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Exchange Rates and Fundamentals: Is there a Role for Nonlinearities in Real Time?

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Abstract

We examine the predictive power of real time linear monetary models with possible nonlinear adjustment in forecast errors for the GBP/USD exchange rates. Real time revisions of UK and US monetary aggregates and output are significant; therefore the use of final data on fundamentals in forecasting exchange rates may yield misleading inferences. By studying recursive forecast errors we claim that in several instances, real time fundamental equilibrium values of exchange rates may be determined in a linear fashion, whereas the adjustment towards fundamentals driven equilibrium values may take a discrete or smooth nonlinear form. Revisions in fundamentals, particularly in the US and UK monetary aggregates and real output, seem to matter mainly for short term forecastability of exchange rates. We find short term forecastability in the form of discrete nonlinear adjustment in some real time vintages. We also document long term forecastability in the form of a smooth nonlinear adjustment towards fundamentals determined equilibrium values of exchange rates.

JEL Classifications: F31, F37.

Keywords: Monetary model, exchange rates, nonlinear adjustment, real time, unit roots, forecasting.

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1 Introduction

One of the most frequently studied puzzles in international monetary economics is the failure of standard linear monetary models of exchange rates to forecast variations in the exchange rates in the short run. Ever since Meese and Rogoff's (1983) work on out-of-sample forecast comparison of varieties of monetary models of exchange rates and naïve random walk model, a consensus view has emerged that monetary models are largely unsuccessful in forecasting exchange rates, at least in the short term. This literature casts doubts about the suitability of economic models based on fundamentals in forecasting exchange rates (see Cheung and Chinn, 2001, or Marsh, Cheung and Chinn, 2004) for evidence based on surveys).

The work of Mark (1995) revived interest in monetary models by focusing on long-term predictability of exchange rates. From this perspective, models based on fundamentals are essentially valid in the long run. That means there is a tendency in the exchange rates to adjust to their long-term values as suggested by the fundamentals. With the use of nonparametric bootstrapping methods he was able to show that monetary models with linear mean reversion are of better use in predicting exchange rates in long horizons than in short horizons. He found some out-of-sample predictability for Japanese Yen, German Mark and Swiss Frank exchange rates vis-à-vis US Dollar at 12 and 16 quarters forecast horizons.

Mark's (1995) work has been subject to criticism on several grounds. Firstly, Berkowitz and Giorgianni (2001) argue that the distribution of the bootstrap test statistic as implemented by Mark depends on the assumption of cointegration between the fundamentals and exchange rates. Given that Mark assumes cointegration between fundamentals and exchange rates to generate bootstrap critical values, if fundamentals and exchange rates are not cointegrated in actual data, critical values and therefore inference from the test would be incorrect. Berkowitz and Giorgianni report very weak evidence of cointegration in the data which is corroborated by Kilian (1999). Kilian finds that even if there is cointegration between fundamentals and exchange rates mean reversion in forecast errors is very slow. Secondly, data generating process and assumed mean reversion has been criticized. Since the work of Neftçi (1984) it has been

increasingly popular to test for nonlinearities and structural instabilities in economic time series. Enders and Granger (1998) show that if nonlinearities are prevalent under the alternative of stationarity, linear tests for unit roots suffer from a lack of power. Not surprisingly, Kilian and Taylor (2003) show that if there is evidence of nonlinear mean reversion standard tests of long-horizon predictability of exchange rates are invalidated. Finally, Faust, Rogers and Wright (2003) argue that data on fundamentals are subject to continuous revisions. They show that Mark's linear adjustment results are mainly the outcome of a certain window of vintages of the real time dataset and therefore not generally valid.¹

Failure of linear versions of monetary models to predict exchange rates even in the long term led a number of researchers to explore the nonlinear data generating process in the long term adjustment of exchange rates towards their equilibrium value given by the fundamentals. In this view, fundamentals based models with an appropriately modelled nonlinear mean reversion will be useful in forecasting exchange rates at least in the long term. The recent work by Balke and Fomby (1997), and Kilian and Taylor (2003) provide evidence of nonlinear adjustments of exchange rates.

Prominent explanations for nonlinear adjustment in exchange rates are related to the existence of transaction costs and heterogeneous beliefs/players. In the case of transaction costs, financial agents are assumed to be rational. Transaction costs in the financial markets create a band within which exchange rates do not respond to small deviations from the long term equilibrium. For large deviations, however, there is a tendency to revert to the fundamental equilibrium to exploit the profitable arbitrage activity. In this view, the speed of mean reversion towards equilibrium increases in deviations from the fundamental equilibrium. The heterogeneity argument is motivated by the existence of heterogeneous agents using different information sets.²

¹ A *data vintage* for a particular date means the data publicly available at this date. For example "1985Q1 data vintage" in our study covers the 57 data points of money supply and output for the period 1970Q3-1984Q4 that is published at the statistical bulletin of the first quarter of 1985. Similarly 1985Q2 vintage has 58 data points for the period 1973Q3-1985Q1. Therefore, the new data vintage, 1985Q2, includes both an additional data point (the new data of the first period of 1985) and any possible revisions on the past 57 data points.

² De Grauwe and Dewachter (1993) make a distinction between chartists and fundamentalists and Kilian and Taylor (2003) between noise traders and rational speculators. Kilian and Taylor (2003) argue that agents cannot form a consensus view over the underlying fundamental equilibrium if the deviations are small. In that case, we can expect to observe random walk behaviour of exchange rates at values close to

Even though observed real time data on exchange rates and interest rates are valid at all time periods, monetary aggregates, output and prices are subject to regular revisions. Given that finance professionals and policymakers possess only real time data at the time of the forecast and are unable to perfectly predict future data revisions of the macroeconomic fundamentals, they will likely form their exchange rates expectations based on the data publicly available at the time when forecasts are made. An econometric study that implements the monetary model based on revised data may therefore yield incorrect inference if time series properties are significantly altered after revisions. Several authors find that revisions to preliminary GDP data are large and in general far from being predictable.³

In this paper, we extend the real time critique of Faust, Rogers and Wright (2003) to capture the dynamic nonlinear adjustment towards fundamental equilibrium values of GBP/USD exchange rates. As there is no consensus in the literature about the likely form of the nonlinear adjustment, we study two different models. For instance, suppose that the transaction cost argument is valid. If these costs are uniformly distributed among financial agents, one can expect a sharp correction in the exchange rate towards the value dictated by the fundamentals, once the uniform transaction costs band is reached. In this case, the threshold autoregressive model (TAR) captures the discrete adjustment in exchange rates (Tong, 1990).⁴ Alternatively, if transaction costs are not uniformly distributed -therefore there exists a continuum of thresholds- a smooth nonlinear adjustment might be expected. In this case, the exchange rate behaviour is possibly more appropriately modelled in the form of an exponential smooth nonlinear adjustment (ESTAR) as suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994).

long term equilibrium value. As the deviations from the long term equilibrium value get large, rational speculators will take a stronger position and prevail. Eventually the mean reversion occurs towards the unobserved long term equilibrium value of exchange rates.

³ Recently, the importance of real time data in macroeconomic evaluations has been addressed by several authors. The debate essentially concentrates about the nature of the revisions, i.e. whether these are news or noise. For instance, Mankiw, Runkle and Shapiro (1984) found that US money data revisions reduce noise. Faust, Rogers and Wright (2005) examined the G-7 countries' output forecasts and found that Italy, Japan and UK output revisions are forecastable in real time, whereas US output revisions are not.

⁴ The TAR model seems to fit well with the observed exchange rate behaviour such as volatility and jumps in the short run (see Coakley and Fuertes, 2001).

A common approach to evaluate forecasting performance of alternative models is to compare the root mean squared errors (RMSEs) obtained from an out-of-sample forecasting exercise. This is often complemented with a series of Diebold-Mariano tests to obtain statistical evidence on the performance of alternative models.⁵ In this paper we also utilize the *forecast consistency* argument as developed by Cheung and Chinn (1998) next to standard forecast assessment methods. The forecast consistency argument imposes no assumption on the long term properties of forecast error time series. The consistency test focuses on the long-run relation between forecasts and the actual series. Secondly, with this evaluation, forecast errors need not be serially uncorrelated. As Cheung and Chinn show this can happen when the model is correctly specified, however fundamentals data may be subject to measurement errors. Finally, due to measurement errors, the assumption of unitary elasticity of the coefficients on the right hand side of the exchange rate equation may be violated, even if forecasts are optimal projections. As measurement errors are the main focus of real time dataset discussions, we find the forecast consistency approach the most relevant method for our purposes. Long term properties of forecast error series is also at the core of the work by Kilian and Taylor (2003). In practice, we will test long-term equilibrium relationship between fundamental based exchange rates and actual exchange rates via a battery of linear and nonlinear integration/cointegration tests.

First, we find that while real output and broad monetary aggregates data are subject to significant revisions, price levels are rarely revised. Output and monetary aggregates revisions contain both news and noise components. Second, we compare the performance of linear models with linear adjustments in the forecast errors with the naïve random walk model, as standard in the literature. We confirm the vast literature that the linear monetary models with linear long term adjustment perform very poorly. Third, we account for nonlinear adjustment in forecast errors in linear models. We implement TAR and ESTAR nonlinear adjustment processes in the forecast errors. We find some evidence of nonlinear mean reversion in forecast errors over 1 to 16 quarters forecast horizons. More specifically, a discrete form of the nonlinear mean reversion is observed in shorter term forecast errors; whereas a smooth (exponential) form of the mean

⁵ See Diebold and Mariano (1995).

reversion is observed in longer term forecast errors. An implementation of the TAR unit root test suggest that up to 25% of the real time vintages exhibit a discrete form nonlinear mean reversion within 1 quarter forecast horizon and up to 44% of the vintages exhibit a discrete form nonlinear mean reversion within 2 quarters forecast horizon, when we take into account revisions in monetary aggregates and real output. Fourth, we do not detect a discrete form of the nonlinear mean reversion in longer term forecast errors. Fifth, an implementation of the ESTAR nonlinear unit root test developed by Kapetanios, Shin and Snell (2003) show that there is indeed some evidence of a nonlinear smooth mean reversion in long term forecast errors when real time price level is used as fundamentals.⁶ In about half of real time estimations a nonlinear mean reversion occurs within 16 quarters forecast horizon at the 5% significance level. Similarly, in about 20% of estimations a nonlinear mean reversion occurs within 4 quarters forecast horizon at the 5% significance level.⁷ Finally, we find very little evidence of a smooth nonlinear mean reversion in alternative monetary models, where revisions in monetary aggregates and real output are taken into account.

We thus claim that at several instances (i) real time fundamental equilibrium values of exchange rates may be determined in a linear fashion, whereas the adjustment towards fundamentals driven equilibrium values may take a discrete or smooth nonlinear form (ii) revisions in fundamentals data matter for the short-term forecastability of exchange rates lending support on the importance of real time data analysis by Faust, Rogers and Wright (2003), (iii) there is some short term forecastability of exchange rates in the form of a discrete nonlinear adjustment, (iv) there is some long-term forecastability in the form of a smooth nonlinear adjustment when real time price level data is used as fundamentals. Here, we lend support on the importance of the smooth nonlinear adjustment in 'real' exchange rates by Kilian and Taylor (2003). We, thus, claim that an accurate description of the exchange rate behaviour has to take into account both real time datasets as fundamentals and possible nonlinear adjustments.

This paper is organised as follows. Section 2 provides a discussion on the importance of data revisions and the real time dataset we have used in the paper. Section 3 presents

⁶ This amounts to estimating a real exchange rate equation.

⁷ For applications of the Kapetanios, Shin and Snell (2003) test in the context of real exchange rates see, for instance, Chortareas and Kapetanios (2004).

results for two possible nonlinear models (TAR and ESTAR) and discusses related nonlinear unit root tests. Finally, Section 4 concludes.

2 Real Time Datasets and Data Revisions

We define the final value of a variable as follows:

$$x_t^f = x_t^{t+1} + r_t^f,$$

where x_t^{t+1} denote a statistical agency's initial announcement (at $t+1$) of a variable that was realized at time t , x_t^f denotes the final or true value of the same variable, and r_t^f is the final revision which can potentially never be observed.

We have quarterly real time *vintages* of the UK and US monetary aggregates, real output and price level data as of period 1977Q1 onwards (see footnote 1). Exchange rates are never revised. We use quarterly end of period Pound Sterling/US Dollar exchange rates made available by the IMF/IFS. We use 1977Q1 to 1984Q4 vintages to construct the *first* operational real time dataset for the 1985Q1 vintage. This means we have 16 real time datasets for the 1985Q1-1989Q1 period (one dataset for each quarter; excluding 1988Q3). Our shortest dataset corresponds to the first dataset (1985Q1) and contains 57 quarterly data points (the first data point is 1970Q3 and the last data point is 1984Q4). Our longest dataset corresponds to the latest dataset (1989Q1), and contains 74 quarterly data points.

There are a few further details about the construction of the real time datasets. First is that the published statistics provided in economic bulletins cover only a limited time period (up to 16 quarters). We therefore conclude that *only* published data was within the reach of financial agents. In other words, we rule out 'privileged access' to revised official data.⁸ This allows us to extend the data backwards with the data published in the previous economic bulletins. Secondly, we use end of period £M3 for the UK and the

⁸ For example January 1986 issue of Economic Trends provides statistics from January 1981 to December 1985, which amounts to 16 data points. We assumed that there is no revision in data covered before January 1981. This assumption rules out finance professionals' possible privileged access to further 'revised' data beyond the data officially 'published' (i.e. made publicly available).

quarterly average of M2 for the US. As this data was published consistently in economic bulletins for the specified time period as such, we assume that finance professionals made use of this real time data.⁹ Further source details are provided in the Appendix.

In Table 1 we report descriptive statistics on the size of revisions for each individual data point over 16 datasets. We also report mean revisions after one, two, four, eight or sixteen quarters after the initial announcement and mean standard deviations in revisions.¹⁰

Typically, well behaved revisions have three characteristics.¹¹ First, revisions are expected to be of zero mean, i.e. initial announcement of the statistical agency is an unbiased estimate of the final value. Second, variance of the final revision should be small compared to the variance of the final value. Third, the final revision should be unpredictable given the information set at the time of the initial announcement.

In our case, data revisions are not well behaved. First, quarter to quarter data revisions are frequent, large and volatile for monetary aggregates and for real output. Revisions have a non-zero mean. Price level revisions are less frequent and small. Secondly, revisions are continuous. Even after 16 quarters revisions are large. Finally, revisions contain both news and noise component.¹²

⁹ We restrict sample period due to data limitations in the UK monetary aggregates. The Office for National Statistics (ONS, formerly known as Central Statistical Office) in the UK published £M3 data continuously up to August 1989 (under the name of M3 after August 1987, whereas the old M3 is renamed as M3c after this date). After August 1989 the ONS ceased to publish M3 and M3c, and started to publish M4c data which is a redefinition of M4 introduced in May 1987. Given that UK joined the Exchange Rate Mechanism between 1990 and 1992, where the monetary policy was effectively delegated to the German Bundesbank, we prefer to use M3 data.

¹⁰ We only report changes vis-à-vis the first announcement. See the Table 1 notes.

¹¹ See for instance Aruoba (2005).

¹² Mincer-Zarnowitz forecast efficiency test results are available upon request. These findings are in line with Mankiw, Runkle and Shapiro (1984) who found US money data revisions reduce noise, while Faust, Rogers and Wright (2005), examining the G-7 countries' output forecasts, found that Italy, Japan and UK output revisions are forecastable in real time, whereas US output revisions are not.

3 Forecasting Exchange Rates

3.1 Monetary Models of Exchange Rates

We assume that financial agents use real time data in forming their exchange rate forecasts. We use superscripts for the date of data announcements and subscripts to indicate the time announced data refers to. The real time *fundamental value* of the log exchange rate at time t based on the initial announcement of fundamentals at time $t+1$ (f_t^{t+1}) is predicted by a simple nested monetary model that takes the following form:

$$f_t^{t+1} = \alpha_1(m_t^{t+1,US} - m_t^{t+1,UK}) + \alpha_2(y_t^{t+1,US} - y_t^{t+1,UK}) + \alpha_3(p_t^{t+1,US} - p_t^{t+1,UK}), \quad (1)$$

where m_t^{t+1} , y_t^{t+1} , and p_t^{t+1} are the logs of money aggregates, output and price levels at time t , based on the initial announcements at time $t+1$. Equation (1) describes a parsimonious relationship between macroeconomic fundamentals and the exchange rate.¹³ We will consider three models: the *MY Model* imposes $\alpha_3 = 0$, the *PY Model* imposes $\alpha_1 = 0$, and finally the *P Model* (the real exchange rate model) imposes $\alpha_1 = \alpha_2 = 0$. These three monetary models form the basis for rational agents in forming their expectations about the future evolution of the exchange rates.

The standard practice in the literature is to restrict parameters on fundamentals equal to 1. Here, we allow financial agents to pursue a slightly more sophisticated statistical strategy. We assume that rational agents have access to a simple OLS estimation technology. Agents estimate a monetary model with real time information about fundamentals up to period $t-1$ and update their forecasts as new information arrives. This parameter updating mechanism ensures that the coefficients of fundamentals reflect optimal projections on past exchange rates and therefore need not necessarily be equal to 1. They use this information about the coefficient estimates in making their exchange rate forecast for $t+k$, k being the forecast horizon.¹⁴ This parameter updating mechanism is in

¹³ For versions of the monetary model with microfoundations see, for instance, Lucas (1982) or Stockman (1987).

¹⁴ Rolling regression coefficient estimates are available from the authors upon request.

line with the forecast consistency argument à la Cheung and Chinn (1998) discussed earlier.

3.2 Calculation of the Forecast Errors

Let \tilde{z}_{t+k}^{t+1} denote the forecast error for forecast horizon $t+k$, where forecasts are based on the real time information available at $t+1$. Equation (2) gives the exchange rate forecast error formulation:

$$\tilde{z}_{t+k}^{t+1} = f_{t+k}^{t+1} - s_{t+k}, \quad (2)$$

where s_{t+k} refers to the actual exchange rate at time t , and f_{t+k}^{t+1} refers to the real time forecast of the fundamental value of the exchange rate based on information available at $t+1$ (real time). Therefore, the difference between the fundamental value of the exchange rate and the actual exchange rate gives the forecast error.

We obtain real time recursive forecast error series for five different forecast horizons, with $k=1, 2, 4, 8$ and 16 . For each combination of model (P, PY, MY), forecast horizon (k), real time datasets (16 datasets), we calculate corresponding forecast error series, where the minimum sample size is set equal to twenty sample points. In other words, we calculate a total of $3 \times 5 \times 16$, i.e. (model) \times (forecast horizon) \times (real time dataset), forecast error series.¹⁵ Gauss programme codes are available upon request.

3.3 Monetary Models with Linear Adjustment in Forecast Errors

In this section we report standard forecast performance evaluation of linear monetary models with linear adjustment towards the fundamentals determined value of the exchange rates vis-à-vis the naïve random walk model of exchange rates. This assessment

¹⁵ Note that instead of looking at the recursive forecast errors, Mark (1995) estimates an error correction specification (ECM) to generate forecast errors. Due to critiques of Berkowitz and Kilian (1999), Giorgianni (2001), and Kilian and Taylor (2003), we opt for the recursive forecast errors instead of relying on the forecast errors based on the error correction specification.

is akin to the work by Meese and Rogoff (1983) and Nelson (1995), among others, and readily comparable to the work by Faust, Rogers and Wright (2003). We compute forecast errors generated by equations (1) and (2) for each individual dataset, model and forecast horizon combination and compare these to the naïve random walk model with the use of the standard Diebold-Mariano test of the forecast accuracy of linear models with a linear adjustment in forecast errors.

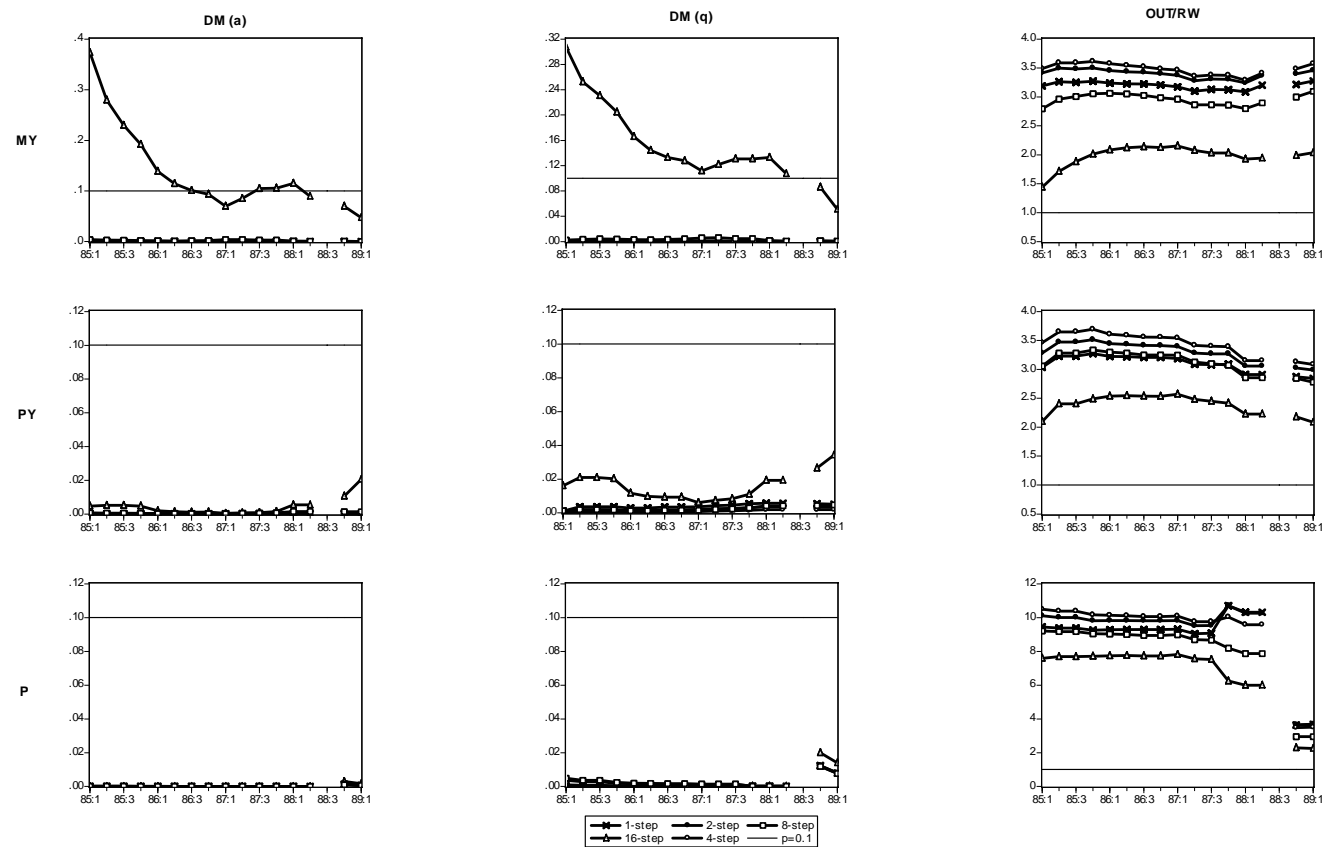
Note that the null hypothesis is equal forecast accuracy, i.e. there is no qualitative difference between the forecasts from two models against the hypothesis that the forecasts are different. Throughout Column 1 and Column 2 in Figure 1, we report p-values of the Diebold-Mariano forecast evaluation. We further report a direct comparison of the root mean squared errors (RMSEs) of the model-based evaluation with the RMSEs based on the random walk model in Column 3. We firmly reject the hypothesis of equal forecast accuracy across most of the linear models with a linear adjustment in real time, corroborating the findings of Faust, Rogers and Wright (2003).

3.4 Accounting for Nonlinear Adjustment in Forecast Errors

Given the recent evidence on nonlinear adjustment of forecast errors (Kilian and Taylor, 2003) we implement a similar exercise. In the following sections we will allow two types of forecast error adjustment dynamics of exchange rates towards the fundamentals based equilibrium evaluated in real time.

We first analyze an immediate transition threshold analysis proposed by Tong (1990) for which unit root tests are developed by Caner and Hansen (2001). The second model we consider is the more realistic exponential smooth transition threshold dynamics model (ESTAR Model) proposed by Granger and Teräsvirta (1993) for which unit root tests are developed by Kapetanios, Shin and Snell (2003).

Figure 1
Forecast Performance of Linear Models vis-à-vis the Random Walk Model



Notes: Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing). The first two columns present the p-values of the Diebold-Mariano tests. DM(a) denotes the test with absolute value of the forecast errors. DM(q) denotes the test with the square of the forecast errors. Third column presents the ratio of RMSE of the linear model with respect to the Random Walk model.

As argued earlier in the Introduction, this evaluation is in line with the forecast consistency argument developed by Cheung and Chinn (1998) that is superior to comparing RMSEs as it imposes no assumption on the long term properties of the forecast error time series. Secondly, with this consistency evaluation, forecast errors need not be serially uncorrelated. As Cheung and Chinn show this can happen when the model is correctly specified, however, fundamentals data may be subject to measurement errors. Finally, due to measurement errors, unitary elasticity of the coefficients on the right hand side of the equation may be violated, even if forecasts are optimal projections. The forecast consistency argument addresses adequately our concerns about the measurement errors in the fundamentals.

3.4.1 TAR Unit Root Tests (Caner and Hansen, 2001)

We postulate the following equation as an appropriate TAR model:

$$\Delta \tilde{z}_{t+k} = I_t \left[\theta_1' \tilde{x}_{t+k-1} + \sum_{j=1}^p \gamma_{1j} \Delta \tilde{z}_{t+k-j} \right] + (1 - I_t) \left[\theta_2' \tilde{x}_{t+k-1} + \sum_{j=1}^p \gamma_{2j} \Delta \tilde{z}_{t+k-j} \right] + \zeta_t, \quad (3)$$

where $\tilde{x}_{t+k-1} = (1 \ t \ \tilde{z}_{t+k-1})$, ζ_t is an *i.i.d.* error, and I_t is the indicator function that takes the form:

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} < \lambda \\ 0 & \text{if } y_{t-1} \geq \lambda \end{cases},$$

where λ is a threshold and the variable y_t is any stationary variable that would determine the change of regime. As in most economic applications we can set $y_t = \tilde{z}_{t+k} - \tilde{z}_{t+k-m}$. That is, we assume that \tilde{z} behaves differently depending on whether past changes in \tilde{z} have been higher or lower than a certain threshold λ . This is a self-exciting M-TAR model with two regimes as in Enders and Granger (1998). The lag length m for the changes in \tilde{z} is determined by the data, as is the search for the optimal threshold λ . The parameter vectors θ_1 and θ_2 can be partitioned as:

$$\theta_1 = \begin{pmatrix} \mu_1 \\ \delta_1 \\ \rho_1 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} \mu_2 \\ \delta_2 \\ \rho_2 \end{pmatrix},$$

where μ_i is an intercept, δ_i is the parameter of the deterministic trend, and ρ_i is the autoregressive parameter with $i = 1, 2$. In order to search for the optimal threshold λ , Caner and Hansen (2001) follow Chan (1993) and find λ as the value of $\Delta\tilde{z}_{t+k-m}$ that minimises the residual sum of squares of the OLS estimation of (3).¹⁶ In order to test for the existence of asymmetry in the adjustment under both regimes they test the null hypothesis $H_0 : \theta_1 = \theta_2$ on the OLS estimation of equation (1), making use of a Wald statistic (W). They propose to choose m to minimise the residual sum of squares of equation (3). Given that the Wald test of asymmetry is a monotonic function of the residual variance, m is chosen as the value which maximizes the Wald test of asymmetry.

The unit root hypothesis involves testing for $H_0: \rho_1 = \rho_2 = 0$. There are two possible alternatives:

$$H_1: \rho_1 < 0 \text{ and } \rho_2 < 0 \quad \text{and} \quad H_2: \begin{cases} \rho_1 < 0 \text{ and } \rho_2 = 0 \\ \text{or} \\ \rho_1 = 0 \text{ and } \rho_2 < 0 \end{cases} .$$

The first alternative corresponds to the stationary case, whilst the second implies stationarity in only one of the regimes, which implies overall non-stationarity but a different behaviour from the classic unit-root. Caner and Hansen (2001) develop an asymptotic theory for the distribution of this unit-root test. However, for finite samples they recommend the use of bootstrapping. As the distribution of the test statistic will depend on whether or not a threshold effect exists, p-values obtained through the bootstrap are not unique. Monte Carlo experiments show that this unit root test has substantial power gains against the linear ADF test as threshold effects become larger. In order to discriminate between the two alternatives in H_2 , Caner and Hansen (2001) recommend another Wald statistic (RI) which is constructed as the sum of the squared values of the individual one sided t -statistics for ρ_1 and ρ_2 .¹⁷

The economic interpretation of this model would be that, for certain macroeconomic variables, positive and negative shocks – or shocks above or below a certain threshold – may have different effects on the mean and speed of convergence of the data.¹⁸ Caner and Hansen (2001) provide further details on the test and inference.

¹⁶ In practice, the outliers are eliminated by trimming the series for the highest and lowest values of Δy_{t-m} .

¹⁷ RI is the one sided Wald test for a unit root, whereas they also propose a two-sided Wald test, $R2$.

¹⁸ See the seminal work of Balke and Fomby (1997) for the analysis of cointegration relations subject to TAR adjustment dynamics. In their case, the threshold is determined by the size of the lagged error correction mechanism.

In testing for the unit root we treat the threshold as unidentified, in which case the bootstrap is based on a linear *AR* model.¹⁹ This test is implemented by choosing the estimated delay parameter m that minimizes the residual variance.²⁰ We report the Wald statistic (W_T) for the threshold effect (for nonlinearity), threshold unit root bootstrap p-values (for nonstationarity), and corresponding t statistics to distinguish between rejection of unit roots and nonstationarity for each series of forecast errors obtained from 16 real time datasets.²¹

First, in Table 2, Columns 1 to 4 we report the fraction of the datasets we can reject the linearity in forecast errors for alternative monetary models under alternative deterministic specifications as regards the trend and the constant. It appears that in a significant fraction of the series we cannot rule out the hypothesis of linearity. We can reject the hypothesis of linearity in the case of the MY model (with trend) up to 36% of the 1-quarter ahead, up to 44% (without trend) of the 2-quarters ahead forecast error series estimated.

¹⁹ The alternative is to treat the threshold as identified, and to base the bootstrap on simulations from a unit root TAR process. Caner and Hansen (2001) show Monte-Carlo evidence that suggests the unidentified threshold bootstrap test suffers from less size distortion than the identified threshold test or a test based on the asymptotic critical values for possible threshold nonlinearities.

²⁰ Caner and Hansen (2001) point out that as the Wald test W_T is a monotonic function of the residual variance, this is tantamount to choosing m as the value that maximizes W_T .

²¹ Bootstrap p-values are calculated using the unidentified threshold bootstrap as described in Section 5.3 in Caner and Hansen (2001).

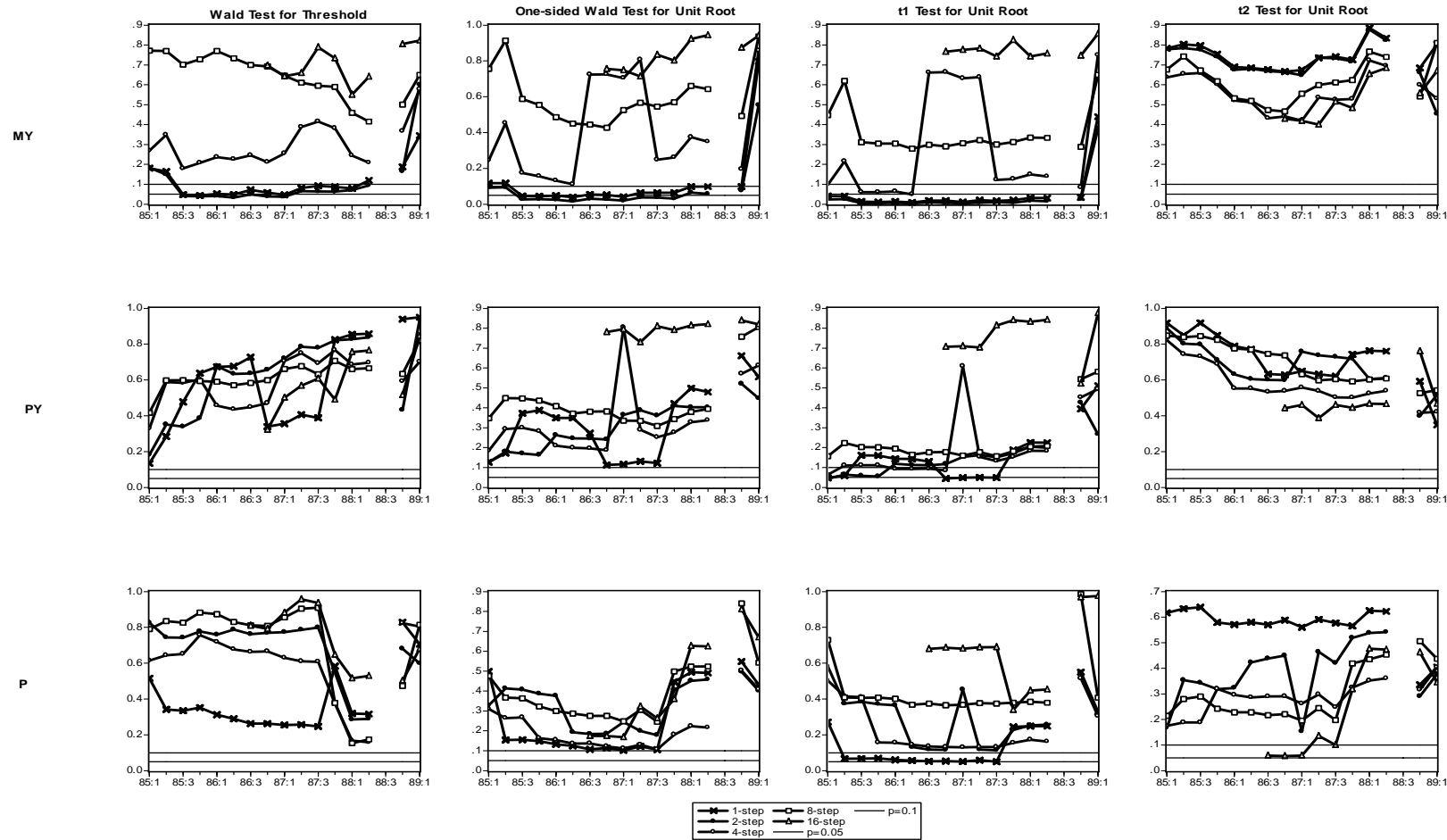
Table 2: TAR estimation - percentage of the vintages that reject the null hypothesis of unit root.

	linearity				unit root								# of vintages used					
	Wald test for threshold effect				One-sided Wald test for unit root				t1 test for unit root				t2 test for unit root				constant	constant and trend
	constant		constant and trend		constant		constant and trend		constant		constant and trend		constant		constant and trend			
	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%
MY Model																		
1-quarter	25%	69%	36%	64%	31%	81%	0%	0%	94%	94%	0%	9%	0%	0%	0%	0%	16	11
2-quarters	44%	75%	36%	64%	63%	94%	0%	0%	94%	94%	0%	9%	0%	0%	0%	0%	16	11
4-quarters	0%	0%	0%	0%	0%	0%	0%	0%	6%	38%	0%	9%	0%	0%	0%	0%	16	11
8-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	16	10
16-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	9	2
PY Model																		
1-quarter	0%	0%	0%	27%	0%	0%	0%	0%	25%	38%	64%	91%	0%	0%	0%	0%	16	11
2-quarters	0%	0%	0%	27%	0%	0%	0%	0%	6%	25%	36%	91%	0%	0%	0%	0%	16	11
4-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%	16	10
8-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	16	4
16-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	9	1
P Model																		
1-quarter	0%	0%	0%	50%	0%	0%	0%	50%	6%	63%	50%	60%	0%	0%	0%	0%	16	10
2-quarters	0%	0%	0%	50%	0%	0%	0%	50%	0%	0%	50%	60%	0%	0%	0%	0%	16	10
4-quarters	0%	0%	0%	13%	0%	0%	0%	13%	0%	0%	0%	50%	0%	0%	0%	0%	16	8
8-quarters	0%	0%	0%	0%	0%	0%	0%	17%	0%	0%	0%	33%	0%	0%	0%	0%	16	6
16-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	30%	0%	0%	10	1

In the case of the PY model for up to 27% of the 1 or 2-quarters ahead forecast error series and in the case of P model for up to 50% of the 1 or 2-quarters ahead forecast error series we can reject the hypothesis of linearity at the 10% significance level when the constant and a trend is included in the estimation. It seems that it is more likely that the hypothesis of linearity is rejected against the TAR alternative when we look at shorter forecast horizons. In the same table we report the fraction of series for which we can reject the hypothesis of unit root by looking at the Wald statistic and individual t-statistics. The results are broadly consistent with linearity tests. The hypothesis of unit root is rejected in a substantial fraction of the shorter horizon forecast errors. As the forecast horizon becomes longer (8 to 16 quarters) there are very few forecast error series obtained under alternative monetary model specifications for which we can reject the hypothesis of unit root. Finally, t-statistics indicate that even if one can reject the hypothesis of a unit root ($\rho_1 = \rho_2 = \mathbf{0}$) in a number of series, this result does not indicate that we can reject the hypothesis of nonstationarity (H_2). Indeed t1 and t2 tests jointly taken into account indicate that it is almost impossible to rule out nonstationarity if the data generating process is assumed to be of TAR type.

For the sake of completeness we report detailed results for each individual dataset in Figures 2 and 3, and in Table 3.

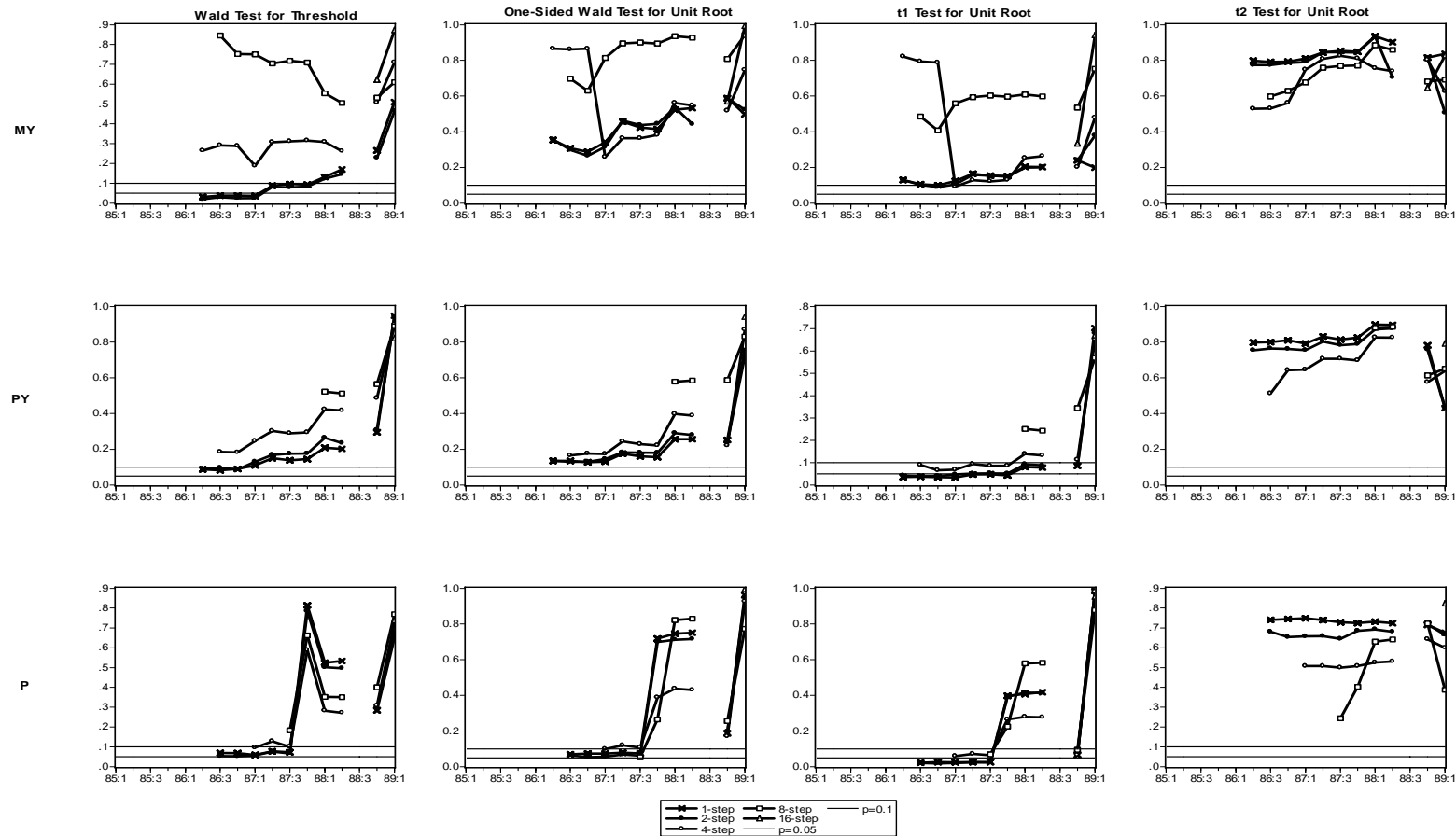
Figure 2
Unit Root and Non-Linearity Tests: TAR Model without Trend



Notes: Vertical axis denotes p-values of estimations from 16 real time dataset. Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing).

Figure 3

Unit Root and Non-Linearity Tests: TAR Model with Trend



Notes: Vertical axis denotes p-values of estimations from 16 real time dataset. Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing).

Table 3: TAR unit root tests - vintages that reject the null hypothesis of unit root.

		MY Model																																				
		Wald test for Linearity				1-Sided Wald Test for Unit Root				t1 test for unit root				t2 test for Unit Root																								
Vintage / Forec. Hor.		Constant				Constant and Trend				Constant				Constant and Trend				Constant				Constant and Trend																
		1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8			
1985Q1		X	X	X	X	X	X	X	X	*	X	X	X	X	X	**	**	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X					
1985Q2		X	X	X	X	X	X	X	X	*	X	X	X	X	X	**	**		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X					
1985Q3	** **	X	X	X	X	X	X	X	X	** **	X	X	X	X	X	** **	*		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X					
1985Q4	** **	X	X	X	X	X	X	X	X	** **	X	X	X	X	X	** **	*		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
1986Q1	* **	X	X	X	X	X	X	X	X	** **	X	X	X	X	X	** **	*		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
1986Q2	** **	X	** **	X	X	** **	X	** **	X	** **	X		X	X	** **	**		X		X	X		X	X		X		X		X		X		X				
1986Q3	* **	X	** **	X	X	*	**	X	** **	X	** **	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X			
1986Q4	* **	** **	X	X	X	X	X	X	X	*	**	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X			
1987Q1	** **	** **	X	X	X	X	X	X	X	** **	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X		X		
1987Q2	* *	* *	X	X	X	X	X	X	X	*	**	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X		X	
1987Q3	* *	* *	X	X	X	X	X	X	X	*	**	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X		X	
1987Q4	* *	* *	X	X	X	X	X	X	X	*	**	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X		X	
1988Q1	* *	X	X	X	X	X	X	X	X	*	**	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X		X	
1988Q2	*	X	X	X	X	X	X	X	X	*	**	X	** **	X	** **	X	** **	X	*	*	X		X		X		X		X		X		X		X		X	
1988Q4										*	*					**	**	*																				
1989Q1																**	**	*																				

		PY Model																																				
		Wald test for Linearity				1-Sided Wald Test for Unit Root				t1 test for unit root				t2 test for Unit Root																								
Vintage / Forec. Hor.		Constant				Constant and Trend				Constant				Constant and Trend				Constant				Constant and Trend																
		1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8			
1985Q1		X	X	X	X	X	X	X	X	*	X	X	X	X	X	**	**	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		
1985Q2		X	X	X	X	X	X	X	X	*	*	X	X	X	X	X	*	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
1985Q3		X	X	X	X	X	X	X	X	*	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
1985Q4		X	X	X	X	X	X	X	X	*	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
1986Q1		X	X	X	X	X	X	X	X	*	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
1986Q2		X	*	*	X	X	X	X	X	*	X		X	X	X	*	X	** **	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
1986Q3		X	*	*	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
1986Q4		*	*	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
1987Q1		X	X	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1987Q2		X	X	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1987Q3		X	X	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1987Q4		X	X	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1988Q1		X	X	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1988Q2		X	X	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1988Q4		X	X	X	X	X	X	X	X	*	X		X	X	X	*	X	** **	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
1989Q1																																						

** denotes 5% and * denotes 10 % significance level
 X denotes a vintage that is not used due to data limitations

Table 3 (continued): TAR unit root tests- vintages that reject the null hypothesis of unit root.

P Model

Vintage / Forec. Hor.	Wald test for Linearity					1-Sided Wald Test for Unit Root					t1 test for unit root					t2 test for Unit Root																									
	Constant					Constant and Trend					Constant					Constant and Trend					Constant					Constant and Trend															
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16						
1985Q1				X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X					
1985Q2				X	X	X	X	X	X	X	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X					
1985Q3				X	X	X	X	X	X	X	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
1985Q4				X	X	X	X	X	X	X	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
1986Q1				X	X	X	X	X	X	X	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
1986Q2				X	X	X	X	X	X	X	X	X	X	X	X	*	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
1986Q3					*	*	X	X	X				*	*	X	X	*	**	**	X	X	X			*				X	X	X						X	X	X		
1986Q4					*	*	X	X	X				*	*	X	X	*	**	**	X	X	X			*				X	X	X						X	X	X		
1987Q1					*	*	*	X	X				*	*	*	X	*	**	**	*	X	X			*				X	X							X	X	X		
1987Q2					*	*		X	X				*	*	X	X	*	**	**	*	X	X						X	X							X	X	X			
1987Q3					*	*		X					*	*	*	X	**	**	**	*	*	X						X									X	X	X		
1987Q4								X													X							X									X	X	X		
1988Q1								X													X							X										X	X	X	
1988Q2								X													X							X										X	X	X	
1988Q4								X										*	*	*	*	X						X											X	X	X
1989Q1																																									

** denotes 5% and * denotes 10 %significance level
 X denotes a vintage that is not used due to data limitations

We report bootstrap p -values for the unit root tests and the nonlinearity tests. In Figure 2 we plot p -values for the threshold effect, bootstrap p -values for the unit root tests and t_1 and t_2 tests for nonstationarity for each k -quarters ahead forecast error series (dataset) estimated with the TAR model without trend. Similarly, in Figure 3 we plot p -values for the threshold effect, bootstrap p -values for the unit root tests and t_1 and t_2 tests for nonstationarity for each k -quarters ahead forecast error series (dataset) estimated with the TAR model with trend. The horizontal axis represents estimated datasets starting in 1985Q1 and ending in 1989Q1 (excluding 1988Q3 as mentioned before).

In Figure 2 (TAR models estimated without trend) we confirm that the results for individual datasets for the linearity test mainly coincide with the tests for unit roots. In the case of the MY model, tests for nonlinearity and unit roots reject the null hypothesis for most of the first and second quarters ahead forecast errors. Specifically, datasets for which both linearity and unit roots are rejected are between 1985Q3 and 1988Q2. P -values for both t - tests indicate that even for those series for which we could reject both linearity and unit root, we are unable to do so for the assumption of nonstationarity. Other monetary models do rather poorly in both linearity and unit root tests at the 5% significance level.

Figure 3 (TAR models estimated with trend) shows that, with the exception of the P model for which we are able to reject the null hypothesis of linearity and unit roots at the 1st and 2nd quarters ahead forecast errors for datasets ranging from 1986Q3 to 1987Q3, we are unable to detect TAR form of nonlinear mean reversion in most of the alternative model/dataset combination.

In summary, TAR unit root tests results suggest that real time monetary aggregates and real output provide quite valuable information about short-term forecastability of exchange rates for 1985Q3-1988Q2 datasets.

As a next step, we assess the implications of another nonlinear dynamic adjustment specification in the forecast errors. The ESTAR model is considered to be more plausible type of nonlinear dynamic adjustment process for exchange rates in the long term.

3.4.2 ESTAR Unit Root Tests (Kapetanios, Shin and Snell, 2003)

The ESTAR model has been very popular recently. As argued earlier transaction cost arguments or existence of heterogeneous traders/beliefs in the financial markets may trigger a smooth asymmetric adjustment of the exchange rate towards its linear fundamental

equilibrium. As discussed in Granger and Teräsvirta (1993) in general and Kilian and Taylor (2003) for the monetary exchange rate models, we postulate a smooth transition autoregressive model of the form:

$$\Delta\tilde{z}_{t+k} = \rho_1\tilde{z}_{t+k-1} + \rho_2\tilde{z}_{t+k-1}G(y_t; \phi, \lambda) + \varepsilon_t, \quad (4)$$

where G is a transition function, ε_t is an *i.i.d.*(0, σ^2) error, y_t is a state variable, ϕ is the speed of transition variable, and λ is a threshold. Because of the particularly interesting properties of ESTAR models for economic applications, Kapetanios, Shin and Snell (2003), focus on tests for a unit root when the DGP follows an ESTAR process under the alternative of stationarity. When we set the state variable as, $y_t = \tilde{z}_{t+k-d}$, it represents a self-exciting ESTAR model. In this case equation (4) becomes:

$$\Delta\tilde{z}_{t+k} = \rho_1\tilde{z}_{t+k-1} + \rho_2\tilde{z}_{t+k-1}[1 - \exp(-\phi(\tilde{z}_{t+k-d} - \lambda)^2)] + \varepsilon_t.$$

The transition function $[1 - \exp(-\phi(\tilde{z}_{t+k-d} - \lambda)^2)]$ determines the degree of nonlinearity as a function of the speed of adjustment coefficient ϕ . In line with most of the literature, we set the delay parameter d equal to 1. (See for instance Teräsvirta, 1994.)

As Kapetanios, Shin and Snell (2003) assume that \tilde{z}_{t+k} is a mean-zero stochastic process, one can set $\lambda = 0$. This makes $G = 1 - \exp\{-\phi\tilde{z}_{t+k-1}^2\}$. As $\tilde{z}_{t+k-1} \rightarrow \pm\infty$, $G \rightarrow 1$, and as \tilde{z}_{t+k-1} gets close to zero, $G \rightarrow 0$. Hence, the process shows three regimes, a middle regime when \tilde{z}_{t+k-1} is close to zero and two symmetric outer regimes when \tilde{z}_{t+k-1} becomes large (either positive or negative). The smoothness of the transition between these regimes depends on the parameter ϕ .

Kapetanios, Shin and Snell (2003) further impose the assumption that $\rho_1 = 0$. This assumption can be justified on the grounds of transaction costs arguments or heterogeneity in beliefs as discussed earlier. The variable displays a mean reverting behaviour towards an attractor when it is sufficiently far away from it, but a random walk representation in the neighbourhood of the attractor. In this case, we have that

$$\Delta\tilde{z}_{t+k} = \rho_2\tilde{z}_{t+k-1}[1 - \exp(-\phi\tilde{z}_{t+k-1}^2)] + \varepsilon_t. \quad (5)$$

And the test for the *joint* null hypothesis of linearity and a unit root can be achieved by testing $H_0: \phi = 0$ against $H_1: \phi > 0$. Using a first order Taylor series approximation to equation (5), one can obtain:

$$\Delta\tilde{z}_{t+k} = \phi\tilde{z}_{t+k-1}^3 + \text{error}. \quad (6)$$

The unit root test is based on the t-statistic for the null $\varphi = 0$ against the alternative $\varphi < 0$ from the OLS estimate of φ ($\hat{\varphi}$). The asymptotic distribution of this test (t_{NL}) is non-standard and Kapetanios, Shin and Snell (2003) derive it and provide asymptotic critical values. We refer to Kapetanios, Shin and Snell (2003, Table 1) for the asymptotic critical values of the t_{NL} .

When the process \tilde{z}_{t+k} is not mean zero, they propose the use of transformations of the data. For the case of a non-zero mean, i.e. $x_t = \mu + \tilde{z}_{t+k}$, they propose the use of de-measured data $\tilde{z}_{t+k}^* = x_{t+k} - \bar{x}$, where \bar{x} is the sample mean. For the case of a non-zero mean and a non-zero deterministic trend, i.e. $x_{t+k} = \mu + \delta t + \tilde{z}_{t+k}$ they propose the use of the de-measured and de-trended data $\tilde{z}_{t+k}^* = x_{t+k} - \hat{\mu} - \hat{\delta}t$, where $\hat{\mu}$ and $\hat{\delta}$ are the OLS estimators of μ and δ . This procedure allows carrying out the test using equation (6) with the de-measured/de-trended data.²² In line with the suggestion of Kapetanios, Shin and Snell (2003) we append to equation (6) 0, 1, 2, or 4 autoregressive lags based on Akaike Information Criterion.

We implement the ESTAR joint linearity and unit root test for 16 available datasets in real time. In Table 4 we report the percentage of datasets for which we can reject the hypothesis of a unit root together with the linearity; therefore, we conclude in favour of nonlinear mean reversion.

A quick inspection of Table 4 suggests that several forecast error series over different horizons obtained from estimations of 16 datasets do not reveal much nonlinear mean reversion at shorter horizons. In the case of PY and MY models with or without trend we do not detect significant ESTAR type mean reversion in short-term forecast errors (1 to 4-quarters ahead). Only in the case of the P model at 4-quarters forecast horizon we find some exchange rate predictability (18.8% of the series at the 5% significance level).

²² Note, however, that this does not ensure a zero mean in the regression, as y_{t-l} ³ may have a mean that is different from zero. Alternative demeaning would involve fully demeaning the left and right-hand side of equation (6). Although this would not affect the distribution of the statistic under the null, it may affect test results. In the empirical application we present both types of demeaning. We denote Kapetanios, Shin and Snell demeaning exercise as ESTAR1- t_{NL} , and full demeaning at the estimation as ESTAR2- t_{NL} .

Table 4: TAR estimation - percentage of the vintages that reject the null hypothesis of unit root.

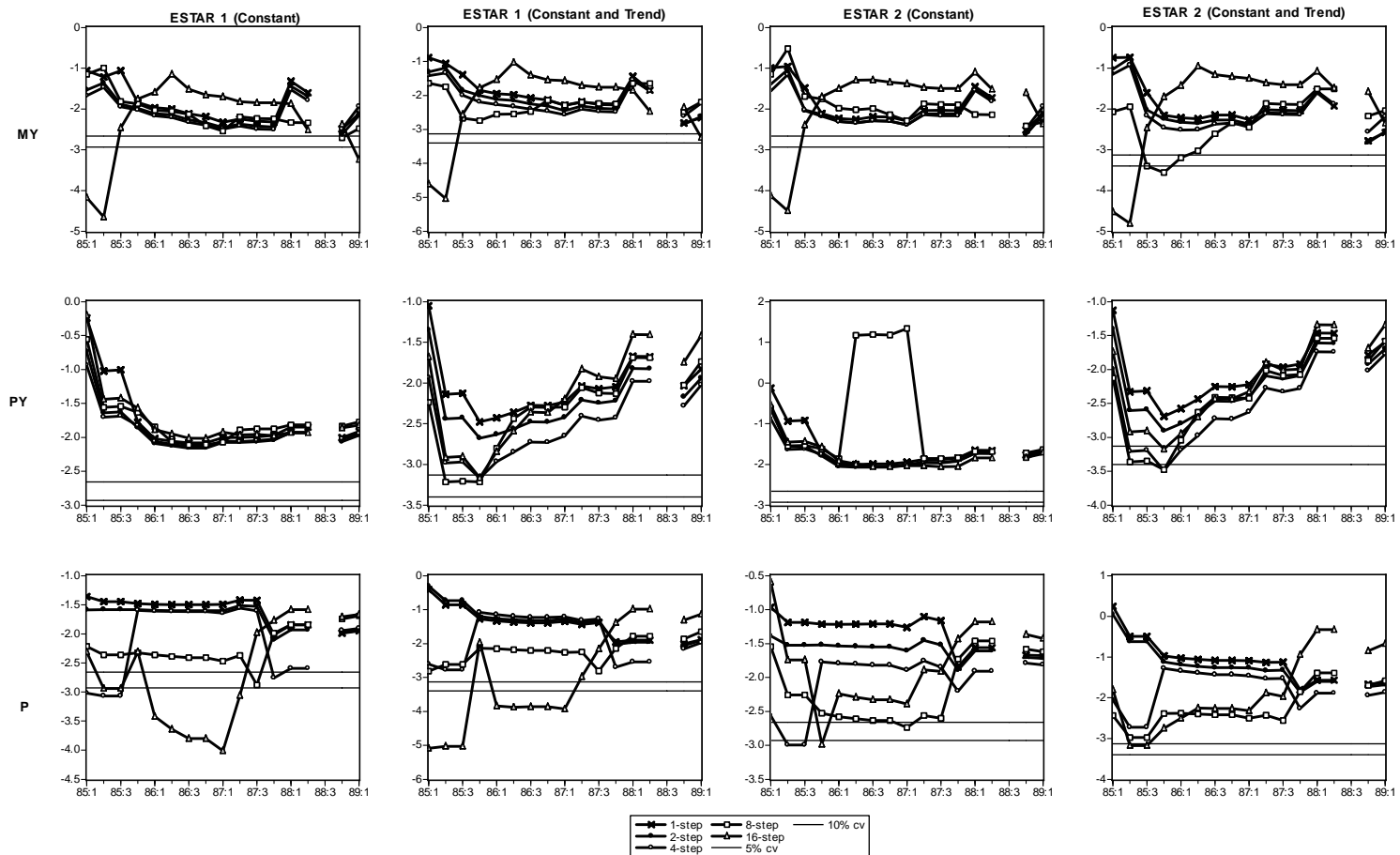
	ESTAR1-t _{NL}				ESTAR2-t _{NL}				
	Constant		Constant and Trend		Constant		Constant and Trend		
	5%	10%	5%	10%	5%	10%	5%	10%	
MY Model									
1-quarter	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8-quarters	0.0	6.3	0.0	0.0	0.0	0.0	6.3	18.8	18.8
16-quarters	18.8	18.8	12.5	18.8	12.5	12.5	12.5	12.5	12.5
PY Model									
1-quarter	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4-quarters	0.0	0.0	0.0	6.3	0.0	0.0	6.3	25.0	25.0
8-quarters	0.0	0.0	0.0	18.8	0.0	0.0	6.3	18.8	18.8
16-quarters	0.0	0.0	0.0	6.3	0.0	0.0	0.0	6.3	6.3
P Model									
1-quarter	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4-quarters	18.8	25.0	0.0	0.0	12.5	12.5	0.0	0.0	0.0
8-quarters	0.0	6.3	0.0	0.0	0.0	6.3	0.0	0.0	0.0
16-quarters	50.0	50.0	50.0	50.0	6.3	6.3	0.0	12.5	12.5

In the long term (16-quarters), however, in the case of the MY model without trend, about 18.8% of the forecast errors series seem to exhibit ESTAR form of mean reversion at the 5% significance level. When we implement the same test for the P model (real exchange rate model) we find that about half of the forecast error series within 16-quarters are mean reverting at the 5% significance level. This corroborates to some extent the findings of Kilian and Taylor (2003) as regards the smooth nonlinear mean reversion of real exchange rates; in our case valid for about half of the real exchange rate models using real time datasets.

Next we present the performance of individual datasets with the ESTAR specification. In Figure 4 we plot the t_{NL} -statistics for each individual series estimated and in Table 5 we report the vintages that reject the null hypothesis of a unit root. Note again that horizontal axis represents datasets that start in 1985Q1 and end in 1989Q1 (excluding August 1988Q3). As we observe, the P model is useful in forecasting exchange rates at 4-quarters forecast horizon in 1985Q1 to 1985Q3 datasets and at 16-quarters forecast horizon in 1985Q2, 1985Q3 and 1986Q1 to 1987Q2 datasets, whereas the MY model is useful in forecasting 1985Q1, 1985Q2 and 1989Q1 datasets.²³

²³ We have also implemented the same series of tests with the use of period average instead of end of period Pound Sterling /US Dollar exchange rates. The percentages of vintages that exhibit nonlinear mean reversion under various forecast horizons are much higher in this case. Results are available upon request.

Figure 4
Unit Root and Non-Linearity Tests: ESTAR Model



Notes: Vertical axis denotes t_{NL} values of estimations from 16 real time dataset. Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing). Critical values for the t_{NL} test statistic at 5% and 10% are given by the straight lines.

Table 5: ESTAR unit root tests - vintages that reject the null hypothesis of unit root.

Vintage / Forec. Hor.	MY Model																PY Model																												
	Constant				ESTAR1-t _{NL}				Constant and Trend				Constant				ESTAR2-t _{NL}				Constant				ESTAR2-t _{NL}																				
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16															
1985Q1					**					**					**																														
1985Q2					**					**					**																														
1985Q3																																													
1985Q4																																													
1986Q1																																													
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1987Q4																																													
1988Q1																																													
1988Q2																																													
1988Q4																																													
1989Q1					*					**					**																														

** denotes 5% and * denotes 10 % significance level

4 Conclusions

In this paper we examine the real time out-of-sample predictive power of fundamentals based linear monetary exchange models with nonlinear adjustments in forecast errors. We extend the analysis of Faust, Rogers and Wright (2003) in the direction of nonlinear mean reversion and Kilian and Taylor (2003) in the direction of accounting for real time revisions in datasets of fundamentals. We utilize the forecast consistency argument next to standard forecast performance evaluation methods.

We claim that in several instances, real time fundamental equilibrium values of exchange rates may be determined in a linear fashion, whereas adjustment towards the fundamentals driven equilibrium values may take a discrete or smooth nonlinear form. Revisions in fundamentals, particularly in the US and UK monetary aggregates and real output, seem to matter mainly for short-term forecastability of exchange rates. Our evidence suggests that in some real time datasets even short-term forecastability can be found in the form of discrete nonlinear adjustment, while long-term forecastability may be present in the form of smooth nonlinear adjustment towards fundamentals determined equilibrium value of exchange rates.

There is a clear potential to extend the model to capture realistic data learning processes in financial markets. In this paper, we focused on the forecasting performance of monetary models based on separate datasets, as is the case in the literature. An obvious alternative would be to calculate forecast errors by allowing financial agents to discard old data and estimate exchange rates (based on fundamentals) based on entirely new datasets each quarter. As this requires a large number of real time datasets to conduct meaningful statistical analysis, we leave this very promising avenue for future research.

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