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Risk-sharing and probabilistic network structure

Marco Pelliccia
Birkbeck, University of London

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(PRELIMINARY AND INCOMPLETE)

Abstract

This paper studies the impact of a probabilistic risk-sharing network structure on the optimal portfolio composition. We show that, even assuming identical agents, we are able to differentiate their optimal risk-choice once we assume the link-structure defining their relationship probabilistic. In particular, the final agent's portfolio composition is function of his location in the network. If we assume positive asset-correlation coefficients, the relative location of a player in the graph influences his risk-behavior as much as those of his direct and undirect partners in a not-straightforward way. We analyze also two potential "centrality measures" able to select the key-player in the risk-sharing network. The findings may help to select the "central" agent in a risk-sharing community and to forecast the risk-exposure of the players. Finally, this paper may explain natural differences between identical rational agents'choices emerging in a probabilistic network setup.

JEL classification: D85, D81, O17.

Keywords: Informal insurance, Risk-sharing, Network.

1 Introduction

The literature on informal insurance schemes between agents is wide. Even if informal insurance networks exist in different forms and environments, for many years social scientists have tried to explain the existence and the sustainability of this phenomenon. It is empirically proved that groups of individuals, under certain conditions, spontaneously form social insurance schemes (see for instance Fafschamps

[*]Email address: mpelliccia@ems.bbk.ac.uk

1992). Moreover, we also observe that the agents do not risk-share their endowments with all the individuals composing the main community[1]. Given these stylized facts many authors have explained theoretically these findings. The main challenge was to prove how mutual insurance relationships between peers could be "stable" in time despite the intuitive risk-sharing norm enforcement problem and the not-always perfect observability of individuals‘ endowments (monitoring problem). As Bloch et al. (2004) shows, there can be precise strategical reasons explaining partial-insurance schemes or the limited-size of the risk-sharing group given certain conditions. Specifically, they suggest that if the links have double roles such as liquidity and information channel, we can expect just specific network structures arising as equilibrum of a strategical problem and guaranteeing the transfer-norm enforcement. Moreover, as Bramoullé and Kranton (2007) notices, individual optimal choices related to the agent's link structure can lead to indirect negative externalities to the rest of the risk-sharing group and differentiate the final outcomes observed by the players. As Stack (1975), De Weerdt and University. United Nations (2002), Dercon and De Weerdt (2002), and Fafchamps and Gubert (2007) have empirically shown, the structure of the risk-sharing groups, in terms of connections between peers, seems far from being randomly formed. A common feature of all the studies about the risk-sharing problem that use a network analysis approach is to consider the transfers between individuals passing through bilateral relationships (represented geometrically by a link between two nodes) so that, indirectly, the action of an agent can also influence the one of another subject not directly connected with him. This intuitive and simple concepts have suggested the social scientists to investigate more the impact of the link-structure between risk-sharing individuals. Particular interest has been focused on the "punishment schemes" enforced by the nodes (through link rewiring strategies) to prevent deviations from the risk-sharing norm. In this paper we are not directly interested on studying the norm-enforcement problem but we focus the attention on the impact of a probabilistic link-structure on the agents‘ risk behavior, i.e. agents computing their optimal risk-exposure and connected in a risk-sharing structure through links consider the existance of the peers‘ connection not certain. The intuition comes from the fact that once the risk-sharing structure between peers is assumed probabilistic, the agents could find optimal to modify their risk-exposure according to their location in the network. Facing a probabilistic link-structure, the impact of this uncertainty can be different between the agents whenever located in "distinct" locations through the network. There are many reasons to assume a probabilistic link-structure in a risk-sharing model using a network approach. The

[1] See for instance Townsend (1995), Udry (1995), Fafchamps and Lund (2003), Murgai et al. (2002), Fafchamps and Gubert (2007).

existence of an agreement between two agents is usually explained by social or geographic factors (family membership, friendship and geographical proximity between many)[2]. However, these exogenous factors could change on time, being strongly dependent on the exogenous agents‘environment (the existance of a peer itself can be considered not perfectly certain in many cases). Moreover, if we reasonably assume that a link between two peers is a bilateral and strategical choice, changes on the exogenous scenarios could let the peers reconsider their previous link-structure. In the interbank market for example we observe a finite number of institutions exchanging liquidity through short-period lending-contracts. This is a particular form of risk-sharing between institutions, foundamental for the diffusion of liquidity in the bank-system. Each bank can bilaterally choose to open one or more liquidity channels as much as the relative partner-institutions. The profitability of these connections can change with the arrival of a new information on the partners‘ risk or more generally with a change on the financial environment. Being enforced by official contracts, the deviation from the risk-sharing norm is rarely observed in this case. However, the individual risk taken by the peers is not easily monitorable by the relative partners. This fact can lead to a classic “moral hazard” problem, in terms of risk-exposure or risk-strategies chosen by the institutions, not solvable by mechanisms of punishment studied using a network analysis approach (the perfect monitoring is a necessary condition to enforce a norm). On the other hand, the choice of a partner-institution seems to be strategical, function of specific market conditions (see for instance Angelini et al. (1996), Angelini et al. (2009),Soramaki et al. (2007), Gabrieli (2011)). Given these facts, each agent may not consider the present bilateral relationships as fixed but dynamic and function of different exogenous factors. In our model we argue that adding uncertainty at level of the link-structure can help to explain difference on optimal risk-choices of identical agents belonging to a risk-sharing community and choosing their optimal portfolio composition. Thus, the probabilistic feature of the graph could help to underline the importance of an agent‘s location analysis to understand relative advantages/disadvantages due to a player‘s position.

As we will extesively describe in the final section, this paper can improve the literature on risk-sharing schemes in different ways. Firstly, the model gives the opportunity and the methods to forecast different agents risk-exposures looking to their locations in the structure and involving two main parameters: The probability to observe in the future the present and existing links, and the correlation of the risks taken by each agent. Secondly, it can help to demonstrate how we can observe different risk-behaviors even assuming identical agents once we add uncertainty on the network structure defining the relationships between the peers. This feature

[2]See for instance Rosenzweig and Stark (1989).

in particular can also refine the literature on network analysis opened by Galeotti and Goyal (2010) and Bramoullé et al. (2010) between many. Finally, this paper can enrich the literature studying the impact of the network structure on the community‘s risk-sharing degree (see for instance Ambrus et al. (2010), Battiston et al. (2009), and Stiglitz (2010)).

The paper is organized as follows. In the first Section we present the main model where risk-sharing agents decide individually and optimally the composition of their portfolio between two assets, a “risk-free” and a “risky” one, assuming zero asset-correlation. In the second part of this section we study the impact on the optimal portfolio choices of a positive asset-correlation. We present in particular two examples of different network structures with specific structural features, underlining the importance of the relative nodes‘ location to explain the different risk-behaviors. In the last part of Section 2 we relax the perfect network observability assumption, assuming myopic nodes. Finally we discuss and test some node “centrality measures” and the implication of the model in terms of the minimizing-risk structure.

2 The model

In this section we introduce the model and the network notation. The model is composed by two main parts. In the first one we study the final equilibria assuming zero-correlation between the risky-assets, while in the second part we relax this assumption and produce two cases of study.

2.1 The network setup

We consider N agents linked each other according the adjacency matrix $G_{n \times n} = [g_{ij}]$, where $g_{ij} = 1$ whenever the nodes i and j are connected by an undirected link while $g_{ij} = 0$ otherwise. We assume also that $g_{ii} = 0$. We define the degree of node i, $d_i = \sum g_{ij} \forall j \in N$. For simplicity we will consider just connected components, i.e. $g_{ij} \neq 0$ for at least one $j \in N$. Payoffs are described by $U_i(x, \delta, G)$ where $\delta \in (0, 1) \equiv$ probability that if $g_{ij} = 1$ for some j at time t, $g_{ij} = 0$ at $t + 1$. The link l_{ij} between the nodes i and j describes the liquidity flow channel between these two nodes. In particular without direct connection, two nodes can transfer liquidity each other just through a different path. We thus keep a generic definition for the link l_{ij} since it has not a precise “physical” meaning. Clearly l_{ij} exists if and only if $g_{ij} = 1$. Finally, we define a path as a non-empty graph $P = (V, L)$ of the form $V = \{x_0, x_1, ..., x_N\}$ and $L = \{x_0x_1, x_1x_2, ..., x_{N-1}x_N\}$, where with $x_{i \in N}$ we define the node i and with x_ix_j the link or edge between i and j. Now we present the definition of "structural symmetry" using the notion of automorphism to describe two or more nodes symmetrically located in a graph. Formally an automorphism is a one-to-one mapping, τ, from N to N of a graph $G(N, L)$ such that $< i \ , j > \in L$ if and only if $<\tau(i) \ , \tau(j) > \in L$. We can define automorphism equivalence between i and j , $i \equiv^{AE} j$, if there exists some mapping τ such that $\tau(i) = j$, and the mapping τ is an automorphism. Thus, if we find such automorphism and we are interested on studying the impact of the link-structure on the agents, we can analyze selected nodes of the graph representing the equivalence classes.

2.2 The zero-correlation case

There are n nodes/players, deciding how much invest of their unit capital on risky asset, x_i , and on risk-free asset, $(1 - x_i)$ at time t. In particular, the risk-free asset has variance $\sigma_{RF} = 0$, while the risky asset, $\sigma_R > 0$. Investing at t on the risk-free asset yelds exactly the amount invested on it at $t+1$, while the expected return from the risky asset is positive and equal to px_ih, where p is the probability of positive return h. The identical agents are risk-averse and maximize their expected profit choosing the optimal portfolio structure. At this stage the assets are identical for all the players but independent each other. The nodes are connected through a network structure describing the liquidity flow-path between the peers. We assume in fact that the agents transfer at each time t liquidity to each other following an equal-sharing rule. Roughly speaking, at each t the agents agree to exchange liquidity such that the final income post-transfer is the same for all the agents belonging to the component. We assume also that the network structure is common knowledge and the players do not deviate from the sharing-rule. This means that the link l_{ij} between two nodes i and j guarantees the respect of the risk-sharing norm, i.e., no strategical deviation from the rule are allowed. As we will see formally, this setting and overall the common knowledge of the whole network structure guarantees the neutrality of the link-structure on the agents' optimal choices.

In absence of any connection (situation that we can define as the "autarky" case), the expected income Y at time t for a generic agent i is

$$E[Y_{it}] = x_{it}hp + (1 - x_{it})$$

for all i at time t . Consequently, the variance observed by player i will be

$$Var[Y_{it}] = \sigma_R^2 x_{it}^2$$

Moreover, we assume that the agents are homogeneous in risk-aversion. This assumption in particular gives us the possibility to exclude any other potential reason to observe heterogeneous choices between the players. The agents' instantaneous preferences are described by

$$u_{it}(C_{it}) = E[C_{it}] - aVar[C_{it}]$$

where a is the coefficient of absolute risk-aversion and C_{it} is the consumption at time t. Notice that if an agent i is not belonging to the risk-sharing group, $C_{it} = Y_{it}$. However, since we are considering just connected components (it does not exist a node such that the degree is equal to zero in our setting), each agent takes into account at each t the transfers to/from the rest of the players. Formally, the expected income post-transfer is

$$E_{it}[I_{it}] = (x_{it}ph + (1 - x_{it}))/n + (ph \sum_{j \neq i} x_{jt} + (1 - x_{jt}))/n \tag{2.1}$$

and since we assume identical agents and no uncertainty on the network structure we expect $x_{it} = x_{jt}$ for all i and j belonging to G. Thus, we can rewrite (2.1) as

$$E_{it}[I_{it}] = x_{it}ph + (1 - x_{it}) \tag{2.2}$$

or the expected income post-transfer is not changed. However, the variance observed by each identical agent at time t is

$$Var[I_{it}] = (x_{it}^2\sigma_R^2)/n^2 + (\sigma_R^2/n^2)\sum_{j \neq i} x_j^2 = (x_{it}^2\sigma_R^2)/n \tag{2.3}$$

so as expected the variance is function of the component size n. Thus, given the expressions above, maximizing the utility function we can find the optimal capital share x_{it}^* invested on risky assets for all i ,

$$x_{it}^* = n(ph - 1)/2a\sigma_R^2$$

The result is in line with the "modern portfolio theory": The optimal capital-share invested on risky-assets is function of the returns of these assets, of their variance, of the risk-aversion coefficient, and finally of the size of the risk-sharing group n. To simplify the notation, from now on we define the rate $(ph - 1)/2a\sigma_R^2$ with k. As expected, given our assumptions, belonging to a risk-sharing group of identical agents helps these to increase their capital share invested on risky-assets from k to nk.

As anticipated in the introduction, next step will be to add uncertainty on the network structure assuming "not certain" the existence of each link at $t+1$ between two generic nodes i and j such that $g_{ij} = 1$ at t. Formally, $Prob[g_{ij,t+1} = 1 \mid g_{ij,t} = 1] = (1-\delta) \in (0,1) \forall i, j \in G$. Notice that we do not allow the "creation" of new links between the agents at this stage. As anticipated, this new model feature gives us the opportunity to describe the case where each agent cannot completely rely on the links he observes at time t. If this is the case, the location of each peer assumes a central role for the optimal individual decision, i.e. we differentiate the nodes‘ final optimal choices. To give an example anticipating the formal results, let‘s think about a star-shaped network structure. A "central" node is connected with many (symmetrically located) peripheral nodes so we can distinguish just two "types" of nodes. It is quite clear in this case that once we assume a probabilistic structure the position of the central node is more "advantageous" than that of the peripheral one. Firstly because the central node observes more direct partners (he can obtain liquidity at $t+1$ with higher probability) and also because of the link‘s "channel feature" he has got, i.e. a peripheral node can receive liquidity from another peripheral node if and only if the liquidity flow pass through the central node. As we will see formally, given the probabilistic feature of the graph, the most "central" node(s) will expect to risk-share with more agents than the peripheral ones. This "advantage" will differentiate the players‘ final choices.

Formally and for a generic network structure, the expected income post-transfer of a generic node i now is

$$E_{it}[I_{it}] = (x_{it}ph + (1-x_{it}))/n_{it} + (ph)/n_{it} \sum_{j \neq i} \theta_{ij,t} x_{jt} + \sum_{j \neq i} \theta_{ij,t}(1-x_{jt})/n_{it} \quad (2.4)$$

where $n_{it} \equiv 1/E_{it}[\frac{1}{n}]$[3] and $\theta_{ij,t}$ is the probability to observe the realization of the partner j. At this stage, we assume zero-correlation between the risky assets, so the variance observed by each node at time t is

$$Var[I_{it}] = (x_{it}^2 \sigma_R^2)/n_{it}^2 + (\sigma_R^2/n_{it}^2) \sum \theta_{ij,t} x_{jt}^2 \quad (2.5)$$

Maximizing the utility function we find the optimal x_i^* at time t

$$x_{it}^* = n_{it}(ph-1)/2\alpha\sigma_R^2 \quad (2.6)$$

We notice that the result is "similar" to the previous one except for the fact

[3] To compute n_{it} we use the formula discussed in the 2.4 Section.

that now the expected component size differentiates the optimal final agent‘s choice. Intuitively a node i will observe the highest n_{it} at time t than the rest of the nodes j if and only if his average distance with the rest of the agents (number of links to reach j) is the smallest one. In the example of the star-shaped network structure, the “central” node of the star will observe the highest n_{it} and consequently he will be able to choose the highest x_{it} between all the nodes belonging to the component. However, anticipating the further analysis, we can say that once we assume agents‘ sight bigger than one (a node i observes js distant more than one link from him), a “simple” analysis of the nodes‘ degree is no more sufficient to define their “centrality degree”. In fact, even if in a star-network the “centrality” of the nodes is intuitively related to their degree, for a generic structure this is not always the case. In particular, at this stage, we can use our measure n_{it} to characterize the agents/nodes‘ centrality scores. The nodes‘ centrality scores are proportional to n_{it} $\forall i \in G$, i.e. the agents‘ centrality is function of the expected number of partners with whom they will share their risk. Finally, notice that the individual peers‘ choices do not affect the single agent i‘ decision at this stage. Without assuming different from zero-correlation between the risky-assets, the key feature differentiating the agents is just n_{it} . In the next section we will relax this assumption and discuss the relative implications.

2.3 The positive-correlation case

In this section we assume positive correlation between the risky-assets. The correlation parameter is defined by $\varphi \in (0, 1)$. We still mantain the previous assumptions and in particular the one defining the graph as probabilistic. Notice that the positive asset-correlation does not influence the post-transfer expected income 2.4, but only the variance observed by the agents at each t. Formally,

$$Var[I_{it}] = (x_{it}^2\sigma_R^2)/n_{it}^2 + (\sigma_R^2/n_{it}^2)\sum_{j\neq i}\theta_{ij,t}x_{jt}^2 + (2\varphi\sigma_R^2 x_{it}/n_{it}^2)\sum_{j\neq i}\theta_{ij,t}x_{jt} \qquad (2.7)$$

Solving the optimization problem we can derive the best reply functions for each node $i \in G$. In particular, we see that the optimal choice now is function also of the partners‘ optimal ones,

$$x_{it}^* = n_{it}(ph-1)/2\alpha\sigma_R^2 - \varphi\sum\theta_{ij,t}x_{jt}^* \qquad (2.8)$$

We notice that two main features affect now the final optimal choice of i. Firstly, as we have seen in the previous section, the measures n_{it} increases the optimal risk-exposure of the agent (this is seen in the first term of the expression above). Secondly, the increasing optimal x^*_{jt} of the peers negatively affects the agent i's optimal risk-exposure. Intuitively, higher is the risk taken by the peers, higher is the risk observed by each node, given the positive asset-correlation. However, the j partners'optimal choices are discounted by their distance with the node i: The i's location affects again his optimal choice but in a reverse way. This is possible since the risk taken by the i's peers, given $\varphi \in (0,1)$, is assumed partially "substitute" of the risk taken by i himself. However, similarly to the discount coefficient used by Bonacich (1987) in his centrality measure, the choices of the i's partners are discounted by θ_{ij} , function of the distance from i to j nodes. Summarizing, the final optimal choice is function of n_{it} , that depends on the relative location of i at time t in the network and on link probability δ, of the correlation coefficient φ, and finally of the peers' optimal choices, function themselves of their relative locations.

To underline the importance of a "structural" analysis to forecast the optimal agents' behaviors, we present below two examples considering different starting network structures. As anticipated we have chosen two specific graphs such that we can easily observe the impact of different locations on the final nodes' choices. The first example considers three nodes connected through a line-shaped link-structure (see Figure 2.1). The second one describes a seven-nodes network in which two symmetric nodes have the highest degree and are respectively connected to two symmetric pairs of peripheral nodes, and finally one node, located as a bridge between the two automorphic subgraphs.

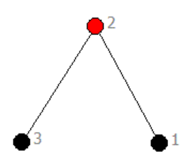

Figure 2.1: 3 nodes Star

In this first example we start with a 3 nodes line-shaped network structure, where the node labelled 2 connects 1 and 3. We can just study the nodes' choices of 1 and 2 since 3 belongs to the same equivalence class of 1. Assuming $k = 0.2$ and for the moment $\varphi = 0$ we find the following optimal x^*_{it} (vertical axis) for δ between 0 and 1 (horizontal axis),

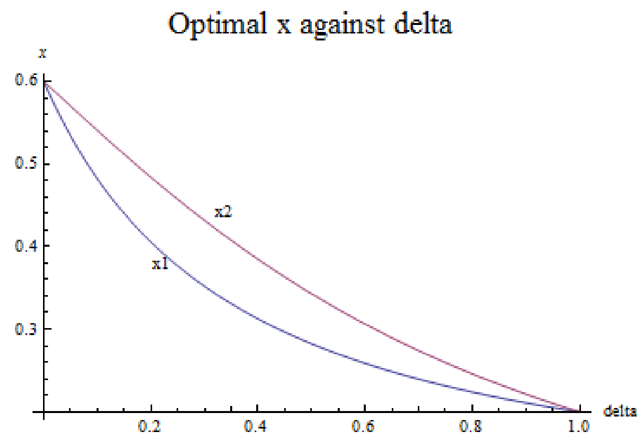

Figure 2.2: 3 nodes Star with zero asset-correlation. We notice the advantage of the node 2 over 1 and 3 for all the $\delta \in (0, 1)$.

As we can see, the node 2 mantains an "advantage" over 1 and 3 for all the values of δ between 0 and 1. Before analyzing the second structure, we study the effect of different positive values of φ on the optimal risky choice. To do this, we fix arbitrary values of δ, (0.3,0.5,0.7,0.8), and compute the optimal x_{it}^* for different φ values (horizontal axis).

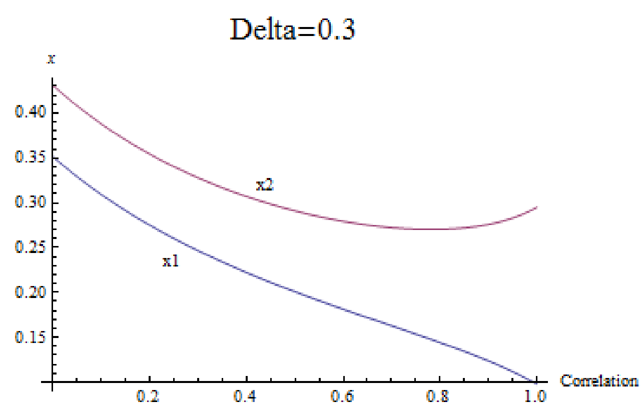

Figure 2.3: 3 nodes Star with positive asset-correlation and $\delta = 0.3$. The node 2 chooses optimal x higher than 1 and 3 nodes for all $\varphi \in (0, 1)$.

In the Fig.2.3 above the link probability is setted to $\delta = 0.3$ and related expected component-size vector $\bar{n} = (2.19, 2.4, 2.19)$. We observe that the node 2 still has an advantage respect to 1 and 3. The probabilistic graph feature simply decreases the optimal agents' choices without reverting the node 2's advantage over 1 and 3. This is possible since, given this specific structure, the node 2 has both an advantage in terms of higher n_{it} and relative location of his partners. As we will see in the next pictures, increasing the probability δ, we observe smaller differences on the optimal x_{it}^* of the nodes, overall for high asset-correlation levels. In the next picture we present the optimal choices for $\delta = 0.5$,

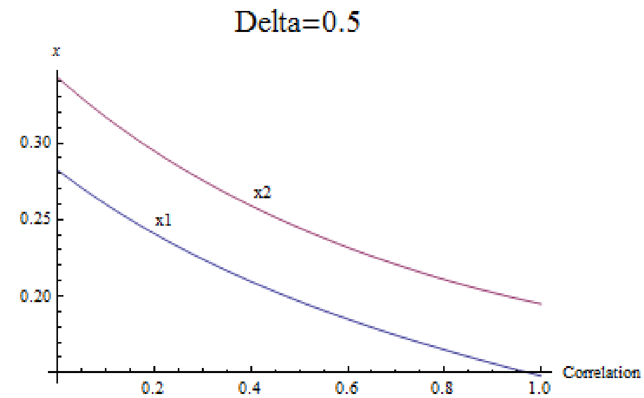

Figure 2.4: 3 nodes Star with positive asset-correlation and $\delta = 0.5$.

As we can see the results do not change qualitatively from the previous one. The node 2 mantains an higher advantage over 1 and 3 than in the previous case. Following, we present other two for $\delta = 0.7$ and 0.8 respectively.

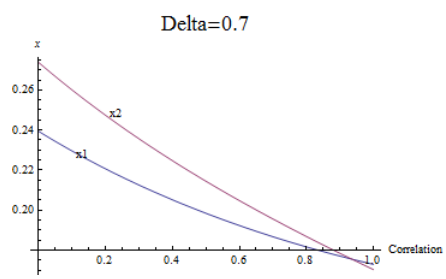

Figure 2.5: 3 nodes Star with positive asset-correlation and $\delta = 0.7$.

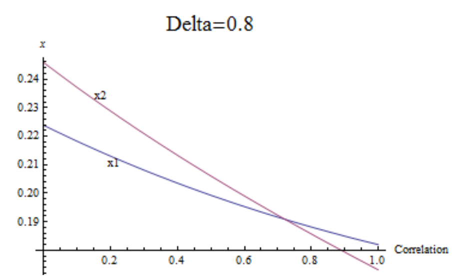

Figure 2.6: 3 nodes Star with positive asset-correlation and $\delta = 0.8$. For $\varphi > 0.7$ we start to see the previous node 2‘s advantage reverted in favour of the peripheral nodes.

Summarizing, increasing the probability δ, given this star-shaped structure, we observe a general decreasing of the nodes‘ optimal risk-exposure but also a decreasing (in favour of the peripheral nodes after certain values of δ) of the relative differences on x_{it}^{*} between the players.

As anticipated above, now we propose the same analysis for a more complex structure composed by seven nodes. As we can see from the Figure 2.7 below, the nodes 1 and 3 are the ones with highest degree, {4,5,6,7} the ones with the lowest (we will define them again as peripheral nodes), and finally the node 2 with a relatively intermediate degree but a particular “bridge position” between two automorphic subcomponents. This classification describes as well the three equivalence classes composing the graph, thus we will analyze just the optimal choices of the representative nodes 1, 2 and 4 of their respective equivalence classes.

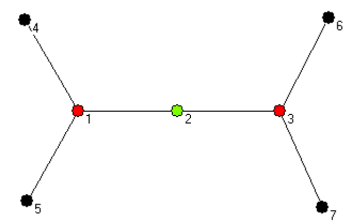

Figure 2.7: 7 nodes structure

We start again analyzing the zero-correlation case. The optimal x_{it}^{*} for δ between 0 and 1 are described below,

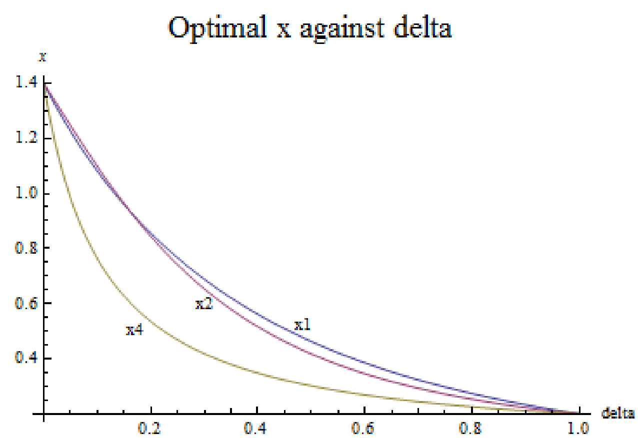

Figure 2.8: 7 nodes structure with zero asset-correlation. Until around $\varphi = 0.1$ the node 2 has the highest optimal x. The peripheral nodes choose the lowest optimal x for all $\varphi \in (0, 1)$.

It is interesting to notice that until a particular level of δ, the most central node is the node 2 (and consequently we expect 2 to choose the highest capital-share on risky assets), but after that, the nodes 1 and 3 appears to have the highest n_{it} scores. This first picture helps us to underline the impact of n_{it}, function of the i's location, and of δ, on the final optimal choice. As we have done previously, let's assume now positive asset-correlation for fixed probability δ levels.

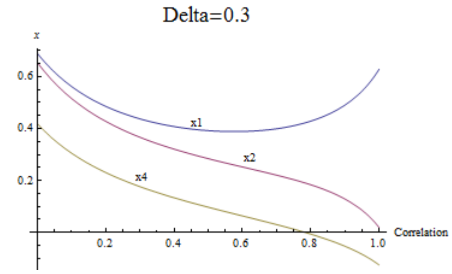

Figure 2.9: 7 nodes structure with positive asset-correlation. The node 1 chooses the highest x. The peripheral nodes choose the lowest optimal x for all $\varphi \in (0,1)$.

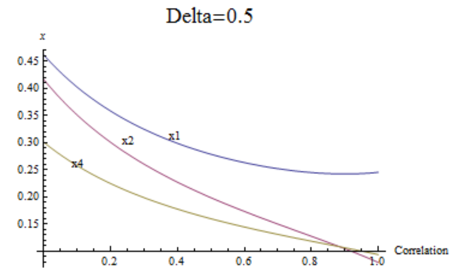

Figure 2.10: 7 nodes structure with positive asset-correlation. We can notice the decreasing differences between the optimal x of the nodes 2 and 4, once we increase the asset-correlation.

It is interesting to observe in the Figure 2.9 that the differences on the optimal choices of the nodes 2 and the peripheral ones become smaller increasing the asset-correlation. This is due to the smaller degree difference between them and also to the "negative" effect given by the relatively advantageous location of the nodes 1 and 3. The nodes 1 and 3 observe with higher probability the peripheral nodes (with low n_{it} scores) while the node 2 observes 1 and 3 as direct partners. This difference explains in part their relative advantage. In the Figure 2.10 we can observe more clearly the impact of the asset-correlation on the nodes' location. For $\delta \geq 0.7$ we notice the advantage of the peripheral node 4 over 2 for $\varphi > 0.5$. The intuition behind is that when the structure is particularly uncertain and the asset-correlation relatively high being more connected or at a shortest distance with other peers is not more advantageous.

Following we present the optimal x^*_{it} for $\delta = 0.7$ and 0.8 respectively,

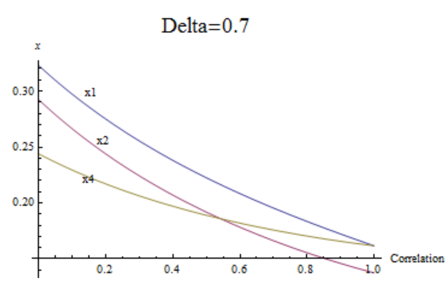

Figure 2.11: 7 nodes structure with positive asset-correlation and $\delta = 0.7$.

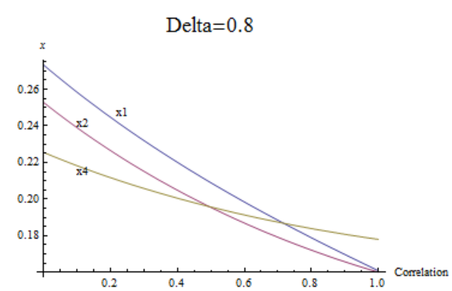

Figure 2.12: 7 nodes structure with positive asset-correlation and $\delta = 0.8$.

In the next section and before discussing about potential structural measures, we discuss in more details the "centrality" measure n_{it}. As anticipated in the introduction, n_{it} relies on the probabilistic feature of the graph.

2.4 The centrality measure n_{it}

We have seen from (8) how the agents' final optimal choices are dependent of their relative location in the network. Moreover, the link between two peers affects in opposite ways the risk taken by them. As we have previously remarked, the positive impact is described by the n_{it} score, while the negative one is catched by the node's partners choices and the positive asset-correlation coefficient. To compute n_{it} we calculate the expected number of peers with whom the agent i is expecting to share his risk. This centrality measure in particular is function of the average distance between the nodes: More links a node i needs to walk through to reach their partners js, lower probability he faces to reach them, assuming δ i.i.d.. Formally,

$$n_{it} = \frac{1}{\sum_{m=1}^{n} p_m(H_m)\frac{1}{m}} \tag{2.9}$$

where m is the number of peers risk-sharing including i, δ is the probability defined between 0 and 1 that an existing link at t will exist at $t+1$, and $p_m(H_m)$ defines the probability to observe the subgraph H_m generated by a specific set of ties (connecting the node i to other $m-1$ nodes) containing m nodes including i. Notice that we can have more than one combination of paths containing i and $m-1$ other nodes. The n_{it} measure, differently from the Bonacich one, takes into account all the paths including i and not only the ones emanated from this. The "density" of the graph and consequently the average distance between the nodes affects directly the measure n_{it}. A more dense graph in fact leads to higher number of subgraphs H_m (increasing the number of combinations of m nodes including i). However, n_{it} is also function of the link-probability δ, so as we have seen from the examples presented above, it is not always the case that for all the values of $\delta \in (0,1)$ the node with lower average distance from the rest of the agents has the highest n_{it} score. Summarizing, the n_{it} centrality measure takes into account all the possible paths passing through the node i at time t, it allows for flowing by parallel duplication, and finally it works on connected graphs (the distance between two nodes belonging to two different unconnected component is assumed infinite). In this model we assume also independent probability over the existance of each distinct path. Finally we want to stress out the fact that this measure counts the expected number of peers a node i is attached to at time t. This means that differently from the mainstream of the network centrality measures, n_{it} is "node-founded" even if the paths to reach each peer directly influence the measure. We can observe a clear example of this "ambiguity" in the second network structure presented above. In that case, the node with highest closeness and betweeness scores is the node 2, given his specific "bridge" location. However, measuring his centrality through n_{it}

we find that just for particularly low link-probability δ the node 2 has the highest score. Intuitively, even if the node can reach in less steps the rest of the peers, with higher probability than 1 and 3 he can find himself in isolation. Following the Sabidussi (1966) criteria qualifying a centrality measure, we can notice that n_{it} fully satisfying them. In particular, adding a tie to a node i increases his n_{it} score, and adding a tie anywhere in the network never decreases his centrality. However, as Borgatti and Everett (2006) underlines, these criteria do not fully explain the centrality of a node. This feature can be also observed in our model: The "total-centrality" of an agent is the result of two centrality measures as we can see from the agent's best reply function. Thus, the n_{it} measure in our model describes the geodesic relative advantage/diadvantage of a node without giving us any information about the relative advantage/disadvantage of the "influence" of a node. We have to remark that the n_{it} measure is directly connected to the Katz (1953) measure and consequently to theBonacich (1987)'s one. Their measures are weighted counts of the number of walks originating (or terminating) at a given node. This means roughly speaking that long walks count less than short ones. However, the n_{it} measure, as previously said, counts the expected number of nodes reached by weighted paths (not necessarely edge or vertex-independent).

We show below, as example, how we can compute the n_{it} measure for the specific case of a star-shaped network structure with one central star-node and n peripheral nodes,

$$n_{it} = \frac{1}{\sum_{c=1}^{n} \frac{1}{c} \frac{(n-1)!}{(c-1)![(n-1)-(c-1)]!}(1-\delta)^{c-1}\delta^{n-c}}$$

$$n_{jt} = \frac{1}{\delta + \sum_{c=2}^{n} \frac{1}{c} \frac{(n-2)!}{(c-1)![(n-2)-(c-1)]!}(1-\delta)^{c-1}\delta^{n-c}}$$

with i labeling the star node and j the peripheral one.

2.5 Myopic nodes and positive asset-correlation

In this section we relax the perfect knowledge of the network structure assumption, assuming that the agents can observe just their direct neighbors (nodes 1-link distant) and neighbors 2-link distant. We mantain the rest of the previous assumptions as much as the positive asset-correlation one. We start presenting the 7-nodes structure previously studied, assuming 1-link nodes' sight and following, the case for 2-links nodes' sight. Notice that for the particular1-link nodes' sight assumption, n_{it} varies between 1 and d_{it} , the degree of node i. Assuming $\delta = 0.3$ we obtain the following results,

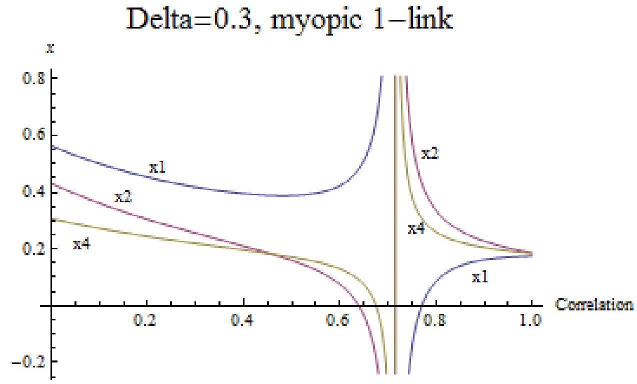

Figure 2.13: 7 nodes structure, positive asset-correlation, myopic nodes (1 link sight) and $\delta = 0.3$.

We can clearly see the "negative" effect of being connected to the nodes {1,3} faced by the rest of the nodes, for low correlation values ($\varphi < 0.7$). However, we observe the reverse situation for correlation values above 0.7: The nodes {2,4,5,6,7}choose higher x^* than 1 and 3, and in particular the node 2 has highest x^*. Notice that assuming 1-link sight the nodes 1 and 3 observe highest number of partners for all the values of δ between 0 and 1. However, the "peer effect" is particularly clear for the node 2 choosing lower x^*_{it} than the peripheral nodes for some asset-correlation level (the same is true for values $\varphi > 0.7$ comparing the nodes {1,3} with {4,5,6,7}). For $\delta = 0.5$ we obtain,

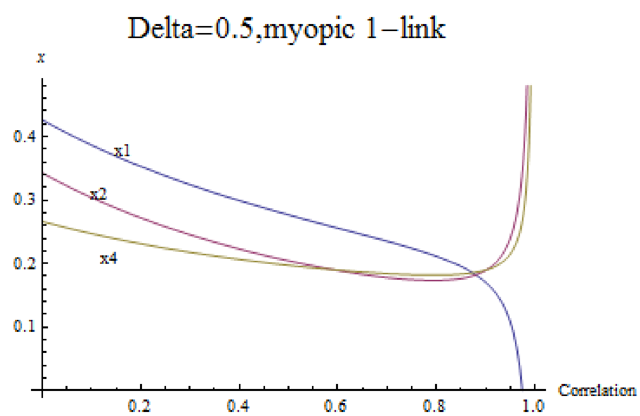

Figure 2.14: 7 nodes structure, positive asset-correlation, myopic nodes (1 link sight) and $\delta = 0.5$.

We notice immediately that for $\delta \geq 0.5$ we don't have the particular result obtained in the previous picture for $\varphi > 0.7$. However, even in this case we observe the "peer effect" reverting the relative advantage of the node 2 on {4,5,6,7} for $\varphi > 0.9$. Following the Figures for $\delta = 0.7$ and 0.8,

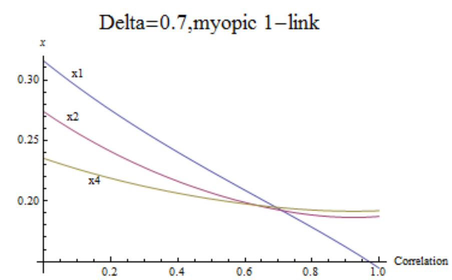

Figure 2.15: 7 nodes structure, positive asset-correlation, myopic nodes (1 link sight) and $\delta = 0.7$.

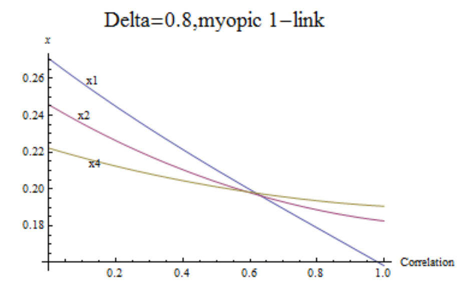

Figure 2.16: 7 nodes structure, positive asset-correlation, myopic nodes (1 link sight) and $\delta = 0.8$.

At this high δ levels, we observe the reverting advantage noticed before for lower asset-correlation coefficients.

Now, as anticipated, we generate the results assuming 2-links distance sight. This case appears to be more interesting than the previous one since the node 2 is the only player able to see the whole structure, i.e. the strategical location of this agent has a strong impact on the final optimal agents'choices. For $\delta = 0.3$ we obtain

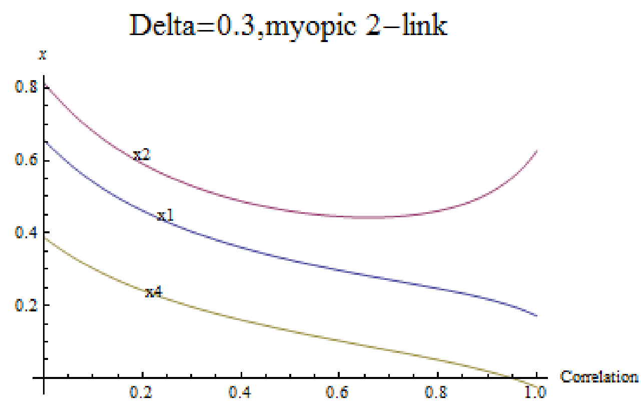

Figure 2.17: 7 nodes structure, positive asset-correlation, myopic nodes (2 link sight) and $\delta = 0.3$.

At this probability level the node 2 observes highest n_{it} than the rest of the nodes. The node 2 mantains an advantage for all the φ values. Even in this case the peer effect is not strong enough to revert the impact of n_{it}. Below the results for higher δ,

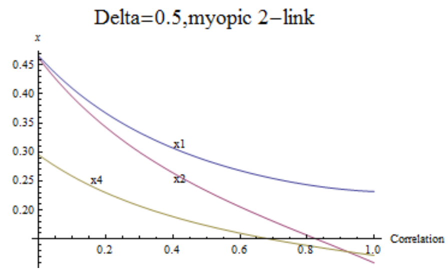

Figure 2.18: 7 nodes structure, positive asset-correlation, myopic nodes (2 link sight) and $\delta = 0.5$.

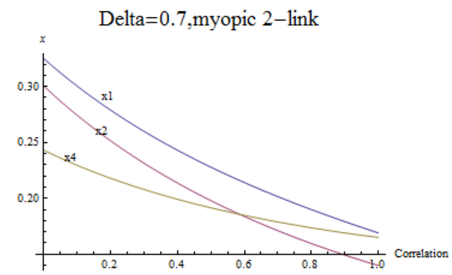

Figure 2.19: 7 nodes structure, positive asset-correlation, myopic nodes (2 link sight) and $\delta = 0.7$.

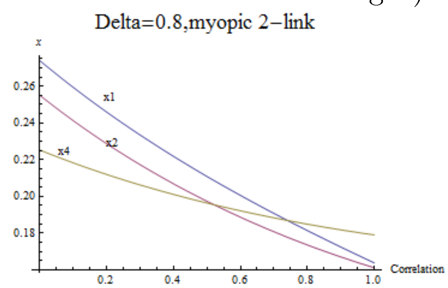

Figure 2.20: 7 nodes structure, positive asset-correlation, myopic nodes (2 link sight) and $\delta = 0.8$.

The results are qualitatively different for $\delta > 0.3$: The peer-effect becomes central to understand the choices‘ differences. In this case, the nodes 1 and 3 have a slightly higher n_{it} score than the node 2 but, for the fact that they are attached to relatively disadvataged located nodes, the difference with node 2'optimal choice is amplified. Notice that if we were studying the centrality "a la Bonacich" it would be not sufficient to understand the results obtained above. This is due to two main features of the model. Firstly, the fact that the results depend mainly on the two parameters δ and φ. Secondly, the fact that the link in our model is channel of two different things. It represents both the liquidity channel between the agents and also a canal through which the peers‘ risk is spread through the network. The models using the β centrality measure or generally the "eigenvectors centralities" assume that the peers‘choices impact on an individual decision is either positive or negative. In this model, both positive and negative effects are present at the same time, as we can see from the 2.6. Having higher n_{it} means for a generic node i to face higher risk-pooling opportunities, while higher Bonacich centrality scores of the direct partners increase the risk observed by i for positive correlation coefficients φ. We can also argue that a simple analysis of the nodes‘degree is not sufficient to understand the optimal agents‘choices. As we argue in the next section to pick up the most central node(s) of the component, we need different "centrality" measures.

2.6 Structural analysis

In this section we try to underline the impact of the starting network structure on the final optimal agents‘ choices. Before starting the proper analysis it is necessary to clarify the meaning of the “link” between the agents as much as the results obtained so far. What we can understand from 2.6 is that the connection between two nodes is both a channel of liquidity and a way to “receive” the peers‘ correlated risk. This double feature represents the central key of the model. Analyzing the literature on risk-sharing using a network analysis approach we see that the negative impact of the peers was usually represented by their risk of default and its related consequences. This implies that with this setting the link between two agents could exist if and only if there has been a previous liquidity flow between the interested parts. In this model is not necessarely the case. The connection represents the existance of a risk-sharing norm between two agents and no previous liquidity exchange is necessary to observe a connection between two nodes. This difference becomes central to explain the agents‘ behaviours observed in our model, overall if we recall the assumption of no-deviation from the risk-sharing norm on the agents‘ strategies. In addition, the double link‘s feature prevents us to use only the Bonacich centrality measures to understand the agent‘ choices. In fact, as underlined before, the Bonacich centrality scores, a “volume measure” using the Borgatti 2006 terminology, could just explain the negative\positive impact of the peer risk-choices but fails to take into account at the same time the two opposite directions. Specifically, the Bonacich measures can help us if and only if we assume one specific “direction” of the peers‘ impact on the agent‘s final decision; In particular, the influence of a node’s choice on the peers’ ones. Looking to the following picture we notice this feature,

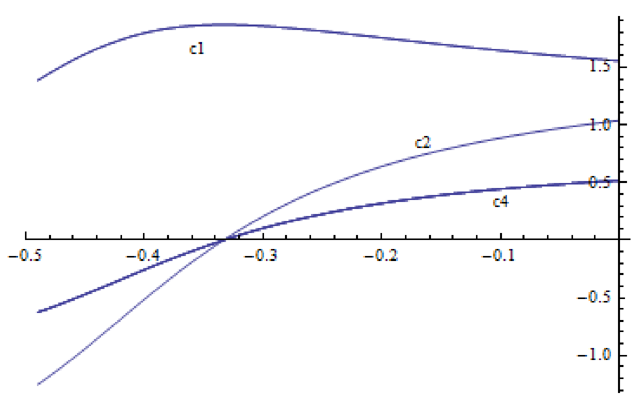

Figure 2.21: Bonacich centrality for the 7 nodes structure.

According to the Bonacich centrality scores, for δ values between 0 and $\mid 0.5 \mid$ the nodes 1 and 3 are the most central. [4].What we can observe in the picture is the

[4]For the fact that δ is between 0 and 1 we know that the Bonacich centrality scores converges

relative advantage given by the location of the nodes 1 and 3, but we miss the negative effect given by the same location once we assume in the model the flow of a positive correlated risk. We notice that the node 2 for example is particularly negatively affected by the close connections with the nodes 1 and 3, respectively linked to two peripheral nodes (located in the "worst" geodesic position). However, as we can see from the b.r.f. above, for particularly high correlation coefficients, and uncertainty on the ties' future existance, the peripheral nodes are those in advantageous location.

Given the "dual-feature" of the link we suggest two different tools, focusing each on one of the two link mentioned characteristics: The $\bar{F}$- and the Intercentrality-measure. The first one measures the fragmentation of the network due to the elimination of a node i. In particular, the proposed $\bar{F}$-measure is a modified version of the F-measure explained by Everett and Borgatti (2010) and takes into account the expected size of the components created after the elimination of i from the main connected component. This measure in particular can help us to understand the impact of the elimination of a specific agent on the n_{it} centrality measure. Formally,

$$\bar{F_{it}} = 1 - \frac{\sum_{j \neq i} n_{jt}(n_{jt} - 1)}{n^2(n-1)}$$

bounded above at 1. This measure catches an important feature of network structure. Intuitively, $\bar{F}$ explains the impact of a node i on the risk-sharing group in terms of his "structural centrality". As example consider a star-shaped structure with the node i as central agent connecting the rest of $n-1$ js nodes through single links with them. The elimination of the node i from the component fragments the whole structure and creates $n-1$ single-node components. In terms of $\bar{F_{it}}$ score, we will obtain $\bar{F_{it}} = 1$, its maximum level. Moreover, $\bar{F_{it}} = 1$ means that $x^*_{jt} = k$, their optimal x when they do not risk-share with other peers (autarky case). On the other hand, suppose the case of a starting graph where the elimination of a node i leaves a component of $n-1$ nodes "perfectly" connected (the resulting component is a connected network where all the j nodes can reach each other with a direct link). In this case, the impact of i-elimination will be relatively the smallest (the exact $\bar{F_{it}}$ score depends also on the δ chosen) since the rest of the j nodes can still enjoy the risk-pooling effect in the most effective way. The $\bar{F}$ scores for the 3-nodes and 7-nodes structures presented above for $\delta \in (0,1)$ are the following,

for $\delta \neq 0.5$ since the inverse of the norm of the largest eigenvalue of the adjacency matrix G is 0.5. Moreover, we added the negative sign in front of δ since we want to analyze the centrality in the case where the j peers' choices are substitute to x_i and consequently have a negative impact on it.

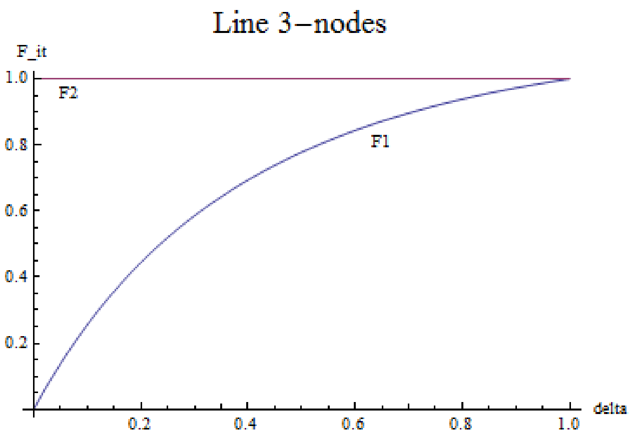

Figure 2.22: $\bar{F}$ centrality for the 3 nodes Star.

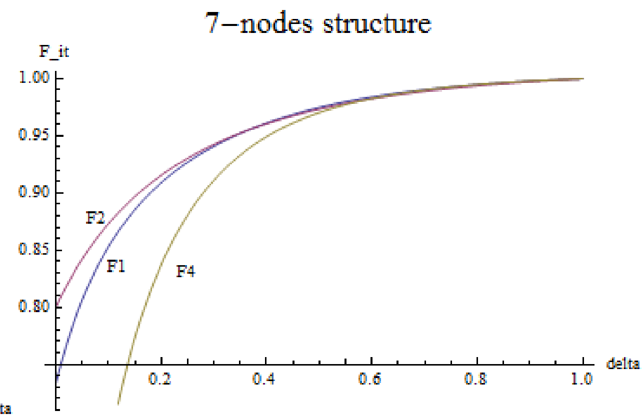

Figure 2.23: $\bar{F}$ centrality for the 7 nodes structure.

We can see that for the 3-nodes structure the node with the highest $\bar{F}$ score is the node 2 for all the $\delta \in (0,1)$ and equal to 1: Taking out this node leads to the maximal "damage" to the structure, leaving the nodes 1 and 3 isolated. For the 7-nodes structure, again the node 2 has the highest $\bar{F}$ score, but as we can see from the picture above his $\bar{F}$ centrality, for δ higher than 0.5, starts to be closer to the node 1 and 4's ones. We will produce now the results for the $\bar{F}$ scores of the "myopic" nodes (observing just their direct neighbors or 1-link distant neighbors and 2-link distant neighbors) composing the 7-nodes structure,

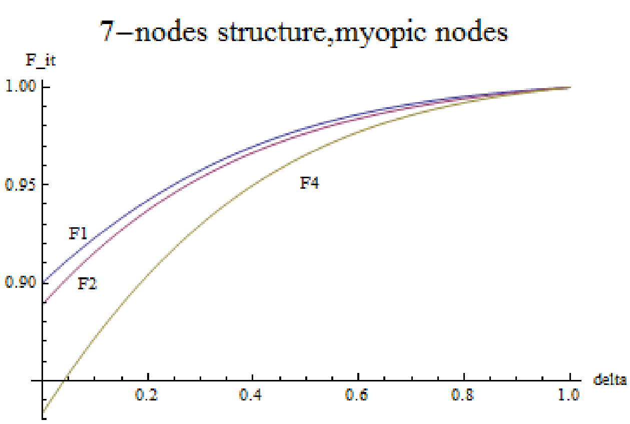

Figure 2.24: $\bar{F}$ centrality for the 7 nodes structure with myopic nodes (1 link sight).

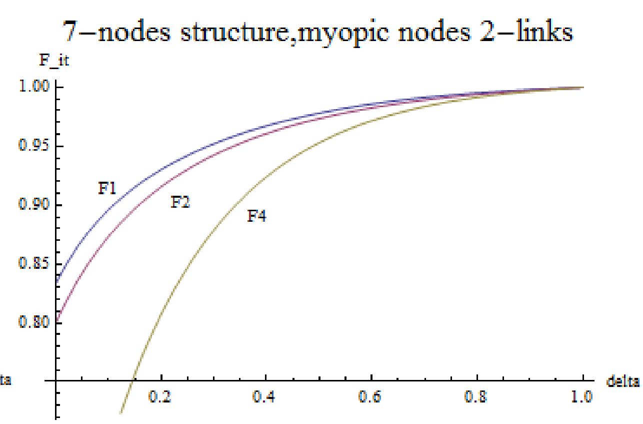

Figure 2.25: $\bar{F}$ centrality for the 7 nodes structure with myopic nodes (2 links sight).

In the first case we notice that the node 1 (and symmetrically the node 3) has the highest $\bar{F}$ score, i.e. the elimination of this node damages more the risk-sharing structure. It is interesting to notice the higher impact of the peripheral node than the node 2. Observe also that at this stage we have analyzed the impact of a single-node elimination and not multiple-nodes one. Assuming 2-links distant sight, the nodes 1 and 3 remain the most "central" according to the $\bar{F}$ measure.

The second measure is a particular centrality tool developed by Ballester et al. (2006) that helps us to understand the impact of a node i on the Bonacich centrality scores of his peers. In particular this measure can tell us the negative impact of the relative peer location. Formally,

$$c_{it}(g, \beta) = b(g, \beta) - b(g^{-i}, \beta) + 1$$

or alternatively,

$$c_{it}(g, \beta) = b_{it}(g, \beta) + \sum_{j \neq i} [b_{jt}(g, \beta) - b_{jt}(g^{-i}, \beta)]$$

with $b_{it}(g, \delta)$ defining the Bonacich score of the node i belonging to the component g at time t. Computing the intercentrality scores for our 7-nodes structure, we find the following results for $\mid \delta \mid \in (0, 1)$,

Nodes	$\beta = -0.2$	$\beta = -0.49$
1	1.775	0.022
2	-0.107	-4.916
4	0.137	-3.482

Figure 2.26: Intercentrality scores for the 7 nodes structure.

As we can see from the table, when we assume relatively small β coefficients, the higher impact is given by the elimination of the nodes 1 and 3. Once we assume higher β, the "central" node is the agent 2.

Summarizing, the parallel analysis of these two measures could explain the impact of each node i in terms of the i's "role" in the network. High $\bar{F}_i$ values tell us that the cohesion of the network strongly depends on the node i, while $c_i(g, \beta)$ scores show the i's peer effect on the other nodes. The net impact on the final agents' choices will depend on the parameters δ and φ assumed.

[................]

Talking about the structure impact on the agents' choices we focus the attention now on the average agent's distance from the rest of the group. The average distance measure d_{it} for a node i at time t gives us the "geographical" information about an agent's location in the network. Formally,

$$d_{it} = \frac{\sum_{j \neq i} d_{ij}}{n - 1}$$

Again, smaller d_{it} does not necessarely mean to choose higher x^*_{it}, as we have seen in the previous examples. However, we argue that, under certain conditions, there exists just a unique structure minimizing the risk taken by the agents, and this is the "line" structure.

We start proving that starting from a line-shaped network structure, any rewiring leads to higher average distance between the nodes.

> **Claim 1.** *Any structure different from the line-shaped one leads to higher average distance between the nodes.*

Proof. Assume a starting line-shaped network structure of $n \geq 4$ nodes as in the Figure (2.27). Let's label the nodes as $j_1, j_2, ..., j_n$, where with the node indexed with 1 we label one of the two peripheral nodes (the other one consequently is indexed with n). For simplicity (but without affecting the final conclusion) we modify the structure, cutting the link between the first node of the line, j_1, with the second one, j_2 , and we activate a new link between j_1 and a generic node j_m , with $m \in [3, n-1]$ as in the Figure (2.28). In this way, the structure now is not more a line but a generic tree with the same number of links between the nodes. Firstly, notice that for the node 1, linking with another node different from j_2 can only be beneficial in terms of average distance with the rest of the nodes, i.e. the net-impact for this node will be positive. The same we can say about the nodes j_{m+a} with $a \in [0, n-m]$. In fact, for all of them it is changed the distance (now shorter) from the node j_1 without affecting the distance with the rest of the nodes. Thus, until now we can say that j_1 and j_{m+a} will have a positive net-impact from the new link-rewiring. Now, let's divide the rest of the nodes, indexed as j_h, with $h \in [2, m-1]$, in two sets: One composed by nodes such that $m - h \leq h - 2$, or $h \geq \frac{m+2}{2}$, and one such that $h < \frac{m+2}{2}$. Notice that doing this we are dividing the j_h nodes between those nodes such that the distance between them and j_m node is smaller/equal than the previous distance (before the rewiring) with j_1 ,with the ones where the converse is true. The intuition behind that comes from the fact that for the nodes such that $h \geq \frac{m+2}{2}$, we expect a positive net impact from the rewiring since they will be linked to the node j_1 in shorter distance, while for the other ones, the new structure will connect them to j_1 with a longer path than before the rewiring. Finally, we need to show that the number of nodes belonging to the latter group is always lower than the number of the rest of the nodes belonging to the network, for any line-shaped structure of n nodes. In particular, it must be true that $\frac{m+2}{2} - 1 \leq 1 + n - \frac{m+2}{2}$ or $m + 2 \leq n + 2$. This is always true with strict inequality, given our initial assumptions on m and n. Thus, we can conclude that any structural modification to the line-shaped network leads to a graph where the average distance between the nodes is lower.

Figure 2.27: Seven nodes Line.

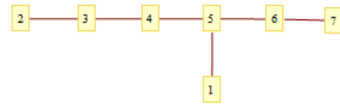

Figure 2.28: Seven nodes Line "rewired".

Given this result we can show that, under certain conditions, the line-shaped network structure is the one minimizing the total risk taken by the agents belonging to the component.

> **Claim** 2. *Given our assumptions and φ such that $\frac{\partial x_i}{\partial d_{it}} < 0$, the network structure minimizing the total risk taken by the agents is unique and it is the line-structure.*

Let's underline the fact that the total risk (variance) observed at each t, is function of the capital shares invested by the agents on the risky-assets. We have previously showed that the optimal x^*_{it} chosen by a generic agent i is a function $f(n_{it})$ and that n_{it} is a function $g(\delta)$ of the probability $\delta \in (0,1)$. Moreover, n_{it} depends on the relative location of the node i at time t. In fact, as we can see from the formula used to compute n_{it}, shorter average distance d_{it} implies higher probability (in average) to observe the partner nodes and consequently higher n_{it} score. In particular, let's define with $l_G = \frac{1}{n(n-1)} \sum d_{it}$ the total average distance between the nodes of the component G. As we have said, $\frac{\partial n_i}{\partial d_i} < 0$ since more a node i is distant from the rest of the nodes (more links between i and any $j \in G$) lower is the expected number of agents observed at $t+1$ (lower n_{it}). Thus, by chain rule we have $\frac{\partial x^*_{it}}{\partial d_{it}} = \frac{\partial f(.)}{\partial n_{it}} \frac{\partial g(.)}{\partial d_{it}} - \varphi \sum_{j \neq i} \frac{\partial \theta_{ij}}{\partial d_{it}} \frac{\partial x^*_{jt}}{\partial d_{it}}$, with $\frac{\partial \theta_{ij}}{\partial d_{it}} < 0$ for the same reason explained for n_{it}, and not clear sign of $\frac{\partial x^*_{jt}}{\partial d_{it}}$ since it depends on the impact of the change of average distance of i, d_{it} , on the average distance d_{jt}, i.e. $\frac{\partial x^*_{jt}}{\partial d_{it}} = \frac{\partial x^*_{jt}}{\partial d_{jt}} \frac{\partial d_{jt}}{\partial d_{it}}$. Thus, the net impact of an increasing of d_{it} on the optimal i's choice depends also on φ and on $\frac{\partial x^*_{jt}}{\partial d_{it}}$. Underlined this fact and given the previous claim on the structure maximizing the distance between the nodes, we can say that, given a specific value of φ small enough or such that $\sum_{j \neq i} \frac{\partial x^*_{jt}}{\partial d_{it}} < 0$, the network structure with highest average distance between the nodes is "the line", and this will be also the structure with lowest individual risk taken by the agents.

We generate an example that shows precisely what we have claimed. In the pictures below we present two graphs composed by 4 nodes: A star-shaped and a line-shaped one. As we can see from the b.r.f. below, given δ we are able to compute the φ such that the line (higher average distance) guarantees to a specific node higher capital shares on risky assets.

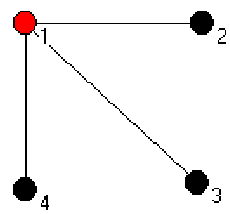

Figure 2.29: 4 nodes Star.

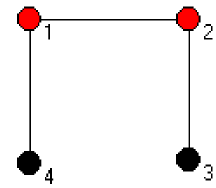

Figure 2.30: 4 nodes Line.

Notice that the previous claim is only valid for φ and δ such that $\frac{\partial x_{it}}{\partial d_{it}} < 0$. This restriction is necessary since for φ and δ such that $\frac{\partial x_{it}}{\partial d_{it}} > 0$ the correlation coefficient is high enough to revert the positive effect on the optimal x_{it} of an higher n_{it} score to a negative one. Intuitively, for high enough asset-correlation levels, being connected with more nodes (in expected value) and overall linked with nodes in "advantageous" locations could become not-beneficial for some agent (see the Figures below).

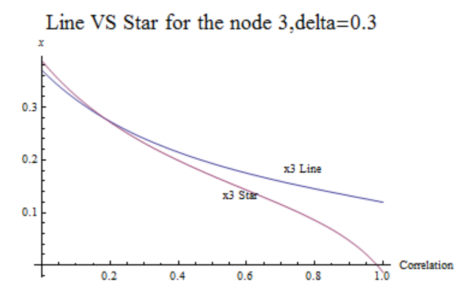

Figure 2.31: 3 node‘s best reply for $\varphi \in (0, 1)$ belonging to the "Star" or the "Line" (as peripheral node) for $\delta = 0.3$.

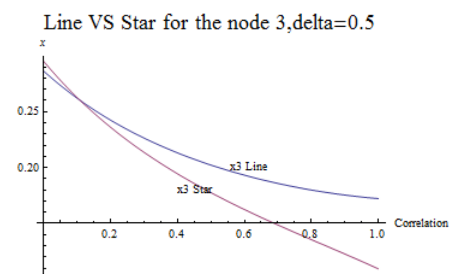

Figure 2.32: 3 node‘s best reply for $\varphi \in (0, 1)$ belonging to the "Star" or the "Line" (as peripheral node) for $\delta = 0.5$.

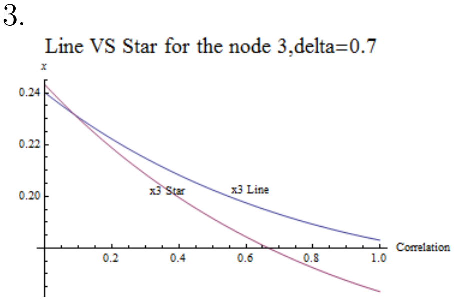

Figure 2.33: 3 node‘s best reply for $\varphi \in (0, 1)$ belonging to the "Star" or the "Line" (as peripheral node) for $\delta = 0.7$.

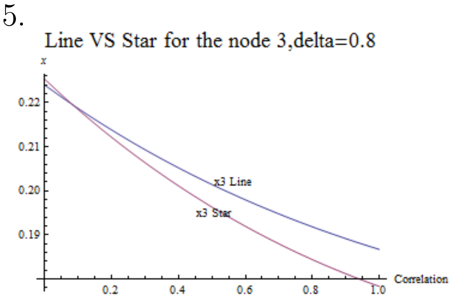

Figure 2.34: 3 node‘s best reply for $\varphi \in (0, 1)$ belonging to the "Star" or the "Line" (as peripheral node) for $\delta = 0.8$.

Without reporting the plots, we remark that for the nodes 1 and 2, we find that the star-shaped network is more beneficial than the line for all the $\delta \in (0, 1)$. This is possible since intuitively these nodes benefit from the star structure in terms of both shorter average distance and peer effect.

Notice that the model does not take into account the leverage between the agents and the default of a peer is not allowed in this model. Even if we assume that the link-structure is probabilistic, the consequence of an exit from the risk-sharing group of a node does not affect directly the income of the other peers. This is possible since we are considering a two-periods game with no “repayment-problem”. For the same reason, we do not study in this model “contagion effects” or related problematics usually underlined in the literature.

[ADD THE CASE FOR THE 3 NODES NET]

3 Conclusions

In this paper we show that, assuming a probabilistic network structure between risk-sharing agents, deciding their optimal capital share to invest on risky assets, we are able to differentiate the agents‘optimal choices, without assuming any difference on agents’degree of risk aversion. In particular, the final optimal choices will be function of their relative location through the network. We analyxe also two measures able to characterize the most “central” agents in our model: One is a modified version of the F measures proposed byEverett and Borgatti (2010), $\bar{F}$, and the other one, the Intercentrality measure discussed in Ballester et al. (2006). In particular, we find that the $\bar{F}$ measure can be useful to pick up the “*key-player*”, or the most influencial node in terms of risk-sharing impact on the expected number of partners to risk-share with. Conversely, the Intercentrality measure can help to forecast the most central node(s) in terms of “risk-influence” on the other peers. The key feature of our model is given by the “double-role” of the links between the agents. If a bilateral link can guarantees liquidity flow between the agents for some exogenous probability, it is also a risk-channel for the peers once we assume positive asset-correlation coefficients. The results obtained underline the importance of a “structural analysis” to understand and forecast potential risk-behaviors of agents belonging to a risk-sharing group. Even if we do not consider “norm-enforcement strategies” in this paper, we are able to explain the optimality of different risk-exposures chosen by identical agents. Assuming a probabilistic link-structure we want to describe a specific scenario where the agents do not consider their present relationships as “certain”. There are many examples catching this feature. One of this could be the interbank market: The present liquidity channels between institutions (in our model represented by links between peers) seem to be dynamic on time and sensible to exogenous changes on the financial market scenarios. Even at “community level”, the link between individuals, representing the existance of a risk-sharing norm between peers, is often explained by previous social-relationships between the partners. However, monitoring constraints and exogenous changes on the social-connections can undermine the existance of the liquidity channels between the parts, i.e. precautionary risk-behaviors could be the optimal startegy in such environment.

The aim of this paper is to improve the risk-sharing literature using a network analysis approach. The previous works studying the structure of the bilateral relationships between agents have focused the attention mainly on the dynamic of the network and in particular on the potential norm-enforcement using links-rewiring as deviation punishment strategy. This paper underlines more the importance of the structure on the optimal risk-choice taken by the connected agents. Further extesions could refine the probabilistic setting, linking the link-probability to the risk-exposures of the agents for example. Doing this way one of the central factors that in this model was assumed exogenous could be endogenized through the players' optimization problem. Moreover, adding the dynamic on the link structure can complete the model opening new questions in terms of expected equilibrium structures and thus explain the dynamic of the network structure observed in the interbank market between many. Given the probabilistic feature of the risk-sharing network and given specific asset-correlation coefficients for example, it could be interesting to study optimal link-rewiring strategies and related equilibrium structures.

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