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Abstract

This paper proposes a model which allows for discrete stochastic breaks in the time-varying transition probabilities of Markov-switching models with autoregressive dynamics. An extensive simulation study is undertaken to examine the properties of the maximum-likelihood estimator and related statistics, and to investigate the implications of misspecification due to unaccounted changes in the parameters of the Markov transition mechanism. An empirical application that examines the relationship between Argentinian sovereign bond spreads and output growth is also discussed.

Keywords: Markov-switching models; Maximum likelihood; Monte Carlo experiments; Time-varying transition probabilities.

JEL Classification: C32.

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1 Introduction

Since the publication of Diebold, Lee, and Weinbach (1994) and Filardo (1994), time-series models with parameters which are subject to changes governed by a finite-state Markov chain with time-varying transition probabilities have attracted considerable attention in the literature. Applications involving such models can be found in many areas of economics and finance. Examples include, among many others, the study of business-cycle fluctuations (Filardo (1994); Filardo and Gordon (1998); Ravn and Sola (1999)), exchange rates (Diebold, Lee, and Weinbach (1994); Engel and Hakkio (1996)), interest-rate dynamics (Bekaert and Harvey (1995); Gray (1996)), asset allocation (Bekaert and Harvey (1995); Ang and Bekaert (2002); Guidolin and Timmermann (2006); Guidolin and Timmermann (2008)), asset returns (Hall, Psaradakis, and Sola (1997); Schaller and van Norden (1997)), and financial/exchange-rate crises (Jeanne and Masson (2000); Peria (2002); Alvarez-Plata and Schrooten (2006); Brunetti, Scotti, Mariano, and Tan (2008)).

A question which is often addressed in empirical studies using regime-switching models with time-inhomogeneous Markov transitions is which variables help to predict the transitions between different regimes (a period of relative calm and a financial crisis, say). In such applications the sample typically includes data from all regimes and one of the objects of the exercise is to separate the regimes on the basis of sample information. An implicit assumption usually made is that the association between the information variables that enter in the transition probabilities and the dependent variable of the model is not altered by the regime changes, so that the parameters associated with the transition probabilities are time invariant.

However, this may not necessarily be a plausible assumption in some empirical applications. Ravn and Sola (1999), for instance, observed that a change in the definition of M2 money stock in the U.S., and therefore in the correlation between M2 and output growth, had a dramatic impact on regime separation. When they considered a short sample which ended before the change in the definition of M2, they found the separation of regimes into booms and recessions to be consistent with the National Bureau of Economic Research (NBER) dating of business-cycle peaks and troughs, and M2 growth to have a statistically significant effect on the transition probabilities. When the sample was extended, M2 no longer had a significant effect on the transition probabilities and the separation of regimes was unrelated to the NBER dating. In view of the fact that in many applications the reason for including exogenous variables in the transition mechanism is to use them as leading indicators of a given event (e.g., a financial crisis), the problem just described is likely to be the rule rather than the exception. As a result, it is likely that potential leading indicators may be found in applications to have no significant effect on the transition probabilities, or may be found to be significant because of the strength of their covariation with the dependent variable after the event (which could not justify their use as leading indicators of the event). In addition, it is likely that the in-sample separation of the regimes will be adversely affected and lead to misleading conclusions.

The contribution of this paper is to propose a regime-switching model which explicitly allows for random breaks in the time-inhomogeneous transition probability matrices of the hidden Markov regime sequence. We conjecture that a change in the (nonlinear) relationship between the variable being modelled and the information variables that determine the evolution of the transition matrices, is likely to result in changes in the covariance between these variables as well as in the relationship between the information variables and the transition probability matrices. Therefore, we propose to consider a multivariate model (e.g., a vector autoregression with Markov regimes) in which both the noise covariance matrix and the transition functions are subject to random breaks driven by an exogeneous finite-state Markov process. In many applications in which an information variable (e.g., money supply) is used as a leading indicator (e.g., of an exchange-rate crisis), although the covariation between the information variable and the main variable of interest (e.g., exchange rate) is likely to have changed over the sample period, the potential resulting changes in the parameters associated with the transition mechanism and the noise covariance matrix are rarely taken into account. The principal idea behind our modelling strategy is to use such breaks to identify changes in the (indirect) relationship between the time series of interest and the transition information variable(s). Our simulation analysis demonstrates that ignoring breaks of this type has significant adverse effects on likelihood-based statistical inference.

The remainder of the paper is organized as follows. Section 1 recalls the structure of a typical Markov-switching autoregressive model and motivates our modelling approach. Section 2 introduces and discusses the proposed multivariate model. Section 3 contains a simulation study that assesses the properties of estimators and test statistics in the presence of parameter changes in the Markov transition functions. Section 4 presents an illustrative empirical application that analyzes the relationship between Argentinian sovereign bond spreads and output growth. Section 5 summarizes and concludes.

2 Markov-Switching Models and Motivation

A prototypical Markov-switching autoregressive model for a univariate time series $\{Y_t\}$ is given by

$$Y_t = \mu(S_t) + \boldsymbol{\phi}'\mathbf{y}_{t-1} + \sigma(S_t)\varepsilon_t, \qquad t = 1, 2, \ldots, \tag{1}$$

where $\mathbf{y}_{t-1} := (Y_{t-1}, \ldots, Y_{t-k})'$ for some positive integer k, $\boldsymbol{\phi} := (\phi_1, \ldots, \phi_k)'$ is an unknown parameter, $\{\varepsilon_t\}$ are independent, identically distributed (i.i.d.) standard normal random variables, and $\{S_t\}$ are random variables which take values in the set $\mathbb{S} := \{0, 1\}$ and indicate the unobservable state, or regime, prevailing at each point t. Here and in the sequel, we write

$$b(V) := b_0 + (b_1 - b_0)\mathbb{I}_{\{V=1\}},$$

for any real-valued quantity $b(V)$ whose values depends on the realization of an $\mathbb{S}$-valued random variable V, $\mathbb{I}_{\{\cdot\}}$ being an indicator variable (with $\mathbb{I}_{\{A\}} = 1$ if condition A is satisfied and $\mathbb{I}_{\{A\}} = 0$ otherwise). The regime-determining variables $\{S_t\}$ in (1) are assumed to be independent of $\{\varepsilon_t\}$ and to form a time-inhomogeneous Markov chain with transition probabilities

$$\mathbb{P}(S_t = 0 | S_{t-1} = 0, Z_{t-1}) =: q_t = \Lambda\left(\alpha_q + \beta_q Z_{t-1}\right), \tag{2}$$

$$\mathbb{P}(S_t = 1 | S_{t-1} = 1, Z_{t-1}) =: p_t = \Lambda(\alpha_p + \beta_p Z_{t-1}). \tag{3}$$

where $\Lambda(u) := 1/(1 + e^{-u})$ is the standard logistic function and Z_{t-1} is an exogenous variable on which the transition probabilities depend (see, e.g., Diebold, Lee, and Weinbach

(1994)). Needless to say, such a model may be straightforwardly generalized to allow for state-dependent autoregressive coefficients $\boldsymbol{\phi}$, more than two regimes, and more than one information variable in (2)–(3); furthermore, $\Lambda(\cdot)$ may be replaced by some other continuous, monotonic function whose range lies in the interval $[0, 1]$.

Under the formulation in (1)–(3), the observable variable Z_{t-1} influences the probability with which $\{Y_t\}$ switches between the two states. This allows for a nonlinear relationship between Y_t and Z_t, since Z_{t-1} affects Y_t indirectly through the transition probabilities p_t and q_t. Furthermore, the effect of Z_{t-1} on the transition probabilities need not be symmetric, in the sense that β_q and β_p are not required to be equal or have the same sign. A Markov-switching autoregressive model with a time-invariant transition mechanism is, of course, a special case of (1)–(3) with $\beta_q = \beta_p = 0$.

An important issue which has not received much attention in the literature concerns the effects on inference of potential changes in the parameters associated with the time-varying transition probabilities. For example, a number of empirical studies have documented that several monetary relationships display instability because of changes in monetary policy and in innovations in the financial sector (see, e.g., Ravn and Sola (1999)). Such changes may lead to instability in the parameters associated with the transition probabilities and may have deleterious effects on inference if they are not accounted for.

A simple set-up, investigated in Psaradakis, Sola, Spagnolo, and Spagnolo (2013), allows the Markov chain $\{S_t\}$ in (1) to be governed by the transition probabilities

$$q_t = \Lambda\left(\alpha_q + \beta_{q,t} Z_{t-1}\right), \qquad p_t = \Lambda\left(\alpha_p + \beta_{p,t} Z_{t-1}\right), \tag{4}$$

where, for some fixed integer $t^* > 1$,

$$\beta_{i,t} = \beta_i + (\beta_i^* - \beta_i)\mathbb{I}_{\{t-t^*>0\}}, \qquad i = p, q, \qquad t \geqslant 1. \tag{5}$$

Under the formulation (4)–(5), the relationship between the transition probabilities and the variable Z_{t-1} undergoes an one-off change at $t = t^*$. In such a case, the transition mechanism (2)–(3) is misspecified when $\beta_q^* - \beta_q \neq 0$ and/or $\beta_p^* - \beta_p \neq 0$, and inference on the parameters of the model and the hidden regimes is affected adversely if (2)–(3) is maintained erroneously.

A more general formulation, which can accommodate stochastic changes in the transition probabilities at unspecified dates, may be obtained by allowing the parameters $(\alpha_q, \alpha_p, \beta_q, \beta_p)$ in (2)–(3) to vary as a time-homogeneous, finite-state Markov chain. Under such a formulation, the relationship between the transition probabilities of $\{S_t\}$ and the variable Z_{t-1} undergoes discrete random changes. A multivariate model which allows for such behavior is discussed next.

3 Modelling Breaks in the Markov Transition Mechanism

For simplicity and clarity of exposition, we will present and discuss in the sequel a bivariate model. The model aims to capture changes in the relationship between Y_t and Z_t by conjecturing that such changes would manifest as changes in the covariance structure between Y_t and Z_t and in the transition mechanism of $\{S_t\}$. This covariance structure, as well as the time-varying transition probabilities of $\{S_t\}$, are then modelled as being subject to random breaks governed by an exogenous finite-state Markov process.

3.1 Model

Let $\{\mathbf{v}_t\}$ be an unobservable sequence of i.i.d. random vectors having a bivariate standard normal distribution and $\{\boldsymbol{\xi}_t := (S_t, X_t)'\}$ be an unobservable sequence of random vectors taking values in $\mathbb{S} \times \mathbb{S}$. We consider the following model for the observable bivariate time series $\{\mathbf{w}_t := (Y_t, Z_t)'\}$:

$$Y_t = \mu(S_t) + \boldsymbol{\phi}'\mathbf{y}_{t-1} + \sigma(S_t)\varepsilon_{yt}, \qquad t = 1, 2, \ldots, \tag{6}$$

$$Z_t = \mu_z + \boldsymbol{\psi}'\mathbf{z}_{t-1} + \sigma_z\varepsilon_{zt}, \qquad t = 1, 2, \ldots, \tag{7}$$

where $\mathbf{y}_{t-1} := (Y_{t-1}, \ldots, Y_{t-k})'$ and $\mathbf{z}_{t-1} := (Z_{t-1}, \ldots, Z_{t-m})'$ for some positive integers k and m, $\boldsymbol{\phi} := (\phi_1, \ldots, \phi_k)'$ and $\boldsymbol{\psi} := (\psi_1, \ldots, \psi_k)'$ are unknown parameters, and the noise $\boldsymbol{\varepsilon}_t := (\varepsilon_{yt}, \varepsilon_{zt})'$ satisfies

$$\boldsymbol{\varepsilon}_t = \mathbf{R}_t^{1/2}\mathbf{v}_t, \tag{8}$$

with

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho(X_t) \\ \rho(X_t) & 1 \end{bmatrix}, \tag{9}$$

and $|\rho_i| < 1$, $i \in \mathbb{S}$. The model is completed by postulating that, conditionally on $\{X_t\}$, $\{S_t\}$ is a time-inhomogeneous Markov chain whose transition probabilities depend on (Z_{t-1}, X_t) and have the following functional form:

$$\mathbb{P}\left(S_t = 0 | S_{t-1} = 0, Z_{t-1}, X_t\right) =: q_t(X_t) = \Lambda\left(\alpha_q(X_t) + \beta_q(X_t) Z_{t-1}\right), \tag{10}$$

$$\mathbb{P}\left(S_t = 1 | S_{t-1} = 1, Z_{t-1}, X_t\right) =: p_t(X_t) = \Lambda\left(\alpha_p(X_t) + \beta_p(X_t) Z_{t-1}\right). \tag{11}$$

In addition, $\{X_t\}$ is a time-homogeneous Markov chain with transition probabilities

$$\mathbb{P}(X_t = 0 | X_{t-1} = 0) = q_x, \qquad \mathbb{P}(X_t = 1 | X_{t-1} = 1) = p_x. \tag{12}$$

Finally, $\{\boldsymbol{\xi}_t\}$, $\{\mathbf{v}_t\}$ and $\overline{\mathbf{w}}_0 := (\mathbf{y}_0', \mathbf{z}_0')'$ are independent of each other, and X_t is independent of $\{S_r : r < t\}$ for all t.

Under the formulation (6)–(12), the covariance structure of $\mathbf{w}_t$, as captured by the covariance matrix $\mathbf{R}_t$, is subject to Markov changes governed by $\{X_t\}$. At the same time, the transition mechanism that governs the Markov process $\{S_t\}$ driving the changes in the conditional mean and variance of $\{Y_t\}$ is also dependent upon the state of nature implied by $\{X_t\}$. The model may, of course, be generalized to allow for more than two states and/or variables, as well as state-dependent parameters $(\boldsymbol{\phi}, \boldsymbol{\psi}, \mu_z, \sigma_z)$.

3.2 Estimation and Inference

Given data $\overline{\mathbf{w}}_0, \mathbf{w}_1, \ldots, \mathbf{w}_T$, statistical inference in the model defined by (6)–(12) can be carried out by using a recursive algorithm analogous to that discussed in Hamilton (1994, pp. 692–694). This entails iterating on the equations

$$\boldsymbol{\delta}_{t|t} = [\mathbf{1}'(\boldsymbol{\delta}_{t|t-1} \odot \boldsymbol{\eta}_t)]^{-1}(\boldsymbol{\delta}_{t|t-1} \odot \boldsymbol{\eta}_t), \qquad t = 1, 2, \ldots, T, \tag{13}$$

and

$$\boldsymbol{\delta}_{t+1|t} = \mathbf{P}_t \boldsymbol{\delta}_{t|t}, \qquad t = 1, 2, \ldots, T, \tag{14}$$

where

$$\boldsymbol{\delta}_{t|\tau} := \begin{bmatrix} \mathbb{P}[\boldsymbol{\xi}_t' = (0,0)|\mathcal{F}_\tau;\boldsymbol{\theta}] \\ \mathbb{P}[\boldsymbol{\xi}_t' = (0,1)|\mathcal{F}_\tau;\boldsymbol{\theta}] \\ \mathbb{P}[\boldsymbol{\xi}_t' = (1,0)|\mathcal{F}_\tau;\boldsymbol{\theta}] \\ \mathbb{P}[\boldsymbol{\xi}_t' = (1,1)|\mathcal{F}_\tau;\boldsymbol{\theta}] \end{bmatrix}, \qquad \boldsymbol{\eta}_t := \begin{bmatrix} f_{00}(\mathbf{w}_t|\boldsymbol{\xi}_t' = (0,0),\mathcal{F}_{t-1};\boldsymbol{\theta}) \\ f_{01}(\mathbf{w}_t|\boldsymbol{\xi}_t' = (0,1),\mathcal{F}_{t-1};\boldsymbol{\theta}) \\ f_{10}(\mathbf{w}_t|\boldsymbol{\xi}_t' = (1,0),\mathcal{F}_{t-1};\boldsymbol{\theta}) \\ f_{11}(\mathbf{w}_t|\boldsymbol{\xi}_t' = (1,1),\mathcal{F}_{t-1};\boldsymbol{\theta}) \end{bmatrix},$$

and

$$\mathbf{P}_t := \begin{bmatrix} q_{t0}q_x & q_{t0}(1-p_x) & (1-p_{t0})q_x & (1-p_{t0})(1-p_x) \\ q_{t1}(1-q_x) & q_{t1}p_x & (1-p_{t1})(1-q_x) & (1-p_{t1})p_x \\ (1-q_{t0})q_x & (1-q_{t0})(1-p_x) & p_{t0}q_x & p_{t0}(1-p_x) \\ (1-q_{t1})(1-q_x) & (1-q_{t1})p_x & p_{t1}(1-q_x) & p_{t1}p_x \end{bmatrix}.$$

Here, $\boldsymbol{\theta}$ denotes the (column) vector of all parameters of the model, $\mathcal{F}_t := \{\mathbf{w}_r : r \leqslant t\}$ is the information set available at time t, $f_{ij}(\mathbf{w}_t|\boldsymbol{\xi}_t' = (i,j),\mathcal{F}_{t-1};\boldsymbol{\theta})$ is the conditional density of $\mathbf{w}_t$ given $\boldsymbol{\xi}_t' = (i,j)$, $i,j \in \mathbb{S}$, and $\mathcal{F}_{t-1}$, $\mathbf{1}$ is a four-dimensional all-ones column vector, and $\odot$ denotes element-wise multiplication.

Note that, under the Gaussianity assumption for $\mathbf{v}_t$, $f_{ij}(\mathbf{w}_t|\boldsymbol{\xi}_t' = (i,j),\mathcal{F}_{t-1};\boldsymbol{\theta})$, $i,j \in \mathbb{S}$, is the bivariate normal density with mean vector

$$\left(\mu_0 + (\mu_1 - \mu_0)i + \boldsymbol{\phi}'\mathbf{y}_{t-1},\ \mu_z + \boldsymbol{\psi}'\mathbf{z}_{t-1}\right)'$$

and covariance matrix

$$\begin{bmatrix} \sigma_0^2 + (\sigma_1^2 - \sigma_0^2)i & \sigma_z\{\sigma_0 + (\sigma_1 - \sigma_0)i\}\{\rho_0 + (\rho_1 - \rho_0)j\} \\ \sigma_z\{\sigma_0 + (\sigma_1 - \sigma_0)i\}\{\rho_0 + (\rho_1 - \rho_0)j\} & \sigma_z^2 \end{bmatrix}.$$

Also note that one may think of the model (6)–(12) as allowing for four Markov states, which may be indexed by the state-indicator variable

$$\zeta_t := \begin{cases} 1, & \text{if } \boldsymbol{\xi}_t' = (0,0), \\ 2, & \text{if } \boldsymbol{\xi}_t' = (0,1), \\ 3, & \text{if } \boldsymbol{\xi}_t' = (1,0), \\ 4, & \text{if } \boldsymbol{\xi}_t' = (1,1). \end{cases}$$

Then, $\{\mathbf{P}_t\}$ may be regarded as the time-inhomogeneous transition probability matrices of the Markov chain $\{\zeta_t\}$, the (i,j)-th element of $\mathbf{P}_t$ being the transition probability $\mathbb{P}\left(\zeta_t = i|\zeta_{t-1} = j\right)$, for $i,j \in \{1,2,3,4\}$.

The algorithm based on the iteration of (13)–(14) yields as a by-product the conditional log-likelihood of $(\mathbf{w}_1, \ldots, \mathbf{w}_T)$, given $(\overline{\mathbf{w}}_0, \boldsymbol{\xi}_0)$:

$$\ell(\boldsymbol{\theta}) := \sum_{t=1}^{T} \ln[\mathbf{1}'(\boldsymbol{\delta}_{t|t-1} \odot \boldsymbol{\eta}_t)]. \tag{15}$$

The maximum-likelihood (ML) estimator $\widehat{\boldsymbol{\theta}}$ of the parameter $\boldsymbol{\theta}$ can then be obtained as the maximizer of (15) with respect to $\boldsymbol{\theta}$. Furthermore, inference about the hidden regimes $\{\boldsymbol{\xi}_t\}$ may be made on the basis of the smoothed state probabilities $\boldsymbol{\delta}_{t|T}$ or the filtered state probabilities $\boldsymbol{\delta}_{t|t}$ (evaluated at $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$).

Asymptotic results relating to consistency and local asymptotic normality of the ML estimator of the parameters of Markov-switching autoregressive models with time-varying transition probabilities were recently established in Pouzo, Psaradakis, and Sola (2016). Under the assumption of correct model specification and suitable stationarity, ergodicity, identification and moment conditions, the usual asymptotic theory for the ML estimator holds in a model like (6)–(12). In particular, $\widehat{\boldsymbol{\theta}}$ is consistent and asymptotically efficient for the true parameter $\boldsymbol{\theta}_0$ (say), and $[-\ddot{\ell}(\widehat{\boldsymbol{\theta}})]^{1/2}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0)$ has a Gaussian limiting distribution with zero mean vector and identity covariance matrix, as T tends to infinity, where $\ddot{\ell}(\widehat{\boldsymbol{\theta}})$ is the Hessian matrix of $\ell(\boldsymbol{\theta})$ evaluated at $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$. In the presence of specification errors, and under certain regularity conditions, $\widehat{\boldsymbol{\theta}}$ is consistent for the parameter that provides the best approximating model, in the sense of minimizing the Kullback–Leibler divergence from the true data-generating process (DGP).

4 Monte Carlo Simulations

Simulation experiments are carried out to assess the properties of the ML estimator and of related test statistics both in correctly specified models and in misspecified models which ignore changes in the parameters associated with the Markov transition functions. We begin by describing the experimental design and simulations, and proceed to report and discuss the results of the experiments.

4.1 Experimental Design and Simulation

The DGP used in the experiments is the bivariate model defined by (6)–(12), with first-order dynamics ($k = m = 1$) and the following parameter values:

$$\mu_0 = 3, \qquad \mu_1 = 0.5, \qquad \phi = 0.9, \qquad \sigma_0 = 0.5, \qquad \sigma_1 = 1,$$

$$\mu_z = 0.1, \qquad \psi = 0.8, \qquad \sigma_z = 1, \qquad \rho_0 = 0.8, \qquad \rho_1 = -0.8,$$

$$p_x = q_x = 0.95, \qquad \alpha_{p0} = \alpha_{p1} = 1, \qquad \alpha_{q0} = \alpha_{q1} = 2,$$

$$\beta_{q0} = -1.5, \qquad \beta_{q1} = 3, \qquad \beta_{p0} = 2.5, \qquad \beta_{p1} = -5.$$

Figure 1 shows the regime-specific marginal densities of $\mathbf{w}_t$, that is, bivariate normal densities having mean vector

$$\left(\frac{\mu_0 + (\mu_1 - \mu_0)i}{1-\phi}, \; \frac{\mu_z}{1-\psi} \right)'$$

and covariance matrix

$$\begin{bmatrix} \dfrac{\sigma_0^2 + (\sigma_1^2 - \sigma_0^2)i}{1-\phi^2} & \{\rho_0 + (\rho_1 - \rho_0)j\}\sqrt{\dfrac{\{\sigma_0^2 + (\sigma_1^2 - \sigma_0^2)i\}\sigma_z^2}{(1-\phi^2)(1-\psi^2)}} \\ \{\rho_0 + (\rho_1 - \rho_0)j\}\sqrt{\dfrac{\{\sigma_0^2 + (\sigma_1^2 - \sigma_0^2)i\}\sigma_z^2}{(1-\phi^2)(1-\psi^2)}} & \dfrac{\sigma_z^2}{1-\psi^2} \end{bmatrix},$$

with $i, j \in \mathbb{S}$.

In each of 1000 Monte Carlo replications, we generate $100 + T$ data points for $\mathbf{w}_t$, with $T \in \{100, 200, 400, 800, 1600, 3200, 6400\}$ and $\mathbf{w}_0' = (0.5, 0.40118)$, but only the last T data points of each realization are used in order to attenuate start-up effects. For each realization, we compute ML estimates of the parameters of three bivariate models for (Y_t, Z_t): (i) a model defined by (6)–(9), with $\rho(X_t) = \rho$ for all t, coupled with the state-independent Markov-switching mechanism associated with (2)–(3) (model M-1); (ii) the model defined by (6)–(12) with both $\{S_t\}$ and $\{X_t\}$ treated as unobservable (model M-2); (iii) a model defined by (6)–(12), but with the additional assumption that the realization of the Markov process $\{X_t\}$ driving the changes in the transition probabilities $q_t(X_t)$ and $p_t(X_t)$ of $\{S_t\}$ and the covariance matrix $\mathbf{R}_t$ is observable (model M-3). We set $k = m = 1$ in all three cases.

Model M-1 is evidently misspecified since it ignores the breaks in the covariance matrix of $\boldsymbol{\varepsilon}_t$ and in the transition probabilities of $\{S_t\}$. Model M-3 accounts for the Markov breaks in $q_t(X_t)$, $p_t(X_t)$ and $\rho(X_t)$ correctly, but an observable regime sequence $\{X_t\}$ is not typically available in situations involving real-world data. However, a comparison of parameter estimates obtained from model M-3 and the empirically relevant model M-2 will reveal whether the ML estimator suffers as a result of the additional randomness introduced by the Markov process $\{X_t\}$.

For all three models, the maximizer of the relevant ML objective function is found by means of a quasi-Newton algorithm that approximates the Hessian according to the Broyden–Fletcher–Goldfarb–Shanno (BFGS) update with numerically computed derivatives. A grid of seven initial values for each parameter is used to initialize the BFGS iterations; those initial values which result in the highest value of the objective function are then selected.[1]

Figure 2 shows plots of: a typical realization of $\{Y_t\}$ and $\{Z_t\}$; the time-varying transition probabilities $p_t(X_t)$ and $q_t(X_t)$ and their estimated values (the latter are computed as $\bar{p}_t = \mathbb{P}(X_t = 0|\mathcal{F}_t)p_{0t} + \mathbb{P}(X_t = 1|\mathcal{F}_t)p_{1t}$ and $\bar{q}_t = \mathbb{P}(X_t = 0|\mathcal{F}_t)q_{0t} + \mathbb{P}(X_t = 1|\mathcal{F}_t)q_{1t}$); the associated realizations of the Markov chains $\{S_t\}$ and $\{X_t\}$; the filtered state probabilities $\mathbb{P}(S_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$ and $\mathbb{P}(X_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$; all quantities depending on the unknown parameter $\boldsymbol{\theta}$ are computed using the ML estimator $\widehat{\boldsymbol{\theta}}$ from model M-2. For this typical realization, model M-2 is remarkably successful at capturing the changes in regime and the movements in the transition probabilities of $\{S_t\}$. It is interesting to compare results for the same realization of $\{Y_t\}$ and $\{Z_t\}$ when model M-1 is used instead. The relevant plots, shown in Figure 3, reveal that, although model M-1 is equally successful in identifying the changes in regime associated with $\{S_t\}$, it fails dramatically in describing the dynamics of $p_t(X_t)$ and $q_t(X_t)$. This suggests that it is unlikely that Z_{t-1} would be found to be useful in predicting the shifts implied by the regime sequence $\{S_t\}$. As we shall see below, this is not a peculiar feature of the particular realization used in Figure 3 but is true in general.

[1] We note that estimation results were found to be fairly robust with respect to the choice of initial values.

4.2 Simulation Results

Table 1a records some of the characteristics of the finite-sample distribution of the ML estimator of the parameters of the equation for Y_t in model M-1. Specifically, we report the estimated bias and conventional measures of skewness and kurtosis based on the standardized third and fourth empirical central moments. We also report the ratio of the sampling standard deviation of the ML estimators to the estimated standard errors (computed from the empirical Hessian) averaged across Monte Carlo replications for each design point. Note that in Table 1a (as well as in Tables 1b and 1c) the figures reported under $\alpha_{p(i)}$, $\beta_{p(i)}$, $\alpha_{q(i)}$ and $\beta_{q(i)}$, $i \in \mathbb{S}$, refer to deviations of the ML estimates of the parameters $(\alpha_p, \beta_p, \alpha_q, \beta_q)$ in (2)–(3) from the true regime-specific values of the corresponding parameters. The most noteworthy finding is the significant bias in the estimation of parameters associated with the transition probabilities, especially β_q and β_p. Such biases are clearly not a small-sample issue and are present even in the largest of the samples under consideration ($T = 6400$). For small sample sizes ($T \leqslant 200$), the distributions of the estimators of many parameters tend to be asymmetric and leptokurtic, but the situation improves with increasing sample sizes. Finally, the estimated standard errors are downwards biased in most cases; however, the bias is not generally substantial and decreases as the sample size increases.

Table 1b contains information on the sampling distributions of conventional t-type statistics, computed as the ratio of the estimation error to the corresponding asymptotic standard error, and are typically treated as having an approximate standard normal distribution (which is true under the assumption of a correctly specified likelihood function). It is immediately obvious that the mean of these distributions can differ substantially from zero, something which is especially true for t-statistics associated with the parameters of the transition functions. The studentized statistics generally tend to have skewed distributions and, in the case of β_q and β_p, highly leptokurtic (especially for the larger sample sizes).

To examine the implications of these results for hypothesis testing, we report in Table 1c the empirical rejection frequencies (based on standard normal critical regions of nominal size 0.05 and 0.10) of: (i) a t-type test of $\mathcal{H}_0 : \vartheta_j = \vartheta_j^*$ versus $\mathcal{H}_1 : \vartheta_j \neq \vartheta_j^*$, where ϑ_j is the j-th element of the parameter vector $\boldsymbol{\vartheta}$ of the model under consideration and ϑ_j^* is its true value; (ii) a t-type test of $\mathcal{H}_0 : \vartheta_j = 0$ versus $\mathcal{H}_1 : \vartheta_j \neq 0$; we refer to the estimated rejection frequencies as 'size' and 'power', respectively.[2] It can be seen that the hypothesis that β_q or β_p is equal to either of the two regime-specific true values is rejected either very rarely or very frequently. Furthermore, tests for the statistical significance of these parameters have very low power. As a result, one may be wrongly led to conclude that a significant leading indicator has no effect on the Markov transition probabilities.

Table 2a records some of the characteristics of the finite-sample distribution of the ML estimator of the parameters of the equation for Y_t in the well-specified model M-2. In sharp contrast to the results obtained under model M-1, the ML estimator exhibits some bias only in small samples and, even for the parameters associated with the transition probabilities, bias is insignificant for $T > 400$. The distributions of the estimators of some parameters tend to be asymmetric and leptokurtic, but the situation improves with increasing sample sizes. Finally, the bias in the estimated standard errors is not generally substantial and decreases as the sample size increases.

Encouraging results are also contained in Table 2b, in which information on the empirical distributions of conventional t-type statistics is recorded. The studentized statistics tend to have distributions with mean and variance that do not differ substantially from their expected values in most cases. Rather surprisingly, the mean of the studentized statistics associated with β_{q1} and β_{p1} is significantly different from zero when $T \geqslant 1600$. This, however, does not appear to have an adverse effect on the size and power properties of t-type tests, the empirical rejection frequencies of which are reported in Table 2c. Unlike the case of model M-1, tests in model M-2 tend to have an empirical Type I error probability which is generally close to the nominal level of the test, especially for $T > 200$. In terms of power, tests involving the parameters associated with the time-varying transition probabilities tend to fare somewhat worse than tests involving other parameters, but rejection frequencies improve with an increasing sample size.

[2] We note that results should be interpreted with caution in the case of $\mathcal{H}_0 : \sigma_j = 0$, $j \in \mathbb{S}$, because the null value of σ_j is on the boundary of the maintained hypothesis.

The simulation results reported in Tables 3a–3c for model M-3 are generally quite similar to those obtained under model M-2. Perhaps the only noteworthy difference concerns the mean of the studentized statistics associated with β_{q1} and β_{p1}, which is closer to zero under model M-3. It is worth emphasizing that model M-3 is not empirically relevant since it assumes that the changes in the Markov transition probabilities and the noise covariance matrix are observable. The simulation results, however, do suggest that not much is lost by treating the aforementioned breaks as unobservable random events driven by an exogeneous Markov process.

To sum up, the results from the Monte Carlo experiments suggest that, in the presence of unaccounted changes in the parameters of the transition functions and the noise covariance matrix, ML produces severely biased estimates, especially for the parameters that appear in the transition probabilities. Such biases are present even for what are very large sample sizes by the standards of empirical applications. Hypothesis tests based on such ML estimates also have unsatisfactory properties. By contrast, the ML estimator in a correctly specified model that allows for hidden breaks in the transition functions and the noise covariance matrix has very good finite-sample properties and performs almost as well as a ML estimator which has full knowledge of the number and location of such breaks.

5 Empirical Application

To illustrate the practical use of the proposed model, we analyze the relationship between Argentinian sovereign bond spreads over U.S. Treasury rates and output growth. These variables are generally expected to be negatively correlated since the higher output growth is, the higher is the capacity of a country to repay debt, leading to lower bond spreads. However, if a structural break occurs as a result of a default on sovereign debt, then correlation will typically change because the economy may start to grow after the default, as resources are no longer used to service the debt, while the country's risk continues to increase, resulting in higher bond spreads. It is likely that such a change in correlation took place at the beginning of 2002 following Argentina's default on sovereign debt.

The data set consists of quarterly observations, from 1995:1 to 2010:1, on the J. P. Morgan Emerging Markets Bond Index (EMBI) of dollar-denominated sovereign bonds issued by Argentina (denoted by Y_t) and the real GDP growth rate (denoted by Z_t). The focus of the analysis is on whether output growth has predictive content for regime changes associated with the bond spread in the presence of a potential break in the relationship between the two variables associated with the sovereign default.

We consider two models. Model 1 is the standard model, à la Diebold, Lee, and Weinbach (1994), given by (1)–(3). Model 2 is given by (6)–(12), and allows for stochastic discrete breaks in the noise covariance matrix and in the transition functions. In both cases, we set $k = m = 4$.

Model 1 is used because of its popularity in applied work. It should be borne in mind, however, that, even in the absence of breaks in the transition mechanism and the noise covariance matrix, the use of such a specification is problematic if Y_t and Z_t are contemporaneously correlated, as the analysis in Pouzo, Psaradakis, and Sola (2016) demonstrates.

ML estimates of the parameters of the two models are reported in Table 4. The estimated coefficients for the Markov transition functions for Model 1, show that an increase (decrease) in output growth increases (decreases) the probability of remaining in a high-volatility regime (high output growth and high spread volatility) since both β_q and β_p are positive and would appear to be statistically significant at the 10% level. We note, however, that inference based on these ML estimates is potentially misleading because of the likely bias of the parameter estimator and the inconsistency of the empirical Hessian covariance estimator in the presence of changes in the parameters of the Markov transition functions and/or endogeneity of the transition information variable.

The results for Model 2 show that the estimated values of β_{q0} and β_{q1} have different signs, so one would expect the evolution of q_t implied by Model 1 and Model 2 to be quite different. A similar result is obtained for β_{p0} and β_{p1}, the main difference being that in this case the estimates are both positive and thus one would not expect the evolution of p_t implied by the two models to be substantially different. Finally, note that that the estimated ρ_0 and ρ_1 have opposite signs, which suggests that the model is capable of identifying the expected characteristic of pre-default and post-default periods mentioned earlier.

Figure 4 shows plots of the estimated time-varying transition probabilities q_t and p_t associated with the regimes $\{S_t\}$, as well as the filtered state probabilities $\mathbb{P}(S_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$, for Model 1. We also compute the estimate of the quantity $\kappa_t := (1 - q_t)/(2 - q_t - p_t)$, which may be thought of as a proxy for the probability that $S_t = 1$, given currently available information on Z_{t-1}, and gives an indication of the contribution of Z_{t-1} in predicting regime changes. It is immediately obvious that the evolution of q_t and p_t mimics the movements in the output growth rate and does not seem to be particularly informative regarding shifts to the regime that is associated with high EMBI. Furthermore, the movements in κ_t appear to be unrelated to the movements in $\mathbb{P}(S_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$, suggesting that output growth does not have much predictive ability for regime changes.

For Model 2, Figure 5 shows plots of the estimated time-varying transition probabilities $\bar{p}_t = \mathbb{P}(X_t = 0|\mathcal{F}_t)p_{0t} + \mathbb{P}(X_t = 1|\mathcal{F}_t)p_{1t}$ and $\bar{q}_t = \mathbb{P}(X_t = 0|\mathcal{F}_t)q_{0t} + \mathbb{P}(X_t = 1|\mathcal{F}_t)q_{1t}$ (evaluated at $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$) associated with the regime sequence $\{S_t\}$, the quantity $\bar{\kappa}_t := (1 - \bar{q}_t)/(2 - \bar{q}_t - \bar{p}_t)$, and the filtered probabilities $\mathbb{P}(S_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$ and $\mathbb{P}(X_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$. The main difference with Model 1 is that both $\bar{q}_t$ and $\bar{\kappa}_t$ now seem to be informative about the regime changes associated with EMBI, tracking the movements of $\mathbb{P}(S_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$ more closely and preceding its changes on several occasions.

The filtered probabilities $\mathbb{P}(S_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$ associated with high EMBI values are very similar for both models. For Model 1 (Model 2), the (implied) variance of EMBI in the regime associated with $S_t = 1$ (which includes the default on sovereign debt) is approximately 12 times (11 times) larger than it is in the regime associated with $S_t = 0$. For Model 1, the (implied) long-run mean of EMBI is 0.7618 (761.8 points since the series was divided by 1000 prior to estimation) in the regime associated with $S_t = 0$ and 3.69997 (3699.9 points) in the regime associated with $S_t = 1$; the corresponding figures for Model 2 are 0.3795 (379.5 points) and 1.9083 (1908.3 points), respectively. Finally, the filtered probabilities $\mathbb{P}(X_t = 1|\mathcal{F}_t; \widehat{\boldsymbol{\theta}})$ shown in Figure 5 reveal several transitions between a regime associated with positive correlation between output growth and EMBI ($X_t = 1$) and a regime that is characterized by negative correlation ($X_t = 0$).

6 Summary

We have considered a class of Markov-switching models with time-varying transition probability matrices in which the parameters associated with the latter are subject to random changes driven by an exogenous Markov process. Such changes will typically be related to changes in the covariance structure between the time series of interest and the information variables which drive the evolution of the Markov transition probabilities. A simulation study has demonstrated the pitfalls of ignoring such changes, pitfalls which include biased parameter estimates and hypotheses tests which exhibit level distortions and low power. The simulations have also shown that the proposed model and related parameter estimator share the same desirable characteristics with a model which incorporates perfect information about the number and location of the breaks associated with the Markov transition functions. As an illustration of the practical use of the proposed class of models, we have analyzed the relationship between sovereign bond spreads and output growth in Argentina. The correlation structure between these two variables, and hence the parameters associated with the time-varying transition probabilities of a related regime-switching model, are likely to have changed as a result of the 2001 economic and financial crisis, a regime shift which the proposed model is well equipped to handle.

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Table 1a: Characteristics of the empirical distribution of ML (Model M-1)

	μ_0	μ_1	$\alpha_{p(0)}$	$\beta_{p(0)}$	$\beta_{p(1)}$	$\alpha_{q(0)}$	$\beta_{q(0)}$	$\beta_{q(1)}$	σ_0	σ_1	ϕ_1
T	Mean										
100	0.080	0.073	0.134	-1.311	-0.689	0.133	-0.689	-1.311	-0.044	-0.044	-0.015
200	0.051	0.048	0.105	-1.293	-0.707	0.090	-0.666	-1.334	-0.018	-0.023	-0.009
400	0.031	0.033	0.069	-1.347	-0.653	0.076	-0.641	-1.359	-0.009	-0.013	-0.006
800	0.020	0.021	0.048	-1.292	-0.708	0.054	-0.685	-1.315	-0.003	-0.007	-0.004
1600	0.012	0.015	0.061	-1.349	-0.651	0.068	-0.651	-1.349	-0.002	-0.002	-0.004
3200	0.012	0.010	0.072	-1.373	-0.627	0.070	-0.634	-1.366	-0.003	-0.002	-0.004
6400	0.008	0.010	0.065	-1.346	-0.654	0.065	-0.658	-1.342	-0.002	-0.002	-0.003
	Skewness										
100	0.267	0.570	2.110	0.389	0.389	1.628	1.245	1.245	0.580	0.274	-0.672
200	0.972	0.086	2.925	3.054	3.054	2.560	-1.173	-1.173	0.386	0.267	-0.066
400	-0.282	-0.017	0.654	-0.016	-0.016	1.506	-0.316	-0.316	0.624	0.388	-0.221
800	0.010	0.001	0.338	-0.003	-0.003	0.441	0.258	0.258	0.154	0.199	-0.236
1600	-0.010	-0.039	0.016	0.292	0.292	0.065	0.125	0.125	0.411	0.250	0.038
3200	0.050	0.120	0.162	0.345	0.345	0.087	0.165	0.165	0.121	0.318	-0.190
6400	0.005	-0.105	0.261	-0.217	-0.217	0.140	-0.086	-0.086	0.216	0.072	-0.034
	Kurtosis										
100	6.808	8.049	12.868	19.083	19.083	8.734	17.098	17.098	6.545	5.148	5.461
200	9.706	4.068	32.717	49.995	49.995	17.849	10.751	10.751	4.545	3.530	3.463
400	4.369	3.310	4.388	3.444	3.444	11.501	4.885	4.885	5.200	3.563	3.374
800	3.217	3.093	3.756	3.315	3.315	3.659	3.109	3.109	2.859	3.634	3.113
1600	3.199	2.957	3.104	3.335	3.335	2.965	3.303	3.303	3.664	3.327	3.541
3200	2.977	2.986	2.967	3.876	3.876	3.251	3.691	3.691	3.137	3.803	2.874
	Ratio of sampling standard deviations to estimated standard errors										
100	1.333	1.487	1.516	1.622	1.622	1.419	1.520	1.520	1.273	1.262	1.409
200	1.248	1.184	1.412	1.587	1.587	1.421	1.326	1.326	1.155	1.100	1.165
400	1.073	1.047	1.157	1.137	1.137	1.231	1.154	1.154	1.102	1.036	1.059
800	1.025	1.048	1.075	1.115	1.115	1.113	1.118	1.118	1.051	1.007	1.032
1600	0.980	0.991	1.075	1.121	1.121	1.045	1.043	1.043	1.037	1.027	0.956
3200	1.056	0.993	0.970	0.934	0.934	1.005	0.962	0.962	1.070	1.051	1.000
6400	0.967	1.001	0.965	0.882	0.882	0.943	0.979	0.979	1.035	1.097	0.982

Table 1b: Empirical moments of t-statistics (Model M-1)

	α_0	α_1	$\alpha_{p(0)}$	$\beta_{p(0)}$	$\beta_{p(1)}$	$\alpha_{q(0)}$	$\beta_{q(0)}$	$\beta_{q(1)}$	σ_0	σ_1	ϕ_1
T	Mean										
100	0.406	-0.368	-0.092	-0.976	0.643	-0.090	0.618	-1.015	-0.551	-0.533	-0.383
200	0.367	-0.356	0.049	-1.516	0.854	0.005	0.874	-1.516	-0.311	-0.373	-0.373
400	0.310	-0.332	0.142	-2.329	1.242	0.161	1.246	-2.382	-0.231	-0.286	-0.339
800	0.282	-0.295	0.211	-3.303	1.923	0.240	1.900	-3.387	-0.134	-0.202	-0.348
1600	0.246	-0.293	0.461	-5.060	2.757	0.537	2.762	-5.064	-0.107	-0.114	-0.470
3200	0.323	-0.289	0.848	-6.918	3.472	0.807	3.630	-6.990	-0.167	-0.141	-0.668
6400	0.327	-0.375	1.114	-9.682	5.095	1.124	5.548	-10.195	-0.132	-0.147	-0.863
	Standard deviation										
100	1.239	1.213	1.137	1.203	1.231	1.096	1.237	1.181	1.174	1.149	1.161
200	1.239	1.213	1.137	1.203	1.231	1.096	1.237	1.181	1.174	1.149	1.161
400	1.070	1.049	1.100	1.389	1.506	1.119	1.641	1.531	1.109	1.051	1.052
800	1.040	1.064	1.064	1.769	1.916	1.071	2.055	1.909	1.064	1.007	1.033
1600	0.978	0.993	1.080	2.736	3.107	1.059	3.274	2.932	1.033	1.029	0.949
3200	1.054	0.999	1.012	3.516	3.824	1.039	4.293	3.878	1.070	1.047	0.995
6400	0.970	0.999	1.019	5.009	5.251	0.989	6.256	6.043	1.036	1.100	0.977
	Skewness										
100	-0.089	0.282	-1.290	0.286	-0.007	-0.591	0.091	0.254	-0.752	-0.963	0.606
200	0.615	0.215	-0.905	0.015	0.378	-0.291	0.302	-0.025	-0.147	-0.256	0.492
400	-0.117	-0.079	-0.420	-1.130	1.794	-0.389	2.483	-1.950	-0.054	-0.095	0.110
800	-0.017	0.020	-0.317	-2.396	3.091	-0.208	4.104	-3.705	-0.197	-0.221	-0.104
1600	-0.019	-0.068	-0.470	-4.605	4.508	-0.356	4.896	-5.040	0.098	-0.089	0.142
3200	0.052	0.093	-0.062	-5.476	6.263	-0.235	6.156	-5.690	-0.053	0.041	-0.149
6400	0.016	-0.106	0.116	-5.510	6.756	0.046	6.444	-5.976	0.102	-0.070	0.012
	Kurtosis										
100	6.559	5.939	11.735	2.357	2.299	3.295	2.185	2.281	5.218	7.661	5.349
200	6.662	4.819	5.697	3.276	3.709	3.027	4.037	3.386	3.841	3.667	4.353
400	3.693	2.972	3.089	11.105	12.300	3.227	16.975	15.445	3.544	3.140	3.007
800	3.128	3.172	3.210	17.651	20.531	2.969	32.036	32.150	2.963	3.294	2.957
1600	3.044	2.881	3.444	38.746	31.972	3.115	35.700	41.810	2.995	3.330	3.375
3200	2.888	2.987	2.998	50.413	54.917	3.498	50.968	49.428	3.122	3.272	2.859
6400	2.996	3.011	3.262	48.070	58.308	2.901	54.950	52.825	2.842	2.964	2.891

Table 1c: Empirical size and power of t-tests (Model M-1)

	α_0	α_1	$\alpha_{p(0)}$	$\beta_{p(0)}$	$\beta_{p(1)}$	$\alpha_{q(0)}$	$\beta_{q(0)}$	$\beta_{q(1)}$	σ_0	σ_1	ϕ_1
T	Size at 0.05										
100	0.058	0.165	0.096	0.318	0.020	0.102	0.020	0.340	0.169	0.172	0.158
200	0.044	0.128	0.069	0.499	0.012	0.066	0.017	0.490	0.125	0.124	0.136
400	0.027	0.102	0.064	0.708	0.003	0.067	0.010	0.712	0.103	0.094	0.101
800	0.031	0.098	0.054	0.828	0.004	0.055	0.001	0.858	0.083	0.077	0.113
1600	0.035	0.082	0.039	0.965	0.000	0.033	0.000	0.971	0.065	0.065	0.104
3200	0.033	0.093	0.014	0.986	0.000	0.016	0.000	0.992	0.082	0.072	0.180
6400	0.022	0.119	0.004	0.993	0.000	0.002	0.000	0.996	0.070	0.088	0.218
	Size at 0.10										
100	0.084	0.249	0.151	0.449	0.045	0.153	0.050	0.472	0.256	0.244	0.253
200	0.079	0.206	0.115	0.596	0.031	0.122	0.042	0.587	0.186	0.198	0.218
400	0.070	0.175	0.106	0.766	0.018	0.109	0.017	0.767	0.175	0.158	0.191
800	0.067	0.173	0.090	0.891	0.009	0.088	0.003	0.910	0.134	0.143	0.188
1600	0.063	0.158	0.064	0.981	0.000	0.060	0.000	0.986	0.124	0.119	0.192
3200	0.059	0.152	0.020	0.989	0.000	0.031	0.000	0.997	0.148	0.145	0.257
6400	0.044	0.181	0.009	0.995	0.000	0.007	0.000	1.000	0.141	0.156	0.346
	Power at 0.05										
100	0.985	0.977	0.678	0.064	0.064	0.677	0.067	0.067	1.000	1.000	1.000
200	0.999	0.997	0.917	0.071	0.071	0.924	0.082	0.082	1.000	1.000	1.000
400	1.000	1.000	0.992	0.117	0.117	0.985	0.107	0.107	1.000	1.000	1.000
800	0.999	0.999	0.999	0.114	0.114	0.999	0.141	0.141	1.000	1.000	1.000
1600	1.000	1.000	1.000	0.228	0.228	0.999	0.228	0.228	1.000	1.000	1.000
3200	1.000	1.000	0.997	0.404	0.404	0.998	0.378	0.378	1.000	1.000	1.000
6400	1.000	1.000	1.000	0.603	0.603	1.000	0.636	0.636	1.000	1.000	1.000
	Power at 0.10										
100	0.989	0.979	0.775	0.150	0.150	0.772	0.160	0.160	1.000	1.000	1.000
200	0.999	1.000	0.952	0.153	0.153	0.952	0.162	0.162	1.000	1.000	1.000
400	1.000	1.000	0.997	0.191	0.191	0.988	0.195	0.195	1.000	1.000	1.000
800	1.000	0.999	0.999	0.210	0.210	0.999	0.226	0.226	1.000	1.000	1.000
1600	1.000	1.000	1.000	0.339	0.339	0.999	0.326	0.326	1.000	1.000	1.000
3200	1.000	1.000	0.997	0.526	0.526	0.998	0.488	0.488	1.000	1.000	1.000
6400	1.000	1.000	1.000	0.603	0.603	1.000	0.636	0.636	1.000	1.000	1.000

Table 2a: Characteristics of the empirical distribution of ML (Model M-2)

	μ_0	μ_1	α_{p0}	β_{p0}	β_{p1}	α_{q0}	β_{q0}	β_{q1}	σ_0	σ_1	ϕ_1
T	Bias										
100	0.017	0.017	0.165	0.130	0.456	0.683	1.101	0.530	-0.019	-0.013	-0.011
200	0.002	-0.001	0.021	0.021	0.113	0.731	0.661	0.118	-0.009	-0.006	-0.007
400	0.003	0.007	0.028	0.013	0.089	0.161	0.210	0.055	-0.003	-0.006	-0.004
800	0.001	-0.001	0.003	0.001	-0.016	0.079	0.062	-0.031	-0.004	-0.003	-0.004
1600	0.001	0.000	0.002	-0.001	-0.056	-0.020	0.022	-0.062	0.000	0.001	-0.001
3200	0.001	0.000	0.004	0.004	-0.066	-0.034	-0.012	-0.063	-0.001	-0.001	-0.002
6400	0.000	0.000	0.002	0.001	-0.067	-0.033	-0.029	-0.066	-0.001	0.000	-0.002
	Skewness										
100	0.049	0.336	1.958	0.113	-0.630	1.252	2.108	-0.195	0.016	-0.244	0.055
200	0.074	0.191	1.175	0.722	2.697	4.039	4.363	-0.331	0.074	0.052	0.104
400	-0.042	-0.058	0.928	0.349	-0.916	4.166	1.162	-0.929	-0.007	0.081	-0.021
800	0.028	-0.015	0.174	0.017	-0.292	1.243	1.095	-0.092	-0.080	-0.090	-0.047
1600	0.003	-0.093	0.184	0.014	-0.197	0.586	0.309	-0.108	-0.029	-0.023	0.083
3200	-0.030	-0.007	0.020	-0.143	-0.088	0.248	0.462	-0.030	-0.041	0.199	-0.085
6400	-0.119	0.078	0.111	0.104	-0.192	0.047	0.207	-0.050	-0.003	0.068	0.128
	Kurtosis										
100	3.572	4.975	12.816	11.959	17.634	12.821	11.870	15.796	4.712	4.854	2.774
200	3.380	3.706	8.914	6.254	93.858	29.665	35.396	24.948	3.468	3.162	3.117
400	3.071	3.035	7.037	3.930	8.137	38.679	5.450	9.473	3.053	3.104	3.033
800	2.981	2.895	3.059	3.047	3.244	9.285	5.939	3.448	3.318	3.113	2.906
1600	3.261	3.016	3.012	2.868	3.192	3.995	3.236	3.042	2.456	2.569	2.828
3200	3.050	2.782	3.199	3.197	3.185	2.960	3.391	2.938	2.472	2.704	3.136
6400	2.634	3.300	3.005	3.178	3.123	3.453	2.860	2.886	2.572	2.778	2.607
	Ratio of sampling standard deviations to estimated standard errors										
100	1.220	1.211	1.327	1.369	1.183	0.881	0.880	1.216	1.141	1.105	1.036
200	1.057	1.088	1.138	1.105	1.538	1.509	1.517	1.257	1.064	1.030	1.026
400	1.032	1.013	1.039	1.019	1.090	1.390	1.092	1.063	0.928	0.951	1.002
800	0.980	1.006	1.016	0.976	0.988	1.059	1.041	0.980	0.948	0.959	0.974
1600	0.965	1.015	0.969	0.946	0.969	1.028	1.023	0.939	0.942	0.868	0.942
3200	0.990	0.989	1.002	0.937	1.010	1.037	0.957	0.951	0.994	1.002	1.006
6400	1.055	1.006	0.939	0.963	0.935	0.893	0.882	0.946	1.015	1.015	1.027

Table 2b: Empirical moments of t-statistics (Model M-2)

	μ_0	μ_1	α_{p0}	β_{p0}	β_{p1}	α_{q0}	β_{q0}	β_{q1}	σ_0	σ_1	ϕ_1
T	Mean										
100	0.158	-0.136	0.089	0.061	-0.084	-0.171	-0.116	-0.096	-0.301	-0.237	-0.228
200	0.017	0.009	-0.079	-0.062	0.027	0.037	0.020	-0.020	-0.199	-0.157	-0.192
400	0.066	-0.129	0.065	-0.005	-0.049	-0.106	-0.040	-0.006	-0.099	-0.172	-0.160
800	0.015	0.033	-0.032	-0.041	0.086	-0.024	-0.074	0.115	-0.151	-0.112	-0.189
1600	0.035	-0.008	-0.021	-0.051	0.205	-0.171	-0.046	0.225	-0.022	-0.007	-0.083
3200	0.070	0.003	0.029	0.048	0.314	-0.165	-0.089	0.299	-0.069	-0.087	-0.189
6400	-0.028	0.039	0.018	0.005	0.434	-0.191	-0.160	0.431	-0.083	-0.068	-0.215
	Standard deviation										
100	1.050	1.085	1.017	1.023	1.042	1.083	1.078	0.989	1.100	1.079	1.052
200	1.050	1.085	1.017	1.023	1.042	1.083	1.078	0.989	1.100	1.079	1.052
400	1.031	1.016	0.966	0.989	1.083	1.056	1.212	0.978	0.982	0.989	1.030
800	0.983	1.011	1.007	0.986	1.031	1.130	1.218	1.057	1.025	1.046	1.019
1600	0.969	1.016	0.971	0.943	0.993	1.466	1.131	0.984	1.082	1.023	1.024
3200	0.989	0.993	0.999	0.936	1.000	1.058	0.954	0.941	1.069	1.055	1.043
6400	1.055	1.006	0.939	0.963	0.935	0.893	0.882	0.946	1.015	1.015	1.027
	Skewness										
100	0.030	0.346	-1.687	-0.560	0.264	-0.539	-0.511	0.214	-0.774	-1.471	-0.339
200	0.074	0.058	-0.198	-0.453	0.341	-0.949	-1.130	0.198	-0.252	-0.363	-0.255
400	-0.054	-0.053	-0.105	-0.221	1.674	-1.008	-2.496	0.495	-0.278	-0.182	-0.249
800	0.020	-0.002	-0.176	-0.234	0.904	-2.152	-3.299	1.465	-0.320	-0.307	-0.225
1600	0.022	-0.092	-0.060	-0.156	0.142	-9.461	-1.950	1.205	-0.156	-0.163	-0.001
3200	-0.037	-0.007	-0.136	-0.259	-0.051	-0.272	0.136	-0.015	-0.124	0.062	-0.171
6400	-0.097	0.072	-0.018	-0.057	-0.174	-0.096	0.046	0.048	-0.091	-0.064	0.059
	Kurtosis										
100	3.145	4.029	16.760	2.981	2.397	2.812	2.896	2.257	5.272	13.632	2.877
200	3.193	3.375	2.833	3.153	3.038	4.812	6.125	2.668	3.336	3.427	3.301
400	2.997	3.015	2.920	3.204	16.777	6.712	20.069	4.767	3.048	3.204	3.233
800	2.988	2.900	2.794	2.986	10.654	19.872	31.557	14.029	3.777	3.349	3.096
1600	3.284	3.074	3.050	2.809	3.791	178.316	19.547	15.149	2.545	2.697	2.893
3200	3.029	2.820	3.204	3.247	3.049	4.576	3.125	2.746	2.459	2.510	3.283
6400	2.632	3.295	3.028	3.079	2.870	2.988	2.855	3.041	2.460	2.529	2.817

Table 2c: Empirical size and power of t-tests (Model M-2)

	μ_0	μ_1	α_{p0}	β_{p0}	β_{p1}	α_{q0}	β_{q0}	β_{q1}	σ_0	σ_1	ϕ_1
T	Size at 0.05										
100	0.073	0.105	0.039	0.063	0.030	0.111	0.094	0.035	0.114	0.112	0.112
200	0.047	0.062	0.062	0.070	0.039	0.079	0.075	0.041	0.093	0.086	0.082
400	0.052	0.068	0.041	0.062	0.054	0.082	0.061	0.042	0.061	0.076	0.079
800	0.046	0.039	0.057	0.057	0.043	0.054	0.043	0.033	0.067	0.068	0.081
1600	0.042	0.052	0.052	0.049	0.031	0.052	0.060	0.022	0.063	0.054	0.064
3200	0.049	0.045	0.056	0.045	0.037	0.075	0.045	0.019	0.077	0.059	0.091
6400	0.072	0.044	0.042	0.042	0.025	0.061	0.050	0.017	0.127	0.125	0.108
	Size at 0.10										
100	0.124	0.167	0.085	0.101	0.119	0.180	0.153	0.115	0.178	0.161	0.167
200	0.103	0.111	0.117	0.118	0.100	0.127	0.110	0.100	0.163	0.137	0.148
400	0.096	0.128	0.073	0.093	0.108	0.115	0.092	0.099	0.121	0.125	0.136
800	0.094	0.099	0.111	0.117	0.085	0.100	0.082	0.077	0.118	0.130	0.138
1600	0.100	0.106	0.096	0.106	0.058	0.128	0.115	0.061	0.149	0.106	0.123
3200	0.086	0.103	0.096	0.075	0.065	0.135	0.101	0.054	0.138	0.138	0.136
6400	0.144	0.094	0.083	0.108	0.042	0.111	0.094	0.039	0.194	0.199	0.188
	Power at 0.05										
100	1.000	0.997	0.757	0.733	0.078	0.039	0.029	0.065	1.000	0.999	1.000
200	1.000	1.000	0.953	0.949	0.168	0.094	0.082	0.160	1.000	0.999	1.000
400	1.000	1.000	1.000	0.999	0.369	0.154	0.197	0.339	1.000	1.000	1.000
800	1.000	1.000	0.999	1.000	0.608	0.445	0.409	0.596	1.000	1.000	1.000
1600	1.000	1.000	1.000	1.000	0.860	0.743	0.750	0.875	1.000	1.000	1.000
3200	1.000	1.000	1.000	1.000	0.983	0.944	0.969	0.986	1.000	1.000	1.000
6400	1.000	1.000	1.000	1.000	1.000	0.989	0.986	0.997	1.000	1.000	1.000
	Power at 0.10										
100	1.000	1.000	0.847	0.822	0.191	0.093	0.101	0.175	1.000	0.999	1.000
200	1.000	1.000	0.976	0.967	0.284	0.194	0.181	0.272	1.000	0.999	1.000
400	1.000	1.000	1.000	0.999	0.513	0.270	0.323	0.465	1.000	1.000	1.000
800	1.000	1.000	0.999	1.000	0.719	0.603	0.573	0.723	1.000	1.000	1.000
1600	1.000	1.000	1.000	1.000	0.919	0.830	0.831	0.936	1.000	1.000	1.000
3200	1.000	1.000	1.000	1.000	0.993	0.963	0.986	0.991	1.000	1.000	1.000
6400	1.000	1.000	1.000	1.000	1.000	0.992	0.994	1.000	1.000	1.000	1.000

Table 3a: Characteristics of the empirical distribution of ML (Model M-3)

	μ_0	μ_1	α_{p0}	β_{p0}	β_{p1}	α_{q0}	β_{q0}	β_{q1}	σ_0	σ_1	ϕ_1
T	Bias										
100	0.010	0.013	0.092	0.929	0.545	0.088	0.333	0.772	-0.011	-0.009	-0.006
200	0.008	0.011	0.044	0.403	0.253	0.029	0.180	0.465	-0.002	-0.006	-0.003
400	0.001	0.004	0.017	0.123	0.119	0.015	0.082	0.189	-0.001	-0.003	-0.001
800	0.001	0.002	0.000	0.074	0.025	0.001	0.039	0.087	0.001	-0.001	0.000
1600	0.000	0.001	-0.004	0.043	0.011	0.000	0.014	0.038	0.001	-0.001	0.000
3200	0.000	0.000	0.003	0.012	0.016	0.002	0.013	0.002	0.000	-0.002	0.000
6400	0.001	0.000	0.000	0.013	0.006	0.001	0.010	0.005	-0.001	-0.001	0.000
	Skewness										
100	-0.027	0.107	1.531	2.889	-1.940	-0.023	-0.066	1.983	0.286	-0.161	-0.391
200	0.163	-0.008	1.246	5.213	-2.769	1.210	-0.762	3.020	-0.028	-0.096	-0.077
400	0.173	-0.082	0.536	1.817	-0.566	0.303	-0.315	3.170	0.061	0.083	-0.049
800	0.047	0.024	0.346	0.449	-0.294	0.219	-0.169	0.698	0.089	0.036	-0.150
1600	0.094	-0.136	0.133	0.436	0.022	0.049	-0.212	0.300	0.012	0.009	-0.017
3200	-0.006	-0.119	0.045	0.402	-0.084	-0.014	-0.108	0.227	-0.010	0.122	-0.082
6400	0.035	-0.175	0.023	0.199	-0.064	-0.004	-0.057	0.188	0.127	-0.048	-0.099
	Kurtosis										
100	3.390	4.456	21.320	16.278	19.050	15.175	25.660	18.308	5.197	3.801	3.857
200	3.366	3.208	13.404	59.420	41.460	7.481	6.852	18.633	2.893	3.357	3.326
400	2.955	2.957	5.208	14.381	5.000	3.378	4.661	34.489	3.119	3.155	2.890
800	3.000	3.013	3.444	3.737	3.706	3.288	3.174	5.248	2.998	2.932	3.435
1600	3.017	3.066	3.287	3.528	2.888	3.203	3.174	3.173	2.602	2.784	3.072
3200	2.914	3.002	3.045	3.539	3.057	3.023	3.092	2.964	3.073	2.930	2.804
	Ratio of sampling standard deviations to estimated standard errors										
100	1.094	1.149	1.342	0.512	1.354	1.183	0.990	0.705	1.083	1.090	1.113
200	1.061	1.017	1.155	0.539	1.441	1.100	1.177	1.440	0.999	1.045	1.059
400	1.016	1.022	1.029	1.169	1.125	1.048	1.054	1.191	1.012	0.993	1.021
800	0.982	1.026	1.016	1.039	1.049	0.998	1.015	1.079	0.988	0.990	1.039
1600	0.991	1.023	1.023	1.011	1.035	1.002	1.022	1.015	0.993	1.004	0.977
3200	1.000	1.024	0.991	1.016	0.995	1.038	1.092	1.039	0.964	0.972	0.969
6400	0.995	1.020	1.016	0.977	1.005	1.034	1.064	0.984	0.985	1.017	1.001

Table 3b: Empirical moments of t-statistics (Model M-3)

	μ_0	μ_1	α_{p0}	β_{p0}	β_{p1}	α_{q0}	β_{q0}	β_{q1}	σ_0	σ_1	ϕ_1
T	Mean										
100	0.095	-0.111	-0.009	-0.048	-0.100	0.037	-0.089	-0.087	-0.215	-0.188	-0.279
200	0.107	-0.154	0.033	-0.039	-0.115	-0.033	-0.061	0.008	-0.084	-0.156	-0.210
400	0.026	-0.083	0.009	-0.031	-0.081	0.004	-0.045	0.031	-0.060	-0.113	-0.141
800	0.044	-0.056	-0.048	0.033	-0.017	-0.042	-0.033	0.025	-0.006	-0.072	-0.035
1600	0.003	-0.050	-0.083	0.044	-0.019	-0.037	-0.025	0.036	0.012	-0.080	-0.025
3200	0.023	0.003	0.019	0.004	-0.061	0.011	-0.047	-0.040	-0.036	-0.113	-0.102
6400	0.055	-0.039	-0.008	0.042	-0.031	-0.002	-0.057	-0.005	-0.109	-0.103	-0.049
	Standard deviation										
100	1.060	1.020	1.029	1.108	1.081	0.963	1.049	1.041	1.021	1.056	1.049
200	1.060	1.020	1.029	1.108	1.081	0.963	1.049	1.041	1.021	1.056	1.049
400	1.022	1.019	0.983	1.026	1.179	1.015	1.120	1.047	1.017	1.001	1.029
800	0.985	1.029	0.993	1.037	1.067	0.987	1.131	1.225	0.989	0.995	1.038
1600	0.991	1.027	1.014	1.016	1.028	1.000	1.010	0.996	0.992	1.006	0.977
3200	1.000	1.026	0.987	0.996	0.986	1.036	1.079	1.033	0.964	0.972	0.969
6400	0.994	1.020	1.018	0.971	1.003	1.033	1.064	0.979	0.985	1.019	1.001
	Skewness										
100	-0.058	0.155	-1.096	-0.435	0.329	-0.349	0.227	-0.459	-0.548	-0.550	0.133
200	0.134	-0.106	-0.398	-1.002	0.555	-0.219	0.563	-0.646	-0.372	-0.348	0.217
400	0.152	-0.047	-0.233	-0.503	1.829	-0.163	1.472	-0.797	-0.215	-0.192	0.099
800	0.048	0.046	0.021	-0.896	0.512	-0.051	2.100	-4.302	-0.092	-0.158	0.005
1600	0.105	-0.149	-0.117	-0.715	0.172	-0.187	-0.024	-0.139	-0.087	-0.092	0.057
3200	-0.013	-0.125	-0.098	0.076	0.020	-0.149	0.021	-0.024	-0.091	0.048	-0.038
6400	0.030	-0.174	-0.076	-0.008	-0.009	-0.117	0.010	-0.021	0.063	-0.106	-0.050
	Kurtosis										
100	3.089	3.266	7.335	2.644	2.668	2.626	2.545	2.683	4.502	3.622	3.235
200	3.204	3.222	3.092	5.353	4.437	2.755	4.865	3.587	3.104	3.065	3.193
400	2.891	2.915	2.875	3.775	17.591	2.815	10.901	4.892	3.138	3.258	2.911
800	2.971	3.060	2.750	7.763	6.274	2.999	21.472	64.288	3.065	2.992	3.475
1600	3.029	3.097	3.162	8.711	3.054	3.176	3.236	2.904	2.599	2.743	3.006
3200	2.886	3.036	3.054	3.185	2.952	3.028	3.048	2.859	3.077	2.901	2.817
6400	2.842	3.042	3.113	3.086	2.970	3.117	2.787	2.933	2.925	3.029	3.116

Table 3c: Empirical size and power of t-tests (Model M-3)

	μ_0	μ_1	α_{p0}	β_{p0}	β_{p1}	α_{q0}	β_{q0}	β_{q1}	σ_0	σ_1	ϕ_1
T	Size at 0.05										
100	0.054	0.088	0.068	0.075	0.048	0.053	0.045	0.091	0.094	0.111	0.098
200	0.047	0.075	0.061	0.075	0.069	0.059	0.055	0.084	0.075	0.092	0.081
400	0.042	0.055	0.058	0.061	0.066	0.064	0.050	0.055	0.057	0.062	0.069
800	0.040	0.058	0.056	0.038	0.057	0.054	0.057	0.053	0.050	0.064	0.058
1600	0.041	0.070	0.064	0.044	0.051	0.056	0.057	0.052	0.046	0.066	0.048
3200	0.041	0.052	0.048	0.043	0.055	0.062	0.077	0.061	0.052	0.053	0.056
6400	0.038	0.059	0.056	0.035	0.054	0.062	0.063	0.052	0.054	0.059	0.063
	Size at 0.10										
100	0.103	0.147	0.104	0.128	0.116	0.099	0.114	0.147	0.156	0.153	0.174
200	0.091	0.139	0.094	0.141	0.140	0.108	0.123	0.133	0.123	0.146	0.148
400	0.100	0.110	0.104	0.094	0.130	0.102	0.115	0.092	0.107	0.115	0.128
800	0.084	0.111	0.116	0.096	0.101	0.113	0.116	0.103	0.103	0.107	0.114
1600	0.094	0.119	0.125	0.089	0.104	0.114	0.105	0.099	0.105	0.113	0.098
3200	0.099	0.105	0.101	0.100	0.104	0.101	0.131	0.115	0.096	0.112	0.111
6400	0.091	0.108	0.106	0.079	0.106	0.110	0.120	0.098	0.110	0.121	0.105
	Power at 0.05										
100	1.000	1.000	0.760	0.038	0.084	0.779	0.103	0.036	1.000	1.000	1.000
200	1.000	1.000	0.962	0.103	0.225	0.978	0.208	0.106	1.000	1.000	1.000
400	1.000	1.000	1.000	0.224	0.416	1.000	0.419	0.260	1.000	1.000	1.000
800	1.000	1.000	1.000	0.559	0.693	1.000	0.695	0.560	1.000	1.000	1.000
1600	1.000	1.000	1.000	0.879	0.927	1.000	0.938	0.864	1.000	1.000	1.000
3200	1.000	1.000	1.000	0.990	0.999	1.000	0.995	0.991	1.000	1.000	1.000
6400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Power at 0.10										
100	1.000	1.000	0.850	0.117	0.177	0.852	0.197	0.101	1.000	1.000	1.000
200	1.000	1.000	0.972	0.217	0.345	0.990	0.321	0.218	1.000	1.000	1.000
400	1.000	1.000	1.000	0.374	0.539	1.000	0.550	0.402	1.000	1.000	1.000
800	1.000	1.000	1.000	0.691	0.777	1.000	0.783	0.674	1.000	1.000	1.000
1600	1.000	1.000	1.000	0.926	0.962	1.000	0.963	0.923	1.000	1.000	1.000
3200	1.000	1.000	1.000	0.996	1.000	1.000	0.999	0.997	1.000	1.000	1.000
6400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4: ML Estimates

Model 1		Model 2			
μ_0	0.0354	μ_0	0.0178		
	(0.0182)		(0.0187)		
μ_1	0.1721	μ_1	0.0895		
	(0.2429)		(0.2022)		
α_q	1.4795	α_{q0}	1.6310	ϕ_1	1.1936
	(0.5250)		(0.8900)		(0.0276)
β_q	0.1028	α_{q1}	1.6310	ϕ_2	-0.1478
	(0.0724)		(0.8900)		(0.0385)
μ_p	0.2243	α_{p0}	0.2072	ϕ_3	-0.1048
	(0.7814)		(0.7846)		(0.0358)
β_p	0.2412	α_{p1}	0.2072	ϕ_4	0.0121
	(0.1117)		(0.7846)		(0.0182)
ϕ_1	1.1497	β_{q0}	-2.0462	μ_z	0.8179
	(0.0234)		(2.9853)		(0.2799)
ϕ_2	-0.1194	β_{q1}	0.2430	ψ_1	1.4522
	(0.0370)		(0.1380)		(0.1107)
ϕ_3	-0.077	β_{p0}	0.1925	ψ_2	-0.5034
	(0.0376)		(0.1931)		(0.1980)
ϕ_4	0.0002	β_{p1}	0.2489	ψ_3	-0.1048
	(0.0198)		(0.1343)		(0.0358)
σ_0	0.0742	σ_0	0.0908	ψ_4	0.0797
	(0.0089)		(0.0142)		(0.1077)
σ_1	1.1046	σ_1	1.0033		
	(0.1713)		(0.1237)	p_x	0.9103
		σ_z	2.1650		(0.0550)
			(0.2023)	q_x	0.8135
		ρ_0	-0.5741		(0.1023)
			(0.1532)		
		ρ_1	0.7078		
			(0.1103)		
Log-likelihood: -9.33121					-129.6105

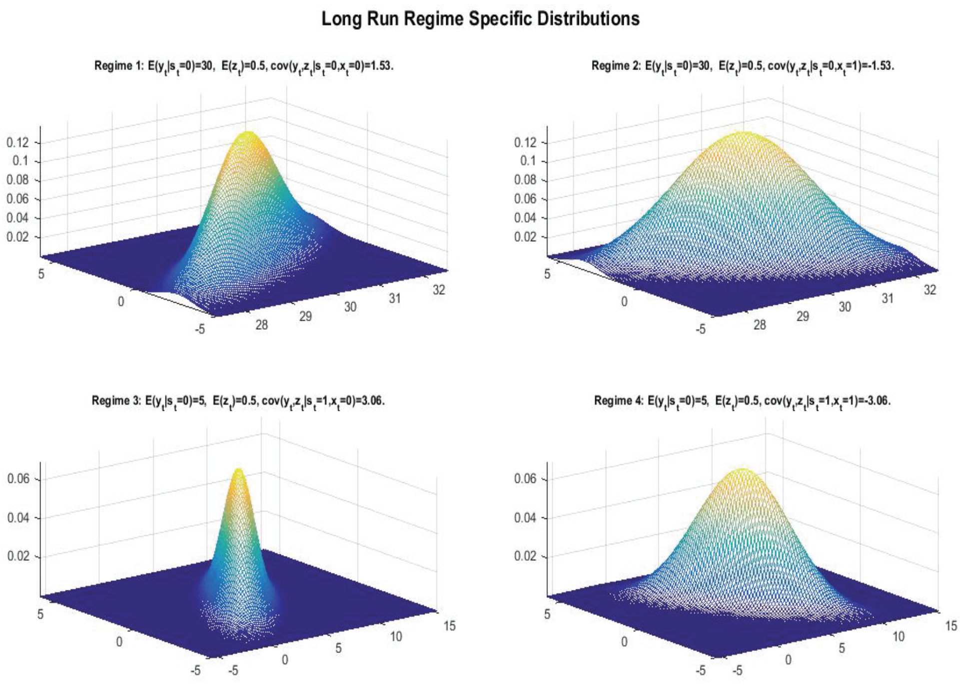

Figure 1: Regime-Specific Marginal Densities

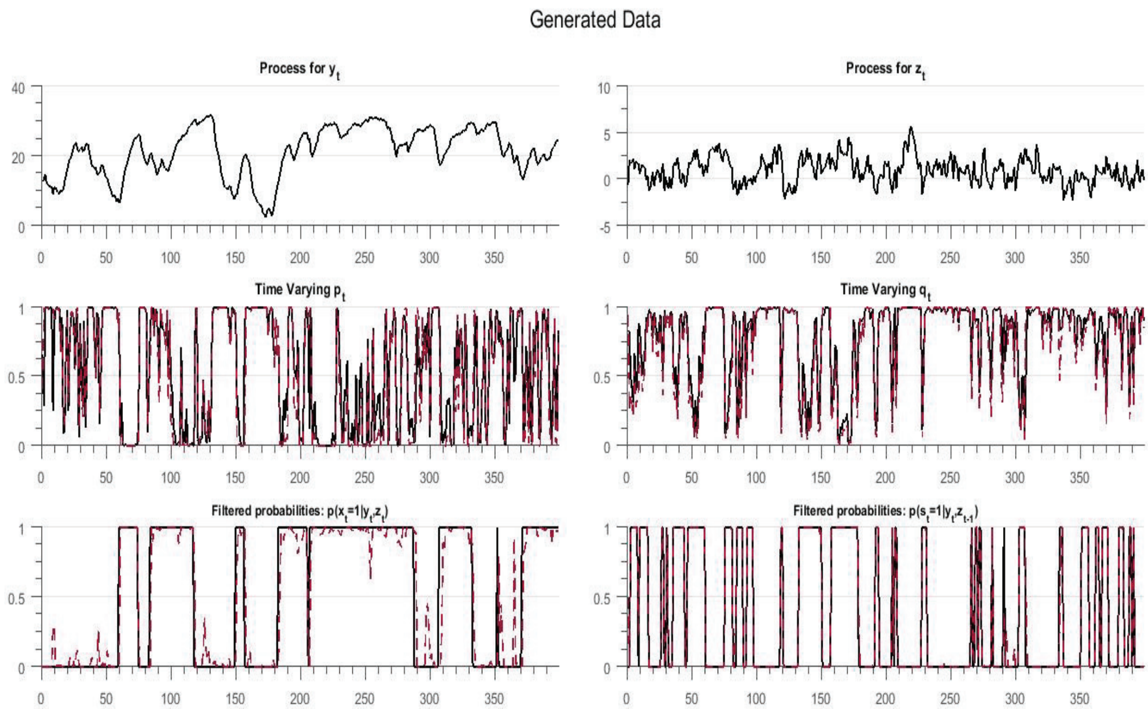

Figure 2: Artificial Data and Estimated Probabilities from Model M-2

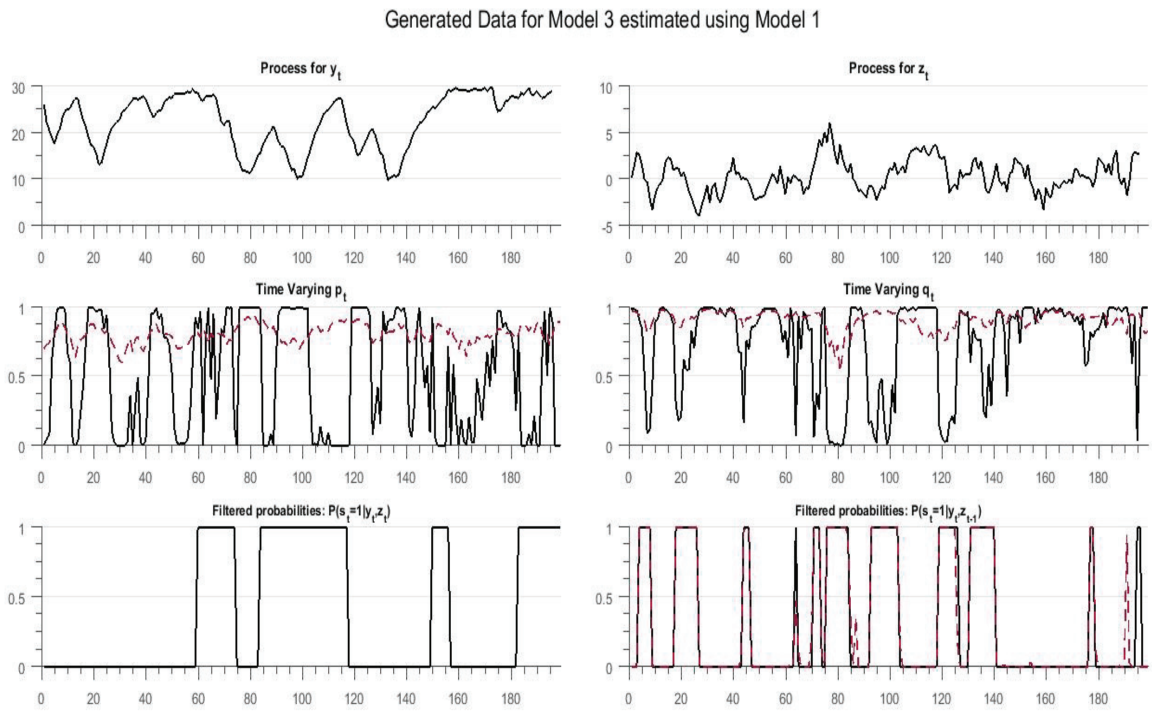

Figure 3: Artificial Data and Estimated Probabilities from Model M-1

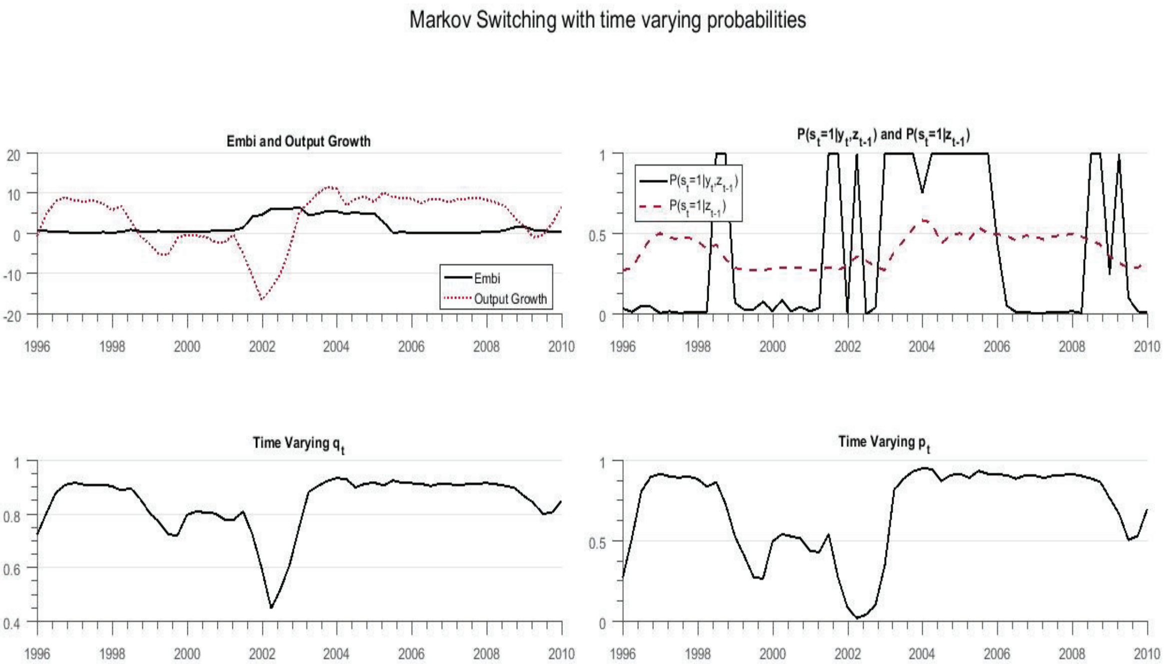

Figure 4: Empirical Results from Model 1

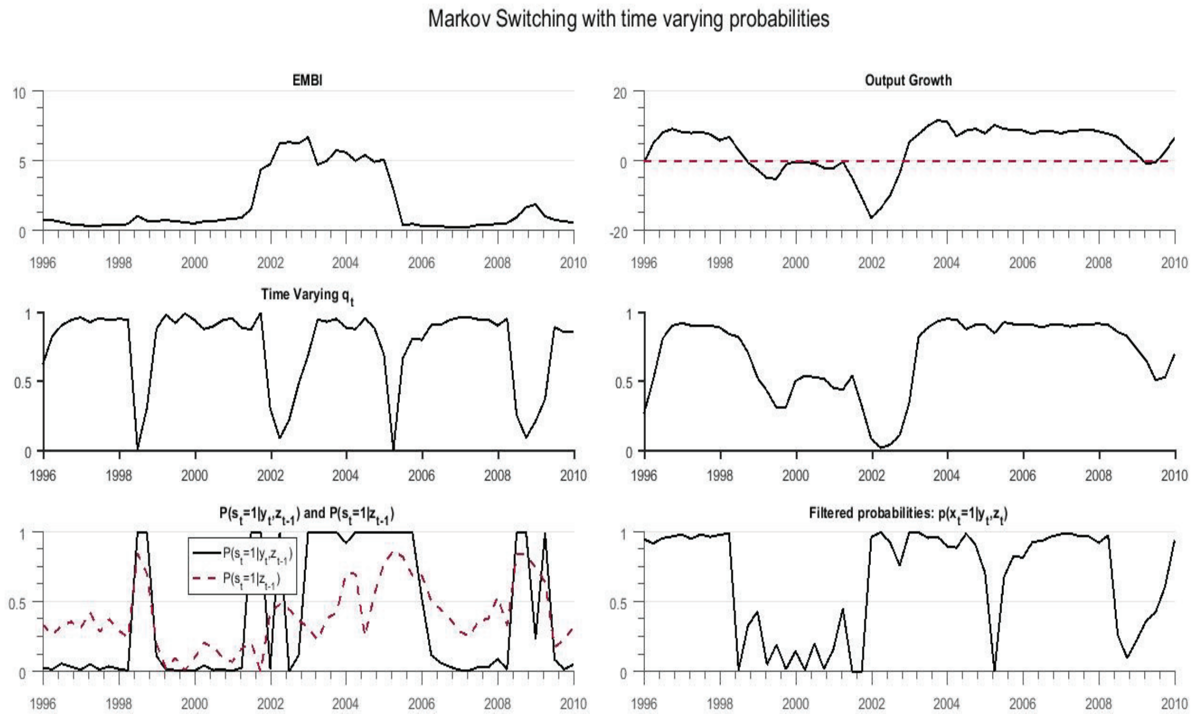

Figure 5: Empirical Results from Model 2