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## **A Golden Rule of Health Care**

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# A Golden Rule of Health Care

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## Abstract

We derive a golden rule for the level of health care expenditures and find that the optimal level of life-extending health care expenditures should increase with rising productivity and retirement age, while the effects of improvement in medical technology are ambiguous.

**Keywords:** Health care, golden rule, productivity.

**JEL Classification:** E62, I12

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## 1. Introduction

Advanced economies devote significant resources to health care. A substantial part of the spending is for treatments that will extend the lives of the elderly.<sup>1</sup> In this paper, we address the question how much a nation should sacrifice in terms of the consumption of individuals in order to extend the life of generations. Clearly, a society could sacrifice the consumption of its population in order to devote large resources to treat the old. Alternatively, a nation could choose to live happily, not worrying about the length of the lifespan, as long as they can.

## 2. Literature

A large body of literature exists on the determinants of health care expenditures. What separates this literature from our approach is that, while the literature mainly approaches the issue as demand for health care by individuals as a function of income and age, we solve the social planner's problem. Our approach is not to describe how individuals form their demand for health care but to derive how much society should spend on extending the lives of its citizens.

The question concerning the optimal level of health care spending is closely related to the golden rule literature that began with Ramsey (1928) on the optimal level of saving and was expanded in papers on the optimal level of research and the optimal level of education (see Phelps, 1966 and 1968, among others). Here, investing in future research ideas, education, or the physical capital stock requires a reduction in current levels of consumption, while better education and technology and a larger stock of capital will make possible an increase in the future level of consumption.

Becker (1964) introduced the concept of human capital. In his framework, individuals have different levels of human capital – which determines their future earning potential – and can add to this stock through education and training. Grossman (1972) expanded Becker's notion of human capital to include health capital and made a distinction between health as an output and medical care as one of many inputs in the production function for good health. Here health is a durable capital stock that gradually depreciates with age but can be augmented through health care and healthy living. Health as capital yields an output of “healthy time” which affects productivity in the workplace, therefore affecting wages, productivity at home,

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<sup>1</sup> According to the World Bank the United States spent on average from 2001-1010 around 16% of GDP on health care (sum of public and private costs).

and utility from leisure. We depart from Grossman in not modeling the investment in good health by individuals but instead modeling the social planner's problem of optimal investment in lower mortality rates among those who have retired from the labor market. The social planner must trade off lower private consumption against a longer lifespan due to better medical care in old age.

The empirical literature on the determinants of health care spending has focused mainly on exploring the income elasticity of the demand for health care. One issue is whether there is a positive relationship between income and the level of health care expenditures. A related question is whether health care is a luxury good; i.e., having elasticity greater than one with respect to income. A part of this literature has used cross-sections of countries having health expenditures per capita as a dependent variable and GDP per capita, the age structure of the population, and sometimes measures of medical technology as explanatory variables; other studies use pooled data for a sample of countries over time; and yet others analyze the time-series properties of the two series for individual countries or groups of countries.

One of the most frequently cited papers in this literature is Newhouse (1977). He studied the relationship between GDP per capita and per capita medical care expenditures for 13 developed countries and found that more than 90% of the variation in the level of health care expenditures across countries could be explained by differences in the level of GDP per capita. Several other studies found similar results using data for different samples of countries and different time periods.<sup>2</sup> Gerdtham *et al.* (1992) extended this analysis to pooled data of 20 OECD countries over the period 1960-1987 and also found a positive income elasticity of health care spending with respect to GDP. The same applies to Parkin *et al.* (1987), who use PPP-adjusted numbers for health care spending and GDP per capita in a cross-section of countries. Hitiris and Posnett (1992) also found a positive effect of income on health care.

Hansen and King (1996) criticized previous studies by pointing out the possibility of non-stationarity of the time series. These authors found non-stationarity in the health care spending and the income series for many countries but failed to detect cointegration between the variables. Blomqvist and Carter (1997) studied individual country series and found both unit roots in the series and a long-run relationship between them. Gerdtham and Lothgren

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<sup>2</sup> See Cullis and West (1979), Maxwell (1981) and Leu (1986), among others.

(2000) found both non-stationarity and cointegration in a panel of countries, allowing for linear trends.

Di Matteo (2005) used state-level data for the United States for the period 1980-1998 and province-level data for Canada for 1975-2000. He found that health care expenditures depended on income, time, and age distribution. While it is not surprising that the ageing of the population increases health expenditures, it does come as a surprise that it is not only those over 65 who tend to increase expenditures but also the proportion aged 18-44. Baltagi and Moscone (2010) also find an effect of the younger population on health care expenditures. They investigate the long-run relationship between health care expenditures and income in 20 OECD countries and find, in addition to a positive long-run relationship between health care expenditures and income, that the proportion of young people has a positive effect on health care spending.

In this paper, we are concerned about the cost of extending lives rather than improving the quality of life. The cost of extending lives has been shown to be a significant component of overall health care expenditures. Seshamani and Gray (2004) use English longitudinal data and find that approaching death affects costs for up to 15 years prior to the time of death. In particular, the tenfold increase in costs from five years prior to death to the last year of life is much greater than the 30% increase from age 65 to 85. Lubitz and Riley (2003) study Medicare payments in the US and find that around 30% of Medicare payments go to people in the last year of life. These authors find that people in their last year of life make up 35% to 39% of the 5% of beneficiaries with the highest costs. Note that this is clearly an underestimate of the cost of extending lives, as the figure omits the cost of treatment of those whose lives were extended beyond one year. Jones (2003) and Miller (2001) also find that US Medicare expenditures rise rapidly in the years preceding death. The former finds that expenditures rise at the rate of 9.4% per year in the 3-10 years before death and then by 45% in the final two years before death.

Jones (2003) argues that the critical determinant of health expenditures as a share of GDP is the willingness of society to transfer resources to those at the end of life. Better medical technology that makes it possible to extend lives makes health care expenditures and life expectancy increase over time. This leads us to the topic of this paper, which is to answer the question how much the working population should pay to extend the lives of the older

generation when realizing that they will also receive the same care funded by the next generation.

### 3. The golden rule of longevity derived

The model is a continuous-time and continuous-age overlapping-generation model where individuals belonging to different generations (at different ages) are alive at each point in time. An individual receives utility from consumption, works during the first part of his life cycle, and retires at a certain age. Furthermore, it is assumed that the economy is a small, open one, so that the interest rate  $r$  is constant for the economy and the current account is balanced at all times.

#### 3.1 Demographics

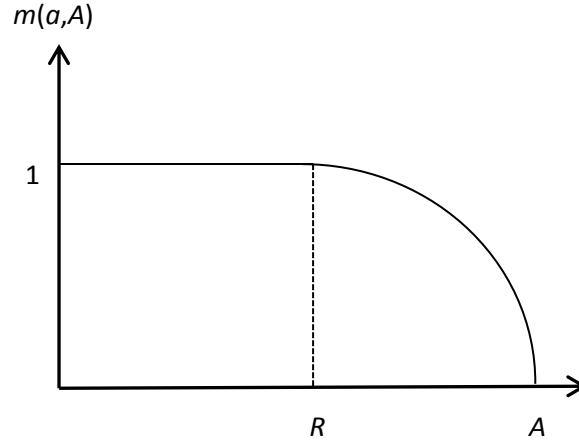
The population at time  $t$  is split into two groups: those working (young), whose ages are between 0 and  $R$  ( $a \in [0, R]$ ), and those retired (old), whose ages are between  $R$  and  $A$  ( $a \in (R, A]$ ), where  $R$  is the retirement age and  $A$  is the maximum age or longevity. Because the main concerns of this paper are the effects of health care expenditure for the elderly, it is assumed that survival probabilities are constant and equal to one when an individual is young, and decreasing and concave in age when he is old. Hence the survival probabilities are:

$$m(a, A) = \begin{cases} 1 & a \in [0, R] \\ f(a, A) & a \in (R, A] \end{cases} \quad (1)$$

where  $\lim_{a \rightarrow R^+} f(a, A) = 1$  and  $f(A, A) = 0$ , which implies that  $0 \leq f(a, A) < 1$  must hold. Further, it is assumed that  $f(a, A)$  is strictly decreasing and strictly concave in  $a$  and strictly increasing concave in  $A$ :

$$\frac{\partial f}{\partial a} < 0, \frac{\partial^2 f}{\partial a^2} < 0, \frac{\partial f}{\partial A} > 0$$

The following figure shows the survival function.



Following Boucekkine *et al.* (2002), the number of individuals born is assumed to grow at a constant rate  $n$  and the number of individuals born at time  $t$  is  $\varphi e^{nt}$ , where  $\varphi > 0$ . The number of individuals aged  $a$  at time  $t$  is therefore:

$$l(t, n, a, A) = \varphi e^{n(t-a)} m(a, A) > 0 \quad (2)$$

Note that  $l(t, n, 0, A) = \varphi e^{nt} > 0$ ,  $l(t, n, A, A) = 0$  and  $\frac{\partial l}{\partial a} < 0$ . Hence the number of individuals born at time  $t$  is  $\varphi e^{nt}$ , the number of individuals exceeding the maximum age  $A$  is zero, and the number of individuals aged  $a$  is strictly decreasing in  $a$  for all  $t$  and  $A$ . In addition,  $\frac{\partial l}{\partial A} > 0$  for all  $a \in (R, A]$ , implying that the number of old individuals at each age level is increasing in longevity. Using (2), the population mass in the economy at time  $t$  can be written:

$$N(t, n, A) = \int_{a=0}^R l(t, n, a, A) da + \int_{a=R}^A l(t, n, a, A) da \quad (3)$$

and the number of young and old individuals, respectively:

$$N_w(t, n, R, A) = \int_{a=0}^R l(t, a, A) da \quad (4)$$

$$N_o(t, n, R, A) = \int_{a=R}^A l(t, a, A) da \quad (5)$$

It follows from (2) and (3) that the population growth rate in the economy follows the growth rate of individuals born:  $\dot{N}/N = n$ , where  $\dot{N}$  is the time derivative.

Furthermore, the number of the young and the old individuals is strictly increasing in the population growth rate:

$$\frac{\partial N_w}{\partial n} = \int_{a=0}^R \varphi (t-a) e^{n(t-a)} m(a, A) da = \int_{a=0}^R (t-a) l(t, n, a, A) da > 0$$



$$\frac{\partial N_o}{\partial n} = \int_{a=R}^A \varphi(t-a)e^{n(t-a)}m(a,A)da = \int_{a=R}^A (t-a)l(t,n,a,A)da > 0$$

### 3.2 Individual utility

Individuals gain utility from consumption:

$$U(c(t,a)) \text{ for } a \in [0,A] \quad (6)$$

where  $c(t,a)$  is consumption for an individual aged  $a$  at time  $t$ . Utility from consumption is standard (strictly increasing and concave). Note that although longevity is uncertain for a given individual, the fraction of individuals reaching a certain age is deterministic for the social planner (see below).

### 3.3 Health care expenditure and longevity

Health care expenditure per old individual at time  $t$   $\tau(t)$  is assumed to affect longevity and thus the health of old individuals (since  $f(R,A)$  is assumed to be strictly increasing in  $A$ ) in the following way:

$$A = \gamma\tau(t) \quad (7)$$

where  $\gamma > 0$  is a parameter measuring efficiency in health care production.

### 3.4 Output

Each young individual produces  $y(t) > 0$  at time  $t$ . National output in the economy at time  $t$  can therefore be written as:

$$N_w(t,n,R,A)y(t) \quad (8)$$

### 3.5 The social planner's problem

We assume a balanced current account growth path and hence balanced budget constraints for the economy at all points in time. The economy-wide budget constraint is therefore in balance at all points in time:

$$\begin{aligned} & \int_{a=0}^R l(t,n,a,A)y(t)da \\ &= \int_{a=0}^R l(t,n,a,A)c(t,a)da + \int_{a=R}^A l(t,n,a,A)c(t,a)da + \int_{a=R}^A l(t,n,a,A)\tau(t)da \end{aligned} \quad (9)$$

Because a balanced budget for the economy is assumed, there is no transfer of resources across time. The social planner's welfare objective can therefore be written as:

$$W = \int_{a=0}^R l(t, n, a, A)U(c(t, a))da + \int_{a=R}^A l(t, n, a, A)U(c(t, a))da \quad (10)$$

The maximization of this objective function subject to the budget constraint in (9) gives the social optimum. Note that the welfare function in (10) is strictly increasing in  $c(t, a)$  for all  $a \in [0, A]$  and  $\tau(t)$  (through  $A$ ). The budget constraint in (9) ensures that a maximum exists to the constrained maximization problem (increased spending on health care per old individual decreases consumption given output and hence raises the marginal utility of consumption, ensuring that a maximum exists).<sup>3</sup>

The Lagrangian for the maximization problem is (after using (1), (2), (6), (7) and (8)):

$$\begin{aligned} \Gamma(t) = & \int_{a=0}^R \varphi e^{n(t-a)}U(c(t, a))da + \int_{a=R}^{\gamma\tau(t)} \varphi e^{n(t-a)}f(a, \gamma\tau(t))U(c(t, a))da \\ & + \lambda(t) \left[ \int_{a=0}^R \varphi e^{n(t-a)}y(t)da - \int_{a=0}^R \varphi e^{n(t-a)}c(t, a)da \right. \\ & \quad - \int_{a=R}^{\gamma\tau(t)} \varphi e^{n(t-a)}f(a, \gamma\tau(t))c(t, a)da \\ & \quad \left. - \int_{a=R}^{\gamma\tau(t)} \varphi e^{n(t-a)}f(a, \gamma\tau(t))\tau(t)da \right] \end{aligned}$$

The first-order conditions are (after some manipulation):

$$\frac{\partial U}{\partial c(t, a)} = \lambda(t) \text{ for } a \in [0, A]$$

$$\left[ \int_{a=R}^A \varphi e^{n(t-a)}U(c(t, a)) \frac{\partial f}{\partial A} da \right] \gamma$$

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<sup>3</sup> In Appendix A1, it is shown that the first-order conditions derived below in fact give a maximum for the welfare function in (10), subject to the budget constraint in (9).

$$\begin{aligned}
& - \lambda(t) \left[ \int_{a=R}^A \varphi e^{n(t-a)} c(t, a) \frac{\partial f}{\partial A} da + \int_{a=R}^A \varphi e^{n(t-a)} \tau(t) \frac{\partial f}{\partial A} da \right] \gamma \\
& - \lambda(t) \int_{a=R}^A \varphi e^{n(t-a)} f(a, A) da = 0 \\
& \int_{a=0}^R \varphi e^{n(t-a)} y(t, 0) da - \int_{a=0}^R \varphi e^{n(t-a)} c(t, a) da - \int_{a=R}^A \varphi e^{n(t-a)} f(a, A) c(t, a) da \\
& - \int_{a=R}^A \varphi e^{n(t-a)} f(a, A) \tau(t) da = 0
\end{aligned}$$

where it has been used that  $f(A, A) = 0$ . The first condition implies that the marginal utility of consumption is independent of age (because  $\lambda(t)$  is independent of age). Hence we have consumption smoothing across generations at each time  $t$ :

$$c(t, a) = c(t) \text{ for } a \in [0, A] \quad (11)$$

Using (11) in the second first order condition gives (and after some manipulation using (1), (2) and (5)):

$$U(c(t)) \frac{\partial N_o}{\partial A} \gamma = \frac{\partial U}{\partial c(t)} (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma + \frac{\partial U}{\partial c(t)} N_o(t, n, R, A) \quad (12)$$

This equation gives optimal spending on health care per old individual. The left-hand side shows increased social welfare in terms of a greater number of old individuals reaching each age level and therefore more individuals receiving utility from consumption. The first term on the right-hand side shows the lost utility for all others, whose consumption is reduced due to the consumption and medical needs of those who reach higher age levels because of the increased provision of health care. The second term denotes the utility effect of their lower consumption that is due to higher health care expenditures.

Finally, the budget constraint must hold, which can be written in the following way after using the result in (11) (and after some manipulation using (1), (2), (4) and (5)):

$$(y(t) - c(t)) N_w(t, n, R, A) = (c(t) + \tau(t)) N_o(t, n, R, A) \quad (13)$$

The output net of consumption of the working-age population must equal the sum of consumption and the health care expenditures of the old generations.

## 4 Implications

The conditions in (12) and (13) give the optimal levels of consumption per capita  $c(t)$  and health care expenditure per old individual  $\tau(t)$  as implicit functions of time  $t$ , population growth  $n$ , productivity  $y(t)$ , the retirement age  $R$ , the parameters  $\varphi$  and  $\gamma$ , and the functional form of the utility function  $U$ . Below we analyze how productivity, both in the private sector  $y(t)$  and in the public sector  $\gamma$ , and the retirement age  $R$  affect the optimal level of health care expenditure per old individual  $\tau(t)$ .

The effect of increased productivity  $y(t)$  is given by:

$$\frac{\partial \tau(t)}{\partial y(t)} = \frac{\frac{\partial^2 U}{\partial c(t)^2} (N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma) N_w}{\Psi} > 0 \quad (14)$$

where

$$\Psi \equiv (N_w + N_o) \frac{\partial U}{\partial c(t)} N_o \left( \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - 2 \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) \gamma + \frac{\partial^2 U}{\partial c(t)^2} \left( N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma \right)^2.$$

Note that  $\Psi$  is negative, as is shown in Appendix A1, the nominator is negative due to diminishing marginal utility, and  $\frac{\partial N_o}{\partial A} > 0$ , as can be seen from (1), (2) and (5), which gives the sign in (14).

Intuitively, the marginal utility of consumption of each member of the working-age population declines as the level of productivity rises. Lower marginal utility reduces the burden of paying taxes to support the currently old and to pay for their health care, which increases the golden rule level of health care provision. It follows that people should live longer in more developed countries.

An increase in the retirement age  $R$  implies an increase in the ratio of working-age population to those retired. The effects of a higher retirement age  $R$  are:

$$\frac{\partial \tau(t)}{\partial R} = \frac{\left[ \frac{(N_w + N_o) \frac{\partial U}{\partial c(t)} \frac{\partial N_o}{\partial R}}{+ \frac{\partial^2 U}{\partial c(t)^2} (N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma) (c(t) + \tau(t)) N_o \left( \frac{1}{N_w} \frac{\partial N_w}{\partial R} - \frac{1}{N_o} \frac{\partial N_o}{\partial R} \right)} \right]}{\Psi} > 0 \quad (15)$$

The sign of the first term in the nominator is negative because  $\frac{\partial N_o}{\partial R} < 0$ , as can be seen from (5), and it has been used that  $f(R, A) = 1$  a constant. The sign of the second term depends on

the sign of  $\frac{1}{N_w} \frac{\partial N_w}{\partial R} - \frac{1}{N_o} \frac{\partial N_o}{\partial R}$ , which is positive because  $\frac{\partial N_w}{\partial R} > 0$  and  $\frac{\partial N_o}{\partial R} < 0$ , as can be seen from (4) and (5). Hence the second term is negative and the nominator is negative. Combining this with a negative denominator gives the sign in (15).

Intuitively, a higher retirement age increases the number of working-age individuals, output, and consumption, which results in increased spending on health care due to diminishing marginal utility from consumption. This result is consistent with Di Matteo (2005) and Baltagi and Moscone (2010), who found a positive relationship between health care expenditures and the share of working-age individuals in the total population.<sup>4</sup>

The effect of increased efficiency in health care provision  $\gamma$  is given by:

$$\frac{\partial \tau(t)}{\partial \gamma} = \frac{\left[ \begin{array}{l} (N_w + N_o) \frac{\partial U}{\partial c(t)} N_o \left( \frac{1}{A} + \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) \tau \\ + \frac{\partial^2 U}{\partial c(t)^2} (N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma) (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \tau \end{array} \right]}{\Psi} \gtrless 0 \quad (16)$$

The second term in the nominator is negative, while the sign of the first term is uncertain. Hence the sign of the derivative is uncertain. According to Jones (2003), this derivative should be positive and should explain a substantial part of the increased expenditures on health care in developed countries.

Intuitively, increased efficiency in health care provision results in a reduction in the health care expenditures per capita needed to maintain unchanged longevity and health, which implies a negative sign for the derivative in (16), while the marginal benefit from spending on health care increases, which implies a positive sign for the derivative. It is not clear which effect is stronger.

## 5 Conclusions

We have derived a golden rule for the amount that a nation should sacrifice in terms of the consumption of individuals in order to extend the life of generations. The results show that more productive societies should spend more, per capita, on health care because the utility

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<sup>4</sup> An increase in the rate of population growth  $n$  will also increase the ratio of the number of working-age individuals to retirees as long as the age of retirement  $R$  is closer to  $A$  than to the date of birth – that is, in the second half of an individual's life (see Appendix A2). This can be shown to increase the optimal level of health care provision  $\tau(t)$  given plausible assumptions.

loss of individuals is smaller due to a lower marginal utility of consumption. Furthermore, societies with a higher retirement age should spend more on health care because of their higher output and consumption levels. However, increased efficiency in health care provision has an ambiguous effect. Empirical studies have confirmed a positive relationship between health care expenditures per capita and GDP per capita, on the one hand, and the share of working-age individuals in total population, on the other, both of which are consistent with our golden rule.

## Appendix

### A1. Second-order conditions

Given consumption smoothing in (11), the first-order derivatives of the Lagrangian can be written as (using (1), (2), (4) and (5)):

$$\begin{aligned}\frac{\partial \Gamma(t)}{\partial c(t)} &= \left( \frac{\partial U}{\partial c(t)} - \lambda \right) (N_w + N_o) \\ \frac{\partial \Gamma(t)}{\partial \tau(t)} &= U(c(t)) \frac{\partial N_o}{\partial A} \gamma - \lambda (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma - \lambda N_o \\ \frac{\partial \Gamma(t)}{\partial \lambda} &= y(t) N_w - c(t) (N_w + N_o) - \tau(t) N_o\end{aligned}$$

And the second-order derivatives are (after evaluating those at maximum and rewriting):

$$\begin{aligned}\frac{\partial^2 \Gamma(t)}{\partial c(t)^2} &= \frac{\partial^2 U}{\partial c(t)^2} (N_w + N_o) \\ \frac{\partial^2 \Gamma(t)}{\partial \tau(t)^2} &= \frac{\partial U}{\partial c(t)} N_o \left( \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - 2 \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) \gamma \\ \frac{\partial^2 \Gamma(t)}{\partial \lambda^2} &= 0 \\ \frac{\partial^2 \Gamma(t)}{\partial c(t) \partial \tau(t)} &= 0 \\ \frac{\partial^2 \Gamma(t)}{\partial c(t) \partial \lambda} &= (N_w + N_o) \\ \frac{\partial^2 \Gamma(t)}{\partial \tau(t) \partial \lambda} &= \left( N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma \right)\end{aligned}$$

Hence, for the bordered Hessian to be positive definite and the first-order conditions being necessary and sufficient for a maximum, the following must hold:

$$\begin{aligned}- (N_w + N_o) \left[ (N_w + N_o) \frac{\partial U}{\partial c(t)} N_o \left( \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - 2 \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) \gamma \right. \\ \left. + \frac{\partial^2 U}{\partial c(t)^2} \left( N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma \right)^2 \right] > 0\end{aligned}$$

or:

$$\begin{aligned}(N_w + N_o) \frac{\partial U}{\partial c(t)} N_o \left( \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - 2 \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) \gamma \\ + \frac{\partial^2 U}{\partial c(t)^2} \left( N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma \right)^2 < 0\end{aligned}$$

or:

$$\frac{\partial^2 N_o}{\partial A^2} < \frac{2}{N_o} \left( \frac{\partial N_o}{\partial A} \right)^2 + \left( -\frac{\partial^2 U}{\partial c(t)^2} \right) \frac{\left( N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma \right)^2 \frac{\partial N_o}{\partial A}}{(N_w + N_o) \frac{\partial U}{\partial c(t)} N_o \gamma}$$

The right-hand side is positive because  $\frac{\partial N_o}{\partial A} > 0$ ,  $\frac{\partial U}{\partial c(t)} > 0$  and  $\frac{\partial^2 U}{\partial c(t)^2} < 0$ . This implies that, for the first-order conditions being necessary and sufficient for a maximum, there must be an upper bound on  $\frac{\partial^2 N_o}{\partial A^2}$ , or an upper bound on how longevity affects the effects of longevity on the number of old individuals  $\frac{\partial \frac{\partial N_o}{\partial A}}{\partial A}$ . Using (1), (2), and (5) gives:

$$\frac{\partial^2 N_o}{\partial A^2} = \varphi e^{n(t-A)} \frac{\partial f}{\partial A_{a=A}} + \int_{a=R}^A \varphi e^{n(t-a)} \frac{\partial^2 f}{\partial A^2} da$$

$\frac{\partial^2 f}{\partial A^2} < 0$  must hold in order to ensure that  $0 \leq f(a, A) < 1$ , as is assumed, and the last term on the right-hand side is therefore negative. The first term on the right-hand side is positive, however, because  $\frac{\partial f}{\partial A} > 0$  for all  $a \in (R, A]$  and all  $A$ . Plugging this into the inequality above gives:

$$\begin{aligned} \frac{\partial f}{\partial A_{a=A}} &< \frac{1}{\varphi e^{n(t-A)}} \frac{2}{N_o} \left( \frac{\partial N_o}{\partial A} \right)^2 \\ &+ \frac{1}{\varphi e^{n(t-A)}} \left( -\frac{\partial^2 U}{\partial c(t)^2} \right) \frac{\left( N_o + (c(t) + \tau(t)) \frac{\partial N_o}{\partial A} \gamma \right)^2 \frac{\partial N_o}{\partial A}}{(N_w + N_o) \frac{\partial U}{\partial c(t)} N_o \gamma} \\ &+ \int_{a=R}^A e^{n(A-a)} \left( -\frac{\partial^2 f}{\partial A^2} \right) da \end{aligned}$$

where all of the terms on the right-hand side are positive. Hence, for the first-order conditions being necessary and sufficient for a maximum, there must be an upper bound to the effects of increased longevity on survival probabilities evaluated at the maximum age  $A$ . We only assume that this is positive  $\frac{\partial f}{\partial A} > 0$  and decreasing in longevity  $\frac{\partial^2 f}{\partial A^2} < 0$ . Hence we have made no assumption about the size of  $\frac{\partial f}{\partial A}$ . Note, however, that assumptions can easily be made such that this inequality holds and the necessary conditions being necessary and sufficient for a maximum.

## A2. Effects from an increase in $n$ on the ratio between working-age population and those retired

Using the results above gives:

$$\frac{\partial N_w}{\partial n} - \frac{\partial N_o}{\partial n} = \int_{a=0}^R (t-a) l(t, n, a, A) da - \int_{a=R}^A (t-a) l(t, n, a, A) da$$



Since  $\frac{\partial l}{\partial a} < 0$  and  $t > a$ , it holds that:

$$\frac{\partial[(t-a)l(t,n,a,A)]}{\partial a} = -l(t,n,a,A) + (t-a)\frac{\partial l}{\partial a} < 0$$

This implies that:

$$\int_{a=0}^R (t-a)l(t,n,a,A)da > (t-R)l(t,n,R,A)R$$

$$\int_{a=R}^A (t-a)l(t,n,a,A)da < (t-R)l(t,n,R,A)(A-R)$$

Hence a sufficient condition for  $\frac{\partial N_w}{\partial n} - \frac{\partial N_o}{\partial n} > 0$  to hold is that:

$$(t-R)l(t,n,R,A)R > (t-R)l(t,n,R,A)(A-R)$$

or:

$$\frac{R}{A} > 1 - \frac{R}{A}$$

*Q.E.D.*

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