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Measuring the Natural Output Gap using Actual and Expected Output Data

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Measuring the Natural Output Gap using Actual and Expected Output Data[*]

by

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Abstract

An output gap measure is suggested based on the Beveridge-Nelson decomposition
of output using a vector-autoregressive model that includes data on actual output
and on expected output obtained from surveys. The paper explains the advantages
of using survey data in business cycle analysis and the gap is provided economic
meaning by relating it to the natural level of output defined in Dynamic Stochastic
General Equilibrium models. The measure is applied to quarterly US data over the
period 1970q1-2007q4 and the resultant gap estimates are shown to have sensible
statistical properties and perform well in explaining inflation in estimates of New
Keynesian Phillips curves.

Keywords: Trend Output, Natural Output Level, Output Gap, Beveridge-Nelson
Decomposition, Survey-based Expectations, New Keynesian Phillips Curve.

JEL Classification: C32, D84, E32.

1 Introduction

The measurement of the output gap is an important and recurring topic in economics despite the well-known difficulties involved in measuring, or even defining, the concept in a straightforward way. Basistha and Nelson (2007) provide a useful review of the literature on gap measurement characterising the approaches found in the literature as 'economic', 'statistical' or a blend between the two. The economic approach typically starts with one or more hypothesised economic relationships that involve a gap concept and 'backs out' the measure of the concept from an estimated or calibrated version of the model. The statistical approach concentrates on capturing the time series properties of the data, usually focusing on the characterisation of the underlying trend and defining the gap with reference to this trend. An advantage of the economic approach is that the gap concept is unambiguously defined by the model within which it is embedded. But it is also reliable only to the extent that the model is a good characterisation of the macroeconomy. The statistical approach typically provides a clearly defined representation of the data but produces gap measures that may not be easy to interpret or use in an economic context. The blended approaches aim to exploit the advantages of both: they attempt to apply statistical techniques in a way that captures the time series properties of the data well but which is also informed by an economic framework.

In this paper, we suggest a measure of the output gap that is obtained using standard vector-autoregressive modelling techniques applied to actual output data and to direct measures of output expectations obtained from surveys. The gap is based on the familiar Beveridge-Nelson (1981) [BN] decomposition and benefits from the advantages of the statistical approaches to measuring the gap. However, the proposed gap measure also has the advantage of having a clear economic interpretation. This

[1]

comes from recognising the economic content implicit in the forward-looking BN trend and also by exploiting information contained in surveys which distinguishes between those parts of output fluctuations that agents expect to be permanent and those they expect to be transitory. It turns out that, empirically, the measure of the gap obtained for the US since the early seventies corresponds reasonably closely to those recently obtained based on DSGE and other elaborate macroeconomic models. But the gap measure proposed here is based on a relatively simple statistical model that avoids the criticisms of measures based on potentially contentious structural models and which can be used easily to generate gap measures for use in real-time decision making.

Despite the variety of approaches taken to measure the gap, a common element, at least among those approaches that makes some reference to economics, is the idea that the trend against which the gap is measured is the output level that would prevail under imperfectly competitive markets but with flexible prices and wages. This follows Friedman's (1968) original description of the *natural* output level as the "level that would be ground out by the Walrasian system of general equilibrium equations provided there is embedded in them the actual characteristics of ... market imperfections, ... costs of gathering information, ..., and so on".[1] This concept has recently been articulated and measured with reference to detailed Dynamic Stochastic General Equilibrium [DSGE] models in, for example, Andres *et al.* (2007), Basistha and Nelson (2007), Edge *et al.* (2008) and Justininao and Primiceri (2008). Woodford's (2003) seminal text also defines the natural rate based on a fully-articulated DSGE framework in which the micro-foundations of the model are made explicit

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[1] The related *potential* output level is sometimes defined as the level that would prevail if product and labour markets were perfectly competitive, matching Okun's (1962) early perception of potential output operating within the constraints of "price stability and free markets".

[2]

based on optimising behaviour on the part of households and firms. The framework accommodates imperfectly competitive markets but draws a distinction between the steady-state and natural output levels. The steady-state output concept describes the economy's output if there were fully-flexible prices and if there was no stochastic variability in individuals' preferences (such as temporary variations in households' impatience to consume or shifts in their disutility of labour), no transitory technological disturbances, and no temporarily high or low levels of government purchases relative to their given target levels, for example. The natural output concept retains the flexible price assumption but acknowledges that these 'transitory real disturbances' would cause output to vary even if prices were entirely flexible. Woodford shows that it is the gap based on the natural level of output that is important in price-setting decisions in a number of variants of micro-founded models and that this is also the concept that should enter into monetary policy decisions if they are to have a micro-founded welfare basis.

The steady-state output concept described above is defined as the level at which an economy with fully-flexible prices would locate in the absence of real transitory shocks. But it is also readily conceived as the level to which the economy would converge when the effect of any real transitory shocks have dissipated and when the effects of any frictions or rigidities hindering output adjustment have been worked through. Seen in this light, there is a very clear relationship between this steady-state output level and the BN trend. The latter is defined in the statistical literature with reference to ARIMA processes in which the first difference of a series is stationary and can, according to the Wold decomposition theorem, be characterised as a linear function of serially uncorrelated random disturbances. The BN decomposition of output is based on comparison of today's output with its forecast profile. The trend can be interpreted as the current observed value of output plus all forecastable future

[3]

changes in the output series. It is precisely the infinite-horizon forecast of the output level that will be achieved when all of the adjustments to the current and historical disturbances have been worked through. While the BN trend is a purely statistical concept, then, its forward-looking nature means it matches closely with the steady-state concept at the heart of Woodford and others' behavioural models.

By focusing on the inifinte-horizon, the BN trend abstracts not only from the transitory dynamics arising from the presence of nominal rigidities but also from the transitory real disturbances highlighted by Woodford as distinguishing the steady-state from the natural level that is important in policy prescription. Identification of these disturbances requires more information on output and its dynamic path to the steady-state. In this paper, we suggest using the extra information that is available in the direct measures of expected future output levels provided by surveys. The argument is that agents are aware of the transitory real disturbances impacting on today's output and purge the series of these transitory elements when they respond to a survey asking what output level they believe will be achieved in the future. The inclusion of direct measures of expected output in the statistical model shifts attention to a multivariate BN definition and the extra sophistication of the model is likely to considerably improve the statistical characterisation of the actual output series compared to that provided by a univariate model say. But a simple infinite-horizon output forecast can still be obtained from the VAR so the links with the economically-meaningful steady-state output concept are retained without the intro-duction of potentially contentious elements to the statistical model. Of course, it is nevertheless interesting to see how the measures compare empirically with those based on more structural models (and with straight statistically-motivated trend measures too) and much of the paper is devoted to these comparisons using recent US data.

The layout of the remainder of the paper is as follows. Section 2 describes the

[4]

modelling framework. It defines the BN trend measures in a multivariate framework and considers these in the context of a vector-autoregressive (VAR) model which accommodates the time series properties of actual output and direct measures of output expectations. The relationships between the theoretical output concepts introduced in the DSGE model and the statistical concepts embodied in the VAR (including the presence of cointegrating relations, the BN decomposition and infinite horizon forecasts) are also described in Section 2. Section 3 describes the application of the methods to quarterly US data over the period 1970q1-2007q4. A VAR is estimated based on data on actual and expected output, inflation and interest rates and the corresponding natural output gap measure is calculated. The properties of the gap measure are discussed and compared to those of other popular gap measures. The performance of the gap measure in explaining US inflation is also explored in the context of various estimated versions of the New Keynesian Phillips curve in Section 4. Section 5 concludes.

2   Measuring BN Trends and the Steady-State and Natural Levels of Output

2.1   A VAR Model of Actual and Expected Output

It is straightforward to describe a statistical model of the joint determination of actual output and direct measures of expected future output using a VAR framework if we assume that actual output is first-difference stationary, and that expectational errors are stationary. The first of these assumptions is supported by considerable empirical evidence, and the latter assumption is consistent with a wide variety of hypotheses on the expectations formation process, including the Rational Expectations hypothesis (REH). Under these assumptions, and if direct measures are available

[5]

for upto two periods ahead, for example, then we can write a statistical model for the series in a variety of different ways. For example, given the assumptions, actual and expected output growth are stationary and have the following fundamental Wold representation:[2]

$$
\begin{bmatrix} y_t - y_{t-1} \\ {}_t y_{t+1}^e - y_t \\ {}_t y_{t+2}^e - {}_t y_{t+1}^e \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} + \mathbf{A(L)} \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{2.1}
$$

where (the logarithm of) actual output at time t is denoted by y_t and the direct measure of (the logarithm of) the expectation of output at time $t + h$, formed by agents on the basis of information available to them at time t, is denoted by ${}_t y_{t+h}^e$, for $h = 1, 2$. Here, α_h is mean expected output growth in $t + h$ for $h = 0, 1, 2$, $\mathbf{A(L)} = \sum_{j=0}^{\infty} \mathbf{A}_j(L)$, the $\{\mathbf{A}_j\}$ are 3×3 matrices of parameters and L is the lag-operator. Actual output growth at time t and the growth in output expected to occur in times $t + 1$ and $t + 2$, based on information at time t, are determined and published in surveys at time t and driven by disturbances ε_{0t}, ε_{1t} and ε_{2t} respectively. The ε_{0t} is interpreted statistically as "news on output growth in time t becoming available at time t", while ε_{ht} is "news on output growth expected in time $t + h$ becoming available at time t" for $h = 1, 2$.[3]

As is shown in detail in the Appendix, the model in (2.1) can be written as a VAR in actual and expected output growth assuming that the lag polynomial $\mathbf{A(L)}$

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[2]Expected growth in output at time $t + 1$, $y_{t+1}^e - y_t$, is stationary as it can be decomposed into actual output growth $(y_{t+1} - y_t)$ and expectational error $(y_{t+1}^e - y_{t+1})$, both of which are stationary by assumption.

[3]It is worth emphasising that all the terms on the left-hand-side of (2.1), other than y_{t-1}, are dated at t and that, for example, ${}_t y_{t+1}^e - y_t \neq {}_t y_{t+1}^e - {}_{t-1} y_t^e = \Delta\, {}_t y_{t+1}^e$.

is invertible or as a cointegrating VAR describing $\Delta \mathbf{z}_t$ where $\mathbf{z}_t = (y_t,\ {}_t y^e_{t+1},\ {}_t y^e_{t+2})'$:

$$\Delta \mathbf{z}_t = \mathbf{a} + \Pi \mathbf{z}_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta \mathbf{z}_{t-j} + \mathbf{u}_t \ , \tag{2.2}$$

and the error terms of $\mathbf{u}_t$ are interpreted as "news on the successive output levels" with $\mathbf{u}_t = (\varepsilon_{0t}, \eta_{1t}, \eta_{2t})' = (\varepsilon_{0t},\ (\varepsilon_{0t} + \varepsilon_{1t}),\ (\varepsilon_{0t} + \varepsilon_{1t} + \varepsilon_{2t}))'$. Both the VAR in actual and expected output growth and the cointegrating VAR are straightforward to estimate. The model can also be written, through recursive substitution of (2.2), as the moving average representation

$$\Delta \mathbf{z}_t = \mathbf{g} + \mathbf{C}(L)\mathbf{u}_t. \tag{2.3}$$

The parameters in Π, Γ_j and $\mathbf{C}(L)$ are functions of the parameters of the model in (2.1) and the assumptions underlying (2.1) translate into restrictions on the parameters of the cointegrating VAR and the moving average representation. Specifically, Π and $\mathbf{C}(1) = \sum_{i=0}^{\infty} \mathbf{C}_i$ take the forms

$$\Pi = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \quad \text{and} \quad \mathbf{C}(1) = \begin{bmatrix} k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \end{bmatrix} \tag{2.4}$$

for scalars k_{ij}, $(i = 1, 2, 3, j = 1, 2)$, k_4, k_5 and k_6. All of these forms will provide an equivalent statistical characterisation of the data. They capture the potentially complicated dynamic interactions between the actual and expected output series but are restricted to reflect the underlying stationarity assumptions that ensure the series, while each growing according to a unit root process, are tied together over the long run.

[7]

2.2 Multivariate BN Trends

The BN trend of a variable is defined as the infinite horizon forecast obtained having abstracted from deterministic growth. For a $n \times 1$ vector process $\mathbf{z}_t$, the BN trends $\overline{\mathbf{z}}_t$ are defined by

$$\overline{\mathbf{z}}_t = \lim_{h \longrightarrow \infty} E[\mathbf{z}_{t+h} \mid I_t] - \mathbf{g}h \qquad (2.5)$$

where $E[.|I_t]$ represents the expectation based on information available at time t, I_t, and $\mathbf{g}$, the element of deterministic growth, is a vector of constants. As Garratt *et al.* (2006) point out, any arbitrary partitioning of $\mathbf{z}_t$ into permanent and transitory components, $\mathbf{z}_t = \mathbf{z}_t^P + \mathbf{z}_t^T$ will have the property that the infinite horizon forecast of the transitory component is zero while the infinite horizon forecast of any permanent component converges on the BN trend; i.e.

$$\lim_{h \longrightarrow \infty} E[\mathbf{z}_{t+h}^T \mid I_t] = 0 \quad \text{and} \quad \lim_{h \longrightarrow \infty} E[\mathbf{z}_{t+h}^P \mid I_t] = \overline{\mathbf{z}}_t. \qquad (2.6)$$

The various alternative measures of trends and cycles provided in the literature effectively represent alternative methods of characterising the dynamic path of the permanent component to the BN steady state therefore.[4]

In the multivariate moving average representation of (2.3), the BN trend can be expressed as

$$\Delta\overline{\mathbf{z}}_t = \mathbf{g} + \mathbf{C}(1)\mathbf{u}_t \qquad (2.7)$$

so the trends are correlated random walks with the change in the trends reflecting the accumulated future effects of the system shock $\mathbf{u}_t$. Given the structure of the

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[4]Alternative approaches to characterising trends and cycles based on the BN decomposition include, for example, Blanchard and Quah (1989), King *et al.* (1991), Crowder *et al.* (1999), Gonzalo and Granger (1993) and Garratt *et al.* (2006).

$\mathbf{C}(1)$ in (2.4) imposed by the initial stationarity assumptions on output growth and expectational errors, (2.7) shows the steady-state value of all three series in $\mathbf{z}_t$ is the same, denoted $\overline{y}_t$, and this is driven by the stochastic term $k_4 \varepsilon_t + k_5 \eta_{1t} + k_6 \eta_{2t}$.

It is worth noting that the BN trend is expressed in terms of currently observable data and is readily obtained on the basis of the estimated parameter values and residuals from (2.2). This is an important feature for any trend that is to be used in real-time decision-making. Papers by Orphanides (2001) and Orphanides and van Norden (2002), for example, have shown that the measurement of the output gap, and the use of these measures in explaining policy decisions, can be substantially distorted by the inappropriate use of 'final vintage' data in constructing gap measures at some earlier mid-sample date. Final vintage data incorporates the effects of revisions to the contemporaneous observations along with data on future outcomes which were unknown to the decision-maker at the time. Many statistically-motivated gap measures are based on 'smoothing' algorithms which use the final vintage datasets to define a trend at t with reference to observations before and after the period. Garratt *et al.* (2008) show that this mistreatment of the end-of-sample issues is particularly detrimental in obtaining gap measures for use in understanding real-time decision-making, showing that the application of smoothing algorithms to *forecast-augmented* series can substantially improve the performance of a gap measure.[5] The trend measures suggested in the present paper, based around the BN trend, are derived entirely using observable current-dated magnitudes and, if the VAR model is re-estimated in each period, it can be readily used in real-time decision making.[6]

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[5]Of course, the application of a 'smoothing' algorithm, such as a centred moving average or the Hodrick-Prescott trend formula, to a forecast-augmented series effectively converts this to a 'filter' since it will then make use only of information available at time t.

[6]In the event, we abstract from these real-time issues in the empirical work of this paper so that we can compare our proposed measures with others found in the literature based on final vintage

[9]

2.3 A Measure of the Natural Level of Output

The BN trend is clearly tied to the steady-state output concept elaborated in Woodford's and others' structural models. But we noted earlier that we can also make use of the direct measures of expected future output gained from the survey to identify those elements of output variations which agents believe will be eliminated over various forecast horizons. We argue that this information can be used to isolate the effects of the transitory real disturbances which should be included in the economically-meaningful natural measure of output.

Our suggested approach assumes that the effect of the real disturbances are known to be relatively short-lived compared to those of monetary disturbances. Certainly the literature on estimating interest rate reaction functions suggests that the effects of monetary policy shocks can be very prolonged (captured empirically through the presence of statistically significant and numerically large coefficients on lagged interest rates when entered as explanatory variables).[7] In contrast, variations in individuals' preferences or deviations from government spending plans or other transitory real disturbances seem less likely to persist. In this case, survey data can be used to identify the separate types of shock because the data provides a direct measure of the output levels expected to be achieved in the future once the effect of the short-lived real disturbances have gone away.

In the simple three variable system of (2.1), for example, we can distinguish between three shocks: the productivity shocks, q_t, assumed to have a permanent effect on output; monetary disturbances, m_t, assumed to have long-lived but transitory effect on output; and real disturbances, s_t, assumed to influence output on impact

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data.

[7]See, for example, Clarida et al. (1999) or Orphanides (2001).

[10]

and for one further period only.[8] The permanent productivity shocks are observed directly as the innovations in the BN trend

$$q_t = \Delta \overline{y}_t = k_4 \varepsilon_{0t} + k_5 \eta_{1t} + k_6 \eta_{2t}.$$

The shocks to the long-horizon expectation $_t y^e_{t+2}$, namely η_{2t}, are influenced by permanent shocks and monetary disturbances only so that we can decompose the η_{2t} as

$$\eta_{2t} = \beta_1 q_t + m_t,$$

while the shocks to $_t y^e_{t+1}$ depend on all three structural shocks

$$\eta_{1t} = \beta_2 q_t + \beta_3 m_t + s_t.$$

Assuming that these structural shocks are independent of each other, the coefficients β_1, β_2 and β_3 can be estimated through simple regressions involving the residuals from the estimated cointegrating VAR model explaining $\Delta \mathbf{z}_t$, (2.2), and the m_t and s_t can be obtained as the residuals from these subsidiary regressions.

The relationships between the VECM residuals in $\mathbf{u}_t$ and the structural shocks $\mathbf{w}_t = (q_t, m_t, s_t)'$ are summarised by

$$
\begin{bmatrix} k_4 & k_5 & k_6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} \varepsilon_{0t} \\ \eta_{1t} \\ \eta_{2t} \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 0 \\ \beta_2 & \beta_3 & 1 \\ \beta_1 & 1 & 0 \end{bmatrix}
\begin{bmatrix} q_t \\ m_t \\ s_t \end{bmatrix} ;
$$

that is

$$\mathbf{u}_t = \mathbf{Q}\mathbf{w}_t,$$

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[8]The assumption that survey data is available for expectations just two periods ahead is made here for the purpose of exposition only; longer-lived real disturbances are accommodated in the empirical model below.

[11]

where $\mathbf{Q} = \left[\begin{bmatrix} k_4 & k_5 & k_6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ \beta_2 & \beta_3 & 1 \\ \beta_1 & 1 & 0 \end{bmatrix} \right]$. Hence, we can rewrite (2.3) as

$$\begin{aligned} \Delta\mathbf{z}_t &= \mathbf{g} + \mathbf{C}(L)\mathbf{u}_t \\ &= \mathbf{g} + \mathbf{C}(L)\mathbf{Q}\mathbf{Q}^{-1}\mathbf{u}_t \\ &= \mathbf{g} + \tilde{\mathbf{C}}(L)\mathbf{w}_t \end{aligned} \qquad (2.8)$$

where $\tilde{\mathbf{C}}(L) = \mathbf{C}(L)\mathbf{Q}$. This is an alternative MA representation for $\Delta\mathbf{z}_t$ in which the shocks have a structural interpretation. It is easily shown that $\tilde{\mathbf{C}}(1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ so that the output series are, of course, driven by the same single stochastic shock, q_t in the long run.

The natural level of output deviates from the steady-state because of the influence of the transitory real disturbances only. Using (2.3) and (2.7), the deviation of output from its long-run level can be written as

$$\mathbf{z}_t - \bar{\mathbf{z}}_t = \mathbf{C}^*(L)\mathbf{u}_t = \tilde{\mathbf{C}}^*(L)\mathbf{w}_t$$

where $\mathbf{C}^*(L) = \sum_{j=0}^{\infty} \mathbf{C}_j^* L^j$, $\mathbf{C}_j^* = -\sum_{i=j+1}^{\infty} \mathbf{C}_i$, and $\tilde{\mathbf{C}}^*(L) = \mathbf{C}^*(L)\mathbf{Q}$. The element of this deviation relating to the transitory real disturbances to y_t is given by $\tilde{C}_{13}^*(L)s_t$ and so the natural level of output can be defined by

$$\tilde{y}_t = \bar{y}_t + \tilde{C}_{13}^*(L)s_t. \qquad (2.9)$$

Hence, the natural output level is influenced by real disturbances although the infinite-horizon forecast of the natural level of output coincides with the steady-state output level. The natural output gap, defined by the difference between the actual and natural levels of output, will be unaffected by the real disturbances.

[12]

3 Estimating Steady-State and Natural Output Gap Measures for the US

This section provides estimates of the steady-state and natural output gap measures defined above based on US data over the period 1970q1-2007q4. In order to capture the macroeconomic dynamics as fully as possible, the model on which we base our estimates makes use of inflation and interest rate series as well as data on actual output and on expected future output at the one-, two-, three- and four-period ahead horizons. Hence we have $\mathbf{z}_t = (y_t, \, _ty^e_{t+1}, _t\, y^e_{t+2}, _t\, y^e_{t+3}, _t\, y^e_{t+4}, p_t, r_t)$ where y_t is (the logarithm of) US real GDP and $_ty^e_{t+h}$, $h = 1, .., 4$ are the corresponding direct measures of expected future output obtained form the Survey of Professional Forecasters. Prices p_t and the short term interest rate, r_t are measured by the GDP deflator and the 3-month Treasury Bill rate respectively. A full description of the data, their sources and the transformations used are provided in the Data Appendix.

3.1 Model Specification and Estimation

The empirical counterpart of the VECM model in equation (2.2) was estimated for the seven variables in $\mathbf{z}_t$ with a lag order of two. The underlying assumptions that actual and expected outputs are difference-stationary but (pairwise) cointegrated with vector $(1, -1)'$ were tested and shown to hold. Prices were also found to be difference-stationary. The interest rate was found to be stationary in levels but this feature can be readily accommodated into the cointegrating VAR framework of (2.2), treating the single variable r_{t-1} as a fifth 'artificial' cointegrating combination of variables.[9]

The model underlying our US output gap measures is simple in form but is complex

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[9]Details of the tests on the order of integration for the variables and those for the choice of lag order in the VAR are available from the authors on request.

in the sense that each of the equations of the system explaining the seven terms in $\Delta \mathbf{z}_t$ includes two lags of all seven variables plus feedback from the five cointegrating vectors plus intercepts; a total of 140 parameters are estimated in total. The estimated model is able to capture very sophisticated dynamic interactions, then, and in the event we find large and statistically significant feedbacks captured both among the actual and expected future output measures and between output, prices and interest rates. In order to illustrate the properties of the estimated system, Table 1 reports the estimated (loading) coefficients on the long-run terms along with the diagnostics for each of the seven equations in our VECM system. These estimated coefficients give a sense of the complexity of the underlying dynamics and the statistical significance of the four equilibrating terms in the individual equations. The diagnostic statistics show that the equations fit the data well and that there are no serious problems of serial correlation, non-normality and heteroskedasticity in the residuals.[10]

Figure 1 illustrates the dynamic properties of the system as they relate to actual and forecast values of the output series, plotting the forecast growth rates of actual and four-period ahead forecasts growth, y_{T+h} and $_T y_{T+4+h}$ for $h = 1, ..., 28$, at the end of the sample, $T = 2007q4$. The plot shows the characteristic smoothness of the four-period ahead expectation series relative to the actual series over the seven years prior to the end of the sample and then shows the gradual convergence of the forecasts of the actual and expected series to close to zero by the end of 2010. The fact that the series converge is, of course, a property of the model that assumes stationarity in the expectational errors. But the rate of convergence is a property of the estimated model dynamics and Figure 1 suggests that the "infinite-horizon" steady-state output level is obtained over a three- or four-year time frame.

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[10]More complete details of the model, including the associated impulse responses describing the system dynamics, are available from the authors on request.

[14]

3.2 Co-movements and Business Cycle Properties of Alternative Gap Measures

The measures of the steady-state output gap and natural output gap obtained using the model described above are plotted in Figure 2. The measures are based, respectively, on the trends defined in (2.7) and (2.9) updated to reflect the dimensions of $\mathbf{z}_t$ in the empirical application. Hence, the steady-state output trend reflects the multivariate BN trend obtained as the infinite-horizon forecast of output from the seven equation cointegrating VAR model, while the natural output trend adds in the effect of four different transitory real shocks identified by their influence on expected output for upto four periods ahead and their lack of influence after that time. The plot shows a strong similarity between the two gap measures, with a contemporaneous correlation of 0.86, although there are also periods when the two diverge by some 1-1.5% and there are times when the two gap measures have different signs. The transitory real disturbances are not trivial, therefore, although the BN trend clearly represents the key determinant of the natural output level.[11]

Table 2 and Figures 3 and 4 compare the natural output gap measure with five other regularly-used gap measures: a gap based on marginal costs, $\widetilde{y}_t^{MC}$; the measure produced by the Congressional Budget Office (CBO), $\widetilde{y}_t^{CBO}$; the gap obtained using a simple linear trend, $\widetilde{y}_t^{LT}$; a gap obtained applying the HP smoother to the output series, $\widetilde{y}_t^{HP}$; and a gap obtained using a trend calculated at each time t by applying the HP smoother to a 'forecast-augmented' output series comprising the actual output

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[11]To be clear, the steady-state and natural output measures are calculated on the basis of the parameters obtained from the model estimated using the whole sample of data 1970q1-2007q4. They could have been obtained on the basis of recursively estimated models to better capture the measure of the gap in real-time but this would have made comparison with other standard measures more difficult.

[15]

data upto time t and the forecast values of y_{t+h}, $h = 1, 2, ...$thereafter, $\tilde{y}_t^{HP-F}$. In this final series, the forecasts are obtained using the same estimated cointegrating model that is used to generate the BN trend. The marginal cost measure is advocated by Gali and Gertler (1999) [GG], Gali, Gertler and Lopez-Salido (2001, 2005) [GGL] and others and, as explained in the Data Appendix, is given by the (logarithm of demeaned) average unit labour costs.[12] The CBO series is the Office's 2007q4 estimate of the maximum level of sustainable output achievable in each period based around a neoclassical production function and calculated levels of factor inputs (see CBO, 2001, for detail of the estimation methods employed). The gap based on the linear and HP trends are standard detrended measures found in the literature (the latter calculated using a smoothing parameter of 1600). The gap based on the forecast-augmented HP trend is a little more unusual. This measure is suggested in Garratt *et al.* (2007) as a means of dealing with some of the end-of-sample issues highlighted by Orphanides and van Norden's (2002) paper on the unreliability of output gaps measured in real time. Although we have abstracted from these real time issues in this analysis, focusing entirely on the data available in 2007q4 throughout, the gap based on the forecast-augmented HP trend is interesting here because it provides an obvious point of comparison with the gap based on the standard HP trend and with the gap based on the natural output level which restricts attention to the infinite-horizon forecasts only.

The summary statistics of Table 2 show that, in terms of the standard deviation

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[12]GG note that, under certain conditions on the form of nominal rigidities and the nature of capital accumulation, there is a proportional relationship between the natural output gap measure derived in a micro-founded DSGE model and the deviation of marginal cost from its steady-state. Although this measure is not directly observable either, GG use theory-based restrictions to propose the demeaned unit labour cost series as an alternative means of measuring the gap and show that this performs well in estimates of the New Keynesian Phllips curve.

[16]

and minimum and maximum values of the series, the size of the natural output gap is broadly in line with the alternatives found in the literature, with output lying between 3.6% below and 5.6% above trend and with mean about zero and standard deviation of 1.96%. The plots show relatively persistent gap dynamics in the natural gap, with a first-order autcorrelation coefficient of 0.81, broadly in line with the corresponding correlations for the other gaps in Table 2. This is an interesting finding that contrasts with gap estimates based on BN trends obtained in univariate exercises. These typically find that much of the variation in output is variation in trend and that the gap is small and noisy; see Morley *et al.* (2003), for example. This feature of the gap measures in Figure 1 is retained even in experiments where the inflation and interest rate variables are dropped from the analysis and the model concentrates on the various output measures only. Hence, it is the complexity of the multivariate model that underlies the finding, not the relationship with the other variables.

The table shows there is a reasonably strong consensus in the size and timing of the four cycles based on the statistical 'smoothing' algorithms underlying the linear trend, CBO and HP-based definitions of trend. The correlations between these four are statistically significant and typically in excess of 0.7 and the agreement on the sign of these gap is also always statistically significant and in the region 65%-75%.[13] The correlations and proportions of agreement between these and the natural output gap and the marginal cost -based gap measure are much lower (and statistically insignificant for the latter). These differences are shown in Figure 3a which plots these two series against the linear trend (chosen as a representative of the four smoothed

---

[13]The measure based on the forecast augmented HP filter is the least closely aligned with the other three series according to the statistics in Table 2. But, as demonstrated in Figure 3, this series is more similar to the statistically-based measures than to $\widetilde{y}_t$ despite the forward-looking nature of the trend underlying $\widetilde{y}_t^{HP-F}$.

[17]

cycles). Indeed, the plot shows that, while there are some clear periods during which $\widetilde{y}_t$ and $\widetilde{y}_t^{MC}$ diverge, they agree on 68% of occasions on the sign of the gap and there appears to be more similarity between these two than between either of these and the other smoothed series.

These similarities are perhaps even more striking in the dynamic cross-correlations provided in Figure 4 which show a statistically-significant positive correlation between $\widetilde{y}_t$ and $\widetilde{y}_{t+s}^{MC}$ at all horizons $s = -2, .., 8$ with the peak at $s = 3$. This stands in stark contrast with the cross-correlations between $\widetilde{y}_t$ and $\widetilde{y}_{t+s}^{LT}$ which show significant positive comovement between the natural output gap and the lagged linear trend gap for upto one year earlier and significantly negative correlations for the future linear trend at one year ahead. A similar picture is given for $\widetilde{y}_t^{MC}$ and $\widetilde{y}_{t+s}^{LT}$ showing that the linear trend gap measure leads the marginal cost gap measure by about a year too, although this relationship is weak.

In brief, then, the proposed natural gap measure has reasonable statistical properties comparable to those of many gap measure found in the literature. The natural gap series is based on a straightforward multivariate BN decomposition of output data. For this reason, it is unsurprising to find that, compared to the marginal cost gap measure, it has a higher contemporaneous correlation with other gap series found in the literature based on simple statistical analyses of output data. But the natural output gap's time series properties are quite distinct from those of the other statistically-based series and appear closer to those of the marginal cost gap measure. This provides some support for the view that the natural output gap measure has the structural interpretation we suggested in the previous section as well as having a clear statistical basis.

[18]

4 Using the Natural Output Gap in a New Keynesian Phillips Curve

A further means of judging the properties of the suggested natural output gap measure is to investigate its usefulness in the analysis of inflation, π_t. Figure 4 shows the dynamic cross-correlations between the gap measure and inflation over the sample period, along with corresponding plots for the marginal cost and linear trend gaps. This shows that the natural output gap measure is highly positively correlated with inflation with correlation coefficients in excess of 0.5 found between $\tilde{y}_t$ and π_{t+s}, $s = -1, .., 5$. Similar patterns are found for the marginal cost gap, with correlation coefficients in excess of 0.5 found between $\tilde{y}_t^{MC}$ and π_{t+s}, $s = -4, .., 4$. In contrast, the smoothed linear trend gap $\tilde{y}_t^{LT}$ is positively correlated, with coefficients in excess of 0.3, only with future inflation at $t + s$, $s = 2, 3, 4, ..$, and negatively correlated with lagged inflation. The patterns found for the natural and marginal cost gap measures are consistent with the forward-looking behaviour underlying the New Keynesian Phillips curve relationships in the DSGE literature. These accommodate the idea that nominal rigidities arise because wages and prices are reset only periodically and, recognising this, firms and households make current decisions based on what is likely to happen between now and the next opportunity to change wages and prices. The pattern in the linear trend gap has the gap leading inflation which is inconsistent with this type of forward-looking behaviour.

The point is made more clearly in the results of Table 3 which reports on the estimation of some "hybrid" New Keynesian Phillips Curve of the form considered in GGL:

$$\pi_t = \lambda \tilde{y}_t^k + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1} + \epsilon_t, \tag{4.10}$$

estimated using three alternative gap measures, $\tilde{y}_t^k = \tilde{y}_t^{LT}$, $\tilde{y}_t^{MC}$, or $\tilde{y}_t$, and subject to

[19]

the restrictions

$$\lambda = (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1}, \qquad \gamma_f = \beta\theta\phi^{-1}, \qquad \gamma_b = \omega\theta^{-1}, \qquad \text{and } \phi = \theta+\omega[1-\theta(1-\beta)],$$

where, in the underlying theoretical formulation based on Calvo pricing, θ represents the degree of price stickiness (proportion of firms who do not re-set prices in each period), ω represents a measure of backwardness (the proportion of firms using a backward-looking rule of thumb in price-setting) and β is a discount factor. This hybrid formulation has the advantage of being able to capture both the forward-looking behaviour of the type suggested in the DSGE literature and any inertia-based backward-looking behaviour.

The measure of inflation used in the empirical work is the change in (the logarithm of) the GDP deflator and the period of estimation is 1970q3-2004q4. The table reports the outcome of four different specifications estimated using each of the three alternative gap measures. The first 'baseline' specification estimates (4.10) using a GMM estimator using as instruments four lags of inflation, two lags of detrended output, marginal costs and wage inflation, matching the instrument set used in GGL. The alternative 'closed form' specification follows the suggestion in Rudd and Whelan (2005) and writes inflation in terms of its discounted sum of current and expected future values of the gap, but still takes into account the cross-parameter restrictions implied by (4.10). We solve forward for upto twelve quarters in this case (and it is for this reason that the sample period ends in 2004q4 in this exercise). In both the baseline and closed form versions, we also estimate the relationship with and without imposing the restriction $\gamma_b + \gamma_f = 1$.

The three columns of the table show first how poorly the smoothed linear trend gap performs in the Phillips curve relations. The coefficient on the gap term is not statistically significant in any of the equations presented and is wrongly-signed in

[20]

three out of the four. The marginal cost and natural gap measures are much more successful in explaining inflation having positive coefficients on the gap in all but one case (namely the unrestricted baseline for $\widetilde{y}_t$) and these are statistically significant for both the marginal cost and natural gaps in the restricted baseline models and the unrestricted closed form models. Both gaps provide similar conclusions on the balance between backward- and forward-looking influences on inflation too, being broadly in the ratio 4:6 across the various specifications using either gap measure. In short, then, the natural gap measure has good explanatory power in the hybrid Phillips curve relationships explaining inflation, providing estimates broadly in line with those of GG and GGL and those obtained here using the marginal cost-based measure.

5   Conclusions

The natural output gap measure suggested in this paper is based on the multivariate BN decomposition of actual and expected output obtained through a simple cointegrating VAR model. As such, it is a clearly-defined measure with a very straightforward statistical motivation. The underlying modelling techniques are easily employed and the gap measure can be readily constructed for use in real-time decision-making. We have argued, though, that the measure also has an economically-meaningful interpretation, matching the natural level described in recent DSGE models. The core of this argument is the recognition of the obvious link between the economically-motivated steady-state concept and the statistically-motivated infinite-horizon forecast concept. The former defines the output level that would be achieved if prices were entirely flexible and in the absence of real disturbances, while the latter focuses on the output level that will be achieved when the full effects of past and contemporaneous shocks have worked through and with no further disturbances occurring.

[21]

The analysis of US data showed that it is this steady-state BN concept that primarily drives the proposed natural gap measure empirically. However, the additional effects of real disturbances, identified using agents' stated views on which parts of output fluctuations they expect to be transitory through survey responses, are not trivial and make a substantial contribution to the natural gap measure estimated for the US in some periods. This gap measure is shown to have sensible statistical properties, more closely resembling those of the marginal cost gap than other statistically-based gap measures, and performs well in explaining inflation over the sample.

Appendix: Alternative Statistical Representations for Actual and Expected Output

The general model in (2.1) gives the Wold representation for actual and expected growth driven by $\mathbf{v}_t = (\varepsilon_{0t}, \varepsilon_{1t}, \varepsilon_{2t})'$, a vector of mean zero, stationary innovations, with non-singular covariance matrix $\Psi = (\psi_{jk})$, $j, k = 1, 2, 3$. This model can be expressed in a variety of alternative ways. For example, assume $\mathbf{A}^{-1}(\mathbf{L})$ can be approximated by the lag polynomial $\mathbf{A}^{-1}(\mathbf{L}) = \mathbf{B}_0 + \mathbf{B}_1 \mathbf{L} + .. + \mathbf{B}_{p-1} \mathbf{L}^{p-1}$, where $\mathbf{B}_0 = \mathbf{I}_2$. In this case, (2.1) can be rewritten to obtain the AR representation

$$
\begin{bmatrix} y_t - y_{t-1} \\ {}_t y^e_{t+1} - y_t \\ {}_t y^e_{t+2} - {}_t y^e_{t+1} \end{bmatrix} = \mathbf{B} - \mathbf{B}_1 \begin{bmatrix} y_{t-1} - y_{t-2} \\ {}_{t-1} y^e_t - y_{t-1} \\ {}_{t-1} y^e_{t+1} - {}_{t-1} y^e_t \end{bmatrix} - ..
$$

$$
. - \mathbf{B}_{p-1} \begin{bmatrix} y_{t-p+1} - y_{t-p} \\ {}_{t-p+1} y^e_{t-p+2} - y_{t-p+1} \\ {}_{t-p+1} y^e_{t-p+3} - {}_{t-p+1} y^e_{t-p+2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (5.11)
$$

where $\mathbf{B} = \mathbf{A}^{-1}(\mathbf{1})\boldsymbol{\alpha}$ and hence

$$
\begin{bmatrix} y_t \\ {}_t y^e_{t+1} \\ {}_t y^e_{t+2} \end{bmatrix} = \mathbf{a} + \Phi_1 \begin{bmatrix} y_{t-1} \\ {}_{t-1} y^e_t \\ {}_{t-1} y^e_{t+1} \end{bmatrix} + \Phi_2 \begin{bmatrix} y_{t-2} \\ {}_{t-2} y^e_{t-1} \\ {}_{t-2} y^e_t \end{bmatrix} + ... + \Phi_p \begin{bmatrix} y_{t-p} \\ {}_{t-p} y^e_{t-p+1} \\ {}_{t-p} y^e_{t-p+2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{0t} \\ \eta_{1t} \\ \eta_{2t} \end{bmatrix}, \quad (5.12)
$$

where $\mathbf{a} = \mathbf{M}_0^{-1}\mathbf{B}$, $\Phi_j = \mathbf{M}_0^{-1}\mathbf{M}_j$, $j = 1, ..., p$, and

$$
\mathbf{M}_0 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \quad \mathbf{M}_p = \mathbf{B}_{p-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
$$

$$
\text{and } \mathbf{M}_j = \mathbf{B}_{j-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \mathbf{B}_j \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}
$$

[23]

for $j = 1, ..., p - 1$. The error terms $\mathbf{u}_t = (\varepsilon_{0t}, \eta_{1t}, \eta_{2t})'$ are defined by

$$
\begin{bmatrix} \varepsilon_{0t} \\ \eta_{1t} \\ \eta_{2t} \end{bmatrix} = \mathbf{M}_0^{-1} \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} = \begin{bmatrix} \varepsilon_{0t} \\ \varepsilon_{0t} + \varepsilon_{1t} \\ \varepsilon_{0t} + \varepsilon_{1t} + \varepsilon_{2t} \end{bmatrix},
$$

and the covariance matrix of the $\mathbf{u}_t$ is denoted $\Omega = (\sigma_{jk})$, $j, k = 1, 2, 3$. Note that ε_{0t} has the interpretation of "news on output level in time t becoming available at time t", which is equivalent to news on output growth given that y_{t-1} is known, while η_{ht} is the "news on the level of output expected in time $t + h$ becoming available at time t". The latter incorporates the news on output levels at time t and the news on growth expected to be experienced over the coming period $(\eta_{ht} = \varepsilon_{0t} + \sum_{j=1}^{h} \varepsilon_{jt})$.

Expression (5.12) can be written

$$\mathbf{z}_t = \mathbf{g} + \Phi_1 \mathbf{z}_{t-1} + \Phi_2 \mathbf{z}_{t-2} + ... + \Phi_p \mathbf{z}_{t-p} + \mathbf{u}_t \tag{5.13}$$

where $\mathbf{z}_t = (y_t,\ {}_t y_{t+1}^e,\ {}_t y_{t+2}^e)'$ and this can also provide the VECM representation

$$\Delta \mathbf{z}_t = \mathbf{a} + \Pi \mathbf{z}_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta \mathbf{z}_{t-j} + \mathbf{u}_t, \tag{5.14}$$

where $\Phi_1 = \mathbf{I}_2 + \Pi + \Gamma_1$, $\Phi_i = \Gamma_i - \Gamma_{i-1}$, $i = 2, 3, .., p - 1$, and $\Phi_p = -\Gamma_{p-1}$.[14] Given the form of the Φ_i described in (5.12), it is easily shown that Π takes the form

$$
\Pi = \begin{bmatrix} k_{11} + k_{12} & -k_{11} & -k_{12} \\ k_{21} + k_{22} & -k_{21} & -k_{22} \\ k_{31} + k_{32} & -k_{31} & -k_{32} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix},
$$

[14]The model at (2.1), and the equivalent forms in (5.11)-(5.15), are quite general and have no implications for the expectations formation process. However, the assumption that expectations are formed rationally can be accommodated in the model through the imposition of restrictions that ensure $y_t = {}_{t-1} y_t^* + \varepsilon_t$ and ${}_t y_{t+1} = {}_{t-1} y_{t+1} + \xi_{1t}$. Hence, the deviation of actual output at time t from the level expected in the previous period is equal to the news on the output level becoming available at that time. This news is, by definition, orthogonal to information available at time $t - 1$.

[24]

where k_{ij}, $i = 1, 2, 3$ $j = 1, 2$ are scalars dependent on the elements of the $\mathbf{B}_j$, $j = 0, 1, .., p-1$. The form of the cointegrating vector captures the fact that actual and expected output cannot diverge indefinitely by assumption and is incorporated through the inclusion of the disequilibrium terms $y_{t-1}-{}_{t-1}y_t^e$ and $y_{t-1}-{}_{t-1}y_{t+1}^e$ in each of the system's equations in (5.14).

Alternatively, through recursive substitution of (5.13), we can obtain the moving-average form given by

$$\Delta \mathbf{z}_t = \mathbf{g} + \mathbf{C}(L)\mathbf{u}_t, \tag{5.15}$$

where $\mathbf{C}(L) = \sum_{j=0}^{\infty} \mathbf{C}_j L^j$, $\mathbf{C}_0 = \mathbf{I}$, $\mathbf{C}_1 = \Phi_1 - \mathbf{I}_n$, and $\mathbf{C}_i = \sum_{j=1}^{p} \Phi_j \mathbf{C}_{i-j}$. The presence of the cointegrating relationships between the y_t, ${}_{t-1}y_t^e$ and ${}_{t-1}y_{t+1}^e$ imposes restrictions on the parameters of $\mathbf{C}(L)$; namely, $\boldsymbol{\beta}'\mathbf{C}(1)=0$, as shown in Engle and Granger (1987). Given the form of $\boldsymbol{\beta}'$ in (5.14), $\mathbf{C}(1)$ takes the form

$$\mathbf{C}(1) = \begin{bmatrix} k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \end{bmatrix} \tag{5.16}$$

for scalars k_4, k_5 and k_6. Hence, the BN trend defined by (2.7) shows the steady-state value of all three series in $\mathbf{z}_t$ is the same and driven by the stochastic trend $k_4 \varepsilon_{0t} + k_5 \eta_{1t} + k_6 \eta_{2t}$.

[25]

Data Appendix

The sources and transformations for the data are as follows:

y_t : the natural logarithm of US real GDP. Source: St Louis Federal Reserve Economic Database [FRED].

p_t : the natural logarithm of the US GDP Price Deflator. Source: FRED.

$_ty^e_{t+h}$, $h = 1, 2, 3$ and 4 : the natural logarithm of expected h quarter ahead US real GDP (corresponding direct measures of output). Source: Survey of Professional Forecasters. The series used in the estimation is defined as $_t\tilde{y}^e_{t+h} = g^y_t + y_t$ where $g^y_t = {}_ty^e_{t+h} - {}_ty^e_{t+h-1}$.

r_t : the annualised US three month treasury bill rate, averaged over the three months in each quarter, expressed as a quarterly rate: $r_t = 1/4 \times \ln[1 + (R_t/100)]$, where R_t is the annualised rate. Source: FRED.

π_t : US GDP price deflator inflation, defined as: $400 * (p_t/p_{t-1})$

mc_t : marginal cost or real (demeaned) unit labour cost, defined as $mc_t = ulc_t + 4.596299$, where 4.596299 is the average unit labor cost (ulc_t) for the sample period 1970q3-2004q4 and $ulc_t = \ln(comnfb_t/ophnfb_t) - \ln(pnfb_t)$ where

(i) $comnfb_t$: non-farm business sector compensation per hour. Source: US Department of Labour, Bureau of Labour Statistics

(ii) $ophnfb_t$: non-farm business sector output per hour of all persons. Source: FRED

(iii) $pnfb_t$: implicit prices deflator in the non farm business sector. Source: FRED.

[26]

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Figure 1: Growth Forecasts of y_{T+h} and $_T y_{T+4+h}$ for $h = 1, ..., 28$
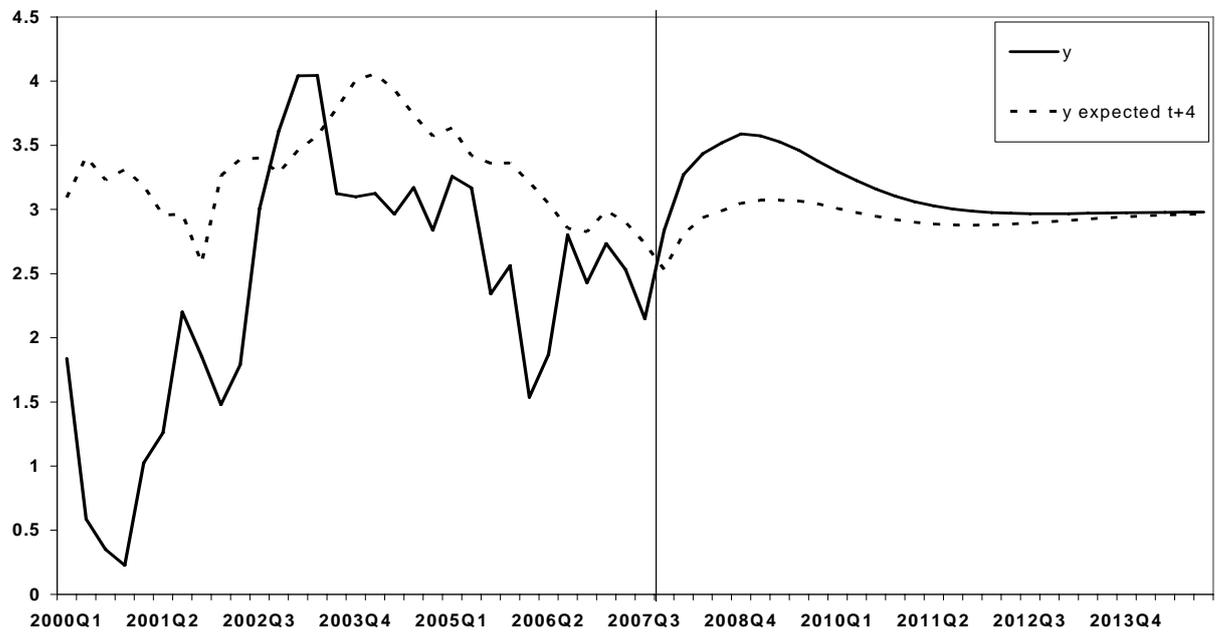

Figure 2: Natural and Steady State Output Gaps

[30]
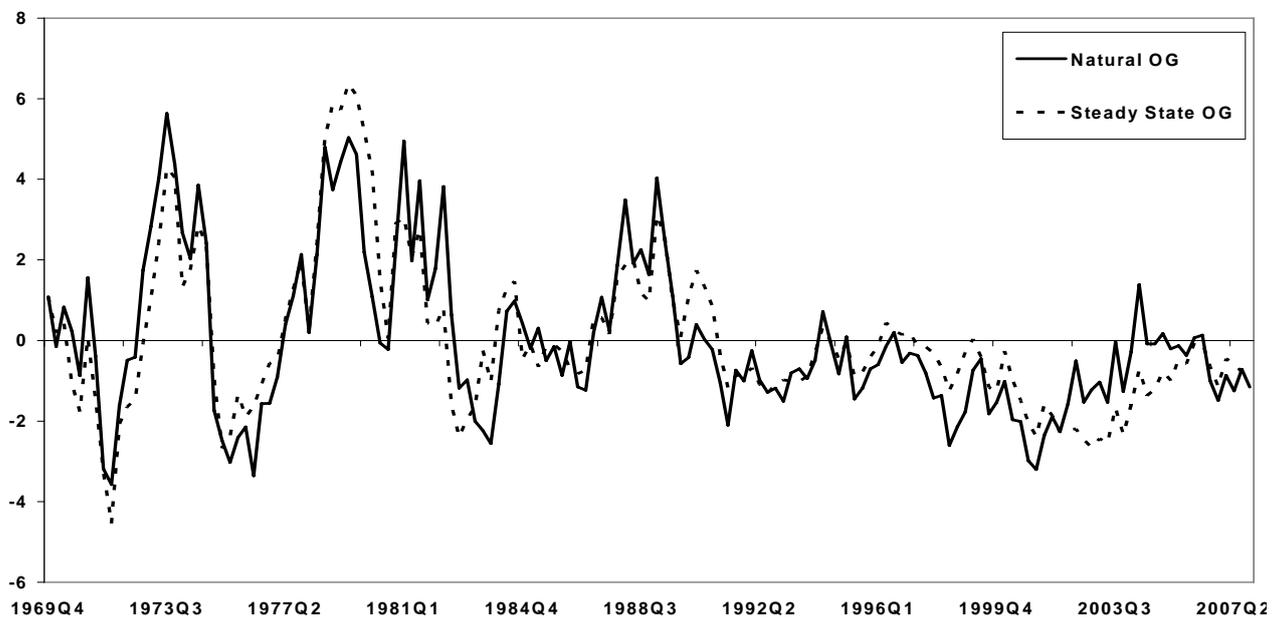
Figure 3a Natural, Marginal Cost and Linear Trend Output Gap Measures
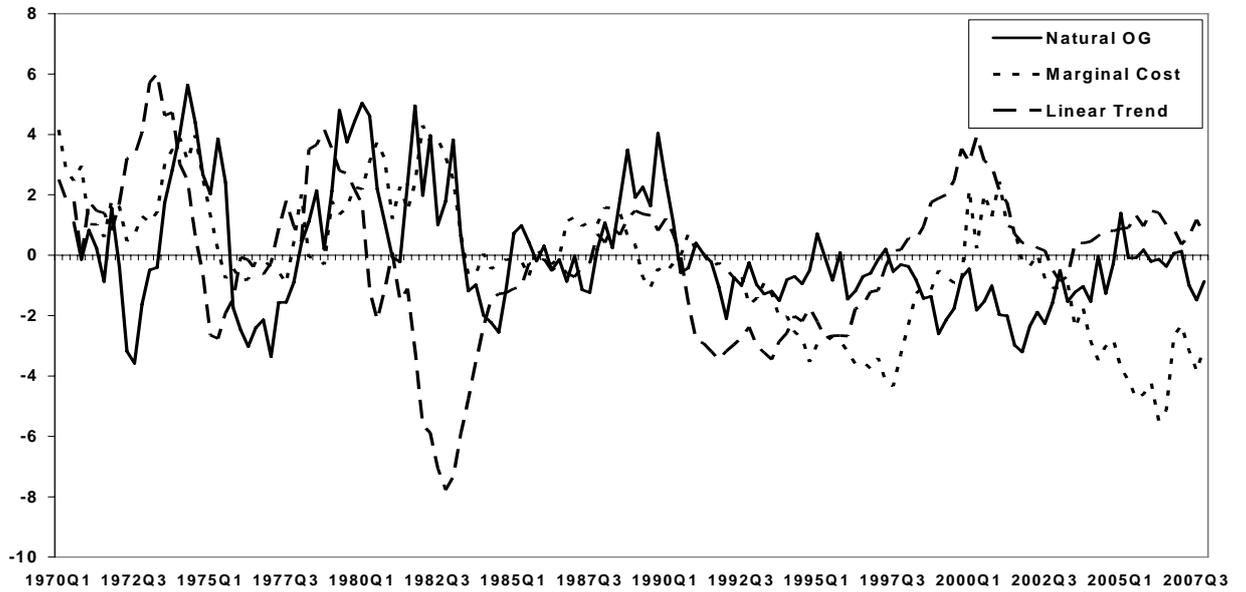
Figure 3b: Linear Trend, CBO and Hodrick-Prescott Output Gap Measures

[31]
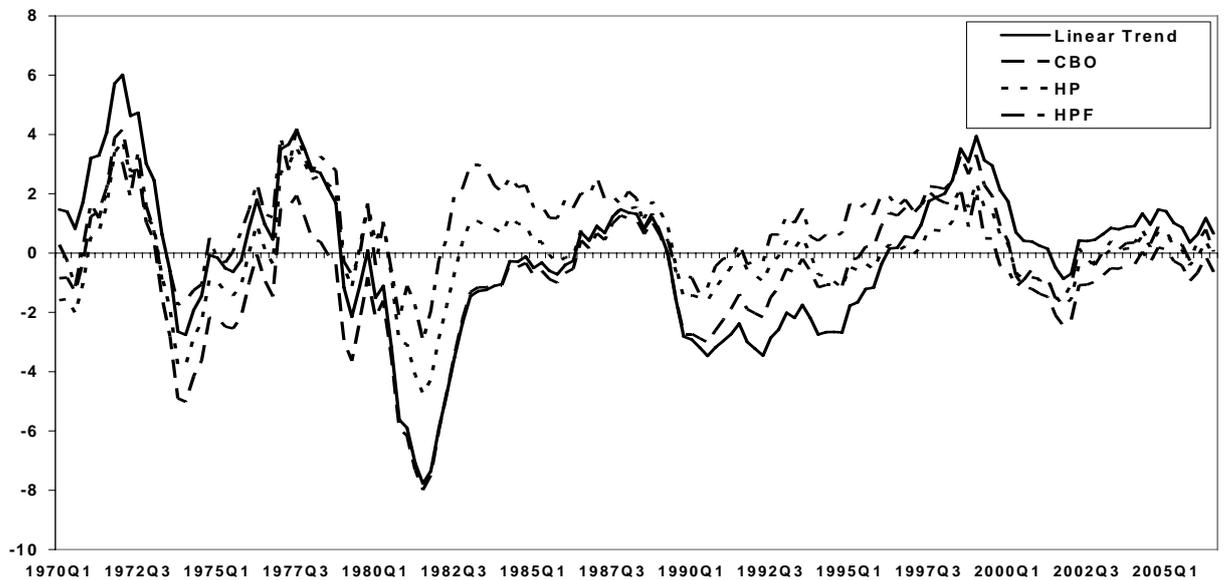
Figure 4: Dynamic Cross-Correlations.

Figure 4a: Natural Output Gap (t), Marginal Cost (t+k)

Figure 4b: Natural Output Gap (t), Linear Trend (t+k)

Figure 4c: Marginal Cost (t), Linear Trend (t+k)

Figure 4d: Natural Output Gap (t), Inflation (t+k)

Figure 4e: Marginal Cost (t), Inflation (t+k)

Figure 4f: Linear Trend Output Gap (t), Inflation (t+k)

[32]
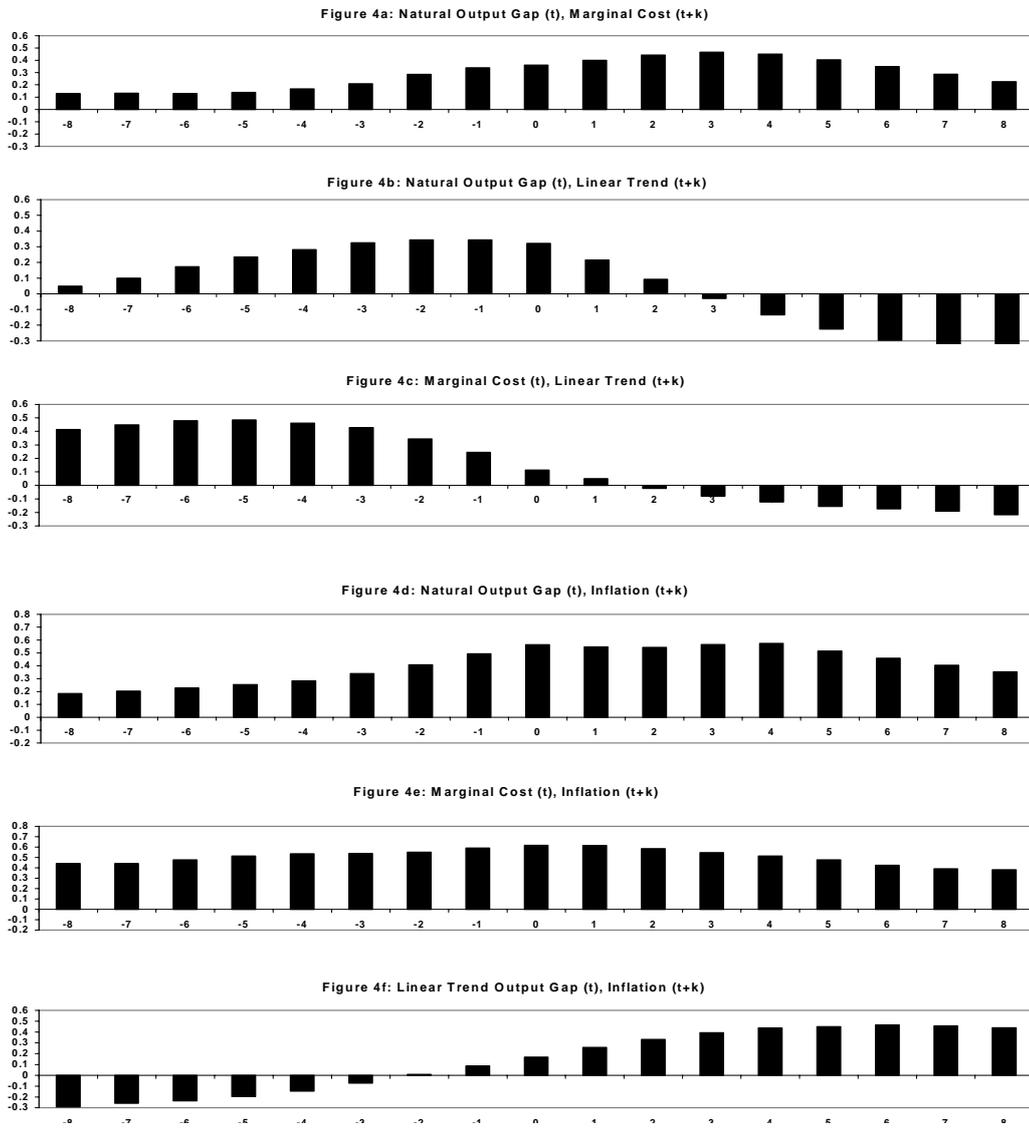
Table 1: VECM Long Run Terms and Diagnostics

Equation	Δy_t	$\Delta_t y^e_{t+1}$	$\Delta_t y^e_{t+2}$	$\Delta_t y^e_{t+3}$	$\Delta_t y^e_{t+4}$	Δp_t	Δr_t
$\widehat{\xi}_{1,t-1}$	−1.38 (1.24)	3.03* (1.35)	2.59 (1.41)	2.80* (1.44)	2.37 (1.45)	−1.02* (0.52)	0.24 (0.28)
$\widehat{\xi}_{2,t-1}$	−6.25* (1.77)	−7.16* (1.94)	−6.28* (2.02)	−7.27* (2.06)	−6.40* (2.08)	0.94 (0.74)	−0.18 (0.40)
$\widehat{\xi}_{3,t-1}$	5.26* (1.86)	5.41* (2.03)	5.39* (2.11)	6.80* (2.15)	5.68* (2.18)	0.40 (0.77)	−0.29 (0.41)
$\widehat{\xi}_{4,t-1}$	−1.22 (0.83)	−1.30 (0.91)	−1.65 (0.94)	−2.25* (0.96)	−1.67* (0.97)	−0.34 (0.35)	0.31* (0.19)
$\overline{R}^2$	.293	.274	.227	.201	.194	.782	.945
$\hat{\sigma}$	.007	.008	.008	.008	.008	.003	.002
$\chi^2_{SC}[4]$	{.749}	{.658}	{.552}	{.413}	{.288}	{.000}	{.001}
$\chi^2_{H}[23]$	{.002}	{.003}	{.007}	{.009}	{.002}	{.033}	{.000}
JB_N	{.179}	{.427}	{.249}	{.254}	{.363}	{.054}	{.000}

Notes: The five long-run terms are given by:

$$\widehat{\xi}_{1,t} = y_t - {}_t y^e_{t+1} + 0.0066,$$

$$\widehat{\xi}_{2,t} = y_t - {}_t y^e_{t+2} + 0.0137,$$

$$\widehat{\xi}_{3,t} = y_t - {}_t y^e_{t+3} + 0.0214,$$

$$\widehat{\xi}_{4,t} = y_t - {}_t y^e_{t+4} + 0.0291.$$

Standard errors are given in parenthesis. "∗" indicates significance at the 5% level and the remaining diagnostics are p-values denoted {.}. $\overline{R}^2$ is the squared multiple correlation coefficient, $\hat{\sigma}$ the standard error of the regression, χ^2_{LM} is a chi-squared test statistic (with 4 d.f.) for serial correlation (SC), χ^2_H the Breusch-Pagan chi-squared test statistic for heteroscedasticity (H) and JB_N Jarque-Bera test for normality (N).

Table 2: Output Gap Measures: 1971q2 − 2007q4

	$\widetilde{y}_t$	$\widetilde{y}_t^{MC}$	$\widetilde{y}_t^{LT}$	$\widetilde{y}_t^{CBO}$	$\widetilde{y}_t^{HP}$	$\widetilde{y}_t^{HP-F}$
Mean	-0.03	-0.34	-0.12	-0.76	0.01	0.77
SD	1.96	2.16	2.48	2.14	1.53	1.39
Min	-3.58	-5.48	-7.78	-7.99	-4.75	-2.75
Max	5.64	4.30	6.01	4.17	3.80	4.19
AR1	0.81	0.93	0.95	0.93	0.88	0.83
$\widetilde{y}_t$	1	0.35*	0.31*	0.24*	0.51*	0.47*
$\widetilde{y}_t^{MC}$	*0.68**	1	0.02	-0.19	-0.03	-0.07
$\widetilde{y}_t^{LT}$	*0.58*	*0.56*	1	0.88*	0.79*	0.58*
$\widetilde{y}_t^{CBO}$	*0.64**	*0.59*	*0.76**	1	0.86*	0.71*
$\widetilde{y}_t^{HP}$	*0.66**	*0.52*	*0.74**	*0.76**	1	0.84*
$\widetilde{y}_t^{HP-F}$	*0.56*	*0.39*	*0.65**	*0.61**	*0.78**	1

Notes: The output gaps measures are: the natural output gap $\left(\widetilde{y}_t\right)$, marginal cost $\left(\widetilde{y}_t^{MC}\right)$, linear trend $\left(\widetilde{y}_t^{LT}\right)$, Congressional Budget Office $\left(\widetilde{y}_t^{CBO}\right)$, Hoderick-Prescott $\left(\widetilde{y}_t^{HP}\right)$ and Hoderick-Prescott forecast augmented $\left(\widetilde{y}_t^{HP-F}\right)$. Summary statistics in the upper panel refer to the mean, standard deviation, minimum and maximum values, and first-order serial correlation coefficient respectively. Figures in the lower panel refer to correlation coefficients between gap measures and, in italics, the proportion of the sample for which there is agreement that the output gap is positive or negative. A '*' indicates significance at the 5% level.

Table 3: Linear trend, Marginal Cost and Natural Gap NKPC Estimates

	Linear Trend Gap $\widetilde{y}_t^{LT}$			Marginal Cost Gap $\widetilde{y}_t^{MC}$			Natural Output Gap $\widetilde{y}_t$		
	γ_b	γ_f	λ	γ_b	γ_f	λ	γ_b	γ_f	λ
Baseline GMM	0.216 (.091)	0.467 (.081)	−0.022 (0.029)	0.330 (.132)	0.591	0.075 (.096)	0.258 (.081)	0.466 (.109)	−0.001 (0.054)
$\gamma_b + \gamma_f = 1$	0.221 (.120)	0.779	0.039 (.039)	0.351 (.102)	0.649	0.094 (.044)	0.313 (.094)	0.687	0.077 (.039)
Closed form GMM	0.267 (.088)	0.742 (.092)	−0.019 (.013)	0.418 (.028)	0.563 (.029)	0.029 (.011)	0.443 (.029)	0.535 (.031)	0.065 (.039)
$\gamma_b + \gamma_f = 1$	0.345 (.056)	0.655	−0.010 (.009)	0.412 (.028)	0.588	0.009 (.005)	0.425 (.028)	0.575	0.004 (.010)

Notes: The table report GMM estimates of the structuiral parameters of equation (4.10) using three alternative measures of the output gap. Estimates are based on quarterly data over the period 1970q1-2004q4. The instruments match those used in GGL as listed in the text.Standard errors are in parentheses..

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