + (1-\pi)E_t[\beta\{U'_e(c^s_{t+1}) - \lambda^s_{t+1}\}] = 0 \\
&\implies \beta E_t[\pi U'_e(c^i_{t+1}) + (1-\pi)U'_e(c^s_{t+1})] = \beta E_t[\pi \lambda^i_{t+1} + (1-\pi)\lambda^s_{t+1}] \\
&\implies \pi U'_e(c^i_{t+1}) + (1-\pi)U'_e(c^s_{t+1}) = \pi \lambda^i_{t+1} + (1-\pi)\lambda^s_{t+1} && (3.64)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mathcal{L}^i_e}{\partial n^i_{t+1}} = -\lambda^i_t q^R_t + \pi E_t[\beta \lambda^i_{t+1}([1-\tau^{rn}_{t+1}] r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q^R_{t+1})] \\
&\qquad + (1-\pi)E_t[\beta \lambda^s_{t+1}(\delta q_{t+1} + [1-\tau^{rn}_{t+1}] r_{t+1})] = 0 \\
&\implies \lambda^i_t q^R_t = \beta\pi E_t\left[\lambda^i_{t+1}([1-\tau^{rn}_{t+1}] r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q^R_{t+1})\right] \\
&\qquad + \beta(1-\pi)E_t\left[\lambda^s_{t+1}(\delta q_{t+1} + [1-\tau^{rn}_{t+1}] r_{t+1})\right] && (3.65)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mathcal{L}^i_e}{\partial m^i_{t+1}} = -\lambda^i_t p_t + \pi E_t[\beta \lambda^i_{t+1} p_{t+1}] + (1-\pi)E_t[\beta \lambda^s_{t+1} p_{t+1}] \le 0, \quad m^i_{t+1} \ge 0, \\
&\qquad \text{and } \{-\lambda^i_t p_t + \pi E_t[\beta \lambda^i_{t+1} p_{t+1}] + (1-\pi)E_t[\beta \lambda^s_{t+1} p_{t+1}]\} m^i_{t+1} = 0 \\
&\implies -\lambda^i_t p_t + \pi E_t[\beta \lambda^i_{t+1} p_{t+1}] + (1-\pi)E_t[\beta \lambda^s_{t+1} p_{t+1}] = 0 \quad \text{or} \quad m^i_{t+1} = 0 \\
&\implies \lambda^i_t p_t = \pi E_t[\beta \lambda^i_{t+1} p_{t+1}] + (1-\pi)E_t[\beta \lambda^s_{t+1} p_{t+1}] \quad \text{or} \quad m^i_{t+1} = 0 \\
&\implies \lambda^i_t = \beta E_t\left[\frac{p_{t+1}}{p_t}\left(\pi \lambda^i_{t+1} + (1-\pi)\lambda^s_{t+1}\right)\right] \quad \text{or} \quad m^i_{t+1} = 0 && (3.66)
\end{aligned}
$$

From Equations (3.11), (3.13) and (3.14), the saver's Lagrangian is

$$
\begin{aligned}
\mathcal{L}_e^s = U_e(c_t^s) - \lambda_t^s\Big(c_t^s + q_t(n_{t+1}^s - \delta n_t) + p_t(m_{t+1}^s - m_t) - (1-\tau_t^{rn})r_t n_t\Big) \\
+ \pi E_t\Big[\beta\Big\{U_e(c_{t+1}^i) - \lambda_{t+1}^i\Big(c_{t+1}^i + q_{t+1}^R n_{t+2}^i - (1-\tau_{t+1}^{rn})r_{t+1}n_{t+1}^i - [\phi_{t+1}q_{t+1} \\
+ (1-\phi_{t+1})q_{t+1}^R]\delta n_{t+1}^i - p_{t+1}(m_{t+1}^i - m_{t+2}^i)\Big)\Big\} + \beta^2\Big\{U_e(c_{t+2}^i) - \lambda_{t+2}^i\Big(c_{t+2}^i \\
+ q_{t+2}^R n_{t+3}^i - (1-\tau_{t+2}^{rn})r_{t+2}n_{t+2}^i - [\phi_{t+2}q_{t+2} + (1-\phi_{t+2})q_{t+2}^R]\delta n_{t+2}^i \\
- p_{t+2}(m_{t+2}^i - m_{t+3}^i)\Big)\Big\} + \dots\Big] \\
+ (1-\pi)E_t\Big[\beta\Big\{U_e(c_{t+1}^s) - \lambda_{t+1}^s\Big(c_{t+1}^s + q_{t+1}(n_{t+2}^s - \delta n_{t+1}^s) + p_{t+1}(m_{t+2}^s - m_{t+1}^s) \\
- (1-\tau_{t+1}^{rn})r_{t+1}n_{t+1}^s\Big)\Big\} + \beta^2\Big\{U_e(c_{t+2}^s) - \lambda_{t+2}^s\Big(c_{t+2}^s + q_{t+2}(n_{t+3}^s - \delta n_{t+2}^s) \\
+ p_{t+2}(m_{t+3}^s - m_{t+2}^s) - (1-\tau_{t+2}^{rn})r_{t+2}n_{t+2}^s\Big)\Big\} + \dots\Big]
\end{aligned}
$$

which gives first order conditions

$$
\begin{aligned}
&\frac{\partial \mathcal{L}_e^s}{\partial c_t^s} = U_e'(c_t^s) - \lambda_t^s = 0 \\
&\implies \lambda_t^s = U_e'(c_t^s) && (3.67)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mathcal{L}_e^s}{\partial n_{t+1}^s} = -\lambda_t^s q_t + \pi E_t[\beta\lambda_{t+1}^i([1-\tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q_{t+1}^R)] \\
&\qquad + (1-\pi)E_t[\beta\lambda_{t+1}^s(\delta q_{t+1} + [1-\tau_{t+1}^{rn}]r_{t+1})] = 0 \\
&\implies \lambda_t^s q_t = \beta\pi E_t[\lambda_{t+1}^i([1-\tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q_{t+1}^R)] \\
&\qquad + \beta(1-\pi)E_t[\lambda_{t+1}^s(\delta q_{t+1} + [1-\tau_{t+1}^{rn}]r_{t+1})] && (3.68) \\
&\implies \lambda_t^s = \beta\pi E_t\left[\frac{1}{q_t}\lambda_{t+1}^i([1-\tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q_{t+1}^R)\right] \\
&\qquad + \beta(1-\pi)E_t\left[\frac{1}{q_t}\lambda_{t+1}^s(\delta q_{t+1} + [1-\tau_{t+1}^{rn}]r_{t+1})\right] && (3.69)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mathcal{L}_e^s}{\partial m_{t+1}^s} = -\lambda_t^s p_t + \pi E_t[\beta\lambda_{t+1}^i p_{t+1}] + (1-\pi)E_t[\beta\lambda_{t+1}^s p_{t+1}] = 0 \\
&\implies \lambda_t^s p_t = \pi E_t[\beta\lambda_{t+1}^i p_{t+1}] + (1-\pi)E_t[\beta\lambda_{t+1}^s p_{t+1}] \\
&\implies \lambda_t^s = \beta E_t\left[\frac{p_{t+1}}{p_t}\Big(\pi\lambda_{t+1}^i + (1-\pi)\lambda_{t+1}^s\Big)\right] && (3.70)
\end{aligned}
$$

From Equations (3.69) and (3.70),

$$
\begin{aligned}
\frac{\lambda_t^s}{\beta} &= \pi E_t\left[\frac{1}{q_t}\lambda_{t+1}^i([1-\tau_{t+1}^{rn}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q_{t+1}^R)\right]\\
&\quad +(1-\pi)E_t\left[\frac{1}{q_t}\lambda_{t+1}^s([1-\tau_{t+1}^{rn}]r_{t+1}+\delta q_{t+1})\right]\\
&= E_t\left[\frac{p_{t+1}}{p_t}\left(\pi\lambda_{t+1}^i+(1-\pi)\lambda_{t+1}^s\right)\right]
\end{aligned}
$$

From Equations (3.63) and (3.67), respectively, $\lambda_{t+1}^i = U_e'(c_{t+1}^i)$ and $\lambda_{t+1}^s = U_e'(c_{t+1}^s)$, and substituting Equation (3.64) gives

$$
\begin{aligned}
&\pi E_t\left[\frac{1}{q_t}([1-\tau_{t+1}^{rn}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q_{t+1}^R)U_e'(c_{t+1}^i)\right]\\
&\quad +(1-\pi)E_t\left[\frac{1}{q_t}([1-\tau_{t+1}^{rn}]r_{t+1}+\delta q_{t+1})U_e'(c_{t+1}^s)\right]\\
&= E_t\left[\frac{p_{t+1}}{p_t}\left(\pi U_e'(c_{t+1}^i)+(1-\pi)U_e'(c_{t+1}^s)\right)\right]
\end{aligned}
$$

Appendix 3.D The portfolio balance equation

The Euler equation (3.15) simplifies as follows:

$$
\begin{aligned}
&\pi E_t\left[\frac{1}{q_t}([1+\tau_{t+1}^{rn}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q_{t+1}^R)U_e'(c_{t+1}^i)\right]\\
&\quad +(1-\pi)E_t\left[\frac{1}{q_t}([1+\tau_{t+1}^{rn}]r_{t+1}+\delta q_{t+1})U_e'(c_{t+1}^s)\right]\\
&\quad = E_t\left[\frac{p_{t+1}}{p_t}\left(\pi U_e'(c_{t+1}^i)+(1-\pi)U_e'(c_{t+1}^s)\right)\right]\\
&\Longrightarrow \pi E_t\left[\left(\frac{p_{t+1}}{p_t}\right)U_e'(c_{t+1}^i)\right]+(1-\pi)E_t\left[\left(\frac{p_{t+1}}{p_t}\right)U_e'(c_{t+1}^s)\right]\\
&\quad = \pi E_t\left[\left(\frac{[1+\tau_{t+1}^{rn}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q_{t+1}^R}{q_t}\right)U_e'(c_{t+1}^i)\right]\\
&\quad +(1-\pi)E_t\left[\left(\frac{[1+\tau_{t+1}^{rn}]r_{t+1}+\delta q_{t+1}}{q_t}\right)U_e'(c_{t+1}^s)\right]\\
&\Longrightarrow \pi E_t\left[\left(\frac{p_{t+1}}{p_t}\right)U_e'(c_{t+1}^i)\right]-\pi E_t\left[\left(\frac{[1+\tau_{t+1}^{rn}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q_{t+1}^R}{q_t}\right)U_e'(c_{t+1}^i)\right]\\
&\quad = (1-\pi)E_t\left[\left(\frac{[1+\tau_{t+1}^{rn}]r_{t+1}+\delta q_{t+1}}{q_t}\right)U_e'(c_{t+1}^s)\right]-(1-\pi)E_t\left[\left(\frac{p_{t+1}}{p_t}\right)U_e'(c_{t+1}^s)\right]\\
&\Longrightarrow \pi E_t\left[\left(\frac{p_{t+1}}{p_t}-\frac{[1+\tau_{t+1}^{rn}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q_{t+1}^R}{q_t}\right)U_e'(c_{t+1}^i)\right]\\
&\quad = (1-\pi)E_t\left[\left(\frac{[1+\tau_{t+1}^{rn}]r_{t+1}+\delta q_{t+1}}{q_t}-\frac{p_{t+1}}{p_t}\right)U_e'(c_{t+1}^s)\right]
\end{aligned}
$$

$$\Longrightarrow \pi E_t\left[\frac{\left(\frac{p_{t+1}}{p_t}-\frac{[1+\tau^{rn}_{t+1}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q^R_{t+1}}{q_t}\right)}{c^i_{t+1}}\right]$$
$$=(1-\pi)E_t\left[\frac{\left(\frac{[1+\tau^{rn}_{t+1}]r_{t+1}+\delta q_{t+1}}{q_t}-\frac{p_{t+1}}{p_t}\right)}{c^s_{t+1}}\right]$$

Then by Equations (3.16) and (3.17), the last line above becomes

$$\pi E_t\left[\frac{\left(\frac{p_{t+1}}{p_t}\right)-\left(\frac{[1+\tau^{rn}_{t+1}]r_{t+1}+\phi_{t+1}\delta q_{t+1}+[1-\phi_{t+1}]\delta q^R_{t+1}}{q_t}\right)}{[1+\tau^{rn}_{t+1}]r_{t+1}n_{t+1}+[\phi_{t+1}q_{t+1}+(1-\phi_{t+1})q^R_{t+1}]\delta n_{t+1}+p_{t+1}m_{t+1}}\right]$$
$$=(1-\pi)E_t\left[\frac{\left(\frac{[1+\tau^{rn}_{t+1}]r_{t+1}+\delta q_{t+1}}{q_t}\right)-\left(\frac{p_{t+1}}{p_t}\right)}{[1+\tau^{rn}_{t+1}]r_{t+1}n_{t+1}+q_{t+1}\delta n_{t+1}+p_{t+1}m_{t+1}}\right]$$

Appendix 3.E Proof of Claim 1

The RHS of Equations (3.65) and (3.68) are identical, thus giving

$$\lambda^i_t q^R_t = \lambda^s_t q_t$$

and from Equations (3.66) and (3.70),

$$\begin{aligned} m^i_{t+1}\neq 0 &\iff \lambda^i_t=\lambda^s_t \\ &\iff q^R_t = q_t \\ &\iff \frac{1-\theta q_t}{1-\theta}=q_t \\ &\iff q_t = 1 \\ \therefore m^i_{t+1}=0 &\iff q_t\neq 1 \end{aligned}$$

□

Appendix 3.F Implication of Assumption 2 for expected portfolio returns

$q_t > 1 \Longrightarrow \theta q_t > \theta$

$$
\begin{aligned}
&\implies 1-\theta q_t < 1-\theta \\
&\implies \frac{1-\theta q_t}{1-\theta} < 1 \\
&\text{i.e. } q_t^R < 1 \\
&\implies q_t^R < q_t \\
&\implies q_{t+1}^R < q_{t+1} \\
&\implies \frac{(1+\tau_{t+1}^{rn})r_{t+1} + (1-\phi_{t+1})\delta q_{t+1}^R}{q_t} < \frac{(1+\tau_{t+1}^{rn})r_{t+1} + (1-\phi_{t+1})\delta q_{t+1}}{q_t} \\
&\implies \frac{(1+\tau_{t+1}^{rn})r_{t+1} + \phi_{t+1}\delta q_{t+1} + (1-\phi_{t+1})\delta q_{t+1}^R}{q_t} < \frac{(1+\tau_{t+1}^{rn})r_{t+1} + \delta q_{t+1}}{q_t}
\end{aligned}
$$

Appendix 3.G The worker's first order conditions

From Equations (3.22) and (3.23) the worker's Lagrangian is

$$
\begin{aligned}
\mathcal{L}_w = E_t\Bigg[\sum_{j=t}^{\infty}\beta^{j-t}U_w(c_j^w, l_j^w) - \lambda_t^w\Big(& c_t^w + q_t(n_{t+1}^w - \delta n_t^w) + p_t(m_{t+1}^w - m_t^w) - (1-\tau_t^{wl})w_t l_t^w \\
&- (1-\tau_t^{rn})r_t n_t^w\Big)\Bigg] \\
= U_w(c_t^w, l_t^w) - \lambda_t^w\Big(& c_t^w + q_t(n_{t+1}^w - \delta n_t^w) + p_t(m_{t+1}^w - m_t^w) - (1-\tau_t^{wl})w_t l_t^w \\
&- (1-\tau_t^{rn})r_t n_t^w\Big) + \beta E_t\Big[U_w(c_{t+1}^w, l_{t+1}^w) - \lambda_{t+1}^w\Big(c_{t+1}^w + q_{t+1}(n_{t+2}^w - \delta n_{t+1}^w) \\
&+ p_{t+1}(m_{t+2}^w - m_{t+1}^w) - (1-\tau_{t+1}^{wl})w_{t+1} l_{t+1}^w - (1-\tau_{t+1}^{rn})r_{t+1} n_{t+1}^w\Big)\Big] \\
&+ \beta^2 E_t\Big[U_w(c_{t+2}^w, l_{t+2}^w) - \lambda_{t+2}^w\Big(c_{t+2}^w + q_{t+2}(n_{t+3}^w - \delta n_{t+2}^w) + p_{t+2}(m_{t+3}^w - m_{t+2}^w) \\
&- (1-\tau_{t+2}^{wl})w_{t+2} l_{t+2}^w - (1-\tau_{t+2}^{rn})r_{t+2} n_{t+2}^w\Big)\Big] + \dots
\end{aligned}
$$

which gives first order conditions

$$
\begin{aligned}
&\frac{\partial \mathcal{L}_w}{\partial c_t^w} = \frac{\partial U_w}{\partial c_t^w} - \lambda_t^w = 0 \\
&\implies \lambda_t^w = \frac{\partial U_w}{\partial c_t^w} = 1 \qquad (3.71)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \mathcal{L}_w}{\partial l_t^w} = \frac{\partial U_w}{\partial l_t^w} + \lambda_t^w(1-\tau_t^{wl})w_t = 0 \\
&\implies \omega(l_t^w)^\nu = \lambda_t^w(1-\tau_t^{wl})w_t \qquad (3.72)
\end{aligned}
$$

$$\frac{\partial \mathcal{L}_w}{\partial n^w_{t+1}} = -\lambda^w_t q_t + \beta E_t[\lambda^w_{t+1}(\delta q_{t+1} + [1-\tau^{wl}_{t+1}]r_{t+1})] \leq 0, \quad n^w_{t+1} \geq 0,$$

$$\text{and} \quad \{-\lambda^w_t q_t + \beta E_t[\lambda^w_{t+1}(\delta q_{t+1} + [1-\tau^{wl}_{t+1}]r_{t+1})]\} n^w_{t+1} = 0$$

$$\implies \lambda^w_t = \beta E_t\left[\frac{\delta q_{t+1} + [1-\tau^{wl}_{t+1}]r_{t+1}}{q_t}\lambda^w_{t+1}\right] \quad \text{or} \quad n^w_{t+1} = 0 \tag{3.73}$$

$$\frac{\partial \mathcal{L}_w}{\partial m^w_{t+1}} = -\lambda^w_t p_t + \beta E_t[\lambda^w_{t+1} p_{t+1}] = 0,\ m^w_{t+1} \geq 0,\ \text{and}\ \{-\lambda^w_t p_t + \beta E_t[\lambda^w_{t+1} p_{t+1}]\} m^w_{t+1} = 0$$

$$\implies \lambda^w_t = \beta E_t\left[\frac{p_{t+1}}{p_t}\lambda^w_{t+1}\right] \quad \text{or} \quad m^w_{t+1} = 0 \tag{3.74}$$

Substituting Equation (3.71) into Equation (3.72) gives

$$\omega(l^w_t)^\nu = (1-\tau^{wl}_t)w_t$$
$$\implies l^w_t = \left[\frac{(1-\tau^{wl}_t)w_t}{\omega}\right]^{\frac{1}{\nu}}$$

Appendix 3.H Wage elasticity of labour demand

The wage elasticity of labour demand is given by

$$\frac{\partial L^D_t}{\partial w_t} \times \frac{w_t}{L^D_t}$$

From the labour demand function (3.49),

$$L^D_t = K_t\left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}}$$
$$= K_t\left[(1-\gamma)A_t\right]^{\frac{1}{\gamma}} w_t^{-\frac{1}{\gamma}}$$
$$\implies \frac{\partial L^D_t}{\partial w_t} = K_t\left[(1-\gamma)A_t\right]^{\frac{1}{\gamma}} \times \left(-\frac{1}{\gamma}\right) w_t^{-\frac{1}{\gamma}-1}$$
$$\implies \frac{\partial L^D_t}{\partial w_t} \times \frac{w_t}{L^D_t} = K_t\left[(1-\gamma)A_t\right]^{\frac{1}{\gamma}} \times \left(-\frac{1}{\gamma}\right) w_t^{-\frac{1}{\gamma}-1} \times \frac{w_t}{L^D_t}$$
$$= K_t\left[(1-\gamma)A_t\right]^{\frac{1}{\gamma}} \times \left(-\frac{1}{\gamma}\right) w_t^{-\frac{1}{\gamma}} \times \frac{1}{L^D_t}$$
$$= K_t\left[(1-\gamma)A_t\right]^{\frac{1}{\gamma}} \times \left(-\frac{1}{\gamma}\right) w_t^{-\frac{1}{\gamma}} \times \frac{1}{K_t\left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}}}$$

$$= \left(-\frac{1}{\gamma}\right) w_t^{-\frac{1}{\gamma}} \times w_t^{\frac{1}{\gamma}}$$
$$= -\frac{1}{\gamma}$$

Given that $\gamma \in (0, 1)$ then labour demand is wage-elastic.

Appendix 3.I Labour market equilibrium

From Equation (3.49) and Equation (3.49), $L_t^S = L_t^D$ implies

$$\left[\frac{(1-\tau_t^{wl})w_t}{\omega}\right]^{\frac{1}{\nu}} = K_t \left[\frac{(1-\gamma)A_t}{w_t}\right]^{\frac{1}{\gamma}}$$
$$\implies w_t^{\frac{1}{\nu}} w_t^{\frac{1}{\gamma}} = \frac{K_t \omega^{\frac{1}{\nu}} [(1-\gamma)A_t]^{\frac{1}{\gamma}}}{(1-\tau_t^{wl})^{\frac{1}{\nu}}}$$
$$\implies w_t^{\frac{\gamma+\nu}{\nu\gamma}} = \frac{K_t \omega^{\frac{1}{\nu}} [(1-\gamma)A_t]^{\frac{1}{\gamma}}}{(1-\tau_t^{wl})^{\frac{1}{\nu}}}$$
$$\implies w_t = \left[\frac{K_t \omega^{\frac{1}{\nu}} [(1-\gamma)A_t]^{\frac{1}{\gamma}}}{(1-\tau_t^{wl})^{\frac{1}{\nu}}}\right]^{\frac{\nu\gamma}{\gamma+\nu}}$$
$$= \frac{K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}}}{(1-\tau_t^{wl})^{\frac{\gamma}{\gamma+\nu}}}$$

Then the quantity of labour is

$$L_t = \left[\frac{(1-\tau_t^{wl})w_t}{\omega}\right]^{\frac{1}{\nu}}$$
$$= \left[\frac{(1-\tau_t^{wl})}{\omega}\right]^{\frac{1}{\nu}} w_t^{\frac{1}{\nu}}$$
$$= \left[\frac{(1-\tau_t^{wl})}{\omega}\right]^{\frac{1}{\nu}} \left[\frac{K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}}}{(1-\tau_t^{wl})^{\frac{\gamma}{\gamma+\nu}}}\right]^{\frac{1}{\nu}}$$
$$= \left[\frac{(1-\tau_t^{wl})}{\omega} \times \frac{K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}}}{(1-\tau_t^{wl})^{\frac{\gamma}{\gamma+\nu}}}\right]^{\frac{1}{\nu}}$$
$$= \left[\frac{K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}-1} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}}}{(1-\tau_t^{wl})^{\frac{\gamma}{\gamma+\nu}-1}}\right]^{\frac{1}{\nu}}$$

$$
\begin{aligned}
&= \left[\frac{K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{-\nu}{\gamma+\nu}} [(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}}}{(1-\tau_t^{wl})^{\frac{-\nu}{\gamma+\nu}}} \right]^{\frac{1}{\nu}} \\
&= \left[K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{-\nu}{\gamma+\nu}} [(1-\tau_t^{wl})(1-\gamma)A_t]^{\frac{\nu}{\gamma+\nu}} \right]^{\frac{1}{\nu}} \\
&= K_t^{\frac{\gamma}{\gamma+\nu}} \omega^{-\frac{1}{\gamma+\nu}} [(1-\tau_t^{wl})(1-\gamma)A_t]^{\frac{1}{\gamma+\nu}}
\end{aligned}
$$

Appendix 3.J Rule (3.34) and the conditional linear relationship between tax rates

Rule (3.34) implies

$$
T_t = \tau_t^{wl} \left(\frac{T}{\tau^{wl}} \right)
$$

which, when substituted into Equation (3.27), becomes

$$
\begin{aligned}
\tau_t^{wl} \left(\frac{T}{\tau^{wl}} \right) &= \tau_t^{wl} w_t L_t + \tau_t^{rn} r_t N_t \\
\Longrightarrow \tau_t^{wl} \left[\frac{T}{\tau^{wl}} - w_t L_t \right] &= \tau_t^{rn} r_t N_t \\
\Longrightarrow \tau_t^{rn} &= \tau_t^{wl} \left(\frac{\left[\frac{T}{\tau^{wl}} - w_t L_t \right]}{r_t N_t} \right) \\
\Longrightarrow \frac{\partial \tau_t^{rn}}{\partial \tau_t^{wl}} &= \frac{\left[\frac{T}{\tau^{wl}} - w_t L_t \right]}{r_t N_t}
\end{aligned} \tag{3.75}
$$

Assuming $r_t N_t \neq 0$ then

$$
\begin{aligned}
\frac{\partial \tau_t^{rn}}{\partial \tau_t^{wl}} > 0 &\iff \frac{T}{\tau^{wl}} - w_t L_t > 0 \\
&\iff T > \tau^{wl} w_t L_t \\
&\iff \tau^{wl} wL + \tau^{rn} rN > \tau^{wl} w_t L_t \\
&\iff \tau^{rn} rN > \tau^{wl} (w_t L_t - wL) \\
&\iff \left(\frac{\tau^{rn}}{\tau^{wl}} \right) rN > w_t L_t - wL
\end{aligned}
$$

Chapter 4

Quantitative strategies

4.1 Introduction

> *"An effort to put reasonable numbers on theoretical relations is harmless and may even be helpful. But it is still theory."*
>
> – Angrist and Pischke (2010), p. 18.

This thesis makes a theoretical contribution to macroeconomics. First, it proposes a novel framework with which to study liquidity shocks, fiscal policy, and large-scale asset purchases. Then, in the tradition of most of the KM-related literature and supported by the quote above by Angrist and Pischke (2010), the thesis performs quantitative experiments. The model is solved and stochastically simulated by the quantitative technique of calibration. Most parameter values are taken from the related literature and empirical microeconomic studies, and the rest are estimated in this chapter. Results are captured by impulse responses, and are reported and analysed in Chapters 5 to 8. A caveat of the calibration methodology is that quantitative results are subject to uncertainty, particular due to uncertainty over the values of the model's parameters. This implies that any fiscal multipliers that are generated from the experiments of the thesis cannot reliably be compared to multipliers from the empirical literature. Nevertheless, Chapter 8 evaluates the robustness of quantitative results by performing a sensitivity analysis of the model to its calibration.

This chapter describes and justifies the procedures that are involved in calibrating the model, performing experiments, and representing the results. The thesis focuses on analysing the results of its experiments, and with this limitation on scope, the chapter does not review or discuss the calibration methodology and perturbation techniques, and it does not pursue an algebraic derivation of the model's solution.

The rest of the chapter is organised as follows. Section 4.2 establishes values for the model's parameters. Table 4.1 summarises various settings for structural parameters; the "baseline" settings are used in Chapters 5 to 7 and the "sensitivity" settings are used exclusively in Chapter 8 for the sensitivity analysis. Table 4.2 summarises the steady state values and shock parameters of exogenous variables. Section 4.3 briefly describes the computational procedures that are involved in solving and stochastically simulating the model. Section 4.4 describes how exogenous stochastic shocks are simulated and how the results are represented. And the chapter's appendices contain some additional algebra and the data used in the calibration exercises.

4.2 Model calibration

4.2.1 Structural parameters

Liquidity constraint parameters, θ and ϕ_t

θ and ϕ_t are the most difficult structural parameters to calibrate. Del Negro et al. (2011) and Ajello (2012) emphasise the problem – the parameters are not directly observable and therefore difficult to estimate from data. KM assume, for simplicity, that θ and ϕ_t are always equal, even when ϕ_t varies stochastically. The majority of the KM-related literature maintains this assumption, and the calibration task is then simplifies to finding an empirical estimate of either parameter.

DEFK propose that ϕ_t is a linear function of the steady state value of a "liquidity share" variable – a ratio of liquid assets (empirically, US government liabilities) to total assets (empirically, net claims of private assets). From US data over the period 1952:1 – 2008:4 the authors obtain an average liquidity share of 12.64%. Then, according to their hypothesised linear relationship, they find that a value of 0.185 for ϕ_t is related to a liquidity share of 13%.

Shi (2012) assumes θ and ϕ_t are equal in steady state, but θ remains fixed when ϕ_t varies stochastically. Shi (2012) associates ϕ_t with the return on liquid assets. He uses DEFK's data on liquid assets, that is, US government liabilities, and estimates a value of 0.273 for ϕ_t in steady state. His idea of a liquidity shock is a fall in ϕ_t to 0.221. Bigio (2012) follows

Lorenzoni and Walentin (2007) and sets θ to match the aggregate moments of coefficients in a regression by Gilchrist and Himmelberg (1998) of the "great ratio" I/K against the return on capital and Tobin's q. Molteni (2014) does not assume $\theta = \phi_t$. He maintains DEFK's value for θ but calibrates ϕ at 0.75, the most relaxed value in the literature, in order to target an empirically observed haircut of 25% on repos that are collateralised by government bonds in Portugal and Ireland.

Besides Shi (2012), Bigio (2012), Molteni (2014), and Ajello (2012), the rest of the KM-related literature, as well as KM, calibrate the liquidity constraints to DEFK's estimated value of 0.185. This thesis applies the same calibration.

Sensitivity analysis in Chapter 8 relies on relaxing the constraints by 10% to a higher value of 0.2035 and symmetrically tightening the constraints by 10% to a lower value of 0.1665. The higher setting is the highest (common) value for θ and ϕ for which the model converges to a unique and stable equilibrium.

Subjective discount factor, β

Frederick et al. (2002) provide an extensive review of the literature on empirical and experimental studies of β, and observe that most arrive at values close to 1, or equivalently, quarterly rates of time preference close to zero, which implies that agents have almost equal preferences for the present and future. More recently, Theodoridis et al. (2012) estimate a time-varying parameter VAR based on the Smets and Wouters (2007) DSGE model, and find that β does not vary over time and is close to, but less than 1. All these results support the standard practice in the DSGE literature to fix β very close to 1. The most popular setting is a quarterly discount factor of 0.99, which means a 1% quarterly rate of time preference. The thesis selects this value. Amongst the KM-related literature, Nezafat and Slavík (2012) share this setting.

Values above and below, but not far away from, the baseline setting for β are chosen for sensitivity analysis. A higher β of 0.999 equates agents' preferences for the present and future. This setting appears in, for example, Fernández-Villaverde (2010), who also investigates fiscal shocks in a calibrated DSGE model with financial frictions. A lower β of 0.98 implies agents are more impatient and prefer the present, and therefore discount future utility by a 2% quarterly rate of time preference.

Capital's share in output, γ

Christensen et al. (1980) estimate an average value of 0.40 for γ in the US between 1947 and 1973. Acemoglu and Guerrieri (2008) obtain a measure over an updated period of 1948 – 2005, and not only confirm that this value still holds, but support the Kaldor (1961) fact that it remains constant over time. The thesis selects this value for the parameter. Furthermore, the setting implies that labour demand is wage-elastic (see Appendix 3.H).

Sensitivity from γ relies exclusively on a lower value of 0.36. Within the KM-related literature, this setting appears in Shi (2012) and Nezafat and Slavík (2012). Jermann and Quadrini (2012) also use this value to calibrate a Real Business Cycle model with a borrowing constraint. Lower values of 0.33 and 0.22 are used by Bigio (2012) and Fernández-Villaverde (2010), respectively. Values above the baseline setting are uncommon in the literature, and are therefore omitted in the sensitivity analysis.

Survival rate after depreciation, δ

The quarterly depreciation rate is set to 2.5%, which implies an annual rate of $1 - (1 - 0.025)^4 \approx 10\%$. This setting is standard in real business cycle studies on the US economy. Since King et al. (1988), who describe 10% as a "more realistic depreciation rate" (p. 218), this value has been widely used in calibrating DSGE models.

Like γ, sensitivity analysis with δ relies on just one alternative setting – a higher value of 0.98. Rates above the baseline are not unusual in the literature. Nezafat and Slavík (2012), Shi (2012), and Bigio (2012), for example, use 0.9774, 0.981, and 0.9873, respectively, and at the extreme end, Fernández-Villaverde (2010) uses 0.99.

Probability of an investment opportunity, π

π is empirically related to the fraction of firms that significantly adjust their capital in a given period. From samples of US manufacturing firms, Doms and Dunne (1998) estimate this fraction at 20% in any given year, from which DEFK set π to a quarterly rate of $1 - (1 - 0.2)^{0.25} \approx 5\%$. The thesis adopts DEFK's setting.

Cooper et al. (1999) and Cooper and Haltiwanger (2006) perform empirical studies similar to Doms and Dunne (1998) and estimate that 14% to 25% of firms significantly adjust their capital in any given year. The difference in estimates between the two sets of studies are due to what the authors consider to be a "significant adjustment" in the capital stock. To Doms and Dunne (1998), a "significant adjustment" means more than 10% of a firm's capital is repaired or replaced, whereas Cooper et al. (1999) and Cooper and Haltiwanger (2006) define the concept as any rate above 20%.[1] The interval estimate for the fraction of firms that invest in a year provide upper and lower alternative settings for π. If 14% of firms are assumed to significantly replace or repair their capital in a year then the implied value of π is $1 - (1 - 0.14)^{0.25} \approx 3.7\%$. If 25% of firms invest then $\pi = 1 - (1 - 0.25)^{0.25} \approx 6.9\%$.

Frisch elasticity of labour supply, $1/\nu$

The value of $1/\nu$ in applied economics is the subject of unresolved debate. On the one hand, micro-econometric studies usually find small estimates, that is, values below 1 (see a review of the literature by Contreras and Sinclair (2008)). Early work by MaCurdy (1981) and Altonji (1986) find a range of estimates from 0 to 0.5 in US data. Most empirical estimates (at least those whose samples are selected from males) have since fallen within this range (see Pencavel (1986) and Domeij and Flodén (2006), for example). On the other hand, macroeconomics needs much larger elasticities for calibrating models in order to match observed business cycle fluctuations in aggregate variables, as Prescott (2006) insists. Peterman (2012), for instance, explains that values between 2 and 4 are required to replicate empirically observed volatility in aggregate labour hours.

According to Peterman (2012) and Chetty et al. (2012), the wide micro-macro disparity on the value of $1/\nu$ is mainly due to sample selection – macroeconomic studies consider an aggregation of all individuals, whereas microeconomic studies rely on narrow samples of the population. For instance, the results of MaCurdy (1981) are drawn from prime-aged males. The DSGE literature is fairly consistent in using elastic values. However, there is a subset of the literature which assumes unitary elasticity. This is done by Christiano et al. (2005), following elasticity estimates in Rotemberg and Woodford (1999), and also by Christiano et al. (2013) and Cesa-Bianchi and Fernandez-Corugedo (2013) in their DSGE models with financial frictions. Within the KM-related literature, Nezafat and Slavík (2012) calibrate with unitary Frisch elasticity. The thesis also adopts the setting.

Sensitivity from $1/\nu$ is assessed with both elastic and inelastic values. A higher value

[1] Alternatively, Gourio and Kashyap (2007) consider a "significant adjustment" as investment which amounts to 35% or more of beginning-of-period capital.

of 2 is used, following the recommendations of macroeconomists; this value is also used in calibrations by Shi (2012) and Bigio (2012). The upper bound of 0.5 from MaCurdy (1981) and Altonji (1986) is used as the lower sensitivity setting.

Relative utility weight on labour, ω

The baseline setting is taken from a DSGE model by Villa and Yang (2011) that is similar in many ways to the model of this thesis. ω is often calibrated in the literature with consideration of ν, since together they form labour's coefficient in the worker's utility function. As Hall (1997) points out, researchers have different ways of representing this coefficient.[2] ω is usually calibrated to match an average or steady state fraction of time spent doing work. According to Villa and Yang, the common assumption in the literature is that individuals spend 8 hours a day working, that is, $\omega = 0.33$. Villa and Yang assume a utility function similar to the one in this model, and they set $\omega = 4.01$. The difference between this model and theirs is that they include habit persistence in consumption.

Villa and Yang's model is based on Gertler and Karadi (2011), who calibrate ω to 3.409 based on estimates by Primiceri et al. (2006). This value is used as a lower sensitivity setting for ω. For a higher sensitivity setting, the value of 8.15 that is set by Nezafat and Slavík (2012) is used. The model in this thesis shares structural similarities with Nezafat and Slavík (2012), including the same utility specification for workers. Their calibration of ω is done to allow the model to produce empirically-observed asset price volatility following simultaneous shocks to productivity and liquidity.

4.2.2 Steady state and AR(1) parameters of exogenous variables

An exogenously determined variable $X_t \in \{A_t, \phi_t, G_t, N^g_{t+1}, M_{t+1}, \tau^{rn}_t, \tau^{wl}_t\}$ that follows a stationary AR(1) process

$$X_t = (1 - \rho_X)X + \rho_X X_{t-1} + u^X_t$$

requires calibrated values for its steady state level, X, shock persistence parameter, ρ_X, and standard deviation of innovations, σ_{uX}. It can easily be shown that the variable's own

[2]Hall (1997), for instance, normalises ω and applies a relative weight to consumption in the worker's utility function.

standard deviation, σ_X, implies (see Appendix 4.A)

$$\sigma_{uX} = \sqrt{(1-\rho_X^2)\sigma_X^2} \tag{4.1}$$

Productivity and liquidity

A is normalised to 1. King and Rebelo (2000) use quarterly US data to estimate an AR(1) process for A_t in natural logarithms and without an intercept, and obtain point estimates of 0.979 for the persistence parameter and 0.0072 for the standard deviation of the residuals. The value of 0.979 for ρ_A is assumed in the thesis, and by Equation (4.1),

$$\sigma_{uA} = \sqrt{(1-\rho_A^2)\sigma_A^2} = \sqrt{(1-0.979^2)0.0072^2} = 0.00147$$

As mentioned earlier in Section 4.2.1, ϕ is assumed to be equal to θ (that is, 0.185). ρ_ϕ is set to a standard value of 0.95. An annual time series of DEFK's liquidity share variable is replicated in the thesis by following the authors' metadata (see Tables 4.4 and 4.5 in Appendix 4.B for the data and metadata, respectively). Figure 4.1 illustrates the liquidity share over time and shows that it is relatively stable for half a decade prior to the recent 2008 global financial crisis. Within this period, the liquidity share has a mean and standard deviation of 0.1110 and 0.0204, respectively. DEFK propose that the liquidity share is a linear function of ϕ_t,

$$LS_t = \phi_0 + 15\phi_t$$

where ϕ_0 is a constant.[3] Then

$$\begin{aligned} \sigma_{LS}^2 &= 15^2\sigma_\phi^2 \\ \implies \sigma_\phi &= \frac{\sigma_{LS}}{15} = \frac{0.0204}{15} = 0.00136 \end{aligned}$$

and by Equation (4.1),

$$\sigma_{u\phi} = \sqrt{(1-\rho_\phi^2)\sigma_\phi^2} = \sqrt{(1-0.95^2)\times 0.00136^2} = 0.00042$$

[3]From Figure 3 on page 43 in DEFK, the straight line appears to travel from 12.5 to 13.25, or 0.75 units along the vertical axis, and from 0.15 to 0.2, or 0.05 units along the horizontal axis, thus giving a slope of 0.75/0.05 = 15.

Policy variables

ρ_G, ρ_{Ng}, $\rho_{\tau rn}$, and $\rho_{\tau wl}$ are all set to a standard value of 0.95; the setting for ρ_M is explained below.

G_t is endogenously determined in Chapter 5's variant of the model. A simulation of that model yields $G = 0.428$. This value is assumed for G in all instances where G_t is exogenously determined. σ_{uG} is set to a very small value of 0.0001 to get the variants of the model in Chapter 6 to converge towards a unique, stable equilibrium after an exogenous shock to government spending; larger shocks in those models are explosive.

The US government started purchasing corporate equities in the third quarter of 2008 as part of the Troubled Asset Relief Program. The natural logarithm of this short time series has a standard deviation of 0.5671 (see Table 4.6 in Appendix 4.B). Then by Equation (4.1),

$$\sigma_{uNg} = \sqrt{(1 - \rho_{Ng}^2)\sigma_{Ng}^2} = \sqrt{(1 - 0.95^2) \times 0.5671^2} = 0.1771$$

Holding privately-issued equity is unconventional behaviour for the government. The government has no motive to save, and it is therefore reasonable to make the following assumption when N_{t+1}^g is exogenously determined:

Assumption 4. In steady state, the government holds no equity, that is, $N^g = 0$.

Equation (3.31) is estimated via least squares from quarterly US data over 1987:1 – 2008:1 (that is, 84 observations); Table 4.3 summarises the estimation results. M_{t+1} is taken as the seasonally adjusted, detrended, natural logarithm of the real monetary base (see Table 4.7 in Appendix 4.B for the data, and the notes therein for a description of how the series is compiled). ρ_M is set to the estimated coefficient 0.952 of the lagged dependent variable in the AR(1) regression. The Dickey-Fuller test on

$$\Delta M_{t+1} = (\rho_M - 1)M_t + u_t^M$$

with standard t-statistic,

$$\frac{\hat{\rho}_M - 1}{SE(\hat{\rho}_M)} = \frac{0.951991 - 1}{0.010284} \approx -5$$

concludes that $|\rho_M| < 1$ and M_{t+1} is a trend-stationary series. Figure 4.2 gives a histogram of the residuals of the regression. The Jarque-Bera test statistic, with p-value of 0.45, does not provide enough statistical evidence to reject a null hypothesis that the regression residuals are normally distributed. σ_{uM} is set to the standard deviation of the regression residuals, 0.004207. The estimated regression coefficients of Equation (3.31) imply a value of 1.95 for M.

Individuals in the US pay tax on income from all sources, not on the type of income earned. Data on dividend and wage taxes is not available from the US. The UK computes taxes by the type of income, including dividend and wage taxes. Parameters related to taxation are therefore estimated from quarterly UK data (see Table 4.8 in Appendix 4.B). Tax rates are computed as ratios of aggregate taxes to aggregate incomes from wages and dividends. Tax liabilities are used instead of actual tax receipts, to avoid the latter's problems with over/underpayments, late payments, etc. Standard deviations $\sigma_{\tau wl} = 0.004$ and $\sigma_{\tau rn} = 0.0112$ are observed from the data. The standard deviations of innovations to tax rates are then computed from Equation (4.1):

$$\sigma_{u\tau wl} = \sqrt{(1-\rho^2_{\tau wl})\sigma^2_{\tau wl}} = \sqrt{(1-0.95^2)\times 0.004^2} = 0.00124$$
$$\sigma_{u\tau rn} = \sqrt{(1-\rho^2_{\tau rn})\sigma^2_{\tau rn}} = \sqrt{(1-0.95^2)\times 0.0112^2} = 0.00349$$

The UK does not have flat rates of tax on wages and dividends. For both types of income the taxpayer enjoys a taxable allowance, and any excess amount earned during the fiscal year is subject to tax. The rate of tax applied on this excess depends on the individual's income for the fiscal year. τ^{wl} and τ^{rn} are set to average ratios, 0.231 and 0.207, of aggregate tax liabilities to aggregate incomes from wages and dividends, respectively (see Table 4.8 in Appendix 4.B).[4]

[4]These rates are very similar to those computed by Gomme and Rupert (2007) from US data and following a methodology set out by Mendoza et al. (1994) and Carey and Tchilinguirian (2000). Gomme and Rupert (2007) compute income tax rates of 0.22 on wages and 0.2868 on capital. These values, however, are not adopted here for two reasons: (i) Gomme and Rupert (2007) use data on *actual* government tax collections, which, as mentioned in the text, may be a less accurate description of the tax burden than tax liabilities data because of errors in tax payments; and (ii) $\sigma_{u\tau wl}$ and $\sigma_{u\tau rn}$ are needed here, and to be consistent, tax rates are obtained from the same dataset. The rates obtained here are also fairly consistent with a dedicated literature that estimates tax rates; Barro and Sahasakul (1983, 1986), Seater (1985), and Stephenson (1998) obtain average wage income tax rates between 0.22 and 0.30 from US data between 1954 and 1994; and Mendoza et al. (1994) obtain average income tax rates between 0.17 and 0.30 for wages and between 0.27 and 0.50 for capital.

4.3 Model solution and stochastic simulation

Nonlinearities in the equilibrium conditions make the model complex enough to not have an analytical solution. Instead, an approximate solution is much simpler to obtain, a strategy first recommended in the seminal work of Kydland and Prescott (1982). Variables are expressed in their levels, which means a linear approximation of the model is obtained. The model is approximated by a second order Taylor series expansion around the non-stochastic steady state, where there are no contemporaneous shocks, but agents give consideration to future shocks. A second order approximation is chosen for two main benefits that are highlighted in a review of perturbation methods by Juillard (2011) – (i) the approximation is usually more accurate at second order than first order; and (ii) certainty equivalence is avoided in a second order approximation, but not in a first order approximation (which is essentially a linearisation); avoiding certainty equivalence means that the variance of future shocks matter to agents.

Solving and simulating the model are handled electronically by the computer software Dynare.[5] The model is first stochastically simulated with a series of random shocks to exogenous variables $\{A_t, \phi_t\} \cup \Omega_t$ over 300 periods.[6] Random shocks are drawn from i.i.d. mean-zero Normal distributions. This first stochastic simulation is therefore approximately the steady state. The stochastic simulation of the model is then replicated, but this time a policy variable (the one being shocked) is increased (or decreased, if the experiment is a negative shock) in period 101 by one standard deviation. The deviations of endogenous variables between the two stochastic simulations are captured. This pair of stochastic simulations is repeated 50 times, and the average deviation in period t for each endogenous variable is computed.[7] This average deviation is taken as a variable's impulse response to an exogenous shock. Time is counted from when the exogenous shock hits, that is, period 101, which is referred to in the thesis as "quarter 1" or the "immediate period of the shock".[8]

[5] Dynare is a pre-processor for Matlab, that is, it generates Matlab code that solves DSGE models. Adjemian et al. (2013) describe the software in detail.

[6] 300 periods is the default setting, but it can be adjusted by users.

[7] 50 repetitions is another default, but adjustable, setting of Dynare.

[8] The first 100 periods is known as the "transient state" or "warm-up" period, and it gives the model enough time to achieve steady state.

4.4 Exogenous shocks and results

4.4.1 Impulse responses and dynamic impact multipliers

Exogenous shocks are simulated to both tax rates simultaneously (a "tax-shock") and individually, to government spending (a "G-shock"), to government equity holdings (an "N^g-shock"), and to the re-saleability constraint (a "liquidity shock").

Hypothetical normalised impulse responses are reported for ease of interpretation and comparison. There are two sets of experiments in which normalised impulse responses are not produced – the tax-shock and the N^g-shock. In these cases, dynamic impact multipliers are computed to obtain a normalised response. For all experiments, impulse responses are reported graphically and numerically. The tax-shock and the N^g-shock report tables of dynamic impact multipliers as additional results. If an endogenous variable's impulse response or impact multiplier in period t is greater than 1 then the response is described as "large", otherwise the response is described as "small".

Since the model is simulated with variables entered in levels, then impulse responses are deviations from steady state in levels. Impulse responses are not reported as percentage deviations from steady state. Percentage deviations depend on the absolute size of the deviation. Normalisation amplifies the level of the deviation, and then percentage deviations will reflect this amplification and give a misleading indication of responses. In other words, the percentage deviation from steady state is $100\Delta Y_t/Y$ without normalisation and $100\Delta Y_t/(Y\sigma_X)$ with normalisation.

When possible, the quarter 1 impulse response of the shocked exogenous variable is normalised to 1 unit, and impulse responses of endogenous variables are then re-scaled by the size of the shock. That is, if an exogenous variable X_t is shocked by a one standard deviation change in its value, σ_X, and the response of an endogenous variable Y_t is ΔY_t, then the changes in X_t and Y_t are represented as 1 unit and $\Delta Y_t/\sigma_X$ units, respectively.

The tax-shock in Chapter 5 comprises a normalised shock to τ_t^{wl} together with a re-scaled shock to τ_t^{rn} that measures 2.8 units. Tax rate cuts alter tax bases and altogether reduce aggregate taxes, T_t, by 3.1 units of general output. A variable T_t^* is constructed to represent government tax collections with tax bases held constant to their steady state levels, or "*ceteris paribus* changes in taxes", that is, from Equation (3.27),

$$T_t^* = \tau_t^{rn} rN + \tau_t^{wl} wL \tag{4.2}$$

where notations without time subscripts represent steady state values. Dynamic impact tax multipliers of a real endogenous variable are then computed as

$$\frac{X_t - X}{T_1^* - T} \tag{4.3}$$

where $(X_t - X)$ is the variable's impulse response in levels in period t. If X_t is not a real variable, that is, not measured in units of general output, then its impulse responses are converted to a real equivalent by being valued at the variable's steady state real price. By using T_t^* instead of T_t, tax multipliers disentangle the discretionary change in taxes (that is, the change in tax rates) from the endogenous component (that is, the changes in tax bases, $w_t L_t$ and $r_t N_t$).[9]

The N^g-shock in Chapter 7 is not in terms of general output, but in units of equity. The shock itself is converted to a real equivalent by being valued at the steady state equity price. Dynamic impact multipliers of a real endogenous variable are then computed as

$$\frac{X_t - X}{q(N_1^g - N^g)} \tag{4.4}$$

As in the tax-shock, if X_t is not a real variable then its impulse responses are valued at the variable's steady state real price.

4.4.2 Speed of convergence to steady state

An endogenous variable is "close to" steady state after a shock when the size of its impulse response in period $t \geq 2$ is less than 10% of the size of the immediate impulse response. An indicator is constructed to describe the speed at which an endogenous variable gets close, or converges, to steady state after being disturbed by an exogenous shock. This convergence indicator is the lowest value of $t \in [2, 201]$ which satisfies

$$X_t < c(X_1 - X)$$

[9]Perotti (2012) highlights the importance of separating discretionary from endogenous changes in taxes by showing they have different effects on output.

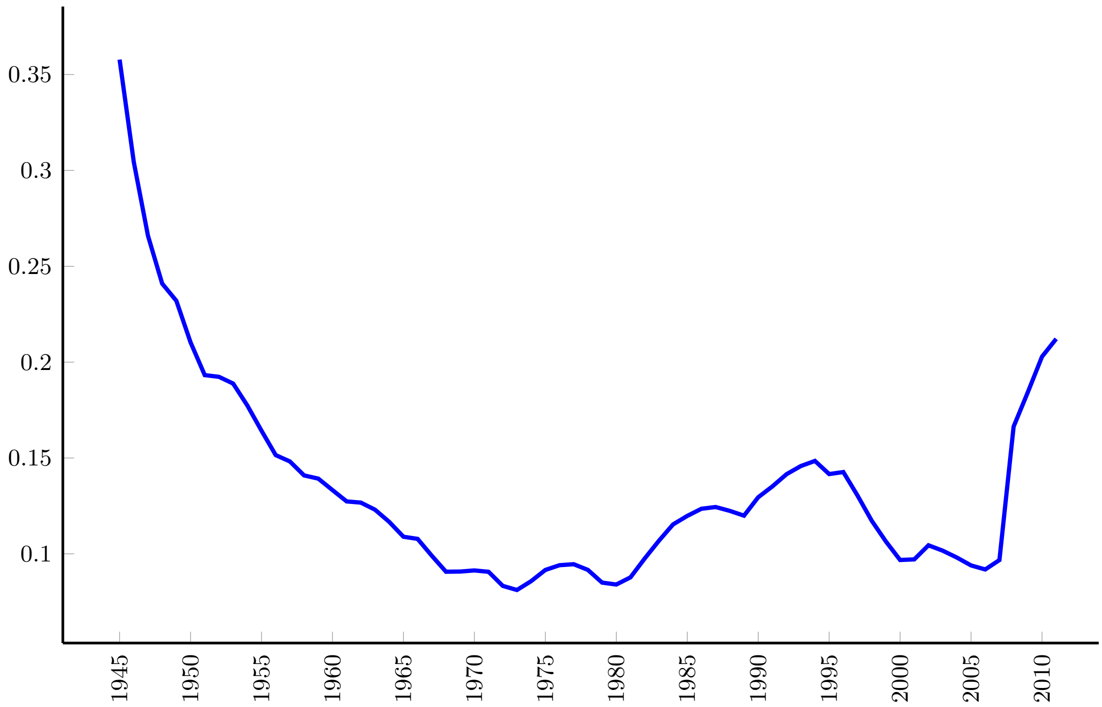
where $t = 201$ means convergence happens some time after 200 quarters, and not in the 201st quarter. The convergence parameter, c, is set to 0.1. The convergence indicator is therefore the time a variable's impulse response takes to fall within 10% of its immediate response.

4.4.3 Parameter elasticity of impulse responses

The sensitivity of shock responses to a change in value of one parameter is assessed by a measure called the "parameter elasticity of impulse responses". This measure is a ratio of the percentage change in a variable's immediate impulse response to the percentage change in a parameter's value.[10] The measure resembles an elasticity, hence the name. The elasticity is used in Chapter 8 for a sensitivity analysis of the model to its calibration. A positive elasticity means that an increase in the parameter's value amplifies the variable's impulse response relative to that of the baseline scenario. A negative elasticity means that an increase (or decrease, respectively) in the parameter's value lowers (or increases, respectively) the variable's impulse response relative to that of the baseline scenario. A variable is considered "sensitive" to a parameter if the elasticity is greater than 1 in absolute value. The model is considered "sensitive" to a parameter if the majority of the variables are sensitive to that parameter.

[10]The parameter elasticity resembles the "elementary effects" ratio of Morris (1991).

FIGURE 4.1: US liquid assets to total assets



Source: Board of Governors of the Federal Reserve System, US Flow of Funds Statistics, various issues; 2012 data is obtained from `http://www.federalreserve.gov/releases/z1/current/z1.pdf` and historical data from `http://www.federalreserve.gov/releases/z1/Current/data.htm`.
NOTES: The liquidity share is calculated according to Del Negro et al. (2011). Tables 4.4 and 4.5 in Appendix 4.B give the data and metadata, respectively.

FIGURE 4.2: Histogram of residuals in the estimation of $M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M$

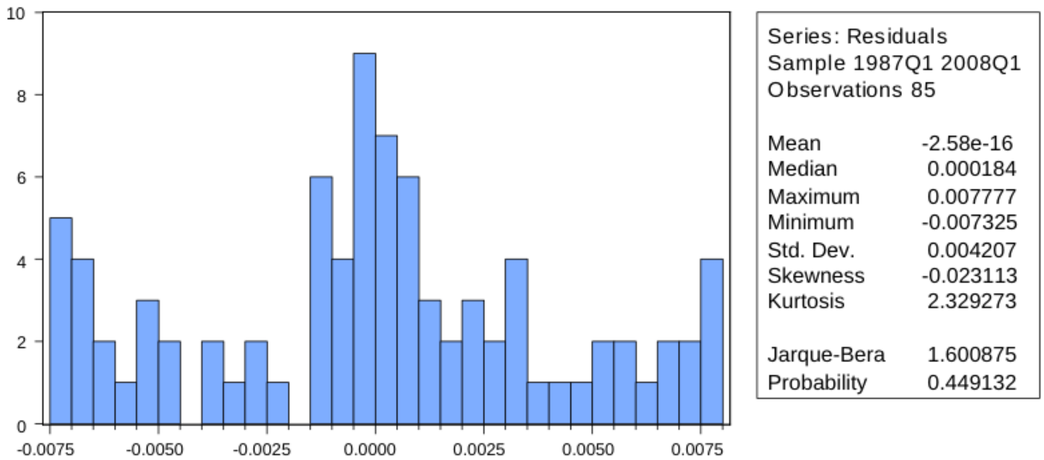


Table 4.1: Structural parameters: baseline and sensitivity settings

Structural parameters	Symbol	Baseline setting	Sensitivity settings	
			Lower	**Higher**
Fraction of investment financed by equity	θ	0.185	0.1665	0.2035
Subjective discount factor	β	0.99	0.98	0.999
Capital's share in output	γ	0.4	0.36	*n.a.*
Survival rate after depreciation	δ	0.975	*n.a.*	0.98
Probability of investment opportunity	π	0.05	0.037	0.069
Inverse Frisch elasticity of labour supply	ν	1	0.5	2
Relative utility weight on labour	ω	4.01	3.409	8.15

Table 4.2: Exogenous variables: steady states and autoregressive (shock) parameters

Variable	Steady state level		Persistence parameter		Standard deviation of innovations	
	Symbol	**Value**	**Symbol**	**Value**	**Symbol**	**Value**
Aggregate productivity	A	1	ρ_A	0.979	σ_{uA}	0.00147
Re-saleable fraction of equity	ϕ	0.185	ρ_ϕ	0.95	$\sigma_{u\phi}$	0.00042
Government spending	G	0.428	ρ_G	0.95	σ_{uG}	0.0001
Government equity	N^g	0	ρ_{Ng}	0.95	σ_{uNg}	0.1771
Money supply	M	1.95	ρ_M	0.952	σ_{uM}	0.00421
Rate of tax on dividends	τ^{rn}	0.207	$\rho_{\tau rn}$	0.95	$\sigma_{u\tau rn}$	0.00349
Rate of tax on wages	τ^{wl}	0.231	$\rho_{\tau wl}$	0.95	$\sigma_{u\tau wl}$	0.00124

NOTES: These values are used to calibrate the stochastic AR(1) processes (3.2), (3.4), (3.29), (3.30), (3.31), (3.32) and (3.33) which have the general form $X_t = (1-\rho_X)X + \rho_X X_{t-1} + u_t^X$ where ρ_X is the persistence parameter and u_t^X are innovations. If a variable does not follow a stochastic AR(1) process then its steady state value is determined endogenously.

Table 4.3: Estimation of $M_{t+1} = (1-\rho_M)M + \rho_M M_t + u_t^M$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$(1-\rho_M)M$	0.093483	0.019623	4.763995	0.0000
ρ_M	0.951991	0.010284	92.57203	0.0000
R-squared	0.990407	Mean dependent var		1.909519
Adjusted R-squared	0.990292	S.D. dependent var		0.042950
S.E. of regression	0.004232	Akaike info criterion		-8.069100
Sum squared resid	0.001486	Schwarz criterion		-8.011626
Log likelihood	344.9367	Hannan-Quinn criter.		-8.045982
F-statistic	8569.580	Durbin-Watson stat		0.024005
Prob(F-statistic)	0.000000			

NOTES: This table gives the results of estimating equation (3.31) via least squares with a sample of 84 observations from 1987:1 to 2008:1. M_{t+1} is the seasonally adjusted, detrended, natural logarithm of the real US monetary base. The data is given in Table 4.7 in Appendix 4.B.

Appendix 4.A Standard deviation of innovations to exogenous variables: derivation of Equation (4.1)

Consider recursive substitutions of the following AR(1) model for X_t:

$$
\begin{aligned}
X_t &= (1-\rho_X)X + \rho_X X_{t-1} + u_t^X \\
&= (1-\rho_X)X + \rho_X \Big[(1-\rho_X)X + \rho_X X_{t-2} + u_{t-1}^X\Big] + u_t^X \\
&= (1-\rho_X)X + \rho_X(1-\rho_X)X + \rho_X^2 X_{t-2} + \rho_X u_{t-1}^X + u_t^X \\
&= (1-\rho_X)X + \rho_X(1-\rho_X)X + \rho_X^2\Big[(1-\rho_X)X + \rho_X X_{t-3} + u_{t-2}^X\Big] + \rho_X u_{t-1}^X + u_t^X \\
&= (1-\rho_X)X + \rho_X(1-\rho_X)X + \rho_X^2(1-\rho_X)X + \rho_X^3 X_{t-3} + \rho_X^2 u_{t-2}^X + \rho_X u_{t-1}^X + u_t^X \\
&= \ldots \\
&= (1-\rho_X)X + \rho_X(1-\rho_X)X + \rho_X^2(1-\rho_X)X + \ldots + u_t^X + \rho_X u_{t-1}^X + \rho_X^2 u_{t-2}^X + \ldots
\end{aligned}
$$

where X is the steady state value of X_t. Then the variance of X_t is given by

$$\sigma_X^2 = \sigma_{uX}^2 + \rho_X^2\sigma_{uX}^2 + \rho_X^4\sigma_{uX}^2 + \ldots = \frac{\sigma_{uX}^2}{1-\rho_X^2}$$

which implies the standard deviation of innovations to X_t,

$$\sigma_{uX} = \sqrt{(1-\rho_X^2)\sigma_X^2}$$

Appendix 4.B Data

Table 4.4: Liquidity share measure: metadata

Item	Line reference in the Flow of Funds Statistics
Liabilities of the federal government	
T-bills	Table L.105, line 21
Treasury securities	Table L.105, line 22
Less: Holdings by the monetary authority	Table L.108, line 12
Less: Holdings by the budgetary agency	Table L.105, line 23
Reserves	Table L.108, line 32
Vault cash	Table L.108, line 33
Currency	Table L.108, line 34
Currency outside banks	Table L.108, line 41
Less: Remittances to the federal government	Table L.108, line 35
Capital (at market value)	
Capital owned by households:	
Real estate	Table B.100, line 3
Equipment and software of non-profit organisations	Table B.100, line 6
Consumer durables	Table B.100, line 7
Capital owned by the non-corporate sector:	
Real estate	Table B.103, line 3
Equipment and software	Table B.103, line 6
Inventories	Table B.103, line 9
Capital owned by the corporate sector:	
Equity outstanding, market value	Table B.102, line 35
Liabilities	Table B.102, line 21
Less: Financial assets	Table B.102, line 6
Less: Government credit market instruments	Table F.105c, line 33
Less: Trade receivables	Table F.105c, line 43

NOTES: This table follows from the appendix of Del Negro et al. (2011).

Table 4.5: US liquid assets, capital, and liquidity

Year	Federal government liabilities ($b)	Capital ($b)	Liquidity share
1945	262.4	471.0	0.3578
1946	238.4	544.9	0.3044
1947	230.1	635.3	0.2659
1948	222.6	701.2	0.2410
1949	222.3	736.0	0.2320
1950	221.2	830.6	0.2103
1951	222.3	928.2	0.1932
1952	229.1	962.1	0.1923
1953	231.1	992.8	0.1888
1954	232.1	1076.3	0.1774
1955	233.6	1188.9	0.1642
1956	228.8	1281.4	0.1515
1957	228.6	1314.0	0.1482
1958	235.2	1434.0	0.1409
1959	245.3	1516.4	0.1392
1960	242.1	1574.6	0.1333
1961	248.2	1700.8	0.1273
1962	253.7	1748.3	0.1267
1963	258.4	1842.1	0.1230
1964	264.1	1999.5	0.1167
1965	264.8	2166.2	0.1089
1966	268.7	2223.8	0.1078
1967	274.6	2498.9	0.0990
1968	281.4	2822.1	0.0907
1969	285.8	2863.0	0.0908
1970	302.4	3008.0	0.0913
1971	333.3	3344.2	0.0906
1972	349.5	3846.3	0.0833
1973	357.0	4042.8	0.0811
1974	377.8	4028.0	0.0858
1975	476.2	4722.8	0.0916
1976	553.2	5327.2	0.0941
1977	611.9	5856.7	0.0946
1978	673.0	6672.2	0.0916
1979	723.4	7787.0	0.0850
1980	823.6	8982.0	0.0840
1981	923.7	9604.3	0.0877
1982	1101.3	10189.0	0.0975
1983	1290.9	10797.8	0.1068
1984	1501.1	11513.4	0.1153
1985	1744.1	12817.3	0.1198
1986	1971.5	13990.6	0.1235
1987	2109.7	14846.2	0.1244

Continued on next page

Table 4.5: *Continued from previous page*

Year	**Federal government liabilities ($b)**	**Capital ($b)**	**Liquidity share**
1988	2242.7	16078.1	0.1224
1989	2404.1	17644.1	0.1199
1990	2671.6	17957.5	0.1295
1991	2985.2	19099.8	0.1352
1992	3276.6	19871.3	0.1416
1993	3543.3	20763.5	0.1458
1994	3710.8	21282.3	0.1485
1995	3868.4	23449.4	0.1416
1996	4048.2	24326.8	0.1427
1997	4097.7	27323.9	0.1304
1998	4089.8	30781.4	0.1173
1999	4232.4	35528.9	0.1064
2000	3808.8	35541.3	0.0968
2001	3855.5	35844.0	0.0971
2002	4118.6	35308.8	0.1045
2003	4549.1	40199.5	0.1017
2004	4930.0	45337.8	0.0981
2005	5273.5	50847.9	0.0940
2006	5475.2	54131.2	0.0919
2007	5797.3	54116.5	0.0968
2008	8980.9	45001.9	0.1664
2009	9990.4	44224.8	0.1843
2010	11707.4	46010.2	0.2028
2011	12444.2	46217.6	0.2121
Average: 1957 - 2007			0.1110
Standard deviation: 1957 - 2007			0.0204

Source: Board of Governors of the Federal Reserve System, US Flow of Funds Statistics, various issues; 2012 data is obtained online from `http://www.federalreserve.gov/releases/z1/current/z1.pdf`, and historical data from `http://www.federalreserve.gov/releases/z1/Current/data.htm`.
NOTES: The liquidity share is calculated according to Del Negro et al. (2011). Table 4.4 gives the metadata. The liquidity share is illustrated graphically in Figure 4.1.

Table 4.6: US federal government stocks of corporate equity

Period	**Equities ($m)**	**ln(Equities)**
2008:4	188,676	12.15
2009:1	223,856	12.32
2009:2	157,566	11.97
2009:3	158,847	11.98
2009:4	67,351	11.12
2010:1	50,234	10.82
2010:2	49,613	10.81
2010:3	50,814	10.84
2010:4	49,928	10.82
2011:1	62,137	11.04
2011:2	65,961	11.10
2011:3	59,282	10.99
2011:4	57,813	10.96
2012:1	48,156	10.78
2012:2	43,618	10.68
2012:3	41,134	10.62
Standard deviation		0.5671

Source: Board of Governors of the Federal Reserve, US Flow of Funds Accounts, Table L.105, line 11.
NOTES: This table gives the value of equities that were purchased by the US government from financial corporations under the Troubled Asset Relief Program. They are valued at market prices.

TABLE 4.7: US real monetary base

Period	M1 ($b)	CPI	CPI s.a.	Real M1	ln (real M1)	ln (real M1), detrended
1987Q1	730.2	111.200	111.4902	6.549456	1.879382	1.794665
1987Q2	743.9	112.700	112.6327	6.604653	1.887774	1.805112
1987Q3	743.0	113.800	113.6317	6.538671	1.877734	1.814580
1987Q4	756.2	115.300	115.1649	6.566237	1.881941	1.823065
1988Q1	756.2	115.700	116.0019	6.518858	1.874699	1.830602
1988Q2	768.1	117.100	117.0301	6.563269	1.881489	1.837261
1988Q3	781.4	118.500	118.3247	6.603862	1.887655	1.843141
1988Q4	783.3	120.200	120.0592	6.524284	1.875531	1.848370
1989Q1	785.7	121.100	121.4160	6.471140	1.867352	1.853101
1989Q2	779.2	123.100	123.0265	6.333595	1.845868	1.857505
1989Q3	777.8	124.400	124.2160	6.261675	1.834448	1.861763
1989Q4	786.6	125.600	125.4528	6.270086	1.835790	1.866049
1990Q1	795.4	127.400	127.7324	6.227079	1.828907	1.870516
1990Q2	806.1	128.900	128.8230	6.257421	1.833768	1.875304
1990Q3	810.1	130.400	130.2071	6.221627	1.828031	1.880521
1990Q4	819.9	133.500	133.3436	6.148778	1.816253	1.886255
1991Q1	827.2	134.600	134.9512	6.129622	1.813133	1.892556
1991Q2	843.1	135.200	135.1193	6.239672	1.830928	1.899433
1991Q3	861.6	136.200	135.9985	6.335363	1.846147	1.906845
1991Q4	878.0	137.400	137.2390	6.397598	1.855923	1.914708
1992Q1	910.4	138.100	138.4604	6.575167	1.883300	1.922900
1992Q2	943.8	139.500	139.4167	6.769633	1.912447	1.931262
1992Q3	963.3	140.500	140.2922	6.866385	1.926638	1.939612
1992Q4	1003.7	141.800	141.6338	7.086583	1.958203	1.947752
1993Q1	1030.4	142.600	142.9721	7.207000	1.975053	1.955481
1993Q2	1047.7	144.000	143.9140	7.280041	1.985136	1.962601
1993Q3	1084.5	144.400	144.1864	7.521514	2.017768	1.968929
1993Q4	1112.9	145.700	145.5293	7.647259	2.034347	1.974292
1994Q1	1131.6	146.200	146.5815	7.719937	2.043806	1.978553
1994Q2	1141.1	147.400	147.3120	7.746144	2.047195	1.981607
1994Q3	1150.6	148.400	148.1805	7.764856	2.049608	1.983394
1994Q4	1150.1	149.500	149.3248	7.702001	2.041480	1.983893
1995Q1	1151.5	150.300	150.6922	7.641404	2.033581	1.983124
1995Q2	1149.2	151.900	151.8093	7.570023	2.024196	1.981144
1995Q3	1145.4	152.500	152.2744	7.521947	2.017825	1.978043
1995Q4	1137.3	153.700	153.5199	7.408160	2.002582	1.973933
1996Q1	1123.5	154.400	154.8029	7.257616	1.982051	1.968957
1996Q2	1124.8	156.300	156.2067	7.200716	1.974180	1.963272
1996Q3	1112.4	157.000	156.7677	7.095847	1.959510	1.957043
1996Q4	1086.3	158.300	158.1145	6.870337	1.927213	1.950445
1997Q1	1081.3	159.100	159.5152	6.778666	1.913780	1.943650
1997Q2	1064.0	160.200	160.1044	6.645666	1.893965	1.936820
1997Q3	1066.3	160.500	160.2626	6.653456	1.895136	1.930096
1997Q4	1065.6	161.600	161.4106	6.601795	1.887342	1.923592

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Table 4.7: *Continued from previous page*

Period	**M1 ($b)**	**CPI**	**CPI s.a.**	**Real M1**	ln (**real M1**)	ln (**real M1**), **detrended**
1998Q1	1074.2	161.600	162.0217	6.629976	1.891601	1.917401
1998Q2	1076.7	162.500	162.4030	6.629804	1.891575	1.911592
1998Q3	1075.0	163.200	162.9586	6.596768	1.886580	1.906220
1998Q4	1086.1	164.000	163.8078	6.630330	1.891655	1.901327
1999Q1	1097.4	164.300	164.7287	6.661861	1.896399	1.896941
1999Q2	1102.0	166.200	166.1008	6.634527	1.892287	1.893085
1999Q3	1098.5	166.700	166.4534	6.599445	1.886986	1.889782
1999Q4	1102.2	168.200	168.0029	6.560601	1.881082	1.887054
2000Q1	1121.6	168.800	169.2405	6.627256	1.891191	1.884922
2000Q2	1114.6	171.300	171.1977	6.510600	1.873432	1.883403
2000Q3	1102.8	172.800	172.5444	6.391399	1.854953	1.882516
2000Q4	1098.7	174.000	173.7961	6.321775	1.844000	1.882278
2001Q1	1097.1	175.100	175.5569	6.249255	1.832462	1.882684
2001Q2	1116.1	176.900	176.7944	6.312983	1.842608	1.883708
2001Q3	1139.0	177.500	177.2374	6.426408	1.860416	1.885292
2001Q4	1166.2	177.700	177.4918	6.570445	1.882582	1.887352
2002Q1	1191.3	177.100	177.5621	6.709201	1.903480	1.889789
2002Q2	1187.7	179.800	179.6927	6.609619	1.888526	1.892500
2002Q3	1199.8	180.100	179.8336	6.671724	1.897878	1.895392
2002Q4	1204.7	181.300	181.0876	6.652583	1.895005	1.898369
2003Q1	1227.1	181.700	182.1741	6.735863	1.907446	1.901336
2003Q2	1250.3	183.800	183.6903	6.806566	1.917888	1.904195
2003Q3	1288.3	183.900	183.6280	7.015816	1.948167	1.906854
2003Q4	1297.3	185.000	184.7832	7.020659	1.948857	1.909229
2004Q1	1306.0	185.200	185.6833	7.033482	1.950682	1.911261
2004Q2	1333.2	188.000	187.8878	7.095726	1.959493	1.912916
2004Q3	1340.6	189.400	189.1198	7.088628	1.958492	1.914185
2004Q4	1360.7	190.900	190.6763	7.136177	1.965177	1.915088
2005Q1	1366.4	190.700	191.1976	7.146532	1.966627	1.915672
2005Q2	1358.2	194.600	194.4838	6.983614	1.943567	1.916017
2005Q3	1366.9	195.400	195.1109	7.005758	1.946732	1.916233
2005Q4	1375.4	199.200	198.9666	6.912718	1.933363	1.916448
2006Q1	1379.6	198.300	198.8175	6.939028	1.937162	1.916809
2006Q2	1380.9	201.500	201.3797	6.857196	1.925299	1.917474
2006Q3	1369.8	203.500	203.1990	6.741176	1.908234	1.918612
2006Q4	1370.2	201.800	201.5635	6.797856	1.916607	1.920401
2007Q1	1372.6	202.416	202.9442	6.763435	1.911531	1.923007
2007Q2	1378.1	206.686	206.5626	6.671585	1.897857	1.926597
2007Q3	1368.8	208.299	207.9909	6.581058	1.884196	1.931331
2007Q4	1379.7	208.936	208.6912	6.611204	1.888766	1.937350
2008Q1	1379.2	211.080	211.6308	6.517010	1.874416	1.944766

Sources: Board of Governors of the Federal Reserve System and Bureau of Labour Statistics (BLS).
NOTES: Nominal seasonally adjusted end-of-quarter base money (M1) is obtained from the Fed. The all-items all-urban-consumers US city average CPI (1982-84 = 100) is obtained from the BLS. The CPI is seasonally adjusted by the multiplicative moving average method. M1 is deflated by the seasonally adjusted CPI (CPI s.a.) to obtain real M1. The natural logarithm of real M1 is detrended by the HP filter.

Table 4.8: UK taxpayers' earnings and tax liabilities

Tax year	**Earnings (£m)**		**Tax liabilities (£m)**		**Tax liabilities / Earnings**	
	Wages	**Dividends**	**Wages**	**Dividends**	**Wages**	**Dividends**
1999-00	382,000	18,300	87,650	3,670	0.229	0.201
2000-01	431,500	20,800	98,580	3,900	0.228	0.188
2001-02	444,900	19,400	102,030	4,053	0.229	0.209
2002-03	451,600	20,800	103,900	4,330	0.230	0.208
2003-04	449,000	25,400	103,100	5,290	0.230	0.208
2004-05	496,000	32,600	113,860	6,070	0.230	0.186
2005-06	539,000	37,100	125,640	7,790	0.233	0.210
2006-07	573,000	41,500	135,370	9,000	0.236	0.217
2007-08	611,000	45,700	145,720	9,950	0.238	0.218
2008-09	n.a.	n.a.	142,000	9,380	n.a.	n.a.
2009-10	614,000	50,000	139,100	10,910	0.227	0.218
2010-11	616,000	36,100	138,600	7,700	0.225	0.213
Average					0.231	0.207
Standard deviation					0.0040	0.0112

Source: HM Revenue and Customs. UK taxpayers' earnings from employment and UK dividends are obtained from Tables 3.6 and 3.7, respectively in HM Revenue and Customs (2012b). Data for the tax year 2008-09 is not available. Income tax liabilities on earnings and dividends are obtained from Table 2.6 in HM Revenue and Customs (2012a). Older issues of these publications are obtained online at `http://webarchive.nationalarchives.gov.uk/20120609144700/http://hmrc.gov.uk/stats/income_tax/table2-6a.pdf` and `http://webarchive.nationalarchives.gov.uk/*/http://hmrc.gov.uk/stats/income_distribution/menu-by-year.htm`.

Chapter 5

Balanced budget tax cuts

5.1 Introduction

This chapter simulates unexpected temporary cuts in tax rates. The model is adapted for this chapter by assuming $\Omega_t = \{N^g_{t+1}, M_{t+1}, \tau^{rn}_t, \tau^{wl}_t\}$ is the set of exogenously determined policy variables. Assumption 4 is made which, together with exogeneity of the money supply, implies that the government balances its fiscal budget in every period by varying its spending to exactly match changes in tax revenue. Within this environment, the objectives of the chapter are to examine (i) the macroeconomic effects of tax cuts and (ii) a policy of tax cuts against a negative liquidity shock.

The chapter is partly motivated by the following.[1] In the basic Kiyotaki and Moore (2012) model, aggregate investment is given by

$$I_t = \frac{(r_t + \phi_t \delta q_t)\pi N_t + \pi p_t M_t - C^i_t}{1 - \theta q_t}$$

which declines after a negative liquidity shock, that is, a fall in ϕ_t. KM propose a government purchase of equity, which reduces the asset's supply on the market, raises its price, q_t, and thereby combats its loss in re-saleability. The effects of such policy are amplified by a portfolio balance effect which increases the price of money, p_t. There is no role for fiscal policy in the basic KM model. The thesis aims to develop such a role by introducing

[1]The current state of the literature also motivates this chapter; see Chapter 2.

distortionary taxes to the model. Here, aggregate investment is given by Equation (3.59),

$$I_t = \frac{([1-\tau_t^{rn}]r_t + \phi_t \delta q_t)\pi N_t + \pi p_t M_t - C_t^i}{1-\theta q_t}$$

Then a cut in the rate of tax on dividends, τ_t^{rn}, offsets the effect that a liquidity shock has on investment. This chapter quantitatively examines this hypothesis. KM's equity purchase programme can be thought of as a direct approach to policy against a liquidity shock – the loss in equity's re-saleability is offset by an increase in the asset's price. Taxation policy can then be though of as an indirect approach – the loss in liquidity is offset by an increase in net income.

The chapter makes three contributions to the macroeconomics literature. First, it contributes to the KM-related literature by proposing a new variant of the basic KM model – one with distortionary taxes. Second, it contributes to the KM-related literature by proposing tax cuts to counteract a liquidity shock. Third, it contributes to the fiscal policy literature by proposing a novel framework – a variant of the KM model, or a neoclassical DSGE model with liquidity constraints – with which to evaluate a balanced budget fiscal stimulus programme via tax cuts. The novelties of the first two contributions are defined in Chapter 2 by a review of the KM-related literature; the novelty of the third contribution is implied by the first two.

The chapter achieves its objective in three stages. The first stage simulates an exogenous across-the-board cut in tax rates (henceforth, a "tax-shock"), that is, simultaneous negative stochastic shocks to τ_t^{wl} and τ_t^{rn}. The cut in τ_t^{wl} is normalised (that is, a 1 percentage point rate cut) and the cut in τ_t^{rn} is re-scaled according to the ratio of standard deviations between both tax rates (that is, a 2.8 percentage point rate cut).[2] The second stage individually simulates exogenous stochastic cuts in τ_t^{wl} and τ_t^{rn} (henceforth, a "τ^{wl}-shock" and a "τ^{rn}-shock", respectively). The τ^{wl}-shock is a 1 percentage point cut in τ_t^{wl} and the τ^{rn}-shock is a 2.8 percentage point cut in τ_t^{rn}. The τ^{wl}-shock and the τ^{rn}-shock are therefore decompositions of the tax-shock. The third stage simulates a negative stochastic liquidity shock in "without policy" and "with policy" counterfactual experiments. The liquidity shock is in the tradition of KM, that is, a one-period fall in the value of ϕ_t followed by an asymptotic return to steady state. The shock is normalised to be a 1 percentage

[2] Specifically, a variable $u_t^{\tau} = (\sigma_{\tau wl}/\sigma_{\tau rn})u_t^{\tau rn}$ is defined, which has a variance of $\sigma_{\tau wl}^2$. Then a 1 unit shock to $u_t^{\tau wl}$ is also a 1 unit shock u_t^{τ} and therefore a $(\sigma_{\tau wl}/\sigma_{\tau rn})^{-1} = 2.8$ unit shock to $u_t^{\tau rn}$.

point decrease in ϕ_t. The policy intervention (henceforth, "tax policy") is a contemporaneous 0.1 percentage point cut in τ_t^{rn}. The reason for selecting a cut in τ_t^{rn} as the policy measure follows from the results of discretionary tax cuts in normal times. The size of the tax policy is chosen to partially offset the liquidity shock, instead of overturn and replace it, and then the dynamic benefits of tax policy can be observed as the economy adjusts to its pre-shock steady state.[3] All shocks are one-period events, and from the second quarter they are gradually phased out with a rate of decay of 5% each period. In all experiments, the model is calibrated with structural parameters set to baseline values that are described in Section 4.2 and summarised in Table 4.1.

The main results of the chapter are as follows. From the tax-shock, output increases on impact and evolves with a hump-shaped trajectory. Decomposing the tax-shock reveals that output's immediate increase comes from the cut in τ_t^{wl} and the hump-shaped trajectory comes from the cut in τ_t^{rn}. More generally, the τ^{wl}-shock affects the supply-side of the economy, that is, the labour market, workers' consumption, output, and the rate of dividends; whereas the τ^{rn}-shock affects the demand-side of the economy, that is, asset markets, investment (and therefore capital), and entrepreneurs' consumption and saving. The liquidity shock affects the same variables as the τ^{rn}-shock. Counteracting the liquidity shock without unnecessary distortions to unaffected variables is best achieved with a cut in τ_t^{rn}. Economic stimulus at any time is best achieved with an across-the-board cut in tax rates, as this brings together the benefits of the τ^{wl}-shock and the τ^{rn}-shock, that is, immediate and persistent increases in output, employment, asset prices, and aggregate demand.

The rest of the chapter is organised as follows. For ease of reference, Section 5.2 summarises the model by its equilibrium conditions. Section 5.3 describes the responses to the tax-shock. Section 5.4 decomposes the tax-shock into the τ^{wl}-shock and the τ^{rn}-shock. Section 5.5 examines a negative liquidity shock without and with tax policy. Section 5.6 examines the role that the borrowing constraint, θ, plays in the transmission of tax cuts; this role is formalised and then quantitatively demonstrated by re-simulating the tax-shock with a higher (that is, a more relaxed) value of θ, while all other parameters, including ϕ, are controlled at their baseline values. Section 5.7 summarises the chapter. Figures and tables appear at the end of the chapter.

[3] The same partially-offsetting approach is taken by KM and Driffill and Miller (2013) for the same reason.

5.2 Model summary

This section summarises the unique variant of the model by the conditions which characterise its dynamic equilibrium. This particular variant differs from those of Chapters 6 and 7 only by the definition of Ω_t and the behaviour of the government.

Government:

$$T_t = \tau_t^{rn} r_t N_t + \tau_t^{wl} w_t L_t \tag{3.27}$$

$$G_t + q_t(N_{t+1}^g - \delta N_t^g) = T_t + r_t N_t^g + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$N_{t+1}^g = (1 - \rho_{Ng})N^g + \rho_{Ng} N_t^g + u_t^{Ng} \tag{3.30}$$

$$M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M \tag{3.31}$$

$$\tau_t^{rn} = (1 - \rho_{\tau rn})\tau^{rn} + \rho_{\tau rn}\tau_{t-1}^{rn} + u_t^{\tau rn} \tag{3.32}$$

$$\tau_t^{wl} = (1 - \rho_{\tau wl})\tau^{wl} + \rho_{\tau wl}\tau_{t-1}^{wl} + u_t^{\tau wl} \tag{3.33}$$

Equity market:

$$N_{t+1}^s - \delta N_t^s = \phi_t \delta \pi N_t + \theta I_t - (N_{t+1}^g - \delta N_t^g) \tag{3.38}$$

$$\begin{aligned} \pi E_t &\left[\frac{\left(\frac{p_{t+1}}{p_t}\right) - \left(\frac{[1-\tau_{t+1}^{rn}]r_{t+1} + \phi_{t+1}\delta q_{t+1} + [1-\phi_{t+1}]\delta q_{t+1}^R}{q_t}\right)}{[1 - \tau_{t+1}^{rn}]r_{t+1}N_{t+1}^s + [\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R]\delta N_{t+1}^s + p_{t+1}M_{t+1}} \right] \\ &= (1 - \pi)E_t \left[\frac{\left(\frac{[1-\tau_{t+1}^{rn}]r_{t+1} + \delta q_{t+1}}{q_t}\right) - \left(\frac{p_{t+1}}{p_t}\right)}{[1 - \tau_{t+1}^{rn}]r_{t+1}N_{t+1}^s + q_{t+1}\delta N_{t+1}^s + p_{t+1}M_{t+1}} \right] \end{aligned} \tag{3.39}$$

$$\phi_t = (1 - \rho_\phi)\phi + \rho_\phi \phi_{t-1} + u_t^\phi \tag{3.4}$$

$$K_t = N_t + N_t^g \tag{3.47}$$

$$q_t^R \equiv \frac{1 - \theta q_t}{1 - \theta} \tag{3.6}$$

$$K_{t+1} = I_t + \delta K_t \tag{3.48}$$

Labour market:

$$w_t = K_t^{\frac{\nu\gamma}{\gamma+\nu}} \omega^{\frac{\gamma}{\gamma+\nu}} (1 - \tau_t^{wl})^{-\frac{\gamma}{\gamma+\nu}} [(1 - \gamma)A_t]^{\frac{\nu}{\gamma+\nu}} \tag{3.53}$$

$$L_t = K_t^{\frac{\gamma}{\gamma+\nu}} \omega^{-\frac{1}{\gamma+\nu}} [(1 - \tau_t^{wl})(1 - \gamma)A_t]^{\frac{1}{\gamma+\nu}} \tag{3.54}$$

General output market:

$$A_t = (1 - \rho_A)A + \rho_A A_{t-1} + u_t^A \tag{3.2}$$

$$C_t = C_t^i + C_t^s + C_t^w \tag{3.55}$$

$$C_t^i = \pi(1-\beta)([1-\tau_t^{rn}]r_t N_t + [\phi_t q_t + (1-\phi_t)q_t^R]\delta N_t + p_t M_t) \tag{3.56}$$

$$C_t^s = (1-\pi)(1-\beta)([1-\tau_t^{rn}]r_t N_t + q_t \delta N_t + p_t M_t) \tag{3.57}$$

$$C_t^w = (1-\tau_t^{wl})w_t L_t \tag{3.58}$$

$$(1-\theta q_t)I_t = ([1-\tau_t^{rn}]r_t + \phi_t \delta q_t)\pi N_t + \pi p_t M_t - C_t^i \tag{3.59}$$

$$Y_t = A_t K_t^{\gamma} L_t^{1-\gamma} \tag{3.60}$$

$$r_t K_t = Y_t - w_t L_t \tag{3.61}$$

$$Y_t = C_t + I_t + G_t \tag{3.62}$$

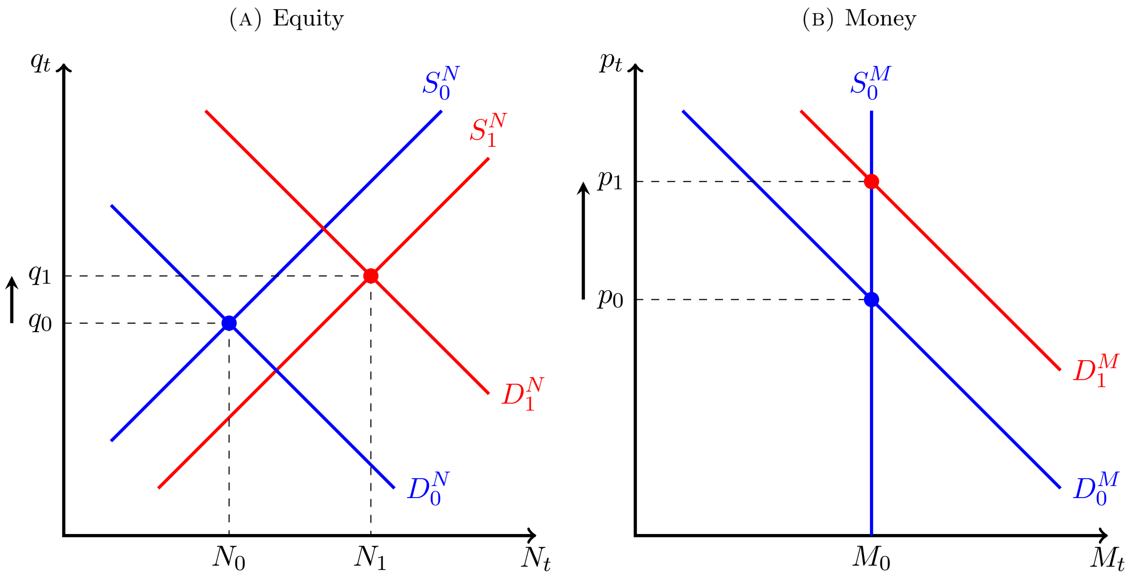
5.3 A tax-shock

Impulse responses to the tax-shock are illustrated in Figure 5.2 and are given numerically in Table 5.1. The tax-shock immediately reduces the government's tax revenue by 3.3 units of general output *ceteris paribus* (as measured by the variable T_t^*) and by 3.1 units of general output with endogenous changes in tax bases (as measured by T_t). By assumption, the government maintains the money supply at its steady state level and continues to hold no entrepreneur-issued equity. The fiscal (and overall) budget (Equation (3.28)) is balanced by reducing spending in tandem with tax revenue. Because government spending does not enter the utility of private agents, there are no consequences for output and private spending. Tax cuts are a one-off event, and from the second quarter tax rates start rising asymptotically towards their pre-shock levels.

The cut in the rate of tax on wages increases the supply of labour at each and every wage rate. Entrepreneurs expand their demand for labour and the market clears with an uptake in employment and a lower wage rate. Given that labour demand is wage-elastic, the tax-shock brings more aggregate gross wages to workers. Workers increase their consumption, which is the largest component of aggregate private consumption (accounting for approximately 80% in steady state). The wage tax base endogenously increases and thereby partially offsets the effect of the tax rate cut on the government's fiscal revenue.

Because of more employment, and with its other determinants unchanged, output has a small increase. This additional general output is matched by higher aggregate demand; as the following paragraphs explain, the tax-shock stimulates private consumption and in-

Figure 5.1: Asset markets and the immediate effects of a tax-shock



NOTES: Money's demand comes from entrepreneurs and its supply is fully controlled by the government. Equity's demand comes from savers and its supply comes from investors; Assumption 4 takes the government out of the equity market. Net worth improvements encourage savers to increase their demand for money and equity, and consequently shift the assets' respective demand curves from D_0^M and D_0^N to D_1^M and D_1^N. Net worth improvements also encourage investors to issue new equity, and consequently shift the asset's supply from S_0^N to S_1^N. The tightly binding borrowing constraint means that the shift in equity's supply is small, which then guarantees a price increase.

vestment sufficiently enough to outweigh the fall in government spending that is required to balance the fiscal budget.

The cut in the rate of tax on dividends directly increases entrepreneurs' net worth. Those without investment opportunities consume and save more, with the latter being translated into an increase in demand for both assets. Those with investment opportunities marginally increase their consumption, and instead feed most of their net worth gains into available investment technologies.

To fulfil new investment plans, investors issue new equity. Because of the tightly binding borrowing constraint, investors sell a small fraction of new issues, and the rest is retained (or "purchased") by liquidating re-saleable equity and stocks of money. The borrowing constraint therefore ensures that the increase in equity's supply is small, and the asset ends up with a higher market price.[4] With no further contemporaneous changes in its market, money also ends up with a higher price. Figure 5.1 illustrates the asset markets and the immediate effects of the shock.

Within the immediate period of the shock, a positive amplifying feedback mechanism,

[4]This hypothesis is formalised and tested in Section 5.6, where it is called the "θ-equity hypothesis".

akin to financial acceleration of Bernanke et al. (1999), is initiated from the changes in the asset markets – higher asset prices improve entrepreneurs' net worth; investment increases; asset demands increase; equity's supply increases to a lesser degree; asset prices rise again; and so on. The effects wear off with each cycle, as only part of investors' net worth improvements are channeled into new investment and the rest is consumed.

The steady state level of investment creates new units of capital that exactly replace depreciated units. Since the shock increases investment above its steady state, the capital stock has a small increase by the end of the first quarter. Investment remains above steady state while it asymptotically converges. This allows the capital stock to continue growing for some time. Capital's depreciation starts to outweigh investment in the 35^{th} quarter and then the stock starts to decline. Capital takes a very long time to converge to its pre-shock level; after 200 quarters it still does not get close to steady state. Investment is much quicker to converge; it gets close to steady state in the 94^{th} quarter.

The labour market's long-term adjustment reflects the net result of two simultaneous forces. First, the rising rate of tax on wages reduces labour supply towards its pre-shock position. Second, growth in the capital stock increases entrepreneurs' demand for labour at each and every wage. Initially, the supply changes are stronger, and the labour market experiences falling employment and a rising wage rate. Eventually the capital stock starts to fall, and the labour demand changes are reversed. When this happens, the real wage starts to fall, while employment continues falling. The wage rate exhibits a hump-shaped trajectory which overshoots its steady level in the 10^{th} quarter and peaks in the 45^{th} quarter, while employment has an asymptotic decline.

Output is initially influenced more by capital than labour, and continues increasing for 27 quarters. Eventually, output is influenced more by falling employment, and begins a gradual asymptotic decline that does not get close to steady state by the 200^{th} quarter. Asset prices fall asymptotically towards steady state and are relatively quick to converge; the price of money gets close to its pre-shock level after 49 quarters, while the price of equity is quicker, at 36 quarters.

5.4 Individual tax cuts: decomposing the tax-shock

Impulse responses to the τ^{wl}-shock and the τ^{rn}-shock are illustrated with the tax-shock's responses superimposed in Figures 5.3 and 5.4, respectively, and are given numerically in Tables 5.2 and 5.3. Table 5.4 compares the τ^{wl}-shock, the τ^{rn}-shock, and the tax-shock by their immediate and largest responses.

The cut in τ_t^{wl} affects the supply-side of the economy – the real wage, employment, output, workers' consumption, and rate of dividends – and accounts for the entire immediate responses of w_t, L_t, Y_t, C_t^w, and r_t in the tax-shock. The cut in τ_t^{rn} affects the demand-side of the economy – asset markets, investment (and therefore capital), and entrepreneurs' consumption and saving – by bringing net worth improvements to entrepreneurs. In the tax-shock, the cut in τ_t^{rn} accounts for 86% of the immediate responses of p_t, q_t, I_t, C_t^i, C_t^s, N_{t+1}^s, and K_{t+1}, as well as T_t. The rest of the responses come from the cut in τ_t^{wl}, which is transmitted to the demand-side of the economy by a higher rate of dividends brining net worth improvements (from Equation (3.61)). The cut in τ_t^{rn} does not immediately affect the supply-side of the economy, but there is a delayed impact when the increase in the capital stock at the end of the first period causes the demand for labour to increase in the second period.

The cut in τ_t^{wl} gives output its immediate increase. But by cutting τ_t^{wl} alone, output loses its hump-shaped trajectory and long-term persistence. By increasing investment and the capital stock, the cut in τ_t^{rn} is what sustains long-term economic growth and gives output its hump-shaped trajectory. The same can be said about employment – the cut in τ_t^{wl} produces an immediate increase and the cut in τ_t^{rn} stimulates persistently high labour demand. Given that workers' consumption is by far the largest component of aggregate private consumption, the cut in τ_t^{wl} accounts for 76% of the immediate change in C_t in the tax-shock.

The τ^{wl}-shock is not the same as a positive shock to aggregate productivity. They are the same in their effects on the labour market (from Equations (3.53) and (3.54)). But the τ^{wl}-shock gives workers' consumption an added boost by reducing the agent's tax obligation (from Equation (3.58)), whereas the productivity shock directly gives output an added boost (from Equation (3.60)). Nevertheless, asset prices, consumption, investment, capital, and output all have the same initial responses and trajectory shapes between the τ^{wl}-shock and a productivity shock in the basic KM model (see Figure 1 in Kiyotaki and Moore (2012), p. 29).

The tax-shock brings together the benefits of both tax cuts, that is, it produces immediate and persistent responses to both the supply and demand sides of the economy. This makes across-the-board tax cuts a superior policy than cutting one tax rate for discretionarily stimulating the economy.

5.5 A liquidity shock and tax policy

5.5.1 Without policy

Impulse responses to the liquidity shock without tax policy are illustrated by the dashed blue lines in Figure 5.5 and are given numerically in Table 5.5. These responses are the same as those of KM's liquidity shock (see Figure 2 in Kiyotaki and Moore (2012), p. 31).

The liquidity shock has no immediate impact on the labour market. Consequently, there are no changes in output, workers' consumption, the rate of dividends, and government tax revenue and spending. The shock lowers investment by making it more difficult for investors to sell their existing stocks of equity; equity's supply falls and price rises. Consequently, there is a positive portfolio balance effect on money's demand. Asset price increases then lead to net worth improvements for entrepreneurs.

There is a substitution effect of the liquidity shock which outweighs the income effect from higher net worth. The shock ruins the appeal of equity and encourages savers to substitute the asset with money in what KM call a "flight to liquidity". The shock makes new investment more difficult to realise, and encourages investors to consume rather than invest. This accounts for the fall in saving and investment in the immediate period of the shock.

Lower investment in the immediate period of the shock reduces the capital stock at the end of that period. In the next period, employment and output initially decline, and they only recover when capital and investment reach levels where investment outweighs depreciation.

5.5.2 With policy

Impulse responses to the liquidity shock with tax policy are illustrated by the solid red lines in Figure 5.5 and are given numerically in Table 5.6. Section 5.4 and the previous section show that the liquidity shock and the τ^{rn}-shock affect the same variables, that is, I_t, K_{t+1}, N_{t+1}^s, C_t^s, C_t^i, p_t, and q_t. This makes a cut in τ_t^{rn} a suitable policy for counteracting the negative liquidity shock without unnecessary impacts on unaffected variables. The cut in τ_t^{rn} partially offsets the loss in asset re-saleability with an increase in net worth (from Equation (3.59)), and thus dampens the decrease in investment brought on by the liquidity shock. Figure 5.5 shows that all the consequences of the liquidity shock described in the previous section are smaller. The only qualitative change that tax policy brings is a contemporaneous decrease in the size of the government. The results of this section suggest that if the cut in τ_t^{rn} is increased sufficiently in magnitude then it can completely offset the liquidity shock.

If, instead, the policy response is a cut in τ_t^{wl} alone then the net worth improvement is small and the offsetting effects on the liquidity shock are weaker than cutting τ_t^{rn}, although the economy will immediately enjoy more employment and output. Alternatively, if the policy response is the tax-shock then not only will the liquidity shock be offset, but there will be the added benefits of immediate increases in employment and output. The cut in τ_t^{rn} precisely targets the liquidity shock, that is, it does not unnecessarily disturb (albeit positively) unaffected variables from their steady states.

5.6 The role of θ in the transmission of tax cuts

This section formalises and quantitatively investigates a hypothesis about the borrowing constraint that is suggested by the results of the tax-shock. From the cut in τ_t^{rn}, in particular, entrepreneurs' net worth improves, investment rises, and the tightness of the borrowing constraint ensures that an increase in equity's supply is small, which thus guarantees a rise in the price of the asset and further net worth improvements. The hypothesis suggests that the tighter the borrowing constraint, the smaller the increase in equity's supply, the higher the increases in asset prices, and the greater the increase in entrepreneurs' net worth; and conversely. The hypothesis is henceforth called the "θ-equity hypothesis". At the same time, the tighter the borrowing constraint, the lower is the level of aggregate investment, because external financing is more difficult to obtain; and conversely. This second hypothesis is henceforth called the "θ-investment hypothesis". Both hypotheses presuppose that θ is a variable and subject to exogenous stochastic shocks.

The hypotheses are compensating. Their policy implication is that if a stimulus programme is underway and involves a cut in τ_t^{rn} then a shock to θ does not significantly interfere with the transmission of the programme. A drop in θ, for instance, at first reduces aggregate investment (the θ-investment hypothesis), but this is compensated by greater net worth improvements from much higher asset prices (the θ-equity hypothesis).

The θ-investment hypothesis is formalised algebraically. From Equation (3.59),

$$\frac{\partial I_t}{\partial \theta} = \frac{\theta\left[([1-\tau_t^{rn}]r_t + \phi_t \delta q_t)\pi N_t + \pi p_t M_t - C_t^i\right]}{(1-\theta q_t)^2}$$

The baseline calibration of β makes C_t^i very small (from Equation (3.56)). Then $\frac{\partial I_t}{\partial \theta}$ is non-negative.

Both hypotheses are verified by re-simulating the tax-shock with a more relaxed borrowing constraint, that is, a higher value of θ, viz. 0.5055. This is the largest possible value that allows the model to converge to a unique stable equilibrium. Such a value is chosen for an illustrative purpose, that is, to exaggerate the difference in responses from baseline. The rest of the model's calibration is controlled at the baseline, including ϕ, which remains at 0.185 by de-coupling its value from θ.[5] Impulse responses are illustrated graphically in Figure 5.6 with original (or "baseline") tax-shock responses superimposed, and are given numerically in Table 5.7.

Results support the θ-investment hypothesis – the re-simulation produces a larger immediate increase in investment, by 2.33 units of general output compared to 1.88 units in the baseline scenario. The results also support the θ-equity hypothesis – the re-simulation produces a smaller immediate increase in equity's price, by 2.24 units of general output compared to 3.25 in the baseline. Via a portfolio balance effect, a weaker equity price increase produces a smaller increase in demand for money relative to the baseline scenario. Consequently, money's price has an insignificantly small increase compared to a large increase of 6.63 units of general output in the baseline.

[5] Section 8.2 relaxes both θ and ϕ_t simultaneously for a sensitivity analysis of the model's calibration.

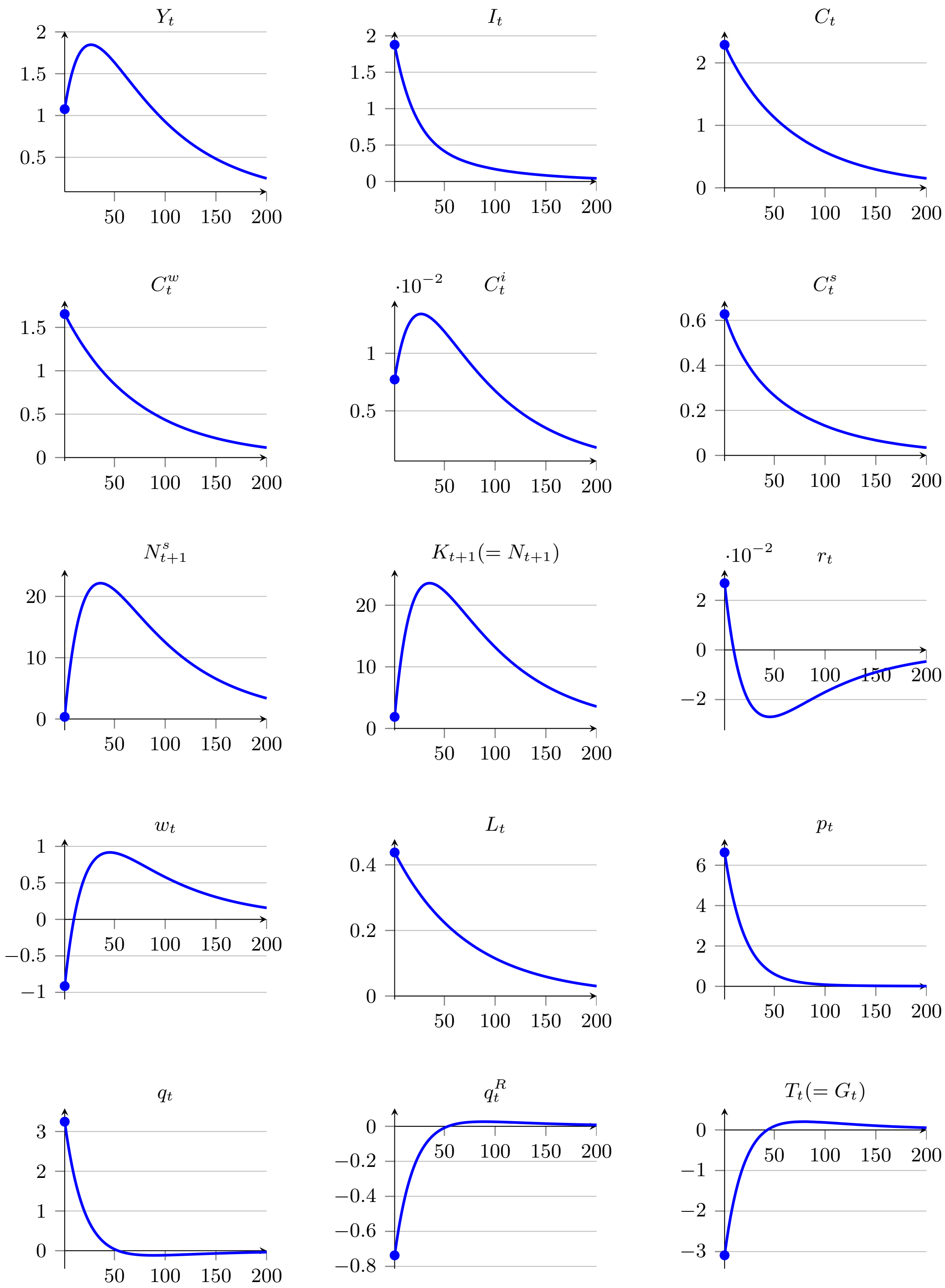
5.7 Chapter summary

This chapter simulates temporary cuts to tax rates in a unique variant of the basic model of Kiyotaki and Moore (2012). There is a government that levies distortionary taxes on wages and dividends, keeps money supply constant, and always balances the fiscal budget by varying spending to match taxes. Simultaneous tax cuts produce immediate and persistent increases in employment, consumption, investment, and output. The cut in the rate of tax on wages is wholly responsible for the immediate supply-side responses (that is, labour and output) and the cut in the rate of tax on dividends is mostly responsible for demand-side responses (most notably, investment) and the long-term persistence of output. The cut in the rate of tax on dividends is the most suitable policy for counteracting a negative liquidity shock; the policy and the shock affect the same variables in opposite ways, and there are no disturbances to variables that are unaffected by the shock. Intuitively, the dividend tax cut restores entrepreneurs' net worth that has been lost to the liquidity shock. This policy prescription is new to the KM-related literature that examines liquidity shocks, and this is the chapter's main contribution to the literature. In normal times, that is, when there is no liquidity shock, the government can achieve long and sustained economic stimulus by cutting both tax rates.

Two hypotheses are supported by a re-simulation of the model – the tighter the borrowing constraint, (i) the smaller the increase in equity's supply, the higher the increases in asset prices, and the greater the increase in entrepreneurs' net worth; and (ii) the lower the level of aggregate investment; and conversely. The hypotheses imply that if a stimulus programme is underway involving a cut in the dividend rate of tax, then an exogenous change in the tightness of the borrowing constraint does not significantly interfere with the transmission of the programme.

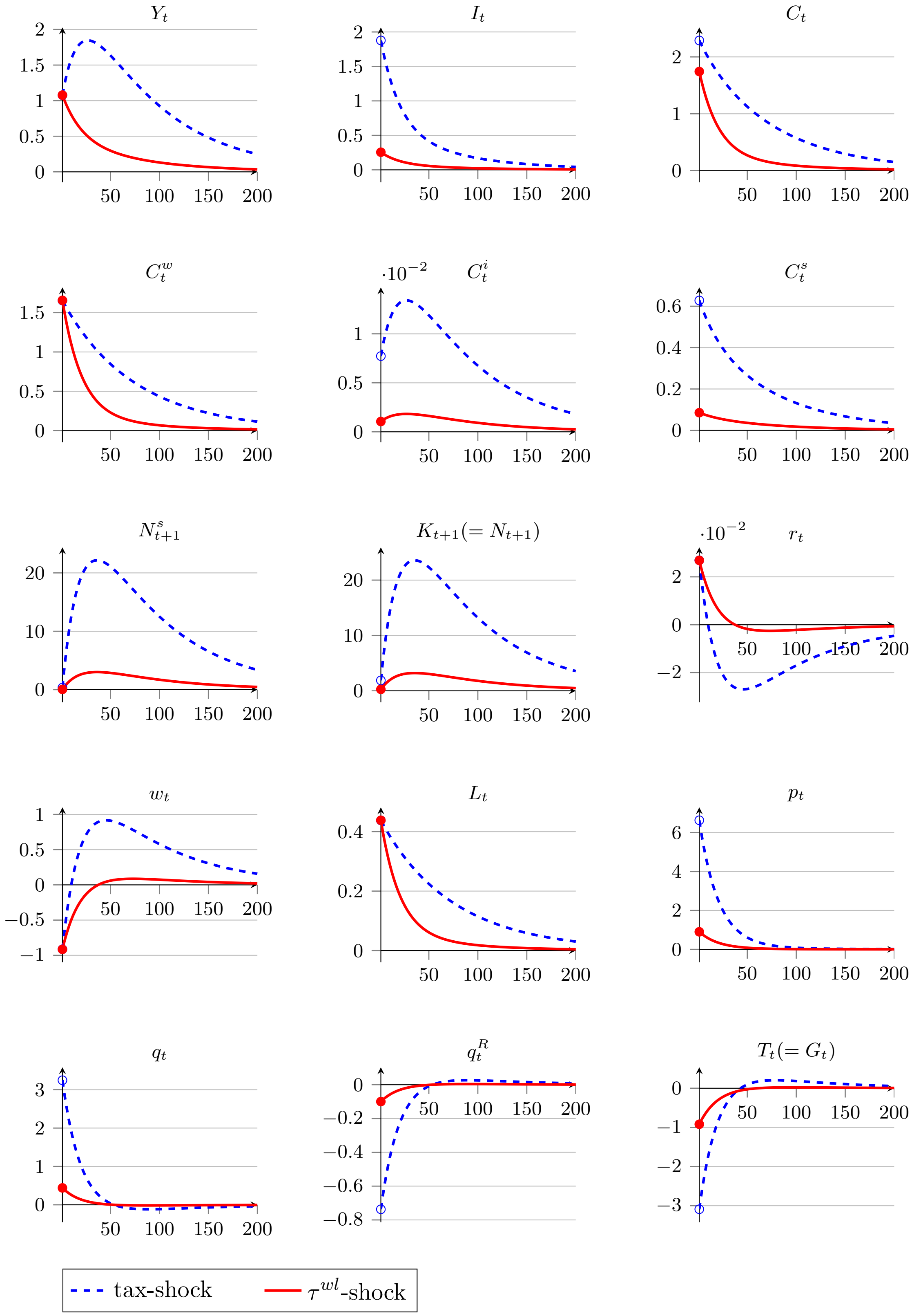
The thesis continues in Chapter 6 by examining expansionary fiscal policy via an increase in government spending. Then Chapter 7 returns to KM's policy prescription of an asset purchase programme. And Chapter 8 examines the sensitivity of the results of the tax-shock to the model's calibration; the analysis finds that tax-shock responses in this chapter are qualitatively robust, but quantitatively sensitive, to a wide variety of alternative structural parameter values, and qualitatively and quantitatively sensitive to implausibly significant variations in the persistence of tax cuts. The conclusion of the thesis evaluates this chapter and proposes extensions which can be pursued as future research.

FIGURE 5.2: Impulse responses to a tax-shock: baseline scenario



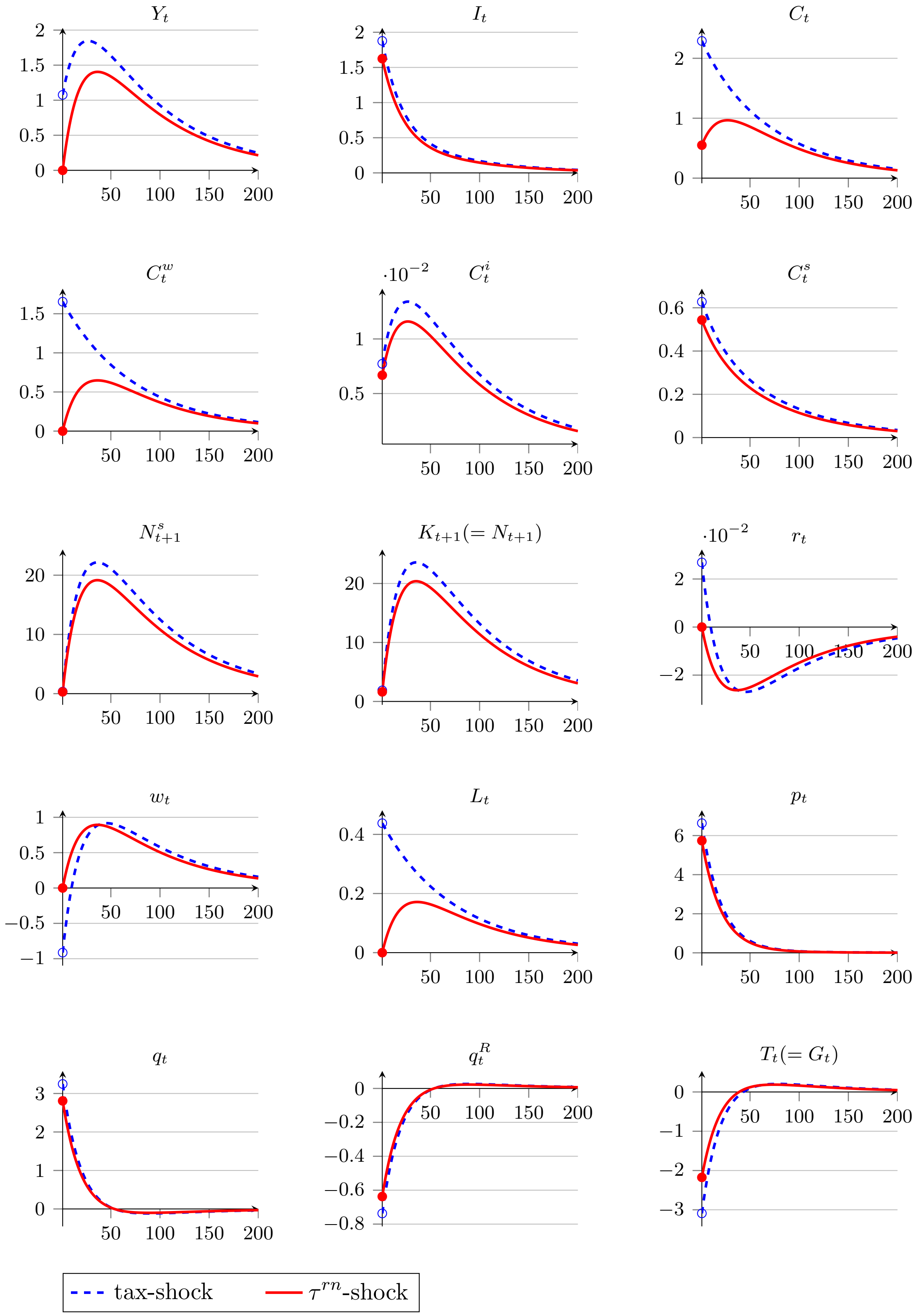
Notes: The tax-shock is a simultaneous 1 percentage point cut in τ_t^{wl} and 2.8 percentage point cut in τ_t^{rn}. Horizontal axes measure quarters after the shock, starting from quarter 1. Vertical axes measure deviations from steady state in levels. Blue dots indicate immediate responses; see the "quarter 1" column of Panel A in Table 5.1 for their values.

Figure 5.3: Impulse responses to a τ^{wl}-shock



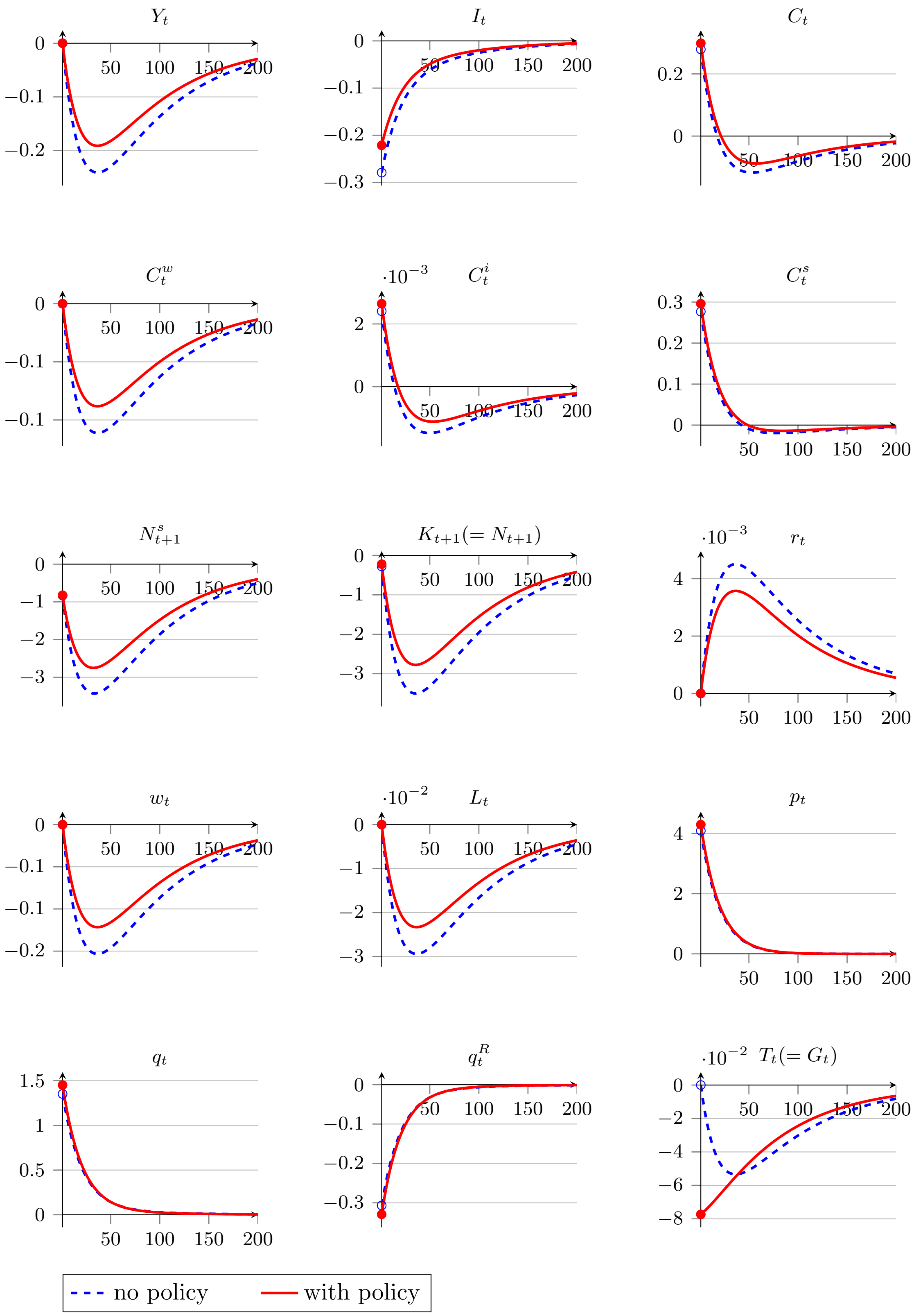
Notes: The τ^{wl}-shock is a 1 percentage point cut in τ_t^{rl}. The tax-shock responses are those from Figure 5.2, from which the same notes apply. Hollow blue dots and solid red dots indicate immediate responses of the tax-shock and the τ^{wl}-shock, respectively; see the "quarter 1" column of Panel A in Table 5.1 and Table 5.2, respectively, for their values.

Figure 5.4: Impulse responses to a τ^{rn}-shock



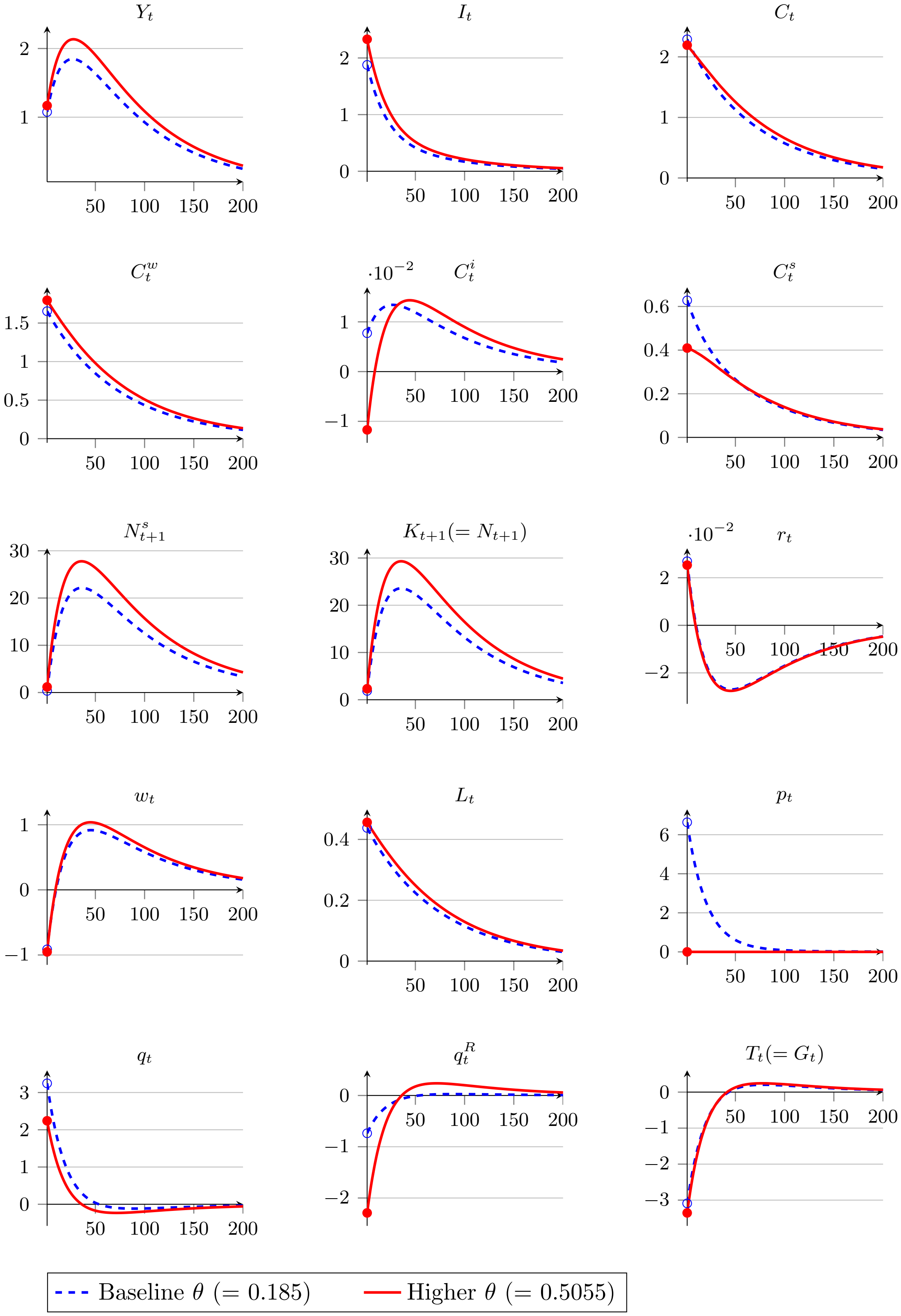
Notes: The τ^{rn}-shock is a 2.8 percentage point cut in τ_t^{rn}. The tax-shock responses are those from Figure 5.2, from which the same notes apply. Hollow blue dots and solid red dots indicate immediate responses of the tax-shock and the τ^{rn}-shock, respectively; see the "quarter 1" column of Panel A in Table 5.1 and Table 5.3, respectively, for their values.

Figure 5.5: Impulse responses to a liquidity shock and tax policy



Notes: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. Tax policy is a 0.1 percentage point cut in τ_t^{rl}. The notes in Figure 5.2 apply. Hollow blue dots and solid red dots indicate immediate responses of the liquidity shock without and with tax policy, respectively; see the "quarter 1" column of Panel A in Table 5.5 and Table 5.6, respectively, for their values.

FIGURE 5.6: Impulse responses to a tax-shock: relaxed θ



Notes: The baseline scenario is from Figure 5.2, from which the same notes apply. Hollow blue dots and solid red dots indicate immediate responses of the baseline and higher θ scenarios, respectively; see the "quarter 1" column of Panel A in Table 5.1 and Table 5.7, respectively, for their values.

TABLE 5.1: Responses to a tax-shock: baseline scenario

	PANEL A: Impulse responses in levels							PANEL B: Dynamic impact multipliers							PANEL C	
	Quarter:						**largest**	**Quarter:**						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**1**	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	1.08	1.15	1.29	1.50	1.81	0.25	1.85	−0.32	−0.35	−0.39	−0.45	−0.54	−0.07	−0.55	27	201
I_t	1.88	1.81	1.67	1.44	0.95	0.04	1.88	−0.56	−0.54	−0.50	−0.43	−0.28	−0.01	−0.56	1	94
C_t	2.29	2.25	2.19	2.05	1.72	0.15	2.29	−0.69	−0.68	−0.65	−0.62	−0.52	−0.05	−0.69	1	169
C_t^w	1.65	1.63	1.59	1.50	1.27	0.11	1.65	−0.50	−0.49	−0.48	−0.45	−0.38	−0.03	−0.50	1	173
C_t^i	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	27	169
C_t^s	0.63	0.61	0.59	0.54	0.43	0.03	0.63	−0.19	−0.18	−0.18	−0.16	−0.13	−0.01	−0.19	1	156
N_{t+1}^s	0.35	2.09	5.25	10.42	19.25	3.38	22.16	−0.12	−0.73	−1.83	−3.64	−6.72	−1.18	−7.74	36	201
$K_{t+1}(=N_{t+1})$	1.88	3.64	6.82	12.03	20.83	3.56	23.58	−0.66	−1.27	−2.38	−4.20	−7.27	−1.24	−8.23	35	201
r_t	0.03	0.02	0.02	0.00	−0.02	0.00	−0.03	−0.01	−0.01	0.00	0.00	0.00	0.00	0.01	46	201
w_t	−0.91	−0.79	−0.55	−0.16	0.55	0.16	0.92	0.27	0.24	0.17	0.05	−0.16	−0.05	−0.27	45	201
L_t	0.44	0.43	0.42	0.40	0.34	0.03	0.44	−0.32	−0.32	−0.31	−0.29	−0.25	−0.02	−0.32	1	173
p_t	6.63	6.31	5.71	4.67	2.57	0.01	6.63	−1.99	−1.89	−1.71	−1.40	−0.77	0.00	−1.99	1	49
q_t	3.25	3.07	2.73	2.16	1.02	−0.04	3.25	−0.97	−0.92	−0.82	−0.65	−0.31	0.01	−0.97	1	36
q_t^R	−0.74	−0.70	−0.62	−0.49	−0.23	0.01	−0.74	0.22	0.21	0.19	0.15	0.07	0.00	0.22	1	36
$T_t(=G_t)$	−3.09	−2.91	−2.57	−1.99	−0.86	0.05	−3.09	0.93	0.87	0.77	0.60	0.26	−0.02	0.93	1	31

NOTES: Panel A gives deviations from steady state in levels. Panel B gives dynamic impact multipliers which are computed according to the methodology outlined in Section 4.4.1. Panel C gives the time at which the absolute largest impulse response occurs, and the convergence indicator which is described in Section 4.4.2.

TABLE 5.2: Responses to a τ^{wl}-shock

	PANEL A: Impulse responses in levels							PANEL B: Dynamic impact multipliers							PANEL C	
	Quarter:						**largest**	**Quarter:**						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**1**	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	1.08	1.04	0.97	0.85	0.60	0.03	1.08	−0.93	−0.90	−0.84	−0.74	−0.51	−0.03	−0.93	1	115
I_t	0.26	0.25	0.23	0.20	0.13	0.01	0.26	−0.22	−0.21	−0.20	−0.17	−0.11	0.00	−0.22	1	93
C_t	1.74	1.67	1.52	1.28	0.77	0.02	1.74	−1.50	−1.44	−1.31	−1.10	−0.67	−0.02	−1.50	1	66
C_t^w	1.66	1.58	1.44	1.20	0.71	0.02	1.66	−1.43	−1.36	−1.24	−1.04	−0.61	−0.01	−1.43	1	61
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	27	66
C_t^s	0.09	0.08	0.08	0.07	0.06	0.00	0.09	−0.07	−0.07	−0.07	−0.06	−0.05	0.00	−0.07	1	156
N_{t+1}^s	0.05	0.28	0.71	1.42	2.62	0.46	3.01	−0.05	−0.29	−0.72	−1.42	−2.63	−0.46	−3.02	36	201
$K_{t+1}(= N_{t+1})$	0.26	0.49	0.93	1.64	2.83	0.48	3.20	−0.26	−0.50	−0.93	−1.64	−2.84	−0.49	−3.22	35	201
r_t	0.03	0.03	0.02	0.02	0.01	0.00	0.03	−0.02	−0.02	−0.02	−0.01	−0.01	0.00	−0.02	1	29
w_t	−0.91	−0.86	−0.75	−0.57	−0.22	0.02	−0.91	0.79	0.74	0.65	0.49	0.19	−0.02	0.79	1	29
L_t	0.44	0.42	0.38	0.32	0.19	0.00	0.44	−0.93	−0.89	−0.81	−0.67	−0.40	−0.01	−0.93	1	61
p_t	0.90	0.86	0.78	0.63	0.35	0.00	0.90	−0.78	−0.74	−0.67	−0.55	−0.30	0.00	−0.78	1	49
q_t	0.44	0.42	0.37	0.29	0.14	0.00	0.44	−0.38	−0.36	−0.32	−0.25	−0.12	0.00	−0.38	1	36
q_t^R	−0.10	−0.09	−0.08	−0.07	−0.03	0.00	−0.10	0.09	0.08	0.07	0.06	0.03	0.00	0.09	1	36
$T_t(= G_t)$	−0.92	−0.87	−0.78	−0.62	−0.31	0.01	−0.92	0.79	0.75	0.67	0.53	0.26	−0.01	0.79	1	38

NOTES: The τ^{wl}-shock represents a 1 percentage point cut in τ_t^{wl}. The notes in Table 5.1 apply.

TABLE 5.3: Responses to a τ^{rn}-shock

	Panel A: Impulse responses in levels							Panel B: Dynamic impact multipliers							Panel C	
	Quarter:							**Quarter:**							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**1**	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	0.00	0.11	0.31	0.65	1.21	0.21	1.40	0.00	−0.05	−0.14	−0.30	−0.56	−0.10	−0.65	36	201
I_t	1.62	1.56	1.45	1.24	0.82	0.04	1.62	−0.75	−0.72	−0.66	−0.57	−0.38	−0.02	−0.75	1	94
C_t	0.55	0.59	0.66	0.78	0.95	0.13	0.97	−0.25	−0.27	−0.30	−0.36	−0.43	−0.06	−0.44	27	201
C_t^w	0.00	0.05	0.15	0.30	0.56	0.10	0.65	0.00	−0.02	−0.07	−0.14	−0.26	−0.05	−0.30	36	201
C_t^i	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	−0.01	0.00	−0.01	27	201
C_t^s	0.54	0.53	0.51	0.47	0.37	0.03	0.54	−0.25	−0.24	−0.23	−0.22	−0.17	−0.01	−0.25	1	156
N_{t+1}^s	0.30	1.81	4.54	9.02	16.64	2.92	19.16	−0.16	−0.97	−2.43	−4.83	−8.91	−1.56	−10.25	36	201
$K_{t+1}(=N_{t+1})$	1.62	3.15	5.90	10.40	18.01	3.08	20.38	−0.87	−1.68	−3.16	−5.57	−9.64	−1.65	−10.91	35	201
r_t	0.00	0.00	−0.01	−0.01	−0.02	0.00	−0.03	0.00	0.00	0.00	0.01	0.01	0.00	0.01	37	201
w_t	0.00	0.07	0.20	0.41	0.77	0.14	0.89	0.00	−0.03	−0.09	−0.19	−0.36	−0.06	−0.41	37	201
L_t	0.00	0.01	0.04	0.08	0.15	0.03	0.17	0.00	−0.02	−0.04	−0.09	−0.17	−0.03	−0.19	37	201
p_t	5.74	5.46	4.93	4.04	2.22	0.01	5.74	−2.64	−2.51	−2.27	−1.85	−1.02	0.00	−2.64	1	49
q_t	2.81	2.65	2.36	1.87	0.88	−0.03	2.81	−1.29	−1.22	−1.08	−0.86	−0.40	0.01	−1.29	1	36
q_t^R	−0.64	−0.60	−0.54	−0.42	−0.20	0.01	−0.64	0.29	0.28	0.25	0.19	0.09	0.00	0.29	1	36
$T_t(=G_t)$	−2.17	−2.04	−1.79	−1.37	−0.55	0.05	−2.17	1.00	0.94	0.82	0.63	0.25	−0.02	1.00	1	29

NOTES: The τ^{rn}-shock represents a 2.8 percentage point cut in τ_t^{rn}. The notes in Table 5.1 apply.

TABLE 5.4: Impulse responses to a tax-shock, τ^{wl}-shock, and τ^{rn}-shock: a comparison

	Quarter 1					**Largest**		
	in levels			**in % of tax-shock**				
Shock:	**tax**	τ^{wl}	τ^{rn}	τ^{wl}	τ^{rn}	**tax**	τ^{wl}	τ^{rn}
Y_t	1.08	1.08	0.00	100	0	1.85	1.08	1.40
I_t	1.88	0.26	1.62	14	86	1.88	0.26	1.62
C_t	2.29	1.74	0.55	76	24	2.29	1.74	0.97
C_t^w	1.65	1.66	0.00	100	0	1.65	1.66	0.65
C_t^i	0.01	0.00	0.01	14	86	0.01	0.00	0.01
C_t^s	0.63	0.09	0.54	14	86	0.63	0.09	0.54
N_{t+1}^s	0.35	0.05	0.30	14	86	22.16	3.01	19.16
$K_{t+1}(= N_{t+1})$	1.88	0.26	1.62	14	86	23.58	3.20	20.38
r_t	0.03	0.03	0.00	100	0	−0.03	0.03	−0.03
w_t	−0.91	−0.91	0.00	100	0	0.92	−0.91	0.89
L_t	0.44	0.44	0.00	100	0	0.44	0.44	0.17
p_t	6.63	0.90	5.74	14	86	6.63	0.90	5.74
q_t	3.25	0.44	2.81	14	86	3.25	0.44	2.81
q_t^R	−0.74	−0.10	−0.64	14	86	−0.74	−0.10	−0.64
$T_t(= G_t)$	−3.09	−0.92	−2.17	30	70	−3.09	−0.92	−2.17

NOTES: See Tables 5.1 to 5.3 for the full sets of responses, from which the same notes apply.

Table 5.5: Responses to a liquidity shock

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	−0.02	−0.05	−0.11	−0.21	−0.04	−0.24	36	201
I_t	−0.28	−0.27	−0.25	−0.21	−0.14	−0.01	−0.28	1	93
C_t	0.28	0.25	0.21	0.13	−0.02	−0.02	0.28	1	184
C_t^w	0.00	−0.01	−0.02	−0.05	−0.10	−0.02	−0.11	36	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	184
C_t^s	0.28	0.26	0.23	0.18	0.08	−0.01	0.28	1	31
N_{t+1}^s	−0.83	−1.05	−1.45	−2.10	−3.15	−0.50	−3.43	33	201
$K_{t+1}(= N_{t+1})$	−0.28	−0.54	−1.01	−1.79	−3.09	−0.53	−3.50	35	201
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	36	201
w_t	0.00	−0.01	−0.03	−0.07	−0.13	−0.02	−0.15	36	201
L_t	0.00	0.00	−0.01	−0.01	−0.03	0.00	−0.03	36	201
p_t	4.09	3.88	3.50	2.85	1.54	0.00	4.09	1	46
q_t	1.35	1.29	1.17	0.96	0.54	0.01	1.35	1	52
q_t^R	−0.31	−0.29	−0.26	−0.22	−0.12	0.00	−0.31	1	52
$T_t(= G_t)$	0.00	0.00	−0.01	−0.02	−0.05	−0.01	−0.05	36	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. These responses are without tax policy. The notes in Table 5.1 apply.

Table 5.6: Responses to a liquidity shock with tax policy

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	−0.02	−0.04	−0.09	−0.17	−0.03	−0.19	36	201
I_t	−0.22	−0.21	−0.20	−0.17	−0.11	0.00	−0.22	1	93
C_t	0.30	0.27	0.23	0.16	0.01	−0.02	0.30	1	161
C_t^w	0.00	−0.01	−0.02	−0.04	−0.08	−0.01	−0.09	36	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	161
C_t^s	0.30	0.28	0.25	0.19	0.09	0.00	0.30	1	34
N_{t+1}^s	−0.82	−0.99	−1.29	−1.77	−2.56	−0.40	−2.75	32	201
$K_{t+1}(= N_{t+1})$	−0.22	−0.43	−0.80	−1.42	−2.45	−0.42	−2.78	35	201
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	36	201
w_t	0.00	−0.01	−0.03	−0.06	−0.11	−0.02	−0.12	36	201
L_t	0.00	0.00	−0.01	−0.01	−0.02	0.00	−0.02	36	201
p_t	4.29	4.08	3.68	2.99	1.61	0.00	4.29	1	46
q_t	1.45	1.38	1.25	1.03	0.57	0.00	1.45	1	50
q_t^R	−0.33	−0.31	−0.28	−0.23	−0.13	0.00	−0.33	1	50
$T_t(= G_t)$	−0.08	−0.08	−0.08	−0.07	−0.07	−0.01	−0.08	1	187

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. Tax policy is a 0.1 percentage point cut in τ_t^{rl}. The notes in Table 5.1 apply.

TABLE 5.7: Responses to a tax-shock: relaxed θ

	PANEL A: Impulse responses in levels							PANEL B: Dynamic impact multipliers							PANEL C	
	Quarter:							**Quarter:**							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**1**	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	1.17	1.26	1.43	1.69	2.08	0.29	2.13	−0.32	−0.35	−0.39	−0.47	−0.58	−0.08	−0.59	28	201
I_t	2.33	2.24	2.08	1.79	1.18	0.05	2.33	−0.64	−0.62	−0.57	−0.49	−0.33	−0.01	−0.64	1	94
C_t	2.19	2.17	2.13	2.05	1.81	0.18	2.19	−0.61	−0.60	−0.59	−0.57	−0.50	−0.05	−0.61	1	183
C_t^w	1.80	1.78	1.74	1.66	1.44	0.14	1.80	−0.50	−0.49	−0.48	−0.46	−0.40	−0.04	−0.50	1	179
C_t^i	−0.01	−0.01	−0.01	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	44	183
C_t^s	0.41	0.41	0.40	0.40	0.36	0.04	0.41	−0.11	−0.11	−0.11	−0.11	−0.10	−0.01	−0.11	1	193
N_{t+1}^s	1.18	3.31	7.18	13.51	24.28	4.27	27.78	−0.36	−1.00	−2.17	−4.08	−7.33	−1.29	−8.39	36	201
$K_{t+1}(=N_{t+1})$	2.33	4.51	8.47	14.93	25.88	4.48	29.30	−0.70	−1.36	−2.56	−4.51	−7.81	−1.35	−8.85	36	201
r_t	0.03	0.02	0.01	0.00	−0.02	0.00	−0.03	−0.01	−0.01	0.00	0.00	0.00	0.00	0.01	45	201
w_t	−0.95	−0.81	−0.56	−0.13	0.64	0.18	1.04	0.26	0.22	0.15	0.04	−0.18	−0.05	−0.29	45	201
L_t	0.46	0.45	0.44	0.42	0.36	0.03	0.46	−0.32	−0.32	−0.31	−0.30	−0.26	−0.02	−0.32	1	179
p_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	201
q_t	2.24	2.10	1.84	1.39	0.52	−0.06	2.24	−0.62	−0.58	−0.51	−0.38	−0.14	0.02	−0.62	1	83
q_t^R	−2.29	−2.15	−1.88	−1.42	−0.53	0.06	−2.29	0.63	0.59	0.52	0.39	0.15	−0.02	0.63	1	83
$T_t(=G_t)$	−3.36	−3.15	−2.78	−2.15	−0.90	0.06	−3.36	0.93	0.87	0.77	0.59	0.25	−0.02	0.93	1	31

NOTES: This table give the results of the tax-shock with a higher calibration for θ, viz. 0.5055, which represents relaxing the borrowing constraint relative to its baseline setting. ϕ is maintained at baseline. Table 5.1 gives the baseline scenario (with $\theta = 0.185$) from which the same notes apply.

Chapter 6

An increase in government spending

6.1 Introduction

This chapter simulates an unexpected temporary increase in government spending. The simulation is performed repeatedly in adaptations of the model in which the government finances its spending in different ways. Three methods of financing are studied – varying tax rates, selling stocks of entrepreneur-issued equity, and issuing money (henceforth, "T-financing", "N^g-financing", and "M-financing", respectively). With each variant of the model, that is, with each financing arrangement, the increase in government spending is simulated on its own, to represent a discretionary stimulus programme in normal times (henceforth, a "G-shock"), and then contemporaneously alongside a temporary negative liquidity shock, to represent policy intervention (henceforth, "G-policy"). The objectives of this chapter are to examine (i) the macroeconomic effects of a G-shock, and (ii) the effectiveness G-policy against a liquidity shock, both under alternative financing arrangements.

The chapter achieves its objectives by first defining Ω in different ways. The various definitions create alternative endogenous mechanisms by which government spending is financed, and thus gives rise to the variants of the model within the chapter. For ease of reference, each model variant is described as a modification the model in Chapter 5 (henceforth, the "benchmark model"). The variants in this chapter and Chapter 5 differ only by the behaviour of the government. Within each variant, there are simulations of a G-shock, a liquidity shock, and a liquidity shock with contemporaneous G-policy. The G-shock is normalised to be an increase in government spending, G_t, by 1 unit of general output. The liquidity shock is normalised to be a 1 percentage point decrease (that is, tightening) in the re-saleability constraint, ϕ_t. And G-policy is an increase in government spending by 0.1 units of general output. The size of the G-policy is chosen to partially offset the liquidity shock, instead of overturn and replace it, and then the dynamic benefits of policy can be observed as the economy adjusts to its pre-shock steady state. All shocks are one-period events, and from the second quarter they are gradually phased out with a rate of decay of 5% each period. The model is calibrated throughout the chapter with structural parameters set to baseline values (which are summarised in Table 4.1).

The chapter is motivated by Chapter 5 and Kara and Sin (2014) (henceforth, KS). Chapter 5 is the first step towards developing fiscal policy in the basic KM model, and it focuses on taxation policy. This chapter expands that research agenda. The thesis is not the first study of fiscal policy in the KM-related literature, but it is the first to do so within the branch of the literature that modifies the KM model. KS is the first quantitative and dedicated study of fiscal policy in the KM-related literature. They modify the model of Del Negro et al. (2011) and study a bond-financed increase in government spending, both on its own and against a liquidity shock. Their bonds take on the same role that money plays in this model. KS is therefore the New Keynesian equivalent of this chapter's M-financing experiment. This chapter complements and expands the work of KS by, respectively, studying the same issues from the perspective of an alternative (neoclassical) framework, and proposing alternative methods to finance government spending (that is, by taxes and selling equity).

In this chapter, M-financing the G-shock and G-policy does not allow the model to re-converge to equilibrium. However, the success of KS in simulating the same experiment suggests that sticky prices may solve the problem. The T-financed G-shock causes consumption and investment to fall, and (contrary to KS) causes employment and output to fall, and all these response are persistent. The N^g-financed G-shock has similar impacts, although employment and output fall from the second period and all the responses are oscillating and highly persistent. A liquidity shock in the T-financing variant causes investment, employment, and output to fall, but (contrary to KS) causes consumption to rise. In the N^g-financing variant, the liquidity shock immediately reduces consumption and investment, and reduces employment and output after a one-period delay. Finally (and contrary to KS),

G-policy exacerbates the liquidity shock in the chapter's successful simulations. Government spending does not enter the utility of private agents, and more of it therefore has no direct impact on private spending and output. Financing this spending, whether by taxes or equity, is what brings these adverse consequences to G-shocks and G-policies. The results of this chapter support the use of balanced budget tax cuts in this model for both economic stimulus in normal times and counteracting liquidity shocks.

The rest of the chapter is organised as follows. Each of the next three sections simulates a G-shock, a liquidity shock, and a liquidity shock with contemporaneous G-policy. The government finances its increase in spending by varying tax rates in Section 6.2, by selling its stocks of entrepreneur-issued equity in Section 6.3, and by issuing money in Section 6.4. Section 6.5 summarises the chapter and relates the results with KS. Figures and tables appear at the end of the chapter.

6.2 Financing with taxes

In this section, the government finances its increase in spending by endogenously varying tax rates to raise more tax revenue, while its equity holdings and the money supply are exogenously determined and unchanged. The benchmark model is modified by (i) including the AR(1) process (3.29) for G_t, (ii) excluding the AR(1) processes (3.32) and (3.33) for τ_t^{rn} and τ_t^{wl}, respectively, and (iii) including rule (3.34) which holds the ratio of the rate of tax on wages to aggregate taxes constant to its steady state level. Accordingly, the set of exogenously determined policy variables is $\Omega_t = \{G_t, N_{t+1}^g, M_{t+1}\}$. The government's behaviour is summarised by the following equilibrium conditions.

$$T_t = \tau_t^{rn} r_t N_t + \tau_t^{wl} w_t L_t \tag{3.27}$$

$$G_t + q_t(N_{t+1}^g - \delta N_t^g) = T_t + r_t N_t^g + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u_t^G \tag{3.29}$$

$$N_{t+1}^g = (1 - \rho_{Ng})N^g + \rho_{Ng} N_t^g + u_t^{Ng} \tag{3.30}$$

$$M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M \tag{3.31}$$

$$\frac{\tau_t^{wl}}{T_t} = \frac{\tau^{wl}}{T} \tag{3.34}$$

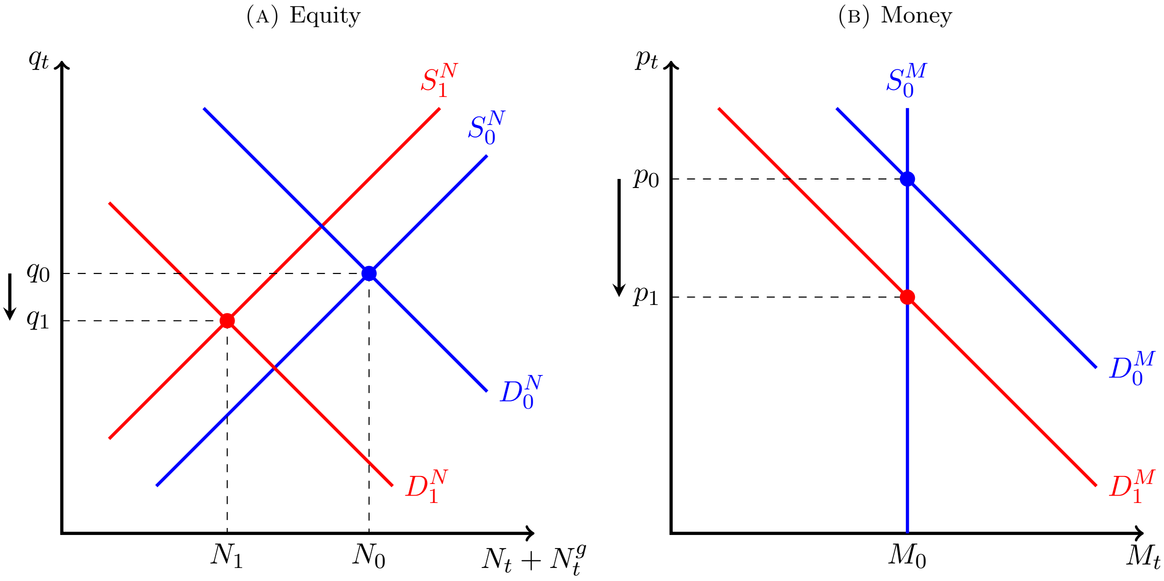
The rest of the model remains the same as in the benchmark model and is summarised in Section 5.2. Assumption 4 is made, and the government therefore balances its fiscal budget ($T_t - G_t$) in steady state and in every period. This section therefore presents the converse of the tax-shock in Chapter 5. However, the G-policy against a liquidity shock involves both tax rates adjusting, and is therefore not the converse of tax policy in Chapter 5 in which only τ_t^{rn} is cut. Impulse responses to the T-financed G-shock are illustrated in Figure 6.3 and are given numerically in Table 6.1. Impulse responses to the liquidity shock without and with T-financed G-policy, respectively, are illustrated in Figure 6.4 and are given numerically in Tables 6.2 and 6.3.

6.2.1 A T-financed G-shock

The G-shock requires the government to increase tax revenues by 1 unit of general output in order to balance its fiscal budget (Equation (3.28)). This requirement is first achieved by raising the tax rate on wages. Workers respond by decreasing their labour supply at each and every wage, and the labour market adjusts to a higher wage rate and lower employment. Since labour demand is wage-elastic (see Appendix 3.H) then workers earn a smaller aggregate gross wage. Being non-Ricardian, they reduce their consumption. The erosion of the tax base from wages is small, and does not overturn the effects of a higher tax rate. The capital stock used in current production is determined before the G-shock hits, and so the fall in employment achieves a reduction in output.

Rule (3.34) together with the fall in gross wages implies an increase in the tax rate on dividends. As a direct result, entrepreneurs' net worth deteriorates, which then causes reductions in private consumption, saving, and investment. These changes have impacts on the asset markets, which Figure 6.1 illustrates. The fall in saving reduces asset demands while the fall in investment reduces equity's supply. The tightly binding borrowing constraint maintains that changes in equity's supply are a small fraction of changes in investment, whether these changes are positive or negative. The equity market ends up with a lower price, which, along with a lower price for money, further reduces entrepreneurs' net worth. Thus begins a financial accelerator feedback loop within the first quarter – net worth deteriorations lead to declines in private spending and saving, then asset prices fall and entrepreneurs' net worth to deteriorate, and so on. With each loop the effects get smaller, since part of net worth deteriorations are passed on as reductions in consumption, which

Figure 6.1: Asset markets and the immediate effects of a T-financed G-shock



NOTES: Money's demand comes from entrepreneurs and its supply is fully controlled by the government. Equity's demand comes from savers and its supply comes from investors. A fall in saving is represented by leftwards shifts in the demands for money and equity, from D_0^M and D_0^N to D_1^M and D_1^N, respectively. A fall in investment is represented by a small leftward shift in equity's supply from S_0^N to S_1^N. Money's price falls from p_0 to p_1, and equity's price falls from q_0 to q_1.

has no direct impact on asset prices.

As the shock wares off, government spending falls asymptotically towards steady state. Consequently, aggregate taxes and tax rates gradually decline and the responses seen in the first quarter are reversed in the second quarter, many of them almost completely so. Output converges very slowly towards steady state and exhibits a gentle hump-shaped trajectory. This happens because employment and capital move in opposite directions for some time. The gradual decline of the tax rate on wages makes employment rise asymptotically. Capital initially falls because investment remains below steady state, and it eventually rises when its depreciation is small enough to be covered by investment.

6.2.2 A liquidity shock and T-financed G-policy

Without policy

A negative liquidity shock produces the same immediate responses as in the baseline model. The liquidity shock has no impact on the labour market, and therefore on output, the rate of dividends, tax rates, and government tax revenue. The shock has an adverse effect on investment by making it more difficult to internally finance. This decreases equity's supply and thereby raises the price of the asset. Money's demand increases, via a portfolio balance effect, which raises its price. Entrepreneurs' net worth then sufficiently improves, and they increase their consumption rather than investment and saving, given the greater difficulty in financing investment and a loss in appeal for equity as a saving instrument.

The effects of the liquidity shock are highly persistent. All variables except asset prices return close to their steady states some time after 200 quarters; asset prices do so after 47 quarters. From the second quarter, the effects of lower investment start to be felt economy-wide. Labour's demand decreases, the capital stock suffers a net depreciation, and output declines. The tax rate on dividends endogenously rises to compensate for the declining equity stock, and this puts further downward pressure on entrepreneurs' net worth.

With policy

The negative liquidity shock is exacerbated by T-financed G-policy intervention. While the liquidity shock does not interfere with the labour market, the financing of G-policy increases the rate of tax on wages and decreases labour's supply at each and every wage. Employment falls, the wage rate rises, aggregate gross wages fall, and workers reduce their consumption. The liquidity shock on its own reduces entrepreneurs' net worth. Financing the G-policy also requires a hike in the dividend tax rate. This brings further reductions in net worth. Consequently, entrepreneurs' consumption, investment, and saving are all lower with G-policy than without. On a somewhat positive note, for variables that are affected by the liquidity shock, G-policy does not alter the shape of their trajectories.

6.3 Financing with equity

In this section, the government finances its increase in spending by selling some of its equity holdings, while the money supply and tax rates are exogenously determined and unchanged. The benchmark model is modified by (i) including the AR(1) process (3.29) for G_t, and (ii) excluding the AR(1) process (3.30) for N^g_{t+1}. Accordingly, the set of exogenously determined policy variables is $\Omega_t = \{G_t, M_{t+1}, \tau^{rn}_t, \tau^{wl}_t\}$. The government's behaviour is summarised by the following equilibrium conditions.

$$T_t = \tau^{rn}_t r_t N_t + \tau^{wl}_t w_t L_t \tag{3.27}$$

$$G_t + q_t(N^g_{t+1} - \delta N^g_t) = T_t + r_t N^g_t + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u_t^G \tag{3.29}$$

$$M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M \tag{3.31}$$

$$\tau_t^{rn} = (1 - \rho_{\tau rn})\tau^{rn} + \rho_{\tau rn}\tau_{t-1}^{rn} + u_t^{\tau rn} \tag{3.32}$$

$$\tau_t^{wl} = (1 - \rho_{\tau wl})\tau^{wl} + \rho_{\tau wl}\tau_{t-1}^{wl} + u_t^{\tau wl} \tag{3.33}$$

The rest of the model remains the same as in the benchmark model and is summarised in Section 5.2. N_{t+1}^g is endogenously determined, so Assumption 4 is not made. In steady state, the fiscal balance is in deficit, but the government holds a large quantity of equity (16.8 units) which earns dividends that amount to the deficit and thus balances Equation (3.28). The value of the re-saleable portion of government equity holdings is greater than the size of the G-shock, so Assumption 3 is not made.

Impulse responses to the N^g-financed G-shock are illustrated in Figures 6.5 and 6.6 and are given numerically in Table 6.4. Impulse responses to the liquidity shock without and with N^g-financed G-policy, respectively, are illustrated in Figure 6.7 and are given numerically in Figure 6.7 and Table 6.5.

6.3.1 An N^g-financed G-shock

The labour market is immediately unaffected by the G-shock. Consequently, there are no immediate changes in output, workers' consumption, the rate of dividends, and taxes. The shock therefore expands the fiscal deficit, which the government finances by selling part of its equity stock. Figure 6.2 illustrates the changes in the asset markets in the immediate period of the G-shock. The government's sale lowers the price of equity, and via a portfolio balance effect, the demand and price of money both decrease. These asset price declines are the source of the adverse effects of the N^g-financed G-shock. Entrepreneurs' net worth falls, and with it investment, saving, and private consumption. Then there are further downward pressures on asset prices. The fall in saving represents a decrease in demand for both assets. The fall in investment represents a decrease in equity's supply, which the borrowing constraint ensures is small and thus leaves the asset with a lower price.[1]

The capital stock decreases by the end of the first quarter after investment falls below steady state. Entrepreneurs respond by optimally lowering their demand for labour, and

[1] This hypothesis about the borrowing constraint is formalised and tested in Section 5.6, where it is called the "θ-equity hypothesis".

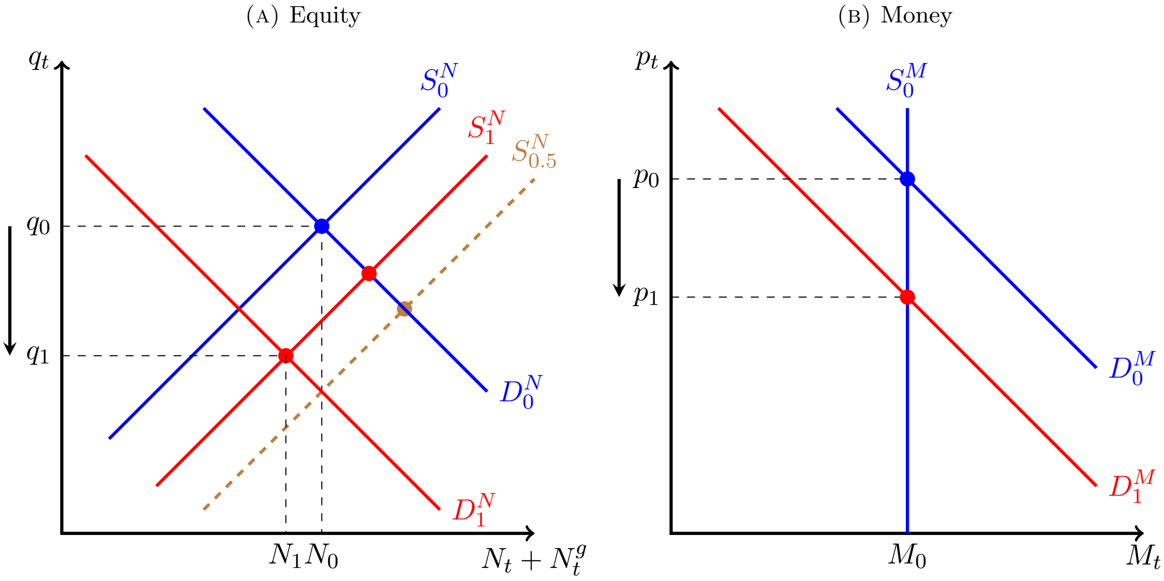
they produce a smaller quantity of general output. Workers earn lower aggregate gross wages, and therefore reduce their consumption. These changes are very small, and they generate a very small decline in government tax revenues, by 0.02 units of general output. Meanwhile, by assumption, government spending falls by 0.05 units in its asymptotic decline to steady state. The fiscal account records a small surplus, and given that tax rates and money supply are exogenously determined and fixed, the government uses the surplus to buy equity. The quantity of equity that the fiscal surplus can afford is not enough to replace depreciation, and so the government's equity stock falls at the end of the second quarter. But the government's purchase turns the asset markets' responses around – equity's supply decreases, its price rises, and money's demand and price both rise. Entrepreneurs enjoy net worth improvements, which lead to more investment, saving, and consumption. And the borrowing constraint ensures any investment-induced increase in equity's supply is small and thus preserves the price drop.

The second quarter's increase in investment is not sufficient to replace all of capital's depreciation for that period, and the capital stock falls again in the third quarter. The responses seen in the second quarter are repeated in the third and subsequent quarters, but as time passes they get smaller. In the 24th quarter the government's equity stock reaches a level such that purchases funded by fiscal surplus start to outweigh depreciation, and the stock then starts to grow. In the 44th quarter the aggregate capital stock starts to grow as depreciation is outweighed by investment; this is when the economy's responses turn around. The impact is first felt in the labour market – entrepreneurs increase their demand for labour and agree to a higher wage rate. Then, with more gross wages being paid out, taxes increase. In the meantime, government spending is asymptotically falling. The fiscal surplus, and hence government equity purchases, start to increase rapidly, and equity's price starts to fall. Eventually money's price falls, and the economy returns to the same type of trajectory seen just after the shock.

The economy oscillates like this for a very long time.[2] Figure 6.6 gives impulse responses over a very long time to illustrate these oscillations. The amplitude gets smaller with each cycle, and eventually the economy converges to steady state. Oscillations are fuelled by government fiscal imbalances which are used to buy equity and thus interfere with asset

[2]Oscillations happen because there are complex eigenvalues with negative real parts in the Blanchard and Kahn (1980) canonical form of the model (Barro and Sala-i-Martin (1995)).

Figure 6.2: Asset markets and the immediate effects of an N^g-financed G-shock



NOTES: Money's demand comes from entrepreneurs and its supply is fully controlled by the government. Equity's demand comes from savers and its supply comes from investors. The government's equity sale shifts supply rightwards from S_0^N to $S_{0.5}^N$. The portfolio balance effect shifts money's demand leftwards from D_0^M; net worth deteriorations take it further leftwards to D_1^M. Net worth deteriorations also produce leftwards shifts in equity's demand from D_0^N to D_1^N and equity's supply from $S_{0.5}^N$ to S_1^N; the borrowing constraint guarantees the size of the supply shift is small. At the end of the first quarter, equity's price falls from q_0 to q_1 and money's price falls from p_0 to p_1.

prices and financial acceleration in the model.[3]

6.3.2 A liquidity shock and N^g-financed G-policy

Without policy

As with the N^g-financed G-shock, the liquidity shock produces a highly persistent and oscillating adjustment to steady state. Attention here is given to immediate responses, and long-term oscillating impulse responses are not graphically illustrated. The liquidity shock has no immediate impact on the labour market, and therefore on output, workers' consumption, the rate of dividends, and taxes. The shock reduces investment by making financing more difficult. Equity's supply falls, and money's price increases by a portfolio balance effect. Accordingly, entrepreneurs' net worth improves.

So far, the effects of the liquidity shock in this variant of the model are the same as in the T-financed G-shock and the tax-shock. However, the government in this framework varies its equity holdings, and this causes the responses to be qualitatively different from

[3] Samuelson (1937) explains such oscillation by a multiplier-accelerator interaction in a simple Keynesian framework.

other liquidity shocks in the thesis so far. With higher asset prices, the government's asset portfolio increases in value, and it sells part of its equity stock to balance its overall budget constraint (Equation (3.28)). The sale is sufficient to leave the asset with a lower price, which causes money's price to fall and entrepreneurs' net worth to deteriorate. Nevertheless, entrepreneurs invest more; this is a residual effect from lowering consumption (from Equation (3.59)) that is, a small amount of net worth is left over and channelled into investment.

With policy

N^g-financed G-policy exacerbates the effects of the liquidity shock. The government contemporaneously increases its spending by 0.1 units of general output, which it finances by selling 0.07 units of equity. The equity sale is what produces the undesired consequences of G-policy. It adds to supply on the market and aggravates equity's price decline due to the liquidity shock. By a portfolio balance effect, money's price falls below that of the no-policy scenario. This is the source of divergence between the policy and no-policy scenarios. G-policy deepens entrepreneurs' net worth deteriorations and causes a small drop in investment, compared to a small increase with no intervention.

Besides investment, no other variable experiences a qualitative change with G-policy in the immediate period of the liquidity shock. However, G-policy changes the short-term direction in subsequent periods for output, workers' and aggregate consumption, capital, employment, the real wage, and the rate of dividends.

In the second period, the capital stock falls, leading to a short-term contraction in output (as opposed to both variables increasing without policy). Investment remains below steady state in the short-term while it recovers, and this leads to continued declines in the capital stock and output. Meanwhile, workers face falling demand for their labour, on account of the capital stock, and they experience lower levels of employment and wage rates. And because of workers, aggregate private consumption falls.

6.4 Financing with money

In this section, the government finances its increase in spending by issuing money, while its equity holdings and tax rates are exogenously determined and unchanged. The benchmark model is modified by (i) including the AR(1) process (3.29) for G_t, and (ii) excluding the AR(1) process (3.31) for M_{t+1}. Accordingly, the set of exogenously determined policy variables is $\Omega_t = \{G_t, N^g_{t+1}, \tau^{rn}_t, \tau^{wl}_t\}$. The government's behaviour is summarised by the following equilibrium conditions.

$$T_t = \tau^{rn}_t r_t N_t + \tau^{wl}_t w_t L_t \tag{3.27}$$

$$G_t + q_t(N^g_{t+1} - \delta N^g_t) = T_t + r_t N^g_t + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u^G_t \tag{3.29}$$

$$N^g_{t+1} = (1 - \rho_{Ng})N^g + \rho_{Ng} N^g_t + u^{Ng}_t \tag{3.30}$$

$$\tau^{rn}_t = (1 - \rho_{\tau rn})\tau^{rn} + \rho_{\tau rn}\tau^{rn}_{t-1} + u^{\tau rn}_t \tag{3.32}$$

$$\tau^{wl}_t = (1 - \rho_{\tau wl})\tau^{wl} + \rho_{\tau wl}\tau^{wl}_{t-1} + u^{\tau wl}_t \tag{3.33}$$

The rest of the model remains the same as in the benchmark model and is summarised in Section 5.2. Assumption 4 is made and the government balances its fiscal budget ($T_t - G_t$) in steady state.

Once disturbed by the M-financed G-shock, the model becomes unstable and does not re-converge to steady state. This result is robust to the calibration of structural parameters.[4]

With tax rates assumed constant, the G-shock does not affect the labour market (from Equations (3.53) and (3.54)). Consequently, output, dividends, and aggregate taxes are all unchanged (from Equations (3.27), (3.60) and (3.61)). Money is then the only policy variable that responds to the shock. General equilibrium cannot be achieved because Equation (3.28) says that the government values its assets *ex post* to the shock, and it implies a collinear relationship between money and its price,

$$M_2 - M = \left(\frac{1}{p_1}\right)$$

where M_2 is the stock of money at the end of period 1, and $1/p_1$ is the quantity of money the government must contemporaneously issue in order to consume 1 unit of general output. One possible solution is to enforce an *ex ante* valuation of assets; this can be achieved by either

[4]Attempts to simulate the M-financed G-shock fails with 311,040 combinations of parameter values from $\theta \in \{0.185, 0.11, 0.2\}$, $\beta \in \{0.99, 0.95, 0.975, 0.98, 0.995\}$, $\gamma \in \{0.4, 0.3, 0.35, 0.375, 0.425, 0.45, 0.475\}$, $\delta \in \{0.975, 0.925, 0.95, 0.98, 0.99\}$, $\pi \in \{0.05, 0.03, 0.04, 0.06, 0.07, 0.08\}$, $\nu \in \{1, 0.5, 0.9, 1.1, 1.5, 2, 3, 4\}$, and $\omega \in \{4.01, 3, 3.5, 4.5, 5, 6, 7, 8\}$. The lack of convergence also holds if terms that have third and higher order effects on the model's solution are excluded (using the pruning algorithm of Kim et al. (2008)) and if the model is approximated by a first order Taylor series expansion.

fixing or pre-determining prices in the government's budget constraint (Equation (3.28)). An alternative solution is to elsewhere determine a target for money supply, perhaps by rule (3.35) for money supply growth.

These solutions are not explored in this chapter. Chapter 7 has the same convergence problem, and there the solution of a monetary rule is examined. KS already show that fixing prices is s valid solution. Their work is the New Keynesian equivalent of this section, with their government-issued bonds taking on the role of money. A G-shock in KS causes consumption and investment to immediately fall, and employment and output to immediately rise. However, because asset prices are not available to transmit the shock into net worth improvements and thereby fuel financial acceleration, the KS model quickly returns to steady state. KS also simulate G-policy, and they find that it ameliorates the effects of a negative liquidity shock, which are immediate declines in consumption, investment, employment, and output.

6.5 Chapter summary

This chapter successfully simulates an exogenous increase in government spending in two variants of the model which differ by the way such spending is financed. In both variants, the spending increase is first simulated on its own, and then contemporaneously alongside an exogenous tightening of private liquidity.

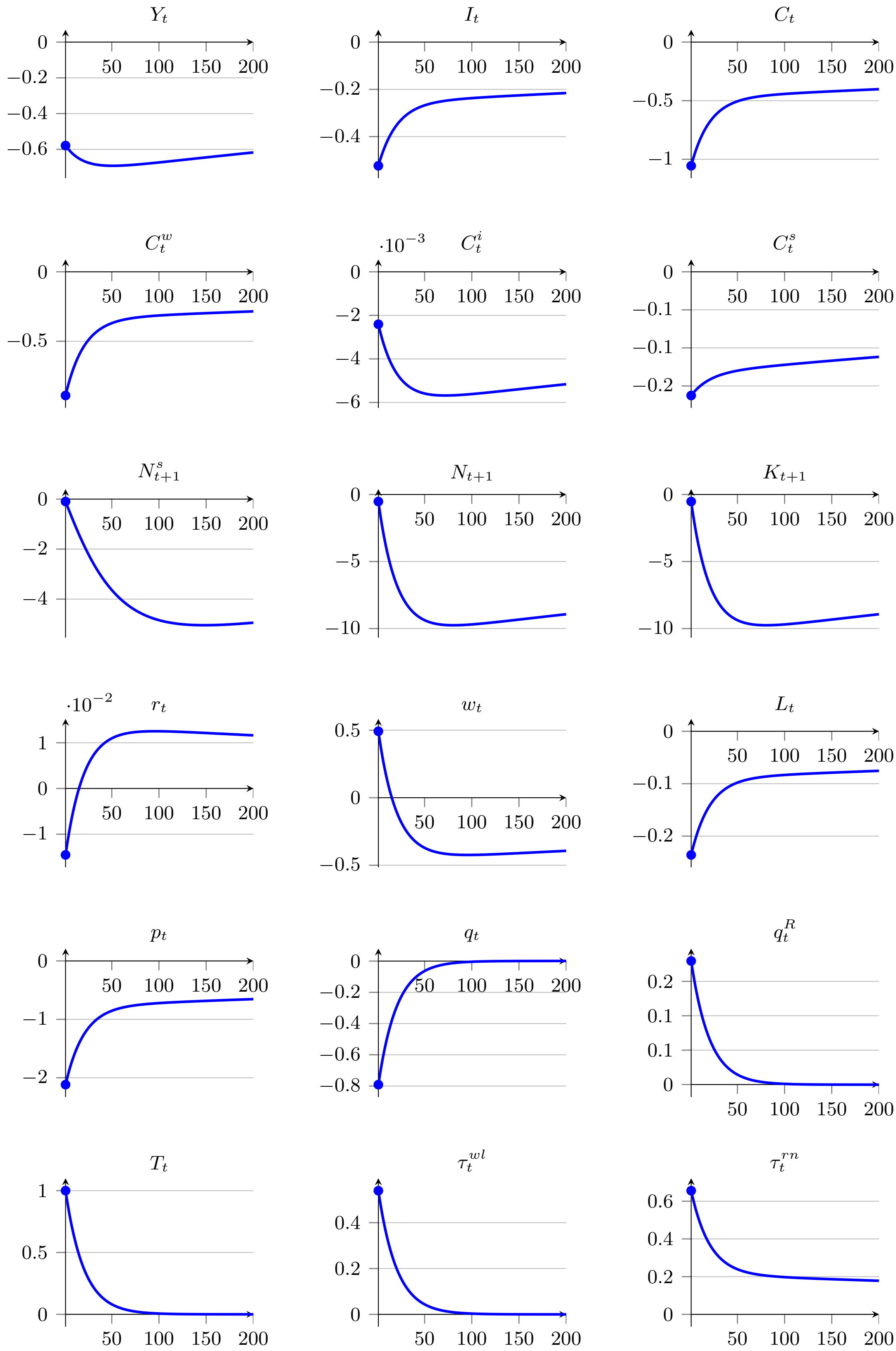
In the first model variant, the government raises tax rates to balance its fiscal budget. A G-shock lowers consumption, investment, employment, and output, and the effects are persistent. This experiment and its results are the converse of the tax-shock in Chapter 5. A liquidity shock causes consumption to increase, investment to fall, and employment and output to decline from the second period. G-policy exacerbates the liquidity shock – the increase in consumption is smaller, the fall in investment is greater, and employment and output now decline on impact. Since G-policy changes both tax rates then it is not the converse of tax policy in Chapter 5 which changes only τ_t^{rn}.

In the second model variant, the government sells part of its equity stock to finance the fiscal deficit. The G-shock causes consumption and investment to immediately fall, and employment and output to fall from the second period. But the economy oscillates over a very long period of time as it converges towards steady state. Oscillations are fuelled by fiscal imbalances that require the government to buy and sell equity and thus interfere with asset prices and the financial accelerator. A liquidity shock also produces oscillating responses; consumption falls and investment rises in the immediate period – which are different responses to liquidity shocks seen in the thesis so far – and employment and output rise from the second quarter. G-policy exacerbates the liquidity shock – the fall in consumption is greater, investment now declines, and employment and output now fall in the second period.

The chapter tries, but fails, to simulate the government financing its spending by monetary expansion. With sticky prices, Kara and Sin (2014) successfully perform such an experiment in a New Keynesian DSGE model. Their G-shock reduces consumption and investment and increases employment and output, and their G-policy ameliorates a liquidity shock. Their responses, however, are short-lived, because sticky prices stifle private net worth improvements which can lead to more investment, consumption, and future output. The results of KS and this chapter together suggest that asset price fluctuations are important for the transmission, amplification, and persistence of shocks.

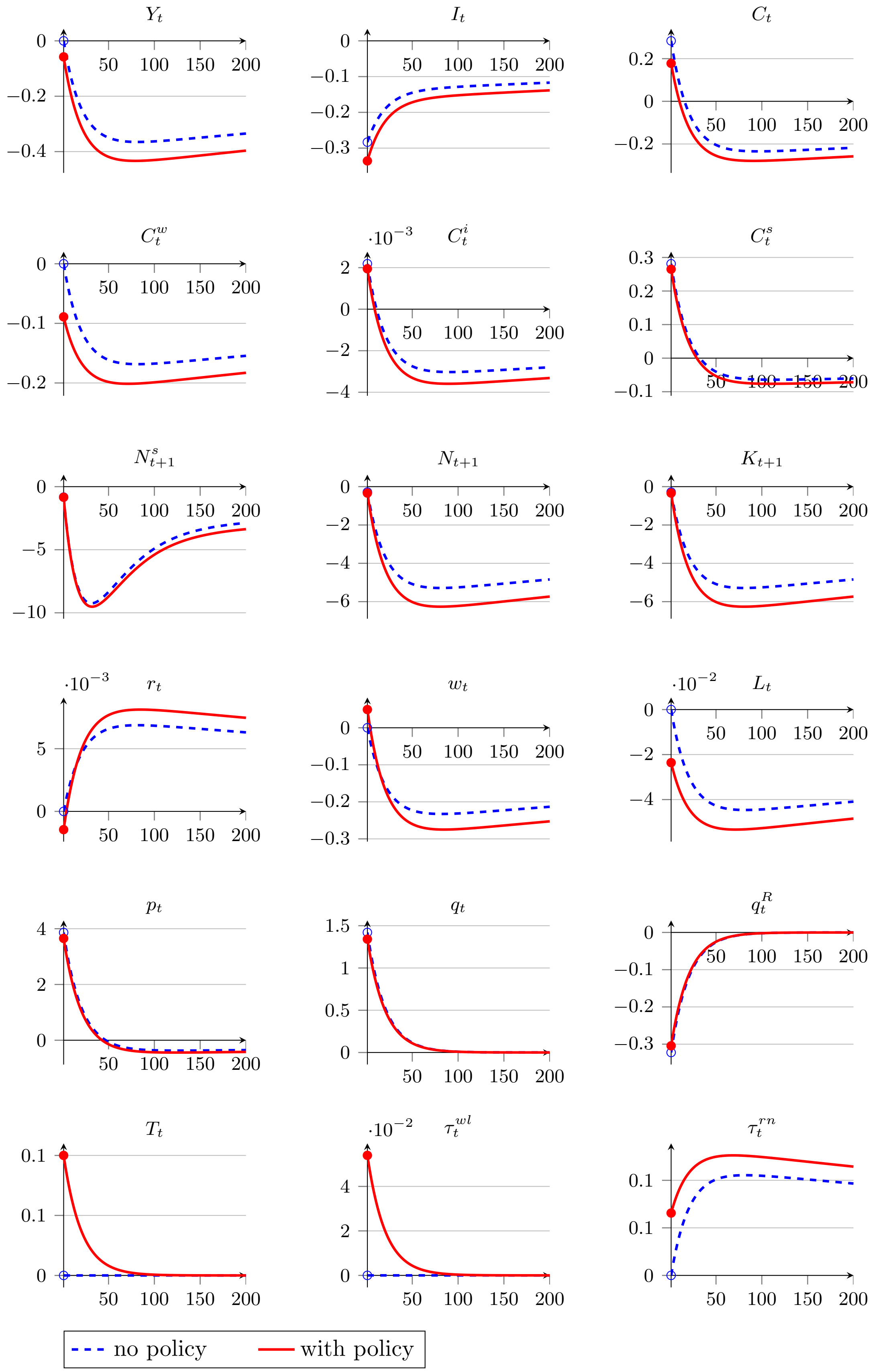
In summary, because an increase in government spending does not alter private agents utility, then the financing side of G-shocks and G-policies bring harmful effects to the model. The conclusion of the thesis evaluates this chapter and proposes worthwhile extensions for future research. Most notably, the conclusion recommends modifying the role of government spending so that it has direct impacts on private agents' behaviour. The next chapter takes a different approach to policy by studying a government purchase of equity, much like the unconventional types of monetary policies that were implemented (in the US and UK, for example) during the 2008 financial crisis.

FIGURE 6.3: Impulse responses to a T-financed G-shock



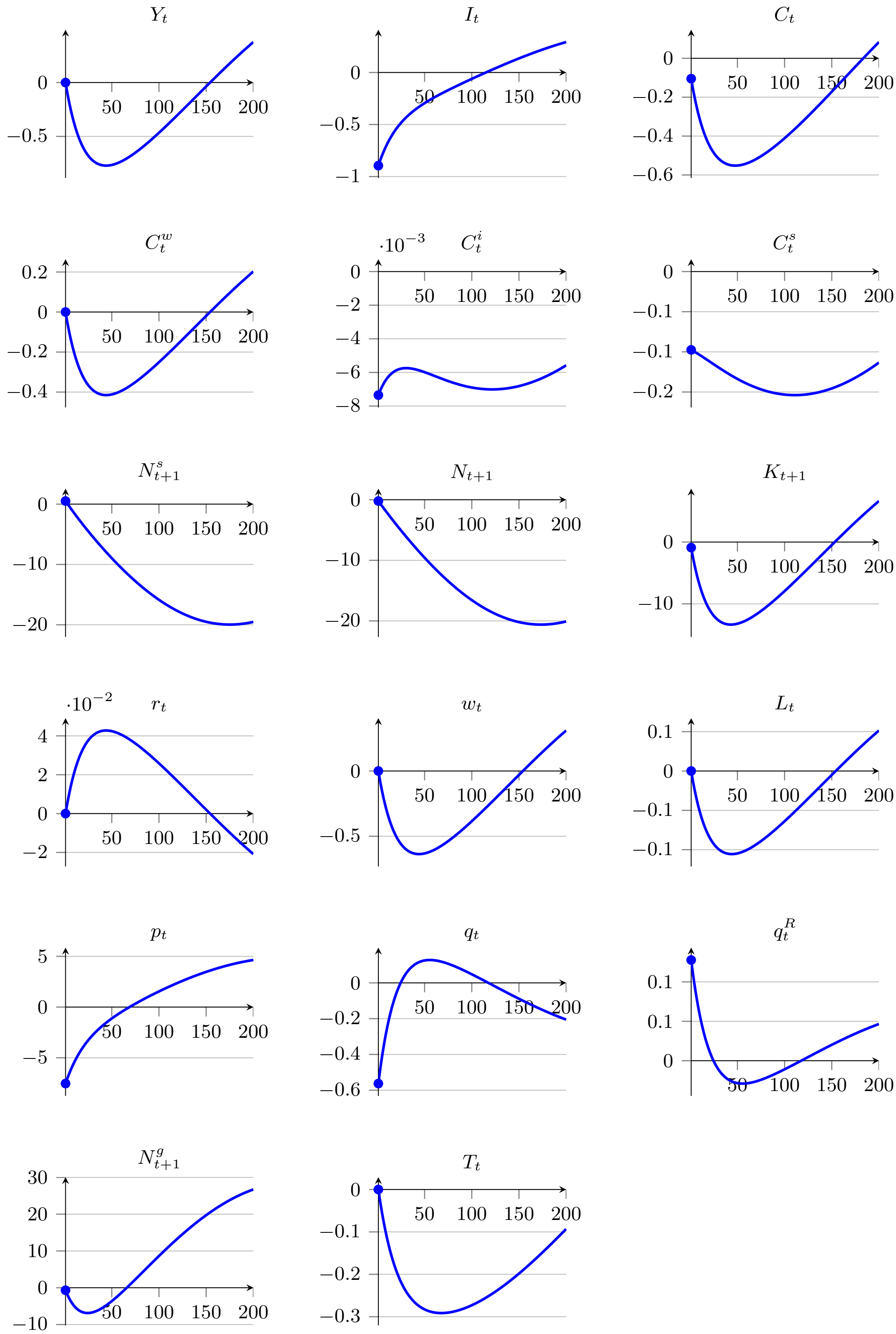
NOTES: Horizontal axes measure quarters after the shock, starting from quarter 1. Vertical axes measure deviations from steady state. Blue dots indicate immediate responses; see the "quarter 1" column of Panel A in Table 6.1 for their values.

FIGURE 6.4: Impulse responses to a liquidity shock and T-financed G-policy



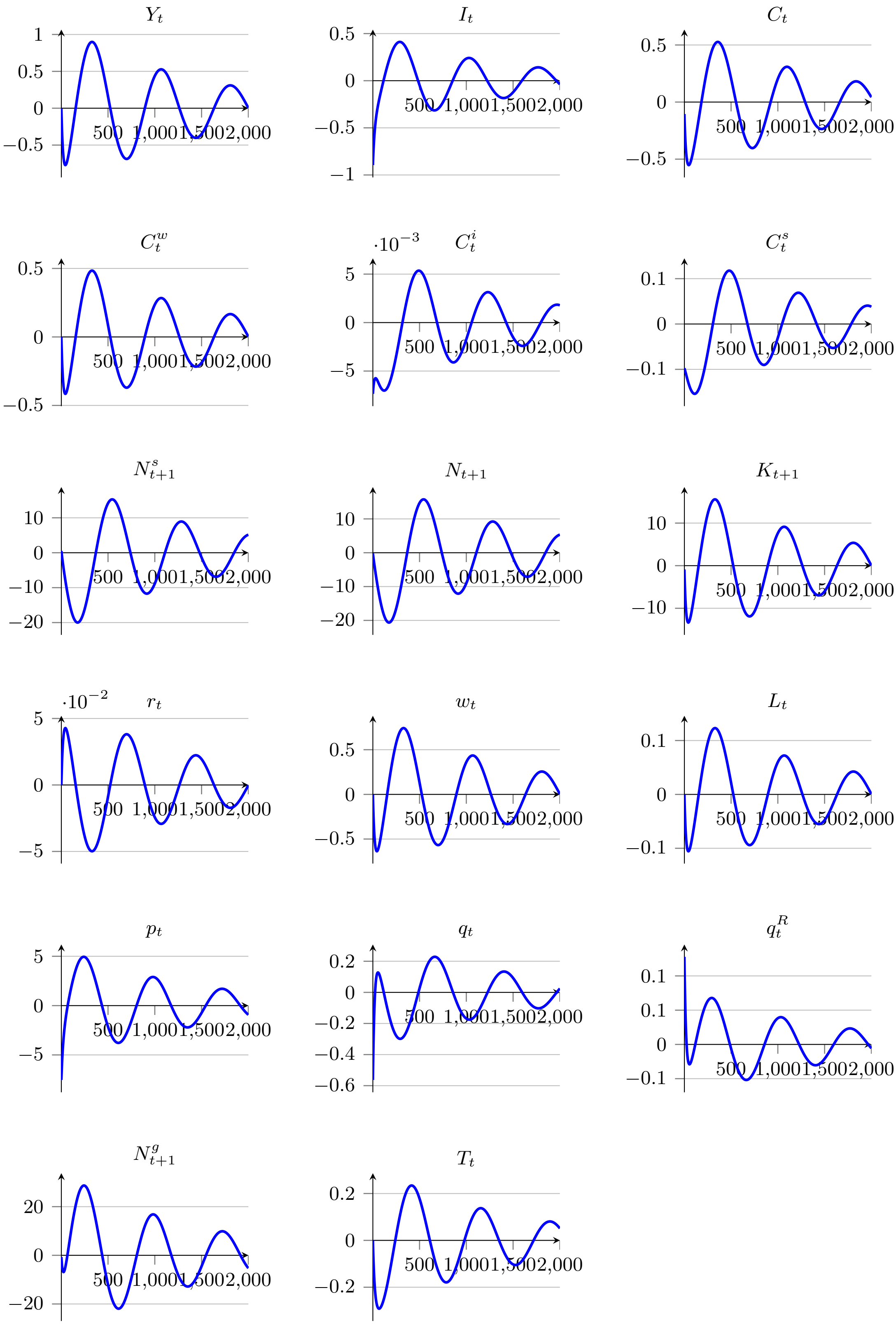
Notes: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. G-policy is a 0.1 unit increase in G_t. The notes in Figure 6.3 apply. See the “quarter 1” column of Tables 6.2 and 6.3 for the values of immediate responses without and with G-policy, respectively.

FIGURE 6.5: Impulse responses to an N^g-financed G-shock



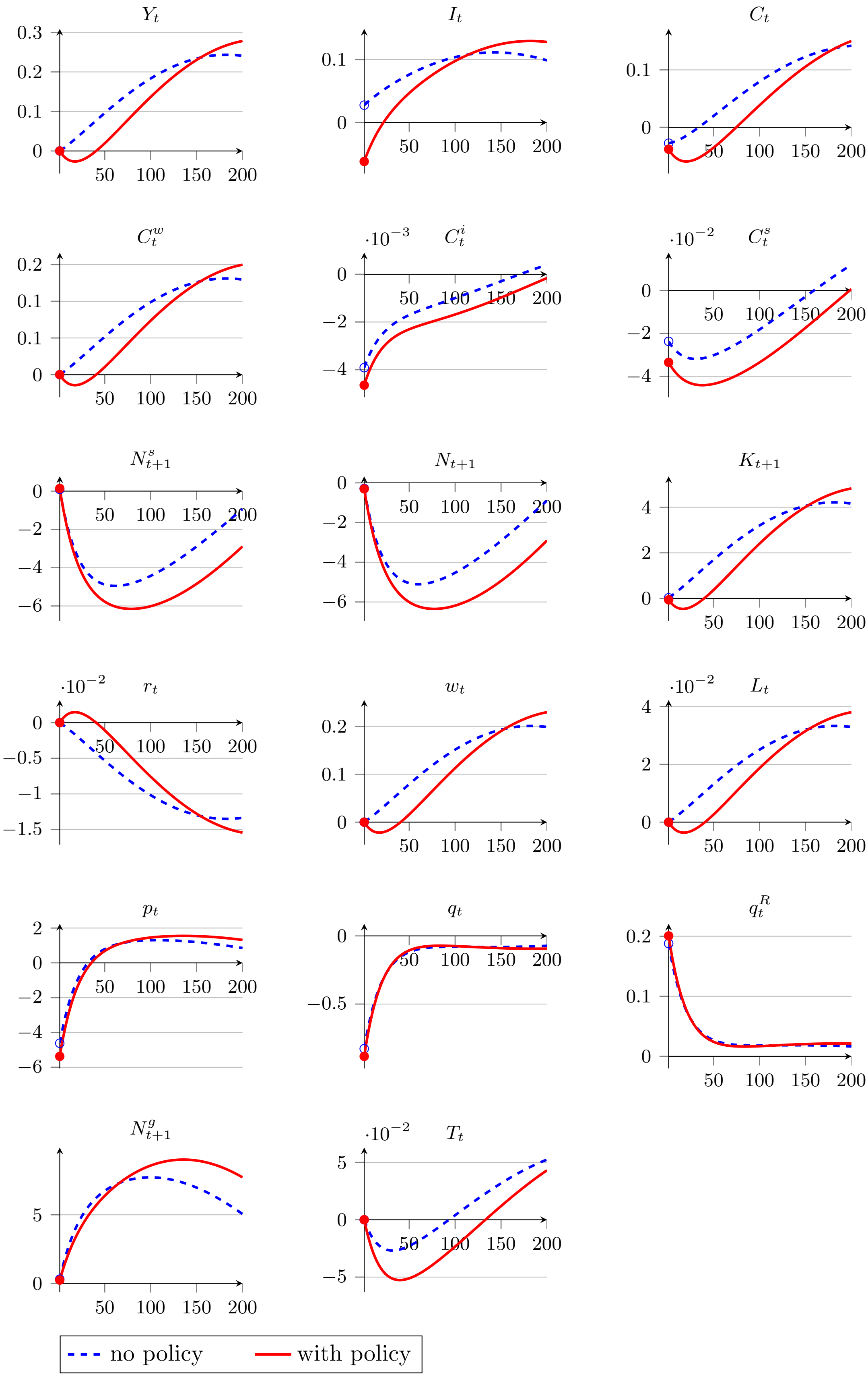
NOTES: The notes in Figure 6.3 apply. See the "quarter 1" column of Table 6.4 for the values of immediate responses.

Figure 6.6: Impulse responses to an N^g-financed G-shock, 2000 quarters



NOTES: These graphs extend Figure 6.5 to 2000 quarters after the G-shock. The same notes apply.

FIGURE 6.7: Impulse responses to a liquidity shock and N^g-financed G-policy



Notes: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. G-policy is a 0.1 unit increase in G_t. The notes in Figure 6.3 apply. See the "quarter 1" column of Tables 6.5 and 6.6 for the values of immediate responses without and with G-policy, respectively.

Table 6.1: Impulse responses to a T-financed G-shock

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						largest	Quarters to:	
	1	2	4	8	20	200		largest	converge
Y_t	−0.58	−0.59	−0.60	−0.62	−0.66	−0.62	−0.69	51	201
I_t	−0.52	−0.51	−0.48	−0.44	−0.35	−0.22	−0.52	1	201
C_t	−1.06	−1.03	−0.97	−0.88	−0.69	−0.40	−1.06	1	201
C_t^w	−0.89	−0.86	−0.81	−0.72	−0.54	−0.28	−0.89	1	201
C_t^i	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	72	201
C_t^s	−0.16	−0.16	−0.16	−0.15	−0.14	−0.11	−0.16	1	201
N_{t+1}^s	−0.10	−0.19	−0.39	−0.76	−1.81	−4.94	−5.04	148	201
N_{t+1}	−0.52	−1.02	−1.94	−3.51	−6.65	−8.93	−9.75	81	201
K_{t+1}	−0.52	−1.02	−1.94	−3.51	−6.65	−8.93	−9.75	81	201
r_t	−0.01	−0.01	−0.01	−0.01	0.00	0.01	−0.01	1	201
w_t	0.49	0.44	0.36	0.21	−0.10	−0.39	0.49	1	201
L_t	−0.24	−0.23	−0.21	−0.19	−0.14	−0.08	−0.24	1	201
p_t	−2.12	−2.05	−1.92	−1.71	−1.27	−0.65	−2.12	1	201
q_t	−0.79	−0.75	−0.68	−0.55	−0.30	0.00	−0.79	1	46
q_t^R	0.18	0.17	0.15	0.13	0.07	0.00	0.18	1	46
T_t	1.00	0.95	0.86	0.70	0.38	0.00	1.00	1	46
τ_t^{wl}	0.54	0.51	0.46	0.38	0.20	0.00	0.54	1	46
τ_t^{rn}	0.66	0.63	0.59	0.52	0.38	0.18	0.66	1	201

NOTES: Panel A gives deviations from steady state in levels. Panel B gives the time at which the absolute largest impulse response occurs, and the convergence indicator which is described in Section 4.4.2.

Table 6.2: Impulse responses to a liquidity shock: T-financed model variant

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	−0.02	−0.06	−0.12	−0.24	−0.33	−0.37	82	201
I_t	−0.28	−0.28	−0.26	−0.24	−0.19	−0.12	−0.28	1	201
C_t	0.28	0.26	0.21	0.12	−0.05	−0.22	0.28	1	201
C_t^w	0.00	−0.01	−0.03	−0.05	−0.11	−0.15	−0.17	82	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	92	201
C_t^s	0.28	0.26	0.23	0.18	0.06	−0.06	0.28	1	201
N_{t+1}^s	−0.83	−1.60	−2.98	−5.17	−8.57	−2.87	−9.25	31	201
N_{t+1}	−0.28	−0.55	−1.05	−1.90	−3.60	−4.84	−5.29	81	201
K_{t+1}	−0.28	−0.55	−1.05	−1.90	−3.60	−4.84	−5.29	81	201
r_t	0.00	0.00	0.00	0.00	0.00	0.01	0.01	82	201
w_t	0.00	−0.01	−0.04	−0.08	−0.15	−0.21	−0.23	82	201
L_t	0.00	0.00	−0.01	−0.01	−0.03	−0.04	−0.04	82	201
p_t	3.86	3.65	3.25	2.57	1.20	−0.35	3.86	1	34
q_t	1.42	1.35	1.22	0.99	0.54	0.00	1.42	1	47
q_t^R	−0.32	−0.31	−0.28	−0.23	−0.12	0.00	−0.32	1	47
T_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	101	201
τ_t^{wl}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	201
τ_t^{rn}	0.00	0.01	0.02	0.03	0.07	0.10	0.11	82	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. These responses are without G-policy. The notes in Table 6.1 apply.

Table 6.3: Impulse responses to a liquidity shock with T-financed G-policy

	Panel A: Impulse responses in levels							Panel B	
	Quarter:							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	−0.06	−0.08	−0.12	−0.18	−0.31	−0.40	−0.43	79	201
I_t	−0.34	−0.33	−0.31	−0.28	−0.23	−0.14	−0.34	1	201
C_t	0.18	0.15	0.11	0.03	−0.12	−0.26	−0.28	91	201
C_t^w	−0.09	−0.10	−0.11	−0.13	−0.17	−0.18	−0.20	71	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	89	201
C_t^s	0.26	0.25	0.22	0.16	0.05	−0.07	0.26	1	201
N_{t+1}^s	−0.84	−1.62	−3.02	−5.25	−8.75	−3.37	−9.52	32	201
N_{t+1}	−0.34	−0.65	−1.24	−2.25	−4.27	−5.73	−6.26	81	201
K_{t+1}	−0.34	−0.65	−1.24	−2.25	−4.27	−5.73	−6.26	81	201
r_t	0.00	0.00	0.00	0.00	0.00	0.01	0.01	85	201
w_t	0.05	0.03	0.00	−0.05	−0.16	−0.25	−0.27	85	201
L_t	−0.02	−0.03	−0.03	−0.03	−0.04	−0.05	−0.05	71	201
p_t	3.65	3.44	3.06	2.40	1.07	−0.42	3.65	1	201
q_t	1.34	1.27	1.15	0.94	0.51	0.00	1.34	1	47
q_t^R	−0.30	−0.29	−0.26	−0.21	−0.12	0.00	−0.30	1	47
T_t	0.10	0.10	0.09	0.07	0.04	0.00	0.10	1	46
τ_t^{wl}	0.05	0.05	0.05	0.04	0.02	0.00	0.05	1	46
τ_t^{rn}	0.07	0.07	0.08	0.09	0.11	0.11	0.13	69	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. G-policy is a 0.1 unit increase in G_t. The notes in Table 6.1 apply.

Table 6.4: Impulse responses to an N^g-financed G-shock

	Quarter: 1	**2**	**4**	**8**	**20**	**200**	**largest**
Y_t	0.00	−0.05	−0.15	−0.31	−0.61	0.38	−0.77
I_t	−0.90	−0.87	−0.82	−0.73	−0.54	0.29	−0.90
C_t	−0.10	−0.13	−0.19	−0.27	−0.45	0.08	−0.55
C_t^w	0.00	−0.03	−0.08	−0.17	−0.33	0.20	−0.42
C_t^i	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
C_t^s	−0.10	−0.10	−0.10	−0.10	−0.11	−0.11	−0.15
N_{t+1}^s	0.51	0.29	−0.16	−1.03	−3.51	−19.52	−19.97
N_{t+1}	−0.21	−0.43	−0.85	−1.69	−4.10	−20.08	−20.61
K_{t+1}	−0.90	−1.74	−3.30	−5.91	−10.82	6.64	−13.35
r_t	0.00	0.00	0.01	0.02	0.03	−0.02	0.04
w_t	0.00	−0.04	−0.12	−0.25	−0.50	0.31	−0.64
L_t	0.00	−0.01	−0.02	−0.04	−0.08	0.05	−0.11
p_t	−7.54	−7.27	−6.77	−5.88	−3.89	4.64	−7.54
q_t	−0.56	−0.52	−0.44	−0.31	−0.06	−0.21	−0.56
q_t^R	0.13	0.12	0.10	0.07	0.01	0.05	0.13
N_{t+1}^g	−0.68	−1.31	−2.44	−4.23	−6.72	26.72	26.72
T_t	0.00	−0.02	−0.04	−0.09	−0.19	−0.09	−0.29

NOTES: The notes in Table 6.1 apply.

Table 6.5: Impulse responses to a liquidity shock: N^g-financed model variant

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	0.00	0.00	0.01	0.03	0.24	0.24	182	201
I_t	0.03	0.03	0.03	0.04	0.05	0.10	0.11	143	201
C_t	−0.03	−0.03	−0.03	−0.02	−0.01	0.14	0.14	200	201
C_t^w	0.00	0.00	0.00	0.01	0.02	0.13	0.13	182	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	201
C_t^s	−0.02	−0.02	−0.03	−0.03	−0.03	0.01	−0.03	31	201
N_{t+1}^s	0.09	−0.19	−0.70	−1.59	−3.37	−0.94	−4.95	61	201
N_{t+1}	−0.28	−0.55	−1.05	−1.90	−3.62	−0.89	−5.11	60	201
K_{t+1}	0.03	0.06	0.11	0.24	0.64	4.16	4.21	181	201
r_t	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	182	201
w_t	0.00	0.00	0.00	0.01	0.03	0.20	0.20	182	201
L_t	0.00	0.00	0.00	0.00	0.00	0.03	0.03	182	201
p_t	−4.62	−4.33	−3.80	−2.89	−1.02	0.85	−4.62	1	201
q_t	−0.83	−0.79	−0.71	−0.59	−0.34	−0.07	−0.83	1	78
q_t^R	0.19	0.18	0.16	0.13	0.08	0.02	0.19	1	78
N_{t+1}^g	0.31	0.61	1.16	2.14	4.26	5.05	7.73	99	201
T_t	0.00	0.00	−0.01	−0.01	−0.02	0.05	0.05	200	201

The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. These responses are without G-policy. The notes in Table 6.1 apply.

Table 6.6: Impulse responses to a liquidity shock with N^g-financed G-policy

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	0.00	−0.01	−0.02	−0.03	0.28	0.28	200	201
I_t	−0.06	−0.06	−0.05	−0.04	0.00	0.13	0.13	182	201
C_t	−0.04	−0.04	−0.05	−0.05	−0.06	0.15	0.15	200	201
C_t^w	0.00	0.00	−0.01	−0.01	−0.01	0.15	0.15	200	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	201
C_t^s	−0.03	−0.03	−0.04	−0.04	−0.04	0.00	−0.04	37	189
N_{t+1}^s	0.14	−0.16	−0.72	−1.69	−3.72	−2.89	−6.15	79	201
N_{t+1}	−0.30	−0.59	−1.13	−2.07	−4.03	−2.90	−6.35	77	201
K_{t+1}	−0.06	−0.12	−0.22	−0.35	−0.45	4.82	4.82	200	201
r_t	0.00	0.00	0.00	0.00	0.00	−0.02	−0.02	200	201
w_t	0.00	0.00	−0.01	−0.02	−0.02	0.23	0.23	200	201
L_t	0.00	0.00	0.00	0.00	0.00	0.04	0.04	200	201
p_t	−5.37	−5.06	−4.48	−3.48	−1.41	1.32	−5.37	1	201
q_t	−0.88	−0.84	−0.76	−0.62	−0.35	−0.09	−0.88	1	201
q_t^R	0.20	0.19	0.17	0.14	0.08	0.02	0.20	1	201
N_{t+1}^g	0.24	0.47	0.92	1.72	3.59	7.73	9.02	136	201
T_t	0.00	0.00	−0.01	−0.02	−0.04	0.04	−0.05	40	201

The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. G-policy is a 0.1 unit increase in G_t. The notes in Table 6.1 apply.

TABLE 6.7: Immediate impulse responses to G-shocks, liquidity shocks, and G-policies: a comparison of T-financing and N^g-financing

	T-financing			N^g-financing		
	G-shock	liquidity shock	G-policy	G-shock	liquidity shock	G-policy
Y_t	−0.58	0.00	−0.06	0.00	0.00	0.00
I_t	−0.52	−0.28	−0.34	−0.90	0.03	−0.06
C_t	−1.06	0.28	0.18	−0.10	−0.03	−0.04
C_t^w	−0.89	0.00	−0.09	0.00	0.00	0.00
C_t^i	0.00	0.00	0.00	−0.01	0.00	0.00
C_t^s	−0.16	0.28	0.26	−0.10	−0.02	−0.03
N_{t+1}^s	−0.10	−0.83	−0.84	0.51	0.09	0.14
N_{t+1}	−0.52	−0.28	−0.34	−0.21	−0.28	−0.30
K_{t+1}	−0.52	−0.28	−0.34	−0.90	0.03	−0.06
r_t	−0.01	0.00	0.00	0.00	0.00	0.00
w_t	0.49	0.00	0.05	0.00	0.00	0.00
L_t	−0.24	0.00	−0.02	0.00	0.00	0.00
p_t	−2.12	3.86	3.65	−7.54	−4.62	−5.37
q_t	−0.79	1.42	1.34	−0.56	−0.83	−0.88
q_t^R	0.18	−0.32	−0.30	0.13	0.19	0.20
N_{t+1}^g	0.00	0.00	0.00	−0.68	0.31	0.24
M_{t+1}	0.00	0.00	0.00	0.00	0.00	0.00
G_t	1.00	0.00	0.10	1.00	0.00	0.10
T_t	1.00	0.00	0.10	0.00	0.00	0.00
τ_t^{wl}	0.54	0.00	0.05	0.00	0.00	0.00
τ_t^{rn}	0.66	0.00	0.07	0.00	0.00	0.00

NOTES: This tables gives first quarter impulse responses in levels for G-shocks, liquidity shocks, and G-polices.

Chapter 7

Government equity purchases

7.1 Introduction

> *"...precisely because of its novelty and the fact that its creation was due to a practical response to circumstances rather than driven by intellectual developments, we lack a clear agreed framework on how unconventional monetary policy impacts the economy."*
>
> – Joyce et al. (2012), p. F275.

> *"Recent research has focused on versions of the New Keynesian model that explicitly incorporate financial intermediation and could thus rationalise quantitative measures taken by central banks since the beginning of the crisis. ... [Cúrdia and Woodford (2011)] is an important step forward in terms of realism, but it is still unsatisfactory ... liquidity ("money") still does not play a role in this model, whereas the provision of liquidity by the central bank plays a crucial role in a financial crisis. ... A good starting point for the explicit modelling of liquidity – surely a crucial element – in the context of a dynamic general equilibrium model of the business cycle is Kiyotaki and Moore (2012)."*
>
> – Zampolli (2012), p. 106.

This chapter simulates an unexpected temporary increase in government holdings of entrepreneur-issued equity. The simulation is performed repeatedly in adaptations of the model in which the government finances the equity purchase in different ways. Five such financing arrangements are examined – issuing money (M-financing), cutting government

Table 7.1: Large-scale asset purchase programmes: a comparison of concepts

Programme	Assets bought	Assets sold/issued	Change in size of central bank balance sheet?
Operation Twist	long-term	short-term	No
Qualitative Easing	less liquid, more risky	more liquid, less risky	No
Quantitative Easing	any	money	Yes
Credit Easing	privately-issued	money	Yes

spending (G-financing), issuing money and cutting government spending (GM-financing), raising taxes (T-financing), and issuing money and raising taxes (TM-financing). With each variant of the model, that is, with each financing arrangement, the government's equity purchase is simulated on its own, to represent discretionary policy in normal times (henceforth, an "N^g-shock"), and then contemporaneously alongside a temporary negative liquidity shock, to represent policy intervention (henceforth, "N^g-policy"). The objectives of this chapter are to examine (i) the model's responses to an N^g-shock, and (ii) the effectiveness of N^g-policy against a liquidity shock, both under alternative financing arrangements.

The N^g-shock and N^g-policy are representative of large-scale asset purchase (LSAP) programmes that were implemented by policymakers (in the US and UK, for example) in response to the 2008 global financial crisis. Four types of LSAPs were performed during the crisis – the Federal Reserve's maturity extension programme or Operation Twist, quantitative easing (QE), qualitative easing, and credit easing.[1] What LSAPs have in common is they involve the central bank buying financial assets from private agents in quantities large enough to influence asset prices. LSAPs differ in terms of what asset is being purchased, how the purchase is financed, and whether or not the central bank's balance sheet changes size. Table 7.1 compares LSAPs along their key attributes.[2]

[1]The economic histories of LSAPs are documented by Joyce et al. (2012) and Zampolli (2012), and by Klyuev et al. (2009) and Lyonnet and Werner (2011) specifically on QE.

[2]The definitions in Table 7.1 have evolved in recent times and have only recently been settled in the literature. In a not-too-distant paper, Klyuev et al. (2009) state that at the time of writing, the literature had no consensus on what the term "quantitative easing" means. The term "quantitative easing" was coined by Richard Werner in the mid-1990 to describe the Bank of Japan purchasing assets from the banking system in exchange for reserves (Lyonnet and Werner (2011)). When first adopted by the Federal Reserve in 2008, QE meant a purchase of privately-issued assets. Then in 2010 the Fed used the term to describe its purchase of long-term government-issued securities from private agents; this is the Bernanke and Reinhart (2004) definition. Buiter (2008) offers a definition which suggests a purchase of any type of asset from private agents; Breedon et al. (2012), for example, use this definition. And a recent paper by Cúrdia and Ferrero (2013) defines QE as purchases of "long-term Treasury and mortgage-backed securities".

LSAPs can be thought of as having two stages of transmission. The first is the financial market impact – a government's purchase decreases the asset's supply to private agents, which bids up the asset's price, or equivalently, lowers its yield. Tobin (1961, 1963, 1969), Brunner and Meltzer (1973), and Friedman (1978) show that lowering the yield on an asset increases the demand for, and therefore the price of, other imperfectly substitutable assets. This is the "portfolio balance effect" or the "portfolio balance channel" through which LSAPs are transmitted to higher asset prices.[3] The second stage of transmission is the macroeconomic impact of higher asset prices; one important channel is that higher prices bring net worth improvements to private agents, which in turn stimulate private investment and consumption.

The literature on LSAPs is divided by which side of the transmission mechanism is being studied. There is a branch which focuses on the financial market impact, that is, whether or not government purchases increase asset prices. There is almost unanimous agreement that LSAPs increase asset prices through a portfolio balance channel.[4] This branch of the literature is burgeoning, and is recently and extensively surveyed by Breedon et al. (2012), Joyce et al. (2012), and Martin and Milas (2012).[5]

The second branch of literature studies the macroeconomics of LSAPs. Joyce et al. (2012) and Martin and Milas (2012) provide excellent and recent surveys. Empirical research has its fair share of statistical models (Chung et al. (2011), Baumeister and Benati (2010), Joyce et al. (2011a), and Kapetanios et al. (2012)) and structural models (Chen et al. (2011) and Chung et al. (2011)). These works rely on observations from the recent 2008 financial crisis, which then presents a great challenge of being able to separate the effects of LSAP from those of other policies that were pursued at the same time.

The first quote at the beginning of the chapter by Joyce et al. (2012) suggests that the theoretical literature should not be lagging in the LSAP research programme. The New

[3]Other channels exist besides portfolio balance. Krishnamurthy and Vissing-Jorgensen (2011) identify seven. The consensus in the literature is that the portfolio balance channel is the most significant; see Gagnon et al. (2011) and Joyce et al. (2011b), for example, for supporting evidence.

[4]Modigliani and Sutch (1966) is a classical paper in which significant effects on asset yields are not found, but their result is overturned by another classical paper from Swanson (2011) who uses an alternative methodology.

[5]Modigliani and Sutch (1966) and Swanson (2011) study asset prices after original Operation Twist in the US in 1961. Bernanke et al. (2004) and Ugai (2007) study quantitative easing in pre-2008 Japan. The financial market impact of QE during the 2008 financial crisis is studied by D'Amico and King (2013), Doh (2010), Neely (2010), Gagnon et al. (2011), Hancock and Passmore (2011), Krishnamurthy and Vissing-Jorgensen (2011), Wright (2011), and Hamilton and Wu (2012) on the US, by Meier (2009), Joyce et al. (2011b), Joyce et al. (2011a), and Breedon et al. (2012) on the UK, and by Rogers et al. (2014) on the US, UK, Euro area, and Japan.

Keynesian DSGE model is currently the workhorse, with modifications of the standard model being calibrated by Del Negro et al. (2011), Caglar et al. (2011), Kara and Sin (2013), and Ellison and Tischbirek (2013). The standard version of the model is not suitable for studying LSAPs because the portfolio balance channel is not present. Eggertsson and Woodford (2003) show that open market operations do not alter private asset portfolios in the standard framework, that is, the irrelevance proposition of Wallace (1981) is upheld. Cúrdia and Woodford (2011) extend the result to a model with heterogeneous agents. However, Andrés et al. (2004) show that LSAPs have significant macroeconomic impacts in a model that has imperfectly substitutable assets. Caglar et al. (2011) suggest that the impacts of LSAPs are inversely related to the degree of asset substitutability. The literature now agrees that models need agent and asset heterogeneity in order to distribute the effects of LSAPS unevenly among the population, and to reduce substitution or arbitrage opportunities among assets. The model readily accommodates modifications, via frictions and assumptions, which introduce asset and agent heterogeneity. This is perhaps why its use is so popular in LSAP research.

Neoclassical DSGE models are less popular in the LSAP literature. The standard framework is not able to produce portfolio balance effects. However, Kiyotaki and Moore (2012) show one way in which this problem can be overcome. They introduce a pair of liquidity constraints to an otherwise standard Real Business Cycle model, and thereby endogenously create a demand for heterogeneous assets. As Zampolli (2012) says in the quote above, the KM model is particularly useful for evaluating LSAPs. KM apply their full model to a theoretical examination of an LSAP against a liquidity shock, in the tradition of Holmström and Tirole (1998). Driffill and Miller (2013) study an LSAP as policy, but in a variant of the KM model and against a liquidity shock that quantitatively matches the 2008 crisis.

This chapter expands the theoretical macroeconomics literature on LSAPs from a neoclassical perspective. It first does this by presenting a unique environment that is based on the basic KM model, as Chapter 2 explains. It then simulates LSAPs that are different to the ones that have appeared in the related literature. The chapter is closely related to KM and Driffill and Miller (2013) through model structure. Among the New Keynesian works, the chapter is also related to Del Negro et al. (2011) and Kara and Sin (2013) through a shared objective of developing policy against liquidity shocks, and to Ellison and Tischbirek (2013) (henceforth, ET) through a shared proposal of conventional LSAPs.

KM and Driffill and Miller (2013) simulate QE. There are no fiscal policy experiments in those papers, although Driffill and Miller speculate about the future fiscal consequences of QE. This chapter studies other types of LSAPs. Before proceeding, some interpretation of the chapter's experiments are required. The monetary authority side of the government is the buyer/holder of equity. When the monetary authority buys equity, it issues an equity-backed instrument which is sold exclusively to the fiscal authority. To buy this equity-backed instrument, the fiscal authority lowers its spending or raises taxes. The fiscal authority can buy all of the equity-backed instrument; this is G-financing and T-financing. The fiscal authority can also buy a fraction of the equity-backed instrument; this is GM-financing and TM-financing, in which case the monetary authority completes its equity purchase with new issues of money. All the chapter's successful N^g-shocks and N^g-policies are credit easing policies that are accompanied by fiscal austerity. Such a policy package is not seen in the theoretical macroeconomics literature that is surveyed in the chapter. The relevance of these experiments is underscored by austerity measures being implemented (in the UK and Euro area, for example) to deal with significant fiscal deficits that were related to the 2008 crisis and its ongoing LSAP policies. With such experiments, the chapter is able to answer two independent questions. When an LSAP is underway, what is the better approach to austerity – cut spending or raise taxes? Should the monetary authority also expand the money supply alongside an LSAP that is already accompanied by austerity?

ET propose making LSAPs a conventional policy tool. Their philosophy is shared by the chapter's N^g-shock, which represents discretionary policy to stimulate the economy, but not in response to some negative shock. ET is different from the chapter in a number of ways. First, ET's model is New Keynesian. Second, ET's LSAP is a central bank purchase of long-term government bonds, which is different from this chapter's privately-issued equity. Third, ET's LSAP is rule-based and depends on the level of output, whereas the programme is discretionary in this chapter. There is another difference which is explained in the next paragraph.

The chapter achieves its objectives by first defining Ω in different ways. This creates different endogenous mechanisms by which the government finances its equity purchase, and thus gives rise to the variants of the model in the chapter. For ease of reference, each model variant is described as a modification the model in Chapter 5 (the benchmark model). The variants in this chapter differ from the benchmark model only in terms of the government's behaviour. Within each variant, there are simulations of an N^g-shock, a liquidity shock, and a liquidity shock with contemporaneous N^g-policy. The N^g-shock and N^g-policy are both normalised to be an increase in government equity holdings, N^g_{t+1}, by 1 unit.[6] The N^g-shock represents a reduction in the supply of equity to private agents (from Equation (3.38)). The equity purchase is a one-off event, and from the second quarter the programme is gradually phased out by the government selling 5% of its depreciated holdings each period *ad infinitum*. The liquidity shock is normalised to be a 1 percentage point decrease (that is, tightening) in the re-saleability constraint, ϕ_t. Throughout the chapter, the model is calibrated with structural parameters set to baseline values. In all of the chapter's experiments, N^g_{t+1} is exogenously determined, and therefore Assumption 4 holds. In steady state there are approximately 16 units of equity, of which saving entrepreneurs hold 15 units and the government holds none. The government therefore balances its fiscal budget in steady state.

The results of the chapter show that all the N^g-shocks immediately raise asset prices, improve entrepreneurs' net worth, increase private spending, and – via investment – increase employment and output after a one-period delay. But these effects are short-lived because investment creates new equity that almost replaces what the government buys. The economy is better off if the government finances its equity purchases by austerity via a cut in government spending rather than more taxes; employment and output are not affected and the fall in government spending is replaced by growth in private demand. By contrast, tax increases immediately reduce employment and output, and dampen, but do not offset, entrepreneurs' net worth improvements. There is no decisive answer as to whether or not an N^g-shock should be partly funded by monetary expansion. Monetary expansion benefits aggregate supply – it reduces the extent to which distortionary taxes must rise; but aggregate demand suffers – the N^g-shock's asset price increases are smaller. Finally, N^g-policy is generally successful at ameliorating a negative liquidity shock, but the benefits are short-lived. This conclusion is supported by the rest of the theoretical macroeconomics literature on LSAPs. The chapter's results differ from ET for one reason – ET's households save, and when the LSAP raises asset prices, these households increase their labour supply

[6]Swanson (2011) computes that, in the original Operation Twist of 1961, the Federal Reserve's purchased long term Treasuries that amounted to 4.5% – 4.6% of the value of total stock of Treasuries outstanding. Swanson argued against describing this transaction as "small". The N^g-shock in this chapter is a purchase of 6.25% of the market and thus earns the description of "large-scale" according to Swanson's criteria.

and the economy produces more output; here, workers are unaffected by higher asset prices because they don't save.

The rest of the chapter is organised as follows. Each of the next five sections simulates an N^g-shock, a liquidity shock, and a liquidity shock with contemporaneous N^g-policy. The government finances its equity purchases by issuing money in Section 7.2, by reducing government spending in Section 7.3, by a combination of reducing government spending and issuing money in Section 7.4, by raising taxes in Section 7.5, and by a combination of raising taxes and issuing money in Section 7.6. Section 7.7 summarises the chapter. Figures and tables appear at the end.

7.2 Financing with money

In this section, the government finances its 1-unit purchase of equity by issuing money, while its spending and tax rates are exogenously determined and unchanged. The benchmark model is modified by (i) including the AR(1) process (3.29) for G_t and (ii) excluding the AR(1) process (3.31) for M_{t+1}. Accordingly, the set of exogenously determined policy variables is $\Omega = \{G_t, N^g_{t+1}\}$. The government's behaviour is summarised by the following equilibrium conditions.

$$T_t = \tau^{rn}_t r_t N_t + \tau^{wl}_t w_t L_t \tag{3.27}$$

$$G_t + q_t(N^g_{t+1} - \delta N^g_t) = T_t + r_t N^g_t + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u^G_t \tag{3.29}$$

$$N^g_{t+1} = (1 - \rho_{Ng})N^g + \rho_{Ng} N^g_t + u^{Ng}_t \tag{3.30}$$

$$\tau^{rn}_t = (1 - \rho_{\tau rn})\tau^{rn} + \rho_{\tau rn}\tau^{rn}_{t-1} + u^{\tau rn}_t \tag{3.32}$$

$$\tau^{wl}_t = (1 - \rho_{\tau wl})\tau^{wl} + \rho_{\tau wl}\tau^{wl}_{t-1} + u^{\tau wl}_t \tag{3.33}$$

The rest of the model remains the same as in the benchmark model and is summarised in Section 5.2.

Without any exogenous shocks, the model converges to a steady state. But once disturbed by the M-financed N^g-shock, the model becomes unstable and does not return to steady state. This result is robust to the calibration of structural parameters.[7]

[7] Attempts to simulate the M-financed N^g-shock fails with 311,040 combinations of parameter values from $\theta \in \{0.185, 0.11, 0.2\}$, $\beta \in \{0.99, 0.95, 0.975, 0.98, 0.995\}$, $\gamma \in \{0.4, 0.3, 0.35, 0.375, 0.425, 0.45, 0.475\}$, $\delta \in$

The problem is the same as what happens when trying to simulate the M-financed G-shock in Chapter 6. General equilibrium cannot be achieved because Equation (3.28) says that the government values its assets *ex post* to the shock, which implies a collinear relationship between money and its price,

$$M_2 - M = \left(\frac{q_1}{p_1}\right) N_2^g$$

where M_2 and N_2^g are the stocks of money and government equity, respectively, at the end of period 1, and q_1/p_1 is the quantity of money the government must contemporaneously issue in order to purchase 1 unit of equity. One possible solution is to enforce an *ex ante* valuation of assets. This can be achieved by either fixing or pre-determining prices in the government's budget constraint (Equation (3.28)). An alternative solution is to elsewhere determine a target for money supply, perhaps by some policy rule. The GM-financed and TM-financed experiments in Sections 7.4 and 7.6 take the latter approach and include rule (3.35) for money supply growth, and in both these experiments the model converges to steady state after the N^g-shock.

7.3 Financing with government spending

In this section, the government finances its 1-unit purchase of equity by reducing its spending, while tax rates and the money supply are exogenously determined and unchanged. The model is exactly the same as the benchmark model in Chapter 5 and is summarised in Section 5.2. Impulse responses to the G-financed N^g-shock are illustrated in Figures 7.1 and 7.2 and are given numerically in Table 7.2. Impulse responses to the liquidity shock without and with G-financed N^g-policy, respectively, are illustrated in Figures 7.3 and 7.4 and are given numerically in Tables 7.3 and 7.4.

7.3.1 A G-financed N^g-shock

Because the steady state price of equity is greater than unity, the government reduces its spending by more than 1 unit of general output in order to finance the N^g-shock. With

$\{0.975, 0.925, 0.95, 0.98, 0.99\}$, $\pi \in \{0.05, 0.03, 0.04, 0.06, 0.07, 0.08\}$, $\nu \in \{1, 0.5, 0.9, 1.1, 1.5, 2, 3, 4\}$, and $\omega \in \{4.01, 3, 3.5, 4.5, 5, 6, 7, 8\}$. The lack of convergence also holds if terms that have third and higher order effects on the model's solution are excluded (using the pruning algorithm of Kim et al. (2008)) and if the model is approximated by a first order Taylor series expansion.

both tax rates unchanged, the labour market remains at steady state in the first quarter. The real wage, employment, and therefore output, gross wages, dividends, and government tax revenue do not change, and a fiscal surplus is therefore created.

The government's equity purchase reduces the asset's supply to private agents, which raises its price and lowers its expected return. Via a portfolio balance effect, money's demand and price both increase. Higher asset prices improve entrepreneurs' net worth, which lead to more investment and private consumption. Because of the tightly binding borrowing constraint, more investment leads to a sufficiently small quantity of new equity issues such that the resulting increase in the asset's supply only partially offsets the shock-induced decrease in supply, and the asset remains with a higher price.

After investment rises above steady state in the first quarter, the capital stock grows by the end of the period. This leads to small delayed increases in output, labour demand, and consequently employment and the real wage in the second quarter. With more gross wages being paid, the rate of return on capital falls, and workers' consumption and aggregate taxes both increase.

The first quarter increase in investment also adds to the equity stock at the end of the period. This addition almost replaces the quantity purchased by government, and in the second quarter equity's price falls. This causes the immediate effects of the shock to almost disappear. By a portfolio balance effect, money's price falls, then entrepreneur's net worth falls, and with it investment.

7.3.2 A liquidity shock and G-financed N^g-policy

Without policy

Immediate and long-term responses to the liquidity shock in this variant of the model are qualitatively the same as in the benchmark model. There are no immediate impacts on labour, and therefore output, as well as workers' consumption, the rate of dividends, and government spending and tax revenue. The shock reduces the supply of equity by making investment financing more difficult, and leaves the asset with a higher price. Money's price then increases, by a portfolio balance effect, and entrepreneurs enjoy higher net worth.

There is a substitution effect of the shock that affects both types of entrepreneurs and outweighs the income effect from higher net worth. First, the shock ruins the appeal of equity and encourages savers to substitute the asset with money, in what KM call a "flight to liquidity". Second, the shock makes new investment more difficult to realise, and encourages investors to consume rather than invest. Accordingly, saving and investment fall in the immediate period of the shock. Lower investment reduces the capital stock at the end of the period. Weak demand for labour propels the economy into a long recession from the second period, which lasts until the capital stock rebounds when its depreciation is overcome by investment.

With policy

G-financed N^g-policy makes noticeable, but short-lived, improvements to the effects of a liquidity shock. The government's equity purchase helps raise the asset's price above that of the no-policy scenario. By a portfolio balance effect, money's price also enjoys a greater increase with policy. Entrepreneurs enjoy improved net worth, and they invest and consume more. However, higher investment increases equity's supply on the market in the second period, and the effects of the government's purchase are almost offset. Asset prices quickly converge to their no-policy trajectories in the second period. N^g-policy improves entrepreneurs' net worth which lead to more investment and consumption. By comparison, without policy there is a fall in investment and a smaller increase in consumption. Both variables then almost converge to their no-policy trajectories in the second quarter.

With N^g-policy, output remains unchanged in the first period, but it has a very small increase in the second period, and remains above steady state for the first year after the liquidity shock. The increase in output is due to the first quarter increase in investment causing the capital stock to expand at the end of the period. As output declines, it is closer to steady state with policy than without policy. But after its trough, output gets further away from steady state than it does in its no-policy scenario. The differences in output's impulse responses between policy and no-policy are very small. Employment, the real wage, and workers' consumption all follow the same pattern as output.

7.4 Financing with government spending and money

In this section, the government finances its 1-unit purchase of equity by a combination of spending reduction and monetary expansion, while tax rates are exogenously determined and unchanged. The benchmark model in Chapter 5 is modified by replacing the AR(1) process

(3.31) for M_{t+1} with the policy rule (3.35) for money supply growth. Accordingly, the set of exogenously determined policy variables is $\Omega = \{G_t, N^g_{t+1}, \tau^{wl}_t, \tau^{rn}_t\}$. The government's behaviour is summarised by the following equilibrium conditions.

$$T_t = \tau^{rn}_t r_t N_t + \tau^{wl}_t w_t L_t \tag{3.27}$$

$$G_t + q_t(N^g_{t+1} - \delta N^g_t) = T_t + r_t N^g_t + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$N^g_{t+1} = (1 - \rho_{Ng})N^g + \rho_{Ng} N^g_t + u^{Ng}_t \tag{3.30}$$

$$\tau^{rn}_t = (1 - \rho_{\tau rn})\tau^{rn} + \rho_{\tau rn}\tau^{rn}_{t-1} + u^{\tau rn}_t \tag{3.32}$$

$$\tau^{wl}_t = (1 - \rho_{\tau wl})\tau^{wl} + \rho_{\tau wl}\tau^{wl}_{t-1} + u^{\tau wl}_t \tag{3.33}$$

$$\dot{M}_{t+1} = (1 - 2\mathbb{1}_t)\left|\dot{Y}_t\right| \tag{3.35}$$

where

$$\mathbb{1}_t = \begin{cases} 1 \text{ if the government buys equity in period } t \\ 0 \text{ if the government sells equity in period } t \end{cases} \tag{3.36}$$

The rest of the model remains the same as in the benchmark model and is summarised in Section 5.2. Once output is supply-determined by Equation (3.60), the rate of monetary growth is established by rule (3.35). Rule (3.35) restrains money supply from unnecessarily financing most of the government's equity purchase. Impulse responses to the GM-financed N^g-shock are illustrated in Figures 7.5 and 7.6 and are given numerically in Table 7.5. Impulse responses to the liquidity shock without and with GM-financed N^g-policy, respectively, are illustrated in Figures 7.7 and 7.8 and are given numerically in Tables 7.6 and 7.7.

7.4.1 A GM-financed N^g-shock

Responses to the GM-financed N^g-shock are almost identical, although marginally smaller, to those of the G-financed N^g-shock. The only difference is that money supply is fixed with G-financing, but it varies as a function of output (from rule (3.35)) with GM-financing.

In this experiment, the government lowers its spending to help finance the N^g-shock. Tax rates are unchanged and the labour market and output therefore remain at their steady states. Then by rule (3.35), money supply remains unchanged and thus leaves the fiscal account to completely finance the shock. Because equity's price is above unity, the government cuts its spending by more than 1 unit of general output.

The government's equity purchase causes asset prices to increase. Because of the differences in elasticity of supply, the increase in money's price is smaller with GM-financing than G-financing, and consequently, so too are the size of responses from the rest of the economy. Entrepreneurs' net worth improvements are smaller in this experiment, resulting in smaller increases in investment and savers' consumption; and the difference in investors' consumption is insignificant. Money's subdued price increase makes the equity purchase program less expensive, and therefore the cut in government spending is smaller.

Because the increase in investment is smaller with GM-financing than G-financing, there are smaller delayed increases in output, labour demand, employment, and the real wage. The increase in output activates monetary expansion via the policy rule (3.35), and this helps drive asset prices further below steady states than in the G-financing experiment.

7.4.2 A liquidity shock and GM-financed N^g-policy

Responses to the liquidity shock, both without and with policy, are almost identical between the GM-financed and G-financed N^g-policies. The only difference is there is a slightly larger increase in money's price and a slightly smaller increase in equity's price with GM-financing. The overall conclusion is the same as that of the G-financed N^g-policy, that is, the GM-financed N^g-policy makes noticeable, but short-lived, improvements to the consequences of the liquidity shock.

7.5 Financing with taxes

In this section, the government finances its 1-unit purchase of equity by raising tax rates, while its spending and the money supply are exogenously determined and unchanged. The benchmark model is modified by (i) including the AR(1) process (3.29) for G_t, (ii) excluding the AR(1) processes (3.32) and (3.33) for τ_t^{rn} and τ_t^{wl}, respectively, and (iii) including the policy rule (3.34) for the rate of tax on wage income. Accordingly, the set of exogenously determined policy variables is $\Omega = \{G_t, N_{t+1}^g, M_{t+1}\}$. The government's behaviour is summarised by the following equilibrium conditions.

$$T_t = \tau_t^{rn} r_t N_t + \tau_t^{wl} w_t L_t \tag{3.27}$$

$$G_t + q_t(N_{t+1}^g - \delta N_t^g) = T_t + r_t N_t^g + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u_t^G \tag{3.29}$$

$$N_{t+1}^g = (1 - \rho_{Ng})N^g + \rho_{Ng}N_t^g + u_t^{Ng} \tag{3.30}$$

$$M_{t+1} = (1 - \rho_M)M + \rho_M M_t + u_t^M \tag{3.31}$$

$$\tau_t^{wl} = \left(\frac{\tau^{wl}}{T}\right) T_t \tag{3.34}$$

The rest of the model remains the same as in the benchmark model and is summarised in Section 5.2. Impulse responses to the T-financed N^g-shock are illustrated in Figures 7.9 and 7.10 and are given numerically in Table 7.8. Impulse responses to the liquidity shock without and with T-financed N^g-policy, respectively, are illustrated in Figures 7.11 and 7.12 and are given numerically in Tables 7.9 and 7.10.

7.5.1 A T-financed N^g-shock

The T-financed N^g-shock begins the same way as the the G-financed and GM-financed experiments – asset prices increase and entrepreneurs invest more. From Equation (3.34), the requirement for more taxes is first achieved by an increase in the rate of tax on wages. This leads to a decrease in labour's aggregate supply and therefore a fall in employment, a rise in the real wage rate, and (because labour demand is wage-elastic) a fall in gross aggregate wages. In aggregate, workers pay more tax to the government, take home a smaller wage, and reduce their consumption. And because workers account for its largest share (approximately 80% in steady state), aggregate consumption falls. With aggregate productivity unchanged and the capital stock being pre-determined, the fall in employment leads to lower production. Furthermore, with a larger share of output going to workers, the returns to capital fall, but only marginally so.

As discussed in Section 3.5, the policy rule (3.34) and the fall in gross wages together imply that the rate of tax on dividends must increase if the tax rate on wages rises. This provides a second channel through which government tax revenue increases. However, the higher dividend tax rate dampens, but does not overturn, entrepreneurs' net worth improvements that are caused by higher asset prices. This is why asset price increases are smaller than in the G-financed experiment where there is no such negative effect.

The rise in investment increases the equity stock at the end of the period, and in the next period the added supply depresses the asset's price. Money's price then falls, entrepreneurs' net worth decline, and the immediate effects of the shock are almost completely reversed. Output receives a boost in the second quarter from an increase in capital. Output does not return to steady state, but it increases and marginally overshoots its steady state in the second quarter. The labour market is likewise affected by the additional capital through aggregate labour demand (Equation (3.49)).

7.5.2 A liquidity shock and T-financed N^g-policy

Without policy

Immediate and long-term responses to the liquidity shock are qualitatively the same as in the benchmark model and in the G-financed and GM-financed experiments. The description that is given to the liquidity shock without policy in the G-financed experiment in Section 7.3 can also be given here. The only difference is that tax rates are endogenously determined in this variant of the model. The tax base from wages does not change, and so the rate of tax on wages does not change. However, when the capital stock falls then the dividends tax base also falls and the rate of tax on dividends rises to keep total tax revenue constant. In contrast, tax revenue falls in the G-financed experiment from the second quarter due to tax bases endogenously adjusting.

With policy

Responses to the liquidity shock with T-financed N^g-policy are not the same as in previous experiments. The differences are due to T-financing distorting the supply-side of the economy, whereas financing with government spending does not. Specifically, T-financing decreases the supply of labour, and then the real wage rises and employment and output fall. Workers' consumption falls, then aggregate private consumption falls (which represents the only qualitative change in responses from the no-policy scenario).

By raising the rate of tax on dividends, T-financing aggravates the reduction in entrepreneurs' net worth that is caused by the liquidity shock. This aggravation at first leads to a sharper fall in investment compared to the no-policy scenario. However, there is now a larger reduction in equity' supply, which implies a larger increase in equity's price and a larger increase in money's price. The resulting improvement in entrepreneurs' net worth helps partially offset the initial fall in investment that is caused by the tax hike. Consequently, the fall in investment with policy is smaller than without policy.

At the end of the immediate period, capital enjoys a smaller decline with policy. In the next period, output recovers quickly and gets close to its no-policy position. The model's other variables also converge to their no-policy responses in the second quarter.

The overall conclusion is that T-financed N^g-policy ameliorates some of the effects of a liquidity shock, and worsens other effects. But the impacts of policy are short-lived, for the same reasons why the T-financed N^g-shock is short-lived, that is, the policy's improvement to investment helps replace the quantity of equity that the government takes out of the market.

7.6 Financing with taxes and money

In this section, the government finances its 1-unit purchase of equity by a combination of raising taxes and issuing money, while its spending is exogenously determined and unchanged. The benchmark model is modified by (i) including the AR(1) process (3.29) for G_t, (ii) excluding the AR(1) processes (3.31), (3.32) and (3.33) for M_{t+1}, τ_t^{rn}, and τ_t^{wl}, respectively, (iii) including the policy rule (3.34) for the rate of tax on wage income, and (iv) including the policy rule (3.35) for money supply growth. Accordingly, the set of exogenously determined policy variables is $\Omega = \{G_t, N_{t+1}^g\}$. The government's behaviour is summarised by the following equilibrium conditions.

$$T_t = \tau_t^{rn} r_t N_t + \tau_t^{wl} w_t L_t \tag{3.27}$$

$$G_t + q_t(N_{t+1}^g - \delta N_t^g) = T_t + r_t N_t^g + p_t(M_{t+1} - M_t) \tag{3.28}$$

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u_t^G \tag{3.29}$$

$$N_{t+1}^g = (1 - \rho_{Ng})N^g + \rho_{Ng} N_t^g + u_t^{Ng} \tag{3.30}$$

$$\frac{\tau_t^{wl}}{T_t} = \frac{\tau^{wl}}{T} \tag{3.34}$$

$$\dot{M}_{t+1} = (1 - 2\mathbb{1}_t)\left|\dot{Y}_t\right| \tag{3.35}$$

where

$$\mathbb{1}_t = \begin{cases} 1 \text{ if the government buys equity in period } t \\ 0 \text{ if the government sells equity in period } t \end{cases} \tag{3.36}$$

The rest of the model remains the same as in the benchmark model and is summarised in Section 5.2. Impulse responses to the TM-financed N^g-shock are illustrated in Figures 7.13 and 7.14 and are given numerically in Table 7.11. Impulse responses to the liquidity shock without and with TM-financed N^g-policy, respectively, are illustrated in Figures 7.15 and 7.16 and are given numerically in Tables 7.12 and 7.13.

7.6.1 A TM-financed N^g-shock

Responses are qualitatively the same as those of the T-financing experiment. While money supply is fixed with T-financing, here it varies with output according to rule (3.35). This is the source of all quantitative deviations between the two experiments.

The government's equity purchase increases asset prices, entrepreneurs' net worth, investment, consumption, and saving. The borrowing constraint ensures that the rise in investment does not feed back to a reduction in equity's price.

The higher rate of tax on dividends reduces the demand for assets. However, the effect is small relative to the increase in demand that is brought on by net worth improvements. Assets are therefore left with net increases in demand. The higher rate of tax on wages lowers employment and output. Rule (3.35) then endogenously expands money supply; this allows the money market to clear after its net increase in demand, and the asset is left with a higher price.

Compared to the T-financing experiment, monetary expansion relieves some of the pressure on tax rates to increase, and thereby dampens the responses to the shock, starting from the changes in asset prices. The smaller increase in investment here means that the capital stock grows by fewer units by the end of the first quarter, and output in the second quarter gets a smaller boost. In fact, while output increases enough to overshoot its steady state in the T-financing experiment, it remains below steady state in this experiment. The labour market is similarly affected by subdued investment.

Response turnarounds in the second quarter are much softer than the sharp changes seen in the T-financing experiment. Trajectories of long-term adjustment are more gradual, and have more pronounced humps. Capital now has a hump-shaped trajectory, because investment slowly declines towards steady state (compared to a sharp drop in the second quarter in the T-financing experiment). In other words, investment is elevated for a longer period of time; it takes 4 quarters to get close to steady state, as opposed to 2 quarters in the T-financing experiment. While investment remains above steady state, the capital stock is able to expand after depreciation. Investment drops below steady state in quarter 5, and capital starts to fall from then. Capital still overshoots its steady state, as it also does in the T-financing experiment, because investment remains below steady state while slowly converging.

7.6.2 A liquidity shock and TM-financed N^g-policy

Without policy

Responses to the liquidity shock are almost identical to those of the T-financed variant of the model in Section 7.5. The only difference is that there is a slightly smaller increase in money's price and a slightly larger increase in equity's price in this experiment.

With policy

Monetary expansion is made possible by rule (3.35) together with an increase in output in the immediate period. Comparing T-financing with TM-financing, the inclusion of money to help finance N^g-policy alleviates the extent to which tax rates increase and thus softens the related consequences. All these consequences are qualitatively the same as the T-financed N^g-policy in Section 7.5. Furthermore, the addition of monetary expansion dampens the increase in money's price. The overall conclusion is the same as that of the T-financed N^g-policy, that is, TM-financed N^g-policy brings both positive and negative, but short-lived, differences to the effects of the liquidity shock. On the positive side, the immediate increases in asset prices, the real wage, and investment are greater. On the negative side, consumption changes direction and immediately falls, and employment and output immediately decline.

7.7 Chapter summary

This chapter successfully simulates a government purchase of equity in four experiments which differ by the way these purchases are financed. The equity purchase programme is simulated in two scenarios, first on its own (an N^g-shock), and then contemporaneously alongside a negative liquidity shock (N^g-policy).

In all experiments, the N^g-shock immediately raises asset prices, which leads to improvements in entrepreneurs' net worth and increases in investment and consumption. The immediate increases in investment are sufficient to create enough new equity that, in the second period after the shock, the additions almost compensate for the stocks taken out by the government. Equity's price falls in the second period, and with it the price of money and entrepreneurs' net worth, investment, and consumption. The direct effects of the N^g-shock are therefore short-lived.

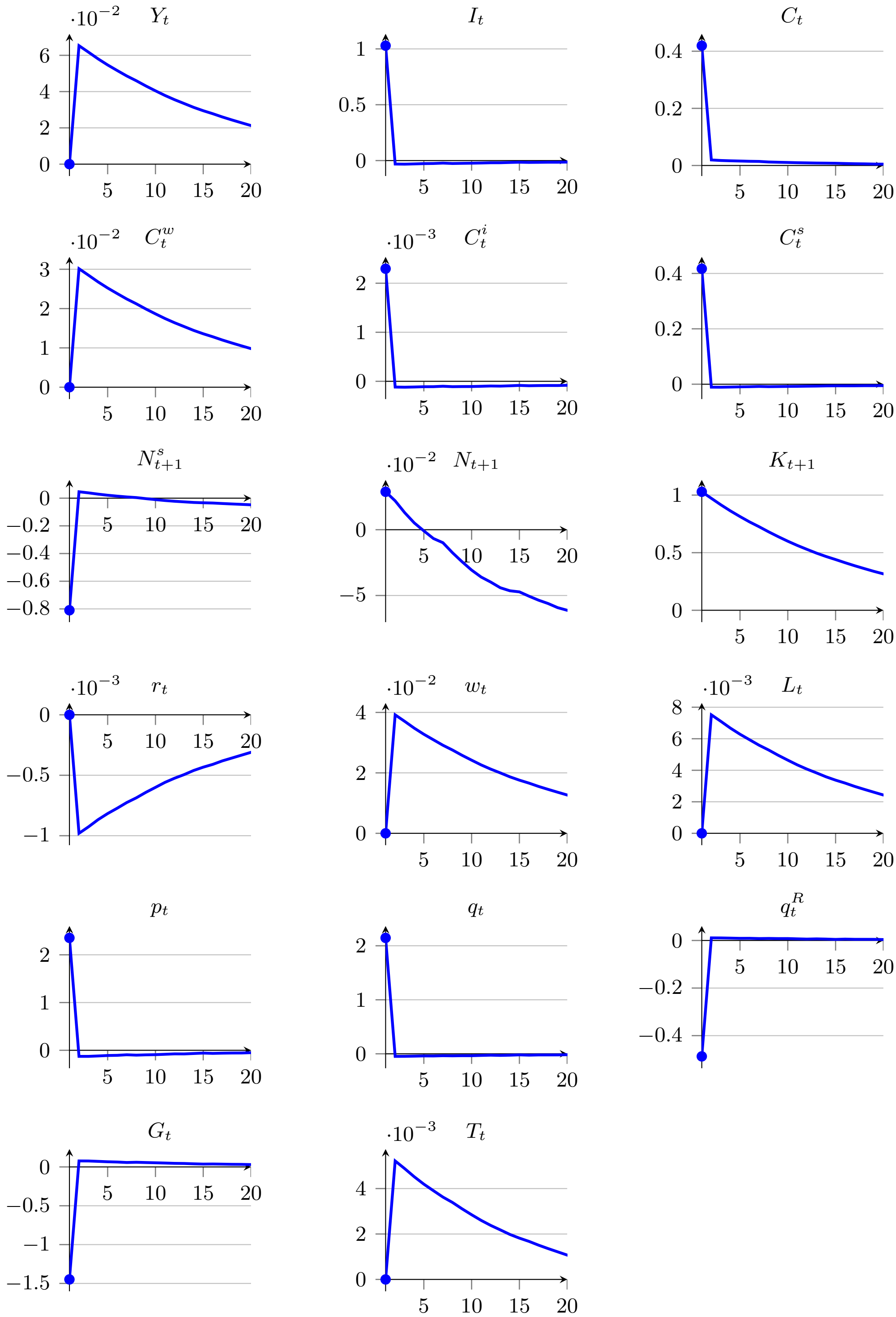
The method of financing compounds the effects of the N^g-shock. All methods of financing involve an austerity measure, either by a reduction in government spending or an increase in taxes. When the government cuts spending, there is no immediate change in output and therefore aggregate demand is re-distributed to private agents who now have more net worth. When taxes are increased, output falls (from the wage tax) and entrepreneurs' net worth falls (from the dividends tax); the latter only partially offsets the initial improvement and thereby dampens the effects of the N^g-shock. The conclusion is that austerity in this model is better (that is, less distortionary) with spending cuts. There is a possibility for the government to additionally finance the N^g-shock with monetary expansion. It is uncertain whether this improves the N^g-shock's response; aggregate supply benefits from monetary expansion, but aggregate demand suffers. First, consider the supply-side of the economy. G-financing vs. GM-financing experiments are not relevant for comparison because the labour market and output do not immediately change. T-financing reduces employment and output, with magnitudes that are smaller with TM-financing because monetary expansion reduces the extent to which tax rates must rise to finance the N^g-shock; a smaller hike in the rate of tax on wages means a smaller decrease in employment. Next, consider the demand-side of the economy. The fundamental difference between fiscal financing alone and fiscal financing plus monetary financing is that money supply is fixed in the former and positively responsive to its own price in the latter. The N^g-shock generates a larger increase in the price of money with fiscal financing alone. Monetary expansion puts additional units of the asset on the market and thus dampens its price increase. Net worth improvements and related consequences are therefore larger with fiscal financing alone.

N^g-policy is generally successfully at ameliorating the effects of a negative liquidity shock. However, for the same reasons given for the N^g-shocks, the benefits of N^g-policy are short-lived and almost disappear after one year. This conclusion is shared by Kiyotaki and

Moore (2012) and the sticky-price models of Del Negro et al. (2011), Kara and Sin (2013), and Driffill and Miller (2013), although in the latter group the short-lived nature of policy is due to the consequences of asset price variations being suppressed.

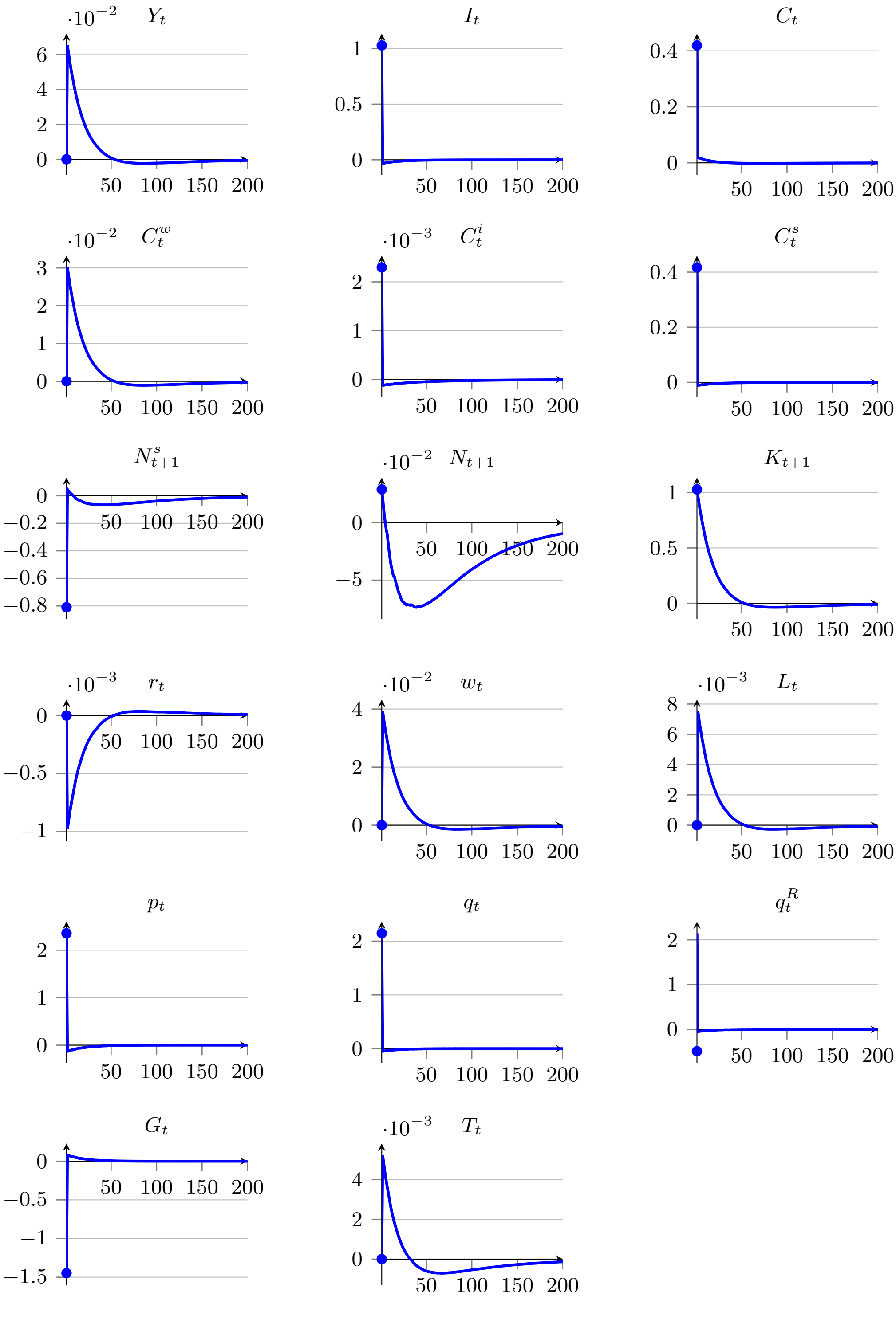
The chapter's G-financed N^g-shock is the converse of the N^g-financed G-shock in Section 6.3. In the former, the government sells equity and reduces it spending. In the latter, the government spends more and sells equity, but then the economy oscillates as it converges to steady state over a very long period. The difference in results suggests a useful lesson for policy design. If the government allows its equity holdings to vary according to some rule then the economy oscillates when the rule is activated. No such oscillation is experienced if the government varies its spending according to a policy rule. In a related paper, Ellison and Tischbirek (2013) install a Taylor rule for asset purchases in which they depend on the level of output. Theirs is a New Keynesian DSGE model, and sticky prices may account for why the economy does not oscillate. It may be useful to extend this chapter with a sticky-price variant of the model, in the tradition of Driffill and Miller (2013) but with distortionary taxes included. This and other extensions of the chapter are outlined in the conclusion of the thesis.

FIGURE 7.1: Impulse responses to a G-financed N^g-shock, 20 quarters



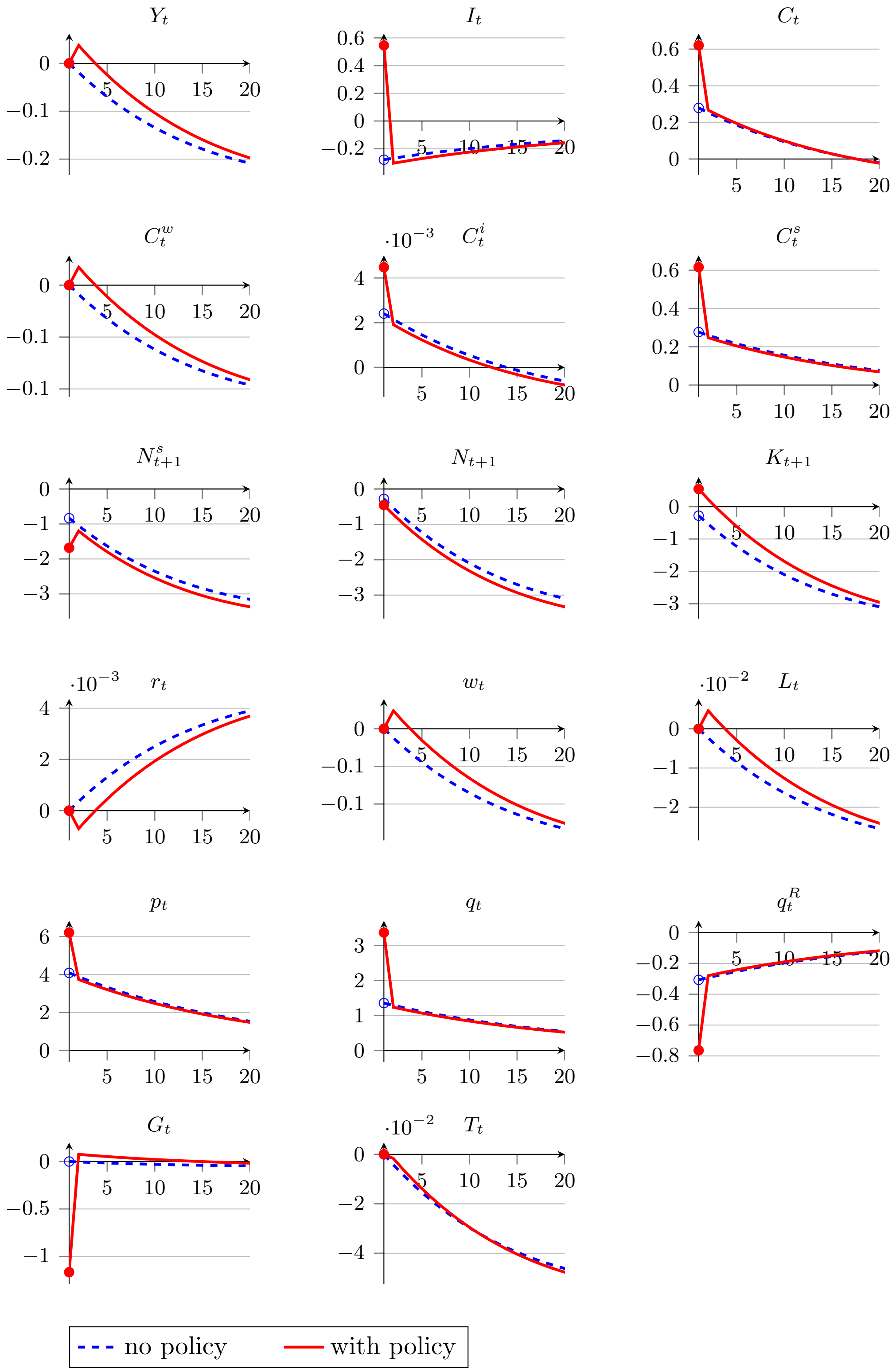
NOTES: Horizontal axes measure quarters after the shock, starting from quarter 1. Vertical axes measure deviations from steady state. Blue dots indicate immediate responses; see the "quarter 1" column of Panel A in Table 7.2 for their values.

FIGURE 7.2: Impulse responses to a G-financed N^g-shock, 200 quarters



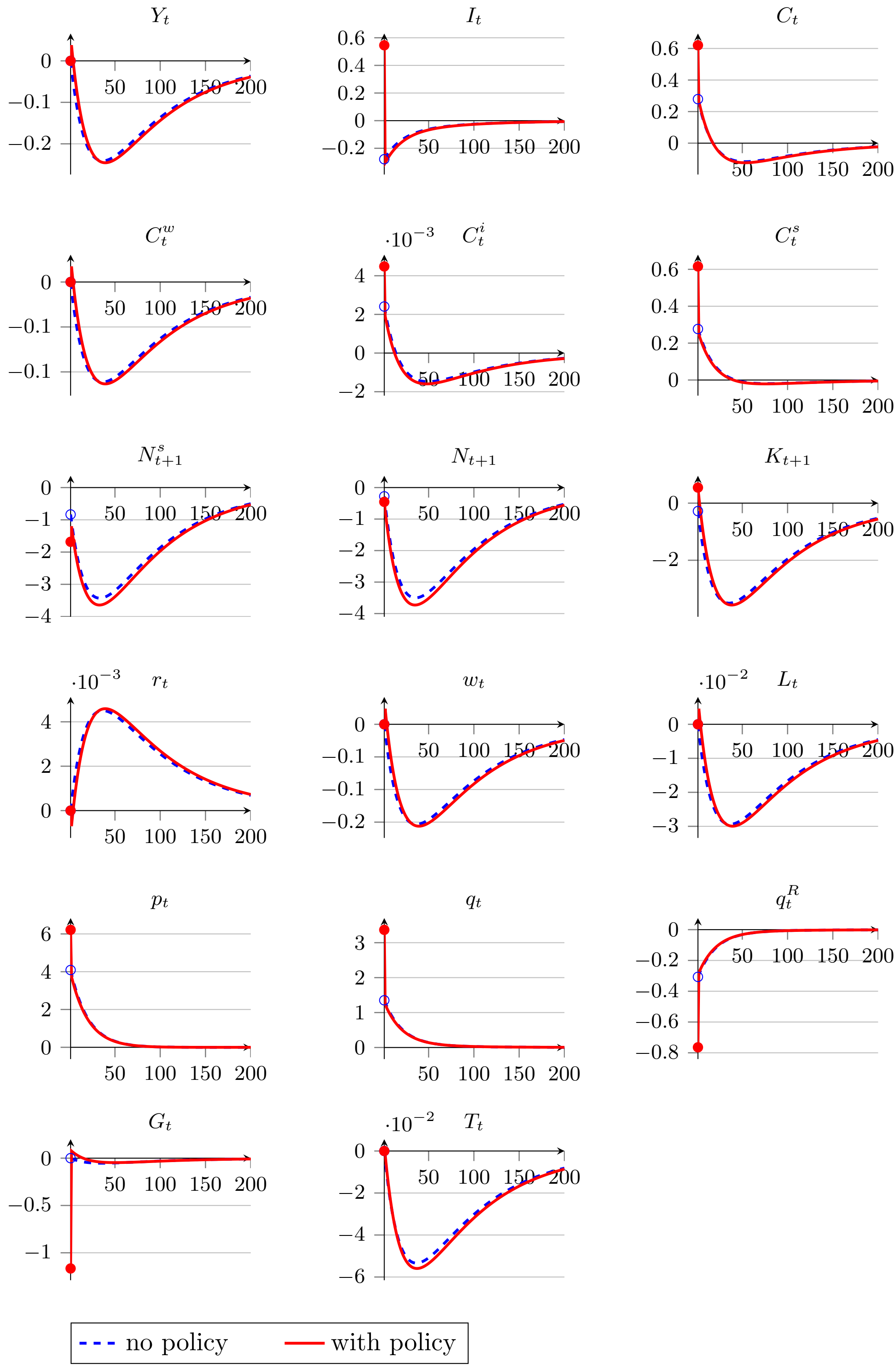
NOTES: These graphs extend Figure 7.1 to 200 quarters. The same notes apply.

FIGURE 7.3: Impulse responses to a liquidity shock and G-financed N^g-policy, 20 quarters



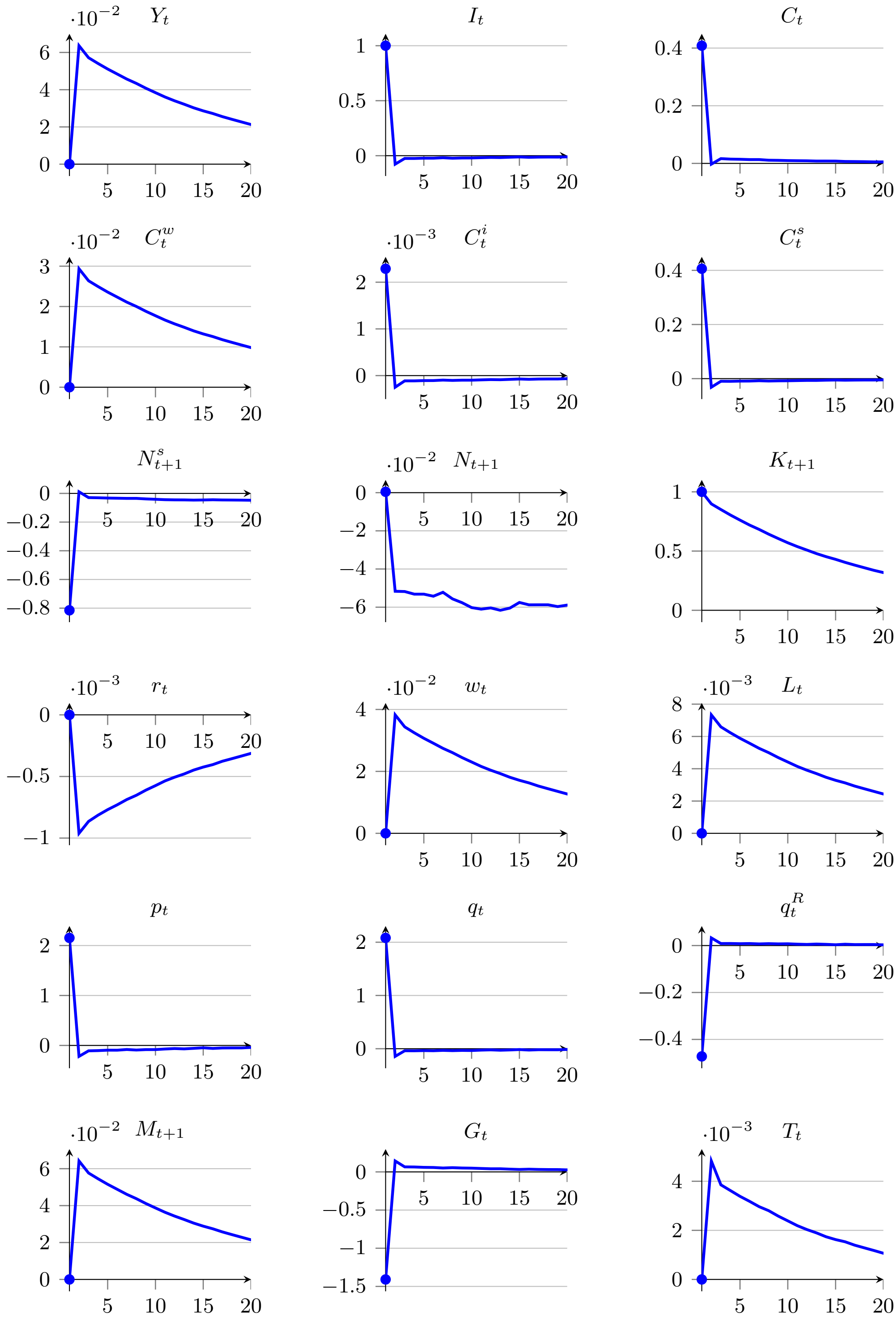
NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Figure 7.1 apply. Hollow blue dots and solid red dots indicate immediate responses of the liquidity shock without and with N^g-policy, respectively; see the "quarter 1" column of Panel A in Table 7.3 and Table 7.4, respectively, for their values.

Figure 7.4: Impulse responses to a liquidity shock and G-financed N^g-policy, 200 quarters



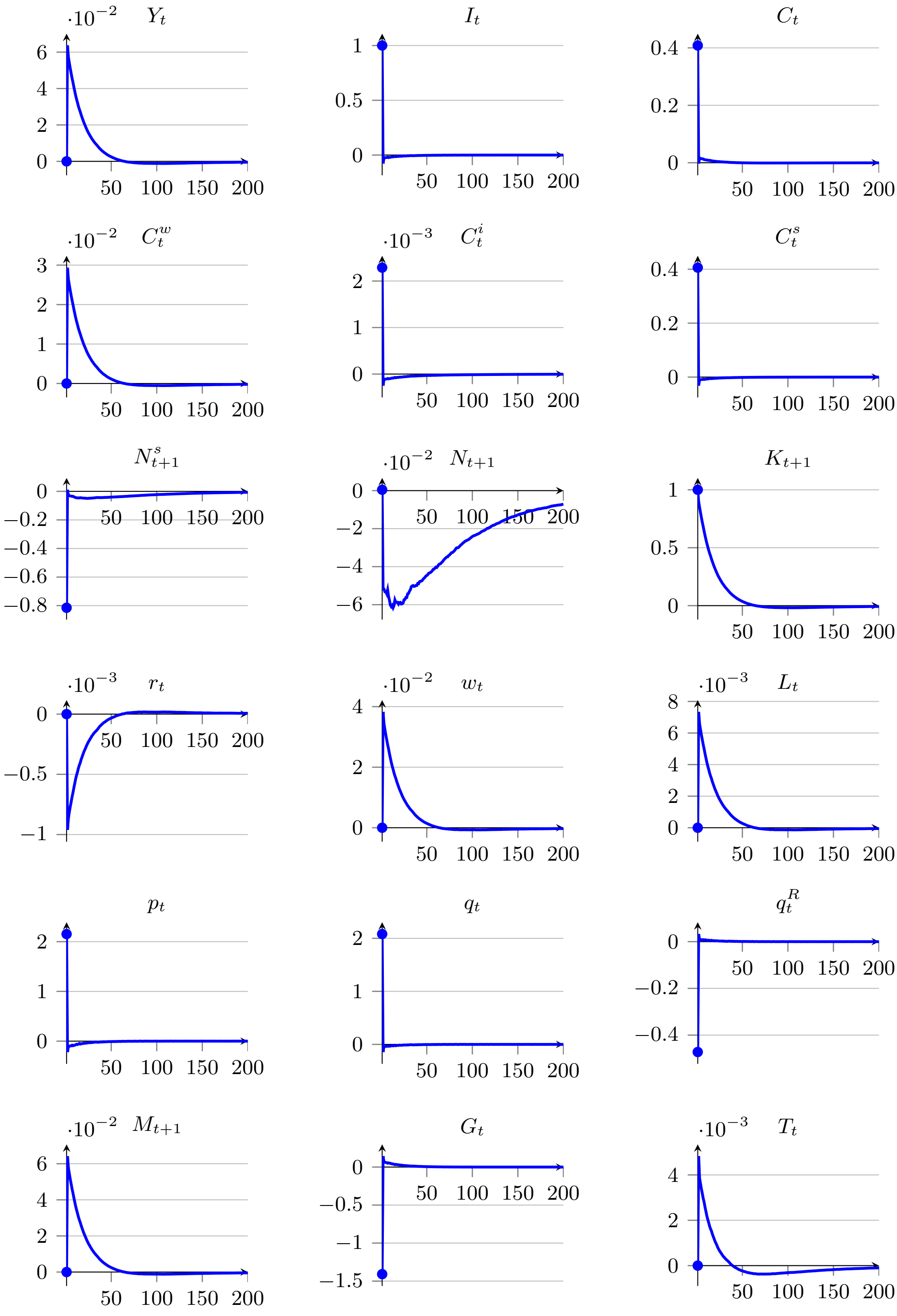
NOTES: These graphs extend Figure 7.3 to 200 quarters. The same notes apply.

FIGURE 7.5: Impulse responses to a GM-financed N^g-shock, 20 quarters



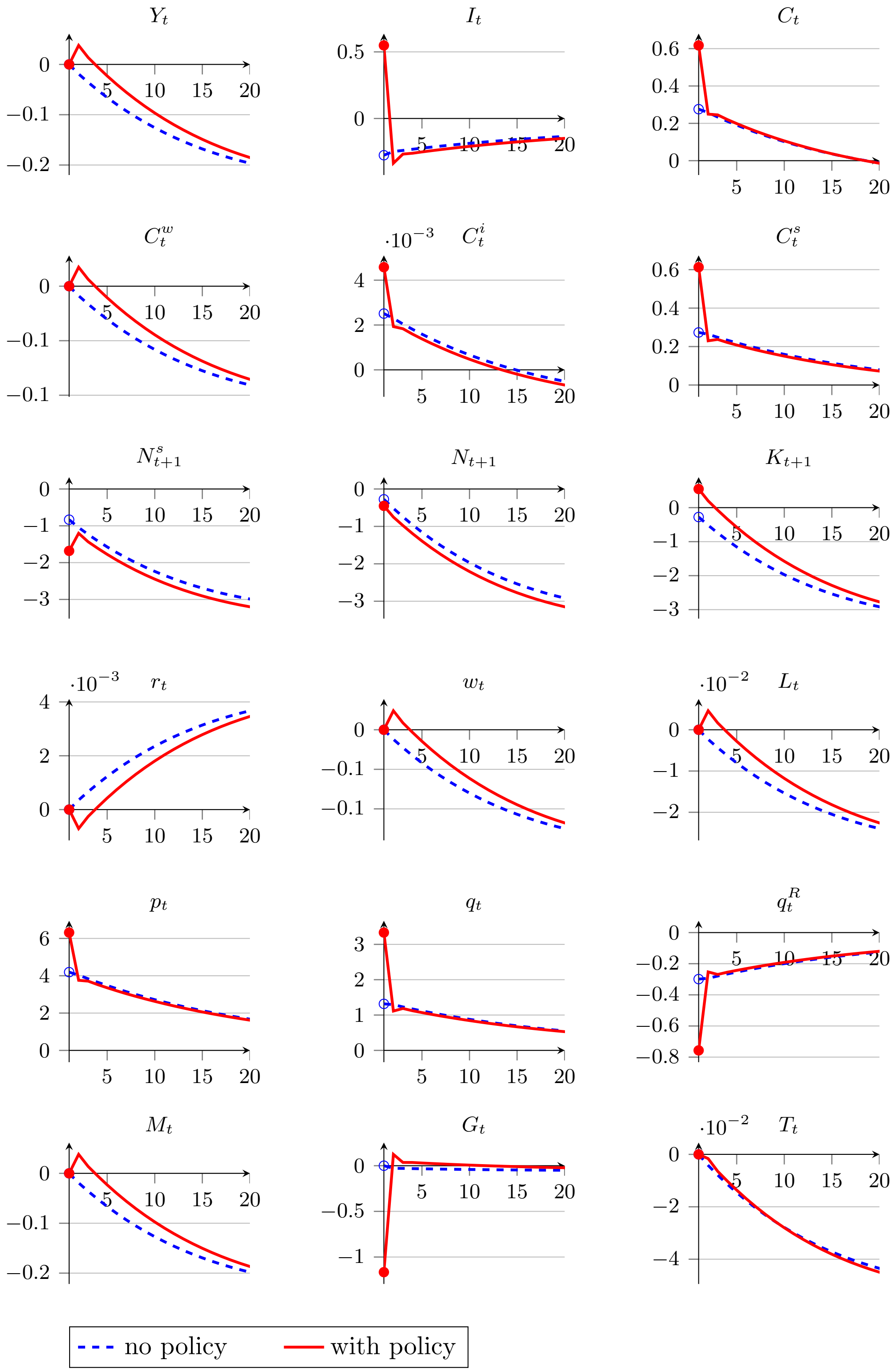
NOTES: The notes in Figure 7.1 apply. See the "quarter 1" column of Panel A in Table 7.5 for the values of immediate responses.

FIGURE 7.6: Impulse responses to a GM-financed N^g-shock



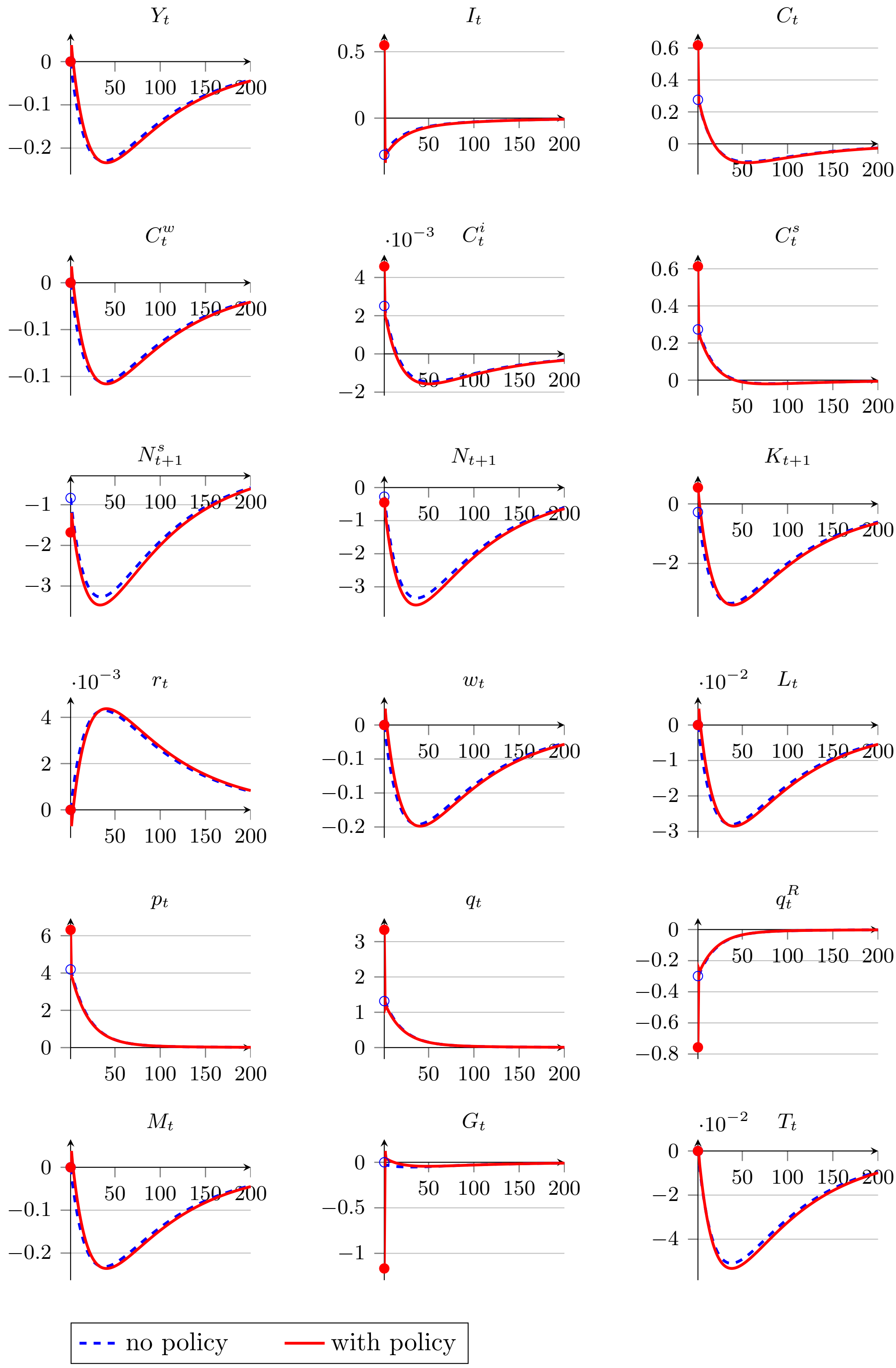
NOTES: These graphs extend Figure 7.5 to 200 quarters. The same notes apply.

FIGURE 7.7: Impulse responses to a liquidity shock and GM-financed N^g-policy, 20 quarters



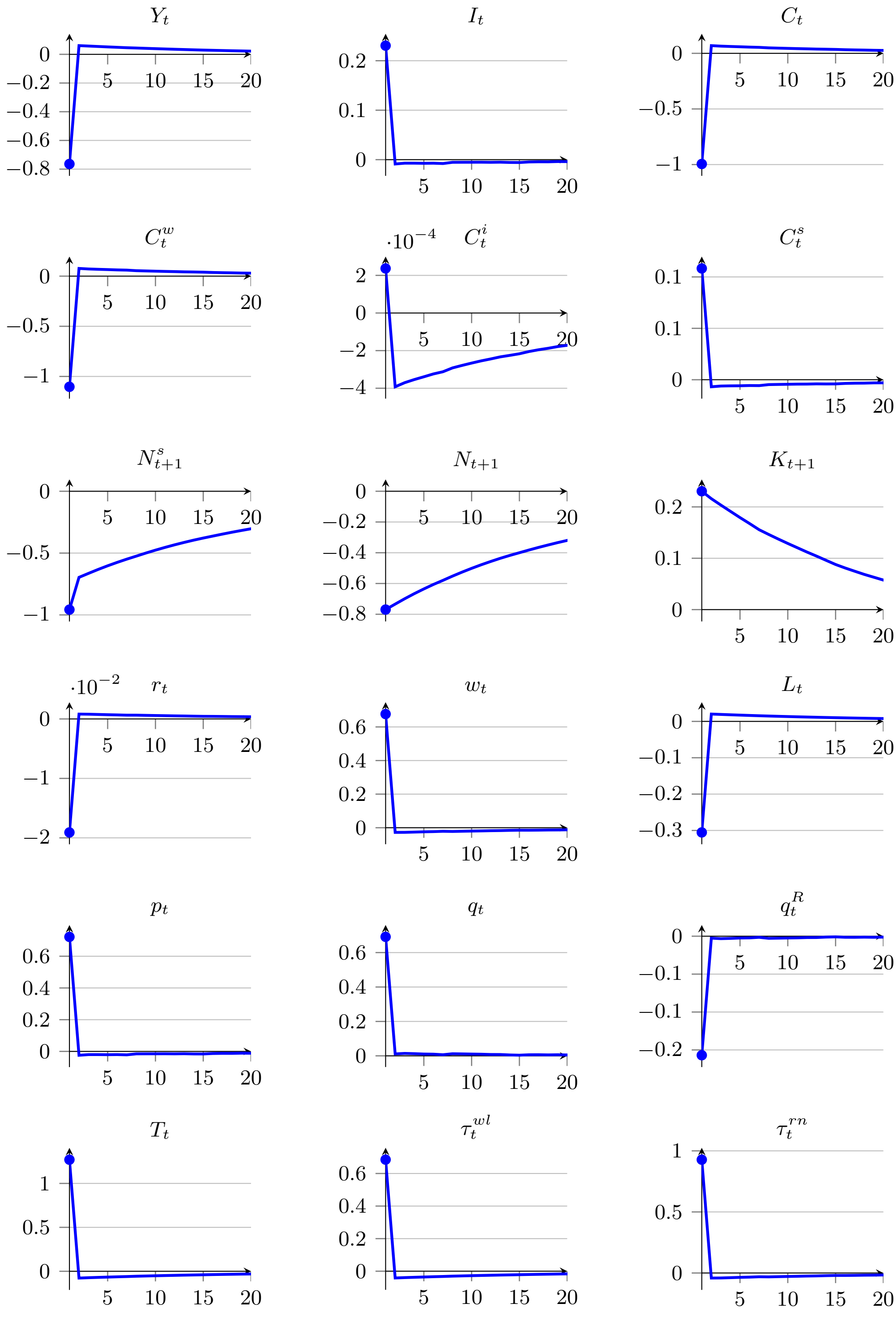
NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Figure 7.1 apply. Hollow blue dots and solid red dots indicate immediate responses of the liquidity shock without and with N^g-policy, respectively; see the "quarter 1" column of Panel A in Table 7.6 and Table 7.7, respectively, for their values.

Figure 7.8: Impulse responses to a liquidity shock and GM-financed N^g-policy, 200 quarters



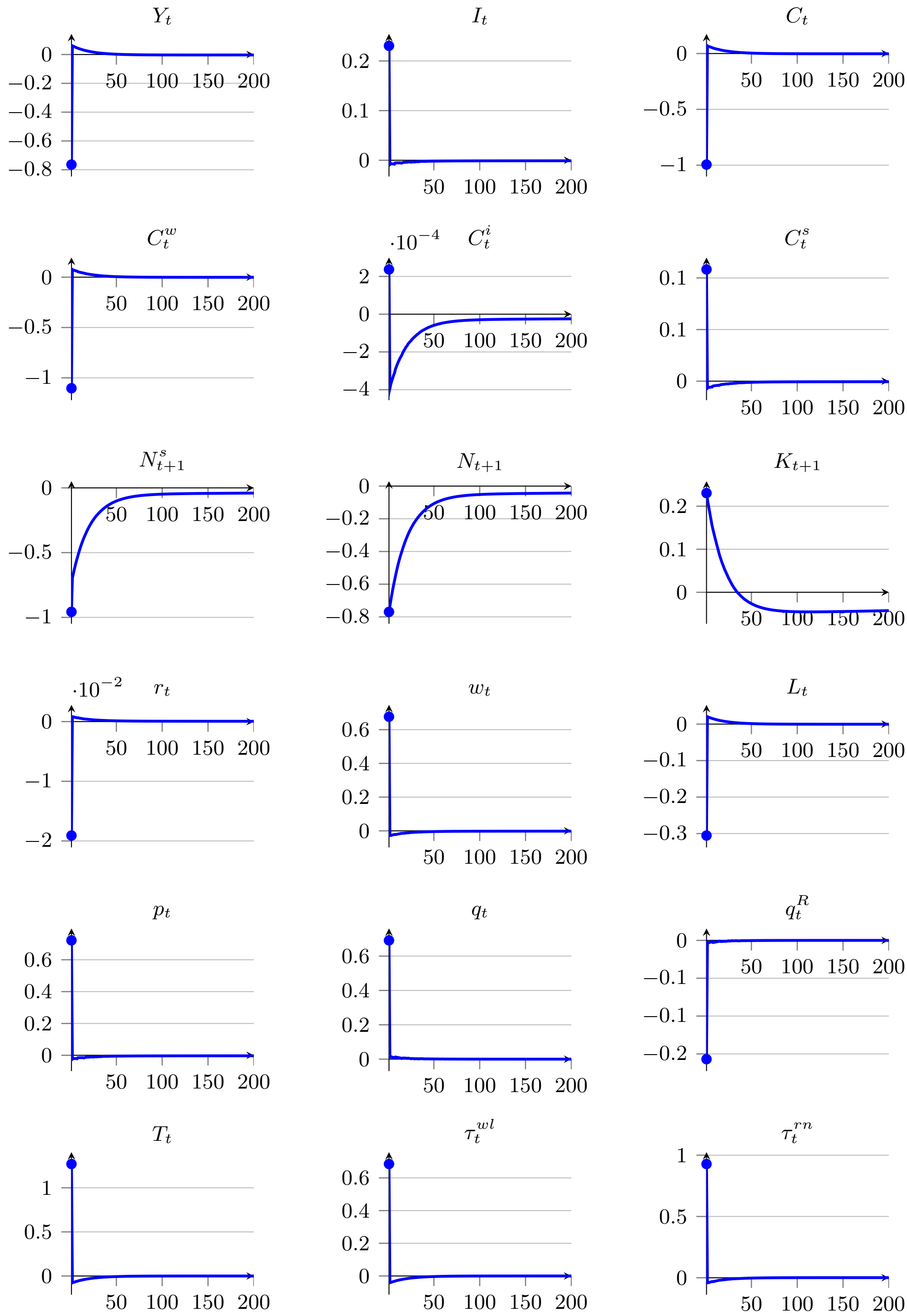
NOTES: These graphs extend Figure 7.7 to 200 quarters. The same notes apply.

FIGURE 7.9: Impulse responses to a T-financed N^g-shock, 20 quarters



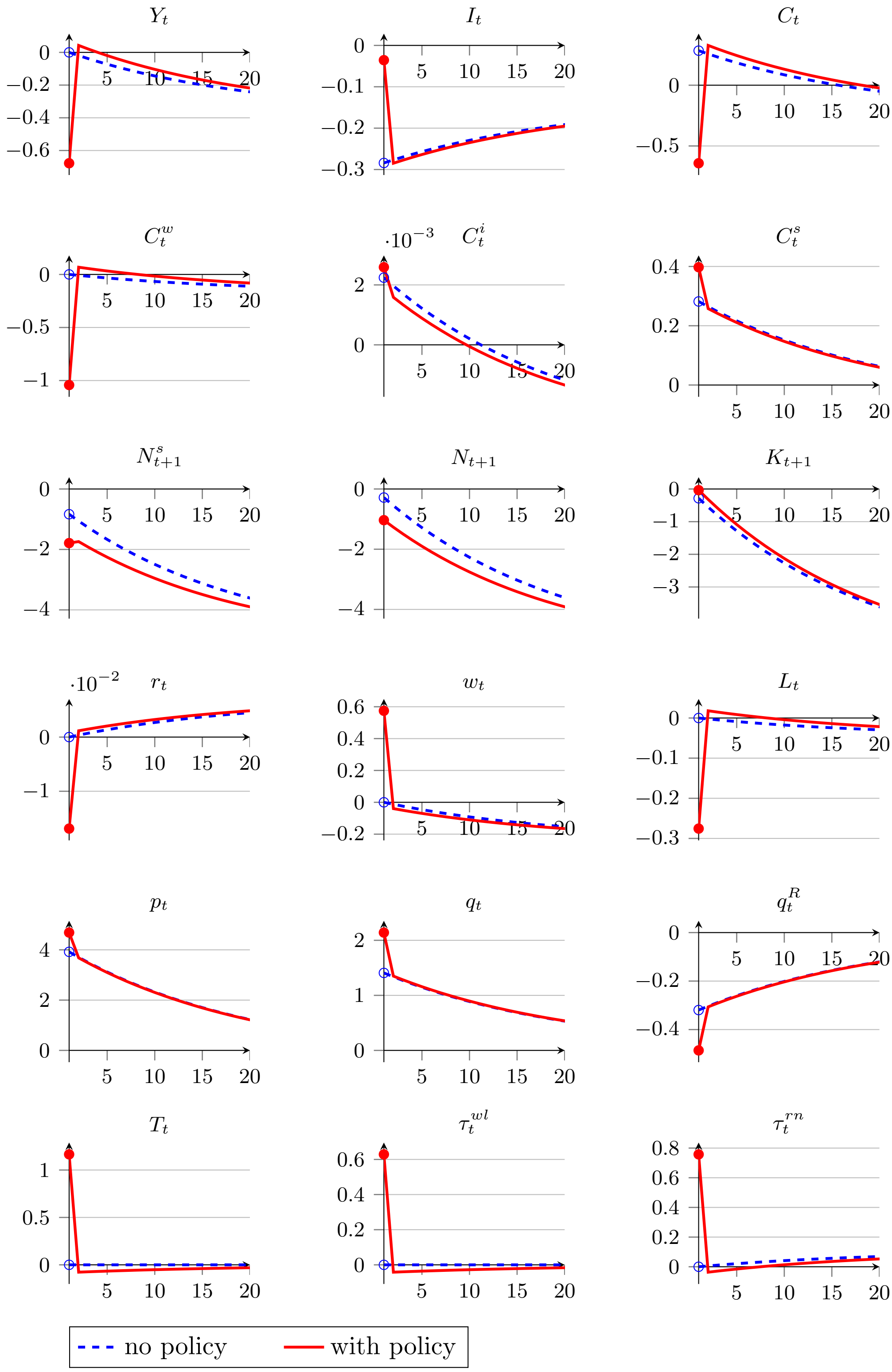
NOTES: The notes in Figure 7.1 apply. See the "quarter 1" column of Panel A in Table 7.8 for the values of immediate responses.

FIGURE 7.10: Impulse responses to a T-financed N^g-shock, 200 quarters



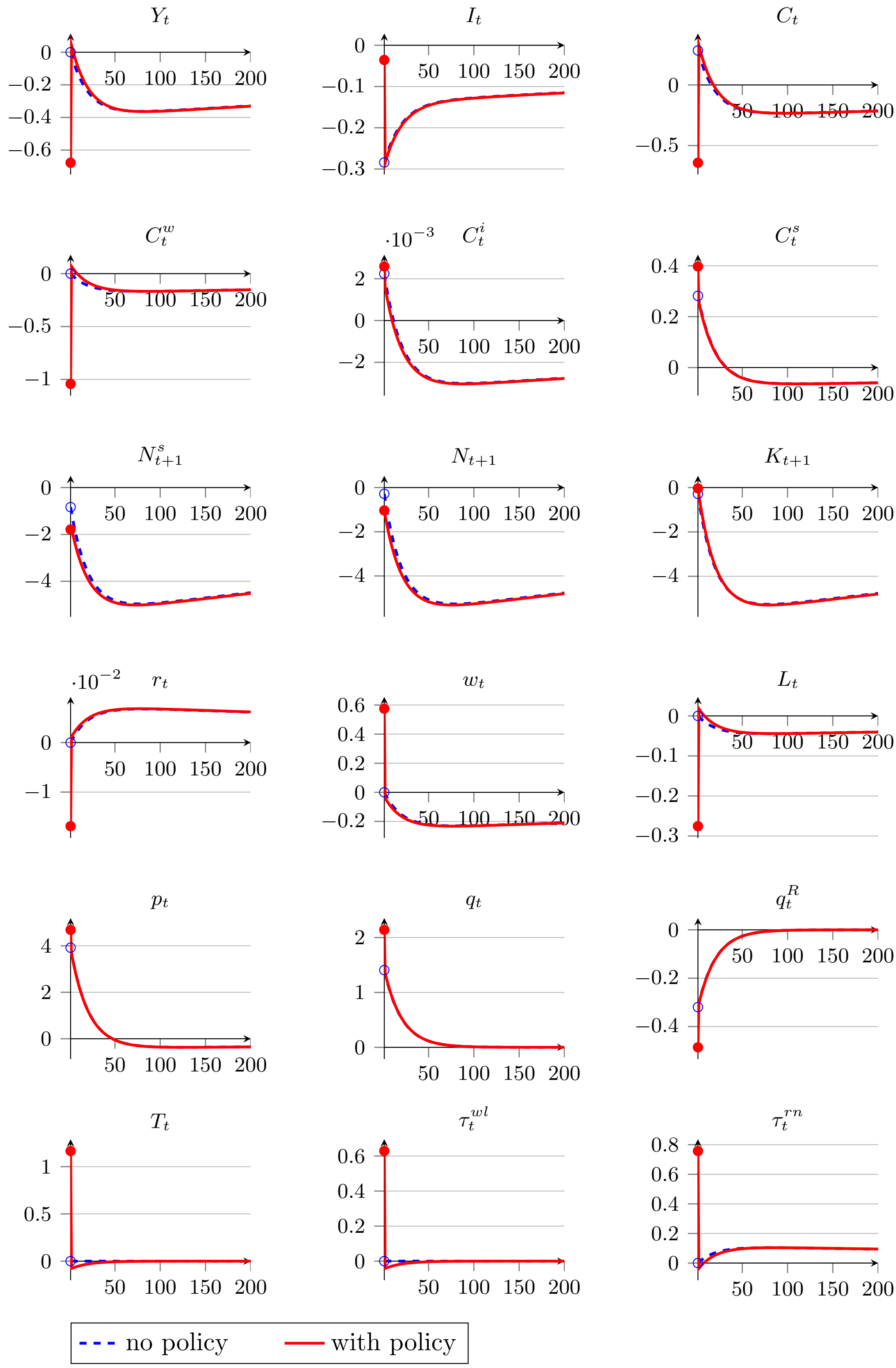
NOTES: These graphs extend Figure 7.9 to 200 quarters. The same notes apply.

FIGURE 7.11: Impulse responses to a liquidity shock and T-financed N^g-policy, 20 quarters



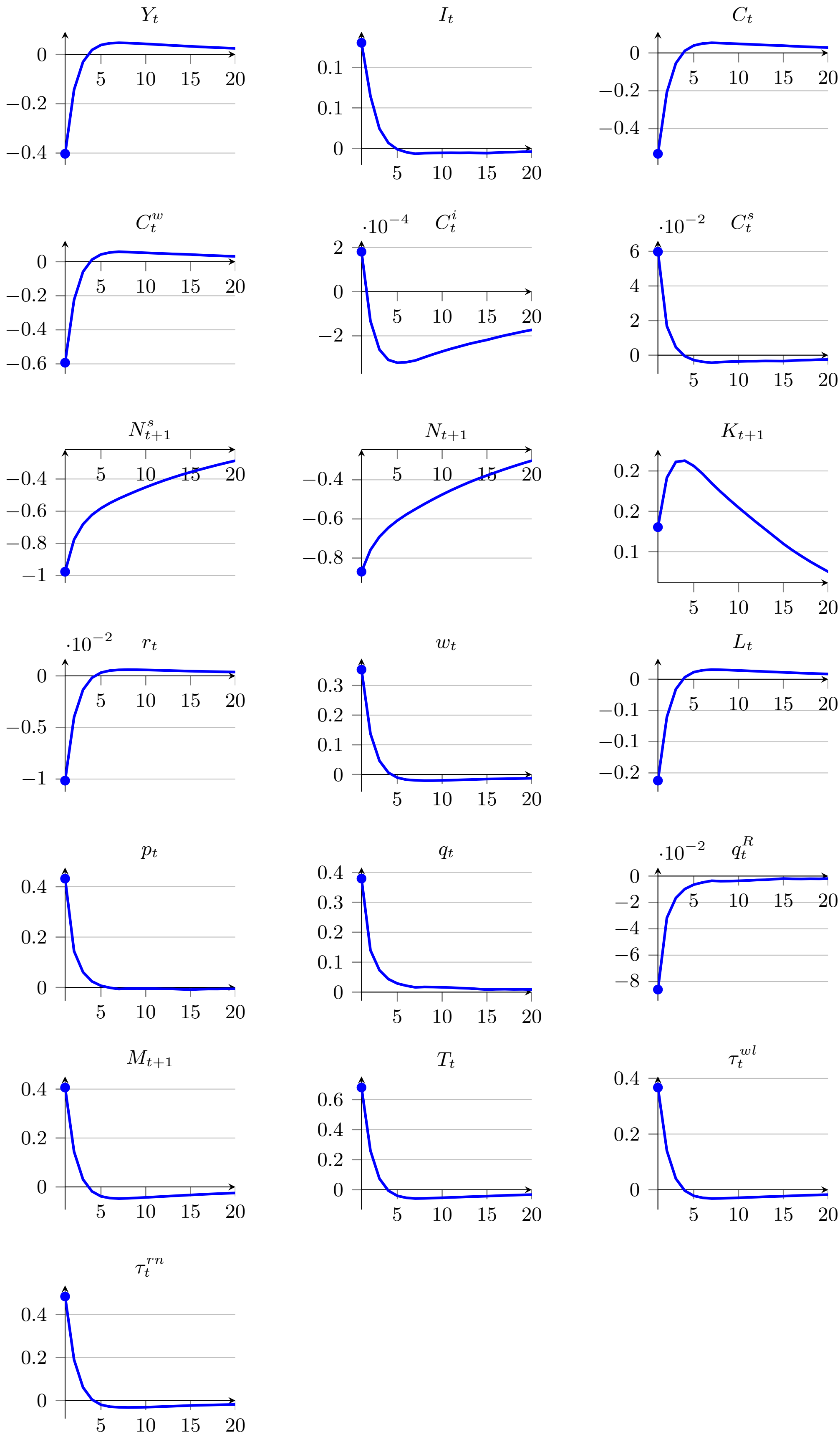
NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Figure 7.1 apply. Hollow blue dots and solid red dots indicate immediate responses of the liquidity shock without and with N^g-policy, respectively; see the "quarter 1" column of Panel A in Table 7.9 and Table 7.10, respectively, for their values.

FIGURE 7.12: Impulse responses to a liquidity shock and T-financed N^g-policy, 200 quarters



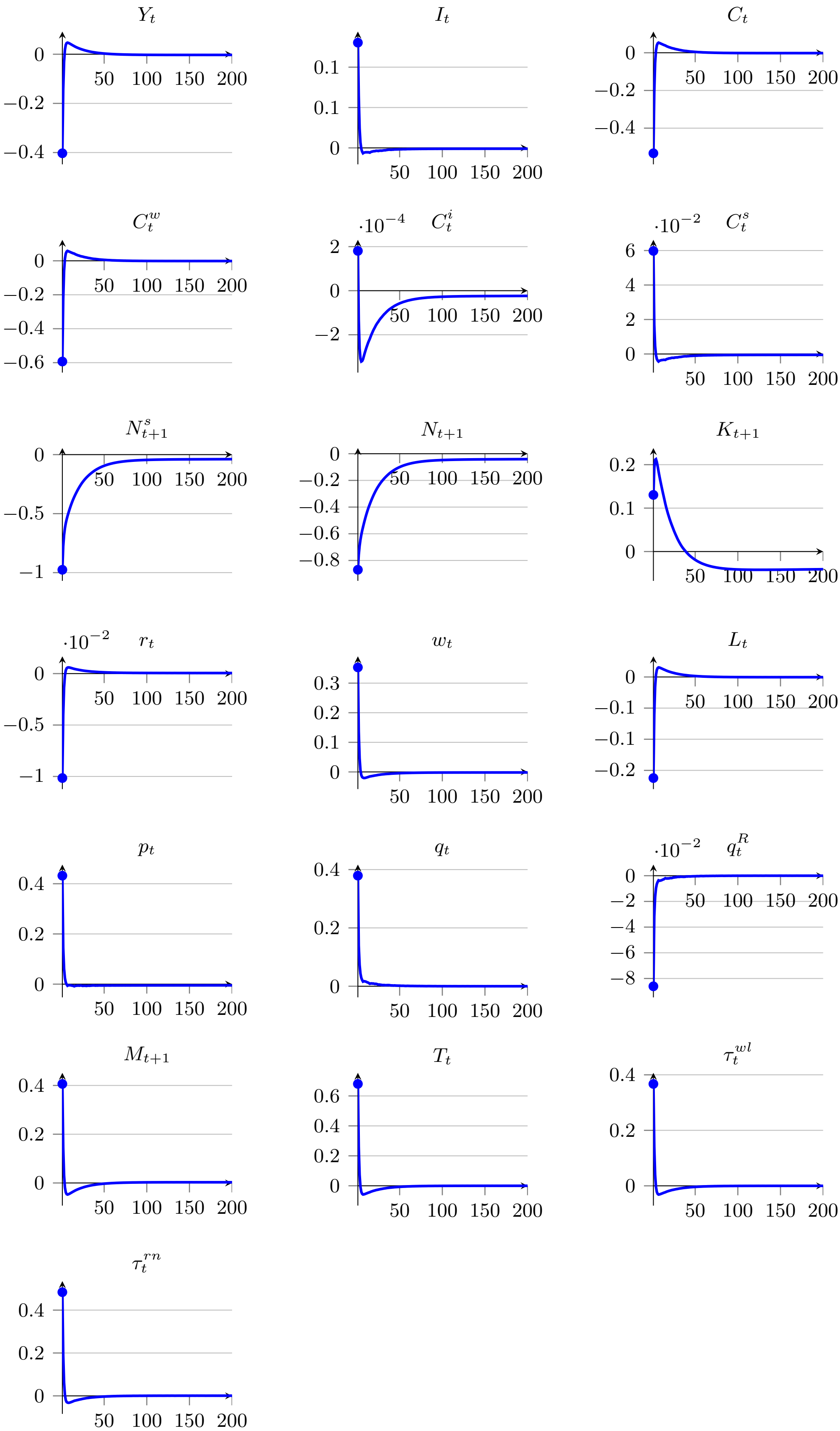
NOTES: These graphs extend Figure 7.11 to 200 quarters. The same notes apply.

FIGURE 7.13: Impulse responses to a TM-financed N^g-shock, 20 quarters



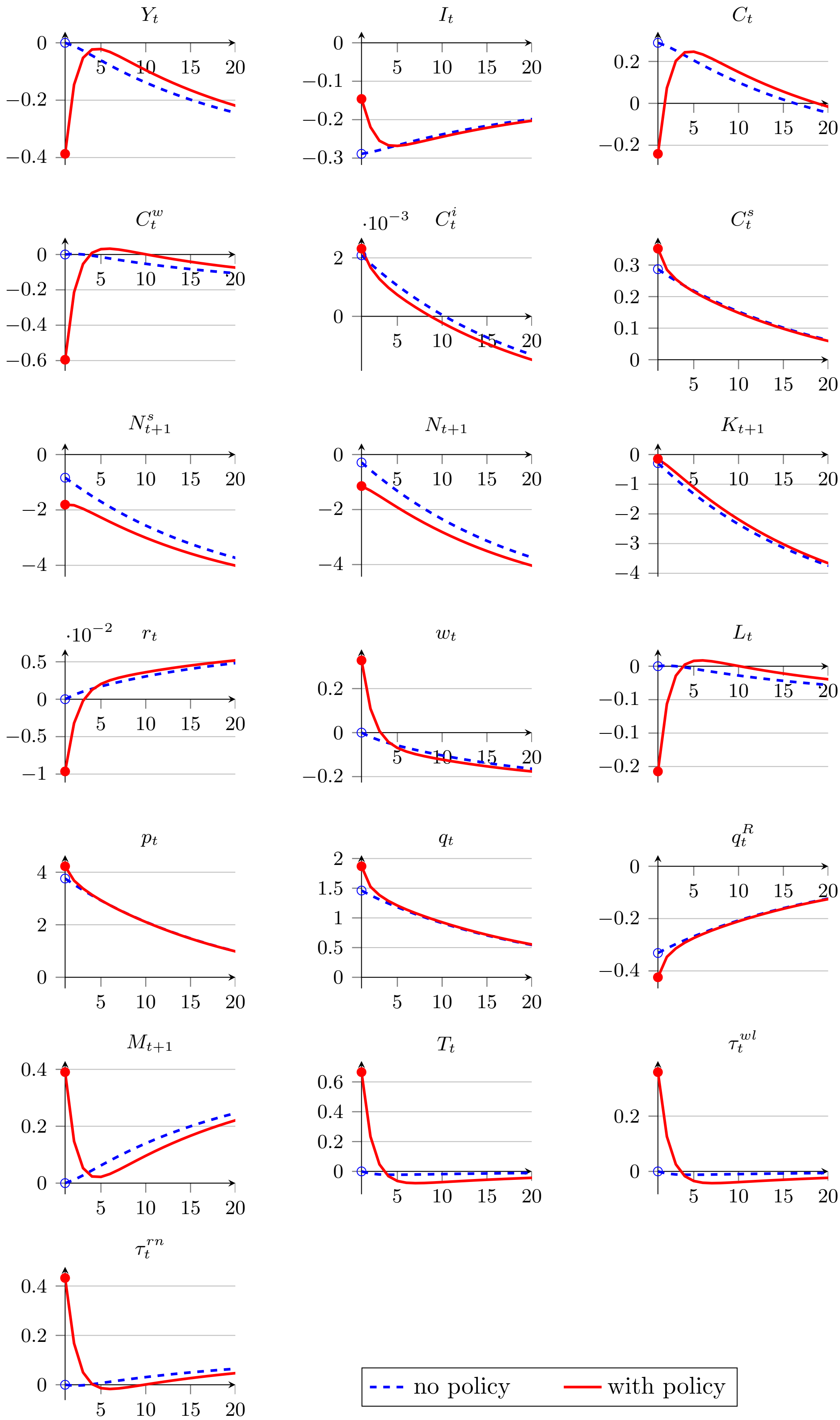
NOTES: The notes in Figure 7.1 apply. See the "quarter 1" column of Panel A in Table 7.11 for the values of immediate responses.

FIGURE 7.14: Impulse responses to a TM-financed N^g-shock, 200 quarters



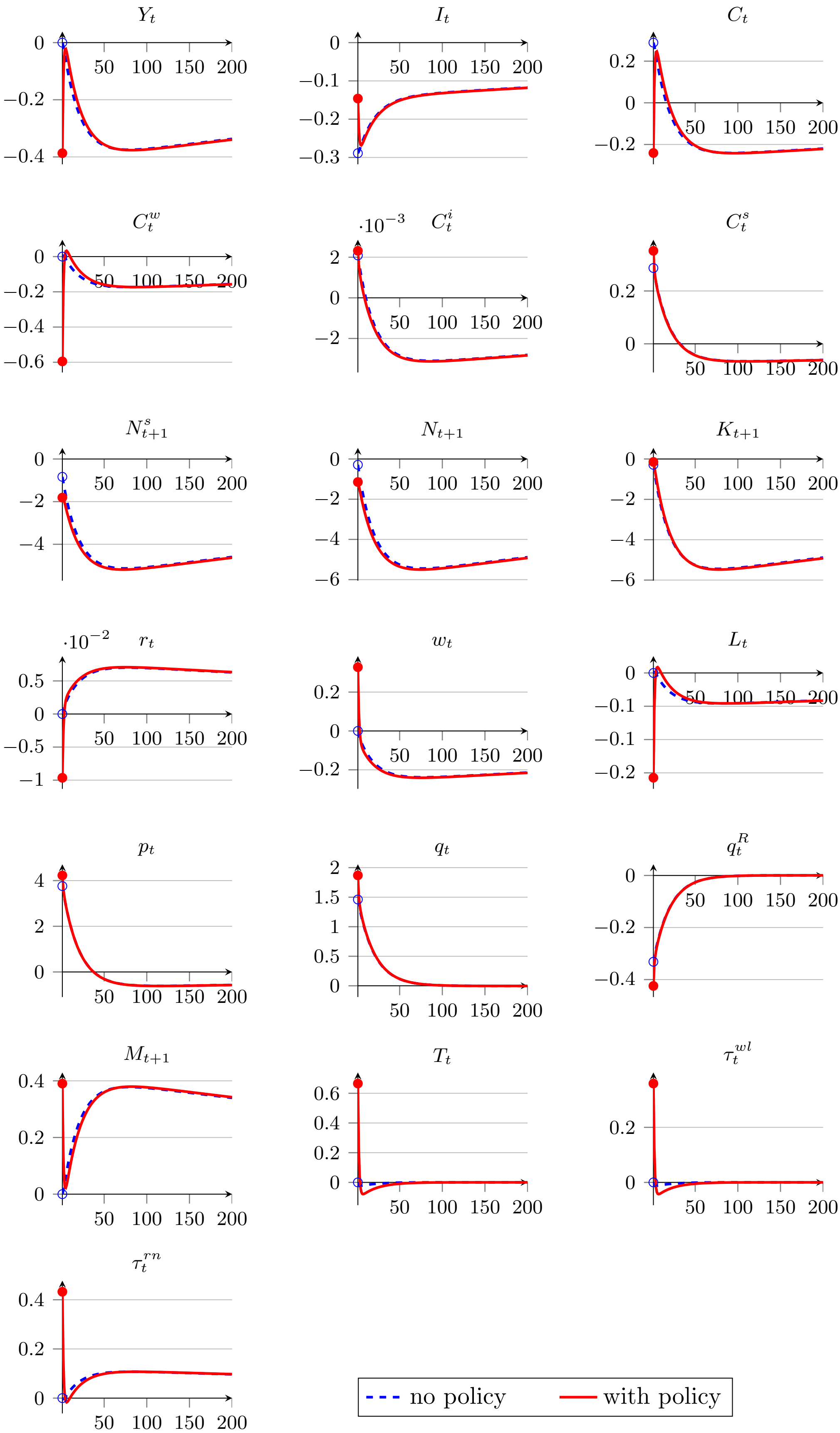
NOTES: These graphs extend Figure 7.13 to 200 quarters. The same notes apply.

FIGURE 7.15: Impulse responses to a liquidity shock and TM-financed N^g-policy, 20 quarters



NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Figure 7.1 apply. Hollow blue dots and solid red dots indicate immediate responses of the liquidity shock without and with N^g-policy, respectively; see the "quarter 1" column of Panel A in Table 7.12 and Table 7.13, respectively, for their values.

FIGURE 7.16: Impulse responses to a liquidity shock and TM-financed N^g-policy, 200 quarters



NOTES: These graphs extend Figure 7.15 to 200 quarters. The same notes apply.

TABLE 7.2: Responses to a G-financed N^g-shock

	Panel A: Impulse responses in levels							Panel B: Dynamic impact multipliers							Panel C	
	Quarter: 1	**2**	**4**	**8**	**20**	**200**	**largest**	**Quarter: 1**	**2**	**4**	**8**	**20**	**200**	**largest**	**Quarters to: largest**	**converge**
Y_t	0.00	0.07	0.06	0.05	0.02	0.00	0.07	0.00	0.06	0.05	0.04	0.02	0.00	0.06	2	36
I_t	1.03	−0.03	−0.03	−0.03	−0.01	0.00	1.03	0.88	−0.03	−0.03	−0.02	−0.01	0.00	0.88	1	2
C_t	0.42	0.02	0.02	0.01	0.00	0.00	0.42	0.36	0.02	0.01	0.01	0.00	0.00	0.36	1	2
C_t^w	0.00	0.03	0.03	0.02	0.01	0.00	0.03	0.00	0.03	0.02	0.02	0.01	0.00	0.03	2	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	2
C_t^s	0.42	−0.01	−0.01	−0.01	−0.01	0.00	0.42	0.36	−0.01	−0.01	−0.01	0.00	0.00	0.36	1	2
N_{t+1}^s	−0.81	0.05	0.03	0.00	−0.05	−0.01	−0.81	−0.81	0.05	0.03	0.00	−0.05	−0.01	−0.81	1	2
N_{t+1}	0.03	0.02	0.01	−0.02	−0.06	−0.01	−0.07	0.03	0.02	0.01	−0.02	−0.06	−0.01	−0.07	39	201
K_{t+1}	1.03	0.97	0.86	0.68	0.32	−0.01	1.03	1.03	0.97	0.86	0.68	0.32	−0.01	1.03	1	36
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2	36
w_t	0.00	0.04	0.03	0.03	0.01	0.00	0.04	0.00	0.03	0.03	0.02	0.01	0.00	0.03	2	201
L_t	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.02	0.01	0.01	0.01	0.00	0.02	2	201
p_t	2.35	−0.13	−0.12	−0.10	−0.05	0.00	2.35	2.02	−0.11	−0.10	−0.09	−0.04	0.00	2.02	1	2
q_t	2.15	−0.05	−0.04	−0.04	−0.02	0.00	2.15	1.84	−0.04	−0.04	−0.03	−0.01	0.00	1.84	1	2
q_t^R	−0.49	0.01	0.01	0.01	0.00	0.00	−0.49	−0.42	0.01	0.01	0.01	0.00	0.00	−0.42	1	2
G_t	−1.45	0.08	0.07	0.06	0.03	0.00	−1.45	−1.24	0.07	0.06	0.05	0.03	0.00	−1.24	1	2
T_t	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2	201

NOTES: Panel A gives deviations from steady state in levels. Panel B gives dynamic impact multipliers which are computed according to the methodology outlined in Section 4.4.1. Panel C gives the time at which the absolute largest impulse response occurs, and the convergence indicator which is described in Section 4.4.2.

Table 7.3: Impulse responses to a liquidity shock: G-financed model variant

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	−0.02	−0.05	−0.11	−0.21	−0.04	−0.24	36	201
I_t	−0.28	−0.27	−0.25	−0.21	−0.14	−0.01	−0.28	1	93
C_t	0.28	0.25	0.21	0.13	−0.02	−0.02	0.28	1	184
C_t^w	0.00	−0.01	−0.02	−0.05	−0.10	−0.02	−0.11	36	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	184
C_t^s	0.28	0.26	0.23	0.18	0.08	−0.01	0.28	1	31
N_{t+1}^s	−0.83	−1.05	−1.45	−2.10	−3.15	−0.50	−3.43	33	201
N_{t+1}	−0.28	−0.54	−1.01	−1.79	−3.09	−0.53	−3.50	35	201
K_{t+1}	−0.28	−0.54	−1.01	−1.79	−3.09	−0.53	−3.50	35	201
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	36	201
w_t	0.00	−0.01	−0.03	−0.07	−0.13	−0.02	−0.15	36	201
L_t	0.00	0.00	−0.01	−0.01	−0.03	0.00	−0.03	36	201
p_t	4.09	3.88	3.50	2.85	1.54	0.00	4.09	1	46
q_t	1.35	1.29	1.17	0.96	0.54	0.01	1.35	1	52
q_t^R	−0.31	−0.29	−0.26	−0.22	−0.12	0.00	−0.31	1	52
G_t	0.00	0.00	−0.01	−0.02	−0.05	−0.01	−0.05	36	201
T_t	0.00	0.00	−0.01	−0.02	−0.05	−0.01	−0.05	36	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. These responses are without N^g-policy. Panel A gives deviations from steady state in levels. Panel B gives the time at which the absolute largest impulse response occurs, and the convergence indicator which is described in Section 4.4.2.

Table 7.4: Impulse responses to a liquidity shock with G-financed N^g-policy

	Panel A: Impulse responses in levels							Panel B	
	Quarter:							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	0.00	0.04	0.00	−0.07	−0.20	−0.04	−0.25	39	201
I_t	0.55	−0.30	−0.28	−0.24	−0.16	−0.01	0.55	1	60
C_t	0.62	0.27	0.22	0.13	−0.02	−0.02	0.62	1	127
C_t^w	0.00	0.02	0.00	−0.03	−0.09	−0.02	−0.11	39	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	127
C_t^s	0.62	0.25	0.22	0.17	0.07	−0.01	0.62	1	22
N_{t+1}^s	−1.68	−1.20	−1.61	−2.28	−3.37	−0.53	−3.64	33	201
N_{t+1}	−0.45	−0.72	−1.21	−2.00	−3.33	−0.56	−3.73	35	201
K_{t+1}	0.55	0.23	−0.35	−1.30	−2.95	−0.56	−3.57	38	201
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	39	201
w_t	0.00	0.02	0.00	−0.05	−0.13	−0.02	−0.16	39	201
L_t	0.00	0.00	0.00	−0.01	−0.02	0.00	−0.03	39	201
p_t	6.22	3.74	3.38	2.75	1.48	0.00	6.22	1	37
q_t	3.37	1.23	1.12	0.92	0.52	0.01	3.37	1	30
q_t^R	−0.76	−0.28	−0.25	−0.21	−0.12	0.00	−0.76	1	30
G_t	−1.17	0.08	0.06	0.03	−0.02	−0.01	−1.17	1	2
T_t	0.00	0.00	−0.01	−0.02	−0.05	−0.01	−0.06	37	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Table 7.3 apply.

TABLE 7.5: Responses to a GM-financed N^g-shock

	Panel A: Impulse responses in levels							Panel B: Dynamic impact multipliers							Panel C	
	Quarter:							**Quarter:**							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**1**	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	0.00	0.06	0.05	0.04	0.02	0.00	0.06	0.00	0.05	0.05	0.04	0.02	0.00	0.05	2	39
I_t	1.00	−0.08	−0.03	−0.02	−0.01	0.00	1.00	0.86	−0.07	−0.02	−0.02	−0.01	0.00	0.86	1	2
C_t	0.41	0.00	0.01	0.01	0.01	0.00	0.41	0.35	0.00	0.01	0.01	0.00	0.00	0.35	1	2
C_t^w	0.00	0.03	0.02	0.02	0.01	0.00	0.03	0.00	0.03	0.02	0.02	0.01	0.00	0.03	2	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	2
C_t^s	0.41	−0.03	−0.01	−0.01	0.00	0.00	0.41	0.35	−0.03	−0.01	−0.01	0.00	0.00	0.35	1	2
N_{t+1}^s	−0.81	0.01	−0.03	−0.03	−0.05	−0.01	−0.81	−0.81	0.01	−0.03	−0.03	−0.05	−0.01	−0.81	1	2
N_{t+1}	0.00	−0.05	−0.05	−0.06	−0.06	−0.01	−0.06	0.00	−0.05	−0.05	−0.06	−0.06	−0.01	−0.06	13	201
K_{t+1}	1.00	0.90	0.80	0.64	0.32	−0.01	1.00	1.00	0.90	0.80	0.64	0.32	−0.01	1.00	1	38
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2	201
w_t	0.00	0.04	0.03	0.03	0.01	0.00	0.04	0.00	0.03	0.03	0.02	0.01	0.00	0.03	2	39
L_t	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.02	0.01	0.01	0.01	0.00	0.02	2	201
p_t	2.15	−0.22	−0.10	−0.09	−0.04	0.00	2.15	1.85	−0.19	−0.09	−0.08	−0.03	0.00	1.85	1	3
q_t	2.08	−0.14	−0.04	−0.04	−0.01	0.00	2.08	1.79	−0.12	−0.03	−0.03	−0.01	0.00	1.79	1	2
q_t^R	−0.47	0.03	0.01	0.01	0.00	0.00	−0.47	−0.41	0.03	0.01	0.01	0.00	0.00	−0.41	1	2
M_{t+1}	0.00	0.06	0.05	0.04	0.02	0.00	0.06	0.00	0.07	0.06	0.05	0.02	0.00	0.07	2	201
G_t	−1.41	0.14	0.06	0.06	0.03	0.00	−1.41	−1.21	0.12	0.06	0.05	0.02	0.00	−1.21	1	3
T_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2	28

NOTES: The notes in Table 7.2 apply.

Table 7.6: Impulse responses to a liquidity shock: GM-financed model variant

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	−0.02	−0.05	−0.10	−0.20	−0.04	−0.23	38	201
I_t	−0.28	−0.25	−0.23	−0.20	−0.13	−0.01	−0.28	1	101
C_t	0.28	0.26	0.21	0.13	−0.01	−0.03	0.28	1	194
C_t^w	0.00	−0.01	−0.02	−0.05	−0.09	−0.02	−0.11	38	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	194
C_t^s	0.27	0.26	0.23	0.18	0.08	−0.01	0.27	1	32
N_{t+1}^s	−0.83	−1.05	−1.41	−2.00	−2.99	−0.58	−3.27	34	201
N_{t+1}	−0.28	−0.52	−0.96	−1.68	−2.92	−0.61	−3.34	37	201
K_{t+1}	−0.28	−0.52	−0.96	−1.68	−2.92	−0.61	−3.34	37	201
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	38	201
w_t	0.00	−0.01	−0.03	−0.07	−0.12	−0.03	−0.15	38	201
L_t	0.00	0.00	−0.01	−0.01	−0.02	−0.01	−0.03	38	201
p_t	4.19	4.03	3.65	3.00	1.67	0.02	4.19	1	51
q_t	1.32	1.29	1.17	0.97	0.55	0.01	1.32	1	54
q_t^R	−0.30	−0.29	−0.27	−0.22	−0.12	0.00	−0.30	1	54
M_{t+1}	0.00	−0.02	−0.05	−0.11	−0.20	−0.04	−0.23	38	201
G_t	0.00	−0.03	−0.03	−0.04	−0.05	−0.01	−0.05	30	201
T_t	0.00	0.00	−0.01	−0.02	−0.04	−0.01	−0.05	38	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. These responses are without N^g-policy. The notes in Table 7.3 apply.

Table 7.7: Impulse responses to a liquidity shock with GM-financed N^g-policy

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	0.04	−0.01	−0.07	−0.19	−0.04	−0.23	41	201
I_t	0.55	−0.34	−0.26	−0.22	−0.15	−0.01	0.55	1	61
C_t	0.62	0.25	0.22	0.14	−0.01	−0.03	0.62	1	131
C_t^w	0.00	0.02	0.00	−0.03	−0.09	−0.02	−0.11	41	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1	131
C_t^s	0.61	0.23	0.22	0.17	0.07	−0.01	0.61	1	23
N_{t+1}^s	−1.68	−1.20	−1.61	−2.21	−3.20	−0.61	−3.47	34	201
N_{t+1}	−0.45	−0.75	−1.19	−1.91	−3.15	−0.64	−3.55	36	201
K_{t+1}	0.55	0.20	−0.33	−1.22	−2.77	−0.64	−3.40	40	201
r_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	41	201
w_t	0.00	0.02	0.00	−0.04	−0.12	−0.03	−0.15	41	201
L_t	0.00	0.00	0.00	−0.01	−0.02	−0.01	−0.03	41	201
p_t	6.31	3.76	3.52	2.90	1.62	0.02	6.31	1	41
q_t	3.33	1.11	1.12	0.93	0.53	0.01	3.33	1	31
q_t^R	−0.76	−0.25	−0.25	−0.21	−0.12	0.00	−0.76	1	31
M_{t+1}	0.00	0.04	−0.01	−0.07	−0.19	−0.04	−0.24	41	201
G_t	−1.17	0.12	0.04	0.01	−0.02	−0.01	−1.17	1	3
T_t	0.00	0.00	−0.01	−0.02	−0.04	−0.01	−0.05	38	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Table 7.3 apply.

TABLE 7.8: Responses to a T-financed N^g-shock

	Panel A: Impulse responses in levels							Panel B: Dynamic impact multipliers							Panel C	
	Quarter:							**Quarter:**							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**1**	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	−0.76	0.06	0.05	0.04	0.02	0.00	−0.76	−0.66	0.05	0.05	0.04	0.02	0.00	−0.66	1	2
I_t	0.23	−0.01	−0.01	−0.01	0.00	0.00	0.23	0.20	−0.01	−0.01	0.00	0.00	0.00	0.20	1	2
C_t	−0.99	0.07	0.06	0.05	0.03	0.00	−0.99	−0.85	0.06	0.05	0.04	0.02	0.00	−0.85	1	2
C_t^w	−1.10	0.08	0.07	0.05	0.03	0.00	−1.10	−0.95	0.07	0.06	0.05	0.03	0.00	−0.95	1	2
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2	2
C_t^s	0.11	−0.01	−0.01	0.00	0.00	0.00	0.11	0.09	−0.01	−0.01	0.00	0.00	0.00	0.09	1	2
N_{t+1}^s	−0.96	−0.70	−0.63	−0.52	−0.30	−0.04	−0.96	−0.96	−0.70	−0.63	−0.52	−0.30	−0.04	−0.96	1	53
N_{t+1}	−0.77	−0.73	−0.67	−0.55	−0.32	−0.04	−0.77	−0.77	−0.73	−0.67	−0.55	−0.32	−0.04	−0.77	1	65
K_{t+1}	0.23	0.22	0.19	0.15	0.06	−0.04	0.23	0.23	0.22	0.19	0.15	0.06	−0.04	0.23	1	201
r_t	−0.02	0.00	0.00	0.00	0.00	0.00	−0.02	−0.02	0.00	0.00	0.00	0.00	0.00	−0.02	1	2
w_t	0.68	−0.03	−0.03	−0.02	−0.01	0.00	0.68	0.58	−0.02	−0.02	−0.02	−0.01	0.00	0.58	1	2
L_t	−0.31	0.02	0.02	0.01	0.01	0.00	−0.31	−0.64	0.04	0.04	0.03	0.02	0.00	−0.64	1	2
p_t	0.72	−0.02	−0.02	−0.02	−0.01	0.00	0.72	0.62	−0.02	−0.02	−0.01	−0.01	0.00	0.62	1	2
q_t	0.69	0.01	0.01	0.01	0.01	0.00	0.69	0.59	0.01	0.01	0.01	0.00	0.00	0.59	1	2
q_t^R	−0.16	0.00	0.00	0.00	0.00	0.00	−0.16	−0.13	0.00	0.00	0.00	0.00	0.00	−0.13	1	2
T_t	1.27	−0.08	−0.07	−0.06	−0.03	0.00	1.27	1.09	−0.07	−0.06	−0.05	−0.03	0.00	1.09	1	2
τ_t^{wl}	0.69	−0.04	−0.04	−0.03	−0.02	0.00	0.69	0.59	−0.04	−0.03	−0.03	−0.01	0.00	0.59	1	2
τ_t^{rn}	0.93	−0.04	−0.04	−0.03	−0.02	0.00	0.93	0.80	−0.04	−0.03	−0.03	−0.01	0.00	0.80	1	2

NOTES: The notes in Table 7.2 apply.

Table 7.9: Impulse responses to a liquidity shock: T-financed model variant

	Panel A: Impulse responses in levels							Panel B	
	Quarter:						**largest**	**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**		**largest**	**converge**
Y_t	0.00	−0.02	−0.06	−0.12	−0.24	−0.33	−0.36	80	201
I_t	−0.28	−0.28	−0.26	−0.24	−0.19	−0.11	−0.28	1	201
C_t	0.28	0.26	0.21	0.12	−0.05	−0.21	0.28	1	201
C_t^w	0.00	−0.01	−0.03	−0.05	−0.11	−0.15	−0.17	80	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90	201
C_t^s	0.28	0.26	0.23	0.18	0.06	−0.06	0.28	1	201
N_{t+1}^s	−0.84	−1.06	−1.48	−2.19	−3.61	−4.49	−4.97	76	201
N_{t+1}	−0.28	−0.55	−1.05	−1.91	−3.61	−4.77	−5.27	79	201
K_{t+1}	−0.28	−0.55	−1.05	−1.91	−3.61	−4.77	−5.27	79	201
r_t	0.00	0.00	0.00	0.00	0.00	0.01	0.01	80	201
w_t	0.00	−0.01	−0.04	−0.07	−0.15	−0.21	−0.23	80	201
L_t	0.00	0.00	−0.01	−0.01	−0.03	−0.04	−0.04	80	201
p_t	3.92	3.70	3.30	2.61	1.23	−0.35	3.92	1	34
q_t	1.41	1.34	1.21	0.98	0.53	0.00	1.41	1	46
q_t^R	−0.32	−0.30	−0.27	−0.22	−0.12	0.00	−0.32	1	46
T_t	0.00	0.00	0.00	0.00	0.00	0.00	0.00	198	201
τ_t^{wl}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	171	201
τ_t^{rn}	0.00	0.01	0.02	0.03	0.07	0.09	0.10	80	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. These responses are without N^g-policy. The notes in Table 7.3 apply.

Table 7.10: Impulse responses to a liquidity shock with T-financed N^g-policy

	Panel A: Impulse responses in levels							Panel B	
	Quarter:							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	−0.68	0.04	0.00	−0.07	−0.22	−0.33	−0.68	1	201
I_t	−0.04	−0.28	−0.27	−0.25	−0.20	−0.12	−0.28	2	201
C_t	−0.64	0.33	0.27	0.17	−0.02	−0.22	−0.64	1	201
C_t^w	−1.04	0.07	0.04	0.00	−0.08	−0.15	−1.04	1	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	88	201
C_t^s	0.40	0.26	0.23	0.17	0.06	−0.06	0.40	1	201
N_{t+1}^s	−1.79	−1.74	−2.09	−2.70	−3.90	−4.52	−5.01	73	201
N_{t+1}	−1.04	−1.27	−1.70	−2.44	−3.91	−4.80	−5.32	76	201
K_{t+1}	−0.04	−0.32	−0.84	−1.74	−3.54	−4.80	−5.30	80	201
r_t	−0.02	0.00	0.00	0.00	0.00	0.01	−0.02	1	201
w_t	0.57	−0.04	−0.06	−0.10	−0.17	−0.21	0.57	1	201
L_t	−0.28	0.02	0.01	0.00	−0.02	−0.04	−0.28	1	201
p_t	4.69	3.68	3.28	2.60	1.22	−0.35	4.69	1	33
q_t	2.14	1.35	1.22	0.99	0.54	0.00	2.14	1	38
q_t^R	−0.49	−0.31	−0.28	−0.23	−0.12	0.00	−0.49	1	38
T_t	1.17	−0.08	−0.07	−0.06	−0.03	0.00	1.17	1	2
τ_t^{wl}	0.63	−0.04	−0.04	−0.03	−0.02	0.00	0.63	1	2
τ_t^{rn}	0.76	−0.04	−0.02	0.00	0.05	0.09	0.76	1	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Table 7.3 apply.

Table 7.11: Responses to a TM-financed N^g-shock

	Panel A: Impulse responses in levels							Panel B: Dynamic impact multipliers							Panel C	
	Quarter:							**Quarter:**							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**1**	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	−0.40	−0.14	0.02	0.05	0.02	0.00	−0.40	−0.35	−0.12	0.02	0.04	0.02	0.00	−0.35	1	12
I_t	0.13	0.06	0.01	−0.01	0.00	0.00	0.13	0.11	0.06	0.01	−0.01	0.00	0.00	0.11	1	4
C_t	−0.53	−0.21	0.01	0.05	0.03	0.00	−0.53	−0.46	−0.18	0.01	0.04	0.02	0.00	−0.46	1	8
C_t^w	−0.59	−0.22	0.01	0.06	0.03	0.00	−0.59	−0.51	−0.19	0.01	0.05	0.03	0.00	−0.51	1	3
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5	8
C_t^s	0.06	0.02	0.00	0.00	0.00	0.00	0.06	0.05	0.01	0.00	0.00	0.00	0.00	0.05	1	3
N_{t+1}^s	−0.98	−0.78	−0.62	−0.50	−0.29	−0.04	−0.98	−0.98	−0.78	−0.62	−0.50	−0.29	−0.04	−0.98	1	49
N_{t+1}	−0.87	−0.76	−0.64	−0.52	−0.30	−0.04	−0.87	−0.87	−0.76	−0.64	−0.52	−0.30	−0.04	−0.87	1	56
K_{t+1}	0.13	0.19	0.21	0.17	0.08	−0.04	0.21	0.13	0.19	0.21	0.17	0.08	−0.04	0.21	4	201
r_t	−0.01	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01	0.00	0.00	0.00	0.00	0.00	−0.01	1	4
w_t	0.35	0.14	0.01	−0.02	−0.01	0.00	0.35	0.30	0.12	0.01	−0.02	−0.01	0.00	0.30	1	4
L_t	−0.16	−0.06	0.00	0.02	0.01	0.00	−0.16	−0.34	−0.13	0.01	0.03	0.02	0.00	−0.34	1	3
p_t	0.43	0.14	0.02	0.00	−0.01	0.00	0.43	0.37	0.12	0.02	0.00	−0.01	0.00	0.37	1	4
q_t	0.38	0.14	0.04	0.02	0.01	0.00	0.38	0.33	0.12	0.04	0.01	0.01	0.00	0.33	1	5
q_t^R	−0.09	−0.03	−0.01	0.00	0.00	0.00	−0.09	−0.07	−0.03	−0.01	0.00	0.00	0.00	−0.07	1	5
M_{t+1}	0.41	0.14	−0.02	−0.05	−0.02	0.00	0.41	0.45	0.16	−0.02	−0.05	−0.03	0.00	0.45	1	12
T_t	0.68	0.26	−0.01	−0.06	−0.03	0.00	0.68	0.58	0.22	−0.01	−0.05	−0.03	0.00	0.58	1	4
τ_t^{wl}	0.37	0.14	0.00	−0.03	−0.02	0.00	0.37	0.32	0.12	0.00	−0.03	−0.02	0.00	0.32	1	4
τ_t^{rn}	0.48	0.19	0.00	−0.03	−0.02	0.00	0.48	0.42	0.16	0.00	−0.03	−0.02	0.00	0.42	1	4

NOTES: The notes in Table 7.2 apply.

Table 7.12: Impulse responses to a liquidity shock: TM-financed model variant

	Panel A: Impulse responses in levels							Panel B	
	Quarter:							**Quarters to:**	
	1	**2**	**4**	**8**	**20**	**200**	**largest**	**largest**	**converge**
Y_t	0.00	−0.01	−0.04	−0.11	−0.24	−0.34	−0.37	79	201
I_t	−0.29	−0.28	−0.27	−0.25	−0.20	−0.12	−0.29	1	201
C_t	0.29	0.27	0.23	0.14	−0.05	−0.22	0.29	1	201
C_t^w	0.00	0.00	−0.01	−0.04	−0.11	−0.16	−0.17	81	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	88	201
C_t^s	0.29	0.27	0.23	0.18	0.06	−0.06	0.29	1	201
N_{t+1}^s	−0.84	−1.07	−1.50	−2.24	−3.73	−4.60	−5.13	75	201
N_{t+1}	−0.29	−0.57	−1.08	−1.97	−3.74	−4.89	−5.45	77	201
K_{t+1}	−0.29	−0.57	−1.08	−1.97	−3.74	−4.89	−5.45	77	201
r_t	0.00	0.00	0.00	0.00	0.00	0.01	0.01	77	201
w_t	0.00	−0.02	−0.05	−0.09	−0.16	−0.21	−0.24	77	201
L_t	0.00	0.00	0.00	−0.01	−0.03	−0.04	−0.05	81	201
p_t	3.76	3.53	3.12	2.41	0.99	−0.57	3.76	1	201
q_t	1.46	1.38	1.24	1.01	0.55	0.00	1.46	1	46
q_t^R	−0.33	−0.31	−0.28	−0.23	−0.12	0.00	−0.33	1	46
M_{t+1}	0.00	0.01	0.04	0.11	0.25	0.34	0.38	79	201
T_t	0.00	−0.01	−0.02	−0.02	−0.01	0.00	−0.02	5	201
τ_t^{wl}	0.00	−0.01	−0.01	−0.01	−0.01	0.00	−0.01	5	201
τ_t^{rn}	0.00	0.00	0.00	0.02	0.06	0.10	0.11	81	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. These responses are without N^g-policy. The notes in Table 7.3 apply.

Table 7.13: Impulse responses to a liquidity shock with TM-financed N^g-policy

	Panel A: Impulse responses in levels							Panel B	
	Quarter: 1	**2**	**4**	**8**	**20**	**200**	**largest**	**Quarters to: largest**	**converge**
Y_t	−0.39	−0.15	−0.02	−0.06	−0.22	−0.34	−0.39	1	201
I_t	−0.15	−0.22	−0.27	−0.26	−0.20	−0.12	−0.27	5	201
C_t	−0.24	0.07	0.24	0.19	−0.02	−0.22	0.25	5	201
C_t^w	−0.60	−0.21	0.01	0.02	−0.07	−0.16	−0.60	1	201
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	86	201
C_t^s	0.35	0.29	0.23	0.17	0.06	−0.06	0.35	1	201
N_{t+1}^s	−1.81	−1.83	−2.11	−2.73	−4.01	−4.63	−5.19	72	201
N_{t+1}	−1.15	−1.31	−1.72	−2.48	−4.04	−4.92	−5.50	75	201
K_{t+1}	−0.15	−0.36	−0.86	−1.79	−3.66	−4.92	−5.48	78	201
r_t	−0.01	0.00	0.00	0.00	0.01	0.01	−0.01	1	201
w_t	0.33	0.11	−0.04	−0.11	−0.18	−0.22	0.33	1	201
L_t	−0.16	−0.06	0.00	0.01	−0.02	−0.04	−0.16	1	201
p_t	4.23	3.68	3.14	2.41	0.99	−0.57	4.23	1	201
q_t	1.87	1.53	1.29	1.03	0.55	0.00	1.87	1	41
q_t^R	−0.42	−0.35	−0.29	−0.23	−0.13	0.00	−0.42	1	41
M_{t+1}	0.39	0.15	0.02	0.06	0.22	0.34	0.39	1	201
T_t	0.67	0.23	−0.03	−0.08	−0.04	0.00	0.67	1	12
τ_t^{wl}	0.36	0.13	−0.02	−0.04	−0.02	0.00	0.36	1	12
τ_t^{rn}	0.43	0.17	0.00	−0.01	0.05	0.10	0.43	1	201

NOTES: The liquidity shock is a 1 percentage point decrease (tightening) in ϕ_t. N^g-policy is a 1 unit increase in N_t^g. The notes in Table 7.3 apply.

Table 7.14: Immediate responses to N^g-shocks: a comparison

	Panel A: Impulse responses in levels				Panel B: Immediate impact multipliers			
	Financing: G	GM	T	TM	**Financing:** G	GM	T	TM
Y_t	0.00	0.00	−0.76	−0.40	0.00	0.00	−0.66	−0.35
I_t	1.03	1.00	0.23	0.13	0.88	0.86	0.20	0.11
C_t	0.42	0.41	−0.99	−0.53	0.36	0.35	−0.85	−0.46
C_t^w	0.00	0.00	−1.10	−0.59	0.00	0.00	−0.95	−0.51
C_t^i	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C_t^s	0.42	0.41	0.11	0.06	0.36	0.35	0.09	0.05
N_{t+1}^s	−0.81	−0.81	−0.96	−0.98	−0.81	−0.81	−0.96	−0.98
N_{t+1}	0.03	0.00	−0.77	−0.87	0.03	0.00	−0.77	−0.87
K_{t+1}	1.03	1.00	0.23	0.13	1.03	1.00	0.23	0.13
r_t	0.00	0.00	−0.02	−0.01	0.00	0.00	−0.66	−0.01
w_t	0.00	0.00	0.68	0.35	0.00	0.00	0.58	0.30
L_t	0.00	0.00	−0.31	−0.16	0.00	0.86	−0.64	−0.34
p_t	2.35	2.15	0.72	0.43	2.02	1.85	0.62	0.37
q_t	2.15	2.08	0.69	0.38	1.84	0.00	0.59	0.33
q_t^R	−0.49	−0.47	−0.16	−0.09	−0.42	−0.41	−0.13	−0.07
M_{t+1}	0.00	0.00	0.00	0.41	0.00	0.00	0.00	0.45
G_t	−1.45	−1.41	0.00	0.00	−1.24	−0.81	0.00	0.00
T_t	0.00	0.00	1.27	0.68	0.00	0.00	1.09	0.58
τ_t^{wl}	0.00	0.00	0.69	0.37	0.00	0.00	0.59	0.32
τ_t^{rn}	0.00	0.00	0.93	0.48	0.00	0.00	0.80	0.42

NOTES: This table gives the first quarter impulse responses in levels and immediate impact multipliers from Tables 7.2, 7.5, 7.8 and 7.11.

Chapter 8

Sensitivity analysis

8.1 Introduction

This chapter analyses the sensitivity of the model's quantitative results to the calibration of its parameters. Sensitivity is analysed using the tax-shock experiment of Chapter 5 and is performed with respect to all of the model's parameters. Throughout the chapter, the results of the tax-shock in Section 5.3 are referred to as "baseline" responses.

The shortfall of the calibration technique is that it carries uncertainty over the model's parameters, which translates into uncertainty over the model's quantitative results. Hansen and Heckman (1996) recommend overcoming this problem with an explicit representation of such uncertainty from a sensitivity analysis. Their recommendation is what motivates this chapter.

A survey by Andronis et al. (2009) concludes that the literature lacks criteria by which results of a sensitivity analysis are to be evaluated. This chapter evaluates sensitivity by any qualitative changes or significant quantitative changes in impulse responses.

The chapter's overall conclusion is that the responses of the tax-shock are (i) qualitatively robust but quantitatively sensitive to the values of structural parameters; (ii) qualitatively and quantitatively robust to small (and plausible) variations in the persistence of shocks to tax rates; and (iii) qualitatively and quantitatively sensitive only to significant (and sometimes implausible) variations in the persistence of shocks to tax rates.

The rest of the chapter is organised as follows. Section 8.2 systematically analyses the sensitivity to structural parameters using three local methods which all involve repeated simulations of the tax-shock with the parameters' "sensitivity settings" that are listed in

Table 4.1. Section 8.2.1 is the first method, which changes one structural parameter at a time; Section 8.2.2 delivers the second and third methods, which change combinations of two or more structural parameters. Section 8.3 analyses sensitivity to the persistence of shocks to tax rates by repeatedly simulating the tax-shock with values of $\rho_{\tau rn}$ and $\rho_{\tau wl}$ that are above and below their calibrated (and fairly standard) settings. Section 8.4 summarises the chapter. Some additional algebra is given in Appendix 3.A. And figures and tables appear at the end of the chapter.

8.2 Sensitivity to structural parameters

8.2.1 One-at-a-time changes

The first approach to structural parameter sensitivity is a one-at-a-time (OAT) method – one parameter is changed to one of its sensitivity settings, and all other parameters remain at their baseline values; this is done for each and every parameter and for each and every sensitivity setting listed in Table 4.1.[1] The OAT method comprises 12 sets of results, which are graphically illustrated by impulse responses in Figures 8.1 to 8.7. The magnitude of immediate impulse responses from all 12 sensitivity simulations, as well as those from the baseline tax-shock, are listed in Table 8.1. Since their differences are due to non-uniform changes in parameter values, impulse responses on their own are unsuitable for comparing different scenarios or for establishing a common criteria to assess sensitivity. Parameter elasticities of impulse responses are computed according to Section 4.4.3 for these purposes. Elasticities from all 12 repeated simulations are given in Table 8.2.

Liquidity constraints

Simultaneously higher and lower calibrations of θ and ϕ are examined.[2] Figure 8.1 shows little variation in impulse responses from the baseline scenario. Moreover, the absolute value of parameter elasticities are less than unity for all variables except C_t^i, N_{t+1}^s, and p_t. The model is therefore not sensitive to the calibration of liquidity constraint parameters.

Varying the value of θ inversely influences the replacement cost of equity (see Appendix 3.A). Either directly or indirectly through q_t^R, the liquidity constraints enter negatively into

[1]The OAT method is similar to the "one-factor-at-a-time" method of Morris (1991), but is different in that is does not randomly select parameter values.

[2]This is unlike the experiment in Section 5.6 in which θ and ϕ are de-linked and only variations in θ are examined.

investors' consumption (Equation (3.56)) and positively into investment and equity's supply (Equations (3.38) and (3.59), respectively). Following a tax-shock, the tighter the liquidity constraints (that is, the lower the values of θ and ϕ), the higher the increase in investors' consumption and the lower the increases in equity's supply and investment; and conversely. This explains why I_t and C_t^i have positive and negative parameter elasticities, respectively. It also explains why money's price is the most sensitive variable to this parameter – from a tax-shock, the tighter the liquidity constraints, the smaller the increase in equity's supply and the greater the increase in equity's price (as if S_1^N is positioned more to the left than it appears in Figure 5.1A); then by a portfolio balance effect, the greater are the increases in money's demand and price. KM observe this movement from equity to money when liquidity constraints tighten, and they call it a "flight to liquidity". *Ceteris paribus*, tightening the liquidity constraints worsens the appeal of the partially liquid asset (equity) and encourages agents to substitute towards the more liquid asset (money), and the price of the liquid asset therefore increases; and conversely.

Subjective discount factor

Parameter elasticities indicate that tax-shock responses of all variables are very sensitive to changes in β; Figure 8.2 illustrates this. All of the parameter elasticities in Table 8.2 indicate that varying β produces the greatest amount of sensitivity among all of the OAT simulations. Elasticities for β are asymmetric, that is, the model is more sensitive to raising the parameter's value than lowering it.

β enters negatively into entrepreneurs' consumption (Equations (3.56) and (3.57)). *Ceteris paribus*, increasing β means entrepreneurs are more willing to delay consumption and spend their net worth more evenly over time.[3] As their patience increase, they consume less in the present. This explains the negative parameter elasticities for C_t^i and C_t^s with higher β. The consequences are higher levels of current saving and investment. Higher β then amplifies the increases in asset demands that are caused by the tax-shock. For money, this means a larger price increase compared to the baseline scenario, hence the positive parameter elasticity for p_t. For equity, there is also a greater supply response; the market adjusts to the shock with a smaller price increase than in the baseline scenario, hence the

[3] Given that workers' optimal behaviour involves them not saving for the future (from Equation (3.58)) then such changes are confined to entrepreneurs.

positive parameter elasticity for p_t. These variations in asset price impulse responses then propagate throughout the economy.

Conversely, *ceteris paribus*, lowering β means entrepreneurs become more impatient and consume more of their net worth in the present; this implies less saving and investment, and lower asset demands and equity supply. Lower β dampens the increase in p_t and amplifies the increase in q_t that are both caused by the tax-shock. Asset price increases feed back into improvements in entrepreneurs' net worth, and investors therefore consume more. This is why C_t^i has a positive parameter elasticity with a lowering of β. In other words, investors increase their consumption because of the net worth improvements they enjoy from the tax-shock's increase in asset prices; varying β up or down does not interfere with this, hence the difference in the sign of parameter elasticities for C_t^i. Net worth improvements also increase saving and investment. However, lowering β only partially offsets the increase in savings but completely offsets the increase in investment, hence the negative and positive parameter elasticities for C_t^s and I_t, respectively.

Capital's share in output

Figure 8.3 suggests that the model is sensitive to γ. Furthermore, elasticities are greater than 1 in absolute value for all variables except L_t and q_t. The only variables with negative elasticities are q_t and r_t. In other words, impulse responses of variables besides q_t and r_t are smaller when γ is lowered. γ enters the aggregate labour demand function and production function (Equations (3.51) and (3.60), respectively). *Ceteris paribus*, lowering γ positions the inverse aggregate labour demand function leftwards from its baseline calibration; this is illustrated in the first graph of Figure 8.12. Lowering γ therefore dampens the shock-induced increases in the real wage and employment, and hence output. This explains why the parameter elasticities of w_t, L_t, and Y_t are all positive. Changes in the goods market then propagate throughout the economy.

Survival rate after depreciation

Figure 8.4 shows small differences in impulse responses between baseline and higher settings for δ. But the change in δ is very small, and parameter elasticities reveal that the change in immediate impulse responses are relatively large. Elasticities are all above 20 in absolute value, making the model very sensitive to the parameter's value. In fact, this parameter is second to β in generating model sensitivity. *Ceteris paribus*, a higher δ means capital and equity stocks retain more of their value after depreciation each period. This effectively provides net worth improvements to entrepreneurs, which amplifies those already brought on by the tax-shock. However, increasing δ raises the appeal of equity. The shock-induced increase in demand for equity is thus amplified and creates a larger fall in q_t (as if D_1^N is further to the right than it appears in Figure 5.1A). This explains the negative parameter elasticity for p_t. Moreover, investors are able to invest more, given their net worth improvements, and they issue more equity, given its greater appeal. Investors also sacrifice consumption for much more investment, hence the negative parameter elasticity for C_t^i.

Probability of investment opportunity

Figure 8.5 suggests that changing the value of π does not significantly alter impulse responses, except for those of p_t. The parameter's elasticities are fairly similar between lowering and raising its value relative to the baseline setting. π enters positively into investors' consumption (Equation (3.56)), investment (Equation (3.59)), and the supply of equity (Equation (3.38)), and enters negatively into savers' consumption (Equation (3.57)). *Ceteris paribus*, raising the value of π increases the population of investors relative to savers; and conversely. Changing the parameter's value therefore shifts aggregate activity from saving to investment. Changing the value of π brings significant changes to the asset markets, but these are outweighed by the effects of the tax-shock. The economy is therefore hardly affected by variations in the parameter's value, hence the very small elasticities for most variables.

Inverse Frisch elasticity of labour supply

Figure 8.6 shows some variation in impulse responses from changes in ν. Parameter elasticities indicate that a minority of variables are sensitive to the parameter, although these elasticities are marginally above 1 and therefore the degree of sensitivity is mild. Elasticities also suggest that the model is more sensitive to lowering the parameter than raising it. Overall, the model is not sensitive to the calibration of ν. Changing the parameter's value affects the economy through the aggregate labour supply function. The baseline setting $\nu = 1$ makes the inverse function (Equation (3.52)) linear in w_t. The inverse function is convex if $\nu < 1$ and concave if $\nu > 1$. These variations in the shape of labour market functions are illustrated in the second graph of Figure 8.12 and they are largely responsible for any deviations of impulse responses from the baseline scenario.

Relative utility weight on labour

Although large deviations in impulse responses are shown in Figure 8.7, these are brought on by large changes in the value of ω. Parameter elasticities provide a more reliable assessment of sensitivity. Since no variable has an elasticity above 1 in absolute value, then the model is not sensitive to raising the parameter's value. However, lowering the parameter produces large elasticities for most variables. Changes in ω in both directions have no effect on w_t, r_t, and q_t and produces the same parameter elasticity with other variables. The overall conclusion is that the model is sensitive to lowering the parameter's value, but not to raising it. ω positively determines the slope of the inverse aggregate labour supply function (Equation (3.52)). The last graph of Figure 8.12 illustrates how varying ω, *ceteris paribus*, affects the labour market, and the remarks said above about changes in ν can also be said about ω.

8.2.2 Combinations of sensitivity settings

The second and third approaches to structural parameter sensitivity both change combinations of two or more parameters to their sensitivity settings. These approaches are called the "Sensitive Combinations" (henceforth, SC) and "All Combinations" (henceforth, AC) methods.

The SC uses combinations of two or more sensitivity settings for only those structural parameters which the OAT method determines the model is sensitive to, that is, β, γ, and δ. There are 10 working combinations of parameter values in this method.[4] The SC builds upon the screening that the OAT method performs, and attempts to capture two or more sensitivity settings from β, γ, and δ which, when combined, produce tax-shock responses that deviate significantly from the baseline. Impulse responses for the SC are illustrated in Figure 8.8. These graphs show that impulse responses to the tax-shock vary only in magnitude, not in direction or trajectory, to the calibration of the model.

The AC uses combinations of two or more sensitivity settings from all structural pa-

[4] An 11^{th} combination with $\beta = 0.999$, $\gamma = 0.4$, and $\delta = 0.98$ does not allow the model to converge to a unique equilibrium.

rameters. There are 754 working combinations of parameter values in this method, within which the 10 combinations in the SC are included.[5] The objective of performing the AC is to capture any combination of two or more parameter values that are outside of those considered by the SC. The AC also avoids any selection bias that the SC may have, despite the OAT method identifying which parameters are key drivers of sensitivity. Impulse responses of the AC resemble those in Figure 8.8, but are more densely populated, and are therefore not reported to avoid repetition. The conclusion of the AC is therefore the same as that of the SC.

Box plots of immediate impulse responses from both SC and AC are presented in Figure 8.9 in which the immediate baseline responses of Section 5.3 are indicated by red crosses. Two conclusions are drawn from inspecting these plots. First, with the exception of C_t^i, baseline responses are not extreme. From Equation (3.56), C_t^i depends on many parameters – π, β, ϕ, θ, and δ. One hypothesis is that combinations of parameter values bring multiple sources of deviation in C_t^i's impulse responses from the baseline, such that the baseline responses appear extreme. Besides C_t^i, other baseline responses fall within the interquartile range of immediate responses from both the SC and AC methods. Second, the AC delivers more extreme immediate impulse responses than the SC. The whiskers of the box plots are much longer from the AC. However, the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles of both methods are very similar. The differences in extreme responses suggest that the model is not only sensitive to the structural parameters that are identified by the OAT method, but also to combinations of any of the parameters.

8.3 Sensitivity to the persistence of shocks to tax rates

Sensitivity to $\rho_{\tau wl}$ and $\rho_{\tau rn}$ is examined by assuming $\rho_{\tau rn} = \rho_{\tau wl}$ and by repeatedly simulating the tax-shock with two values that are above (0.99 and 0.96) and three values that are below (0.94, 0.88, and 0.10) the baseline setting (0.95) of the parameters. Baseline values of structural parameters are maintained. Results are illustrated graphically by impulse responses in two ways – Figure 8.10 gives the usual 200-quarter graphs and shows the variation in long-term trajectories, and Figure 8.11 gives a close-up of the first 20 quarters and shows the divergence of trajectories after the shock's initial impact. Table 8.3 reports

[5] There are 217 additional combinations of parameter values that do not allow the model to converge to a unique stable equilibrium.

convergence indicators that are computed according to Section 4.4.2.

Very small changes (by ±1 basis point) in $\rho_{\tau wl}$ and $\rho_{\tau rn}$ from the baseline do not significantly alter the responses of any variable in the model. When $\rho_{\tau wl}$ and $\rho_{\tau rn}$ are both increased and decreased to 0.96 and 0.94, respectively, Figure 8.10 shows that the shape and speed of adjustment paths hardly change. Figure 8.10 also shows that an increase by 4 basis points (which brings the parameters close to unity) significantly amplifies adjustment paths. All variables except aggregate taxes and asset prices then exhibit hump-shaped trajectories and very long shock persistence, and those variables that have hump-shapes in the baseline now have exaggerated humps. For any setting below 0.88, output loses its hump-shaped trajectory. At this range of persistence, investment falls rapidly towards steady state and is quickly outpaced by rising depreciation.

Reducing the persistence parameters down to very low levels reveals those variables whose shock propagations are driven by intrinsic features of the model. At a persistence calibration of 0.10, the slowest variables to adjust are N^s_{t+1}, K_{t+1}, C^i_t, and Y_t in that order. I_t and K_{t+1} take more than 200 quarters to converge and Y_t takes more than 5 years. This result suggests that the tax-shock's financial acceleration is the reason why these variables' responses are persistent. Recall, the shock leads to higher asset prices, which support entrepreneurs' net worth improvements, investment, saving, and further asset price increases, and output expands via the capital stock.

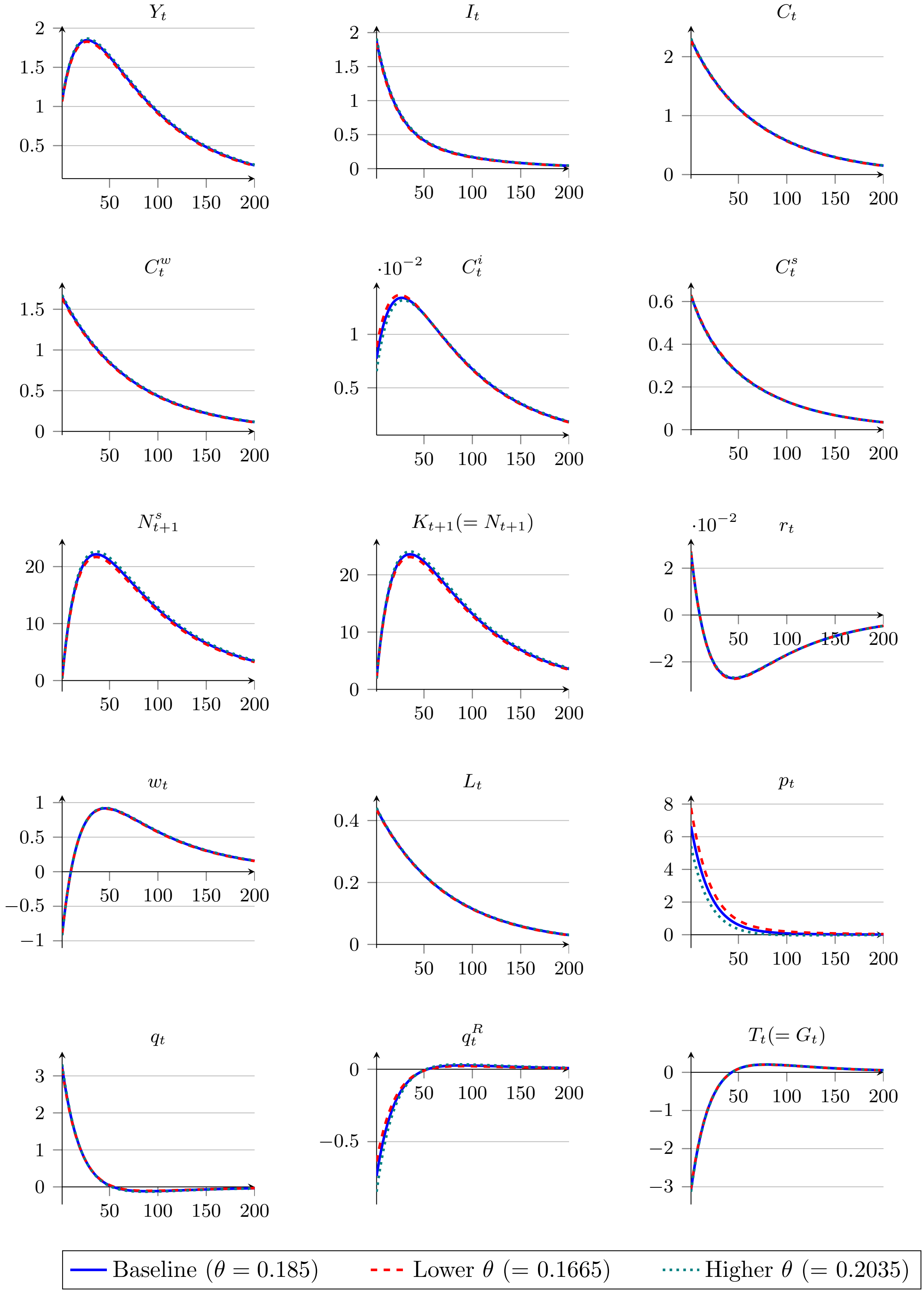
8.4 Chapter summary

This chapter analyses the sensitivity of tax-shock responses to the calibration of structural parameters and the persistence of shocks to tax rates.

Structural parameter sensitivity analysis is performed systematically by three local methods, all involving repeated simulations of the tax-shock with "sensitivity settings" for the parameters. The first method changes one structural parameter at a time; the second and third methods change combinations of two or more structural parameters. Results indicate that responses to the tax-shock (that are described in Chapter 5) are quantitatively sensitive to one-at-a-time variation of three structural parameters – the subjective discount factor (β), capital's share in output (γ), and the survival rate of capital after depreciation (δ). Tax-shock responses are also sensitive to combinations of alternative parameter settings, more so when these settings go beyond those of β, γ, and δ. Nevertheless, from changing parameter values either one-at-a-time or in combinations, tax-shock responses vary only in magnitude, and not in direction or adjustment trajectories. The analysis also shows that, with the exception of investors' consumption (C_t^i), baseline responses are not extreme when compared against the alternative calibrations. C_t^i depends on five parameters, and combinations of parameter values bring significant variations in impulse responses from baseline that make the baseline responses appear extreme.

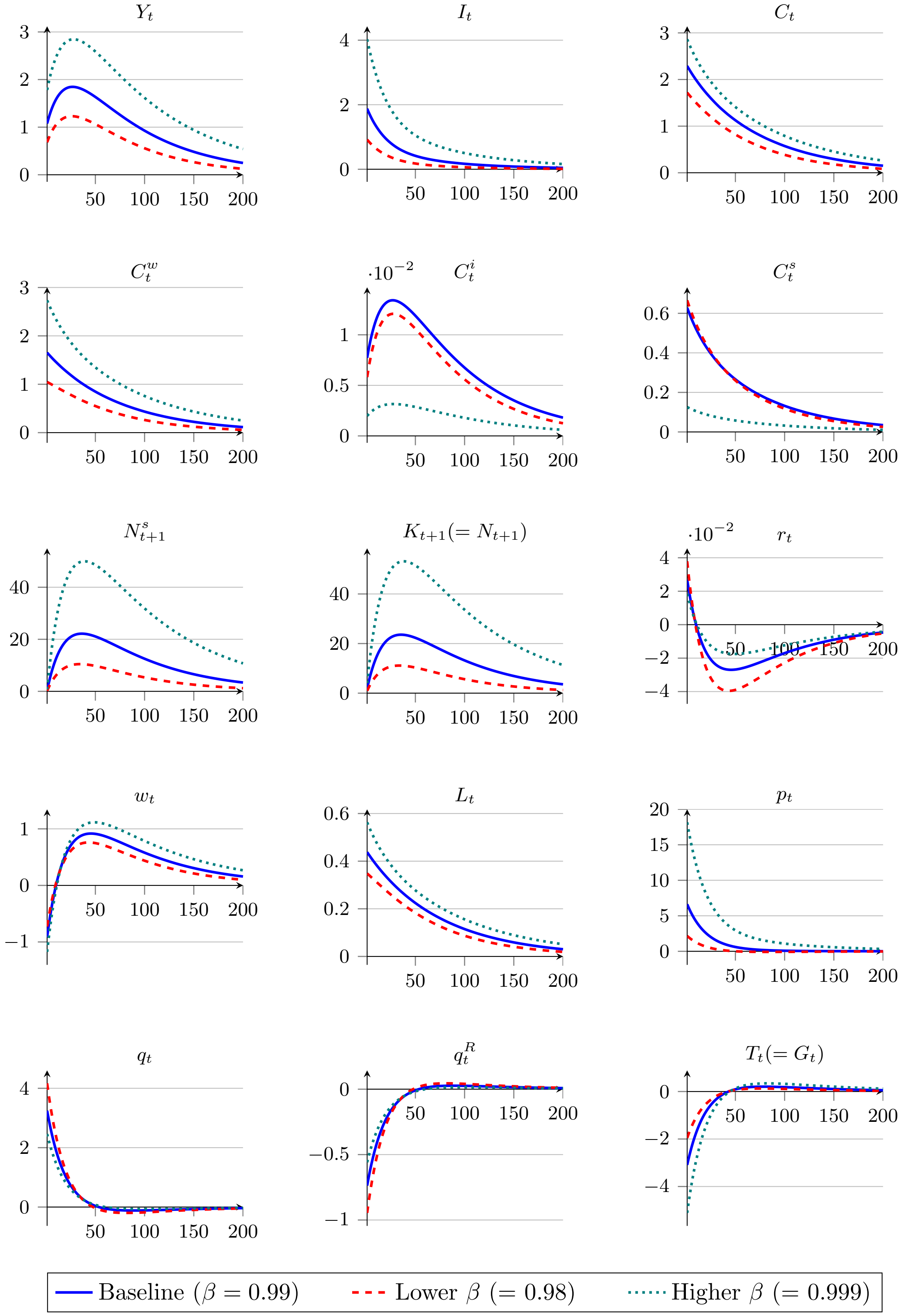
The tax-shock results of Chapter 5 are quantitatively and qualitatively sensitive to the calibration of persistence parameters. Very small (and plausible) changes in parameter values do very little to alter responses. But with larger (and sometimes implausible) parameter variations, there are significant changes in trajectory and convergence. Lowering the level of persistence reveals that savers' equity, capital, investors' consumption, and output are still slow to converge to steady state. This suggests that financial acceleration in the model is the cause of these variables' persistence.

FIGURE 8.1: Impulse responses to a tax-shock: sensitivity to θ



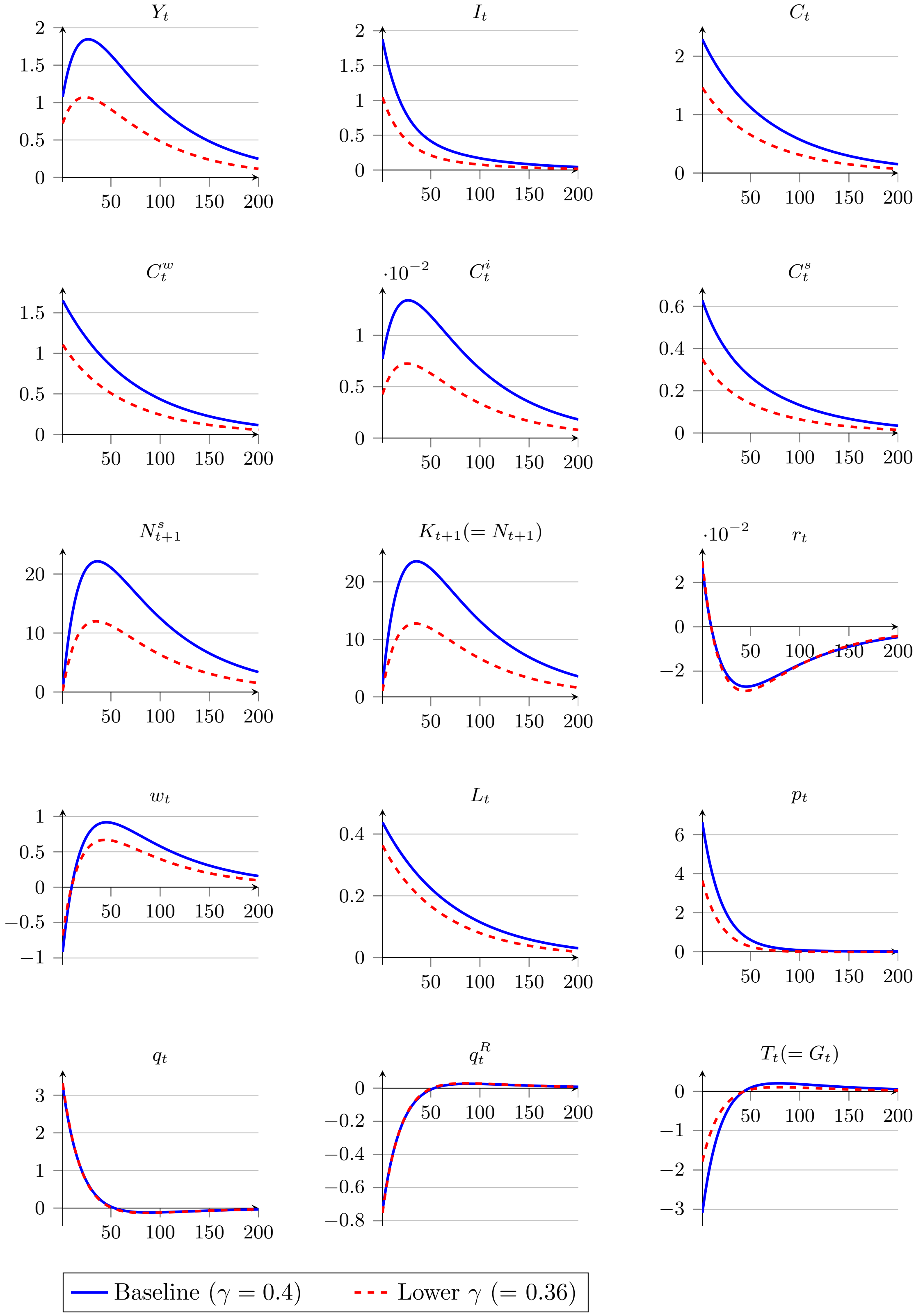
NOTES: Horizontal axes measure quarters after the shock, starting from quarter 1. Vertical axes measure deviations from steady state in levels.

FIGURE 8.2: Impulse responses to a tax-shock: sensitivity to β



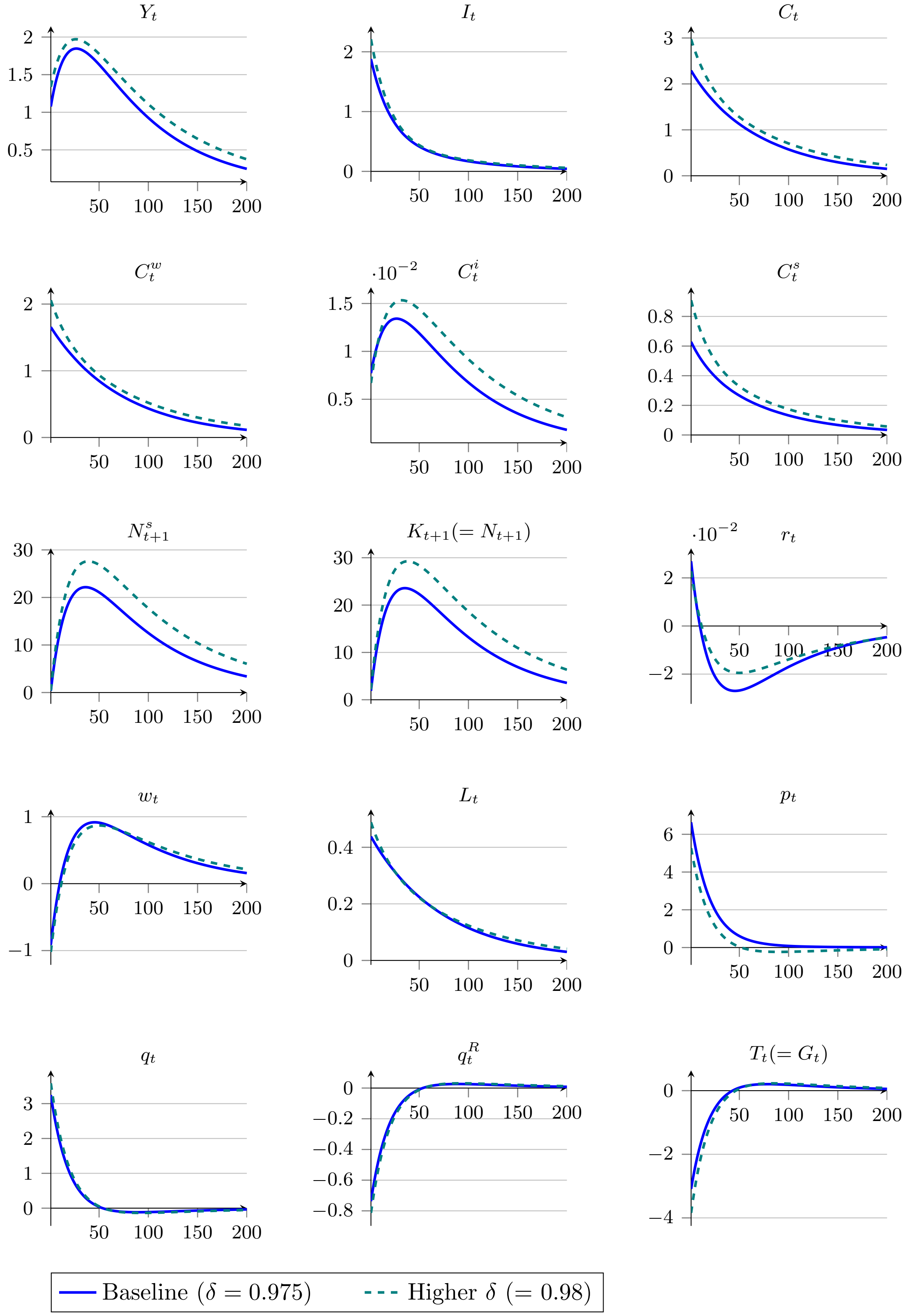
NOTES: The notes in Figure 8.1 apply.

FIGURE 8.3: Impulse responses to a tax-shock: sensitivity to γ



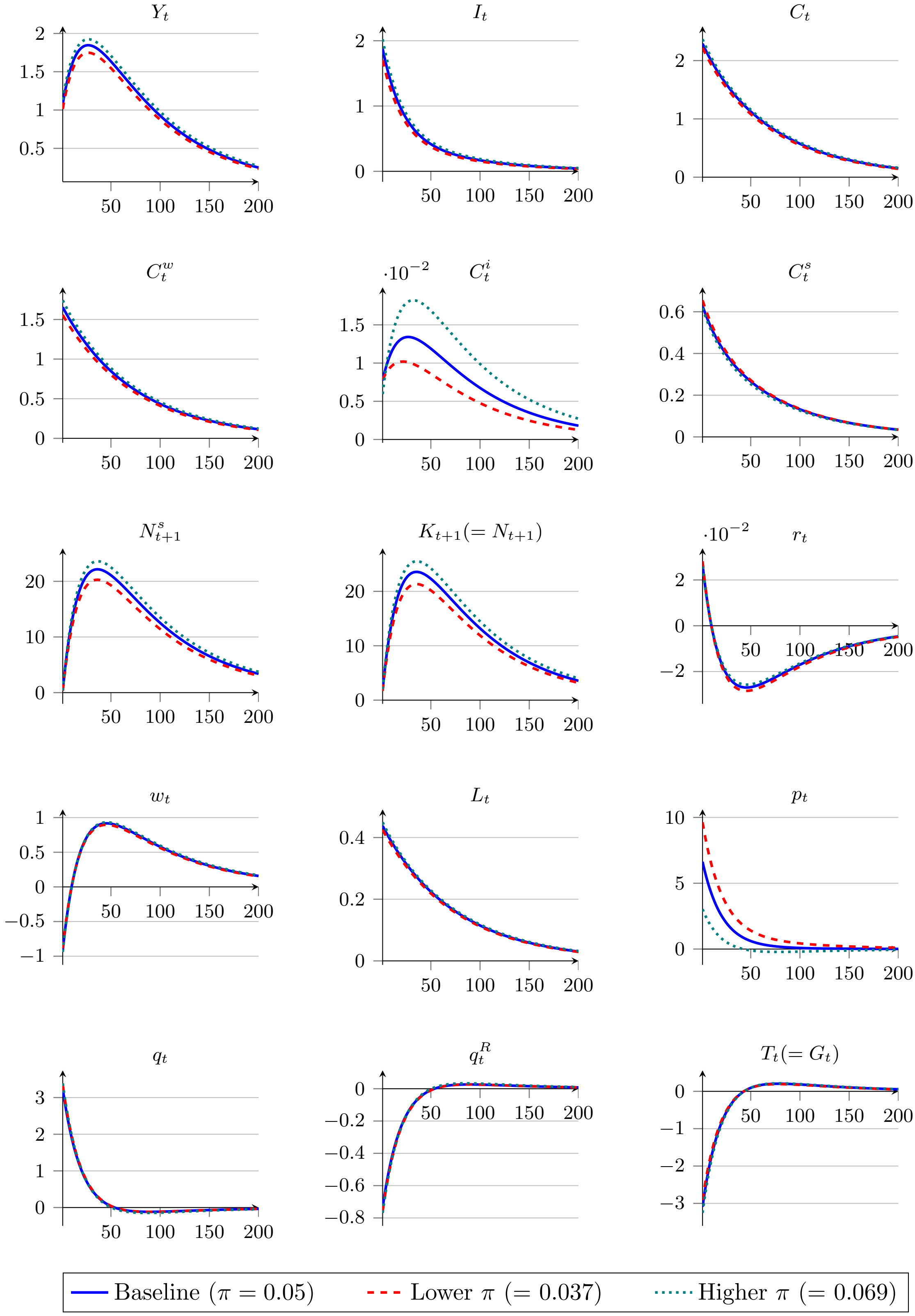
NOTES: The notes in Figure 8.1 apply.

Figure 8.4: Impulse responses to a tax-shock: sensitivity to δ



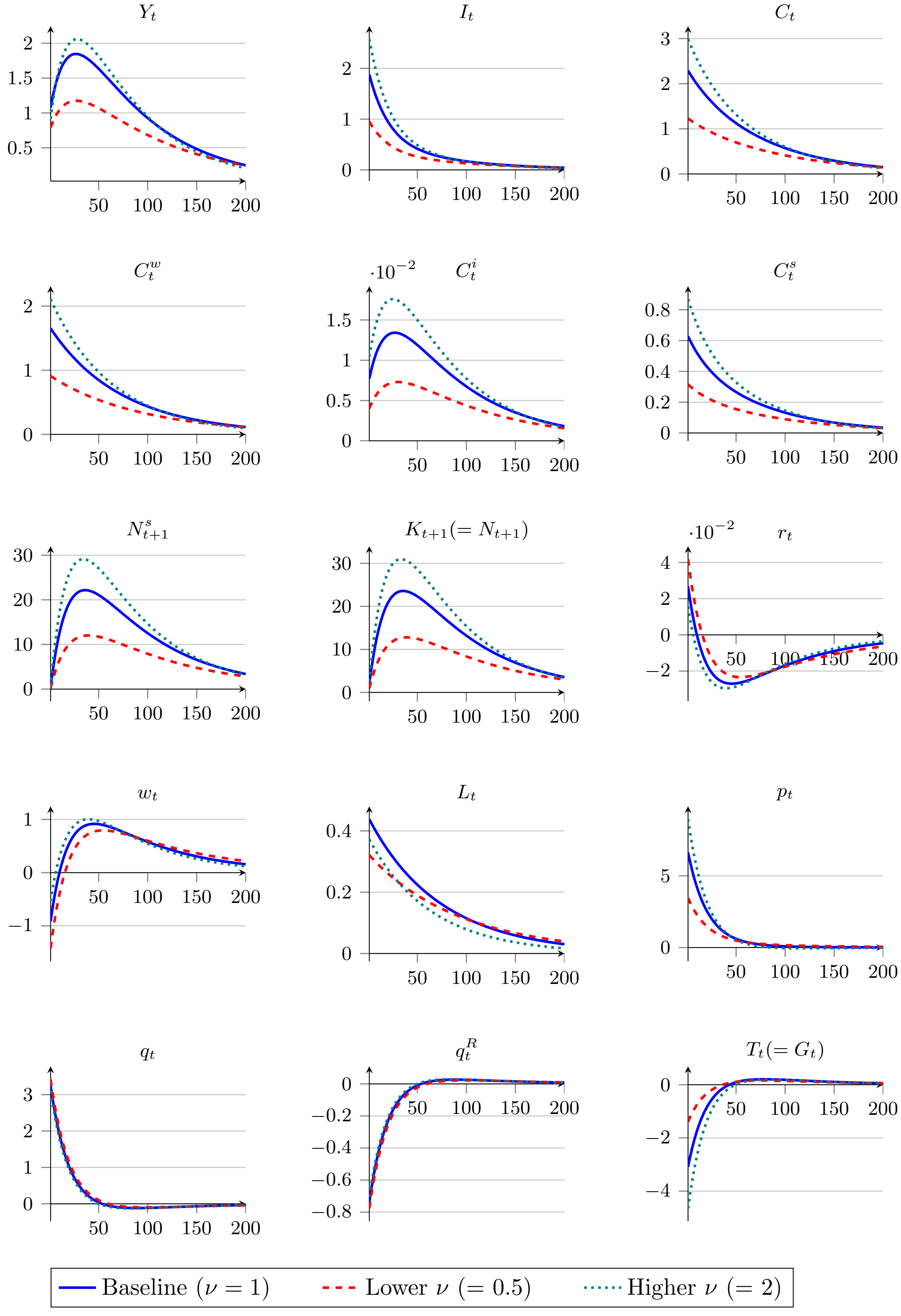
NOTES: The notes in Figure 8.1 apply.

FIGURE 8.5: Impulse responses to a tax-shock: sensitivity to π



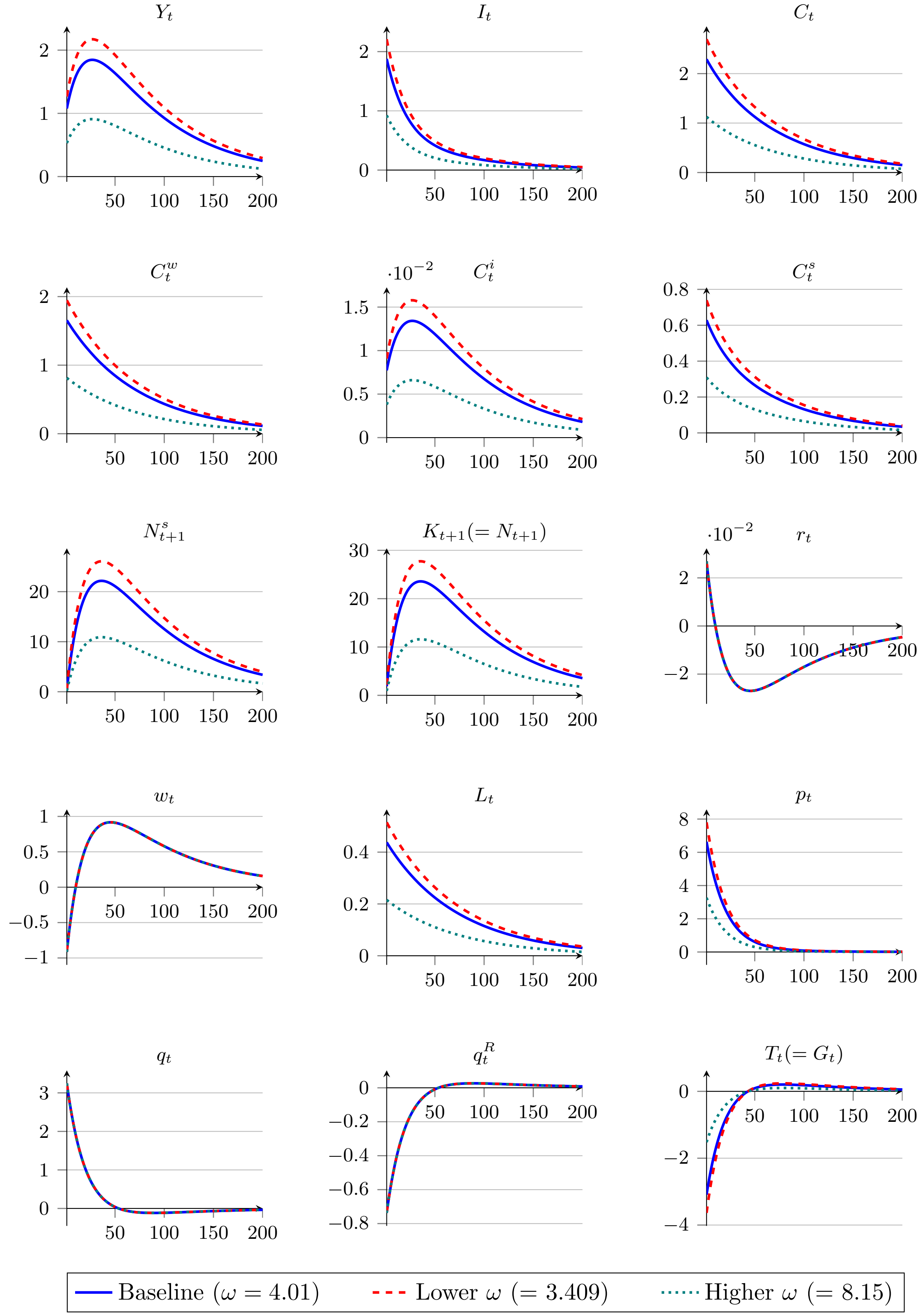
NOTES: The notes in Figure 8.1 apply.

Figure 8.6: Impulse responses to a tax-shock: sensitivity to ν



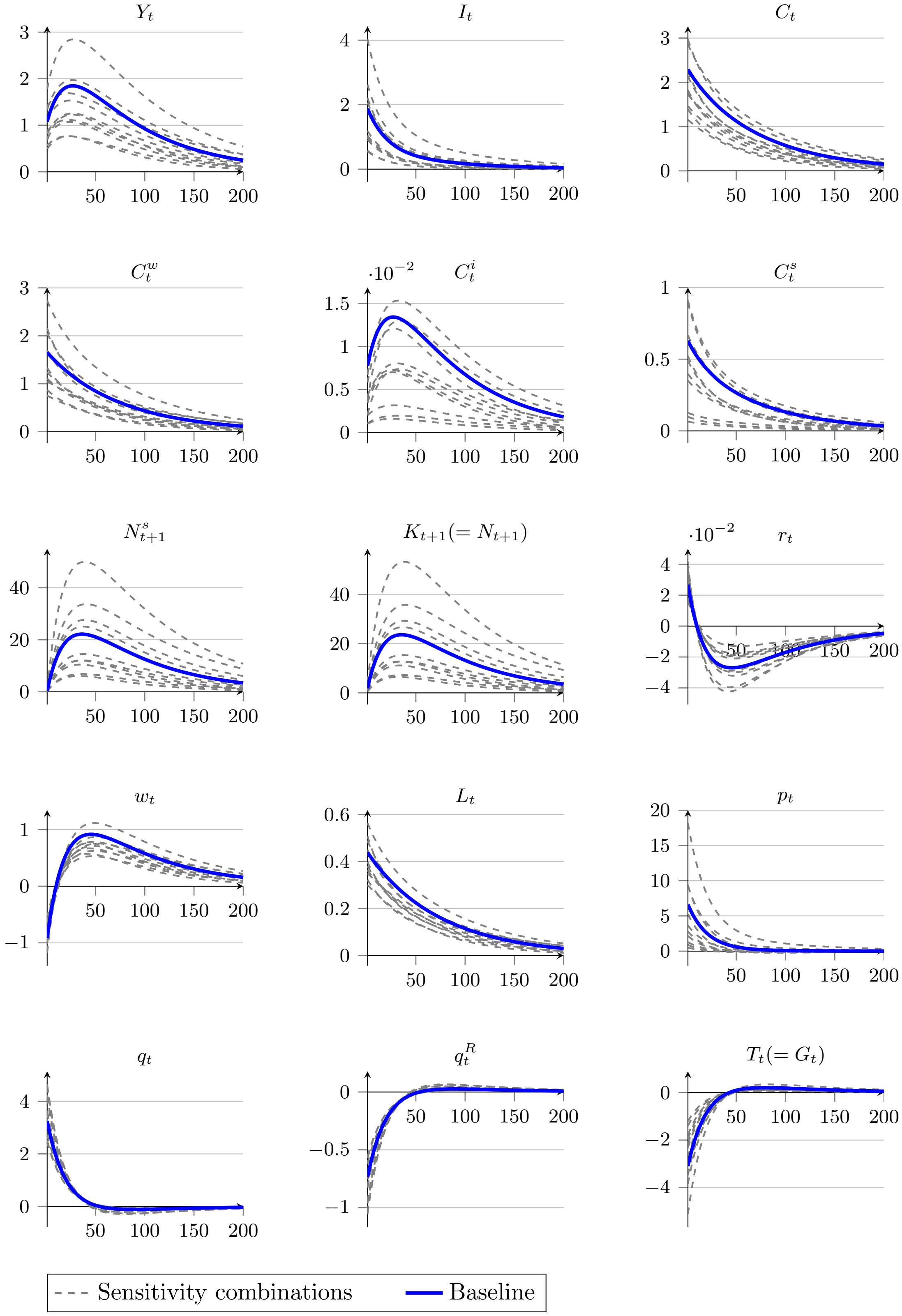
NOTES: The notes in Figure 8.1 apply.

FIGURE 8.7: Impulse responses to a tax-shock: sensitivity to ω



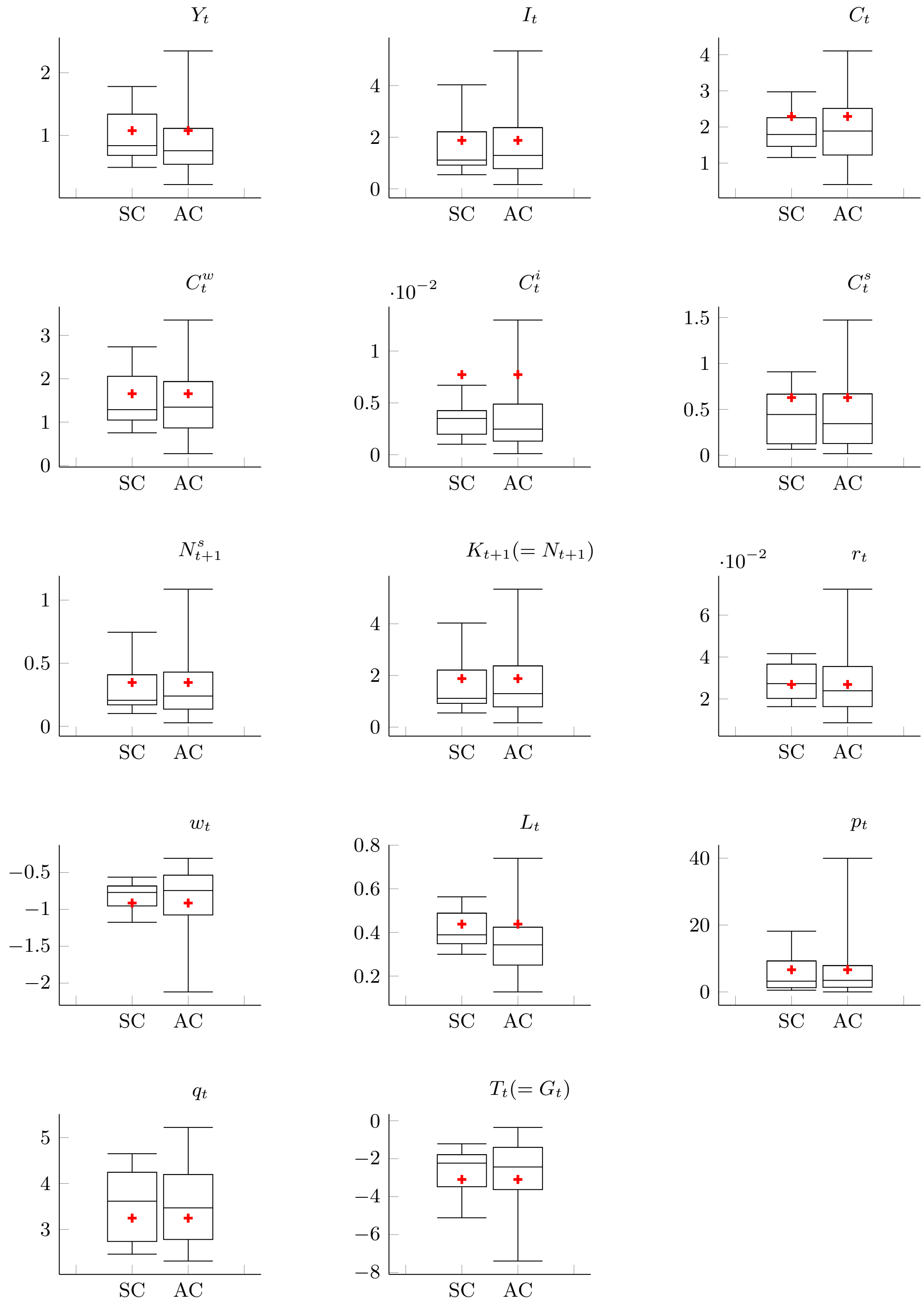
NOTES: The notes in Figure 8.1 apply.

FIGURE 8.8: Impulse responses to a tax-shock: sensitivity to combinations of β, γ, and δ



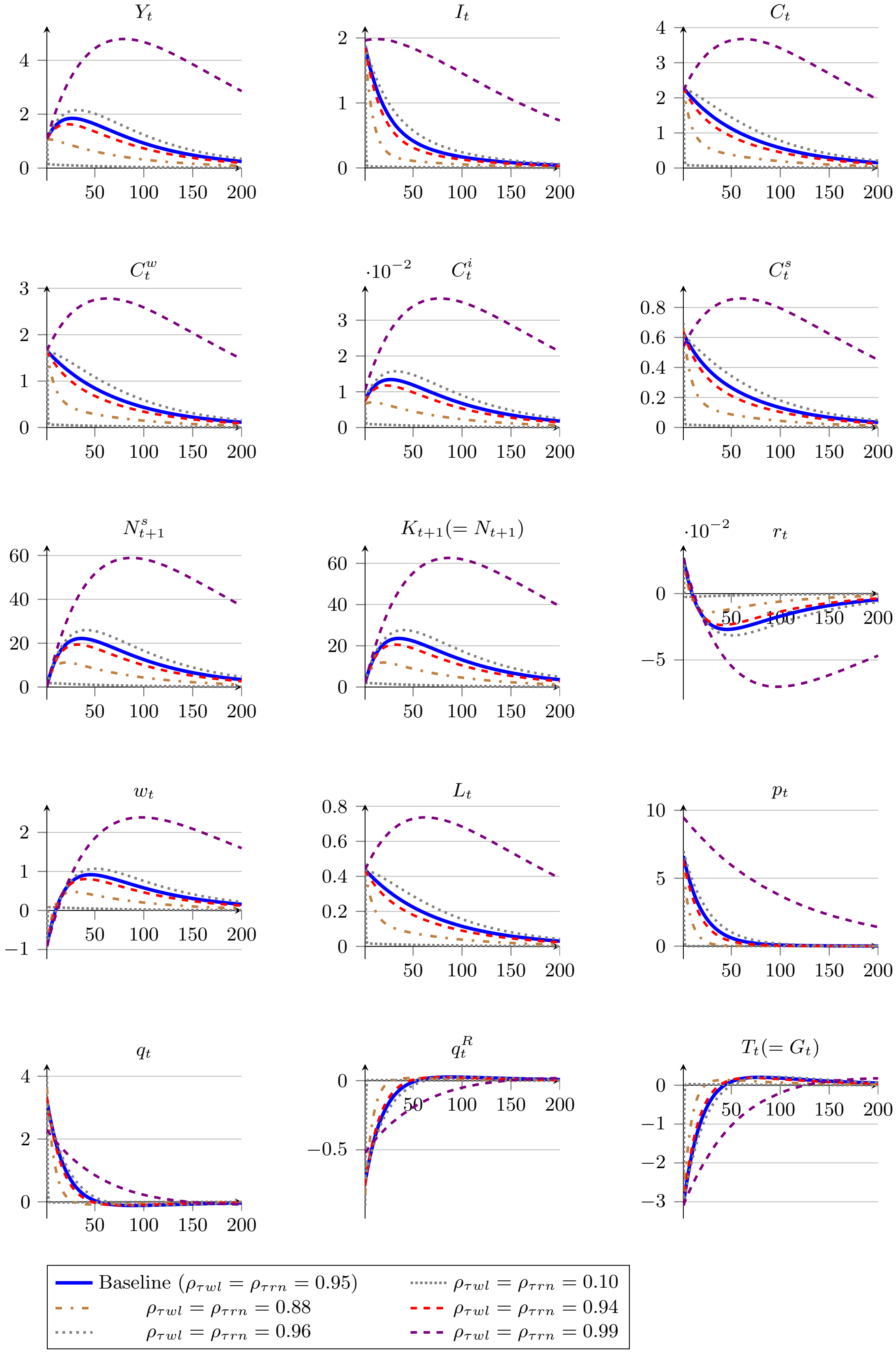
NOTES: These graphs show impulse responses to the tax-shock from 10 repeated simulations with combinations of sensitivity settings for β, γ and δ that are listed in Table 4.1. An 11^{th} combination with $\beta = 0.999$, $\gamma = 0.4$, and $\delta = 0.98$ does not allow the model to converge to a unique equilibrium. The notes in Figure 8.1 apply.

FIGURE 8.9: Box plots of immediate impulse responses to a tax-shock: combinations of sensitivity settings



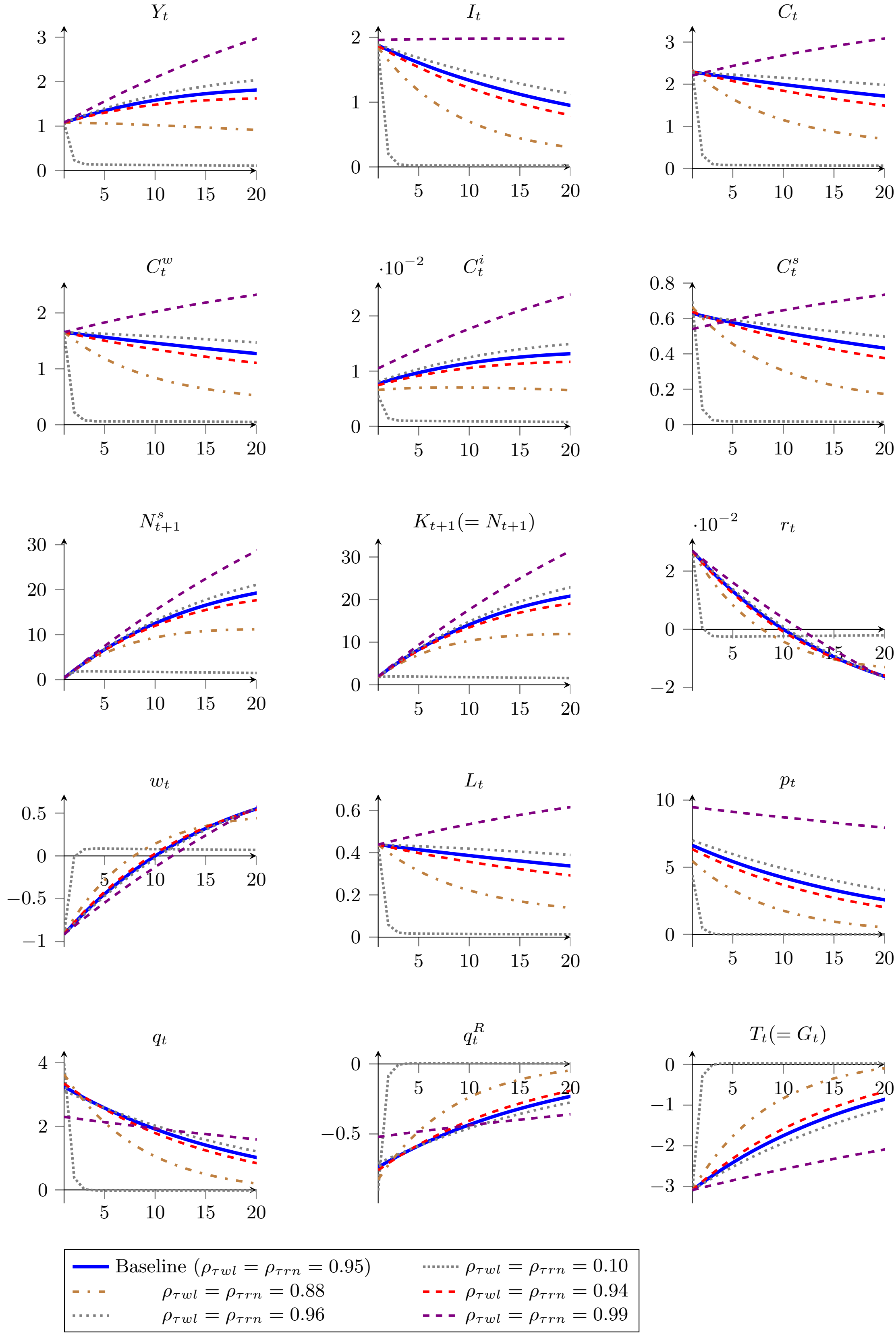
NOTES: These box plots show the 25th and 75th percentiles, median, largest, and smallest immediate impulse responses from the Sensitive Combinations (labelled “SC”) and All Combinations (labelled “AC”) approaches to structural parameter sensitivity. Red crosses indicate immediate impulse responses in the baseline scenario; see the “quarter 1” column of Panel A in Table 5.1 for their values.

FIGURE 8.10: Impulse responses to a tax-shock: sensitivity to $\rho_{\tau wl}$ and $\rho_{\tau rn}$, 200 quarters



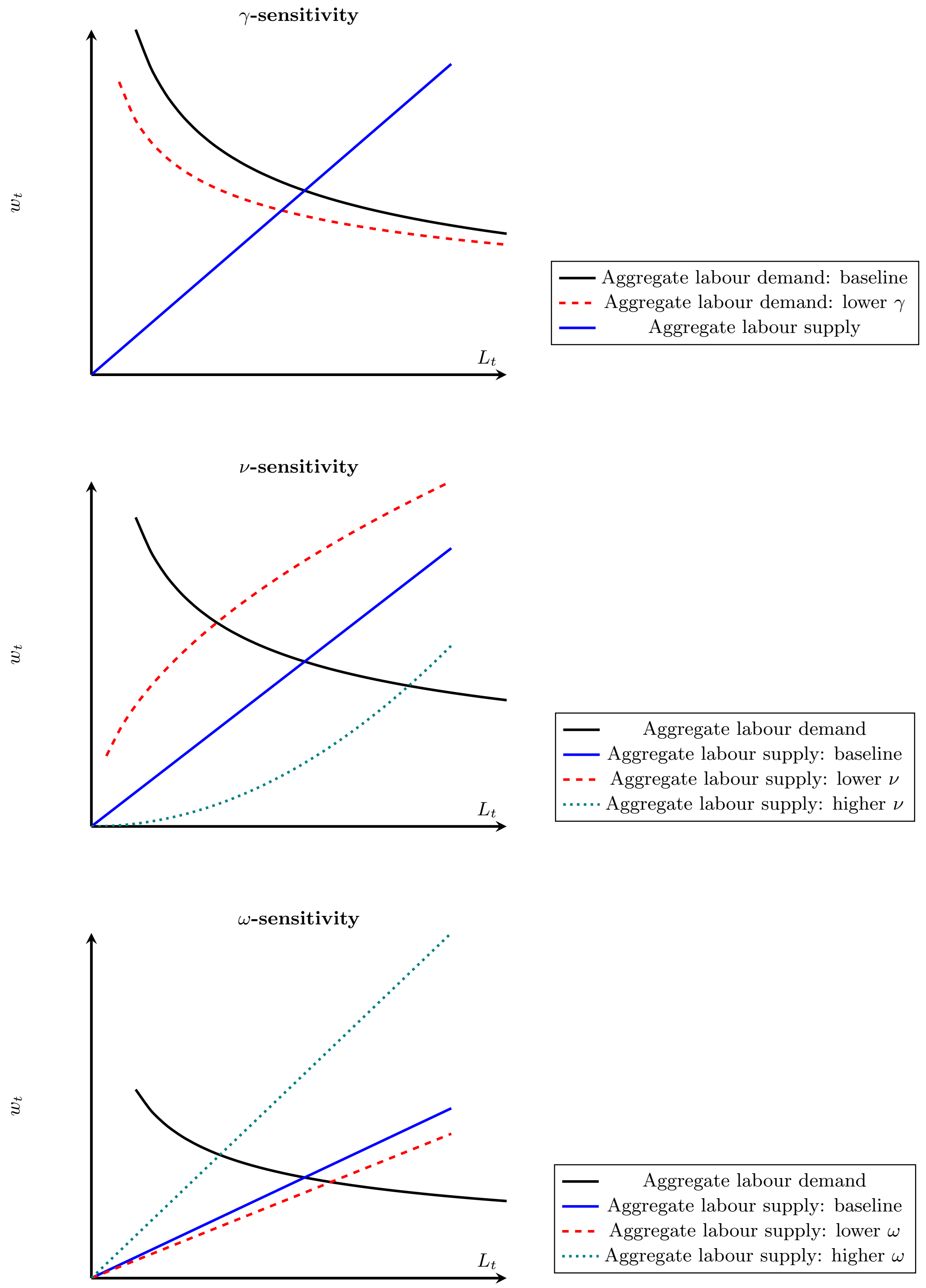
NOTES: These graphs show impulse responses to the tax-shock from repeated simulations with lower-than-baseline settings for persistence parameters, $\rho_{\tau wl}$ and $\rho_{\tau rn}$. The notes in Figure 8.1 apply.

Figure 8.11: Impulse responses to a tax-shock: sensitivity to $\rho_{\tau wl}$ and $\rho_{\tau rn}$, 20 quarters



NOTES: These graphs show the first 20 quarters of Figure 8.10. The same notes apply.

Figure 8.12: Labour market sensitivity to γ, ν, and ω



NOTES: These graphs plot Equations (3.51) and (3.52) using steady state levels of the capital stock and total factor productivity and sensitivity settings for γ, ν, and ω that are listed in Table 4.1.

TABLE 8.1: Immediate impulse responses to a tax-shock: one-at-a-time parameter sensitivity

	baseline	θ		β		γ	δ	π		ν		ω	
		L	**H**	**L**	**H**	**L**	**H**	**L**	**H**	**L**	**H**	**L**	**H**
Y_t	1.1	1.1	1.1	0.7	1.8	0.7	1.3	1.0	1.1	0.8	0.9	1.3	0.5
I_t	1.9	1.8	1.9	0.9	4.0	1.0	2.2	1.7	2.0	1.0	2.6	2.2	0.9
C_t	2.3	2.3	2.3	1.7	2.9	1.5	3.0	2.2	2.4	1.2	3.0	2.7	1.1
C_t^w	1.7	1.6	1.7	1.0	2.7	1.1	2.1	1.6	1.7	0.9	2.1	1.9	0.8
C_t^i	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C_t^s	0.6	0.6	0.6	0.7	0.1	0.4	0.9	0.7	0.6	0.3	0.9	0.7	0.3
N_{t+1}^s	0.3	0.3	0.4	0.2	0.7	0.2	0.4	0.3	0.4	0.2	0.5	0.4	0.2
$K_{t+1}(=N_{t+1})$	1.9	1.8	1.9	0.9	4.0	1.0	2.2	1.7	2.0	1.0	2.6	2.2	0.9
r_t	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_t	−0.9	−0.9	−0.9	−0.7	−1.2	−0.7	−1.0	−0.9	−0.9	−1.4	−0.5	−0.9	−0.9
L_t	0.4	0.4	0.4	0.3	0.6	0.4	0.5	0.4	0.4	0.3	0.4	0.5	0.2
p_t	6.6	7.8	5.4	2.2	18.2	3.6	5.3	9.6	3.0	3.5	8.9	7.8	3.3
q_t	3.2	3.2	3.3	4.2	2.5	3.3	3.6	3.3	3.4	3.4	3.1	3.2	3.2
q_t^R	−0.7	−0.6	−0.8	−0.9	−0.6	−0.8	−0.8	−0.8	−0.8	−0.8	−0.7	−0.7	−0.7
T_t^*	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3	−3.3
$T_t(=G_t)$	−3.1	−3.1	−3.1	−2.0	−5.1	−1.8	−3.8	−2.9	−3.3	−1.4	−4.6	−3.6	−1.5

NOTES: This table gives the quarter 1 deviations from steady state in levels, for baseline and one-at-a-time sensitivity scenarios. "L" and "H" refer to the lower and higher sensitivity parameter settings, respectively, that are listed in Table 4.1.

TABLE 8.2: Parameter elasticities of impulse responses to a tax-shock

	θ		β		γ	δ	π		ν		ω	
	L	**H**	**L**	**H**	**L**	**H**	**L**	**H**	**L**	**H**	**L**	**H**
Y_t	0.1	0.1	36.3	71.9	3.3	47.4	0.2	0.1	0.5	−0.2	−1.2	−0.5
I_t	0.2	0.2	50.5	126.0	4.4	34.4	0.4	0.2	1.0	0.4	−1.2	−0.5
C_t	0.1	0.1	24.7	27.5	3.6	58.0	0.1	0.1	0.9	0.3	−1.2	−0.5
C_t^w	0.1	0.1	36.3	71.9	3.3	47.4	0.2	0.1	0.9	0.3	−1.2	−0.5
C_t^i	−1.4	−1.5	24.5	−81.8	4.5	−25.9	0.0	−0.6	1.0	0.4	−1.2	−0.5
C_t^s	0.0	0.0	−5.9	−88.0	4.4	87.1	−0.2	0.0	1.0	0.4	−1.2	−0.5
N_{t+1}^s	1.2	1.2	50.5	126.0	4.4	34.4	0.4	0.2	1.0	0.4	−1.2	−0.5
$K_{t+1}(= N_{t+1})$	0.1	0.1	20.2	31.4	2.5	22.4	0.1	0.1	−1.1	−0.4	0.0	0.0
r_t	0.1	0.1	20.2	31.4	1.7	22.4	0.1	0.1	0.5	−0.2	−1.2	−0.5
w_t	−0.1	−0.1	−40.5	−34.6	−1.0	−29.4	−0.2	−0.1	−1.1	−0.4	0.0	0.0
L_t	0.2	0.2	50.5	126.0	4.4	34.4	0.4	0.2	1.0	0.4	−1.2	−0.5
p_t	−1.7	−1.8	66.7	191.1	4.5	−40.1	−1.7	−1.4	1.0	0.3	−1.2	−0.5
q_t	0.2	0.2	−28.0	−26.6	−0.2	20.2	−0.1	0.1	−0.1	0.0	0.0	0.0
q_t^R	1.4	1.5	−28.0	−26.6	−0.2	20.2	−0.1	0.1	−0.1	0.0	0.0	0.0
T_t^*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$T_t(= G_t)$	0.1	0.1	36.3	71.9	4.2	47.4	0.2	0.1	1.1	0.5	−1.2	−0.5

NOTES: Elasticities are computed according to the methodology outlined in Section 4.4.3. "L" and "H" refer to the lower and higher sensitivity parameter settings, respectively, that are listed in Table 4.1.

TABLE 8.3: Convergence of impulse responses to a tax-shock: sensitivity to $\rho_{\tau wl}$ and $\rho_{\tau rn}$

			(B)							
$\rho_{\tau wl} = \rho_{\tau rn} =$	**0.99**	**0.96**	**0.95**	**0.94**	**0.90**	**0.89**	**0.88**	**0.80**	**0.50**	**0.10**
Y_t	201	201	201	201	198	189	182	139	67	22
I_t	201	117	94	77	39	34	30	15	5	3
C_t	201	194	169	151	103	94	87	44	6	3
C_t^w	201	197	173	155	107	99	92	49	6	3
C_t^i	201	201	201	201	201	201	195	157	91	47
C_t^s	201	181	156	136	87	78	70	28	6	3
N_{t+1}^s	201	201	201	201	201	201	201	201	201	201
$K_t(= N_{t+1})$	201	201	201	201	201	201	201	201	201	182
r_t	201	201	201	201	176	168	160	118	45	2
w_t	201	201	201	201	176	168	160	118	45	2
L_t	201	197	173	154	107	99	91	49	6	3
p_t	201	61	49	40	24	22	20	12	5	3
q_t	100	42	36	31	20	19	17	11	5	2
q_t^R	100	42	36	31	20	19	17	11	5	2
T_t^*	201	58	46	39	23	21	20	12	5	3
$T_t(= G_t)$	91	37	31	28	19	17	16	10	5	2

NOTES: This table gives convergence indicators which are described in Section 4.4.2.

Chapter 9

Conclusion

9.1 Overview

Can discretionary policy relieve the effects of liquidity constraints that limit investment, and thereby stimulate economic activity in normal times, that is, when there are no other exogenous shocks? Can discretionary policy ameliorate the effects of an exogenous tightening of the liquidity constraints (a liquidity shock)? The thesis answers these questions by cutting tax rates, increasing government spending, and increasing government holdings of privately-issued equity in a calibrated version of the neoclassical DSGE model of Kiyotaki and Moore (2012). These policies are exogenous and temporary, and carry various endogenous and contemporaneous financing arrangements that do not involve debt, but rather changes in the composition of the government's flow of funds.

The basic KM model is modified by introducing a government which levies distortionary taxes on wages and dividends, consumes general output, issues money, and holds entrepreneur-issued equity. Including distortionary taxes is a unique modification within a branch of a burgeoning literature that extends the work of KM and studies liquidity shocks. One branch of this literature introduces KM's simultaneously binding liquidity constraints to fairly standard New Keynesian DSGE models (Del Negro et al. (2011), Ajello (2012), Kara and Sin (2013, 2014), and Molteni (2014)). The thesis belongs to a second branch of the literature which modifies KM's basic model (Bigio (2010, 2012), Nezafat and Slavík (2012), Shi (2012), and Driffill and Miller (2013)). None of the papers in the latter group have distortionary taxes, and they do not examine fiscal policy in KM's liquidity constrained environment. The discretionary polices in this thesis are therefore new approaches to ameliorating a liquidity shock in the KM model.

The relevance of the thesis is supported by the 2008 financial crisis, at the heart of which is a negative liquidity shock. The KM model is particularly suited for studying liquidity shocks, as its novel feature – a pair of liquidity constraints – endogenously creates a demand for assets of varying liquidity and allows government asset purchases to avoid the irrelevance proposition of Wallace (1981) which plagues standard models. In crisis-afflicted countries, such as the US and UK, conventional monetary policy was immovable with the nominal policy interest rate at or near 0%. Policy packages to combat the crisis were dominated by unconventional purchases of partially liquid assets from private agents and by fiscal policy. Asset purchase programmes have varied by the type of asset that the policymaker offered in exchange, and whether or not the policymaker's balance sheet changed size. Fiscal policies have made complete U-turns as part of consolidation strategies; the UK government, for example, attempted fiscal expansion in the early aftermath of the crisis, and then (albeit with political change) switched to an "age of austerity" to deal with the debt overhang caused by low aggregate demand and prior stimulatory measures.[1] The crisis created new debates on the effectiveness of asset purchase programmes, and reignited old debates about the effectiveness and choice of fiscal policy. The thesis contributes to all these debates, from a theoretical and neoclassical perspective.

The thesis does not make policy recommendations because its results are conditional on a model that is highly simplified by assumptions that divorce its agents' behaviour from reality. For instance, government spending in the model does not affect private agents' utility, but in reality such spending affects private utility, for example, by welfare payments. The thesis merely begins a programme of work on fiscal policy in the KM model. This conclusion outlines extensions of the thesis for a future work programme which will help develop a comprehensive understanding of fiscal policy in the KM model, and from which policy advice can be obtained.

[1]The "age of austerity" is a phrase used by the UK Prime Minister David Cameron at a pre-election speech in 2009. See Cameron (2009).

9.2 Summary of results

Can discretionary policy relieve the effects of liquidity constraints that limit investment, and thereby stimulate economic activity in normal times?

If the government keeps money supply constant and varies its spending to match tax revenues and thereby balance its fiscal budget in every period, then simultaneous cuts in both tax rates successfully stimulate economic activity – consumption, investment, employment, and output all immediately increase and remain above their steady states for a very long time. Of all the simulations in the thesis, this is the most effective discretionary policy for economic stimulus in normal times. These results vary only in magnitude, but not in direction or trajectories, to alternative calibrations of structural parameters. The results, however, are quantitatively and qualitatively sensitive to the calibration of the persistence of shocks to tax rates; very small (and plausible) changes in persistence parameter values do very little to alter responses, but with larger (an sometimes implausible) variations there are significant changes in variables' trajectories and speeds of convergence.

The worst set of discretionary policies involve an increase in government spending. Government spending does not create employment and output and does not enter the utility of private agents. If the government finances its spending with more taxes then it balances its fiscal budget, but there are immediate and persistent declines in consumption, investment, employment, and output. If, instead, the government runs a fiscal deficit and finances its spending by selling part of its equity stock then consumption and investment immediately fall and employment and output fall after a one-period delay.

Intermediately, if the government (unconventionally) buys equity from entrepreneurs and finances this purchase with austerity measures, and regardless of whether or not the purchase is accompanied by monetary expansion, then the economy is stimulated for a brief period of time. The accompanying austerity measure is better (that is, less distortionary) when government spending is cut rather than when taxes are increased. With a cut in government spending, the equity purchase programme does not immediately change output and employment, but private consumption and investment both increase. With an increase in taxes, the programme immediately reduces output and employment, as well as private consumption, but investment increases. The increase in investment in both scenarios adds new equity to the market which almost replaces the stock taken out by the government. This is why the programme has short-lived effects. Results are unable to conclude whether or not adding monetary expansion to the policy mix improves the outcome of the programme with austerity; aggregate supply benefits from monetary expansion, but aggregate demand suffers.

Can discretionary policy ameliorate the effects of an exogenous liquidity shock?

Responses to a liquidity shock (without policy) are the same across most variants of the model (the exception is the N^g-financed variant in Chapter 6) and in the KM model – consumption immediately rises, investment immediately falls, and employment and output do not immediately change, but from the second period the economy enters a long recession.

The most successful policy against the liquidity shock is cutting the rate of tax on dividends and lowering government spending to balance the fiscal budget. The rate cut and the liquidity shock both affect entrepreneurs' liquidity in opposite ways, and variables that are not affected by the liquidity shock are not disturbed by the tax cut. Intuitively, the dividend tax drives a wedge between investors' gross net worth and liquid funds. These agents are already liquidity constrained and are therefore investing in sub-optimal quantities. When the tax is cut, they do not save more or consume more, but instead channel their gains into more investment, from which they enjoy more utility in the next period and the economy benefits in the long-term from more capital.

By contrast, increasing government spending, financed either by an increase in taxes or by selling some stocks of equity, exacerbates the liquidity shock.

Intermediately, an equity purchase programme is generally successful at ameliorating the liquidity shock, regardless of how it is financed, but the benefits are short-lived and almost disappear after one year. This result is shared by KM and the sticky-price models of Del Negro et al. (2011), Kara and Sin (2013), and Driffill and Miller (2013).

9.3 Limitations and directions for future research

Policymakers cannot be advised by this thesis to undertake balanced budget tax and spending cuts. Notwithstanding that balanced budgets are very difficult to achieve in practice, this model has simplifying assumptions that lead to results which may differ from reality. The balanced budget policy is best for *this* model, and further research with replications of the policy in various environments is recommended before generalising an outcome and adopting the measures in practice.

Government employment and welfare

One of the major digressions from reality concerns government spending. In reality, spending cuts mean reductions in employment for those who work for the state, reductions in output from state enterprises, and reductions in welfare and consumption by those who receive transfers from the government. These effects imply a trade-off between spending cuts and tax cuts, and this trade-off is absent in the model. The absence explains why the economy is stimulated by policies in Chapters 5 and 7 that involve spending cuts, and why it contracts from policies in Chapter 6 that involve more government spending. For future research, this model should be modified to allow the government to employ labour and produce general output and/or make transfers to workers and thereby positively influence their utility. Then all the experiments in the thesis that involve variations in government spending should be repeated, and the net effects after trade-offs should be analysed to formulate policy advice.

Alternative balanced budget tax cuts

One type experiment that is not performed in Chapter 5, but which would complete all the possible policy combinations with a balanced fiscal budget, is a cut in one rate of tax and a simultaneous increase in the other rate of tax, while government spending and other policy variables are left unchanged. Such an experiment is useful for studying the relative merits of one tax cut versus another, which is not the objective of the thesis, and is therefore ommitted.

Ricardian workers

Because asset returns are sufficiently low, workers in this model prefer not to save, and instead consume all their wages in each period. Then any policy which targets asset prices has no direct impact on labour supply and output. This is why there is no trade-off between the direct effects of equity purchases and the financing of these purchases in Chapter 7. If, however, workers save (as they do in the related work of Kara and Sin (2014)) then asset price policy has direct impacts on output. The model can therefore be extended, either by exogenous assumptions or otherwise, to encourage workers to participate in asset markets. Such a model faces the challenge of overcoming Ricardian equivalence from tax cuts, that is, expansionary policy may lead to workers adjusting their saving and leaving (a large component of) aggregate demand unchanged. Another complication of such a model, one that is already expressed by Driffill and Miller (2013), is understanding the complex inter-temporal effects on workers' saving and consumption due to expectations of tax changes.

Storage technology

Responses to a liquidity shock in all the variants of the model match those of the basic KM model. The thesis therefore suffers from the same flaw that plagues KM and is pointed out by Shi (2012) – the liquidity shock lowers output and raises asset prices, but such a countercyclical response by prices is not observed in actual recessions. KM construct a "full" model to address this criticism by including a perfectly liquid storage technology, and thereby achieve endogenous price stability for money. Driffill and Miller (2013) resolve the problem by assuming sticky prices for both assets. KM and Driffill and Miller (2013) obtain an added bonus of amplified investment responses to the liquidity shock. But Driffill and Miller (2013) and the rest of the New Keynesian literature experience small and short-lived responses to policy because sticky prices for both assets stifle financial acceleration. So instead of assuming sticky prices, a useful extension of this thesis is to include KM's storage technology, or equivalently, introduce distortionary taxes to KM's full model. The aim would be to see whether simulated asset price movements can be replicated to match actual observations in liquidity shocks (for example, in the 2008 crisis).

Quantitative easing

Once the storage technology makes money's price sticky then the M-financed G-shock and M-financed N^g-shock can be successfully simulated. The latter experiment represents quantitative easing, and is precisely the policy that KM, Del Negro et al. (2011), Kara and Sin (2013), Driffill and Miller (2013), and Molteni (2014) perform against a liquidity shock.

Risky projects

KM and the rest of the related literature assume that investment technologies produce capital with certainty. A novel approach is to assume the technologies are heterogeneous with non-zero probabilities of default. Then equity becomes a risky asset and there are two negative implications of government equity purchases. First, the government introduces default risk to its balance sheet, and losses sustained from private investment failures may eventually result in more taxes, spending cuts, or monetary expansion. Second, government policy introduces moral hazard; entrepreneurs may be encouraged to undertake riskier investments with higher payoffs while knowing that any losses to their liquid net worth will be refunded by government policy.

New Keynesian and empirical versions

Finally, it may be worthwhile to replicate the experiments of this thesis in New Keynesian DSGE models. The outcome may help identify whether any of the real or nominal rigidities affect the transmission of fiscal shocks. Ajello (2012) already presents a New Keynesian model with KM's liquidity constraints and distortionary taxes, but he does not apply his model to a study of fiscal policy. It may be worthwhile to use his model, instead of design a new one from scratch. Alternatively, Del Negro et al. (2011) and Kara and Sin (2013, 2014) already develop New Keynesian models with KM's liquidity constraints, and it may not be difficult to introduce distortionary taxes to their frameworks. Eventually, the ideas of this thesis should be translated into empirical research by estimating these New Keynesian DSGE models, and the results can contribute to debates on fiscal multipliers. Given its parameter uncertainty, the thesis cannot compute multipliers that are reliable for comparison with the empirical literature.

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