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Sports Policy Analytics for Professional Sports Leagues

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Abstract

The organisers of most professional sports leagues now employ one or more forms of policy intervention such as revenue sharing and salary capping schemes. The focus of the sports economic literature was initially directed towards the theoretical effects of these policies on competitive balance, wage rates and owner profits in the context of Major US sports leagues. That work has since been broadened in the literature to include other types of policy intervention and other model assumptions such as ‘win maximising’ owners and ‘open’ labour markets that characterise other professional leagues such as for association football.

This paper consolidates the analytical treatment in the sports economics literature of both product market and labour market policy interventions by league sports organisers to form a standardised set of techniques presented in a generally accessible format. This is intended to provide the reader with the method and approach for similar analysis of other combinations of assumptions and policy specification as appropriate to particular professional sports leagues.

More recent policy intervention has included the regulation of financial performance of professional (association) football clubs. This paper adds to the literature by also providing a framework for analysing the effect of this policy intervention on professional sports clubs.
1. Introduction

In 1885, when the Football Association allowed professional playing contracts, it also intervened in the operation of the league to prevent players from playing for more than one team during a season and to limit the wages that could be paid to players. These are examples of sports policy interventions. Other types of policy intervention include revenue sharing schemes and various salary capping schemes. In 2003 (for the 2004/05 season) the Union of European Football Associations (UEFA) introduced a Club Licencing system and in 2010 this was broadened to include a further form of policy intervention with Financial Fair Play (FFP) regulations for competitors in its competitions. The organisers of most professional sports leagues now employ one or more forms of policy intervention.

A body of the sports economics literature has analysed the effects of these policy interventions. The focus of this body of work is primarily on the change in competitive balance that results from each policy intervention although, in some cases, it is also on the effects on wage rates and owners’ profits. Rottenberg (1956) analyses the effect of the policy intervention known as the ‘reserve clause’ in professional baseball leagues. El-Hodiri and Quirk (1971) provides a mathematical model of a professional sports league to examine the justification of the exemption of some policy interventions in Major US sports leagues from some aspects of antitrust statutes. Quirk and Fort (1992) provides a graphical approach to model professional sports leagues with analysis of policy interventions in Major US sports leagues. That work has since been broadened by the literature, in particular by Vrooman (1995 and 2007), Szymanski (2004) and Késenne (2000, 2003 and 2007), to include other types of policy intervention and other model assumptions such as ‘win maximising’ owners and ‘open’ labour markets that characterise professional association football leagues.

For the financial regulation introduced by UEFA, Müller et al (2012) provides the empirical and theoretical foundation. Sass (2012) presents a multi period mathematical model which shows the increasing competitive dominance of large market teams being dampened by the FFP regulations. Peeters and Szymanski (2013) take the approach of simulating the impact of FFP on the finances and sports results of European clubs and estimating its effect empirically.

The first part of this paper introduces the fundamental relationships that are used to analyse sports policy interventions. Initially the relationships are examined for a single club and then this is extended to the case of two clubs in a league. The second part consolidates the application of these relationships to the analytical treatment of both product market and labour market policy interventions by league sports organisers in the sports economics literature to form a standardised set of techniques presented in a generally accessible format. This is intended to provide the reader with the method and approach for similar analysis of other combinations of assumptions and policy specification as appropriate to particular professional sports leagues. Supporting mathematical detail is relegated to appendices. It is then shown how the framework can be used to also analyse the effect of the more recent regulation of financial performance of professional (association) football clubs.
2. Sports policy analytics framework

This section introduces the functional relationship model and the extension to a two club model for the analysis of sports policy interventions.

2.1 The single club functional relationship model

The four key fundamental relationships are represented in four quadrant diagram shown in Figure 1.

Figure 1. Sports Policy Analytics Functional Relationships Model

Figure 1 shows the four main variables that matter to a sports club: their revenue (income), win proportion (output), talent and cost (expenditure). Each quadrant shows the relationship between two of these four variables.

Quadrant 1 relates the two main variables in the economic market in which the sports club operates: win proportion and revenue. Quadrant 2 defines the relationship between the win proportion of a sports club and the quantity of talent employed by the club. This incorporates an assumption as to the effectiveness of the talent employed relative to the quantity of talent chosen by competitive clubs. Quadrant 3 relates the two main variables in the labour (talent) market: quantity and cost of talent employed. Quadrant 4 specifies the budget constraint that the club operates with. These relationships are elaborated below. The dynamic connection between the quadrants in considered in the context of the FFP regulation.
Quadrant 1. Market Revenue Function

Quadrant 1 in Figure 1 depicts the relationship between the revenue of a club and its output. The output of each team is represented by the proportion of games they win in the league. To simplify the analysis the output is expressed as a proportion of the total number of wins.

Clubs may derive income from multiple sources. These include fans, commercial interests, owners and other financial backers and competition prize money. In the simplest form, it may be assumed that there is a single source of income and that this depends only on the proportion of the total number of wins in a league competition.

Figure 3 extends quadrant 1 of Figure 1. In the policy analysis literature it is assumed that the total revenue function is concave (as shown in Figure 2a which corresponds to quadrant 1 of Figure 1). The rationale for this is that the market demand is a function of the uncertainty of outcome and that there is a proportion of wins that, if attained, would produce negative marginal revenue and consequently total revenue would start to fall. This point is represented by the line connecting Figures 2a and 2b.

In this case the (inverse) market demand (average revenue, AR) function (shown in Figure 2b) is assumed to be of the form:

\[ P = a - bw \]

Where:
- \( P \) = Price of unit output
- \( w \) = Win proportion of team
- \( a \) and \( b \) are parameters of the market demand function

The total revenue (\( TR \)) function is given by the product of \( P \) and \( w \) and is therefore of the form:

\[ TR = aw - bw^2 \]

The marginal revenue (of winning) (\( MR^w \)) function is given by differentiating the total revenue function with respect to output (\( w \)) and is consequently given by:

\[ MR^w = a - 2bw \]

This is also shown in Figure 2b.

The graphical representations show that the total revenue function attains a maximum value at the output level where the value of the marginal revenue function is zero.

Note that the output axis for a team (shown in Figure 2) is restricted to the range 0-1.
Quadrant 2. Talent Effectiveness (Production / Contest Success) Function

Quadrant 2 in Figure 1 depicts the relationship between the output that results from differing levels of resource input. To simplify the analysis it is assumed that there is only one, homogeneous resource input. It is convenient to represent this in terms of units of talent rather than units of individual players.

This function determines the ability of a club to convert that talent into competitive success. It depends on factors such as the managerial skill, training and ability of the talent to perform as a team. It is also a ‘contest success’ function because it takes into account the competitive balance resulting from the effectiveness of talent deployed by others clubs.

In the policy analysis literature the basic assumption is generally that the output of a team \((i)\) is determined by the following relationship form:

\[
   w_i = \frac{t_i}{\sum_{j=1}^{n} t_j}
\]

Where:

- \(w_i\) = Win proportion of team \(i\)
- \(t_i\) = Talent quantity deployed by team \(i\)
- \(n\) = Number of teams in the league

That is, the win proportion of team \(i\) is given by the share of the total talent in the league that is deployed by team \(i\).

In the simplest form, it may be assumed that the club has no impact on the effectiveness of the talent employed and that there is no competitive reaction to the talent choice of a club. A more general version would allow for variations in both of these assumptions.

Supply of talent

The initial assumption (included in the production function) is that there is a fixed supply of talent. This is known as a closed labour market and implies that an increase in talent by one team must correspond to an equal reduction in talent by the other team.

Quadrant 3. Talent Deployed Cost Function

Quadrant 3 in Figure 1 depicts the relationship between the quantity of talent and the cost of talent. To simplify the analysis it is assumed that the teams have no other costs.

The cost of talent depends on factors such as the effectiveness of the scouting and youth development activity of a club. In the simplest form, talent is considered to be homogenous and it is assumed that the only cost of talent is the wage rate. A more general version would allow for variations in talent which could correspond to the requirement of different positions or roles in a team and other costs of acquiring talent.
Figure 4 extends quadrant 3 of Figure 1. In the policy analysis literature the wage rate is assumed to be constant, regardless of the output level, and the same for all teams in the league.

In this case the total cost (\(TC\)) function for a team is given by:

\[ TC = w \times c \]

Where:
- \(w\) = Win proportion of team
- \(c\) = Wage rate

This is shown graphically in Figure 3a.

The marginal cost (\(MC\)) function for a team is given by differentiating the total cost function with respect to output (\(w\)) and is consequently equal to the wage rate, hence:

\[ MC = c \]

Where:
- \(c\) = Wage rate

This is shown graphically in Figure 3b.

Note that the wage rate is the rate paid above the ‘reservation wage’ required to keep talent in the market.

**Demand for talent**

The demand for talent function can be added to quadrant 3 of Figure 1. Its derivation is shown in Figure 4. The market demand function (Figure 4a) is taken from quadrant 1 of Figure 1 and combined with the marginal cost relationship from Figure 4b. It shows an initial position for profit maximising owners with a win proportion of \(w^1\) which is sold at a price of \(p^1\) and a wage rate of \(c^1\).

The production function (Figure 4b) is taken from quadrant 2 of Figure 1. For the win proportion \(w^1\), the corresponding quantity of talent is shown as \(t^1\).

The marginal revenue (product) of talent (Figure 4c) shows the resulting relationship between marginal changes in the quantity of talent and the revenue produced at a given market price. This is shown for the initial market price (\(p^1\)) by the line \(MRP^t(p^1)\) with point A showing the relationship between revenue and the initial quantity of talent at that market price.

The demand for talent function (Figure 4d and quadrant 3 of Figure 1) shows the relationship between the quantity of talent and the wage rate. The initial wage rate of \(c^1\) is shown to correspond to the initial quantity of talent of \(t^1\) at point A. The demand for talent function is then
derived by considering further initial positions. A second position is shown by considering, for example, an (exogenous) increase in the wage rate (i.e. cost of talent). This results in a marginal revenue product of talent function (Figure 4c) for the second market price \( p^2 \) shown as the line \( MRP^t(p^2) \) and a second corresponding wage rate of \( c^2 \) shown to correspond to the initial quantity of talent of \( t^2 \) at point B. The points formed as A and B (Figure 4d) are connected (by repetition of this procedure) to form the demand for talent function.

It is important to note that:

a) In equilibrium, profit maximising owners have the same marginal revenue of talent.
b) For the demand for talent to be the same as the marginal revenue of winning it is necessary that:
   - the market price is invariant to changes in the win proportion. This corresponds to an assumption of perfect competition in the product market. It equates the marginal revenue of talent to the demand for talent.
   - the marginal physical product of talent is equal to one.
Figure 4. Product market and Demand for talent

Profit maximising owners

(a) Marginal Revenue of WINS
Marginal Revenue depends on market demand.

(b) Production function
Converts units of talent into units of wins.

(c) Marginal Revenue (Product) of TALENT ($MRP^t$)
Marginal Physical Product of talent.

(d) Demand for Talent
Depends on
(a) Marginal revenue of talent
(b) Market demand
Quadrant 4. Owner Objective (Budget constraint)

Quadrant 4 in Figure 1 relates the revenue of a club to its cost.

a) Budget constraint and win maximisation

Professional sports clubs operating as commercial entities face a budget constraint. In the simplest form, it may be assumed that this is a requirement to, at least, break-even each year.

Figure 5 shows quadrant 4 of Figure 1 with the horizontal axis reversed. The break-even line shown is at an angle of forty five degrees denoting equality between revenue and cost.

A win maximising owner would choose a talent quantity with a total cost equal to the total revenue.

A more general version would accommodate a range of financial tolerance.

b) Profit maximisation

In the policy analytics literature the basic assumption is that the objective of owners is to maximise profit.

This is shown in Figure 6 by combining extensions of quadrants 1 and 3 from Figures 3b and 4b into a single diagram.

The profit maximising equilibrium for a club is given where:

\[ MR^w = MC \]

At this point, with the wage rate shown at \( c^* \), the club would choose a talent quantity to achieve the win proportion shown at \( w^* \).
2.2 Two club relationship model extension

Historically, the sports policy analysis literature has focussed on the effect of interventions in the product market (mainly in the form of revenue sharing schemes) and in the labour market (in the form of both quantity and price restrictions) in particular, on the competitive balance between clubs in a league competition. The two club model presented in the literature by Quirk and Fort (1992, Chapter 7) amongst others can be seen as an extension of the one club functional relationship model. Before introducing the sports policy interventions, it is instructive to consider the two club relationships with alternative assumptions for the owners’ objective and for the supply of talent. The analytical cases are shown below.

**Profit maximising owners**

The analytical model presented is based on a league comprising two teams (numbered with suffix subscripts 1 and 2). This simplification facilitates a graphical representation of the analytics. It is assumed that the structure of the league is fixed (i.e. the number of teams is fixed) and that the league is closed in the sense that these are the only two teams. As both teams produce an output represented by the proportion of games they win in the league, they have a combined total output of one. Furthermore, from Figure 6, as the total output is fully shared between the two teams, it enables one of the teams (Team 2) to be represented as its vertical mirror image on the same graph as the other team (Team 1) shown in Figure 7.

![Figure 7. Profit maximising league equilibrium](image-url)
It is assumed that the market demand (for wins) is fixed for both teams (so they cannot affect this, for example, by relocating to a different market area) and that the only difference between the teams is that Team 1 faces a greater market demand than Team 2. This is represented by the intercept of the marginal revenue function with the own team vertical access being greater for Team 1 (at \( a_1 \)) than for Team 2 (at \( a_2 \)). It is also assumed that the market demand (for wins) decreases as the output of wins increases at the same rate for both teams (i.e. for the parameter \( b = b_1 = b_2 \)).

Equilibrium for the league with profit maximising owners then achieved where:

\[
MR_1^w = MR_2^w
\]

This corresponds to a market clearing equilibrium wage (above the reservation wage) shown as \( c \) and an equilibrium win proportion output shown as \( \bar{w} \) in Figure 7. At the equilibrium the teams maximise their joint revenue.

**Win maximising owners**

Sloane (1971) highlights alternative owner objectives. Késenne (2007, Chapter 3.3.2, pp. 37-47), for example, provides the method of adopting the alternative assumption of win maximising owners with a break-even profit constraint in the model. This is shown in Figure 8 together with the comparative profit maximisation assumption.

**Figure 8. Win maximising owners**

![Figure 8. Win maximising owners](image-url)
With the win maximisation assumption owners seek to maximise average revenue (subject to the break-even constraint) and hence equilibrium in the model is shown at A in Figure 8 where:

\[ AR_1^w = AR_2^w \]

This corresponds to a market clearing equilibrium wage (above the reservation wage) shown as \( \tilde{c} \) and an equilibrium win proportion output shown as \( \bar{\nu} \) in Figure 8.

Note that, compared to the profit maximising owner assumption, with equilibrium at B in Figure 8, the model shows that the equilibrium wage rate is higher and the competitive balance has a higher win proportion for team 1 which is the team with the larger market.

**‘Closed’ v ‘Open’ labour market**

Whilst the basic model assumption of a closed labour market may be appropriate in some cases it is not generally applicable. Szymanski (2004), for example, provides the method of adopting the alternative assumption of an open (i.e. unlimited supply of talent assumption) labour market in the model.

The labour market supply assumption is incorporated in the production function. It is assumed that a unit increase in talent employed by a team resulted in a unit decrease in talent employed by the other team, with corresponding unit changes in the win proportion for both teams, as, with a closed labour market, the talent increase for a team must come from the other team. In this case the marginal revenue of wins functions for each team is the same as their respective marginal revenue of talent functions.

However, if a team is able to change its quantity of talent without affecting the quantity of talent employed by the other team, as in the case of an open labour market, the team’s marginal revenue of wins function is not the same as their marginal revenue of talent function.

The effect on the marginal revenue of talent of an open labour market is shown in Figure 9 together with that for the comparative closed labour market assumption. This shows that the effect of the open labour market assumption is to increase the equilibrium level of competitive balance (shown at win proportion \( \bar{\nu} \)) and decrease the equilibrium wage rate (shown at \( \tilde{c} \)) compared to the closed labour market assumption. The mathematical derivation of the marginal revenue of talent and the related production functions with both closed and open labour market assumptions is explained in Appendix A.
Figure 9. Open labour market

\[ MR_1^w = MR_1^t \text{ Closed} \]

\[ MR_2^w = MR_2^t \text{ Closed} \]

\[ MR_1^t \text{ Open} \]

\[ MR_2^t \text{ Open} \]

Win proportion (w)
3. Policy interventions

3.1 Product market schemes
Interventions in the product market considered here are revenue sharing schemes. To incorporate these schemes in the two club model it is necessary to consider both the source of the revenue (to be shared) and the distribution planned for the revenue (i.e. the share scheme). The analytical approach is to specify a team’s revenue after the share in terms of the revenue before the share. The specification has two components. One defines the revenue retained (i.e. not shared) and the other defines the revenue gained (i.e. from the revenue share scheme).

It is necessary to identify whether the source is from teams in the model or exogenously produced from, for example, the collective selling of broadcast rights. In the first case it is necessary to specify the revenue collection rule for the model. In the second, it is not. Consequently, the examples that follow will address the enogenous source case.

It is also necessary to specify the rule for the share scheme. The possibilities considered in the following examples address the cases where the revenue is distributed to:

- One team only
  This is the case with gate revenue sharing. It is used, for example, in the NFL.
- All teams equally
  This type of scheme was used by the Football League which imposed a 4% levy on revenue.
- All teams on merit
  This type of scheme is used, for example, with part of the distribution of the broadcast media rights sold collectively by the Premier League.

The analytic approach is shown below for three types of scheme. In each case the total revenue function is shown for team 1 and the total revenue function for team 2 can be obtained by symmetry. As such, the analytical approach is based on the two team extension of the product market quadrant from Figure 1.

(i) Gate (i.e. mutual) revenue sharing
With this type of scheme the revenue is collected from the ‘home’ team and distributed to the ‘away’ team. In this case the total revenue function for team 1 after the revenue share is:

\[ TR'_1 = \alpha TR_1 + (1 - \alpha) TR_2 \]

Where:
- \( TR'_1 \) = Total revenue of team \( i \) after the revenue share
- \( TR_1 \) = Total revenue of team \( i \) before the revenue share
- \( TR_2 \) = Total revenue of team \( j \) before the revenue share
- \( \alpha \) = The share proportion

The analytical approach is shown below for the cases where it is assumed that:

a) Profit maximising owners and a closed labour market
b) Win maximising owners
c) Open labour market
a) Profit maximising owners and a closed labour market

The analysis of this case is presented by, for example, Quirk and Fort (1992, Chapter 7, pp. 273-276). Equilibrium with two clubs before the introduction of the gate revenue sharing scheme (from Figure 7) is shown at point A in Figure 10.

Figure 10. Gate revenue share with profit maximising owners and closed labour market

Gate revenue sharing has the effect of shifting the marginal revenue of winning downward for both teams by the same amount and hence the equilibrium win proportion remains unchanged. The mathematical proof of this result is given in Appendix B. Vrooman (1995, p. 977) called this the “revenue sharing paradox”.

However, the gate revenue sharing scheme acts as a proportionate tax on winning and makes both teams less incentivised to invest in talent. Consequently, the demand for talent is reduced and the wage rate falls. The market clearing equilibrium wage rate falls to the reservation wage (shown at point B) if the gate revenue share is increased to 50 per cent of the home gate revenue (i.e. all revenue is shared equally between the teams).

Note that with the win proportion unchanged, revenue is unchanged but with the reduction in the wage rate the owners’ profit increases for both teams. In this case there is an incentive for the teams to collude in selecting their respective talent levels because their joint revenue is maximised at the relative win proportion where the marginal revenue of a win is the same for both teams.
b) Win maximising owners

The analysis of this case is presented by, for example, Késenne (2007, Chapter 6.2.2, pp. 110-114). Equilibrium with two clubs before the introduction of the gate revenue sharing scheme (from Figure 8) is shown at point A in Figure 11. Note that in equilibrium the average revenues of each team are equal to the total revenue of both teams combined.

![Figure 11. Gate revenue share with win maximising owners](image)

Gate revenue sharing has the effect of shifting the average revenue non-linearly for both teams (as shown in Figure 11). In equilibrium the average revenues of each team is still equal to the total revenue of both teams combined. However, as the revenue share is increased the equilibrium win proportion shifts towards increased competitive balance. It reaches perfect balance when the gate revenue share is increased to 50 per cent of the home gate revenue (shown at point B).

As the revenue share is increased the equilibrium wage rate increases until it reaches a maximum where total revenue is maximised (shown at point C). This is achieved at the win proportion that is the equilibrium for profit maximising owners. Vrooman (2007, p. 326) notes that in this case “intuition prevails over paradox”.

However, as the gate revenue share is increased further towards 50 per cent of the home gate revenue, whilst the competitive balance increases, the equilibrium wage rate decreases. The wage
rate with equal revenue share compared to the wage rate without the revenue share scheme is determined as follows:

- If slope of $AR_2 < \text{slope of } AR_1$, (i.e. $b_2 < b_1$) when revenue share reaches ‘equal shares’, the wage rate remains higher than ‘no share’ level ($\overline{c}$) at C.
- If slope of $AR_2 = \text{slope of } AR_1$, (i.e. $b_2 = b_1$) when revenue share reaches ‘equal shares’, the wage rate is equal to the ‘no share’ level ($\overline{c}$) at C.
- If slope of $AR_2 > \text{slope of } AR_1$, (i.e. $b_2 > b_1$) when revenue share reaches ‘equal shares’, the wage rate is below the ‘no share’ level ($\overline{c}$).

Hence, with win maximising owners and a gate revenue share that results in an equilibrium win proportion that is more competitively balanced than the profit maximising owners’ equilibrium, the wage rate could be higher, equal or lower than before the scheme is introduced.

The derivation of the equilibrium wage rate is shown at Appendix C.

c) Open labour market

The analysis of this case is presented by, for example, Szymanski (2004) and shown in Figure 12. With no gate revenue sharing, teams choose talent level to maximise own profits and equilibrium is where:

$$MR^t_1 = MR^t_2 \ [\text{shown at point A}]$$

With equal revenue sharing, teams choose talent level to maximise joint revenue (and hence profits) and equilibrium is where:

$$MR^w_1 = MR^w_2 \ [\text{shown at point B}]$$

Hence, as gate revenue sharing is introduced, equilibrium is a weighted average of the talent choices, competitive balance is reduced ($0.5 > \overline{w} > \overline{\overline{w}}$) and the wage rate is increased ($\overline{c} < \overline{\overline{c}}$).

---

**Figure 12. Gate revenue share with open labour market**

- $MR^w_1 = MR^t_1 \ [\text{Closed}]$
- $MR^w_2 = MR^t_2 \ [\text{Closed}]$
- $MR^t_1 \ [\text{Open}]$
- $MR^t_2 \ [\text{Open}]$
- $0.5 \overline{w} \overline{w}$
- $0 \overline{1}$

Win proportion (w)
(ii) Pool (i.e. equal) revenue sharing

The analysis of this case is presented by, for example, Vrooman (2007, pp. 320-321). With this type of scheme the revenue is collected from the ‘home’ team and distributed equally to all teams in the league. In this case the total revenue function for team 1 after the revenue share is:

\[ TR'_1 = \alpha TR_1 + (1 - \alpha) \left( \frac{TR_1 + TR_2}{2} \right) \]

Where:
- \( TR'_1 \) = Total revenue of team 1 after the revenue share
- \( TR_1 \) = Total revenue of team 1 before the revenue share
- \( TR_2 \) = Total revenue of team 2 before the revenue share
- \( \alpha \) = The share proportion

Profit maximising owners will be in equilibrium at the relative win proportion where the marginal revenue of a win is the same for both teams. This is shown before the introduction of the gate revenue sharing scheme at point A in Figure 13.

![Figure 13. Pool (i.e. equal) revenue share scheme](image)

For the two team model, the effect of the scheme is the same as the gate revenue sharing scheme except that in this case the market clearing equilibrium wage rate falls to the reservation wage (shown at point B) if the pool revenue share is increased to 100 per cent of the home gate revenue (i.e. all revenue is shared equally between the teams).
(iii) (Pool) Merit (‘winner-takes-all’) revenue sharing

The analysis of this case is presented by, for example, Vrooman (2007, p. 321). With this type of scheme the revenue is collected from the ‘home’ team and distributed equally to all teams in the league in proportion to their own win proportion. In this case the total revenue function for team 1 after the revenue share is:

\[ TR'_1 = \alpha TR_1 + (1 - \alpha)(TR_1 + TR_2) w_1 \]

Where:
- \( TR'_1 \) = Total revenue of team 1 after the revenue share
- \( TR_1 \) = Total revenue of team 1 before the revenue share
- \( TR_2 \) = Total revenue of team 2 before the revenue share
- \( \alpha \) = The share proportion
- \( w_1 \) = Win proportion of team 1

Profit maximising owners will be in equilibrium at the relative win proportion where the marginal revenue of a win is the same for both teams. This is shown before the introduction of the gate revenue sharing scheme at point A in Figure 14.

Figure 14. (Pool) Merit revenue share scheme
Pooled merit revenue sharing has the effect of shifting the marginal revenue non-linearly for both teams (as shown in Figure 14). As the revenue share is increased the equilibrium win proportion remains unchanged. The mathematical proof of this result is given in Appendix D.

However, the incentive provided by the scheme for teams to invest in talent has the effect of increasing the demand for talent and consequently the wage rate rises. The market clearing equilibrium wage rate rises to the total revenue of the teams combined (shown at point B) as the revenue share is increased to 100 per cent. At this point all revenue is allocated to wages (shown at ̄c) and the teams have a zero (break-even) profit.

If all the revenue is shared (i.e. α = 0) the ‘winner-takes-all’ (in proportion to wins).

3.2 Labour market schemes

Intervention in the labour market can be aimed at the allocation of talent between teams (i.e. controls over the quantity of talent employed by teams) or the price of talent (i.e. controls over the wage rate paid to players). This section considers the following type of policy intervention:

I. Restricted allocation of talent schemes (i.e. Quantity (of talent) controls)
   - ‘Reserve clause’ / ‘Retain and transfer’ / ‘Reverse order draft’
     The ‘reserve clause’ was used in the MLB and ‘retain and transfer’ system was used in professional association football until the late 1970’s. The ‘reverse order draft’ has operated in all Major US sports leagues since the 1950’s.

II. Restricted price of talent schemes (i.e. salary caps)
   (i) % of Average team revenue
     This type of scheme has been introduced as the Short Term Cost Protocol by the Premier League in 2013. It is used in the NBA (since 1984/85) and in the NFL (since 1994).

   (ii) % of Own team revenue
     This type of scheme was proposed by the G14 in 2002 and has been introduced as the Salary Cost Management Protocol by the Football League in League 2 in 2004/05 and in League 1 in 2011/12.

   (iii) Luxury tax (‘soft’ cap)
     This type of scheme was introduced for the MLB in 1996 (withdrawn in 1999 and reintroduced in 2003).

   (iv) Individual player (maximum wage)
     This type of scheme was in operation in English professional football from the acceptance of professional contracts by the Football Association in 1885 until it was abolished in 1961.

In each case, the analytical approach is based on the two team extension of the labour market quadrant (3) from Figure 1.
I. Restricted allocation of talent schemes (i.e. Quantity (of talent) controls)

Reserve clause / Retain and transfer / Reverse order draft

These three types of scheme all restrict the quantity of talent that is available to teams. As such the analytical approach is the same for each and is shown in Figure 15.

![Figure 15. Restricted allocation of talent schemes](image)

The profit maximising equilibrium (shown at point A) has a talent share of $\bar{t}$. If the policy is intended to increase the competitive balance between the teams it will aim to restrict the talent available to team 1 and increase the talent available to team 2. Note that, assuming that the supply of talent is fixed (i.e. with the closed labour market assumption of the basic model) the quantity difference will be the same for both teams.

Consider the case where the policy is intended to result in an equal balance between the teams. This is shown by the vertical line at the talent share of 0.5. At this allocation of talent the wage rate is at $\bar{c}$ as, although team 1 would prefer to offer a higher wage rate (shown at $\bar{c}_1$) and buy more talent (and win more), it is not able to buy more talent and $\bar{c}$ is the maximum that team 2 will pay for talent at the level of the policy.
The difference between the wage rate that the teams are prepared to pay at the talent share of the policy creates an incentive for the teams to trade talent (‘cheat’) until the equilibrium talent share returns to the profit maximising equilibrium (shown at point A) with a talent share of $\tilde{\ell}$.

This result was noted by Rottenberg (1956) and is known as the ‘invariance principle’. He argued that, in this case, if the economic rights to the players’ talent were owned by the teams (rather than the players) the wage rate would not be bid up to the market equilibrium at $c$ but that team 1 would transfer the benefit gained from the increased win proportion to team 2 as the incentive to redistribute talent to team 1.

II. Restricted price of talent schemes (i.e. salary caps)

The analytic approach is shown below for four types of salary cap scheme.

(i) % of Average team revenue

The analysis of this case is presented by, for example, Késenne (2000). With this type of scheme the total wage cost ($c \times t_1$) capped to a fixed percentage ($\gamma$) of the average revenue of all the teams in the league. In this case, with two teams in the league, the total wage cost maximum for each team ($cap_i$) is:

$$cap_i = Max \left\{ \gamma \left( \frac{TR_1 + TR_2}{2} \right) \right\}$$

Hence, the capped wage cost function can be included in the analytical model for team 1 (and by symmetry for team 2) as:

$$c = \gamma \left( \frac{TR_1 + TR_2}{2} \right) t_1$$

This is shown for both team 1 and team 2 in Figure 14. Before the cap is introduced the profit maximising equilibrium is shown at point A. The cap is binding on a team if it is below the team’s demand for talent function. In Figure 16 the level of the cap set is shown as binding on team 1 for a range of its demand for talent function whilst it is not binding on team 2 at any quantity of talent demanded. Consequently the equilibrium with this level of salary cap will be where the cap function for team 1 is equal to the (uncapped) demand for talent function for team 2 (shown at point B). The salary cap has the effect of increasing the competitive balance (from $\tilde{\ell}$ to $\tilde{\ell}$) and reducing the wage rate (from $\tilde{c}$ to $\tilde{c}$).

Note again that the difference between the wage rate that the teams limited by the salary cap are prepared to pay at the talent share resulting from the policy creates an incentive for the teams to trade talent (‘cheat’) until the equilibrium talent share returns to the profit maximising equilibrium (shown at point A) with a talent share of $\tilde{\ell}$.  

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If the cap is set at a level where it binds both teams, or if it also imposed as minimum spend, the equilibrium will be where the share of talent (and competitive balance) is equal (i.e. 0.5) and both teams have same salary spend (shown, for example, at point C).

(ii) % of Own team revenue

The analysis of this case is presented by, for example, Késenne (2003). With this type of scheme the total wage cost \((c \times t_i)\) capped to a fixed percentage \((\delta)\) of each team’s own revenue \((TR_i)\). The total wage cost maximum for each team \((cap_i)\) is:

\[
cap_i = \delta TR_i
\]

Hence, the capped wage cost function can be included in the analytical model as:

\[
c = \frac{\delta TR_i}{t_i}
\]

Or equivalently for team 1 (and by symmetry for team 2) as:

\[
c = \delta AR_1
\]

As the level of the cap is increased (i.e. a lower limit is applied) in equilibrium the cap binds on team 2 before it binds on team 1. This is shown in Figure 17.
Note that the cap (shown by $\delta AR$ curve) for each team is below its own average revenue function and that, as it is a constant proportion of the team’s revenue, the divergence decreases in absolute terms as the level of talent employed by the team increases.

The demand for talent for team 1 is shown by the kinked line efg and the demand for talent for team 1 is shown by the kinked line rAs. The cap becomes binding for team 2 at the profit maximising talent share $\tilde{\ell}$ and equilibrium with team 1 shown at point A.

As the cap is increased further the equilibrium follows the path from point A, along the demand for talent curve for team 1, to point B in Figure 18 where it also becomes binding on team 1.
At the level of salary cap where it also becomes binding for team 1, the demand for talent for team 1 is shown by the kinked line $e'bg$ and the demand for talent for team 1 is shown by the kinked line $r'u's$.

As the cap is increased from the level where it is binding on team 2 to the level where it is binding on both teams it can be seen that, in equilibrium, the talent share for team 1 increases and hence competitive balance decreases. Consequently the policy tends to ossify the league. The reduction in competitive balance reaches a limit at $\tilde{t}$, which is the win maximising owners’ equilibrium (where average revenue is equal for both teams). It can also be seen that the wage rate is reduced. Note that the equilibrium wage rate is lower than would result with win maximising owners if there were no salary cap.

As the cap is increased further (i.e. binding on both teams) the equilibrium follows the path from $B$ to $\tilde{t}$. Hence there is no further effect on competitive balance but the wage rate continues to be reduced.
(iii) Luxury tax (‘soft’ cap)
The analysis of this case is presented by, for example, Késenne (2007, Chapter 7.3, pp. 134-136).
With this type of scheme a proportional tax rate \( (\tau) \) is applied to total wage costs \((c \times t_1)\) in excess of the ‘soft’ cap \((C)\). To show the analytical treatment of this scheme, consider the case where it becomes binding on team 1 only. This is shown in Figure 19.

When the total wage spend of team 1, for example, exceeds the cap its marginal cost becomes:
\[
MC_1 = (1 + \tau)c
\]

The profit maximising condition \((MR_1 = MC_1)\) for team 1 becomes:
\[
\left(\frac{\partial R_1}{\partial t_1}\right) = (1 + \tau)c
\]

Hence, the demand for talent function for team 1 becomes:
\[
c = \left(\frac{\partial R_1}{\partial t_1}\right) \left(\frac{1}{1 + \tau}\right)
\]

The demand for talent for team 1 becomes the kinked line shown as kBm with the constraint becoming effective as the talent share increase above point B. Then the equilibrium with team 2 is where:
\[
\left(\frac{\partial R_1}{\partial t_1}\right) \left(\frac{1}{1 + \tau}\right) = \left(\frac{\partial R_2}{\partial t_2}\right)
\]

This is shown at point C. The scheme has the effect of increasing the competitive balance (to a talent share of \(\tilde{c}\) and of reducing the equilibrium wage rate to \(\tilde{c}\).

**Figure 19. Luxury tax**
(iv) Individual player (maximum wage)

The analysis of this case is presented by, for example, Késenne (2002). With this type of scheme the maximum wage cost for an individual player has a limit (\(c^I\)). This is shown in Figure 20 by a horizontal line at the wage rate limit. It is effective if it is set below the market wage rate (shown at \(\tilde{c}\)). In this case the profit maximising equilibrium for team 1 i.e. where: \(c^I = D_1^t\) is shown at point A but the profit maximising equilibrium for team 2 i.e. where: \(c^I = D_2^t\) is shown at point B. Hence, the equilibrium share of talent is not determined by the model.

Késenne (2002) suggests that it may be expected that the actual outcome is closer to A, if the wage capped players prefer to play for the more successful team, and hence competitive balance will be reduced.

Note, again, that there is an incentive for teams to ‘cheat’ until equilibrium is reached with the talent share at \(\tilde{\tau}\).

Figure 20. Individual player salary cap
3.3 Financial regulation

FFP is a more recent form of policy intervention. The UEFA (2010) FFP regulations address two issues of concern for professional football competition. In both cases the risk is that the integrity of sporting competition is damaged. This negative externality may reduce interest, and hence demand, for the competition and this can adversely affect spectator attendance, broadcast media value, other commercial income and investor interest in the competition.

One issue is the concern that clubs overspend and are not able to finance the deficit. This may result in the direct economic failure of the sporting entity with unpaid creditors and possibility an inability to fulfil competition fixtures. It also has an indirect risk of contagion of economic impact on other sporting entities in the competition resulting from a failure to fulfil fixtures (given the ‘joint product’ inherent in competition) and more widely due to non-payment of transfer fees to other sporting entities (Lago et al., 2006).

The other issue is the concern that clubs overspend and, whilst they are able to finance the deficit, the amount is sufficiently large to be considered as resulting in an ‘undue’ advantage in the sporting competition. In this case the integrity of the sporting competition is undermined by the ‘off field’ economic competition for financial resource. The term ‘financial doping’ was used by UEFA Chief Executive Lars-Christer Olsen to describe this situation (1).

Figure 21 illustrates the concerns considered by UEFA (2010) in the FFP regulations.

**Figure 21. Club Overspend Competition Consequences Diagram**
Club Licencing was initially introduced by UEFA in 2004 with the aim of raising the standards of professional football clubs in five categories given as,

i. Sporting
ii. Infrastructure
iii. Personnel and administrative
iv. Legal
v. Financial

An ‘overdue payments’ rule was introduced \(^{[2]}\) as a financial criteria to limit the systemic risk of contagion between clubs resulting from unpaid transfer fees by a club that experiences economic failure.

The UEFA (2010) FFP regulations introduced a ‘break-even’ requirement for clubs participating in its competitions \(^{(3)}\). In essence, this measure is the requirement that the ‘relevant income’ of a club is at least equal to its ‘relevant expense’ \(^{(4)}\). UEFA recognise that the clubs may incur debt requiring external finance and that, from time to time, they may incur losses. The UEFA regulations deem that up to five million euros in the monitoring period is acceptable. They also allow a limited amount of further loss to be offset in the break-even calculation if it is financed by equity (i.e. the issue of new shares) and / or related parties (i.e. unconditional gifts). It is important to note that debt financing (i.e. with the obligation of repayment) is not allowed to offset any loss. Further allowance is given for expenditure on infrastructure, youth development and community development activities.

The model presented in Figure 1 can be used to analyse the FFP break-even sports policy intervention. Consider, for example, the situation shown in Figure 1 where a club that intends to spend \(x^1\) to deploy an amount of \(t^1\) of talent in competition (quadrant 3). This is expected to result in the win proportion \(w^1\) (quadrant 2) which would produce an income of \(y^1\) for the club (quadrant 1). In the simple case where a club has no other costs, and it has an objective to break-even the constraint is represented (quadrant 4) by a line at an angle of (minus) forty five degrees. It would achieve this if the level of income was greater or equal to the corresponding expenditure shown at \(x^1\) in quadrant 4 of Figure 1.

It can readily be seen that a change to one of the specific functions could result in an expected profit or loss. In the latter case a club may assess that, in order to comply with the break-even requirement, it is necessary to reduce its wage cost. In the short term the club may face downward rigidity in wages (quadrant 3) due to fixed term contracts with players and so it may be necessary for the club to sell some of its players. Since the strength of the playing squad has been shown to be closely correlated with competitive results \(^{(5)}\), this may be expected to cause the club to achieve a lower win proportion (quadrant 2). This would result in a reduced income (quadrant 1). In some cases, the income reduction could be greater than the wage cost reduction (quadrant 4) and the club could experience the perverse result of a worse financial situation than they would have achieved if they had taken no action. In the case the effect of the financial regulation could be a ‘death spiral’ for a club.

This illustrates the dynamic relationship between the main variables in the context of the FFP regulation.
4. Conclusion

The functional relationship model consolidates the techniques presented in the literature to analyse both the product and labour market policy interventions with particular focus on their effects on the sporting competitive balance. It also provides the capability to analyse the effects of the more recent financial regulation introduced for clubs in some (association) football competitions.
Appendix A

**Deriving the effect of the production function and marginal revenue of talent functions**
(for 2-team model) with closed and open labour market assumptions

Every team chooses the talent quantity \( t \) they want. The choices of both teams determine the win proportion of each team as follows:

\[
w_1 = \frac{t_1}{t_1 + t_2} \quad w_2 = \frac{t_2}{t_1 + t_2}
\]

This relationship is called the **Contest Success Function**.

The production function shows the relationship between the win proportion of a team and its own quantity of talent employed. The effect of the talent choice on the win proportion for say, team 1 \( \left( \frac{\partial w_1}{\partial t_1} \right) \) is given by the partial differentiation of \( w_1 \) with respect to \( t_1 \) (i.e. on the assumption that \( t_2 \) is held constant).

In general, the rule for the partial differentiation of a quotient is:

\[
\frac{\partial}{\partial x} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}
\]

Here:

\[ f(x) = t_1 \quad \text{and} \quad g(x) = t_1 + t_2 \]

So:

\[ f'(x) = 1 \]

But, note that \( t_2 \) depends on \( t_1 \) so:

\[ g'(x) = 1 + \frac{\partial t_2}{\partial t_1} \]

Then:

\[
\frac{\partial w_1}{\partial t_1} = \frac{1 	imes (t_1 + t_2) - t_1 \times (1 + \frac{\partial t_2}{\partial t_1})}{[t_1 + t_2]^2}
\]

\[
\frac{\partial w_1}{\partial t_1} = \frac{t_2 - t_1 \left( \frac{\partial t_2}{\partial t_1} \right)}{[t_1 + t_2]^2}
\]

This shows how the effect of the talent choice by team 1 on the win proportion \( \left( \frac{\partial w_1}{\partial t_1} \right) \) depends on the talent choice of team 2 \( \left( \frac{\partial t_2}{\partial t_1} \right) \).

If the total talent supply is normalised so that: \( t_1 + t_2 = 1 \)

And

a) The closed labour market assumption is made (i.e. \( \frac{\partial t_2}{\partial t_1} = -1 \))

The production function becomes:

\[
\frac{\partial w_1}{\partial t_1} = 1
\]
Or

b) The open labour market assumption is made (i.e. $\frac{\partial t_2}{\partial t_1} = 0$)

The production function becomes:

$$\frac{\partial w_1}{\partial t_1} = (1 - w_1)$$

Since the two team model has the following ‘adding up’ constraint:

$$w_1 + w_2 \equiv 1$$

The production function is equivalently:

$$\frac{\partial w_1}{\partial t_1} = w_2$$

Finally, since:

$$MR^t = MR^w \times \frac{\partial w}{\partial t}$$

a) With the closed labour market assumption:

$$MR^t = MR^w \times 1$$

$$MR^t = MR^w$$

And

b) With the open labour market assumption:

$$MR^t = MR^w \times w_2$$

The equivalent results for team 2 can be obtained by symmetry.
Appendix B

Proof of independence of gate revenue share on competitive balance
(for 2-team model) with profit maximising owners and closed labour market assumptions
Based on Quirk and Fort (1992, p. 273-276)

1) Let:
   $TR'_i = \text{Revenue of team } i \text{ after sharing}$
   $TR_i = \text{Revenue of team } i \text{ from home game}$
   $\alpha = \text{Retained share (of home gate)}$

2) For team 1:
   a) Total revenue (after sharing) for team 1 is:
      $$TR'_1 = \alpha TR_1 + (1 - \alpha) TR_2$$
   b) Marginal revenue of winning for team 1 is:
      $$\frac{\partial TR'_1}{\partial w_1} = \alpha \frac{\partial TR_1}{\partial w_1} + (1 - \alpha) \frac{\partial TR_2}{\partial w_1}$$
   c) From the adding up constraint for win proportions:
      $$w_1 + w_2 \equiv 1$$
      $$\partial w_1 = -\partial w_2$$
   d) Then:
      $$\frac{\partial TR'_1}{\partial w_1} = \alpha \frac{\partial TR_1}{\partial w_1} - (1 - \alpha) \frac{\partial TR_2}{\partial w_2}$$

Hence, marginal revenue of winning for team 1 is:
$$MR'_1 = \alpha MR_1 - (1 - \alpha) MR_2$$

3) For team 2:
   By symmetry, marginal revenue of winning is:
   $$MR'_2 = \alpha MR_2 - (1 - \alpha) MR_1$$

4) In revenue share equilibrium:
   $$MR'_1 = MR'_2$$

Hence:
$$\alpha MR_1 - (1 - \alpha) MR_2 = \alpha MR_2 - (1 - \alpha) MR_1$$

Expanding:
$$\alpha MR_1 - MR_2 + \alpha MR_2 = \alpha MR_2 - MR_1 + \alpha MR_1$$

$\alpha$ terms cancel, leaving:
$$MR_1 = MR_2$$

Hence, competitive balance is independent of $\alpha$
Appendix C

Derivation of equilibrium wage rate with gate revenue sharing
(for 2-team model) with win maximising owners assumption

1) Let:
\( TR_i' = \text{Revenue of team } i \text{ after sharing} \)
\( TR_i = \text{Revenue of team } i \text{ from home game} \)
\( \alpha = \text{Retained share (of home gate)} \)
\( c = \text{Wage rate} \)

2) Total revenue (with sharing scheme) for team \( i \) is:
\[ TR_i' = \alpha TR_i + (1 - \alpha) TR_j \]

3) Average revenue of winning
   a) For team 1:
   \[ AR_i' = \frac{1}{w} [\alpha TR_i + (1 - \alpha) TR_j] \]
   \[ AR_1' = \frac{1}{w} [\alpha w AR_1 + (1 - \alpha)(1 - w)AR_2] \]
   b) For team 2:
   \[ AR_2' = \frac{1}{(1 - w)} [\alpha TR_2 + (1 - \alpha) TR_1] \]
   \[ AR_2' = \frac{1}{(1 - w)} [\alpha(1 - w)AR_2 + (1 - \alpha)w AR_1] \]

4) Selecting a target level of competitive balance determines:
   a) Required revenue retained share given by setting \( AR_1' = AR_2' \):
   \[ \alpha = \frac{w^2 AR_1 - (1 - w)^2 AR_2}{w AR_1 - (1 - w)AR_2} \]
   b) Equilibrium wage rate given by substituting for \( \alpha \) in \( AR_1' \) or \( AR_2' \):
   \[ c^* = w AR_1 + (1 - w)AR_2 \]

5) Wage rate \( c^* \) reaches a maximum where:
   \[ \frac{\partial c^*}{\partial w} = 0 \]
   i.e.
   \[ [AR_1 + \frac{\partial AR_1}{\partial w} w] - [AR_2 + \frac{\partial AR_2}{\partial w} (w - 1)] = 0 \]
   Which is identical to the profit maximising owner equilibrium where:
   \[ MR_1 w - MR_2 w = 0 \]
   Where: \( MR_i = \text{Marginal revenue of winning for team } i \)
   Solving for \( w^* \) and substituting the value into equation 4a (above) yields the retained revenue share (\( \alpha^* \)) that produces the maximum equilibrium wage rate.
Appendix D

Proof of independence of merit (pool) revenue share on competitive balance
(for 2-team model) with profit maximising owners and closed labour market assumptions

1) Let:
\[ TR'_i = \text{Revenue of team } i \text{ after sharing} \]
\[ TR_i = \text{Revenue of team } i \text{ from home game} \]
\[ \alpha = \text{Retained share (of home gate)} \]

2) For team 1:
   a) Total revenue (after sharing) for team 1 is:
   \[ TR'_1 = \alpha TR_1 + (1 - \alpha)(TR_1 + TR_2)w_1 \]
   b) Marginal revenue of winning for team 1 is:
   \[
   \frac{\partial TR'_1}{\partial w_1} = \alpha \frac{\partial TR_1}{\partial w_1} + (1 - \alpha)\left[ \left( \frac{\partial TR_1}{\partial w_2}w_1 + TR_1 \right) + \left( \frac{\partial TR_2}{\partial w_1}w_1 + TR_2 \right) \right]
   \]
   [Note: \( TR_xw_1 \) differentiated as product terms]
   c) From the adding up constraint for win proportions:
   \[ w_1 + w_2 \equiv 1 \]
   \[ \partial w_1 = -\partial w_2 \]
   \[ MR'_1 = \alpha MR_1 + (1 - \alpha)(MR_1 - MR_2)w_1 + (1 - \alpha)(TR_1 + TR_2) \]

3) For team 2:
   By symmetry, marginal revenue of winning is:
   \[ MR'_2 = \alpha MR_2 + (1 - \alpha)(MR_2 - MR_1)w_2 + (1 - \alpha)(TR_1 + TR_2) \]

4) In revenue share equilibrium
   \[ MR'_1 = MR'_2 \]
   Hence
   \[ \alpha MR_1 + (1 - \alpha)(MR_1 - MR_2)w_1 = \alpha MR_2 + (1 - \alpha)(MR_2 - MR_1)w_2 \]
   [Note: \((1 - \alpha)(TR_1 + TR_2)\) terms cancel]
   From the adding up constraint for win proportions:
   \[ w_2 = 1 - w_1 \]
   And expanding
   \[
   \alpha MR_1 + MR_1w_1 - MR_2w_1 - \alpha MR_1w_1 + \alpha MR_2w_1
   = \alpha MR_2 + MR_2 - MR_1 - \alpha MR_2 + \alpha MR_1 - MR_2w_1 + MR_1w_1 + \alpha MR_2w_1
   - \alpha MR_1w_1
   \]
   \[ \alpha \text{ terms and } w_1 \text{ terms cancel, leaving:} \]
   \[ MR_1 = MR_2 \]

Hence, competitive balance is independent of \( \alpha \)
Notes


(2) This was enhanced by UEFA, 2012, Articles 65-66.

(3) This is updated in UEFA 2012, Articles 58-63.

(4) ‘Relevant income’ and ‘relevant expense’ is defined in UEFA, 2012, Article 58 and Annex X.


References


