BIROn - Birkbeck Institutional Research Online

Geman, Hélyette and Scheiber, Matthias (2014) Spot price modelling of industrial metals – an heterogeneous agent based model for Copper. Working Paper. Birkbeck College, University of London, London, UK.

Downloaded from: https://eprints.bbk.ac.uk/id/eprint/15283/

Usage Guidelines:
Please refer to usage guidelines at https://eprints.bbk.ac.uk/policies.html or alternatively contact lib-eprints@bbk.ac.uk.

ISSN 1745-8587

Birkbeck, University of London

Department of Economics, Mathematics and Statistics

BWPEF 1404

Spot Price Modelling of Industrial Metals – An heterogeneous agent based model for Copper

Helyette Geman
Birkbeck, University of London

Matthias Scheiber
Birkbeck, University of London

May 2014

Birkbeck Working Papers in Economics & Finance

Spot Price Modelling of Industrial Metals – An heterogeneous agent based model for Copper

Professor Helyette Geman
Director, Commodity Finance Centre
Birkbeck, University of London
London
United Kingdom
h.geman@bbk.ac.uk

Matthias Scheiber
Birkbeck, University of London
London
United Kingdom

Version: 1

May 2014

Draft version
Status: Not submitted for publication elsewhere.

"The authors" here: Helyette Geman, Matthias Scheiber
Copyright: The authors.

Abstract

We will show in this paper the role of inventories in explaining copper price volatility. Using a three factor model we derive a fundamental long-term value for copper. Second, we emphasis the significance of this fundamental long-term value by considering an agent based model approach in which mean-reversion focused fundamental investors trade with chartists who follow price trends. We show that fundamental investors take increasing positions in copper when the spot price of copper deviated from its fundamental value (i.e. the fundamental value is higher than the spot price) and chartists loose relative significance.

Keywords

Heterogeneous agent based modelling, copper spot price modelling, 3 factor stochastic volatility model, Runge Kutta, Kalman Filter

Introduction

We propose an empirical model based on the heterogeneous agents literature. Price changes are induced by fundamental as well as technical demand. The model is estimated for Copper.

In this type of model, the market price is formed by the trading behaviour of heterogeneous agents, who condition their buying and selling on a number of forecast rules. The relative weights put on these rules are determined by the past performance error of the different forecasts, so agents can change their strategy of how to behave. The model is based on an approach proposed by Lux&Marchesi (1999, 2000) and Lux (1998) which we will follow throughout this document. Additionally we will explicitly model the fundamental value that will be used by the experts as input variable for their recommendation. This fundamental value – the long-term equilibrium spot price - against which fundamentally driven experts make their recommendation, is calculated out of the forward curve of Copper.

The model to calculate the long-term equilibrium price is combining two different strings of literature. One is that commodity prices follow a “random walk” described by geometric Brownian motion. This is the model of stock price uncertainty underlying the famous Black-Scholes option pricing formula and it leads to closed-form solutions in some interesting cases. In this model, prices are expected to grow at some constant rate with the variance in future spot prices increasing in proportion to time. If prices increase (or decrease) more than anticipated in one time period, all future forecasts are increased (or decreased) proportionally. The other direction of authors has been focusing on the use of mean-reverting price models and argued that these models are more appropriate for many commodities. Intuitively, when the price of a commodity is higher than some long-run mean or equilibrium price level, the supply of the commodity will increase because higher cost producers of the commodity will enter the market—new production comes on line, older production expected to go off line stays on line—thereby putting downward pressure on prices. Conversely, when prices are relatively low, supply will decrease since some of the high-cost producers will exit, putting upward pressure on prices. When these entries and exits are not instantaneous, prices may be temporarily high or low but will tend to revert toward the equilibrium level. There are elements of truth in each of these simple models of commodity prices. For most commodities, there appear to be some mean reversion in prices but there is also uncertainty about the equilibrium price to which prices revert.

In this article, we develop a simple three-factor model of commodity prices that captures all of the effects mentioned before; In our model, the equilibrium price level is assumed to evolve according to geometric Brownian motion with drift reflecting expectations of the exhaustion of existing supply, improving technology for the production and discovery of the commodity, inflation, as well as political and regulatory effects. The short-term deviations—defined as the difference between spot and equilibrium prices—are expected to revert toward zero following an Ornstein-Uhlenbeck process. These deviations may reflect, for example, short-term changes in demand resulting from variations in the weather or intermittent supply disruptions, and are tempered by the ability of market participants to adjust inventory levels in response to changing market conditions.
Although neither of these factors is directly observable they can be calculated indirectly if forward curve data (especially for long-term contracts) are available via a recursive technique like the Kalman Filter.

Related Work

This paper adds to a debate that models commodity spot prices oscillating around a long-run trend rather than showing mean reversion.
Finding mean-reversion in commodity spot prices has some crucial implications like return variance that does not increase linearly with time or the implications on the value of real options such as mines as well as the consequences on monetary policy as higher trending commodity prices will have direct impact via higher inflation. In another example Casassus et al (2005) show that if commodity prices revert to a constant mean, the prices of options on commodity futures will be significantly smaller than in case of a random.
Most commodity pricing papers on commodity futures use a mean-reverting process to a constant level to model the spot prices of commodities like the one-factor model of Schwartz (1997) or Geman and Nguyen (2005).

However there has been a series of papers aiming to show that this reversion to a constant mean has often to be rejected as Cashin, Liang and McDermott (2000) have shown that shocks to commodity prices can be persistent while Grilli and Yang (1988), use a dataset from 1900 to 1986 to proof that commodity prices (real prices in this case) show a positive trend over time.

We follow Geman and Nguyen (2005) and introduce a three factor stochastic volatility model for copper prices. In contrast to Geman and Nguyen (2005) the long-term trend to which the mean reversion process for the copper spot price reverts over time is modelled by a geometric Brownian motion with drift. This is based on the work of Geman (2000) which takes a long-term trend around which the commodity price oscillates over time. This is in contrast to most three factor commodity models which use a second mean reversion process to explain the price trajectory of a fundamental value or convenience yield (see Schwartz 1996 as an example). In order to introduce fat tails into the distribution of copper prices we use the fundamental value gained from our three factor model and introduce financial investors, labelled fundamentalists who use this price input as an internal benchmark or fair value to assess the value of copper prices. These fundamentalists interact with what we will label as technical investors who follow a trend-following approach to assess the fair value of copper prices.

A simple heterogeneous agent based model will be presented in this paper to assess the effect of fundamental as well as technical traders on the price of copper. As the model is based on the fundamentals of copper it makes sense to take supply and demand into consideration. Rapidly growing countries like India or China are dominant on the demand side yet this cannot fully explain the dramatic moves of copper since 2007. A possible cause of this larger price volatility is the existence of speculators in the copper market (Geman, 2005). Similar observations have been made for other commodities like oil over the last couple of years where inventory speculation caused a run-up in oil prices during the 1970s (Danielsen, 1979). This poses a strong challenge towards the Efficient Market Hypothesis of Fama (1970). The Efficient Market hypothesis assumes rational expectations and thus the current price of copper should reflect all available information.
One of the deviations from the Efficient Market Hypothesis is represented by heterogeneous agents. Brock and Hommes (1997) account for different types of investors. A cobweb type demand-supply model was used by Brock and Hommes (1997) where agents choose between

naive and rational expectations. Investors switch between different forecasting strategies based on the past performance of these strategies. The switching of investors introduces non-linearity into the system and thus local instability and complicated dynamics can be observed in a fully rational notion of equilibrium.
Frankel and Froot (1988) classified two types of investors, fundamentally based investors and technical traders in an environment of exchange rates. Later models like Foellmer et al. (2004) also introduce liquidity traders to account for volatility in an equity market which is close to equilibrium. In general, fundamental traders are comparing the current price of a financial asset with their fair value and thus have a stabilizing effect as they would buy in case of a lower current price compared to the spot price and vice versa. Technical traders in contrast base their investment decision on past prices, e.g. trend followers buy if they observed a price increase in the past and sell in case of falling prices. Hommes (2006) gives a detailed overview of Heterogeneous Agent Models. Reitz and Westerhoff (2007) and Reitz and Slopek (2009) have been some of the recent authors to estimate Heterogeneous Agent Models for commodities though research in the 1960s by Smidt (1965) already indicated the existence of speculation in commodity markets.

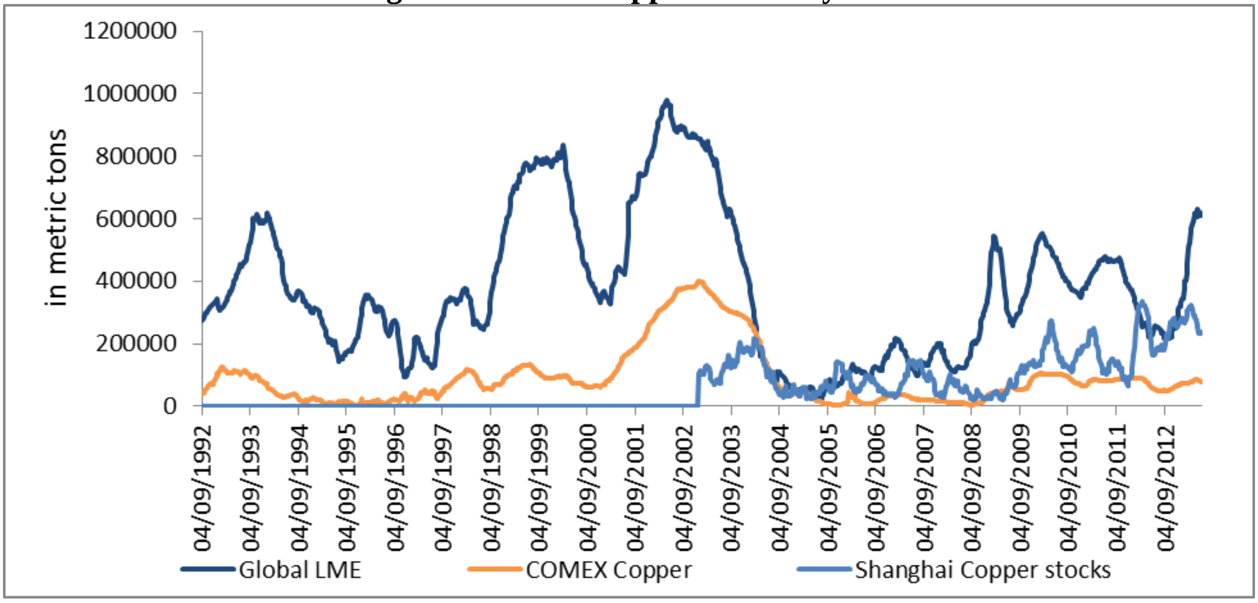
The aim of this paper is to wrap a three-factor commodity model into a Heterogeneous Agents Models where the Fundamental price input is directly derived from the three-factor model for copper. The Heterogeneous Agents Model is based on the work of Lux&Marchesi (1999,2000) who introduced an algorithm combining fundamental and noise traders (which will be denoted chartist in our paper). The input of the fundamental price (which is used by fundamentally driven investors to compare with the current price of copper) is directly generated via a three-factor copper price model where it is calculated out of the model spot price. The spot price is based on a long-term price (which is represented by a Geometric Brownian motion with positive drift) of copper combined with short-term fluctuations (via a mean-reversion process) and considering stochastic volatility. It is this long-term fundamental price of copper calculated via the modelled spot price which serves as an input in the Heterogeneous Agents Model.

Chile, the United States, Peru and China represent the largest producers of Copper (http://www.bgs.ac.uk/mineralsuk/statistics/worldStatistics.html). With the rapid expansion of Chinese economic growth over the last decade China now represents also the largest importer of Copper followed by the United States and Europe (Source: International Trade Center). For years the London Metal Exchange (LME) and the Chicago Mercantile Exchange (COMEX) in the US have been the main locations for trading Copper forward contracts. A very small fraction of futures contracts at COMEX and LME are physically unwound at maturity (as in all commodity futures markets). Over the years as Chinese significance in copper trading rose the Shanghai exchange became the third major player in the copper space as can be seen on the chart below.

Weekly copper inventory data for the LME and COMEX are available since September 1992 while Chinese weekly copper inventory data are available from January 2003. Figure 1 shows Global copper inventory data since 1992 (in metric tons) and the gain in significance of China on copper inventories.

The chart also shows the cyclical nature of copper, often labelled “Dr. Copper” in financial markets. The period of 2000/2001 and the 2008 recession have both led to a sharp rise in copper inventories as demand collapsed.

Figure 1: Global Copper Inventory data



The aftermath of the 2001/2002 recession has resulted in a sharp decrease in overall copper inventories while from 2005 onwards a general upward trend could be observed despite the volatility around the financial crisis in 2008.

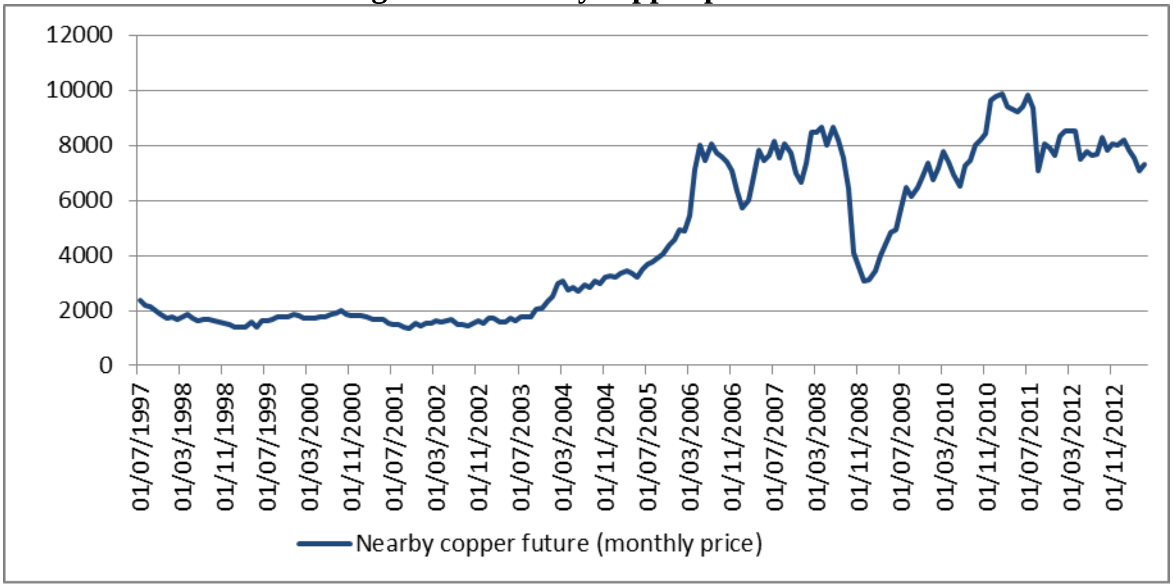
Figure 2: Global Copper Inventory statistics

Test Statistics	LME Copper Inventory	COMEX Copper Inventory	Shanghai Copper Inventory
Average	380,104.65	90,451.36	60,732.90
Median	342,125.00	65,211.00	23,731.00
Maximum	980,075.00	399,368.00	336,387.00
Minimum	25,525.00	1,478.00	-
Standard Deviation	11,695.20	3,178.62	13,074.37
Dickey Fuller Test Statistic	- 6.23	- 5.60	- 9.94
p-value	0.01	0.01	0.01
Lag order	10	10	10

The table above summarizes key statistics for the three inventory markets (LME, COMEX and Shanghai). The Dickey Fuller Test statistics are statistically significant and reject the null hypothesis of a unit root.

Turning to copper prices, our analysis comprises monthly copper future data since 1997. We use Chicago Mercantile Exchange data (COMEX). The nearby contract represents the proxy for the cash price and we also obtained data for the 3 month, 6 month, 9month, 12 month, 15 month and 21 month forward contract since 1997. The chart below shows the price development of the nearby contract over time.

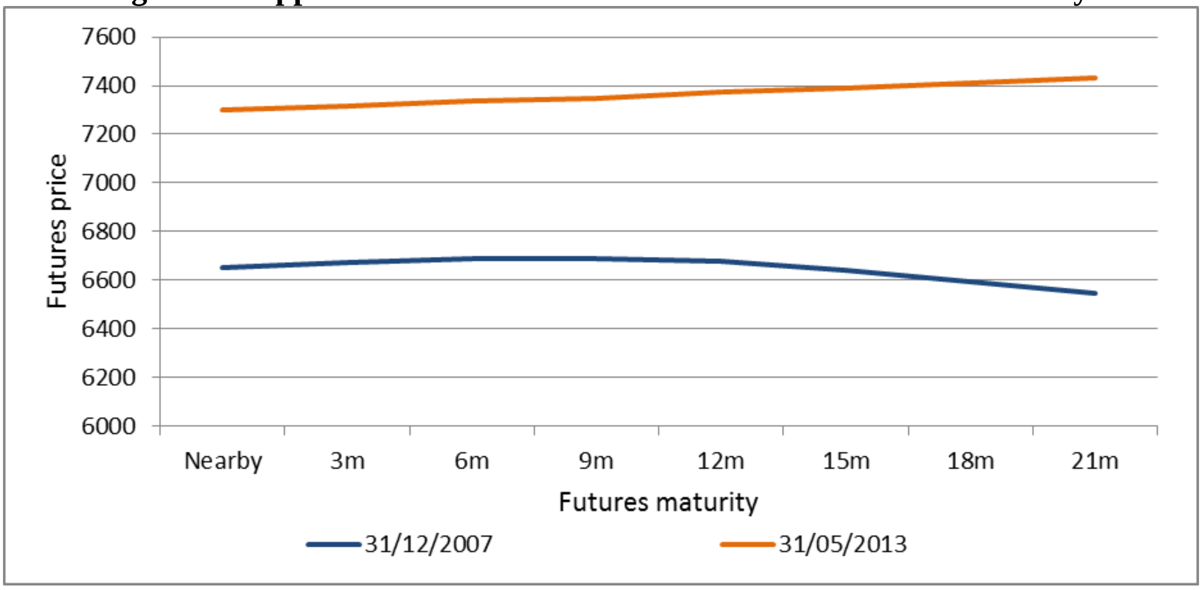
Figure 3: Monthly copper price since 1997



The sharp increase in commodity prices from 2002 to 2007 was also observed in copper prices with a quadrupling of prices over this period. The Financial Crisis in 2008 led to a sharp correction with more than -50%.

Financial investors (as in so many other commodities) have not only contributed to the higher volatility in commodity prices over the last couple of years but also influenced the shape of the forward curve. The chart below shows that the copper forward curve was inverted before the Financial crisis but has been in Contango lately.

Figure 4: Copper forward curve over time – December 2007 versus May 2013



We also look more in detail on the statistical properties of each copper future maturity over the period 1997 to 2013 using monthly log prices. As with many commodity markets all maturities of copper futures observed show a negative skewness (relatively few low values) while the excess Kurtosis is positive and thus indicates fait tail behaviour.

Figure 5: Copper futures statistical analysis 1997 to 2013 (using log prices)

	Nearby Contract	3m Future	6m Future	9m Future	12m Future	15m Future	18m Future	21m Future
Mean	8.18	8.18	8.18	8.17	8.17	8.16	8.16	8.15
Standard Deviation (annualized)	27.66%	27.45%	26.85%	26.22%	25.66%	25.27%	24.84%	24.47%
Standard Error	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Skew	- 0.89	- 0.90	- 0.94	- 1.00	- 1.05	- 1.08	- 1.09	- 1.10
Excess Kurtosis	2.54	2.32	2.62	2.81	2.92	2.87	2.96	2.96

The table above also picks up the so called “Samuelson effect” which states that futures price volatility decreases with increasing maturity.

Next to looking at copper future prices over various maturities we also analyse in more detail the volatility behaviour of nearby copper futures. We introduce the scarcity variable, here denoted as $s_t$and defined as inverse inventory at time t (see Geman et al. 2005). We take the copper stock for the US and Global markets at the end of period t (here monthly) and calculate the scarcity as the inverse of inventories. In order to understand the impact of inventories on spot price volatility (σ_t)we run the following multi-variant regression that includes a constant, a variable that accounts for possible trends over time (δ) as well as the sensitivity β which should be positive if high inventories reduce nearby copper future volatility.

$$\sigma_t = \alpha + \delta t + \beta s_{t-1} + \epsilon_t \tag{1}$$

We run this regression for different inventory data, namely a global inventory proxy which includes LME, COMEX as well as Shanghai data and for a US inventory proxy (COMEX only). σ_t is the monthly volatility of nearby copper futures based on daily data. s_{t-1} represents the scarcity at the end of the previous month (inverse of inventories).
Using the entire data series (from 1997 to 2013) as input the F-Test and the T-statistics for each input variable show that the scarcity variable cannot be rejected at a 1% significance level as driver of copper price volatility.

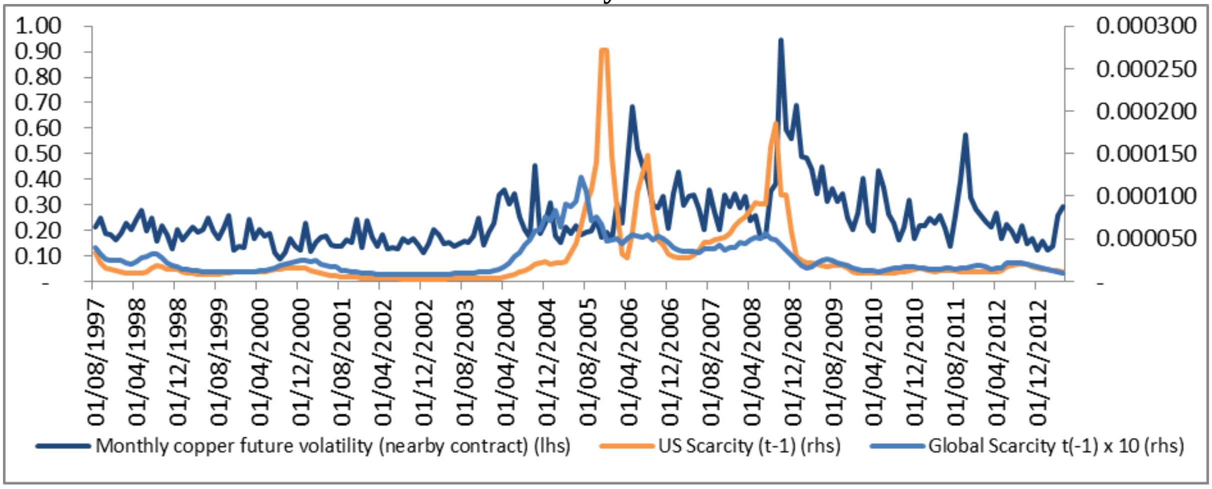
Figure 6: Monthly copper price volatility regression output (based on monthly inventory data)

	Global Inventory Analysis	US Inventory Analysis
	Coefficients	*Coefficients*
Multiple R	0.39	0.41
R Square	0.15	0.17
Adjusted R Square	0.14	0.16
Standard Error	0.11	0.11
Observations	190.00	190.00
F-Test	16.58	18.49
Intercept	0.15	0.16
T-stat	7.92	9.99
Time effect	0.00	0.00
T-stat	4.91	4.55
Global Scarcity t(-1)	10,279.48	631.62
T-stat	2.62	3.20

Despite adding Chinese stock data to the global stock variable US inventories show a higher R2 in explaining copper nearby futures volatility. It is worth mentioning that the time effect does not seem to influence spot price volatility because the coefficient is close to 0 for both, global copper inventories as well as US inventories only.

In the next chart we contrast the 2 derived scarcity variables versus daily copper spot price volatility on a monthly basis. The scarcity variables show 2 event of significant spikes, namely in 2005 and just before the Financial crisis. The positive beta confirms that our scarcity proxy shows a positive relationship to subsequent copper price volatility. Thus when inventories get lower a rise in copper spot price volatility is more likely. Beside these 2 events the scarcity proxies did not signal tight inventories for most of the last 15 years.

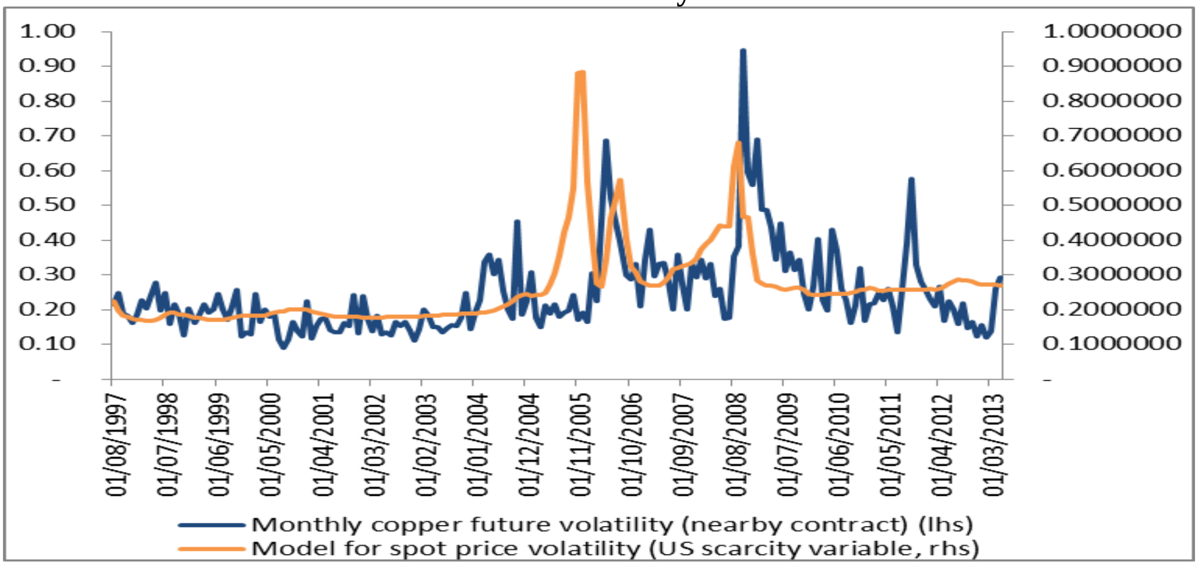
Figure 7: Monthly copper price volatility based on daily data versus Global and US scarcity variables



Looking further into the historical behaviour of copper spot prices we use the regression outputs for an in-sample period (1997 to 2004) and approximate copper spot price volatility for the out-of-sample period (2005-2013) and compare it with realized volatility.

$$\widehat{\sigma_t} = \hat{\alpha} + \hat{\delta}t + \hat{\beta}s_{t-1} \tag{2}$$

Figure 8: Monthly copper price volatility based on daily data versus modelled volatility



As could have been suspected from the previous chart looking at the scarcity variable over time the estimated volatility anticipates the 2 spikes in copper future volatility in late 2005 and during the financial crisis while the spot price volatility spike in 2011 seems having been driven by financial market volatility (Euro-Zone debt crises) rather than fundamental (supply/demand) reasons.

Reference level or fundamental spot price of copper for fundamentally driven investors

In this section we describe the role of the financial players present in our model more in detail. We described the general market characteristics of copper markets in the previous chapter while this chapter will provide a framework for modelling the demand and supply relationship via the fundamental value of copper prices. We will distinguish between two different types of market participants, fundamental agents and technical agents. The agent of our model takes the expected price for the next time interval [t; t+], called the reference level, from a financial expert. Indeed, we need to describe how these experts choose this reference level. We consider a finite set of financial experts I = {1,2,......,M}.

The fundamental value or benchmark of each expert, denoted L_t^i, is the value, on a logarithmic scale, at which this expert $i \in$ expects the price to return in the long run. The long-run price of Copper is based on a three-factor model for commodity prices where the equilibrium price level is assumed to evolve according to a geometric Brownian motion with drift, reflecting themes like exhaustion of existing supply, improved technology for production or inflation (Geman, 2000) This long-term equilibrium price is not directly observable in the market though in the case of long-term futures prices this information can be estimated over time. Additionally, the long-term equilibrium price level will be refined with a short-term deviation term which is expected to revert to zero following an Ornstein-Uhlenbeck process. These short-term deviations are representing short-term changes in demand or supply e.g. because of weather. The third factor introduced will be a scarcity parameter for copper which represents the inverse of global copper stocks at the end of each time period $t \in T$

As often done in financial literature we define returns as changes in log prices. We denote the spot price of Copper by S_t at time t. First we introduce a scarcity variable similar to Geman and Nguyen (2005) denoted as s_t and defined as inverse inventory at time t. The same notation as in Geman and Nguyen (2005) is used to denote the world stock of copper at the end of period t, It and thus the scarcity $s_t=1/I_t$ follows. To gain more insight into the effects of inventories on volatility have run the following equation in the previous section.

$$\sigma_t = \alpha + \delta t + \beta s_{t-1} + \varepsilon_t, \qquad (18)$$

where t denotes the time period (month), s_{t-1} is the scarcity variable at time t-1, and σ_t is the standard deviation of the nearby returns over period t. α accounts for the possibility of a trend in either the volatility or scarcity series. The constant β, the sensitivity of volatility to past inventory data is positive if high inventories reduce volatility. If on the other side inventories are very low then an additional unit of inventory will have a greater effect on volatility.

The Copper spot price is decomposed into three stochastic factors

$\ln(S_t) = X_t$ (19)

where X_t will be the short-term deviations from the Copper equilibrium price modelled as a mean reversion process with a stochastic long-term equilibrium price L_t.

The variance of the spot return is assumed to be stochastic and represented by the following equation:

$var_t = (\alpha + \beta s_t)^2$ (20)

where α as well as β are constant and β being positive.

The dynamics of the stochastic component of the spot price under the real probability measure are driven the following stochastic differential equations:

$dX_t = (\kappa(L_t - X_t) + \lambda_x v_t)dt + \sqrt{var_t}dz_t^x$ (21)

where L_t is represented by a geometric Brownian motion with constant drift μ and λ_χ represents the market price of commodity risk:

$dL_t = (\mu + \lambda_L \sigma_L v_t)dt + \sigma_L dz_t^L$ (22)

where λ_L represents the risk premium on the long-term mean uncertainty. The variance of the stochastic component of the spot price can thus be written by

$dvol_t = (a(b - v_t) + \lambda_v \sigma_v var_t)dt + \sigma_v \sqrt{var_t}dz_t^v$ (23)

We thus assume that the two state variables X_t and v_t follow a mean-reversion process and L_t a geometric Brownian Motion respectively. Further there exists a correlation ρ_{xL} (respectively ρ_{vL} and ρ_{xv}) assumed between the Brownian Motions z_x and $z_L$and z_v.We assume no arbitrage opportunities because we have more instruments than sources of risk and hence the market is complete. The variables a,b and σ_v are positive and λ_v is the market price of volatility risk.

The existence by arbitrage does hold. (of a risk-neutral probability measure Q can be assumed)

$dX_t = \kappa(L_t - X_t)dt + \sqrt{var_t}d\hat{z}_t^x$ (24)

$dL_t = \mu dt + \sigma_L d\hat{z}_t^L$ (25)

$dvol_t = a(b - var_t)dt + \sigma_v \sqrt{var_t}d\hat{z}_t^v$ (26)

The choice of the square root process in equation (25) ensures positivity of the solution while mean reversion implies bounded values. The fact that var_t is observable means that the Kalman Filter procedure used to calculate the parameters involved is based on normally distributed quantities.

Our representation of the spot price has the features of a mean-reverting behaviour with a stochastic trend and stochastic volatility.

The assumptions above imply the following dynamics of the copper spot price under the Q measure:

$$dS_t = k[(L_t - \ln S_t) + \frac{1}{2k} var_t]S_t dt + \sqrt{var_t} S_t d\hat{z}_t^x \tag{27}$$

Because the future price is a Q-martingale based on assumptions (18) and (19) the price F_t^T at time t of a future contract maturing at time T can be written as

$$F_t^T = E_Q\left(\frac{S_T}{\mathcal{F}_T}\right) = e^{A(t,T)+B(t,T)\ln S_t + C(t,T)L_t + D(t,T)var_t} \tag{28}$$

The solution of this form yields the system of the following ordinary differential equations:

$$B' + kB = 0 \tag{29}$$

$$C' - kB = 0 \tag{30}$$

$$-D' + \frac{\sigma_v^2}{2}D^2 - D(a + \rho_{xv}\sigma_v B) + \frac{\sigma_L^2}{2}C^2 + \frac{1}{2}B^2 = 0 \tag{31}$$

$$A' - \mu C - abD = 0 \tag{32}$$

with initial conditions A(T,T)=0, B(T,T)=1, C(T,T)=0 and D(T,T)=0. The solutions to (29) and (30) are elementary and plugging into (27) and (28) results in the following expressions:

$$B(t,T) = e^{-\kappa(T-t)} \tag{33}$$

$$C(t,T) = e^{\kappa(T-t)} \tag{34}$$

$$A(t,T) = \mu\frac{1-e^{\kappa(T-t)}}{k} + ab\int_t^T D(u,T)du \tag{35}$$

Where D(t,T) is the solution to the following ordinary differential equation

$$D'(t,T) = -\frac{\sigma_v^2}{2}D^2 + D(a - \rho_{xv}\sigma_v B) - \frac{\sigma_L^2}{2}C^2 - \frac{1}{2}B^2 - \rho_{LX}\sigma_L BC \tag{36}$$

The integral in (35) will be solved using the numerical procedure of the trapezoidal rule. The solution to equation (36) is not available in closed form but will be solved numerically with high precision by methods like Runge-Kutta.

The Kalman Filter approach for a three-factor copper price model

The Kalman Filter will be used to calculate unobserved state variables (long-term equilibrium price and stochastic component of the spot price) based on observations (in this case the log of Future prices for Copper) that depend on these state variables. We will work in a discrete setting and given a prior distribution on the initial values of the state variables and a model

describing the likelihood of the observations as a function of the true values, the Kalman Filter will generate updated posterior distributions for these state variables.
In the three-factor model only the stochastic component of the spot price and its stochastic long-term equilibrium mean are unobservable whereas the scarcity variable is directly obtained by taking the inverse of the inventory numbers.
Two equations are crucial for the Kalman Filter, namely the measurement equation and the transition equation. The measurement equation relates the observable vector Y_t^F to the state vector Z_t where Z_t is defined as $Z_t = [X_t , L_t]$ via the following relationship:

$$Y_t^F = M_j + L_j Z_j + \omega_j, j = 1, \ldots, J \tag{37}$$

where

$$M_j = [A\left(t_j, T_i\right) + D\left(t_j, T_i\right) var_t]\Big], i = 1, \ldots. N, N \times 1 \; vector$$

$$L_j = [B\left(t_j, T_i\right), C\left(t_j, T_i\right)]\Big], i = 1, \ldots. N, N \times 2 \; vector$$

ω_j is a $N \times 1 \; vector$ of serially uncorrelated disturbances with E[$\omega_j] = 0, Var[\omega_j] = \Omega$ where Ω is a diagonal matrix.

Equations (21) and (22) can be used to derive the transition equation for the copper spot price as

$$dZ_t = (U_t + HZ_t)dt + V_t dW_t \tag{38}$$

where

$U_t = [\lambda_x v_t, \mu + \lambda_L \sigma_L var_t]'$ and $H = \begin{bmatrix} -\kappa & \kappa \\ 0 & 0 \end{bmatrix}$ and is V_t such that

$$V_t V_t' = \begin{bmatrix} 1 & \rho_{XL}\sigma_L \\ \rho_{XL}\sigma_L & \sigma_L^2 \end{bmatrix} var_t$$

and the discrete-time transition equation for the Kalman Filter is obtained as:

$$Z_j = e^{H\Delta} Z_{j-1} + G_{12}\left(\Delta_j\right) + \tilde{V}_j \epsilon_j \tag{39}$$

ϵ_j is a 2 x 1 vector of serially uncorrelated disturbances with $E\left(\epsilon_j\right) = 0$ and $Var\left(\epsilon_j\right) = IdentityMatrix(2x2)$. $G_{12}\left(\Delta_j\right) = \int_0^{\Delta} e^{H(\Delta_j - u)} U_j du$ is the approximate discrete-time version of U_t in the transition equation.
The Kalman Filter allows estimating the state variables over time by updating the estimator $\widetilde{Z_{J/J}}$ but this is assuming a specific assumption about the parameters of the process. The equations above assumed the prior knowledge of these parameters. In practice however the parameters are unknown so they have to be estimated, e.g. by Maximum likelihood:

$$\log Likelihood = -\frac{NJ}{2} log 2\pi - \frac{1}{2}\sum_{j=1}^{J} log\left|R_j\right| - \frac{1}{2}\sum_{j=1}^{J} \hat{v}_j' R_j^{-1} \hat{v}_j \tag{40}$$

Where the conditional distribution of $\hat{v}_{j+1}$is normal with mean zero and a covariance matrix R_j.

Three-factor copper price model - results

In this section we estimate the model parameters developed earlier. We apply the Kalman filtering procedure to the time series of nine maturity copper futures prices (N=9) for up to 2 years out the forward curve. We use monthly data from July 1997 to May 2013.

The Kalman filter is a recursive method for computing the unobserved state variables and works best for normally distributed data. These state variables are described a transition equation while the link between the observable futures prices and the state variables is explained by the measurement equation. The Kalman filter optimizes a log-likelihood function that minimizes the error between the model output and the real-world data used as input. In our three-factor model only the stochastic component of the spot price and its long-term mean (we use this long-term mean as the fundamental value input for our financial agent model later) are unobservable while the scarcity variable is directly obtained as the inverse of the inventory numbers.

The table below gives the estimated parameters and standard errors of the three-variable model and shows the estimated values of the common parameters.

Table 1: Parameter output – three factor model

Parameter	Three-factor model	Standard error
kappa	0.78	0.03
μ	8.62	0.05
σ_L	0.80	0.26
a	2.34	0.17
b	0.23	0.04
σ_v	2.95	0.34
corr_xv	0.51	0.08

Variable a is much higher than kappa indicating that the stochastic volatility process shows a stronger mean-reversion behaviour compared to the stochastic component of the spot price. The correlation between the stochastic component of the spot price and spot price volatility is positive and statistically significant, in conformity with the theory of storage.

Heterogeneous agent based model considering co-movement for Copper prices

In the previous section amongst the variables derived was the long-term fundamental value for copper. We will use this value as a fair value proxy in an agent based model approach. A simple heterogeneous agent based model will be presented in this chapter to assess the effect of fundamental as well as technical traders on the price of copper.

As mentioned earlier the aim of this paper is to wrap a three-factor commodity model into a Heterogeneous Agents Models where the Fundamental price input is directly derived from the three-factor model for copper.

This section describes the approach towards the Heterogeneous agent model chosen while the next section applies the output from the three-factor model to the agents model and calibrates the parameters to copper prices observed.

Demand/Supply relationship

Before going into detail about the various strategies that investors can apply this section focuses on simple demand and supply functions in order to characterize the copper market. Similar to Geman and Nguyen (1995) who use a linear regression to see the impact of inventories on volatility it makes sense to evaluate the overall demand and supply for copper in a simplified linear regression which takes into account exogenous factors as well as endogenous price-sensitive factors.

In the case of copper as with most commodities we can distinguish between real demand and investment demand, namely demand of fundamental investors and demand of technically driven investors.

The link of real and speculative demand with the price dynamics of copper will be modelled via the following equation:

$$\dot{P} = \beta D = \beta D_c + D_f = \beta[(\dot{n}_+ - n_-)s_c + n_f s_f(p_f - p)] \qquad (41)$$

where $\dot{P}$ represents the price change of copper which is a function of excess demand from fundamentalist (ED_f) and chartists (ED_c). We further follow Lux (1995) and distinguish between optimistic chartists (their absolute number is n_+) and pessimistic chartists (their absolute number is n_-). When we multiply the absolute number of chartists (N=n_-+n_+) with the amount of shares they hold (denoted s) then we derive the total demand of chartists. In this setting we are interested in the excess demand of chartists which according to equation (3) represents the difference of positive minus pessimistic chartists.

The interaction between chartists and fundamentalists is defined by two ratios, namely:

$$x := \frac{n_+ - n_-}{n_c} \qquad (42)$$

describing the excess of optimistic chartists over pessimistic chartists and

$$z := \frac{n_c}{N} \qquad (43)$$

where z represents the fraction of chartists amongst the entire population of traders.

Fundamentalists

Fundamentalists base their demand for copper on the difference between the current expectation (at time t) of the future spot price (at time t+1) and the current price of copper. The expected excess return of fundamentalists is thus given by d$|(p_f - p)/p|$ where d is a discount factor. This represents the present value of the trading profit expected by the fundamentalist which would occur when the price p has reverted back to the fundamental value p_f.

Chartists

The second type of strategies which is considered in this paper is chartists. Chartists base their investment decision on past price patterns. In contrast to fundamentalists, who tend to have a stabilizing effect on financial assets via contrarian trades chartists are more likely to invest with the trend and thus encourage current trends in the market further. In line with what has been proposed in previous Heterogeneous Agents Models (e.g. Hommes 2006) we focus on a pure trend following approach where chartists look at the past price at t-1 and thus try to assess short-term trends.

The distinction between optimists and pessimists adds further refinement to the price dynamics of our agent-based model approach. Both benefit from a price move $\dot{p}$ above a refinancing rate r_f (we are focusing on excess returns above risk free rate r_f). Thus the profit of an optimistic chartist can be modelled as:

$$optimists\ trading\ profit = \frac{r + \dot{p}/w}{p} - r_f \tag{44}$$

where w represents a speed of transition parameter between the chartists and the fundamentalists.

$$pessimists\ trading\ profit = r_f - \frac{r + \dot{p}/w}{p} \tag{45}$$

Optimists are long the stock and thus pay the refinancing rate r_f whereas pessimists are short the stock and thus receive r_f.
After we defined the trading profits of both groups, fundamentalists and chartists, the next step is to combine both in a systematic interaction approach. This is based on the utility function of both groups which is in simple terms a function of performance.

Interaction between fundamentalists and chartists

The transition probability of a chartist from positive to negative and vice versa is determined by Utility

$$U_1 = \alpha_c x + \beta_c \frac{\dot{p}}{v} \tag{46}$$

which directly feeds into the transition probability of moves between optimists and pessimists where v is a variable for the speed of change from optimists to pessimists

$$p_{\mp\pm} = v(\frac{n_c}{N}\exp(\pm U_1)) \tag{47}$$

The utility is thus a function of the relative weight of optimists versus pessimists (x) and the price change. The transition probability function is an exponential function (Lux 1995) and considers moves from pessimists to optimists in the case of rising prices and a higher transition probability of optimists switching to pessimist in the case of falling copper prices in equation (47).

The transition probability of moving from fundamentalists to chartists is modelled in a similar way as

$$p_{f\pm} = w\frac{n_+}{N}\exp(U_{2,\pm}) \tag{48}$$

And vice versa as

$$p_{\pm} = w\frac{n_f}{N}\exp(-U_{2,\pm}) \tag{49}$$

where the utility of moving from fundamentalist group to the optimist group $U_{2,\pm}$ and from the optimist to fundamentalists -$U_{2,\pm}$ is given by

$$U_{2,+} = \alpha_3(\left(\frac{r+\frac{p}{w}}{p} - r_f\right) - d\left|\frac{p_f - p}{p}\right|) \tag{50}$$

The Utility of moving from the optimist to the fundamentalist is derived as the difference of the performance of the optimist (who is long copper and short cash) and the discounted expected performance of the fundamentalist.

Similarly the utility from pessimists to fundamentalists, denoted $U_{2,-}$ can be described as the difference of the performance of a pessimist (who is short copper and long cash) minus the discounted expected profit of the fundamentalist

$$U_{2,-} = \alpha_3(\left(r_f - \frac{r+\frac{p}{w}}{p}\right) - d\left|\frac{p_f - p}{p}\right|) \tag{51}$$

As mentioned in the previous section this paper will explicitly derive the input for the fundamental investors, namely the long-run equilibrium price of copper which was labelled L_t. The next section will give a brief overview of the derivation of this long-term equilibrium spot-price for copper.

$$\frac{dx}{dt} = \frac{\frac{dn_+}{dt} - \frac{dn_-}{dt}}{n_c} - (\frac{n}{n_c^2})dn_c/dt$$

$$n_+ = \frac{(1+x)zN}{2}$$

$$n_- = \frac{(1-x)zN}{2}$$

$$\frac{dx}{dt} = z[(1-x)p_{\mp} - (1+x)p_{\pm}] + \frac{(1-z)(1-x^2)(p_{f+} - p_{+f} + p_{-f} - p_{f-})}{2} \tag{52}$$

As $p_{\pm -}$ and $p_{\mp}$ are of the form of exponential functions (as shown in equations (47-49) above) we can use the following trigonometric identities:
$$\mathrm{Sinh}(x) = \frac{(e^x - e^{-x})}{2}, Cosh(x) = \frac{(e^x + e^{-x})}{2} \ and\ Tah(x) = \frac{Sinh(x)}{Cosh(x)}$$

From the definition of z we can follow that $\frac{dz}{dt} = \frac{\frac{dn_c}{dt}}{N}$ which can be expressed as

$$\frac{dz}{dt} = \frac{(1-z)z(1+x)(p_{+f} - p_{f+})}{2} + \frac{(1-z)z(1-x)(p_{-f} - p_{f-})}{2} + a(1-z) \tag{53}$$

We assumed at the start that chartists adjust their position for a fixed amount t_c (of shares). Chartists who are bullish try to increase their stake while those who are bearish will try to decrease their shares. This leads to an excess demand from chartists

$$D_c = (n_+ - n_-)t_c = nt_c = xzNt_c = xzs_c \qquad \text{where } s_c = Nt_c \tag{54}$$

Fundamentalists on the other side will buy copper when the price has fallen below their fair value proxy and sell when the price is above their fair value. We can thus formulate the excess demand of fundamentalists as

$$D_f = t_f\delta(p_f - p) = (1-z)N\delta(p_f - p) = (1-z)s_f(p_f - p) \qquad \text{where } s_f = N\delta \tag{55}$$

Combining equations (54) and (55) for dp/dt results in equation (43) and we have thus proofed the aggregate demand equation.

Agent based modelling approach - results

In this section we present the results of the agent based model introduced before. This Poisson-type dynamics of updating strategies and opinion index will be approximated within a simulation framework. We chose small time increments in order to avoid synchronicity of decisions and because the phenomenon of volatility bursts requires higher precision between the time steps modelled. We are using the long-term fundamental value derived from the three-factor model as benchmark for fundamental traders. We assume for the simulation a total number of 500 agents. In order to make sure the system is able to calibrate and in order to avoid degenerate situations in which either the group of chartists or fundamentalists has declined to zero we ensure a minimum number of 4 agents in each group, fundamental and technical agents. Despite the fact that this scenario of an absorbing state decreases with a sufficient number of agents it still has a positive probability of occurring and thus we prefer to apply a lower limit on each agent category.

We show in the table below the fixed parameter values for dividends and average rate of return. It is worth noting, as shown in several academic studies applying this model, that this approach is not very sensitive to those parameters and we have thus chosen values that are in line with previous applications (Lux, 1998, Lux et al. 2000).

Table 2: Fixed parameters for agent based model

Fixed Parameter	Assumed Value
Number of steps per integer time step	50
Number of microsteps for dp/dt	100
Number of agents	500
Minimum number of agents in a strategy	4
Nominal dividends of the asset	0.4%
Risk free rate	0.04%
Frequency of optimist/pessimist revaluation	3
Frequency of chartist/fundamentalist revaluation	2
Discount factor	0.75
Imprecision in excess demand perception	0.05

We estimate the importance of the opinion index for chartists (α_c), the importance of price changes for chartist expectations (β_c), the importance of profit differentials for a switch between chartists and fundamentalists (α_3) as well as the reaction speed of auctioneers (β) with the help of the Generalized method of moments technique (GMM). This method requires that a certain number of moment conditions (g(X, θ) are specified for the model for which we show the generalized form below.

$$m(\theta_0) \equiv E[g(X_t, \theta_0)] = 0 \tag{56}$$

These moment conditions are functions of the model parameters and data such that their expectation is zero at the true values of the parameters. The GMM method minimizes a certain norm of the sample average of our moment conditions and can thus be written as:

$$\hat{\theta} = arg\ \min_{\theta \epsilon \Theta} (\tfrac{1}{T} \textstyle\sum_{t=1}^{T} g(X_t, \theta))' \hat{W} (\tfrac{1}{T} \textstyle\sum_{t=1}^{T} g(X_t, \theta)) \tag{57}$$

where W represents the positive-definite weighting matrix.

Based on 14 years of monthly data we derive estimates for the parameters shown in the table below. All four parameters are positive as expected and statistically significant. It is worth noting that the importance of profit differentials for a switch between chartists and fundamentalists (α_3) as well as the reaction speed of auctioneers (β) are reasonably small. The importance of the opinion index of chartists and the importance of price changes for chartist expectations are closely linked and show a high positive correlation. The importance of profit differentials for a switch from chartists to fundamentalists is negatively correlated to the importance of price changes for chartist expectations as well as to the opinion index of chartists. Thus the higher the optimism (pessimism) amongst chartists the lower (higher) the probability of chartists to move to the fundamentalist group and the less (more) attention chartists are paying to past profit differentials for their strategy assessment.

Table 3: Agent based model output

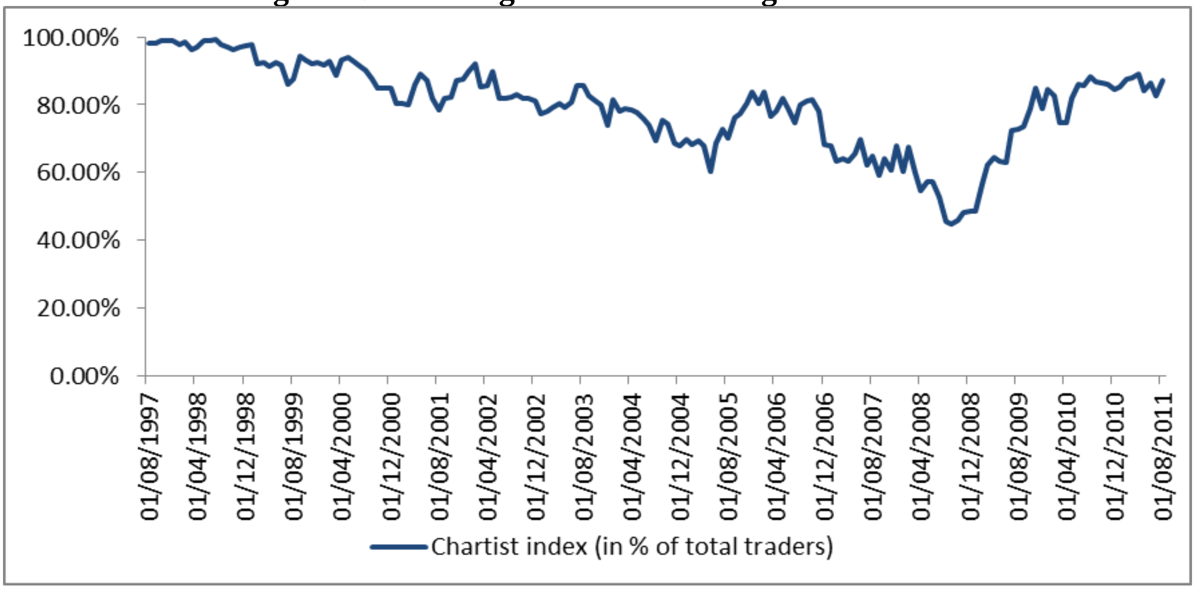
Number of observations:	169
Number of parameters:	4
Number of degrees of freedom:	165
Orthogonality conditions:	5

Parameter	Estimate	Standard Error
α_c	0.550	0.01
β_c	0.233	0.01
α_3	0.100	0.01
β	0.103	0.01

correlation of parameters	α_c	β_c	α_3	β
α_c	1	0.885	-0.996	0.326
β_c	0.885	1	-0.923	0.597
α_3	-0.996	-0.923	1	-0.390
β	0.326	0.597	-0.390	1

In order to provide some further insight into the model over time we show below the fraction of chartists modelled over time based on our approach. Based on the copper price data evaluated we can see that the number of chartists started to drop during the financial crisis. Copper prices retreated sharply and fell well below the fair value assumed by fundamental traders. Thus those fundamental traders gained in relative performance (mean reversion was more profitable) while a strong price recovery after the Financial crisis caused an increase in chartist traders as their trades became more profitable (momentum was more profitable).

Figure 9: Percentage of chartist amongst trader universe



Conclusion

We have shown in this paper that inventory plays a role in explaining copper price volatility. Using a three factor model we derived a fundamental long-term value for copper. The addition of a stochastic component in the spot price shows a positive correlation to copper spot price volatility. Second, we emphasis the significance of this fundamental long-term value by considering an agent based model approach in which mean-reversion focused fundamental investors trade with chartists who follow price trends. We showed that fundamental investors take increasing positions in copper when the spot price of copper deviated from its fundamental value (i.e. the fundamental value is higher than the spot price) and chartists loose relative significance.

References

Amilon, H., 2008. Estimation of an adaptive stock market model with heterogeneous agents, Journal of Empirical Finance 15, pp. 342-362

Boswijk, H.P., C.H. Hommes, and S. Manzan, 2007. Behavioral heterogeneity in stock prices, Journal of Economic Dynamics & Control, pp. 1938-1970

Brock, W.A., and C.H. Hommes, 1997. A Rational Route to Randomness, Econometrica, 65 (5), pp. 1059–1096

Brock, W.A., and C.H. Hommes, 1998. Heterogeneous beliefs and routes to chaos in a simple asset pricing model, Journal of Economic Dynamics and Control 22, pp. 1235-1274

Danielsen, A.L., 1979. The role of speculation in the oil price ratchet process, Resources and Energy 2, pp. 243-263

De Jong, E., W.F.C. Verschoor, and R.C.J. Zwinkels, 2009. Behavioural heterogeneity and shift-contagion: Evidence from the Asian Crisis, Journal of Economic Dynamics & Control 33 (11), pp. 1929-1944

Ellen, t.S., and R.C.J. Zwinkels, 2010. Oil price dynamics: A behavioural finance approach with heterogeneous agents, Energy Economics

Fama, E., 1970. Efficient capital markets: A review of theory and empirical work, The Journal of Finance 25 (2), pp. 28-30

Foellmer, H., U. Horst, and A. Kirman, 2004. Equilibria in Financial Markets with Heterogeneous Agents: A Probabilistic Perspective, Working Paper

Foellmer, H., and M. Schweizer, 1993. A Microeconomic Approach to Diffusion Models for Stock Prices, Mathematical Finance, 3, pp. 1-23

Foellmer, H., 1994. Stock Price Fluctuations as a Diffusion in a Random Environment, Philos.Trans R. Soc. Lond., Ser. A, 1684, pp. 471-483

Forbes, K.J., and R. Rigobon, 2002. No Contagion, Only Interdependence: Measuring Stock Market Comovements, Journal of Finance, Vol. 57, No.5, pp. 2223-2261

Frankel, J.A. and K.A. Froot, 1988. Chartists, fundamentalists and the demand for dollars, Greek Economic Review 10, 49-102

Geman, H., 2000. Scarcity and Price Volatility in Oil Markets, EDF Tading Technical Report

Geman, H., and V.N. Nguyen, 2005. Soybean inventory and forward curve dynamics, Management Science, Vol. 51, No. 7, pp. 1076-1091

Geman, H., 2005. Commodities and Commodity Derivatives, Modelling and Pricing for Agriculturals, Metals and Energy, John Wiley & Sons Ltd. West Sussex, England

Karolyi, G.A. and R.M. Stulz, 1996. Why Do Markets move together? An Investigation of US-Japan Stock Return Comovements, Journal of Finance, Vol. 51, No.3, pp. 951-986

Kirman, A. and G. Teyssiere, 2002. Microeconomic Models for Long-Memory in the Volatility of Financial Time Series, Studies in Nonlinear Dynamics and Econometrics, 5, pp. 281–302.

Longin, F., and B. Solnik, 2001. Extreme Correlation of International Equity Markets, Journal of Finance, Vol. 56, No.2 (April 2001), pp. 649-676

Lux, T., 1995. Herd Behavior, Bubbles and Crashes, The Economic Journal, Vol. 105, No. 431 (Jul. 1995) pp.881-896

Lux, T., 1998. The socio-economic dynamics of speculative markets: interacting agents. Chaos, and the fat tails of return distributions, Journal of Economic Behavior & Organization, Vol. 33 (1998), pp. 143-165

Lux, T. & M. Marchesi, 2000. Volatility clustering in financial markets: A microsimulation of interacting agents. International Journal of Theoretical and Applied Finance 3:4, pp. 675—702

Pindyck, R.S., and J.J. Rotemberg, 1990. The Excess Co-Movement of Commodity Prices, The Economic Journal, Vol. 100, No. 403, pp. 1173-1189

Pindyck, R.S., 1990. Inventories and the short-run dynamics of commodity prices, MIT's center for Energy Policy Research

Pindyck, R.S., 1990. The long-run evolution of energy prices, The Energy Journal, 20, pp.1-27

Schwartz, E. and J.E. Smith, 2000. Short-Term Variations and Long-Term Dynamics in Commodity Prices, Management Science, Vol. 46, No. 7

Pindyck, R.S., 2004. Volatility and Commodity Price Dynamics, The Journal of Futures Markets, Vol.24, No.11, pp. 1029-1047

Reitz, S., and U.Slopek, 2009. Nonlinear oil price dynamics – a tale of heterogeneous speculators?, German Economic Review 10 (3), pp. 270-283

Reitz, S. And F.H.Westerhoff, 2007. Commodity price cycles and heterogeneous speculators: a STAR-GARCH model, Empiricial Economics 33 (2), pp. 231-244

Smidt, S., 1965. Amateur Speculators: A survey of trading strategies, information sources and patterns of entry and exit from commodity futures markets by non-professional speculators, Cornell University, Cornell Studies in Policy and Administration.

Steinsaltz, D., 1999. Locally Contractive Iterated Function Systems, Annals of Probability, 4, pp. 546–588