ISSN 1745-8587

Birkbeck UNIVERSITY OF LONDON

Department of Economics, Mathematics and Statistics

BWPEF 1602

Factor models in panels with cross-sectional dependence: an application to the extended SIPRI military expenditure data

Ron P. Smith
Birkbeck, University of London

Elisa Cavatorta
King's College London

April 2016

Birkbeck Working Papers in Economics & Finance

Factor models in panels with cross-sectional dependence: an application to the extended SIPRI military expenditure data *

Elisa Cavatorta
King's College London

Ron P. Smith
Birkbeck, University of London

April 4, 2016

Abstract

Strategic interactions between countries, such as arms races, alliances and wider economic and political shocks, can induce strong cross-sectional dependence in models of military expenditures using panel data. If the assumption of cross-sectional independence fails, standard panel estimators such as fixed or random effects can lead to misleading inference. This paper shows how to improve estimation of dynamic, heterogenous, panel models of the demand for military expenditure allowing for cross-sectional dependence in errors using two approaches: Principal Components and Common Correlated Effect estimators. Our results show that it is crucial to allow for cross-section dependence and there are large gains in fit by allowing for both dynamics and between country heterogeneity in demand models of military expenditures. Our estimates show that mean group estimation of error correction models using the Common Correlated Effect approach provides an effective modelling framework.

JEL Category: C33, C82, H56
Keywords, Military Expenditures, Panel Data, Factor models.

*This paper is part of a larger SIPRI project supported by Sweden's Riksbanken Jubileumsfond.

1 Introduction

Models of military expenditures have to deal with strategic interactions between countries that induce cross-section dependence in panel data. These can arise for a variety of reasons: arms races between hostile countries; burden sharing within alliances; security-web interactions within regional networks; and economic and political shocks, like terrorist attacks, affecting the general perception of threat for all units. These strategic interactions generate unobserved common factors in empirical models of military expenditures which make the errors correlated across cross-sectional units. If the unobserved strategic factors that cause the cross-sectional dependence are correlated with regressors of interest, such as income, then the estimated coefficients of those regressors will be biased and inconsistent.

While the importance of cross-section dependence has been widely recognised in empirical arms race and alliance models, the issue has usually been addressed in the context of quite restrictive models. For instance, Dunne et al. (2008) rely on qualitative judgements about a country's security-web to allow the military expenditures of a country's neighbours to be aggregated. This approach relies on identifying allies and enemies and deciding ad-hoc weights in the aggregation procedure. An alternative approach, recently adopted in the panel literature, assumes that the cross-sectional dependence can be characterised by a finite number of unobserved common factors affecting all units with different intensities. This approach can be implemented empirically if there exist proxies for the common factors. One way to obtain proxies is by extracting cross-sectional commonalities in military expenditures using Principal Component Analysis (PCA). PCA estimates the linear combinations of military expenditures and factor loadings that account for most of the variation in the data. A number of these linear combinations can be included in a demand model to control for cross-sectional dependence and estimation can be achieved using standard least squares or maximum likelihood estimation. A second way to obtain proxies, known as Common Correlated Effect (Pesaran, 2006), consists of approximating the unobserved common factor using cross-section averages of the dependent and independent variables. One advantage of this approach is that is easily implementable, it yields consistent estimates under a variety of conditions – e.g. serial correlation in errors, contemporaneous correlation between regressors and unobserved factors, spatial and temporal correlations as shown by Coakley et al. (2006); Kapetanios et al. (2011); Pesaran and Tosetti (2011)– and the estimates have easy interpretation.

These approaches require panel data with large N and large T. SIPRI provides the most reliable and widely used series of military expenditures data in academic research on defence economics, but until recently had the disadvantage that it started in 1988. The new release of SIPRI military expenditure data since 1945 enables the application of large T panel techniques and the implementation of factor models that allow for cross-sectional dependence in errors.

In this paper, we develop a dynamic model of the demand for military expenditure where there is error cross-sectional dependence due to unobserved common factors generated by strategic interactions. This analysis develops the approach used in Cavatorta (2010) for the MENA region and builds on recent panel time-series procedures surveyed by Chudik and Pesaran (2015*b*) and given a text-book treatment in Pesaran (2015).

We show how to improve estimation of dynamic models of military expenditures with cross-sectional dependence in errors using Principal Components (PCs) and Common Correlated Effect (CCE) estimation procedures and compare the estimates with standard estimation techniques which ignore the issue. Unlike the standard fixed effect approach, we allow for heterogeneity between countries, both in regression coefficients and in the impact of the unobserved strategic factors. We also examine differences between the Cold War period and the post-Cold War period and between different regions. Our results show that it is crucial to allow for cross-section dependence, doing so, whether by PCs or CCE methods, substantially improves the fit. The results also show that it is crucial to allow for both dynamics and between country heterogeneity, both of which also substantially improve the fit. The gain in fit in the dynamic model from allowing for a structural break at the end of the Cold War is less substantial.

This paper makes two distinct contributions to the existing literature in defence economics. Firstly, it provides a pedagogical guide for empirical researchers interested in estimating dynamic, heterogeneous, panel models of the demand for military expenditure allowing for cross section dependence induced by unobserved common factors. This dependence is likely to be an issue at different levels of analysis: changes in military expenditures can be the result of global strategic interactions and world-wide economic or technological shocks as well as regional threat level and economic changes. Secondly, the paper suggests that it is important for researchers using the extended SIPRI dataset to take account of dynamics, heterogeneity and cross-section dependence when choosing their specification. In particular, mean group estimation of error correction models augmented by cross-section averages seems to provide an effective modelling framework.

The paper proceeds as follows: Section 2 discusses the data issues and provides summary statistics. Section 3 sets out the basic theoretical framework of factor models. Section 4 uses the PCs and CCE procedures to estimate static factor models that attempt to determine the relative influence of economic and strategic factors on the shares of military expenditure. Section 4.1 provides estimates of the principal components to provide an indication of the number of common factors driving relative investments in military expenditure at the level of the world and the individual regions and Section 4.2 discusses the results. Section 5 allows for dynamics both in PCs and CCE models. Section 6 provides some conclusions and suggestions for further research.

2 Data Issues

The SIPRI military expenditure data are the most widely used series in academic research on military expenditures. They have the disadvantage that, up to now, SIPRI has only provided consistent data from 1988. This is quite a short post Cold War sample and researchers have tried to extend the data either by splicing to earlier, unapproved, SIPRI series or to Correlates of War, COW, series, neither of which are quite consistent with the later SIPRI authorised series. Brauner (2015) discusses the combination of SIPRI and COW series. The availability of a database with a longer alpha-test version of the authorised SIPRI series thus enables some more interesting explorations of the data.

SIPRI provides three series. D_{it} a domestic measure of military expenditure in local currency at current prices; M_{it} real military expenditure in constant US prices and exchange rates of a base year; and the share (or military burden) $S_{it} = M_{it}/Y_{it}$ military expenditure as a percentage of Gross Domestic Product, GDP. The GDP series that SIPRI use to construct the share, Y_{it}, is also available in the database.

There are two ways that one could convert D_{it} into M_{it}. Set $t = b$ for the base year and use P_{it} for the domestic price index and P_t^* for the US price index and E_{it} for the countries exchange rate against the US dollar, with E_{ib} being the value in the base year. Then the constant price and exchange rate series is

$$M_{it} = \frac{D_{it}}{P_{it}E_{ib}}.$$

This is the procedure that SIPRI uses and it has the advantage that M_{it} maintains the time-series properties of D_{it}/P_{it} real military expenditure in the country, since it is only scaled by a constant the base year exchange rate. However changes in the base year can cause large apparent movements in the estimate of military expenditure in constant dollars for a particular year given the volatility of exchange rates.

An alternative procedure would convert the domestic military expenditure into US dollars for each year and deflate by a US price index to give

$$\widetilde{M}_{it} = \frac{D_{it}}{E_{it}P_{it}^*} = M_{it}/R_{it}.$$

These two will be equivalent only if the real exchange rate

$$R_{it} = \frac{E_{it}P_{it}^*}{E_{ib}P_{it}} = 1,$$

is equal to unity, that is if purchasing power parity held.

The focus of discussions of data quality is usually on the problems of measurement of military expenditure and in their discussion of sources and methods SIPRI comment on the limitations of the data in terms of reliability, validity and comparability. However, it should be recognised that there are substantial revisions in measured GDP, price indexes and in purchasing power parity exchange rates as well. Thus these are also sources of measurement error and data revisions.

We chose to use a balanced panel which gave a large T for a large set of $N+1$ countries. The 50 year period 1965-2014 with some interpolation, gave a sample of 70 countries (or 72 for some calculations).

We give summary statistics, for military expenditure, GDP and the share of military expenditure for the full sample and for the balanced panel in Table 1.

In terms of the share of military expenditure the full and balanced samples were very similar: means of 2.96% in the full sample and 2.89% in the balanced sample. The full sample, with a standard deviation of 3.5%, was more dispersed than the balanced sample, with a standard deviation of 2.9. In the balanced sample 80% of the variance came from the between-country cross-section dimension. Both distributions are highly skewed. While there is a minimum of zero to the left, there is no maximum to the right, military expenditure can be over 100% of GDP. The maximum in the full sample was 117% in the balanced sample 30%. The distributions of military expenditure and GDP are also skewed. Military expenditure in the full sample, is about 60% of the balanced sample. However, GDP in the full sample is twice that of the balanced sample. Notice that the mean of the ratios of military expenditure to GDP is not the same as the ratio of the means.

The bottom panel of Table 1 gives summary statistics for the shares of military expenditure for the five regions. Four of the regions, Africa, Americas, Asia, and Europe, have very similar mean shares of military expenditure between 2-3%, the Middle East is much higher at 8.7%.

3 Modelling Framework

3.1 Strategic Interactions

The model we use to provide a framework is very standard in the literature and is reviewed in Dunne and Smith (2007). It assumes that military expenditures are determined by both economic and strategic factors. The economic factors are typically measured by GDP to approximate the budget constraint and the strategic factors are usually measured by the military expenditures of other countries, allies or potential enemies, which represent the threat or fear factor. Starting from a simple static logarithmic model determining m_{it} the logarithm of real military expenditures of country $i = 0, 1, 2, ..., N$ in year $t = 1, 2, ..., N$, by the logarithm of their real GDP, y_{it}, and other countries military expenditures, m_{jt}, for $j \neq i$, then the model can be written as a system of $N+1$ equations of the form:

$$m_{it} = \alpha_i + \eta_i y_{it} + \sum_{j \neq i} \gamma_{ij} m_{jt} + u_{it}. \tag{1}$$

The income elasticity of demand for military expenditure is η_i the feedback from other countries military expenditures is given by γ_{ij}. Smith (1995) discusses how equations of this sort can be derived from optimising a social welfare function, which depends on security and consumption, subject to a budget constraint. Other variables could be added such as indicators of political regime as in Brauner (2015);

indicators of internal or external conflict as in Dunne et al. (2008) or other sources of income, such as aid, as in Collier and Hoeffler (2007). But given our focus is on cross-section dependence we will just use a simple model relating military expenditures to GDP.

Clearly it is not possible to freely estimate the N feedback coefficients.[1] Various ways to deal with this curse of dimensionality have been adopted in the military literature. It is common to focus on just a few countries, where one can estimate action-reaction models of the Richardson arms-race type. But even in classic arms races, like Greece and Turkey or India and Pakistan, the actors are responding to other threats than from their antagonist; the Soviet Union in the case of Greece and Turkey and China in the case of India and Pakistan, and the expenditures of possible allies would also matter. One might expect that for enemies $\gamma_{ij} > 0$, reflecting arms races; for allies $\gamma_{ij} < 0$, since their military expenditures that can substitute for yours; and for uninvolved pairs $\gamma_{ij} = 0$. In the literature on alliances, surveyed in Murdoch (1995), considerable attention is paid to how the military expenditures of allies should be aggregated. The technology may be that strength depends on the simple sum, the best shot or the weakest link. Another common procedure to reduce this curse of dimensionality is to use *ad-hoc* weights to sum the military expenditures of potential allies to give a measure of friends spending and sum that of potential adversaries to give a measure of foes spending. These *ad-hoc* weights can be based on qualitative judgements about the security web, the nature of the linkages with the other countries, as in Dunne et al. (2008).

3.2 Principal Component Approach

If a set of allies are all responding to a common threat, they are likely to all move their military expenditures together generating a positive correlation between them as well as between them and their potential enemies. Such positive correlations between allies are common in the literature. This positive correlation among all the military expenditures of a group of interacting nations can be represented by a common unobserved threat factor driving the military expenditures of the interacting countries. Among a large group of countries, there are likely to be more than one strategic interaction, so the military expenditures may be driven by more than one threat factor. Assume that there are K such interactions and with K such unobserved latent factors, f^*_{kt}, $k = 1, 2, .., K$. Then we can write the model as

$$m_{it} = \alpha_i + \eta_i y_{it} + \sum_{k=1}^{K} \lambda_{ik} f^*_{kt} + e_{it} \tag{2}$$

Where the λ_{ik} are non-zero if country i is involved in interaction k. The f_{kt} can be estimated by the method of principal components, PCs, as linear combinations of

[1] If the dependence was on own and others lagged military expenditure, (1) would correspond to the infinite VAR discussed by Chudik and Pesaran (2011).

the military expenditures:

$$f_{kt}^{*}=\sum_{j=1}^{N}a_{kj}m_{jt}\qquad k=1,2,..,K \tag{3}$$

and one might hope that a few PCs would account for a lot of the variation in military expenditures. One would expect K to be much smaller than N, so estimating (2) will be much easier than estimating (1) . One can recover the coefficients on other countries military expenditure as

$$\gamma_{ij}=\lambda_{ik}a_{kj}.$$

If we define the logarithm of the share of military expenditure in GDP as $s_{it}=m_{it}-y_{it}$ then we can write the model in log shares as

$$s_{it}=\alpha_i+\beta_i y_{it}+\sum_{k=1}^{K}\lambda_{ik}f_{kt}+e_{it} \tag{4}$$

where $\beta_i=\eta_i-1$. If the income elasticity of demand for military expenditures is unity, as is often assumed, $\beta_i=0$ and log GDP drops out of the equations. Shares may also be a better indicator of threat perceptions than military expenditures, not being dominated by size, so one could estimate the factors as the PCs of the shares of military expenditure:

$$f_{kt}=\sum_{i=1}^{N}a_{ki}s_{it}\qquad k=1,2,..,K. \tag{5}$$

Estimating the factors using (5) rather than (3) implies that the threat from enemies or support from allies is represented not by the level of their military expenditures but by their share of military expenditure in output, perhaps as an indication of commitment or intent. This is not implausible given the importance attached by NATO to the commitment made at the 2014 Cardiff summit to spend at least 2% of GDP on defence. The shares model can be compared with the levels model using some model selection criterion like the BIC,[2] which can be used even though the models are not nested. To nest model (4) in model (2) , one would need to add the log GDP of the other countries to (2).

The strategic factors may not be global but regional so the model could be applied not over all countries N, but over the number of countries in the region N_r, for $r=1,2,...R$. One could also allow for the US being a dominant unit, as discussed in Chudik and Pesaran (2013) that appears as an explanatory variable in every region. The Principal Components allow us to measure how much of the variance of the shares or military expenditure is accounted for by these strategic factors and how they influence each country.

[2]The BIC seems more appropriate than the AIC because it is more parsimonious and with large data sets it is easy for parameters to proliferate.

Model (4) is heterogeneous, the coefficients differ across countries and we can report the mean group estimator of Pesaran and Smith (1995) reporting the average and standard error of the coefficients. A special case of model (4) is the two way fixed effect model, which imposes slope homogeneity, $\beta_i = \beta$, and the factors having the same effect on each country $\sum_{k=1}^{K} \lambda_{ik} f_{kt} = \alpha_t$. The model is then:

$$s_{it} = \alpha_i + \alpha_t + \beta y_{it} + e_{it}. \tag{6}$$

A model which is intermediate between the mean group and two way fixed effects is the interactive fixed effects model of Bai (2009), which assumes slope homogeneity but allows the effect of the factors to differ over countries.

Model (4) is a very parsimonious model. Clearly, there are many other variables that one might think are omitted from the model. These might include measures of conflict and of the institutions in the country, and many other economic and political variables. Using a parsimonious model has the advantage that we can use the maximum number of observations for which we have SIPRI share data, not losing data because of missing observations on other variables. It also allows us to focus on the role of cross-section dependence in a simple case. Denote these omitted variables by the vector z_{it}, so that the correct model is

$$s_{it} = \alpha_i^* + \beta_i^* y_{it} + \sum_{k=1}^{K} \lambda_{ik}^* f_{kt} + \phi_i' z_{it} + e_{it}.$$

Notice that β_i^* is measuring a different parameter of interest, from β_i. The parameter β_i^* measures the effect of a change in income holding z_{it} constant, while β_i measures the effect of a change in income allowing z_{it} to adjust as it does in the sample. The effect of this omission depends on the correlation between income, the global factors, f_{kt}, and the country specific omitted variables z_{it}. Consider the case of democracy. It seems a relevant variables since democracies spend less than autocracies on the military and are richer. If the country is a democracy throughout the period, $z_{it} = z_i$, it does not vary over time. The effect of any time invariant variable is picked up by the intercept α_i. After the Cold War many country undertook a process of democratisation. This time-varying factor will be correlated across countries and hence with the global factors, which pick up the reduction in the share of military expenditure. To the extent that the country specific variables are correlated with GDP and the factors, these variables will pick up the effects of the z_{it} and may be a parsimonious representation of many influences that are correlated with income or across countries.[3] To the extent to which the country specific variables vary over time in a way that is uncorrelated with income or the factors, this will increase the unexplained variance. How big an improvement in fit results from including these possible omitted variables is a subject for future research.

[3]Pesaran and Smith (2014) make a similar argument.

3.3 Common Correlated Effect Approach

Above we assumed that the unobserved factors were estimated by principal components, but they can also be allowed for using the correlated common effect, CCE, estimator. Reparameterise (2) in terms of shares, so $\beta_i = \eta_i - 1$, and for exposition initially assume that there is only a single threat factor, though in estimation we allow for a multi-factor model (e.g. there there may be separate factors for enemies and allies). Then

$$s_{it} = \alpha_i + \beta_i y_{it} + \lambda_i f_t + e_{it}. \tag{7}$$

Average (7) over the countries to give

$$\overline{s}_t = \overline{\alpha} + \overline{\beta}\overline{y}_t + \overline{\lambda} f_t + \overline{e_t} + \overline{\eta}_t \tag{8}$$

where

$$\begin{array}{lll} \overline{s}_t = \sum_{i=1}^N s_{it}/N, & \overline{\lambda} = \sum_{i=1}^N \lambda_i/N, & \overline{e}_t = \sum_{i=1}^N e_{it}/N, \\ \overline{\beta} = \sum_{i=1}^N \beta_i/N, & \overline{y_t} = \sum_{i=1}^N y_{it}/N, & \overline{\eta}_t = \sum_{i=1}^N \left(\beta_i - \overline{\beta}\right) y_{it}/N. \end{array}$$

Assuming $\overline{\lambda} \neq 0$, we can write (8) as

$$f_t = \overline{\lambda}^{-1}\left(\overline{s}_t - \overline{\alpha} - \overline{\beta}\overline{y}_t - \overline{e_t} - \overline{\eta}_t\right)$$

thus we can approximately filter out the effect of the factor by including $\overline{s}_t$ and $\overline{y}_t$ in (7) instead of the factor

$$\begin{aligned} s_{it} &= \alpha_i + \beta_i y_{it} + \lambda_i \left[\overline{\lambda}^{-1}\left(\overline{s}_t - \overline{\alpha} - \overline{\beta}\overline{y}_t - \overline{e_t} - \overline{\eta}_t\right)\right] + e_{it} \\ s_{it} &= a_i + \beta_i y_{it} + \delta_{1i}\overline{s}_t + \delta_{2i}\overline{y}_t + u_{it} \end{aligned} \tag{9}$$

where

$$\begin{aligned} u_{it} &= \lambda_i \overline{\lambda}^{-1}(-\overline{e_t} - \overline{\eta}_t) + e_{it}, \\ a_i &= \alpha_i - \lambda_i \overline{\lambda}^{-1}\overline{\alpha}. \end{aligned}$$

Pesaran (2006) provides more details and the generalisation to the dynamic case, which we use below, is provided in Chudik and Pesaran (2015*a*). Note that the covariance of $\overline{s}_t$ with u_{it} declines with N, so for N large we can treat $\overline{s}_t$ as exogenous.

We can compare (7) , using two factors, estimating the f_t by the principal components of the shares, with (9) and see which fits better. If we use two PCs both the CCE and PC equations will have the same degrees of freedom. In this case, we can just sum the Log-likelihoods over the N countries, treating the equations as independent, which seems reasonable given that we will have accounted for any strong factors.

Both (7) and (9) assume heterogeneous relationships, different for every country. We could also see whether there is any evidence of homogeneity by comparing the BIC of the heterogeneous models with the BIC of the two way fixed effect model (6) , above which imposes slope homogeneity $\beta_i = \beta$ and that the factor has the same effect everywhere so $\lambda_i f_t = \lambda f_t = \alpha_t$.

4 Estimation of Static Factor Models

4.1 Estimation of the Common Factors (PCA)

The PCs were estimated from the shares of military expenditure and from the logarithms of military expenditures, the logarithms of GDP and the logarithms of the shares for the 70 countries in our balanced panel over the whole period, 1965-2014; for the two sub-periods 1965-88 and 1989-2014; and for the five regions. The cumulative share of the variance of military burden explained by the first 5 Principal Components is given in Table 2 for each case.

For all 72 countries and the whole period, the first PC explains 50% of the variance of the share, the second 17% and the third 7%, so the first three explain 74% of the variance and the first 5, 82%. Clearly, there are strong common factors that drive these 72 series. In the Cold War the first PC explains less of the variation than in the post Cold War periods, though the total explained by the first 5 is similar. The strength of the common factors differ across regions. For Africa and the Americas, the first PC explains a much smaller part of the variance than it does in the other regions and the second a larger part relative to the other regions. The first PC explains 37% in Africa and 43% in the Americas, compared to 60% or more in the other regions. It seems plausible that more idiosyncratic factors drive the shares of military expenditures in Africa and the Americas. Notice that the shares are not weighted, so in the measured variance of the shares in the Americas, the US gets equal weight to any other country. Because of this, including the US in the other regions did not change the results very much, since it has a small weight. In Asia, Europe and the Middle East the first PC explains a larger proportion of the variance than in the full sample, in Europe a striking 79% of the variance is explained by the first PC. Europe was in the front line of the Cold War and it is plausible that European shares of military expenditure were driven largely by the Cold War factor.

For all 72 countries and the whole period, the first PC is plotted against the second PC in Figure 1. The bulk of the observations lie in a vertical column, with a value for PC1 of just over 0.1. This roughly corresponds to the mean share, giving an an equal positive weight to most countries. There are a group of countries that have negative values for PC1, these are Algeria, Burundi, Colombia, Congo, Ecuador, Japan, Liberia, Sierra, Leone, Sri Lanka. LYA, Mexico and Uganda have very small positive values. All these countries except Japan and, perhaps, Mexico have seen substantial civil wars. It is interesting that this PCA procedure does identify these as outliers even though it was not designed to do so. There is also a pattern for countries in the main group with a high or low value for PC2, to have a value of PC1 closer to zero, introducing some curvature.

When one looks at it by sub-period, the post-Cold War period figure looks very similar to Figure 1, with most countries having a positive weighting. However, for the Cold War period the pattern is very different. Figure 2 shows a large block of countries having a positive weighting, and a large block having a negative weighting.

The curvature apparent in Figure 1 is now apparent in both blocks. It looks almost like a circle.

Figure 3 plots PC1 and PC2. PC1 is roughly constant till about 1985 and then trends steadily downwards. It reflects the high shares of the cold war, then the downward trend. PC2 trends upwards to the early 1980s then trends downwards. It is not so clear what the interpretation of PC2 is. This is a limitation of PCs, it is often difficult to interpet them.

Table 3 gives the proportion explained by the first five PCs for the log share, the log of military spending and the log of GDP. The proportions explained were higher for the log of share than share, the first PC explaining 55% of log share as compared to 50% of share. Not surprisingly the common factors in log military expenditure and log GDP are much higher than for log shares because the level variables have more variance, and log GDP has a strong trend. For log GDP the first PC explains 95% of the variance.

4.2 CCE and PC static model estimates

We report the Pesaran and Smith (1995) mean group estimates for these models. Estimates are given for the full sample, two sub-periods 1965-88 and 1989-2014 and for the five regional groupings. The tables give the Wald test for the hypothesis that the means of the three slope coefficients are zero. It is a test on the averages of the coefficients. The LL is the sum of the maximised log likelihood over the N countries thus reflects the fit of the individual regressions not the means.

The mean group estimates using PCs MG-PC are given in Table 4, together with the estimates with no factors in the bottom panel. Adding the factors clearly improves the fit and changes the estimate of the coefficient of GDP. There is clear evidence of a structural break. Splitting the data into two sub-periods improves the fit substantially, increasing the log-likelihood for the model including PCs from 1980 to 3301. The pattern of coefficients is quite different in the whole period, where log GDP is insignificant and the two factors significant, and the two sub-periods, where GDP is significant and, with the exception of the first factor in the pre-Cold War period, the factors are not significant on average. Notice that while the average of the coefficients of the factors may not be significant, they may be significant in individual countries: some responding positively and some negatively to the factors. Although the coefficients of GDP are significant in the sub-periods they are not large, indicating that the common assumption that the income elasticity of demand for military expenditure is close to one, is not unreasonable. Disaggregation by regions has a much smaller effect, raising the LL from 1980 to 2138. In the regions, the first factor is significant and positive, but the second factor is insignificant.

The mean group estimates using cross-section means MG-CCE are given in Table 5. As with the PC estimates, splitting the sample into sub-periods improves the fit substantially increasing LL from 1757 to 3023. It is also clear that including two PCs works better than including the means of the share and log GDP. As in the PC case, the coefficient of log GDP is insignificant in the whole period but significant in the two sub-periods. Whereas the pattern of GDP only being significant when the sample is split into two sub-periods is the same in the PC and CCE cases, the size of the coefficient are not. The CCE estimates suggest that military expenditure is very inelastic, with an elasticity of 0.49 in the first period compared with the PC estimate of 0.79. In the second period the difference is smaller, with the CCE estimate of 0.77 and the PC of 0.86. The fact that the coefficient of GDP is larger in the CCE estimator suggests that it is a countries GDP relative to the world average that matters. In all cases the mean of the coefficients of log GDP, $\bar{\beta} = N^{-1} \sum \beta_i$ is of opposite sign to the mean of the coefficients of GDP average, $\bar{\delta}_2 = N^{-1} \sum \delta_{1i}$. In the two sub-periods when $\bar{\beta}$, the coefficient of log GDP, is significant that on average GDP, $\bar{\delta}_2$, is also significant. The PC estimate based on the shares cannot capture this relative feature whereas the CCE estimator does.

It is also noticeable that in almost all the cases the mean of the coefficients of the mean share, $\bar{\delta}_1 = N^{-1} \sum \delta_{1i}$ from (9) is close to one and is very significant. In the two way fixed effect estimator the coefficient of the mean of the dependent variable, δ_1, would be one and the coefficient on the mean of the independent variable, δ_2 would be equal and opposite the coefficient of the independent variable, since (6) can be written as

$$s_{it} - \bar{s}_t = \alpha_i + \beta(y_{it} - \bar{y}_t) + \varepsilon_{it}.$$

Figure 4 plots the histogram of the coefficients of income for both procedures, PCA and CCE. The CCE estimates seem more dispersed, though in both cases the range is quite wide (income elasticities between -0.5 and almost 2: there is clearly considerable heterogeneity in the income coefficients. Figure 5 plots the scatter diagram between the two sets of income coefficients. There is a positive though not very strong relationship, the fitted regression line being dominated by an outlier, where both estimates are close to -1.5. The average income coefficients differ between the two methods, primarily because of the high variance of the CCE estimates.

5 Dynamic Factor Models

5.1 PCs and CCE dynamic models

Dynamics can be important in models of military expenditures, since the variables may be I(1) and possibly cointegrated.[4] Below, for exposition, we present the equations using the CCE estimator is used, but the equations for the PC estimator have the same form.[5]

[4]Breitung and Pesaran (2008) discuss unit roots and cointegration in panels. Kapetanios et al. (2011) discuss panels with non-stationary multifactor error structures.

[5]The model was also estimated allowing for the possibility that the US is a dominant unit, following Chudik and Pesaran (2013). To do this, the change and lagged level of the US military shares was added to the error correction model. However, they added relatively little to the fit, so the results are not reported.

The dynamic CCE equation is:

$$\begin{aligned} s_{it} &= \alpha_{0i} + \alpha_{1i} s_{i,t-1} + \beta_{0i} y_{it} + \beta_{1i} y_{i,t-1} + \delta_{10,i} \overline{s}_t + \delta_{11,i} \overline{s}_{t-1} \\ &\quad + \delta_{20,i} \overline{y}_t + \delta_{21,i} \overline{y}_{t-1} + u_{it}. \end{aligned} \tag{10}$$

This can also be written in error correction form

$$\begin{aligned} \Delta s_{it} &= a_{0i} + a_{1i} s_{i,t-1} + b_{0i} \Delta y_{it} + b_{1i} y_{i,t-1} + d_{10,i} \Delta \overline{s}_t + d_{11,i} \overline{s}_{t-1} \\ &\quad + d_{20,i} \Delta \overline{y}_t + d_{21,i} \overline{y}_{t-1} + u_{it}. \end{aligned} \tag{11}$$

The error correction form is useful to capture both long and short-term dynamics in a single model. The dynamic PCs model specifications are similar, substituting $\Delta \bar{f}1$ for $\Delta \overline{s}_t$ and $\bar{f}1$ for $\overline{s}_{t-1}$ and $\Delta \bar{f}2$ for $\Delta \overline{y}_t$ and $\bar{f}2$ for $\overline{y}_{t-1}$. The number of estimated parameters remains the same, so the models are directly comparable using log-likelihoods.

5.2 PCs and CCE dynamic estimates

The mean-group estimates of the dynamic model (11) are reported in Table 6. Controlling for cross-sectional dependence substantially improves the fit of the models: the log-likelihoods from CCE models (columns 1 to 3) and PCs models (columns 4 to 6) increase significantly from those of models ignoring cross-sectional dependence. Based on the log-likelihood, the model using PCs (column 4) performs slightly better than the model using CCE (column 1). However, the magnitude of the factors' coefficients are difficult to interpret as they do not have clear-cut units of measurement. This makes the interpretation of long-run effects of the general common threat level difficult.

In the CCE models, the mean group estimate of the long-run effect of a common increase in the level of threat, represented by the global average share of military expenditure, is easily calculated by $\sum -(d_{11,i}/a_{1i})/N$. In this case, the two coefficients have the same units, in PCs models the two coefficients have different units. We show the heterogeneity of long-run effects of military expenditures to common threat estimated using CCE in Figure 6. The distribution of long-run effects using PCs model is almost uniformly centered at zero because the estimated coefficient of the first factor, d_{11}, is almost zero in any country. The US is an extreme outlier because the coefficient a_{1i} is almost zero making the ratio very large. The heterogeneous estimator, on which the mean-group estimates are based, also perform much better than the fixed effect, FE, estimator which imposes homogeneity. Thus we do not report the FE results. The coefficient on $\overline{s}_{t-1}$ in the FE model (0.0965) and that of the mean-group estimator (0.206) are different, suggesting that country heterogeneity is important.

Splitting the CCE model by pre- and post-1988 period does not improve the fit as measured by the BIC. This is because there is a trade-off between fit and number of estimated parameters. The time-series by period are too short and the increase in estimated parameters imposes a large penalty on the fit. Interestingly, the long-run effects of a common increase in the level of threat pre- and post-1988 are different and there is a modest heterogeneity. This suggests that using a simple Cold War dummy variable for pre- and post-1988 is not sufficient to capture that variation. In addition, there is little correlation between the estimated coefficient on y_{t-1} in the Cold War and post-Cold War period (these correlations are -0.09 and 0.03 using the CCE and PCs models, respectively.

6 Conclusions

In this paper, we consider a model of the demand for military expenditure where there is cross-sectional dependence in the errors due to unobserved common factors generated by strategic interactions. If such omitted factors are correlated with the regressors, as they well may be, they will cause the coefficients of those regressors to be biased. Using either cross-section means or principal components to proxy the factors we find that allowing for cross-section dependence significantly improves the fit and changes the values of the coefficients. Clearly there are strong strategic factors driving the shares of military expenditure, so it is important that one allow for these factors. There is also evidence of substantial heterogeneity across countries, so that assuming slope homogeneity as is done in fixed effect models may be misleading. The procedures to allow for cross-section dependence, which we describe in detail, are relatively straightforward to implement: add cross-section means or principal components.

There are a range of natural extensions. Firstly, to emphasise the role of cross-section dependence and to obtain the largest possible sample, we have used a very simple model with income as the only independent variable. There are many other economic and political measures that have been used to explain military expenditures in the literature and their role could be investigated. Secondly, we have assumed that income is exogenous despite the fact that there is a large literature investigating the effect of military expenditure on growth. This issue could be investigated by estimating a VAR in military expenditures and GDP and testing for the pattern of Granger causality. Thirdly, our results show that there is considerable heterogeneity and there is scope to examine the factors that determine that heterogeneity: are there characteristics of the countries that explain the differences in coefficients? This could be linked to case studies of the individual countries. The extended SIPRI data set opens up the possibility of a large range of quantitative and qualitative studies.

References

Bai, Jushan (2009), 'Panel data models with interactive fixed effects', *Econometrica* **77**(4), 1229–1279.

Brauner, Jennifer (2015), 'Military spending and democracy', *Defence and Peace Economics* **26**(4), 409–423.

Breitung, Jörg and M Hashem Pesaran (2008), 'Unit roots and cointegration in panels', *The Econometrics of Panel Data* pp. 279–322.

Cavatorta, Elisa (2010), 'Unobserved common factors in military expenditure interactions across mena countries', *Defence and Peace Economics* **21**(4), 301–316.

Chudik, Alexander and M Hashem Pesaran (2011), 'Infinite-dimensional vars and factor models', *Journal of Econometrics* **163**(1), 4–22.

Chudik, Alexander and M Hashem Pesaran (2013), 'Econometric analysis of high dimensional vars featuring a dominant unit', *Econometric Reviews* **32**(5-6), 592–649.

Chudik, Alexander and M Hashem Pesaran (2015*a*), 'Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors', *Journal of Econometrics* **188**(2), 393–420.

Chudik, Alexander and M Hashem Pesaran (2015*b*), Large panel data models with cross-sectional dependence: a survey, *in* B.Baltagi, ed., 'The Oxford Handbook on Panel Data', Oxford University Press, chapter 1.

Coakley, Jerry, Ana-Maria Fuertes and Ron Smith (2006), 'Unobserved heterogeneity in panel time series models', *Computational Statistics & Data Analysis* **50**(9), 2361–2380.

Collier, Paul and Anke Hoeffler (2007), 'Unintended consequences: Does aid promote arms races?', *Oxford Bulletin of Economics and Statistics* **69**(1), 1–27.

Dunne, J Paul and Ron P Smith (2007), The econometrics of military arms races, *in* T.Sandler and K.Hartley, eds, 'Handbook of Defense Economics', Vol. 2, Elsevier, pp. 913–940.

Dunne, J Paul, Sam Perlo-Freeman and Ron P Smith (2008), 'The demand for military expenditure in developing countries: hostility versus capability', *Defence and Peace Economics* **19**(4), 293–302.

Kapetanios, George, M Hashem Pesaran and Takashi Yamagata (2011), 'Panels with non-stationary multifactor error structures', *Journal of Econometrics* **160**(2), 326–348.

Murdoch, James C (1995), 'Military alliances: Theory and empirics', *Handbook of Defense Economics* **1**, 89–108.

Pesaran, M Hashem (2006), 'Estimation and inference in large heterogeneous panels with a multifactor error structure', *Econometrica* **74**(4), 967–1012.

Pesaran, M Hashem (2015), *Time-series and panel data econometrics for macroeconomics and finance*, Oxford University Press.

Pesaran, M Hashem and Elisa Tosetti (2011), 'Large panels with common factors and spatial correlation', *Journal of Econometrics* **161**(2), 182–202.

Pesaran, M Hashem and P. Ron Smith (1995), 'Estimating long-run relationships from dynamic heterogeneous panels', *Journal of econometrics* **68**(1), 79–113.

Pesaran, M Hashem and Ron P Smith (2014), 'Signs of impact effects in time series regression models', *Economics Letters* **122**(2), 150–153.

Smith, Ron (1995), The demand for military expenditure, *in* K.Hartley and T.Sandler, eds, 'Handbook of defense economics', Vol. 1, Elsevier, pp. 69–87.

Table 1: Descriptive Statistics

	N group	Obs	Mean	Std.Dev.	Min	Max
Entire panel (1965-2014)						
burden	172	6,420	0.029608	0.035163	0	1.173498
milex (billion)	172	6,372	9.108008	45.84714	0	720.2188
GDP (billion)	172	8,600	32652.5	511791.7	0	2.72E+07
Balanced panel						
burden	70	3,500	0.028917	0.029275	0.001399	0.304638
milex (billion)	70	3,412	14.75106	60.63815	0.001655	720.2188
GDP (billion)	70	3,500	15802.79	92983.97	1.55E-11	150286
Military burden by region						
Africa	19	950	0.023637	0.022778	0.001399	0.267347
Americas	16	800	0.020704	0.013485	0.003482	0.090634
Asia	11	550	0.027417	0.015566	0.005447	0.069917
Europe	17	850	0.022885	0.011169	0.004746	0.059623
Middle East	6	300	0.087121	0.057432	0.015873	0.304638

Table 2: Cumulative Proportions of Military Burden explained by Principal Components

	All	Cold War period	Post-Cold War	Africa	Americas	Asia	Europe	Mid East
N	72	72	72	20	16	11	17	7
T	65-14	65-88	89-14	65-14	65-14	65-14	65-14	65-14
PC1	0.4982	0.4195	0.5711	0.374	0.4293	0.6654	0.7929	0.5961
PC2	0.672	0.5976	0.7068	0.6008	0.6414	0.783	0.8827	0.8021
PC3	0.7397	0.7323	0.7993	0.6854	0.7479	0.8475	0.9389	0.8963
PC4	0.7899	0.7928	0.8432	0.7531	0.8134	0.8974	0.9569	0.9518
PC5	0.8241	0.8398	0.8659	0.8102	0.863	0.9282	0.9711	0.9776

Table 3: Cumulative Proportions of (log) Military Burden/Expenditure/GDP explained by Principal Components

	log(burden)	log(milex)	log(GDP)
N	70		
T	1965-2014		
PC1	0.548	0.6141	0.9491
PC2	0.7146	0.7782	0.9717
PC3	0.7775	0.849	0.9913
PC4	0.8263	0.8897	0.995
PC5	0.861	0.9206	0.9972

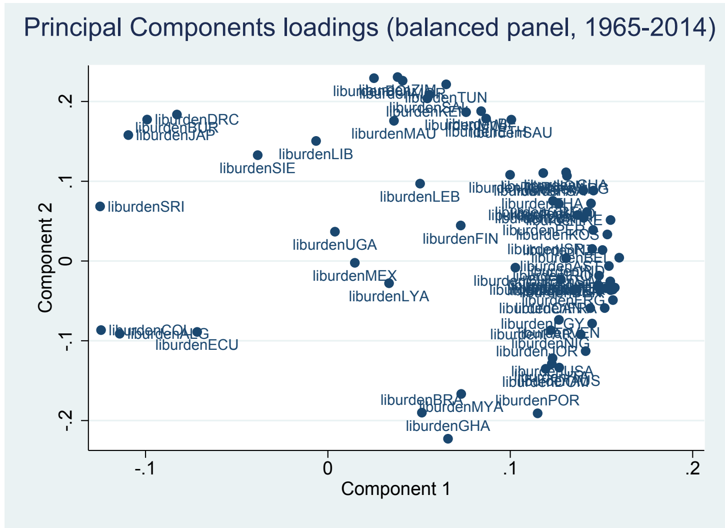

Figure 1: Principal Components (balanced panel, 1965-2014)

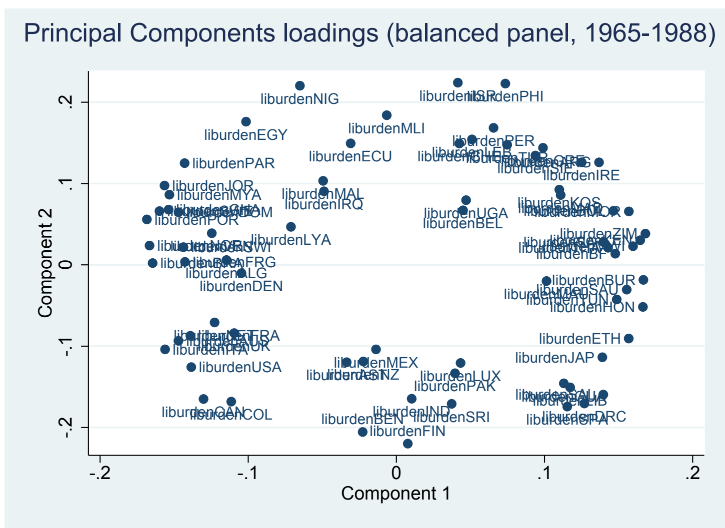

Figure 2: Principal Components pre-Cold War period (balanced panel, 1965-1988)

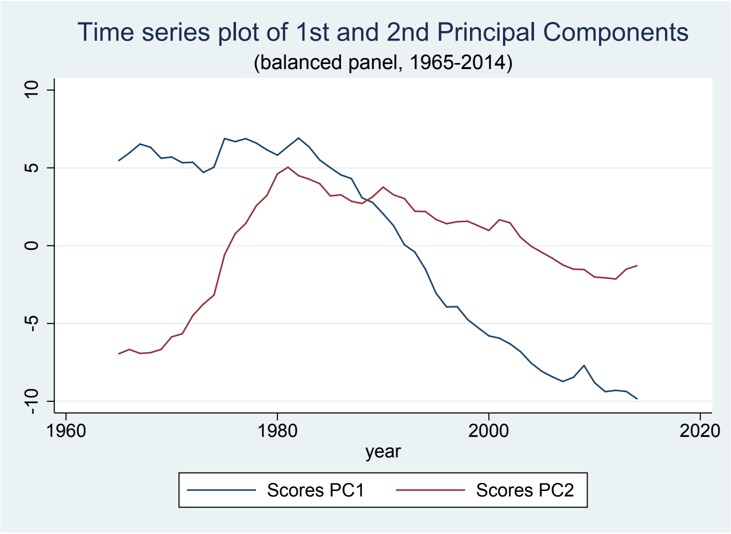

Figure 3: Times series of Principal Components (balanced panel, 1965-2014)

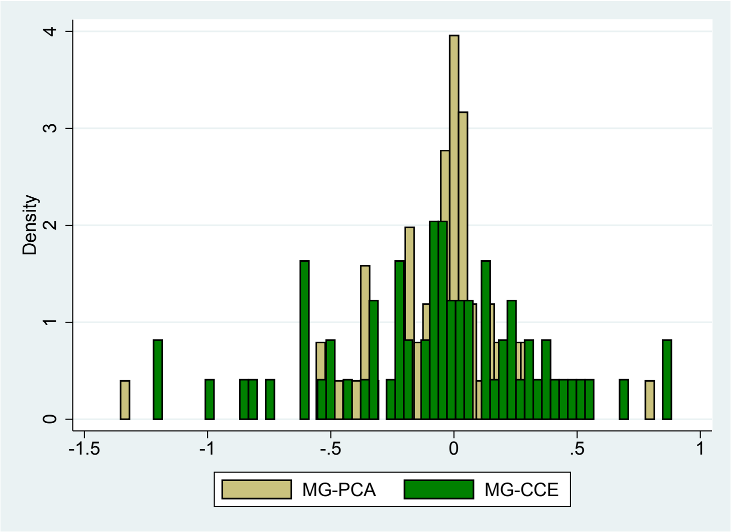

Figure 4: Density of income coefficients by methods (PCA and CCE)

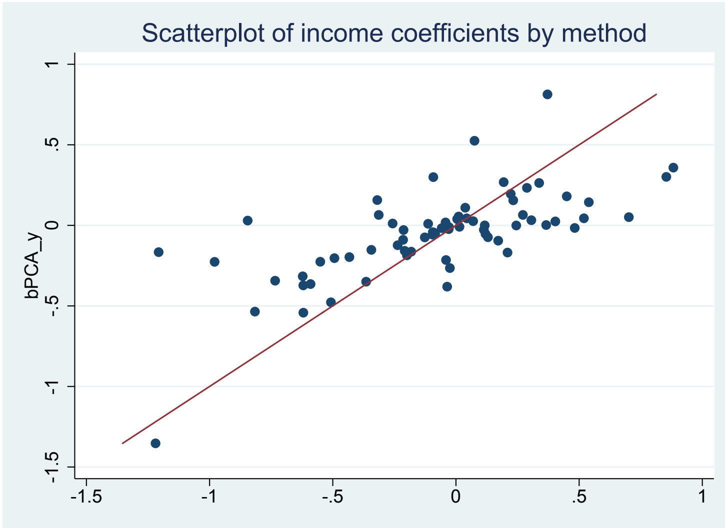

Figure 5: Scatterplot of income coefficients by methods (PCA on y-axis and CCE on x-axis)

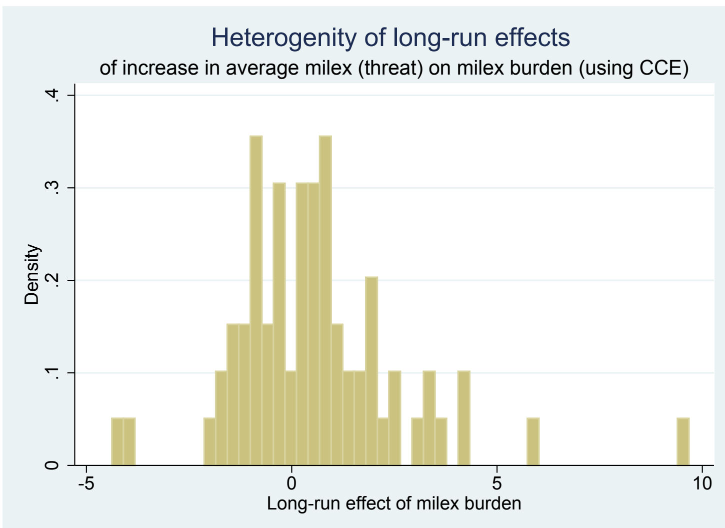

Figure 6: Histogram of log-run burden effect

Table 4: Mean-group PCA

	All	Cold War: 1965-1988	Post Cold War: 1989-2014	Africa	Americas	Asia	Europe	Middle East
log(GDP)	-0.052572	-0.2089***	-0.1416**	-0.078859	0.034248	0.010462	-0.055326	-0.043979
	(0.033)	(0.043)	(0.055)	(0.059)	(0.031)	(0.027)	(0.034)	(0.047)
Factor 1	0.024793***	0.0344***	0.0135*	0.064745***	0.083522***	0.062855*	0.060953***	0.154592***
	(0.008)	(0.008)	(0.006)	(0.020)	(0.025)	(0.037)	(0.007)	(0.026)
Factor 2	0.018434**	-0.00531	0.0021	0.035109	-0.000844	0.018428	0.018975	-0.037736
	(0.008)	(0.005)	(0.005)	(0.033)	(0.028)	(0.037)	(0.024)	(0.028)
constant	1.903050**	5.9158***	4.2782***	2.208523*	-0.305319	0.471629	2.150944**	2.923907***
	(0.807)	(1.008)	(1.493)	(1.261)	(0.753)	(0.775)	(0.899)	(1.084)
N	3500	1680	1820	950	800	550	850	300
N groups	70	70	70	19	16	11	17	6*
Periods	50	24	26	50	50	50	50	50
Wald	43.556	25.89	31.95	13.484	15.120	8.477	132.585	95.902
Sum LL	1980	1489	1812	116	291	474	1104	153
Baseline mean-group estimator without factors								
log(GDP)	-0.124173***	0.04817	-0.289774***	-0.085656	-0.0734***	-0.0827*	-0.235051***	-0.144943**
	(0.022)	(0.034)	(0.038)	(0.059)	(0.024)	(0.043)	(0.035)	(0.058)
constant	3.887287***	-0.106126	8.204896***	2.512179**	2.408351***	3.034025**	6.861455***	5.360349***
	(0.530)	(0.777)	(0.961)	(1.262)	(0.679)	(1.191)	(0.959)	(1.340)
N	3500	1680	1820	950	800	550	850	300
N groups	70	70	70	19	16	11	17	6
Periods	50	24	26	50	50	50	50	50
Wald	32.988	2.02	56.965	2.122	9.269	3.714	45.227	6.286
Sum LL	236	763	1172	-392	2.66	178	436	35

Notes: Factor 1 and Factor 2 are the first and second principal component scores obtained from the military expenditure shares within each group. LYA is unclassified.

Table 5: Mean-group CCE

	All	Cold War: 1965-1988	Post Cold War: 1989-2014	Africa	Americas	Asia	Europe	Middle East
log(GDP)	-0.07508	-0.511968***	-0.234229**	-0.14737	-0.080137*	0.031771	-0.18895	-0.1293
	(0.052)	(0.081)	(0.098)	(0.173)	(0.047)	(0.110)	(0.118)	(0.087)
log(burden) avg	1.008558***	1.273064***	0.917549***	0.997181***	1.074476**	1.328976***	0.977802***	0.938505***
	(0.187)	(0.300)	(0.333)	(0.251)	(0.486)	(0.359)	(0.162)	(0.208)
log(GDP) avg	0.023524	0.352208***	0.215003**	0.055344	0.046473	-0.03157	0.175827	0.082387**
	(0.041)	(0.064)	(0.094)	(0.174)	(0.041)	(0.131)	(0.119)	(0.033)
constant	1.036694	3.914157***	0.24876	1.836215	0.960657	-0.0326	0.409344	1.269784
	(0.689)	(1.029)	(2.241)	(1.324)	(1.158)	(1.312)	(1.505)	(2.058)
N	3500	1680	1820	950	800	550	850	300
N groups	70	70	70	19	16	11	17	6
Periods	50	24	26	50	50	50	50	50
Wald	43.35	55.64	16.05	17.457	9.437	14.815	97.338	73.763
Sum LL	1757	1348	1673	-139	191	398	1066	154

Notes:

Table 6: Dynamic models correcting for cross-sectional dependence, Mean-Group estimator

	Common Correlated Effect				Principal Components		
	All period	Cold-War: 1965-1988	Post Cold-War:1989-2014		All period	Cold-War: 1965-1988	Post Cold-War:1989-2014
	(1)	(2)	(3)		(4)	(5)	(6)
$shares_{t-1}$	-0.346182***	-0.537727***	-0.535553***	$shares_{t-1}$	-0.360873***	-0.600078***	-0.567210***
	(0.020)	(0.036)	(0.034)		(0.021)	(0.033)	(0.035)
Δy_t	-0.575561***	-0.773903***	-0.609458***	Δy_t	-0.453167***	-0.743175***	-0.483785***
	(0.062)	(0.102)	(0.074)		(0.048)	(0.078)	(0.067)
y_{t-1}	0.041528*	-0.053314	-0.033627	y_{t-1}	-0.012754	-0.122054***	-0.048003
	(0.024)	(0.085)	(0.057)		(0.014)	(0.033)	(0.041)
$\Delta \bar{shares}_t$	0.841572***	1.098419***	0.883816***	Δf_1	0.025485***	0.011801	0.024136***
	(0.157)	(0.222)	(0.254)		(0.006)	(0.010)	(0.007)
$\bar{shares}_{t-1}$	0.206372**	0.729032***	0.596947***	$f1, t-1$	0.010922***	0.021640***	0.008144
	(0.081)	(0.229)	(0.231)		(0.004)	(0.006)	(0.005)
$\Delta \bar{y}$	0.183700***	0.477572***	0.301562**	Δf_2	0.018661**	0.007614	0.008434*
	(0.068)	(0.131)	(0.141)		(0.007)	(0.005)	(0.005)
$\bar{y}_{t-1}$	-0.032691*	0.009545	0.069088	$f2, t-1$	0.005316*	0.000372	0.000927
	(0.019)	(0.066)	(0.066)		(0.003)	(0.004)	(0.003)
constant	-0.180541	1.324848	-1.044626	constant	0.634145*	3.651121***	1.764053
	(0.313)	(0.879)	(1.501)		(0.345)	(0.784)	(1.106)
Observations	3,430	1,610	1,820		3,430	1,610	1,750
LL	3511	1991	2501		3576	1978	2480
LL no correction for CS dependence $*$	2974	1545	2086		2974	1545	2086
LL ECM Fixed Effect	1606	739	951		1591	723	917
Periods	49	23	26		49	23	25♮

Notes: Mean-Group estimator, standard errors of coefficients in parenthesis. Notes: ♮– we loose one observation since we need the first difference in the factors, Δf_1 and Δf_2, which are computed from PCA by Cold War period. In CCE models the first difference of the cross-section averages by Cold War period is always available. $*$ indicates log-likelihood from a dynamic model without correction for cross-sectional dependence.