Liquidity, Term Spreads and Monetary Policy *

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Abstract

We propose a model with segmented markets that delivers endogenous variations in term spreads driven by banks’ portfolio decisions while facing maturity risk. Future profitability influences the term premium banks require to carry this risk. When expected profitability is relatively high (low) spreads are low (high). Spread fluctuations feed back into the macroeconomy through investment decisions. Econometric evidence corroborates this link between expected financial profitability and yield spreads. Finally, we analyse unconventional monetary policy by allowing banks to sell assets to the central bank. These interventions exploit a new channel of policy transmission through banks’ portfolio choice affecting the yield curve.

The presence of term spreads, or alternatively term premia, has implications on financial transactions, macroeconomic outcomes and policy design. The existing literature finds that the slope of the yield curve has significant predictive power in explaining US business cycle fluctuations (see, for instance, Estrella and Hardouvelis (1991) or more recently Rudebusch and Williams (2009)). This predictive power stems from the relation between the slope of the yield curve and the future path of short-term interest rates, and also, from variations in term spreads. While there is

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a growing literature on term spreads and macroeconomic outcomes, there is little consensus on the determinants of time varying term premia. Moreover, recent large scale purchase programmes adopted by the Federal Reserve Bank, the Bank of England and the European Central Bank had a significant impact on the shape of the yield curve, increasing the importance of understanding and modelling those variations. In this paper, we propose a theoretical model with segmented markets that provides an explanation for endogenous variations in term spreads driven primarily by changes in banks’ balance sheets, their expected profitability and their appetite to bear the risk of maturity transformation.

Early structural models that look at the term structure of interest rates rely on the expectations hypothesis; thereby limit the analysis to cases of constant risk premium. There is evidence, however, that the term premium is time varying (see for instance Piazzesi and Swanson (2008) and references therein). Consequently, macro-finance models focus on the variability of the stochastic discount factor and its links to macroeconomic variables. The literature is extensive. For instance, recent work by Piazzesi and Schneider (2007) and by Rudebusch and Swanson (2012) model risk premium as an outcome of the negative covariance between inflation and consumption growth. In this framework financial investors demand a higher risk premium as a hedge against (long-term) inflation risk.¹

Although supporting the view that long run inflation risk is an important determinant of term spread fluctuations, three main empirical findings motivate the search for additional factors external to monetary policy. First, as stressed by Gürkaynak and Wright (2012), the US treasury inflation protected securities’ (TIPS) forward rate dynamics have not been that different from their nominal counterparts, indicating the term premia are also influenced by real factors. Second, Benati and Goodhart (2008) observe that during the 2000’s the marginal predictive content of term spreads to future output increased, although monetary policy uncertainty remained low. Finally, dynamics of short-run rates and inflation expectations do not explain all the variability of long-term rates, particularly in the last decade (De Graeve et al. (2009)). In view of that, next to nominal

¹Other relevant contributions to the literature focus on the effects of learning about long-term inflation targets of the central bank (Kozicki and Tinsley (2005)) or on the possible segmentation of short and long-term bond markets (Vayanos and Vila (2009)), identifying other drivers of time varying risk premia.

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factors, also real factors must be playing a role in explaining variations in term spreads.

In order to provide an alternative real frictions view, we focus on the role of financial inter-
mediation in the determination of term spreads. We develop a DSGE model with endogenous
term spreads derived from banks’ portfolio choice and their risk assessment of potential liquidity
shortages. The main feature of the banking structure in our model is the introduction of poten-
tial liquidity risks and relates to the contributions of Holmstrom and Tirole (1998) and Diamond
and Rajan (2001; 2005). In our framework, banks hold a portfolio of equities, short and long-
term lending funded by short-term borrowing, thus they bear the risk of maturity transformation.
The presence of short and long-term lending opportunities is a result of the assumption of market
segmentation in capital investments. We assume banks’ long-term investments may suffer from
potential liquidity shortages such that liquidity injections are required to maintain those assets in
the banks’ books. This formalises the maturity mismatch risk. Term spreads, which ultimately
denote the cost of hedging this potential risk, are determined by the volatility of future short-term
rates, as usual in the macro finance literature, and the additional element introduced here, the pre-
mium for bearing the maturity risk. We show that cash-flow patterns impact banks’ profitability
and hence their balance sheets, altering the risk premia derived from their portfolio decisions. We
label this additional mechanism that produces endogenous term spread movements due to maturity
mismatch risk, the bank’s portfolio channel.\textsuperscript{2}

A key feature of our framework, that is absent in canonical macro-finance models that look
at the yield curve (e.g. Rudebusch and Swanson (2008), see also Rudebusch \textit{et al.} (2007) for
a general discussion), is that endogenous movements in term spreads feed back to the macroe-
conomy. This interaction occurs since long-term rate’s influence investment in capital such that
entrepreneurs demand for loans is downward sloping in long-term interest rates.

In line with the existing empirical evidence, our theoretical model generates term premia mo-
vements that are good predictors of future real output. As the economy approaches the peak of the
business cycle, spreads tend to be at their lowest and will tend to increase thereafter. Similarly,

\textsuperscript{2}The portfolio choice introduces a real friction that affects the yield curve. However, any nominal factor that alters
the risk of maturity transformation would also impact the bank decision and lead to term spread variations. Hence, in a
nominal model the proposed mechanism reflects the interaction between nominal and real factors.

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as the economy approaches the bottom of the business cycle, spreads tend to be at their highest. We show that the endogenous movements in long-term rates and term spreads are linked to the fluctuations in the expected profitability of banks’ portfolios. Banks rely on the overall profitability of their portfolio to cover any liquidity needs. Hence, future profitability relates directly to their appetite to bear maturity risk and thus to the risk premia they require to commit to provide long-term funding to firms. As output growth falls during a recession, profitability is expected to remain low and increase in the future and hence spreads are high; it is costly to pay variable funding costs in the short-term to increase long-term earnings. As the economy approaches the peak of the business cycle, profitability is expected to remain high and decrease in the future and thus spreads tend to be low; risks of funding costs can be undertaken while profits remain high to improve long-term earnings.

Our model, by creating a new policy channel, allows us to analyse the impact of unconventional monetary policies similar to the recent quantitative easing (QE) adopted in the US and the UK. In our setting, the monetary authority, by altering bank’s balance sheet conditions, can influence long-term rates and investment without relying on base-rate changes. The channel through which QE affects the economy in our framework is distinct to the one stressed in some of the recent theoretical papers. For instance, in Gertler and Kiyotaki (2010) QE works through a direct replenishing of banking capital, covering for current shortages. In contrast, unconventional monetary policy in our framework aims to protect banks from potential liquidity shortages in the future, offering a mechanism for the relaxation of future balance sheet constraints. This in turn leads to an increase in banks’ willingness to carry maturity transformation risk and to a reduction in term spreads. We find that allowing banks to sell long-term assets to the central bank after a liquidity shock leads to a sharp decrease in term spreads matching the results presented by several empirical studies on the recent QE policies in the US, the UK and the Eurozone. Furthermore, such interventions have significant impact on long-term investments, decreasing the amplitude of output responses after a liquidity shock. The base rate does not need to decrease as much as when only conventional policies are implemented and the resulting inflation turns out to be higher.

Empirically, the importance of bank balance sheets and bank risk taking has recently been
stressed by Adrian and Shin (2010a; 2010b), and Adrian et al. (2010b). These authors show that financial intermediary balance sheets contain strong predictive power for future excess returns on a broad set of equity, corporate, and Treasury bond portfolios; higher banking asset growth is related to decreasing risk premia. This link between banks balance sheets and risk premia is indeed present in our model. As a bank expects higher profits, it increases asset holdings and consequently spreads decrease. While their work focuses more on leveraging and asset growth rates, the mechanism behind the link between balance sheets and risk premia in our model is distinct; we look particularly at the role of the variability of expected bank profits and the maturity transformation risk.

Focusing on bank profitability and the yield curve, we present three distinct sets of empirical evidence. First, we document bank level microeconometric evidence on the link between expected profitability (obtained from survey data) and yield spreads giving support to our main theoretical channel. We find that an increase in US bank level expected financial business profitability (as measured by the expected mean forecast in earnings per share for major US financial institutions) leads to a significant decline in yield spreads next to variations in real output and inflation. Second, using a vector autoregression (VAR) and employing the identification of Barsky and Sims (2011) to separate contemporaneous and expected future innovations (news) in bank profitability, we report a negative correlation between spreads today and expected changes in bank profitability. Third, we document the impact of shocks to monetary policy, output and consumer expectations (used as a proxy to an anticipated productivity shock) on the main macroeconomic variables. The theoretical model’s dynamic properties are largely in line with the VAR impulse responses in all three cases.

The paper is organised as follows. Section 1 presents a simple partial equilibrium model to highlight the relationship between banks’ profits, balance sheets and term spreads. Section 2 describes the general equilibrium model of endogenous term spreads. We start presenting our results in Section 3, focusing on the main drivers of the endogenous movements in term spreads, their potential to predict future output growth and the main dynamic properties of the model after monetary and technology shocks. Section 4 discusses unconventional monetary policies. Eco-
nometric evidence on the link between expected financial business profitability and spreads, in support of our channel, and a VAR estimation to study term spreads’ dynamic responses to the key macroeconomic shocks are presented in Section 5. Finally, Section 6 concludes.

1. A Simple Model of Bank’s Portfolio Choice

We start by presenting a simple partial equilibrium model of the banks’ portfolio decision aiming at explaining the basic link between banks’ balance sheet (portfolio), their profits and the inclination of the yield curve. In the next section, we analyse a similar decision problem in a general equilibrium model to explore the effects of endogenous fluctuations of spreads on economic activity and monetary policy.

The simple model has three periods. At period zero, the bank selects a portfolio of assets and holds them until maturity. While assets are in the balance sheet, the bank must fund them with short-term deposits. We assume bank’s portfolio may contain three assets: a long-term asset \( X_L \), a short-term asset \( X_S \) and equity (or a portfolio of the rest of risky short-term assets available in the economy), denoted by \( Z \). The short-term asset pays out a certain return of \( R_S \) one period after the portfolio has been set. Equities also pay out in period 1, however, their return \( R_Z \) is uncertain. Long-term assets mature and pay out a certain return of \( R_L \) two periods after the portfolio decision, however, the bank might be forced to make an injection of liquidity (\( \varrho \)) in its own balance sheet to keep the long-term assets that were funded with short-term borrowing in the portfolio during period 1.

This liquidity injection, similar to the one used in Holmstrom and Tirole (1998),\(^3\) effectively implies that the bank may be exposed to cash flow shocks at period 1, leading to an ex-post revaluation of the overall return on long-term asset holdings, replicating problems of balance sheet funding.\(^4\) Thus, \( \varrho \) can be understood as a reduced form shock to the maturity mismatch exposures.

\(^3\)We simplify their framework, excluding moral hazard problems and the potential for bank failures, to focus particularly on term spreads.

\(^4\)Note that ex-post revaluations might occur if a portion of long-term assets must be sold due to lack of funding. Hence, although the liquidity injection in the model occurs on the asset side of the bank’s balance sheet, it can also be understood as a reduced form liability shortage shock.

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in the bank’s balance sheet. The bank fully funds its portfolio with deposits ($D$) that provide the holder with a gross return of $R_D$.

In order to depict the basic channel as clearly as possible, we simplify a number of features that are part of the bank’s portfolio choice in our main model (Section 2). Firstly, we focus on the portfolio decision at time zero only, with short-term assets and equity remaining in the balance sheet for one period and long-term assets for two periods. Secondly, we assume deposits are in infinite supply at the equilibrium short-term rate and $R_D$ is exogenously set and constant for the two periods the long-term assets are held.\(^5\) These simplifications allow us to concentrate on the bank’s decision of how much long-term asset to hold at time zero, the point when long-term rates, and thus term spreads, are set. The main factors affecting this decision will then be the bank’s expected profitability and the liquidity risk the long-term asset holder bears.

In order to study term spread fluctuations banks must care about risk. For simplicity we assume banks are risk averse.\(^6\) The bank problem, formally, is

$$
\max_{(X_S, X_L, Z, D_0, D_1)} \quad E \left[ \Pi^B \right]
$$

s.t. \( \Pi^B = \frac{(\Pi^B_1)^{1-\sigma_B}}{1-\sigma_B} + \beta \frac{(\Pi^B_2)^{1-\sigma_B}}{1-\sigma_B} \)

\( \Pi^B_1 = (R_Z - 1)Z + (R_S - 1)X_S - \rho X_L - (R_D - 1)D_0 \)

\( \Pi^B_2 = (R_L - 1)X_L - (R_D - 1)D_1 \)

\( D_0 = Z + X_L + X_S \)

\( D_1 = X_L \) \hspace{1cm} (1)

where $\Pi^B_i$ is the bank profits at time $i = 1, 2$.

It is straightforward to see that short-term rates will be equal to the return on deposits. The key equation to determine the long-term exposure in the bank’s portfolio and therefore the long-term

\(^5\)Note that volatility of short-term funding costs could also generate increased maturity transformation risk. In the general equilibrium model in Section 2 short-term rates will be endogenous and thus we incorporate this risk there. Moreover, changes in short-term rates also result in valuation gains or losses on long-term assets in the portfolio. These will also be included in the general equilibrium model.

\(^6\)In Aksoy and Basso (2012) we show that assuming banks are subject to a (constant) Value-at-Risk constraint yields similar results.

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If banks do not care about risk ($\sigma_B = 0$) then term spreads will be constant (the slope of the yield curve will depend only on $E[\Pi]$). If banks are risk averse then term spreads will depend on the expected path of profits. When expected profits in period 1 increase relative to the profits in period 2, due to higher expected return on equity, then the first term of equation (2) decreases. Consequently, in equilibrium, long-term rates also decrease while the bank’s demand for long-term assets increases. Bank’s portfolio selection implicitly determines a structure of cash flows in periods 1 and 2. Banks are more willing to increase long-term exposures that will provide cash-flows in the future if profits are relatively high during the interim period when these exposures are funded and liquidity shortages may occur. In short, higher profits on investments allow banks to cover for potential liquidity shortages and cash-flow mismatches.

The portfolio choice that implicitly determines the term premia is made at time zero, but depends on profits in periods 1 and 2 while the long-term asset is in the balance sheet. Hence, the portfolio channel highlighted here is forward looking in nature; what matters are expected profits and not how they respond in the current period. Note that the covariance between profits in the interim period ($\Pi_1^B$) and funding costs ($R_D - 1 + \varrho$) and the covariance between final profits ($\Pi_2^B$) and gains from long-term asset holding ($R_L - R_D$) will also be important to determine the extent to which an increase in relative profitability affects the first term of equation (2) relative to its second term. We will discuss the role of these covariance terms in more detail in the general equilibrium model.

2. General Equilibrium Model

In the previous section we presented a partial equilibrium model of bank’s portfolio choice, establishing a link between bank’s appetite to bear maturity transformation risk, its balance sheet

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Equity holdings will be determined by $E \left[ (\Pi_1^B)^{-\sigma_B}(R_Z - R_D) \right] = 0$. 

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holdings and its expected profits from portfolio investments. During periods in which the assets held in their balance sheets are expected to pay higher returns, banks are willing to increase exposure to maturity transformation risk, increasing long-term asset holdings and charging lower risk premia. In this section, we generalise and extend our simple portfolio choice model. Specifically, we endogenise term spreads embedding a banking sector’s portfolio choice into a DSGE framework. We then study their effects on economic activity and monetary policy.

The model economy is populated by a continuum \( i \in [0,1] \) of intermediate good producers, a final good producer, a continuum of households, banks, entrepreneurs and the central bank. Entrepreneurs borrow funds from a bank and transform consumption goods into capital. There are two types of entrepreneurs, one with access to a short-run investment project and one with a long-run investment opportunity available. This introduces a segmentation of short and long-term funding requirements, similar to the one stressed by Vayanos and Vila (2009). As a result, banks can invest in short and long-term assets in distinct markets. Intermediate good producer \( i \) hires labour from households, produces a differentiated input using labour and the current capital stock. At the end of the period it sells the inputs to the final good firm and buys new capital from entrepreneurs. The final good firm combines all inputs to produce consumption goods that are then sold to households and entrepreneurs. We assume households (workers) receive the profits from banks and entrepreneurs, which are all of unit mass. Thus, only households consume.\(^8\)

The bank receives deposits from households, provides loans to both entrepreneur types and buys equity from the intermediate firms. Note that long-term loans are issued at every period but do last for two periods, thus banks’ balance sheets will contain three loan agreements. These are: a short-term loan, a long-term loan and another long-term loan issued in the previous period. Finally, we assume that during the current period the bank might need to make a liquidity injection into its balance sheet to keep long-term assets that are funded with short-term borrowing in its portfolio. Figure 1 shows the production and financial flows of the model.

\(^8\)An equivalent alternative would be to follow a model structure similar to Gertler and Kiyotaki (2010), where a family is split into banks and consumers but consumption is done at the family level.
Figure 1: Model Structure

2.1. Households

The household maximises its expected discounted lifetime utility given by

\[
\max_{C_t,H_t,D_t} E_0 \sum_{t=0}^{\infty} \beta^t \left( C_t^{1-\sigma} - \frac{H_t^{1+\eta}}{1+\eta} \right), \quad \beta \in (0, 1), \quad \sigma, \eta, \chi > 0
\]  

(3)

where \(C_t\) denotes the household’s total consumption and \(H_t\) denotes the composite labour index. The curvature parameters \(\sigma, \eta\) are strictly positive. \(\beta\) is the discount factor. The household faces the following budget constraint

\[
C_t + D_t = W_t H_t - \frac{R_{t-1,CB} D_{t-1}}{P_t} + \frac{\bar{\Pi}_t}{P_t}
\]  

(4)

where \(W_t\) is the wage index and \(R_{t,CB}\) is the rate of return on deposits \(D_t\). The central bank sets \(R_{t,CB}\) according to a monetary policy rule to be specified. We assume the only asset available to the “worker” is a deposit made directly to the financial intermediary, thus only banks invest in equities issued by the intermediate good firms and lend to entrepreneurs. Although not modelled here, one reason for that would be the existence of higher household-firm agency costs relative to bank-firm agency costs.

Finally, nominal profits realised at period \(t\) for entrepreneurs with short-term projects, with

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long-term projects and the bank, respectively, are passed on to the household ($\Pi_t = \Pi_t^{ES} + \Pi_t^{EL} + \Pi_t^B P_t$).

### 2.1.1. Optimal Wage Setting

Households supply a continuum of labour types $j \in [0, 1]$. The composite labour index $H_t$ is then given by

$$H_t = \left( \int_0^1 H_{jt}^{\varepsilon_W} \frac{\varepsilon_W}{\varepsilon_W - 1} \right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}. \quad (5)$$

From the subsequent intermediate firms minimisation problem we have that the demand for each labour type and the wage index are given by

$$H_{jt} = \left( \frac{W_{jt}}{W_t} \right)^{\varepsilon_W} H_t, \quad (6)$$

$$W_t = \left( \int_0^1 W_{jt}^{1-\varepsilon_W} \right)^{\frac{1}{1-\varepsilon_W}}. \quad (7)$$

Households, when allowed (Calvo scheme with parameter $\omega_w$), set wages $W_{jt}$ to maximise expected utility subject to the budget constraint and the labour demand equation. The main reason to include both price and wage rigidity is to ensure firms’ dividends are pro-cyclical after a productivity shock (see Carlstrom and Fuerst (2007)).

### 2.2. Entrepreneurs

Entrepreneurs are responsible for capital formation. There are two types of entrepreneurs, one who invests in short-term projects and one who invests in long-term projects. This ensures a separation between the markets for short and long-term funding. As such, we assume a set of mass unit of entrepreneurs has a short-term investment opportunity available, at each period. Another set of mass unit of entrepreneurs has a long-term (two periods) investment opportunity available, at each period. Thus, there are always three mass units of active entrepreneurs in the economy.

Short-term entrepreneurs borrow funds from the bank ($X_{S,t}$), buy consumption goods and

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transform these into capital next period using the following production function

\[ y_{k_{t+1}}^S = \gamma_S \ln(1 + X_{S,t}). \]  

(8)

The capital produced is then sold to the intermediate good firms. The profits of these entrepreneurs are given by

\[ \Pi_{t+1}^{ES} = P_{t+1} q_{t+1} \gamma_S \ln(1 + X_{S,t}) - R_{t,S} P_t X_{S,t}. \]

(9)

where \( R_{t,S} \) is the gross interest rate on short-term borrowing and \( q_{t}^S \) is the price of short-term capital denominated in consumption goods. Short-term entrepreneurs select \( X_{S,t} \) to maximise expected profits.

Long-term entrepreneurs also borrow from the bank \((X_{L,t})\), buy consumption goods and transform these into capital after two periods with the following production function

\[ y_{k_{t+2}}^L = \gamma_L \ln(1 + X_{L,t}). \]

(10)

where \( \gamma_L > \gamma_S \). The capital produced is then sold to the intermediate good firms. Long-term entrepreneurs profits are given by

\[ \Pi_{t+2}^{EL} = P_{t+2} q_{t+2} \gamma_L \ln(1 + X_{L,t}) - R_{t,L} P_t X_{L,t}, \]

(11)

where \( R_{t,L} \) is the gross interest rate on long-term borrowing and \( q_{t}^L \) is the price of long-term capital denominated in consumption goods. Long-term entrepreneurs select \( X_{L,t} \) to maximise expected profits.

The production function for short and long-term capital output \((y_{k+m})\) is assumed to take the form \( \gamma_m \ln(1 + X_{m,t}) \) for \( m = \{S, L\} \), for two key reasons. Firstly, we need capital production to have decreasing returns (concave function) such that movements in borrowing rates influence the marginal propensity to invest. That ensures the demand of long-term and short-term borrowing is downward sloping on their respective interest rates (see equations (B.12) and (B.13) in Appendix.

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B). Secondly, that functional form ensures each unit of consumption good invested \((X_{mt})\) is turned into one unit of capital plus an increment, which decreases as the amount invested increases and whose overall size depends on the parameter \(\gamma_m\) (this interpretation holds as long as \(X_{mt2}\) is small and \(\gamma_m\) close to one, which will be the case in our calibration). We also assume that \(\gamma_L > \gamma_S\) or that, for the same level of investment, long-term capital is more productive than short-term capital due to the potential liquidity problem banks may face on long-term asset holdings.\(^9\)

2.3. Banks

At every period \(t\), a bank, representing all financial business in the economy, acquires three types of nominal assets: a short-term debt \((P_X S_t)\), a long-term debt \((P_X L_t)\) and equity \((Z_t)\).\(^10\) Furthermore, the bank has a long-term asset it carries over from last period \((P_{t-1}X_{L,t-1})\). The bank funds these investments with short-term deposits \((D_t)\) from households. Diamond (1989; 1991; 2004), amongst others, provide agency-based explanations for the usage of short-term debt as the optimal funding instrument for banks, theoretically supporting the balance sheet structure employed here.\(^11\) Equities are acquired from the intermediate good producers. The investment in equity made at time \(t\), \(Z_t\), pays off a return (dividend plus capital gains) at period \(t+1\), denoted by \(TR^{E}_{t+1}\) (presented in detail in the intermediate good producer problem). Short-term entrepreneurs pay back the loan made at time \(t\) in period \(t+1\), providing a return to the bank of \((R_{S,t} - 1)P_X S_t\).

Long-term entrepreneurs pay back the loan made at time \(t-1\) in period \(t+1\), providing a return to the bank of \((R_{L,t-1} - 1)P_{t-1}X_{L,t-1}\) where \(R_{L,t-1}\) is the nominal long-term rate set at time \(t-1\).

Finally, long-term asset holdings, acquired at time \(t\), may change in value at \(t+1\) due to changes in short-term rates \((\Delta V_{t+1})\) and may require a liquidity injection at time \(t+1\) of \(\varphi_{t+1}P_{t+1}\) per unit of the long-term assets that is funded by short-term borrowing in the balance sheet (which

\(^9\)This assumption effectively ensures that at steady state \(X_{L,t}\) and \(X_{S,t}\) are not far apart.\n
\(^10\)We include equity as the additional asset since New Keynesian models already embed firm equity and thus we keep the framework as simple as possible. More importantly, 68% of total market value of shares in the US is held by institutional investors (Lewellen (2011)), which are themselves integrated to or part of the various financial businesses we aim to capture. Nonetheless, extending the variety of assets is an interesting avenue for future research, particularly including risky loans to incorporate credit risk fluctuations. As long as default risk is pro-cyclical, as is the total return in equity in our model, the results we obtain on term spreads movements remain qualitatively the same.\n
\(^11\)This balance sheet structure has been prevalent in the financial sector (at least) until the most recent crisis (see Brunnermeier (2009) for a discussion)
in our framework is simply equal to $X_{L,t}$. We assume that the liquidity shock is given by $\varrho_t = (1 - \rho L)\bar{\varrho} + \rho L\varrho_{t-1} + e_L$, where $e_L$ is an i.i.d. normal shock.

As in our simple model $\varrho_t$ is understood as a reduced form shock to the maturity mismatch exposures in the bank’s balance sheet (originating from either the liability or asset side). It is important to dissociate $\varrho_t$ from an aggregate shock that increases the default rate in the economy.\footnote{12} A change in $\varrho_t$ occurs due to funding or balance sheet problems and is thought to be unrelated to the fundamental value of long-term asset holdings. Acharya et al. (2011), for instance, present a model highlighting how a funding problem might occur even when the fundamental value of asset holdings remains high.

Note that although it is natural to think of $\varrho_t > 0$, or potential liquidity shortages, we could also have liquidity gains with $\varrho_t < 0$, for instance due to an increase in the prospective gains from securitisation of long-term assets in the banks’ balance sheets. In this case, instead of liquidity shortages, the banking sector is characterised by excess liquidity, which could give rise to an inverted yield curve with long-term rates lower than short-term rates. That might have been the case in the UK during the few years preceding the 2007 crisis. An interesting extension of the model left for future research is to make $\varrho_t$ endogenous based on the potential for securitisation vis-a-vis the expected need for liquid funds/provisions or shortage of funding.

Bank’s total profits at period $t+1$ will therefore have six main components: (i) the total return on equity ($TR^E_{t+1}$), which includes both dividends and capital gains; (ii) payments from long-term positions that matured; (iii) payments from short-term positions that matured; (iv) costs of deposit funding; (v) liquidity costs (or gains) from maturity mismatch; and finally (vi) gains or losses from the re-valuation of long-term positions held in the balance sheet, denoted $\Delta V_{t+1,t}$. Formally,

$$\Pi^B_{t+1} = \frac{1}{P_{t+1}} \left[ TR^E_{t+1} + (R_{L,t-1} - 1)P_{t-1}X_{L,t-1} + (R_{S,t} - 1)P_tX_{S,t} - D_t(R_tCB - 1) - \varrho_{t+1}X_{L,t}P_{t+1} + P_tX_{L,t}\Delta V_{t+1,t} \right].$$

\footnote{12} Although such a shock might be a trigger for the potential liquidity problems we model through a change in $\varrho_t$.

\footnote{13} In fact we do assume $\bar{\varrho} > 0$, or liquidity shortages are expected at steady state, to ensure we obtain an upward sloping yield curve, matching the data in the US.

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Re-valuation of long-term positions is given by

$$\Delta V_{t+1,f} = \frac{t+1f_{t+2} - E_t[\Omega_{t+1}]}{R_{t+1, CB}} - 1 = \frac{R_{t+1, LB}}{R_{S,t+1}} - 1$$

(13)

where the forward rate is given by $t+1f_{t+2} = \frac{R_{t+1, LB}}{R_{S,t+1}}$. Therefore, if interest rates are constant from $t$ until $t+1$, it follows that $\Delta V_{t+1,f} = 0$.

We assume banks are risk averse. The only risk involved in the banking business in our model is the maturity transformation risk. The bank commits to lend money to long-term investment opportunities having to acquire funds next period to re-finance this balance sheet commitment plus any additional liquidity injection needed. Risk aversion here implies that the bank does not only care about the return on short and long-term assets, requiring them simply to pay the same expected return on average. The bank weighs these returns according to the expected profits of the entire portfolio, requiring higher premium to bear risk when overall profitability is low but accepting lower risk compensation when overall returns are high. Effectively, the bank cares about the covariance between the returns of each asset and the returns of the overall portfolio.

Note that even if banks were risk neutral, the limits on Value-at-Risk (VaR) banks normally abide to would effectively imply that overall profitability of assets would influence banks’ required premium to bear maturity risk through the VaR constraint. Hence, the assumption that banks are risk averse reflects that some measure of overall riskiness and expected profits affect their long-term asset demand decision or implicitly the premium they require for bearing maturity transformation risk. The bank’s profit maximisation problem is given by\(^\text{14}\)

$$\max_{\{X_{S,t},X_{L,t},D_t,Z_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Pi_t^{1-\sigma_B}}{1-\sigma_B} \quad \text{s.t.} \quad D_t = P_tX_{S,t} + P_tX_{L,t} + Z_t + P_{t-1}X_{L,t-1},$$

(14)

\(^{14}\)Although we include $Z_t$ in the set of choice variables in the maximisation for completeness, given that equity is the best asset in the portfolio, paying a gross return higher than the short-term rate, banks will always demand the total amount of equity supplied by intermediate firms.
where $\sigma_B$ controls the degree of risk aversion.

Based on the bank’s demand for $\{X_{S,t}, X_{L,t}\}$ and the respective supply of assets from entrepreneurs we obtain the equilibrium values for short-term and long-term interest rates $\{R_{S,t}, R_{L,t}\}$. Given the potential for liquidity shocks or the risk of maturity transformation, the bank will require a premium to hold long-term positions, or $R_{L,t} > R_{S,t}$. If the bank were to hedge this maturity risk they could go long on short-term assets (for instance short-term government bonds) and go short on long-term assets (for instance long-term government bonds). The overall cost of such a hedging portfolio would be the difference between short-term and long-term government yields or the term spread.\(^{15}\) Thus, assume the bank is considering whether to increase its holdings of long-term assets which are funded by short-term borrowing. At the margin, this additional position could be offset by buying the hedging portfolio such that no additional return is gained nor risk undertaken. If the bank decides to hold that additional position unhedged, it must receive the premium adequate to the risk undertaken. As a result, at the margin, the difference $R_{L,t} - R_{S,t}$ is directly related to the term premium in the government bond market. As such, we define term spreads (annualised in percentage points) between long and short-term rate in the same fashion as the macro-affine literature

$$tp_t = \frac{1}{2} \left[ (R_{L,t} - 1) - (R_{t, CB} - 1) - (R_{t+1, CB} - 1) \right],$$

\((15)\)

2.4. Firms

The final good representative firm combines a continuum of intermediate inputs $i \in [0, 1]$ with the following production function

$$Y_t = \left( \int_0^1 y_{i,t} \right)^{\frac{1}{\epsilon}}.$$

\(^{15}\)This occurs since our model does not include default or credit risk. If that were to be the case we could decompose the bank’s overall risk by the term spread exposure plus the credit exposure, which could be hedged by credit default derivatives or default insurance.

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As standard this implies a demand function and aggregate price level given by

\[ y_t = \left( \frac{P_i}{P_t} \right)^{-\varepsilon} Y_t, \quad \text{and} \quad P_t = \left( \int_0^1 p_{1,t}^{-\varepsilon} \right)^{1/\varepsilon}. \] (17)

The intermediate sector is constituted of a continuum of firms \( i \in [0, 1] \) producing differentiated inputs with the following constant returns to scale production function

\[ y_{i,t} = A_t K_{i,t}^S \alpha \zeta K_{i,t}^L \alpha (1 - \zeta) H_{i,t}^{1-\alpha}, \] (18)

where \( A_t \) denotes the productivity level at time \( t \) (which follows the standard AR(1) process \( \log(A_t) = \rho A \log(A_{t-1}) + e_A \), where \( e_A \) is an i.i.d. normal shock), \( K_{i,t}^S \) is the capital stock originated from short-term projects, \( K_{i,t}^L \) is the capital stock originated from long-term projects and \( H_{i,t} \) is the household composite labour used in production. Each firm \( i \) hires labour and invests in both stocks of capital. Implicit here is the assumption that short-term and long-term capital are not perfect substitutes, which reflects the fact that long-term projects might have a distinct technological enhancement compared to capital based on short-run investments. This assumption ensures the price of each capital type are potentially different. As such, fluctuations in economic activity can have distinct effects on short and long-term credit demand influencing the determination of the yield curve.

To characterise the problem of the intermediate firms, we split their decision into a pricing decision (given their real marginal cost) and a cost minimisation decision. Following the standard Calvo pricing scheme (\( \omega \)), firm \( i \), when allowed, sets prices \( P_{i,t} \) according to

\[
\max_{P_{i,t}} E_t \left[ \sum_{s=0}^{\infty} P_{t+s} Q_{i,t+s+1} \omega s Y_{i,t+s+1} \left( \frac{P_{i,t}}{P_{t+s}} - \Lambda_{t+s,i} \right) \right],
\] (19)

subject to the demand function (equation (17)), where \( Q_{i,t+s+1} \) is the economy’s stochastic discount factor, defined in the Appendix B, and \( \Lambda_{t+s,i} \) is the firm’s i real marginal cost at time \( t+s \). To obtain the real marginal cost, we need to solve the firm’s intertemporal cost minimisation problem. That is

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\[
K^S_{i,t+1}, K^L_{i,t+1}, H_{i,t} \text{ subject to the production function (equation (18)) and investment equation } I^m_{i,t} = K^m_{i,t+1} - (1 - \delta)K^m_{i,t}
\]

for \( m = \{S, L\} \), where \( \delta \) is the depreciation rate.\(^{16}\)

Finally, total return on equity is given by

\[
TR^E_{i,t} = P_{i,t}Y_{i,t} - W_{i,t}H_{i,t} - P_i(q_t^S I^S_t + q_t^L I^L_t) + P_i(q_{i,t+1}^S K^S_{i,t+1} + q_{i,t+1}^L K^L_{i,t+1}) - Z_t,
\]

where \( Z_t = P_{t-1}(q_{t-1}^S K^S_t + q_{t-1}^L K^L_t) \). The first three terms comprise the flow of profits (dividends) and the last two the capital gains (due to changes in amount and price of capital held in the intermediate firms). We assume equities are bought (or evaluated) by banks at the end of time \( t - 1 \). Thus, the value of the intermediate firms at the beginning of time \( t \), denoted \( Z_t \), is equal to the value of its capital holdings (before production and investment in (new) capital takes place during period \( t \)) times their prices, all set at period \( t - 1 \).

### 2.5. Market Clearing Conditions

The capital market clearing conditions are given by

\[
I^S_t = yk_t^S = \gamma S \ln(1 + X_{S,L-1}), \text{ and } I^L_t = yk_t^L = \gamma L \ln(1 + X_{L,L-2}).
\]

The good market clearing condition, or the aggregate demand, is given by

\[
Y_t = C_t + X_{S,t} + X_{L,t} + \varphi_t X_{L,t-1}.
\]

\(^{16}\)Note that the demand for each type of labour stated in the household wage setting problem can be obtained by minimising the total cost of labour \( \int W_{i,j} H_{i,j} d j \) subject to the labour composite index.
Capital and labour markets across intermediate firms are aggregated such that

\[ K^m_t = \int_0^1 K^m_i di \text{ for } m = S, L \text{ and } H_t = \int_0^1 H_j d j. \]  

(25)

The credit market clearing condition is

\[ \frac{D_t}{P_t} = X_{S,t} + X_{L,t} + \frac{Z_t}{\pi_t} + \frac{X_{L,t-1}}{\pi_t}, \]  

(26)

where \( Z_t = \frac{(q^S_{t-1}K^S_{t-1} + q^L_{t-1}K^L_{t-1})}{\pi_t} \) and \( \pi_t = \frac{p_t}{\bar{p}_{t-1}}. \)

Finally, we assume the central bank sets monetary policy (conventional) according to a standard monetary rule given by\(^{17}\)

\[ \frac{R_{t, CB}}{\bar{R}_{CB}} = \left[ \left( \frac{\pi_t}{\bar{\pi}} \right)^{\epsilon_x} \left( \frac{Y_t}{\bar{Y}} \right)^{\epsilon_y} \right], \]  

(27)

where \( \bar{X} \) is the steady state value of a variable \( X_t. \)

### 2.6. Equilibrium and Calibration

The equilibrium of the economy is defined as the Lagrange multiplier \( \{\Lambda_t\} \), the allocation set \( \{C_t, H_t, K^S_{t+1}, K^L_{t+1}, X_{S,t}, X_{L,t}, Y_t, D_t, r^S_t, r^L_t, TR^E_t, \Pi^B_t\} \) and the vector of prices \( \{P_t, \pi_t, w_t, w_{1,t}, R_{L,t}, R_{t, CB}, q^S_t, q^L_t, R_{s,t}, p_t\} \) such that the household, the final good firm, intermediate firms, entrepreneurs and the bank maximisation problems are solved, and the market clearing conditions hold.

Details of the equations that determine the recursive equilibrium and the steady state of the economy are shown in Appendix B. Before discussing the results we quickly present the main parameter values used for the benchmark version of our model. As standard we set the goods market mark-up to 20%, thus \( \epsilon = 6 \). The labour market mark-up is set to 7.5% or \( \epsilon^w = 14. \)\(^{18}\) We set the discount factor \( \beta = 0.99, \) the intertemporal elasticity of substitution in consumption \( \sigma = 1 \) \(^{17}\) As spreads move endogenously, one could verify whether the monetary rule should be augmented to include term spreads. In this model the addition of term spreads to the rule does not yield higher welfare (results available upon request). In an alternative model with lending relationships we showed that inserting endogenous banking spreads into the rule improves welfare (see Aksoy et al. (2013)).\(^{18}\)

While some contributions to the DSGE literature set \( \epsilon^w = 21 \) others set \( \epsilon^w = 2. \) Our results are unchanged when we vary \( \epsilon^w \) within this range.

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and the Frisch elasticity of labour supply $\eta = 1$. The Calvo price and wage parameters are $\omega = 0.5$ and $\omega^w = 0.6$.\textsuperscript{19} The depreciation rate is set to $\delta = 0.05$ and the share of capital in production to $\alpha = 0.36$. Fan et al. (2012) report that the debt maturity ratio, (that is, long-term interest bearing debt over total debt) is about 80% in the US, 60% in the UK, 55% in Germany and 40% in Japan during the period 1991-2006. They found that the median long-term debt ratio across 39 different countries is estimated to be around 60%. Hence we set the share of short run capital to $\zeta = 0.4$ ensuring the steady state share of long-term loans in total loans to be 60%. We set the degree of risk aversion of banks to $\sigma_B = 1$, which is the same as the one for the household.

The steady state long-term rate is given by $R_L = \frac{1}{\rho_A} + \frac{\bar{\rho}}{p}$ and thus depends on the liquidity shortage at steady state ($\bar{\rho}$). We set $\bar{\rho} = 0.0025$, such that the 10 year term premium is roughly 100 basis points matching the US data (Rudebusch and Swanson (2008)). We initially assume that the central bank follows a simple Taylor Rule with inflation parameter $\epsilon_\pi = 2.5$ and output gap parameter $\epsilon_Y = 0.125$. Note that higher values of $\epsilon_Y$ and lower values of $\epsilon_\pi$ easily lead to indeterminacy issues in models with cost channels (see Aksoy et al. (2011)). Finally, we set the persistence of the productivity process $\rho_A = 0.8$ and the persistence of liquidity shocks $\rho_L = 0.8$, while setting the standard deviation of their respective disturbances to $\nu_A = 0.01$ and $\nu_L = 0.0001$.

In order to incorporate the covariance effect on the bank’s portfolio decision, the model is solved to a third order approximation using Dynare++ (without centralisation).

### 3. Term Spreads and Economic Activity

In this section we analyse the mechanism that drives term spread fluctuations in our model, the link between spread movements and future output growth and finally present the impulse responses to contemporaneous productivity and monetary shocks.

\textsuperscript{19}These are a bit smaller than the ones obtained in DSGE-based Bayesian estimations. However, most of these studies have assumed wage and price indexation decreasing the effect of nominal rigidity on economic activity, while here for simplicity we do not. Our results are unchanged when lower degrees of price and wage rigidity are assumed.
3.1. Endogenous Term Spreads

We start by focusing on the channel that governs the fluctuations of term spreads in our model. The key equilibrium condition (see Appendix B for details) that determines the long-term rate, and consequently term spreads, comes from the bank’s portfolio decision. The bank will set long-term assets holdings \(X_{L,t}\) such that

\[
E_t \left[ \prod_{t+1}^{t+2} \frac{-\sigma_B}{\pi_{t+1}} \left( R_{t,CB} - 1 - \Delta V_{t+1,t} + Q_{t+1} \right) \right] = \beta E_t \left[ \frac{1}{\pi_{t+1} \pi_{t+2}} \prod_{t+2}^{t+3} \frac{-\sigma_B}{\pi_{t+2}} \left( R_{L,t} - R_{t+1,CB} \right) \right]. \tag{28}
\]

As we stressed in Section 1, this is intrinsically a forward looking condition. A portfolio decision made at time \(t\) is influenced by the relative path of expected profits in periods \(t + 1\) and \(t + 2\). In order to highlight this feature we look at the dynamic responses in our model economy when agents expect productivity to be higher in the next period.

Figure 2 shows impulse responses for output, bank profits, the short-term rate and term spreads to a one period ahead positive shock in productivity. For all variables the percentage deviation from steady state is shown except for term spreads movements where the change in the percentage rate is reported (thus a 0.2 deviation implies a 20 basis point change in term spreads).

As expected, a positive news about productivity increases output initially as investment in capital increases. When high productivity materialises, output increases further and is expected to converge back to steady state thereafter. Bank profits jump with the shock as high price of capital imply high equity valuation. Bank profits are expected to stay high in the following period and then become negative with the equity value decreasing as the economy converges back to equilibrium. We note that short-term interest rates increase and term spreads decline in the period the news is known; equivalently long-term rates do not increase as much as short-term rates.

The movements in spreads can be understood by looking at the endogenous response of bank profits after the shock. Long-term asset exposures undertaken in the current period must be funded during the next period, since these are kept in the balance sheet, when liquidity injections might be needed. After two periods these assets mature and pay excess returns. The bank is less likely to suffer from balance sheet problems if, when liquidity injections are needed, profits are rela-
very high as compared to the period in which the long term assets mature. This is because cash
flows from high profits can be used to cover for these required injections. As a result, bearing
maturity risk in the interim period becomes relatively cheaper, bank’s demand for long-term assets
increases, generating lower term spreads. Thus, after receiving the news that economic activity is
going to increase in the future, the bank sets term spreads lower since it expects profitability to
be high during the period funding costs must be paid. When expected profitability is increasing
(e.g. from period 2 onwards), profits in the funding periods will be relatively smaller. As a result,
bearing maturity risk becomes relatively more expensive and the opposite occurs.

In summary, endogenous spread movements arise in our model since changes in economic
conditions (as reflected by the paths of future profits) alter the relationship between the bank’s
shadow value of funding costs (paid as the long-term assets are held in the balance sheet) and
their payoffs (materialised when these assets mature). When relying on a high order solution to
the model, these variations in shadow value will not only depend on relative profits at time $t + 1$
and $t+2$, but also on how they relate with the size and the variations of funding costs, given by 
\[
\left(\frac{(R_{t+1}R_{t+2}-1)\Delta Y_{t+1}}{\bar{\sigma}_{t+1}} + Q_{t+1}\right) .
\]

In order to further investigate endogenous movements in term spreads, we present three variants of our model when agents receive positive news on future productivity. In the first case, we set a higher steady state liquidity shortage ($\bar{\sigma} = 0.0125$), increasing the expected value of funding costs. We denote this case as high\textsubscript{liq}. In the second case, we set a high variance of liquidity shock ($\nu_t = 0.04$), which increases the variability of funding costs. We denote this case as high\textsubscript{vl}. Finally, in the third case, we assume that the bank is more risk-averse and set $\sigma_B = 2$. We denote this case as high\textsubscript{lr}. Figure 3 compares the dynamic responses of spreads and bank profits for these three cases, against the benchmark model.

![Fluctuations in Term Spreads](image)

In all three cases bank profits move in a very similar way after the shock. Therefore, in order to distinguish the three cases, we need to uncover how the same response in profits leads to different dynamics in term spreads, exploring the relationship between profits and funding costs. In the first variant, we observe that spread movements are amplified under higher steady state liquidity.
shortages. The intuition for this result is as follows: at the steady state, the bank sets long-term rates higher than short-term rates to offset potential liquidity shortages and hence, the higher $\bar{\varrho}$ or the higher the average need for liquidity injection, the higher will the steady state long-term rates be. Given that long-term rates are relatively high, an equivalent increase in bank profits (when compared to the benchmark case) induces a stronger adjustment in long-term rates, which in turn implies long-term rates falling (relative to short-term rates) by a greater amount than under the benchmark case.

The opposite occurs when the variance of the liquidity shock is high. The bank is willing to bear more maturity risk in periods of high profits since they know that high profits can be used to offset liquidity shortages. However, the more volatile are these shortages, the less certain the bank will be that high profits will be enough to offset them. Therefore, an equivalent movement in bank profits leads to smaller movements in term spreads after a positive productivity shock.

The third variant illustrates that the term spreads is more responsive to shocks as the degree of bank risk aversion increases. $\sigma_B$ effectively determines how fluctuations of bank profits influence the bank’s long-term rates decision. When $\sigma_B \to 0$, the bank will set long-term rates to be a discounted sum of short-term rates and term spreads will be constant. This mechanism is the same as the one explored in the macro-finance literature where Epstein-Zin preferences are used to increase risk aversion in order to match volatility of risk/term premia (see Rudebusch and Swanson (2012)).

The same conclusions on the effects of changes in expected bank profits on term spread fluctuations can be analytically drawn by exploring a higher order approximation to the portfolio
choice equation (28) (the derivation is shown in Appendix C).  

$$E_i[\tilde{p} + 0.5(\tilde{p})^2] = E_i[\sigma_B(\tilde{\Pi}^B_{t+1} - \tilde{\Pi}^B_{t}) + 0.5\sigma_B(\tilde{\Pi}^B_{t+1} - \tilde{\Pi}^B_{t})^2 + 0.5(\tilde{\varphi}_{t+1} + 0.5\tilde{\varphi}_{t}^2) + \text{CovTerms} + \Xi]$$  

$$\text{CovTerms} = -\sigma_B\tilde{\Pi}^B_{t+1} + \tilde{\varphi}_{t+1} - \sigma_B\tilde{\Pi}^B_{t} + \frac{1}{\beta}(\tilde{R}_{t+1} - \Delta\tilde{V}_{t+1}) + \sigma_B\tilde{\Pi}^B_{t} - \tilde{\varphi}_{t+1}$$  

$$+ \left[\frac{1}{\beta}(\tilde{R}_{t+1} - \Delta\tilde{V}_{t+1}) + \tilde{\varphi}_{t+1} + \sigma_B\tilde{\Pi}^B_{t} - \tilde{\varphi}_{t+1} \right] \tilde{R}_{t+1} - \frac{1}{\beta} \frac{\tilde{r}_{t+1}}{\tilde{r}_{t+2}} - \frac{\tilde{r}_{t+2}}{\tilde{r}_{t+1}}.$$  

The first term on the right hand side of the variations in term premium equation shows how relative profits impact spreads to a first order. As easily verified, setting $\sigma_B = 0$, or assuming bank’s utility is linear on profits, eliminates all the effects of movements of profits on term spread decisions. Looking at the covariance terms we observe that the higher the covariance between profits, $\tilde{\Pi}^B_{t+1}$, and the liquidity shortage, $\tilde{\varphi}_{t+1}$, (see the first term of CovTerms), the lower term spreads will be, with the strength of the effect being positively associated with $\sigma_B$ and $\tilde{\varphi}$. Hence, as we increase $\tilde{\varphi}$, keeping the expected changes in profits constant, movements in spreads are amplified.

Finally, the increase in the variance of the liquidity shock $\nu_i$ has two opposing effects. Firstly, it tends to raise spreads since $E(\tilde{\varphi}_{t+1})^2$ has a positive impact on spreads. Secondly, it becomes a stronger driver of the expected covariance between $\tilde{\Pi}^B_{t+1}$ and $\tilde{\varphi}_{t+1}$. The latter implies that the expected positive movement in profits due to the shock will have little effect on the covariance term and as such one of the drivers of the endogenous movements of spreads loses its significance. As a result of this effect on the covariance, higher volatility of liquidity shocks dampens the impact of future productivity shocks on term spreads.

Our empirical results, presented in Section 5, confirm the relationship between spreads and financial sector profitability obtained here. When expected profitability of financial business increases, spreads fall. The model also generates a negative link between bank’s asset holdings and term premia. Adrian et al. (2010b) present empirical evidence of this negative relationship across

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various asset classes. They stress that the relationship is a result of time variation in discount rates and not due to the potential link between bank’s asset growth and expected future cash flows (of the market portfolio). Note that the mechanism we highlight depends on the expected future cash flows, but the cash flows of the banks, who are selecting asset holdings and influencing risk premia, and not those of the equity market portfolio. We model the bank’s portfolio decision in order to uncover the potential causes for changes in discount rates, which in turn lead to changes in asset holdings. We propose that these variations may relate to the future path of bank profits. Thus, although we focus on expectations of bank cash flows, we see our mechanism as complementary to the one of Adrian et al. (2010b), who stress the importance of variable discount rates in their empirical work. This is further attested by our focus on the Euler equation (28) from the bank problem as a driver of term premia fluctuations.

Although we focus on term spreads, augmenting our model to include different assets in the bank’s portfolio would possibly allow us to further explore the links between financial intermediation and the macroeconomy, extending the analysis to other risk premia. In fact, if risky loans were included into the model, variation in default probability would reinforce the channel we presented. Positive news on economic activity would not only lead to high price of equities but also to an increase in the expected value of risky loans as a result of lower default probabilities. Thus, expected bank profits would increase due to both assets, allowing the bank to increase its overall exposure and depressing risk premia.

3.2. Yield Spreads and Output Growth

As stressed by Gürkaynak and Wright (2012), term structure models should generate a high slope of the yield curve at the beginning of recoveries from recessions and a relatively flatter yield curve towards the end of booms, feature which is related to the predictive power of yield spreads. Hamilton and Kim (2002) conclude that lower term premiums predict slower GDP growth, although this effect appears to be strong only in the short-run, while Wright (2006) shows that lower term premium raises the odds of a recession.

In order to verify if the dynamics of term spreads in our model are consistent with this feature,
Figure 4 plots the impact of a four quarters anticipated technology shock. Thus, at time t agents learn that there will be a productivity shock at time t+4. Based on the information at time t, the bank forms an expectation of future growth and profits which will affect long-term rates and thus term spreads. These then feed back to the economy influencing long-term investment and output.

Figure 4: Anticipated Productivity Shock

We observe that output and long-term investment increase from t until t + 4 (time of the realisation of the productivity shock). Therefore, if one is regressing output gains ($\tilde{y}_{t+3} - \tilde{y}_t$) on $\tilde{i}_p$ (a variant of Hamilton and Kim (2002) estimation) getting a positive parameter estimate must imply that $\tilde{i}_p > 0$, which is what we obtain. The future path of bank profits allow us to explain the observed term spread fluctuations. Bank profits will initially become negative, but are expected to increase making it relatively more costly to bear maturity risk. Hence, long-term rates and spreads increase in period t. Spreads are at their highest when output is at its lowest. At $t + 3$, productivity is about to reach its peak, output is approaching its highest point and spreads their lowest point; bank profits are high but are expected to decrease in a few periods. This alters the shadow value

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of funding costs versus gains from long-term asset holdings, such that spreads reach their lowest point (is relatively cheaper to bear maturity risk). Therefore, as observed in the data, high sloped yield curve indicates future output is increasing while a flatter yield curve indicates that output is reaching its peak.

Rudebusch et al. (2007) refer to a potential contradiction while discussing the intuition behind the results of regressions of output growth on the level of term spreads. The level regression suggests that low spreads lead to lower output in the future. However, under a standard IS curve, low term spreads should result in higher investment (assuming it depends on long-term rates) and thus higher output in the future. They confirm this view in the data by estimating output differences ($\Delta \bar{y}$) on spread differences ($\Delta \bar{r}_p$), obtaining the expected negative parameter estimate. As opposed to the standard models in the macro-finance literature where the yield curve is built based on the stochastic discount factor, the term premium here has a direct effect on long-term investment and output and thus this mechanism is in place. Hence, our model also confirms the prediction that decreasing spreads lead to higher output. As the economy moves from $t$ to $t + 4$, bank profits are increasing at a decreasing rate, implying the shadow price of bearing maturity risk is decreasing and thus spreads are decreasing. That leads to increasing long-term capital investment and consumption. However, at the time the anticipated shock is known, period 1, bank profits are expected to be low and increasing sharply forcing the bank to initially charge more for long-term commitments. Thus, term spreads are high but decreasing.

Note that Adrian et al. (2010a) assess empirically the link between bank’s assets, spreads and output growth. They propose that lower term spreads, holding riskiness constant, leads to lower net interest margins, which in turn leads to lower banking asset growth and hence, lower output. Effectively, this mechanism would occur for movements of spreads that are exogenous to the bank’s balance sheet decision proposed here, which is based on riskiness. We can obtain a similar link using our model if we were to alter $\gamma_L$ (the parameter that controls entrepreneurs demand for loans given the long-term interest rate - see equation (B.13) in Appendix B). As $\gamma_L$ decreases (exogenously), spreads and bank’s long-term asset holdings decrease, leading to lower

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22See also Hamilton and Kim (2002).
output. Nonetheless, in the framework presented here, endogenous fluctuations of spreads are intrinsically linked to the riskiness of bank’s portfolio or asset holdings, preventing us from fully analysing changes in spreads holding riskiness constant, as their mechanism suggests.

3.3. Contemporaneous Shocks and Impulse Responses

In this subsection we look at the dynamic properties of our benchmark model after standard productivity (technology) and monetary shocks. As previously discussed, the bank portfolio channel stressed relates to the effect of the future path of profits on term premia. As a result, the key insight to understand spread movements is not the immediate impact on profits after the shocks but the expected path of bank profits from period 2 onwards.

![Figure 5: Monetary Shock](image-url)

We start with a monetary contraction. Figure 5 presents the impulse responses. Output, investment and consumption decrease as the short-term rate increases. Long-term asset values and equity returns initially decrease, depressing profits at the time of the shock. Nonetheless, equity

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returns increase next period and bank profits rebound. As profits are expected to be higher in period 2 relative to period 3, the shadow price of the cost of funding decreases and spreads will fall. Profits are expected to fall back and increase towards the steady state from period 3 onwards, pushing term spreads upward.

Positive technology shocks have the opposite effect (see Figure 6). Output, consumption and investment all increase. Inflation decreases leading to a decline in the base rate. Bank profits jump as both long-term assets and equity gain in value. However, as the price of capital decreases from this period onwards, total equity returns are expected to be lower for the following periods. Hence, bank profits will be negative and increase from period 2 onwards. As a result, the shadow price of funding costs increase, pushing term premia up.

In Section 5 we compare the dynamic properties presented here to the impulse responses obtained from our vector auto-regressive estimations. In general our model does a good job in matching the empirical correlations. Spreads movements are largely in line with their empirical counterparts.
although showing less persistence than in the data (see Section 5.3 for details).

4. Unconventional Monetary Policy

Our model, by creating a new policy channel, allows us to analyse the impact of unconventional policies during periods of large shocks to liquidity shortages. The Federal Reserve Bank (FED) conducted two purchase programmes of long-term Treasuries and other long-term bonds, known as QE1 in 2008-2009 and QE2 in 2010-2011. These QE policies comprised of the purchase of mortgage backed securities, Treasuries and “Agencies” from the private sector. Gagnon et al. (2011) analyse the effectiveness of the Large-Scale Assets Purchases conducted by the FED. They find that the purchase programme led to reductions in long-term interest rates on a range of securities, including some securities that were not included in the purchase programme, indicating that portfolio balancing effects were in play. They argue that the reductions in interest rates primarily reflect lower risk rather than lower expectations of future short-term rates. Krishnamurthy and Vissing-Jorgensen (2011) find that these QE policies in the US led to a significant decline in nominal rates on long-term safe assets (Treasuries and “Agencies”, assets which were more heavily traded by the FED) and only a small effect on less safe assets such as corporate rates and mortgage rates (assets which were less heavily influenced by FED market activity). Their results suggest that the effects of asset purchases on the duration of risk premium are small, while effects on liquidity-safety premium are substantial.

Beirne et al. (2011) report on the effectiveness of the Covered Bond Purchase Programme (CBPP), which started in July 2009 for a period of twelve months in the Eurozone, and show that covered bond yields decreased by 12 basis points. They also argue that the programme increased the liquidity of the secondary market and managed to encourage lending. Joyce et al. (2011) report that the QE interventions in the UK led to a 100 basis point decrease in Gilt yields. They observe that QE affects other asset classes as well, although the purchase programme has been overwhelmingly of government securities. Thus, they also highlight the importance of portfolio balancing effects. Finally, Borio and Disyatat (2010) provide a survey of different forms of
possible unconventional monetary policies and argue that the main balance sheet channel operates through the central bank’s ability to reduce yields and ease financing constraints by altering the risk profile of private portfolios. Overall, a constant theme in these studies is the effect on long-term rates through lower term spreads being crucial for the effectiveness of the interventions and for allowing the financial market to continue funding economic activity.

Two main features of our model are particularly important in formalising these types of interventions. First, fluctuations in term spreads are relevant factors in determining output fluctuations, as long-term rates influence investment decisions. Thus, a QE intervention aimed at lowering long-term rates stimulates economic activity. Second, given that term spreads or long-term rate decisions are directly determined by fluctuations in future bank profits and changes in their balance sheet holdings, our model provides a new channel through which these effects arise.\(^\text{(23)}\)

In order to study the main effects of QE policies we first introduce two types of unconventional monetary policies and then analyse their impact after a liquidity shortage shock. The first is a simple liquidity injection \(\text{(QE}_t\text{)}\) to the bank, financed by a lump-sum tax collected from households. Liquidity injection, which is costless to the receiving bank, is set such that \(\text{QE}_t = \xi_t X_{L,t-1} P_t \rho_t\), where \(\xi_t = \phi_t \left( \frac{X_L}{X_S} - \frac{X_{L,t}}{X_{S,t}} \right) \frac{X_S}{X_L} \) and \(\frac{X_{L,t}}{X_{S,t}}\) is the ratio of long to short run funding that would be in place without QE intervention. The liquidity injection is a proportion \(\xi_t\) of the bank’s liquidity shortage and its intensity depends on how skewed current investment funding is towards short-term relative to long-term funding. Note that this relative difference will be a direct function of future liquidity conditions.

The second unconventional policy is the existence of favourable conditions for the bank to borrow funds from the central bank using their long-term asset positions as collateral. Favourable conditions in our context imply a lower rate of borrowing relative to the short-term funding currently available. The bank now decides the fraction of long-term assets \(\Theta_t\) they want to pledge as collateral to obtain funds from the central bank. Effectively, at time \(t\), the bank makes a two period investment. At period \(t + 1\) they sell a portion \(\Theta_t\) of these assets to the central bank to

\(^{23}\)However, an important caveat, which underlines a promising path for future research, is the fact that our bank portfolios are fairly simple, with only three assets. They do not include, for instance, housing debt/mortgages, thereby restricting the analysis of some of the portfolio balancing effects mentioned.

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receive additional funds, promising to buy the assets back at $t + 2$ before they mature. The total cost of central bank funding will be $\Theta_t X_{L,t-1} P_{t-1} (R_{t,QE} - 1) + \frac{\phi_{re}}{2} \Theta_t^2$, where $R_{t,QE}$ is the borrowing rate. The term $\frac{\phi_{re}}{2} \Theta_t^2$ is included such that the marginal cost of this type of funding is increasing as usage increases.

Note that, although this type of intervention appears to be more complex, since it is implemented through the market, it requires less information about the bank’s balance sheets by the central bank. Hence, these types of policies are rather straightforward to implement in practice. In fact, a significant portion of the QE interventions in the last few years were in that spirit.

The bank’s problem in this case becomes (first order conditions are shown in the Appendix B)

$$\max_{\{X_{S,t}, X_{L,t}, \Theta_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta_t \frac{\Pi_t^{B^{1-\sigma_B}}}{1 - \sigma_B}$$

s.t.  
$$D_t = P_t X_{S,t} + P_t X_{L,t} + Z_t + P_{t-1} X_{L,t-1} - \Theta_t X_{L,t-1} P_{t-1}$$

where

$$\Pi_{t+1}^B = \frac{1}{P_{t+1}} [TRE^E_{t+1} + (R_{L,t-1} - 1) P_{t-1} X_{L,t-1} + (R_{S,t} - 1) P_t X_{S,t} - D_t (R_{S,t} - 1) -$$

$$-\phi_{re} X_{L,t} P_{t+1} + X_{L,t} P_t \Delta V_{t+1,t} - \Theta_t X_{L,t-1} P_{t-1} (R_{Q,E,t} - 1) - \frac{\phi_{Q,E}}{2} \Theta_t^2].$$

Finally, we assume the central bank sets $R_{Q,E,t} = R_{S,t} \left[1 - \phi_{re} \left(\frac{Z_t}{X_S} - \frac{Z_t}{X_S} - \frac{\delta_S}{X_S} - \frac{\delta_L}{X_L}\right)\right]$. Thus, the lower the long-term funding relative to short-term without intervention, the more favourable central bank funding will be.

Figure 7 illustrates the results for a liquidity shock of 0.02 with $\phi_\xi = 4.5, \phi_{Q,E} = 0.005$ and $\phi_{re} = 0.15$. That means the central bank covers roughly 80% of the liquidity shortage under the first intervention after the shock and, in the second case, buys roughly 75% of long-term assets from the bank’s balance sheets (see the bottom left graph in Figure 7).

We first look at the effect of a liquidity shock without the presence of unconventional po-
Figure 7: QE Policies

licies (depicted by impulse responses with a circle). The shock first leads to a sharp increase in long-term rates and spreads. As a result, long-term investment drops significantly reducing output. Conventional monetary policy works overtime, reducing the base-rate substantially sustaining output through consumption. After a few periods, the base rate starts returning to its steady state level and lower capital stock due to the low investment in the previous periods materialises. Output, then, declines further reaching its lowest point two years after the shock. Note that conventional policies are quite important to sustain output in the immediate periods after the shock. This assumes nominal rates can be decreased.\textsuperscript{24} If however, rates are at a lower point at the time of the shock and conventional policies cannot be effectively used, the liquidity shock may have a larger impact.

When unconventional policies are used we see a different pattern. We observe that both QE policies do have a significant impact on term spreads. These policies lead to dampened responses

\textsuperscript{24}The zero bound is not reached in our simulations.
of long-term investment and output relative to the case where only conventional monetary policy is used. Both types of QE interventions are equally efficient in stabilising the economy, showing that market interventions are a good alternative to basic liquidity provisions which by design aim directly to neutralise the disturbances but are much harder to implement in practice. As unconventional policies are used conventional interventions become less important with the policy rate decreasing only mildly. In other words, while we do not model the nominal zero bound restriction, our calibrations show that short rates variations are minimal, indicating that after such shocks unconventional policy clearly dominates conventional policy. An important result is that inflation turns out to be significantly higher after QE interventions. Thus, even if nominal rates are close to 1, QE interventions could lead to lower real rates.

The channel through which these interventions affect our model economy also highlights how central bank programmes of buying long-term government bonds and selling short-term bonds (e.g. “Operation Twist”) could impact the yield curve. In offering the banks the possibility to exchange long-term commitments to shorter term assets, the central bank effectively allows financial institutions to decrease their exposure to maturity transformation risk. As a result, banks may decrease the premia they require to continue providing long-term funding, depressing long-term yields.

One of the important debates in policy making is about the timing of unwinding the large-scale purchases. Although not completely suited to provide a definite answer to such a question, we can use our model to verify the effectiveness of short-term asset purchase agreements, which sell securities back to banks after one period, and the interventions that allow banks to move long-term assets away from balance sheets for longer periods. In order to do that we modify the model such that long-term investments now require one year (four quarters) commitments from entrepreneurs and hence from banks. Appendix D shows the details of the model and each of the two QE interventions: one period asset purchases and three periods asset purchases. Figure 8 displays our results.\textsuperscript{25} We set $\phi_{QE}$ and $\phi_{re}$ for each of these two interventions such that the

\textsuperscript{25}Impulse responses for long-term capital stock are shown after the fourth period since that is the point changes to long-term investment done at time $t = 1$ start having effects.
We observe that when the central bank holds assets for longer periods, the same intervention in terms of assets purchases leads to lower levels of term spreads/long-term yields and to a smaller decline in long-term investments after a liquidity shock. There is a gain for the central bank to hold the securities bought in such interventions for longer periods of time, since they are more effective in freeing up the bank’s balance sheet, amplifying their effect on long-term funding. Obviously, these securities remain in the central bank balance sheet for longer and thus the monetary authority is taking significantly more risks than when it keeps securities for only one period.

5. Empirical Evidence

As argued in the introduction there is strong evidence that US term spreads help to predict US real output growth (see, for instance, Rudebusch and Williams (2009)). Furthermore, Adrian et al.

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(2010b) highlight the importance of financial sector variables, particularly the growth in financial intermediary asset holdings, in predicting several asset prices and risk measures. Finally, Kurmann and Otrok (forthcoming) establish a link between changes in the slope of the yield curve and news on total factor productivity. We complement this macro empirical evidence by looking particularly at the linkages between financial sector profitability, term spreads and output growth. For this purpose we conduct three sets of empirical analyses. We start by analysing the microeconomic link between the changes in the expected bank profitability at the bank level and the evolution in term spreads. Second, given the importance of the expected forward looking path of profits in explaining term spreads movements we look at the response of the main economic variables to anticipated innovations (news) to banking profitability as identified using the methodology developed by Barsky and Sims (2011) (adapted from Uhlig (2003)). These two sets of estimations aim at testing whether the theoretical channel we present is reflected in the data. Third, we employ a standard VAR to look at the impulse responses to contemporaneous productivity and monetary shocks, and a VAR in levels to analyse the effect of a consumer expectations shock, which is used as a proxy to changes in expected productivity growth, following Barsky and Sims (2012) closely. These VAR evidence is largely used to assess whether the broad empirical contours are in line with the dynamic properties of the theoretical model.

5.1. Spreads and Expected Financial Business Profitability: Bank Level Evidence

Our main theoretical mechanism postulates that as expected profits at time $t+1$ increase spreads at time $t$ tend to decrease. The opposite occurs when expected profits decrease. In this subsection we investigate whether there is empirical support for our claims.

The theoretical measure of profits includes both cash flows or net interest rate earnings and capital gains or book value changes. An equivalent measure of profits at the micro-level, which includes both net interest gains and book value changes, is the bank’s total earning per share. This restricts our sample to banks with publicly traded equities. We thus select the financial companies in the S&P 500. In order to obtain a measure of expected profits we use the forecast of earnings per share as reported by Thomson-Reuters I/B/E/S. We convert the information into quarterly
frequency\textsuperscript{26} for the period of 1990Q1 and 2007Q2, obtaining a dataset of 22 large US financial institutions for which we have longer time series data available.\textsuperscript{27} We define the changes in the expected profitability (mean expected earnings per share) as

\[
\Delta E_{t-1} \Pi^B_{i,t} = \left[ \frac{E_{t-1}(EPS)_{i,t} - EPS_{i,t-1}}{E_{t-2}(EPS)_{i,t-1} - EPS_{i,t-2}} \right]
\]

The remaining data used consists of seasonally adjusted US real GDP expressed in billions of chained 2005 Dollars (\(y\)), term spreads (\(spread\)), computed using the effective Federal Funds rate (\(ffr\)) in percent per annum as reported by the US Federal Reserve and ten year government bond rate in percent per annum. \(\pi\) is annualised quarterly inflation rate calculated from the CPI index (\(P\)) as reported by the IMF/IFS.

We estimate the following unbalanced fixed (cross-section) effects panel data specification given by

\[
spread_{t} = \alpha_i + \alpha_{TS}(spread_{t-1}) + \alpha_y(\Delta y_{t-1}) + \alpha_{\pi}(\pi_{t-1}) + \alpha_{\Pi}(\Delta E_{t-1} \Pi^B_{i,t}) + \varepsilon_{i,t}
\]

While we report estimation results based on the model with cross-section weights/panel corrected standard errors and covariance, we also estimate the model with specifications that account for various patterns of correlation between the residuals (Robust Coefficient Covariances). Our results are not affected by the specification of basic variance structures. We report four main estimations (Table 1): first an OLS estimation without controlling for the changes in the expected banking profitability; second a panel data estimation that controls for the expected banking profitability (benchmark); third the same panel including short-term interest rates (\(ffr\)) as additional control; and finally the benchmark panel with data extended until 2011Q2 to verify the sensitivity

\textsuperscript{26}Given that there are more than one forecast done in each quarter we take a simple average of these, hence obtaining a mean forecast for the quarter.

\textsuperscript{27}They are: JPMorgan Chase & Co (JPM), Chubb Corp (CHUBB), Lincoln National Corp (LINCOLN), Marsh and McLennan Cos (MARSH), PNC Financial Services Group Inc (PNC), Suntrust Banks Inc (SUN), Torchmark Corp (TORCH), Loews Corp (LOEWS), Morgan Stanley (MS), Comerica Inc (COMER), Fifth Third Bancorp (5TH), Progressive Corp-Ohio (PROG), Huntington Bancshares (HUNTING), Northern Trust Corp (NORTH), Franklin Resources Inc (FRANK), Equity Residential (EQRESID), Goldman Sachs (GOLDMAN), Prudential Financial Inc (PRUD), Apartment Inv and Mgmt (APINV), Federated Investors Inc (FED), Ameriprise Financial Inc (AMFIN) and Cincinnati Financial Corp (CINNFIN).

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of our results to the inclusion of the credit crunch period. We start by looking at the comparison between the simple OLS and the benchmark panel results (second block column). We observe that expected increases in the bank profitability leads to a statistically significant decline in term spreads. Furthermore, the coefficient for $\Delta E_{i-1}\Pi_{i}^B$ is economically significant. A marginal increase in the growth rate of earnings per share leads to a decline in term spreads by about 14 basis points. These results provide indirect support to the theoretical bank portfolio channel we propose. As we show when banks expect higher profitability in the forthcoming period, the shadow price of funding costs decrease, leading to a decrease in term spreads. Furthermore, although we focus on the aggregate channel we observe a large degree of heterogeneity across financial institutions, fixed effects ranging from -.20 to .03 (these are not reported here).

We perform two robustness exercises. First, we include the base interest rate (federal funds rate) as an additional regressor. Spreads become less persistent, the coefficient on interest rates is significant, but most importantly, expected bank profitability remain both statistically and economically significant. Second, we extend the sample period to include the recent financial crisis (up to 2011Q2); we find that the coefficient of growth in expected profits decreases by 50% but remains statistically significant (it becomes significant at 1% confidence level).

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5.2. Spread and Expected Financial Profitability: Macroeconometric Evidence

In this section, we present new macroeconometric evidence on the connectedness between changes in the expected banking profitability, term spreads and other key macroeconomic variables. Recent research such as Barsky and Sims (2011) and Kurmann and Otrok (forthcoming) uses a modification to Uhlig’s (2003) method to identify total factor productivity (TFP) news as the innovation that explains most of the forecast error variance (FEV) of TFP over a ten year horizon but is orthogonal to contemporaneous TFP movements. Using this identification scheme, Barsky and Sims (2011) show that a TFP news shock can explain over 40% of US business cycle variations over a ten year horizon. Kurmann and Otrok (forthcoming) also employ the same identification but look at the link between TFP news and variations in the slope of the US yield curve.

Given that our interest is to uncover a relationship between spreads and expected bank profits, we employ the same empirical strategy to extract the innovation (news) that explains most of the FEV of bank profitability but is orthogonal to its contemporaneous innovation. Following these contributions closely, we set up a VAR in levels utilising quarterly US data covering the period 1970Q1-2007Q2. Our data consists of the same GDP ($y$), interest rate ($\text{ffr}$), spread and price ($P$) measures used in the previous analysis plus seasonally adjusted gross private domestic investment ($I$) as reported by the Bureau of Economic Analysis. In order to incorporate a variable that reflects future news about the evolution of economic activity (see Barsky and Sims (2012) for details) we also include the index derived from a five years forward looking question on confidence from the Michigan Index of Consumer Expectations, which we denote $E5Y$.\footnote{The actual question is “Turning to economic conditions in the country as a whole, do you expect that over the next five years we will have mostly good times, or periods of widespread unemployment and depression, or what?” The variable is then constructed as being the percentage giving a positive answer minus the percentage giving a negative answer plus one hundred. For further discussion see Barsky and Sims (2012). They suggest that the correlation of this confidence measure is at least over 85% with alternative questions along similar dimensions.}

Finally, we include a measure of bank profits using the actual earnings per share for the 204 major financial institutions included in S&P 1500 as reported by Compustat.\footnote{More financial institutions are included here relative to the panel estimation in Section 5.1 since sufficiently long time series data of actual EPS is readily available while data on EPS forecasts is available only for a limited sample of financial institutions.} We calculate an aggregate measure of financial business profitability ($\Delta \Pi^B$) using the changes in actual earnings

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per share ($EPS$ - excluding extraordinary items) as follows:\footnote{Our bottom-up measure of financial profitability ($\Delta \Pi_B^t$) based on actual earnings per share is preferred to financial business undistributed corporate profits (FBP), as reported by the Flow of Funds Statistics of the US Federal Reserve, since the former measure is directly associated with the expected earnings per share used in the pervious section. As reported by the US Federal Reserve, FBP is part of the distribution of national income and includes inventory (book-value) valuations as does our $EPS$ measure. Our $EPS$ measure closely reflects the concept of banking profits used in the theoretical model that not only reflects cash flows but also changes in the market value of assets.}

$$\Delta \Pi_B^t = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{EPS_{i,t} - EPS_{i,t-1}}{|EPS_{i,t-1}|} \right),$$

(35)

where $i$ indexes financial firms in our sample after adjusting for outliers.\footnote{$\Delta \Pi_B^t$ is calculated excluding the financial firms with the highest and the lowest EPS change for each quarter.} Detailed data descriptions are provided in Appendix A. The moving average VAR representation is given by

$$z_t = B(L)u_t,$$

(36)

where $u_t$ is a vector of reduced form errors and $z_t' = [\Delta \Pi_B^t, E5Y_t, spread_t, I_t, y_t, P_t, ffr_t]$. The variable of interest is $\Delta \Pi_B^t$. Based on the identification proposed by Barsky and Sims (2011) we assume bank profitability can be decomposed into two uncorrelated innovations $\varepsilon_t^{current}$ and $\varepsilon_t^{news}$, such that

$$\Delta \Pi_B^t = v(L)\varepsilon_t^{current} + d(L)\varepsilon_t^{news},$$

(37)

where $v(L)$ and $d(L)$ are lag polynomials with the restriction that $d(0) = 0$. This provides the key distinction between the two innovations allowing the second to be related to an anticipated or news component. As standard one can assume there is a relationship between reduced form errors ($u_t$) and fundamental shocks ($\varepsilon_t$), such that $u_t = A\varepsilon_t$, and thus the structural moving average representation of the VAR is given by $z_t = B(L)A\varepsilon_t = C(L)\varepsilon_t$. Matrix $A$ must satisfy $AA' = \Sigma$, where $\Sigma$ is the variance covariance matrix of innovations, and is not unique. Thus, starting from any permissible matrix $A$ (we select a simple Choleski decomposition) one can then obtain an alternative impact matrix $AQ$, where $Q$ is an orthogonal matrix. Using a VAR with bank profitability ordered first, the identification method selects a column $q$ of $Q$ that maximises the FEV of bank profitabi-
Figure 9: Impulse Responses to News on Bank Profitability

Figure 9 presents impulse responses of output, bank profitability, spread and interest rate to the bank profitability news shock. We note that short term rates increase and spreads decline upon impact of the news on the future profitability. The negative co-movement between spreads and expected banking profitability result is the key implication in our theoretical model and is consistent with the microeconometric evidence presented in the previous subsection. We also

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32 See Barsky and Sims (2011) for more details.
33 Confidence bands are calculated as suggested by Kilian (1998).
34 The negative co-movement between spreads and expected banking profitability is robust to extending the sample to include the recent financial crisis (1970Q1-2010Q2). Note that while real output also follows a hump-shaped response

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obtain the share of FEV for each variable that is attributable to the news shocks over a twenty quarters horizon. The news shock explains 5% of the movements in spreads, 23% of variations in investment, around 17% of variations in real GDP, 4% of variations in the CPI and about 33% of variations in consumer expectations. Given that most variations in spreads are attributable to shocks in spreads (75%), the contribution of the news shock to spreads FEV appears to be nontrivial.

A necessary caveat on these results is in order. The decomposition of bank profitability into a contemporaneous and a news component as uncorrelated innovations works well when the variable is exogenous. Bank profits are clearly endogenous in the short-run and hence the identification of the news component might reflect this endogeneity. The strong link between the identified innovation (news) and consumer expectations together with the hump-shaped response of output to this innovation indicate that the bank profitability news might be related to overall news about economic activity. Nonetheless, results clearly reveal a negative correlation between anticipated movements in bank profits and term spreads and corroborate the microeconometric evidence previously presented.

5.3. VAR estimations

In the previous two subsections we have presented statistical evidence of a relation between expected banking profitability at the micro and macro-levels and term spreads. Here, as a final set of empirical support to our model and by utilising a VAR framework, we assess the impact of various shocks on term spreads and other macroeconomic variables at the aggregate level. Our aim is to verify whether the dynamic properties of our theoretical model are in line with their empirical counterparts. Firstly, we look at a case that attempts to capture the effects of an anticipated productivity shock by analysing the impact of a shock on consumer expectations. Secondly, despite the fact that our theoretical channel is forward looking in nature, given that contemporaneous shocks affect the future path of bank profits, we also analyse the dynamic properties of the main economic variables after a monetary and a real output shock.

in this extended sample, it declines upon the impact of the shock and recovers thereafter.
In order to study the impact of monetary, real output and consumer expectations shocks, we follow the identification structure of Christiano et al. (1999). The VAR specification is given by

\[
\mathbf{z}_t = \mathbf{B}(L)\mathbf{u}_t,
\]

where \(\mathbf{z}_t = [\mathbf{x}'_{1t}, \mathbf{w}_t, \mathbf{x}'_{2t}]\), \(\mathbf{w}_t\) is a \((j \times 1)\) vector of the variables of interest, consisting of \([\mathit{ffr}_t, \mathit{y}_t]\) for monetary and real output shocks in the VAR in differences and \([E5Y_t]\) for consumer expectations shocks in the VAR in levels. Note that \(\mathbf{x}_{1t}\) is a \((k_1 \times 1)\) vector with elements whose contemporaneous and lagged values influence the variables of interest at time \(t\) and \(\mathbf{x}_{2t}\) is a \((k_2 \times 1)\) vector with elements whose only values are affected by an innovation to the variables of interest at time \(t\).

Finally, \(\mathbf{u}_t\) is a \((k \times 1)\) vector of reduced form errors with \(k = k_1 + j + k_2\). As before, we are interested in obtaining the impact matrix \(\mathbf{\bar{A}}\) linking reduced form errors \(\mathbf{u}_t\) to fundamental shocks \(\epsilon_t\) such that \(\mathbf{u}_t = \mathbf{\bar{A}}\epsilon_t\). We, thus, assume that \(\mathbf{\bar{A}}\) has a block triangular structure with zero in its upper diagonal as in Christiano et al. (1999).

For our benchmark analysis we assume \(\mathbf{x}_{1t}\) is empty, thus the variable of interest is ordered first, being unaffected by the other variables contemporaneously. For robustness we also estimate the model with all variables in \(\mathbf{x}_{1t}\) and \(\mathbf{x}_{2t}\) empty. Finally, the estimation of the benchmark case assumes a four-lags structure to capture sufficient dynamics in the system. We focus on the impulse responses to a shock on the variable of interest (monetary, real output or consumer expectations) and calculate the one standard error bias-corrected bootstrap confidence bands as suggested by Kilian (1998). Our benchmark specification for all VAR estimation covers the period 1970Q1-2007Q2. As robustness checks we also run estimations for the period of 1970Q1 to 2010Q2 including the recent financial crisis and for the 1985Q1 to 2007Q2 focusing on the period after the Volcker disinflation. Results do not change qualitatively and hence we only report the benchmark estimation results.\(^{35}\)

\(^{35}\)We also estimate the VARs with FBP instead of EPS to check for robustness. The dynamic responses do not change significantly.

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5.3.1. Consumer Expectations Shocks

Beaudry and Portier (2004; 2006) propose business cycle models where consumers become aware of changes in future productivity that is uncorrelated to current productivity. Barsky and Sims (2012) show that these shocks to consumer expectations are not Granger caused by income or consumption and help to predict long lasting variations in consumption and real output. Thus, innovations on consumer expectations can be characterised as noisy measures of changes in expected productivity growth, serving as a proxy to study the effect of news on productivity, which is the focus of this subsection. We follow their contribution closely and set up the VAR with $z_t = [E5Y_t, x_{2t}']$ with $x_{2t}' = [\Delta \Pi^B_t, spread_t, I_t, y_t, P_t, ffr_t]$. Thus, the variable of interest is consumer expectations ($E5Y$) and the variables in $x_{2t}'$ are respectively, financial business profitability ($\Delta \Pi^B$), term spreads ($spread$), real investment ($I$, in logs), real GDP ($y$, in logs), CPI ($P$, in logs) and the Federal Funds rate ($ffr$).

The upper panel of Figure 10 shows impulse responses to shocks to our consumer expectations measure. After a positive consumer expectations shock, spreads respond as the model predicts; they increase on impact and decrease as output increases, reaching their minimum level as output peaks, establishing the countercyclical term spreads result. Financial business profitability as measured by variations in actual earnings per share responds positively followed by a decline and then follows a hump-shaped pattern. As expected, both investment and real output also follow a hump-shaped pattern. Fed Funds rate appears to accommodate the inflationary pressures that are being built up in the real economy. Spreads are very persistent and the variance decomposition of the spread fluctuations shows that 75% of its variation is explained by its own shocks.\textsuperscript{37} Investment and the Federal Funds rate explain around 6% while GDP and bank profits explain around 4% of spreads FEV.

\textsuperscript{36}Barsky and Sims (2012) report similar results in a five variable VAR that studies $E5Y$, consumption, real output, real rates and inflation. We also estimate a VAR model with consumption, instead of investment, results are largely unchanged.

\textsuperscript{37}Results reported are for a five period horizon; the decompositions follow a similar pattern for longer horizons.
Figure 10: Impulse Responses
The empirical impulse responses shown above are closely matched by the ones obtained from our theoretical model after a one year ahead anticipated shock (see Section 3.2). Spreads initially increase, with investment pushing output up. As economic conditions improve with output and bank profits increasing, spreads tend to fall, reaching the lowest point as the economy reaches the peak of the business cycle and converging back to steady state thereafter. As in the data, investment, output and bank profitability follow a hump-shaped pattern.

The effects of an anticipated productivity shock (or more precisely news on TFP) is analysed by Kurmann and Otrok (forthcoming). They find that the main driver of fluctuations in the slope of the term structure is news about future innovations to TFP. They report that term spreads increase after a news shock, while real rates decrease, matching the results presented here. They decompose the responses of the slope of the term structure after a TFP news shock and show that two thirds is explained by movements in expected future short-term rates and one third by time variation in term spreads. They, thus, conclude that monetary policy plays an important role in determining movements in the yield curve. Our theoretical model attempts to highlight a channel through which these movements in term spreads occur in response to news about future economic activity that affect the future path of bank profits. Hence, although acknowledging its importance, our focus is on the additional drivers that are not directly related to monetary policy. The negative correlation observed between expected profitability and the slope of the yield curve obtained in Section 5.2 gives indirect support to the bank portfolio mechanism as one explanation for the observed variation in term spreads.

5.3.2. Monetary Policy Shock

In order to study the impact of monetary and real output shocks, we order the VAR with $z_t' = [ffr_t, \Delta y_t, x_{2t}']$ and with $x_{2t}' = [\Delta \Pi^B_t, spread_t, \Delta I_t, \pi_t]$. These assumptions mean that i) $x_{1t}$ is empty, ii) $ffr$ does not respond contemporaneously to other shocks and real GDP contemporaneously responds only to the Federal Funds rate. We also check for robustness with respect to

38Note that their spread movements are more persistent than the ones reported here, which might indicate that using a TFP series to extract the news component provides a better identification than using a contemporaneous innovation to consumer expectations.

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alternative orderings.

The middle panel in Figure 10 displays impulse responses to a one standard deviation monetary policy shock. We first note that a positive monetary policy shock leads to an instantaneous, statistically significant sharp decline in earnings per share ($\Delta \Pi_B$) as predicted by our model in Section 3.3 and confirming den Haan et al. (2007) findings on the effects of a monetary policy shock on bank equity. Second, spreads immediately decrease significantly, recovering smoothly thereafter. Real investment, output and inflation respond as expected. As a robustness check we also estimate the VAR with an alternative ordering where $x_{2t}$ is empty and all variables other than $ffr_t$ populate $x_{1t}$. We note that our results are not affected by this alternative assumption.

The impulse responses after a monetary shock in our theoretical model show a similar pattern to the empirical ones obtained from the unrestricted VAR. Both output and bank profits fall, with bank profits bouncing back and becoming positive soon after the shock. Although the model is able to account for the fall in spreads at the period when the shock occurs, the empirical results also show that spreads actually fall persistently, while in the theoretical model they become positive after the first period. Recall that the main motivation for looking at financial intermediation as a driver of spread movements was to uncover an additional channel that may work in parallel to the key component that is already identified in the literature, namely, inflation expectations or inflation risk. As Gürkaynak et al. (2010) show monetary policy changes may influence inflation risk and expectations, which after a contractionary intervention tend to fall, depressing term spreads. Adding this element to our model could help in delivering a persistent decline in spreads as observed in the data.

5.3.3. Real Output Shock

The lower panel in Figure 10 shows impulse responses with respect to a positive one standard deviation shock to real output. While we do not attempt to identify the nature of the real output shock (demand or supply) our results suggest that the pattern of empirical impulse responses tends to match the behaviour of the impulse responses with respect to a contemporaneous technology shock as depicted in Section 3.3. We find that spreads initially increase significantly. That is...
followed by a steady decline eventually turning negative and recovering thereafter. The main distinction is that the theoretical model is unable to replicate the amplitude of the fall in spreads after period 3.

Perhaps not surprisingly, a positive real output shock is associated with an instantaneous increase in both real investment and real GDP which takes about 8 quarters to stabilise while inflation declines. As a robustness check to the ordering of variables, we alternatively assume that i) variables other than $\Delta y_t$ are populating $x_{1t}$, thus $x_{2t}$ is empty and ii) $ffr_t$ is ordered right after $\Delta y_t$ instead of being before. Our qualitative results are not affected significantly by these alternative assumptions. Finally, we observe that more than 90% of the FEV in spreads in the VAR in differences (which was used to identify both monetary and real output shocks) is explained by the short term interest rate and the spread’s own shocks. Each of the other variables (GDP, investment, inflation and bank profitability) account individually for less then 3% of the fluctuations.

Overall, the empirical impulse responses of spreads are more persistent than the ones from the theoretical model after contemporaneous shocks. The main reason is that in our model asset prices move sharply in the current period returning very smoothly back to the steady state. We modify the theoretical model in two ways to explore its ability to generate more persistent spread movements. Firstly, we assume long-term assets lasting for four periods and secondly, we allow for variable capital utilisation.\(^{39}\) In both cases we obtain more persistent spread movements. The main downside is that spreads are then less volatile to news shocks since asset prices do not move as sharply. We conjecture that other mechanisms that generate greater propagation in asset prices during the subsequent periods after a shock (e.g. incorporating learning) may lead to more persistent spread responses as they potentially have greater impact on the future path of bank profits.

\(^{39}\)Results for these model extensions are available from the authors upon request.
6. Conclusions

Fluctuations in term spreads have relevant implications for macroeconomic outcomes and may predict output growth. Undoubtedly, inflation expectations or more generally long-term inflation risks are important determinants of these fluctuations. However, the observation that nominal and real yield curves move together in many instances suggests that other factors are in play. We propose a model that delivers endogenous variations in term spreads driven primarily by changes in banks’ portfolio decision and their appetite to bear the risk of maturity transformation. We show that fluctuations in future profitability of banks’ portfolio affect their ability to cover for any liquidity needs and hence influence the premium they require to carry maturity risk.

While we present a model in which bank portfolios are fairly simple, we are able to match important features of the data. Our model suggests that factors predominantly external to monetary policy may contribute not only to the marginal predictive power of spreads but also to the understanding of the linkages between banks, spread movements and the macroeconomy.

Embedding this banking sector framework into a DSGE model allows us to analyse the interaction between spread movements and unconventional policies. Unconventional policies are shown to have a strong impact on spread movements fomenting long-term investment and helping reduce output losses after negative liquidity shocks, matching the general view on the effects of recent asset purchases programmes. Next, we show that asset purchases programmes that keep the assets in the central bank’s balance sheet for longer are more effective in offsetting a liquidity shock. These policy measures allow the central bank to restore short-term rates to steady state levels more quickly. This result supports the ECB’s decision to intervene in the market and hold the assets purchased under the CBPP programme until maturity. Finally, we complement the theoretical analysis with empirical evidence confirming the link between expected banking profitability and terms spreads.

Bank portfolio choices and balance sheet conditions matter in determining risk premia in the macroeconomy. Further research looking at constraints on risk taking and trading of instruments that allows banks to manage balance sheet exposures such as asset backed securities and collatera-
lised funding may improve our understanding of their importance. More specifically, the present work highlights at least three areas in which future projects, exploring the role of bank’s portfolio decisions in affecting risk premia and the macroeconomy, may be fruitful. First, increasing the complexity of banks’ portfolios will provide a better understanding of this important channel, most notably, including (workers) housing investment funded by financial intermediation. That would mean fluctuations in term spreads would not only influence investment but also consumption, potentially amplifying the effects of spread movements, since as we observe, consumption and investment move in opposite directions compensating each other. Moreover, including other long-term asset classes may potentially allow us to study portfolio balancing effects after QE interventions. Second, making liquidity shortages endogenous based on the potential for securitisation of long-term assets may be crucial to fully understand those factors behind spread movements and their marginal predictive power. Finally, final investors’ (after securitisation) and banks’ risk assessment could also be time varying affecting the linkage between long-term funding risks and economic activity.

Appendix A. Data

This Appendix provides a description of the data used in the empirical study.

- Treasury Bill Rate (Units: Percent per Annum), (Series ID: 60C..ZF ), Source: International Financial Statistics/IMF.
- Federal Funds Effective Rate (H15/H15/RFSPFF_N.M), Source: US Federal Reserve.
- Government Bond Yield: 10 year (Units: Percent per Annum), (Series ID: 61...ZF ), Source: International Financial Statistics/IMF.
- CPI All Items City Average (Units: Index Number), (Series ID: 64...ZF ), Source: International Financial Statistics/IMF.
- Real Gross Domestic Product, Seasonally Adjusted Annual Rate , (Series ID: GDPC96), Source: US Department of Commerce: Bureau of Economic Analysis.

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- Gross Private Domestic Investment, Seasonally Adjusted at Annual Rates, (Table 5.1.),
  Source: Bureau of Economic Analysis.

- Financial Business; undistributed corporate profits excluding CCAdj, (FOF Code: FA796006403.Q),
  Source: Flow of Funds Accounts, Board of Governors of the Federal Reserve.

- Earnings Per Share (Basic) - Excluding Extraordinary Items, Source: Compustat.

- Expected Earnings Per Share, Meanest, Source: Thomson/Reuters I/B/E/S.

- E5Y, Michigan Index of Consumer Expectations.

Appendix B. Equilibrium Conditions and Steady State

The household maximisation routines yield the following equilibrium conditions

$$\beta E_t \left( \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right) = \frac{C_t^{-\sigma}}{R_t CB}$$

(B.1)

and

$$w_{jt} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \left[ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega_t \beta)^{s} \frac{H_{t+s}}{C_{t+s}} H_{t+s} \right]}{E_t \left[ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega_t \beta)^{s} H_{t+s} \left( \prod_{k=1}^{s} \pi_{t+k} \right)^{-1} \right]}.$$  

(B.2)

This equation can be conveniently expressed in recursive form as such

$$0 = f_{1,t}^w \varepsilon_w - f_{2,t}^w w_{jt},$$

$$f_{1,t}^w = H_t \frac{\chi H_t^0}{C_t^{-\sigma}} + E_t \left[ \beta \omega_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} f_{1,t+1}^w \right],$$

$$f_{2,t}^w = H_t + E_t \left[ \beta \omega_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1} \frac{f_{2,t+1}^w}{\pi_{t+1} f_{2,t+1}} \right],$$

where, $w_{jt} = W_{jt}/P_t$ and $w_t = W_t/P_t$, is given by

$$w_t^{1-\varepsilon_w} = (1 - \omega_t) w_{jt}^{1-\varepsilon_w} + \omega_t w_{t-1}^{- \varepsilon_w}. \quad (B.3)$$

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We assume intermediate firms discount future payoffs using the household’s stochastic discount factor given by

\[ Q_{\ell,t+1} = \beta E_t \left( \frac{C_{\ell,t+1}^{-\sigma}}{\pi_{t+1}C_t^{-\sigma}} \right) = \frac{1}{R_{CB,t}}. \]

Given that the purpose of our analysis is not to look at the effects of firm-specific capital we assume that there exists a capital market within firms. That way all intermediate firms will have the same labour-capital ratio and \( \Lambda_{t,i} = \Lambda_t \) for all \( i \), as in the case where a capital rental market is available. The net aggregate investment in (new) capital is then acquired from entrepreneurs. Note that, as shown by Sveen and Weinke (2007), the relevant difference of considering firm-specific capital is that the parameter on the marginal cost in the Phillips curve would be lower, increasing effective price stickiness. Our results are not qualitatively affected by this change.

Based on that, \( p_{i,t} \) is determined by solving the price setting maximisation, substituting for the stochastic discount factor and using \( \Lambda_{t+s,i} = \Lambda_{t+s} \). That gives

\[
p_{i,t} = \frac{1}{\epsilon} E_t \left[ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega \beta)^s \Lambda_{t+s} Y_{t+s} \left( \prod_{k=1}^{s} \pi_{t+k} \right)^\epsilon \right]. \tag{B.4}
\]

The recursive formulation is given by

\[
0 = f_{1,t} \frac{\epsilon}{\epsilon - 1} - f_{2,t} p_{i,t}, \\
f_{1,t} = Y_t \Lambda_t + E_t \left[ \beta \omega \frac{C_{t+1}^{-\sigma} \pi_{t+1}^{-1} f_{1,t+1}}{C_t^{-\sigma} \pi_{t+1} f_{2,t+1}} \right], \\
f_{2,t} = Y_t + E_t \left[ \beta \omega \frac{C_{t+1}^{-\sigma} \pi_{t+1}^{-1} f_{2,t+1}}{C_t^{-\sigma} \pi_{t+1} f_{1,t+1}} \right],
\]

where, \( p_{i,t} = P_{i,t} / P_t \) and \( \pi_t = P_t / P_{t-1} \), is given by

\[
1 = (1 - \omega)p_{1,t}^{1-\epsilon} + \omega \pi_{t}^{\epsilon-1}. \tag{B.5}
\]

From the firm cost minimisation problem we obtain the demand for capital and labour. After rearranging the first order conditions and substituting for the stochastic discount factor \( Q_{\ell,t+1} \), we
obtain the following equilibrium conditions

\[
Y_t = A_t K_t^{S(1-\zeta)a} K_t^{L(1-\eta)} H_t^{1-\alpha}, \quad (B.6)
\]

\[
\Lambda_t = \frac{w_t H_t}{Y_t(1-\alpha)}, \quad (B.7)
\]

\[
q^S_t = \beta E_t \left\{ \frac{C_{i+1}^{S^\alpha}}{\pi_{i+1} C_t} \left[ \Lambda_{t+1} \frac{\alpha \zeta Y_{i+1}}{K_t^S} + (1-\delta) q^S_{i+1} \right] \right\}, \quad (B.8)
\]

\[
q^L_t = \beta E_t \left\{ \frac{C_{i+1}^{L^\alpha}}{\pi_{i+1} C_t} \left[ \Lambda_{t+1} \frac{\alpha(1-\zeta) Y_{i+1}}{K_t^L} + (1-\delta) q^L_{i+1} \right] \right\}. \quad (B.9)
\]

Using the capital aggregation conditions, investment evolves according to

\[
I^S_t = K^S_{t+1} - (1-\delta) K^S_t, \quad (B.10)
\]

\[
I^L_t = K^L_{t+1} - (1-\delta) K^L_t. \quad (B.11)
\]

From entrepreneurs maximisation problems we obtain

\[
X^S_{S,t} = \frac{\gamma S E_t(q^S_{t+1} \pi_{t+1})}{R^S_{S,t}} - 1, \quad (B.12)
\]

\[
X^S_{L,t} = \frac{\gamma L E_t(q^L_{t+1} \pi_{t+1} \pi_{t+2})}{R^L_{L,t}} - 1. \quad (B.13)
\]

Finally, from the bank maximisation problem we have that

\[
R^S_{S,t} = R_{t+1,CB}, \quad \text{and} \quad (B.14)
\]

\[
E_t \left[ \Pi_{t+1}^{B \sigma} \left( \frac{R_{t+1,CB} - 1 - \Lambda V_{t+1} }{\pi_{t+1}} + \xi_{t+1} \right) \right] = E_t \left[ \frac{\beta \Pi_{t+2}^{B \sigma} (R_{L,t} - R_{t+1,CB})}{\pi_{t+1} \pi_{t+2}} \right]. \quad (B.15)
\]

where

\[40\]Once again we have used the fact that marginal costs are the same across intermediate firms.

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\[
\Pi_t^B = \frac{TR_t^E}{P_t} + \frac{(R_t-2L-1)X_{L,t-2} + (R_t-1, CB-1)}{\pi_t} \left( X_{S,t-1} - \frac{D_{t-1}}{P_{t-1}} \right) - q_tX_{L,t-1} + \frac{X_{L,t-1} \Delta V_{t-1}}{\pi_t}, \tag{B.16}
\]

\[
\frac{TR_t^E}{P_t} = Y_t - w_tH_t - q_t^I I_t^S - q_t^L I_t^L
\]
\[+ q_t^S K_{t+1}^S + q_t^L K_{t+1}^L - \frac{q_{t-1}^S K_t^S + q_{t-1}^L K_t^L}{\pi_t}, \tag{B.17}
\]

\[
\frac{D_t}{P_t} = X_{S,t} + X_{L,t} + \frac{(q_{t-1}^S K_t^S + q_{t-1}^L K_t^L)}{\pi_t} + \frac{X_{L,t-1}}{\pi_t}, \text{ and} \tag{B.18}
\]

\[
\Delta V_{t+1,t} = \frac{R_{t+1} - E_t[Q_{t+1}]}{R_{S,t+1}}. \tag{B.19}
\]

We define the term spreads (annual rate in percentage points) between long and short-term rate as
\[
tp_t = \frac{1}{2} [(R_{L,t} - 1) - (R_{t,CB} - 1) - (R_{t+1,CB} - 1)] \cdot 400. \tag{B.20}
\]

Finally, the central bank sets monetary policy according to
\[
\frac{R_{t,CB}}{R_{CB}} = \left[ \frac{\pi_t \xi (Y_t)^{\xi_y}}{\bar{X}} \right], \tag{B.21}
\]

where \( \bar{X} \) is the steady state value of \( X_t \). This is the standard monetary rule whereby the base rate responds to deviations in inflation and output. The recursive equilibrium is determined as the solution to equations (22)-(24) and (B.1) - (B.21).

**Steady State** From pricing equation (normalising prices at steady state to 1) we have that
\[
\Lambda = \frac{\epsilon - 1}{\epsilon}. \tag{B.22}
\]
From wage pricing equation we have that

\[ \tilde{w} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\chi H}{C^{1-\sigma}}. \]  

(B.23)

From the intermediate firm problem we have that

\[ \Lambda = \frac{\tilde{w} \tilde{H}}{\tilde{Y}(1 - \alpha)}; \]  

(B.24)

\[ \bar{q}_s = \frac{\beta \Lambda \alpha \zeta \tilde{Y}}{K^s [1 - \beta (1 - \delta)]}, \]  

and

(B.25)

\[ \bar{q}_L = \frac{\beta \Lambda \alpha (1 - \zeta) \tilde{Y}}{K^L [1 - \beta (1 - \delta)]}. \]  

(B.26)

From entrepreneurs problems we have that

\[ \bar{X}_s = \gamma_s \beta \bar{q}_s - 1, \]  

and

(B.27)

\[ \bar{X}_L = \gamma_L \bar{q}_L - 1. \]  

(B.28)

From the bank problem we have that

\[ \left( \frac{1}{\beta} - 1 \right) + \varphi = \beta \left( R_L - \frac{1}{\beta} \right) \]  
or  \[ R_L = \frac{1}{\beta^2} + \frac{\varphi}{\beta}. \]  

(B.29)

The term spread at the steady state is given by

\[ \tilde{r}_p = \frac{1}{2} (R_L - 1) - (1/\beta - 1) - (1/\beta - 1.400). \]  

(B.30)
Clearing conditions and investment flow equation determine that

\[ Y = \bar{C} + \bar{X}_S + (1 + \rho)\bar{X}_L, \]  
(B.31)

\[ \tilde{Y} = \tilde{H}^{1-\omega} \bar{K}_S^{\alpha \xi} \bar{K}_L^{\alpha(1-\zeta)}, \]  
(B.32)

\[ \delta \bar{K}_S = \gamma_S \ln(1 + \bar{X}_S), \text{ and} \]  
(B.33)

\[ \delta \bar{K}_L = \gamma_L \ln(1 + \bar{X}_L). \]  
(B.34)

**Unconventional Monetary Policy - Benchmark Model**

The bank’s problem under the unconventional monetary policy with Repo of Long-term Assets is

\[
\max_{\{X_{S,t}, X_{L,t}, \Theta_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t \frac{\Pi_t^{B1-\sigma_B}}{1 - \sigma_B}
\]
conditioned on

\[
\text{s.t. } D_t = P_t X_{S,t} + P_t X_{L,t} + Z_t + P_{t-1} X_{L,t-1} - \Theta_t X_{L,t-1} P_{t-1}
\]

where

\[
\Pi_{t+1}^B = \frac{1}{P_{t+1}} [TR_{t+1}^E + (R_{L,t-1} - 1)P_{t-1} X_{L,t-1} + (R_{S,t} - 1)P_t X_{S,t} - D_t(R_{S,t-1} - 1) -
- \varrho_t X_{L,t} P_{t+1} + X_{L,t} P_t \Delta V_{t+1,t} - \Theta_t X_{L,t-1} P_{t-1}(R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_t^2].
\]

(B.36)

The first order conditions in this case are

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Appendix C. Second Order Approximation of Long-term Rate Decision

The bank equilibrium condition is given by

\[ E_t \left\{ \Pi_{t+1}^B \right\} \approx E_t \left\{ -\sigma_b \Pi_{t+1}^B \right\} \approx \beta E_t \left\{ \Pi_{t+1}^B \right\} \approx \beta E_t \left\{ \frac{\Pi_{t+2}^B}{\pi_{t+1} \pi_{t+2}} (R_{L,t} - R_{CB,t+1}) \right\} \]  

(C.1)

and at the steady state

\[ \left( \frac{1}{\beta + \bar{\sigma}} \right) \frac{1}{\beta} = R_L \text{ and } R_{CB} = \frac{1}{\beta} \]  

(C.2)

Let \( W_1 = (R_{CB,t} - 1 - \Delta V_{t+1,t} + \tilde{\eta}_{t+1}) \) and \( W_2 = (R_{L,t} - R_{CB,t+1}). \)

**Approximation of Left-Hand Side (LHS)**

\[ E_t \left\{ \frac{\Pi_{t+1}^B}{\pi_{t+1}} (R_{CB,t} - 1 - \Delta V_{t+1,t} + \tilde{\eta}_{t+1}) \right\} \approx \beta E_t \left\{ \frac{\Pi_{t+2}^B}{\pi_{t+1} \pi_{t+2}} (R_{L,t} - R_{CB,t+1}) \right\} \]

\[ E_t \left\{ \frac{\Pi_{t+1}^B}{\pi_{t+1}} \right\} \approx \beta E_t \left\{ \frac{\Pi_{t+2}^B}{\pi_{t+1} \pi_{t+2}} \right\} \approx \beta E_t \left\{ \frac{\Pi_{t+2}^B}{\pi_{t+1} \pi_{t+2}} (R_{L,t} - R_{CB,t+1}) \right\} \]

(C.3)

41 Note that \( \tilde{\sigma}_t = \sigma_t * \pi_{t+1}. \) For analytical simplicity we look at deviations of \( \tilde{\sigma}_t. \) This do not alter the intuition of the results.

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From the definition of $W_1$

$$W_1 = \frac{1}{\beta \Gamma} R_{CB,J} - \frac{1}{\beta \Gamma} \Delta V_{r+1,t} + \frac{\rho}{\Gamma} \delta_{r+1}, \text{ where } \Gamma = \left( \frac{1}{\beta} - 1 + \tilde{\rho} \right) \text{, and}$$

$$\bar{w}_{1} + 0.5 \left( \bar{w}_1 \right)^2 = \frac{1}{\beta \Gamma} \left[ R_{CB} + 0.5 \left( R_{CB} \right)^2 \right] + \frac{\rho}{\Gamma} \left[ \Delta V_{r+1,t} + 0.5 \left( \Delta V_{r+1,t} \right)^2 \right] - \frac{1}{\beta \Gamma} \left[ \Delta V_{r+1,t} + 0.5 \left( \Delta V_{r+1,t} \right)^2 \right] \quad (C.7)$$

Hence, LHS becomes

$$E_t \left\{ \begin{array}{l}
-\sigma_b \left( \bar{W}_1^0, \delta W_1^0 \right) + 0.5 \sigma_b \left( \bar{W}_1^0 \right)^2 + \frac{\rho}{\Gamma} \left[ R_{CB} + 0.5 \left( R_{CB} \right)^2 \right] + \frac{\rho}{\Gamma} \left[ \Delta V_{r+1,t} + 0.5 \left( \Delta V_{r+1,t} \right)^2 \right] \\
-\bar{V}_{r+1} + 0.5 \left( \bar{V}_{r+1} \right)^2 + \sigma_b \left( \bar{V}_{r+1} \right)^2 - \frac{1}{\beta \Gamma} \left[ \Delta V_{r+1,t} + 0.5 \left( \Delta V_{r+1,t} \right)^2 \right] \\
-\sigma_b \left( \bar{W}_1^0 - \frac{\rho}{\Gamma} \Delta V_{r+1,t} + \frac{\rho}{\Gamma} \Delta V_{r+1,t} \right) - \left\{ \frac{\rho}{\Gamma} \left( R_{CB} - \frac{\rho}{\Gamma} \Delta V_{r+1,t} + \frac{\rho}{\Gamma} \Delta V_{r+1,t} \right) \right\} \\
\end{array} \right\} \quad (C.5)$$

Approximation of Right-Hand Side (RHS)

$$\beta E_t \frac{1}{\pi_{r+2} \pi_{r+2}} \left[ \left( \bar{W}_2 \right)^{-\sigma_b} \left( R_{L,t} - R_{CB,t+1} \right) \right] =$$

$$\beta E_t \frac{1}{\pi_{r+2} \pi_{r+2}} \left[ \left( \bar{W}_2 \right)^{-\sigma_b} \left( R_{L,t} - R_{CB,t+1} \right) \right] \approx$$

$$E_t \left\{ \begin{array}{l}
-\sigma_b \left( \bar{W}_2^0, \delta \bar{W}_2^0 \right) + 0.5 \sigma_b \left( \bar{W}_2^0 \right)^2 + \bar{W}_2 + 0.5 \left( \bar{W}_2 \right)^2 - \bar{V}_{r+1} + 0.5 \left( \bar{V}_{r+1} \right)^2 - \bar{V}_{r+1} + 0.5 \left( \bar{V}_{r+1} \right)^2 \\
-\sigma_b \left( \bar{W}_2^0, \delta \bar{W}_2^0 \right) + 0.5 \sigma_b \left( \bar{W}_2^0 \right)^2 + \bar{W}_2 - \bar{V}_{r+1} + 0.5 \left( \bar{V}_{r+1} \right)^2 - \bar{V}_{r+1} + 0.5 \left( \bar{V}_{r+1} \right)^2 \\
\end{array} \right\} \quad (C.6)$$

From the definition of $W_2$

$$\bar{W}_2 = \left( \frac{\beta + \rho}{\Gamma} R_{L,t} - \frac{1}{\beta \Gamma} R_{CB,t+1} \right) \text{ where } \Gamma = \left( \frac{1}{\beta} - 1 + \tilde{\rho} \right) \text{ and}$$

$$\bar{W}_2 + 0.5 \left( \bar{W}_2 \right)^2 = \frac{\beta + \tilde{\rho}}{\Gamma} \left[ R_{L,t} + 0.5 \left( R_{L,t} \right)^2 \right] - \frac{1}{\beta \Gamma} \left[ R_{CB,t+1} + 0.5 \left( R_{CB,t+1} \right)^2 \right] \quad (C.7)$$

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Hence, RHS becomes
\[
E_t \left( \frac{\bar{\rho} + 0.5\bar{\rho}^2}{2} \right) = E_t \left[ \sigma_\beta \left( \tilde{\Pi}_{\gamma+2} - \tilde{\Pi}_{\gamma+1} \right)^2 + 0.5\sigma_\beta \left( \tilde{\Pi}_{\gamma+2} - \tilde{\Pi}_{\gamma+1} \right)^2 \right] + E_t \left[ \frac{1}{\tilde{\rho}} \left( R_{t+1} - \tilde{\rho} \bar{R}_{t+1} \right) \right] + E_t \left[ \frac{1}{\tilde{\rho}} \left( \rho_{t+1} + 0.5 \rho_{t+1}^2 \right) \right]
\]

From the definition of term premium we have that \( tp = 0.5(R_{t+1} - \rho_{t+1}) + 1 \), hence
\[
E_t \left( \frac{\bar{\rho} + 0.5\bar{\rho}^2}{2} \right) = E_t \left[ \sigma_\beta \left( \tilde{\Pi}_{\gamma+2} - \tilde{\Pi}_{\gamma+1} \right)^2 + 0.5\sigma_\beta \left( \tilde{\Pi}_{\gamma+2} - \tilde{\Pi}_{\gamma+1} \right)^2 \right] + E_t \left[ \frac{1}{\tilde{\rho}} \left( R_{t+1} - \tilde{\rho} \bar{R}_{t+1} \right) \right] + E_t \left[ \frac{1}{\tilde{\rho}} \left( \rho_{t+1} + 0.5 \rho_{t+1}^2 \right) \right].
\]

We can now combine the LHS and RHS to get
\[
E_t \left[ \frac{\bar{\rho} + 0.5\bar{\rho}^2}{2} \right] = E_t \left[ \sigma_\beta \left( \tilde{\Pi}_{\gamma+2} - \tilde{\Pi}_{\gamma+1} \right)^2 + 0.5\sigma_\beta \left( \tilde{\Pi}_{\gamma+2} - \tilde{\Pi}_{\gamma+1} \right)^2 \right] + E_t \left[ \frac{1}{\tilde{\rho}} \left( R_{t+1} - \tilde{\rho} \bar{R}_{t+1} \right) \right] + E_t \left[ \frac{1}{\tilde{\rho}} \left( \rho_{t+1} + 0.5 \rho_{t+1}^2 \right) \right] + CovTerms + \Xi
\]

where,
\[
CovTerms = -\sigma_\beta \delta \left( \tilde{\Pi}_{\gamma+1} - \tilde{\Pi}_{\gamma+2} \right) \left( R_{t+1} - \bar{R}_{t+1} \right) + \sigma_\beta \delta \left( \tilde{\Pi}_{\gamma+1} - \tilde{\Pi}_{\gamma+2} \right) \left( \rho_{t+1} + 0.5 \rho_{t+1}^2 \right)
\]

\[
\Xi = \rho_{t+2} - 0.5 \rho_{t+2}^2 - \frac{1}{\tilde{\rho}} \left( \rho_{t+1} + 0.5 \rho_{t+1}^2 \right). \tag{C.10}
\]

### Appendix D. Long-term Investment with 1Y maturity

If we assume long-term investments are done at period \( t \) but mature at \( t + 4 \) then \( X_{L,t} \) becomes
\[
X_{L,t} = \frac{\gamma L E_t [q_{L+4} \pi_{t+4} \pi_{t+2} \pi_{t+3} \pi_{t+4}]}{R_{L,t}} - 1. \tag{D.1}
\]

Note that the approximated signed is also used here since the denominator should be \( \left( \frac{1}{\beta} - 1 + \rho \right) + \beta - 1 \) and not \( \gamma = \left( \frac{1}{\beta} - \rho \right) \).
And the long-term rate is set such that

\[ \beta^2 E_t \left[ \frac{\Pi_{t+1}^B - \sigma (R_{t-4,L} - R_{t+3,CB})}{\pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}} \right] = E_t \left[ \frac{\Pi_{t+1}^B - \sigma (R_{t-1,CB} - 1 - \Delta V_{t+1,t} + q_{t+1})}{\pi_{t+1}} \right] + \beta E_t \left[ \frac{\Pi_{t+2}^B - \sigma (R_{t+1,CB} - 1 - \Delta V_{t+2,t} + q_{t+2})}{\pi_{t+1} \pi_{t+2}} \right] + \beta^2 E_t \left[ \frac{\Pi_{t+3}^B - \sigma (R_{t+2,CB} - 1 - \Delta V_{t+3,t} + q_{t+3})}{\pi_{t+1} \pi_{t+2} \pi_{t+3}} \right]. \] (D.2)

Where

\[ \Pi_t^B = t_{t}^E + \frac{(R_{t-4,L} - 1)}{\pi_{t+1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_{t}} + \frac{-q_t(X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + X_{L,t-1} \Delta V_{t-1,t-1} + X_{L,t-2} \Delta V_{t-2,t-2}}{\pi_{t}} + \frac{X_{L,t-3} \Delta V_{t-3,t-3}}{\pi_{t+1} \pi_{t-2}}, \] (D.3)

\[ d_t = X_{S,t} + X_{L,t} + \frac{X_{L,t-1}}{\pi_{t}} + \frac{X_{L,t-2}}{\pi_{t+1} \pi_{t-1}} + \frac{X_{L,t-3}}{\pi_{t+1} \pi_{t-2}} + z_t, \] (D.4)

\[ \Delta V_{t+1,t} = \frac{\left[ t_{t+1} f_{t+4} - E[q_{t+1}/(\beta^2) + q_{t+2}/(\beta) + q_{t+3}] \right]}{R_{t+1,CB} R_{t+2,CB} R_{t+3,CB}} - 1, \] (D.5)

\[ \Delta V_{t+2,t} = \frac{\left[ t_{t+2} f_{t+4} - \beta E[q_{t+1}/(\beta^2) + q_{t+2}/(\beta) + q_{t+3}] \right]}{R_{t+2,CB} R_{t+3,CB}} - 1, \] (D.6)

\[ \Delta V_{t+3,t} = \frac{\left[ t_{t+3} f_{t+4} - \beta^2 E[q_{t+1}/(\beta^2) + q_{t+2}/(\beta) + q_{t+3}] \right]}{R_{t+3,CB}} - 1. \] (D.7)

We define the term premium (annual rate in percentage points) between long and short-term rate as

\[ t_p = \frac{1}{4} (R_{L,t} - R_{t,CB} + R_{t+1,CB} - R_{t+2,CB} - R_{t+3,CB} + 3) \times 400. \] (D.8)

Finally, the good market clearing condition is

\[ Y_t = C_t + X_{S,t} + X_{L,t} + q_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}). \] (D.9)

**Short-term asset purchase agreements**

We assume banks can only repo the long-term asset that is about to mature.

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Profits and deposits are given by

\[
\Pi_t^B = \rho_t^E + \frac{(R_{t-4,L} - 1)}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} - q_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + \frac{\Theta_{t-1} X_{L,t-4}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} (R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_{t-1}^2
\]

\[
+ \frac{X_{L,t-1} \Delta V'_{L,t-1}}{\pi_t} + \frac{X_{L,t-2} \Delta V'_{L,t-2}}{\pi_t \pi_{t-1}} + \frac{X_{L,t-3} \Delta V'_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}}, \quad \text{and}
\]

\[
d_t = X_{S,t} + X_{L,t} + \frac{X_{L,t-1}}{\pi_t} + \frac{X_{L,t-2}}{\pi_t \pi_{t-1}} + \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}} + z_t - \Theta_t \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}}. \tag{D.11}
\]

Which implies

\[
\Pi_{t+1}^{B \rightarrow B} \left( (R_{C,B,t} - R_{QE,t}) \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} - \phi_{QE} \Theta_t \right) = 0. \tag{D.12}
\]

\[
\beta^3 \mathbb{E} \left[ \Pi_{t+1}^{B \rightarrow B} \left( (R_{C,B,t} - R_{QE,t}) \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} - \phi_{QE} \Theta_t \right) \rho_{t+1} \right] = \mathbb{E} \left[ \Pi_{t+1}^{B \rightarrow B} \left( (R_{C,B,t} - R_{QE,t}) \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} - \phi_{QE} \Theta_t \right) \rho_{t+1} \right]^+. \tag{D.13}
\]

**Long-term asset purchase agreements**

We assume banks can sell the long-term asset with the longest maturity and buy back before maturity.

Profits and deposits are given by

\[
D_t = P_t X_{S,t} + P_t X_{L,t} + P_{t-1} X_{L,t-1} + P_{t-2} X_{L,t-2} + P_{t-3} X_{L,t-3} + Z_t
\]

\[
-\Theta_t P_{t-1} X_{L,t-1} - \Theta_t P_{t-2} X_{L,t-2} - \Theta_t P_{t-3} X_{L,t-3}, \quad \text{and}
\]

\[
\Pi_t^B = \rho_t^E + \frac{(R_{t-4,L} - 1)}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} - q_t (X_{L,t-1} + X_{L,t-2} + (1 - \Theta_{t-1}) X_{L,t-3})
\]

\[
+ \frac{\Theta_{t-3} X_{L,t-4}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} (R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_{t-3}^2
\]

\[
+ \frac{X_{L,t-1} \Delta V'_{L,t-1}}{\pi_t} + \frac{(1 - \Theta_{t-1}) X_{L,t-2} \Delta V'_{L,t-2}}{\pi_t \pi_{t-1}} + \frac{(1 - \Theta_{t-2}) X_{L,t-3} \Delta V'_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}}, \tag{D.14}
\]

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That implies

\[
\beta^3 E_i \left\{ \Pi_{t+3}^{-\sigma_B} \left( R_{CB,t-3} - \frac{\sigma_B}{\sigma_B} R_{CB,t-3} + \frac{\rho}{\rho} R_{CB,t-3} - \frac{\theta_{QE}}{\theta_{QE}} \right) \right\} = E_i \left\{ \Pi_{t+1}^{-\sigma_B} \left( R_{CB,t-1} - \frac{\sigma_B}{\sigma_B} R_{CB,t-1} + \frac{\rho}{\rho} R_{CB,t-1} - \frac{\theta_{QE}}{\theta_{QE}} \right) \right\} 
\]

\[
+ \beta E_i \left\{ \Pi_{t+2}^{-\sigma_B} (1 - \theta_{t+1}) \left( R_{CB,t+2} - \frac{\sigma_B}{\sigma_B} R_{CB,t+2} + \frac{\rho}{\rho} R_{CB,t+2} - \frac{\theta_{QE}}{\theta_{QE}} \right) \right\} 
\]

\[
+ \beta^2 E_i \left\{ \Pi_{t+3}^{-\sigma_B} (1 - \theta_{t+1}) \left( R_{CB,t+3} - \frac{\sigma_B}{\sigma_B} R_{CB,t+3} + \frac{\rho}{\rho} R_{CB,t+3} - \frac{\theta_{QE}}{\theta_{QE}} \right) \right\} \quad (D.16)
\]

\[
\Pi_{t+1}^{-\sigma_B} \left( R_{CB,t-1} - \frac{\sigma_B}{\sigma_B} R_{CB,t-1} + \frac{\rho}{\rho} R_{CB,t-1} - \frac{\theta_{QE}}{\theta_{QE}} \right) + \Pi_{t+2}^{-\sigma_B} \left( R_{CB,t-1} - \frac{\sigma_B}{\sigma_B} R_{CB,t-1} + \frac{\rho}{\rho} R_{CB,t-1} - \frac{\theta_{QE}}{\theta_{QE}} \right) + \Pi_{t+3}^{-\sigma_B} \left( R_{CB,t-1} - \frac{\sigma_B}{\sigma_B} R_{CB,t-1} + \frac{\rho}{\rho} R_{CB,t-1} - \frac{\theta_{QE}}{\theta_{QE}} \right) = 0. \quad (D.17)
\]

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References


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