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Hart, Sarah and Rowley, P.J. (2014) Corrigendum to "Involution products in Coxeter groups" [J. Group Theory 14 (2011), no. 2, 251–259]. Journal of Group Theory 17 (2), pp. 379-380. ISSN 1435-4446.

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**Corrigendum**  
**Involution Products in Coxeter Groups**  
**J. Group Theory 14 (2011), no.2, 251 - 259**

S.B. Hart and P.J. Rowley

In [1], Theorem 2.4 states a well-known result on Coxeter groups which gives conditions under which the stabilizer of a nonzero vector is a proper parabolic subgroup. However the hypothesis of this result is incorrectly stated in our paper: it holds for finite Coxeter groups but is not true in general for infinite Coxeter groups. We are grateful to an anonymous referee of a subsequent paper for pointing this out. As a consequence, the proof of Theorem 1.1 in [1], which uses Theorem 2.4, is incomplete. Here we complete the proof of Theorem 1.1 without recourse to Theorem 2.4.

Theorem 1.1 states that if  $X$  is a strongly real conjugacy class of a Coxeter group  $W$  (not necessarily finite), then there exists  $w_* \in X$  such that  $e(w_*) = 0$ . That is to say, there are involutions  $\sigma, \tau$  of  $W$  such that  $w_* = \sigma\tau$  and  $\ell(w) = \ell(\sigma) + \ell(\tau)$ . At the point in the proof where Theorem 2.4 is used, we have established that  $zy$  is an element of  $X$  where  $z$  and  $y$  are involutions with the following properties. First,  $y$  is the central involution of some standard parabolic subgroup  $W_J$  of  $W$ . The involution  $z$  has the property that  $\ell(gzg^{-1}) \geq \ell(z)$  for all  $g \in W_J$ . It follows that if  $\ell(zr) < \ell(z)$  for any  $r \in J$ , then  $rzs = z$  and  $z \cdot \alpha_r = -\alpha_r$ .

Now let  $K = \{r \in J : \ell(zr) < \ell(z)\}$ . From the above we know that  $z \cdot \alpha_r = -\alpha_r$  for all  $r \in K$ . If  $K$  is nonempty then, as  $\Phi_K^+ \subseteq N(z)$ ,  $\Phi_K^+$  is finite. Therefore  $W_K$  has a unique longest element  $w_K$ , which is an involution, and  $N(w_K) = \Phi_K^+$ . If  $K = \emptyset$  we set  $w_K = 1$ . In all cases, since  $y$  is central in  $W_J$  and  $w_K \in W_J$ , we see that  $w_Ky = yw_K$  is an involution. Moreover  $zr = rz$  for all  $r \in K$ , and thus  $zw_K$  is also an involution. Let  $\sigma = zw_K$  and  $\tau = w_Ky$ . Then  $\sigma\tau = zy \in X$ . Moreover  $z$  and  $y$  both act as  $-1$  on  $\Phi_K^+$ . Thus, by Lemma 2.2,

$$N(\sigma) = N(z) \setminus [-z \cdot N(w_K)] = N(z) \setminus N(w_K)$$

and

$$N(\tau) = N(y) \setminus [-y \cdot N(w_K)] = N(y) \setminus N(w_K) = \Phi_J^+ \setminus N(w_K).$$

Consider  $r \in J$ . If  $r \in K$ , then  $\alpha_r \in N(w_K)$  and so  $\alpha_r \notin N(z) \setminus N(w_K) = N(\sigma)$ . On the other hand if  $r \in J \setminus K$  then by definition of  $K$ ,  $\alpha_r \notin N(z)$  and hence  $\alpha_r \notin N(\sigma)$ , which is after all a subset of  $N(z)$ . Hence for all  $r \in J$  we have  $\alpha_r \notin N(\sigma)$ . This implies that  $N(\sigma) \cap \Phi_J^+ = \emptyset$ , because every positive root in  $\Phi_J^+$  is a positive linear combination of some  $\alpha_r, r \in J$ . But  $N(\tau) \subseteq \Phi_J^+$  and therefore  $N(\sigma) \cap N(\tau) = \emptyset$ . Hence, by Lemma 2.2,  $\ell(\sigma\tau) = \ell(\sigma) + \ell(\tau)$ . Setting  $w_* = \sigma\tau$  we have  $w_* \in X$  and  $e(w_*) = 0$ , so completing the proof of Theorem 1.1.  $\square$

## References

- [1] S. B. Hart and P. J. Rowley *Involution Products in Coxeter Groups*, J. Group Theory 14 (2011), no.2, 251 - 259.