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# Learning about the Prospects for Mobility: Economic and Political Dynamics following Fundamental Policy Reform<sup>☆</sup>

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## Abstract

The political left turn in Latin America, which lagged its transition to liberalized market economies by a decade or more, challenges conventional economic explanations of voting behavior. This paper generalizes the forward-looking voter model to a broad range of dynamic, possibly non-concave income processes. Under full information, the model implies support for redistributive policies materializes rapidly if few prospects of upward mobility are present. In contrast, modeling voters' evolving beliefs about an unknown income dynamic process shows a slow and polarizing shift toward redistributive preferences. Simulation using fitted income dynamics suggests that this imperfect information perspective accounts for Latin America's right-to-left political shift, and offers additional insights about political dynamics in the face of economic polarization.

*Keywords:* Income dynamics, redistributive politics, polarization, Bayesian learning, Latin America.

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## 1. Introduction

Most Latin American countries had transitioned to market economies by the early 1990's. The largely center-right political leadership that instituted these transitions continued to win national elections and persisted in into the early

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2000's. After that, electoral politics turned sharply left. Presidential elections saw left-leaning candidates defeat more conservative opponents in Brazil, Bolivia, Chile, Ecuador, El Salvador, Honduras, Nicaragua, Paraguay, Peru and Venezuela.<sup>1</sup> Not only did these elections usher in a political shift, but in many instances they were tightly contested between candidates offering fundamentally different economic visions. The goal of this paper is to provide a theoretical framework to help us understand the economic forces that underlie this political dynamic.

We start from the perspective that by the early 1990s, Latin America and other, what are now called, transitional economies, experienced the exhaustion of (left) political alternatives. In Peru, for example, the 1980s ended with hyperinflation and a general economic malaise that had been overseen by a left-of-center government. Our goal in this paper is not to explain the rightward shift from the interventionist, putatively left-wing policies of the 1980s, but instead to explain the subsequent return to left-leaning governments following a decade or more of conservative, economically liberal rule.

The influential body of political economy literature that focuses on economic inequality as a force that determines both political institutions and voting patterns would seem to offer a window into this right to left shift Acemoglu and Robinson (2006); Boix (2003). However, the fact that inequality measures tend to be remarkably stable over time makes it unlikely that inequality can explain Latin America's right-left voting dynamics. Robert Kaufman Kaufman (2009) confirms the inconvenient empirical fact that contemporaneous measures of economic inequality do a very poor job of explaining political institutions and voting patterns in Latin America.<sup>2</sup>

Although we could abandon the search for economic explanations of contemporary voting patterns, we instead take our cue from Benabou and Ok (2001) and Moene and Wallerstein (2001) who model voters as forward-looking agents who look beyond current income inequality and focus on how policies will influence their future economic prospects. From this starting point, we offer the following contributions:

- We analyze forward-looking political preferences under a variety of income dynamics, including dynamics that offer prospects for upward mobility ('POUM' as in Benabou and Ok), as well as empirically based dynamics which are neither concave or convex that offer no prospects of upward mobility ('No-POUM').

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<sup>1</sup>While the contemporary Latin American left cannot be defined by a single, shared economic model, this new left does share an impulse and desire to shift resources and opportunity to those at the bottom of the income distribution. For instance, Greene and Baker (2011) construct vote revealed leftism (VRL) from ideological ratings of presidents and parliamentary parties in Latin America from 1996-2008, showing that the left has an economic policy mandate to halt or partially reverse neoliberal economic policies.

<sup>2</sup>Fields (2007) makes this point even more strongly by showing how inequality can increase during the early stages of a period of upward mobility that would dampen political preferences for redistribution.

- We also consider political dynamics when voters lack full information and must live and learn about the income dynamics that characterize their economy. This is particularly relevant to transition countries that have fundamentally altered their economic model, as happened across Latin America circa 1990.
- We show that not only does the incorporation of learning provide a richer suite of possible political dynamics, it also reveals that perceptions of the dead weight loss associated with redistribution can, in surprising ways, further fuel political instability.
- To draw out the implications of our model, we estimate income distribution dynamics for two Latin American countries, Chile and Peru, and show that the learning, forward-looking voter model is broadly consistent with the recent political histories of both countries.

In their seminal paper, Benabou and Ok show that concave income distribution dynamics that offer the prospect of upward mobility can account for surprising conservatism by voters below the mean income who would benefit in the short run from redistributive policies.<sup>3</sup> While this POUM model has little to say about the right-left political dynamics observed in contemporary Latin America, we show here that the income transition functions suggested by poverty trap theory, which offer limited or no prospects of upward mobility can result in a surprisingly and increasingly pro-redistribution electorate.<sup>4</sup> Specifically, we show that forward-looking political preferences are determined by the smoothed envelopes drawn around income transition functions, where the transitions themselves need not be concave or convex. This finding generalizes the connection between redistribution and income beyond the usual concepts in the literature.

In an effort to corroborate this theoretical intuition, we calibrate income dynamics for Latin American countries. These reveal for some countries the sort of No-POUM dynamics that would be expected to generate an increasingly pro-redistribution electorate. Applying these dynamics to a full information, forward-looking voting model indicates that the demand for redistribution should have been stronger and should have occurred well in advance of the suite of Latin American presidential elections that moved governments to the left in the early 2000's. This result presents a puzzle that questions fundamental assumptions about how economic voters perceive and react to their material prospects.

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<sup>3</sup>Complementary endogenous explanations for anti-redistributive positions include disincentives for labor supply Meltzer and Richards (1981), asset formation Persson and Tabellini (1994), inefficient levels of public goods Alesina and Rodrik (1994), multidimensional policy spaces in which non-economic preferences conflict with pocketbook voting Roemer (2001). To highlight the roles of income dynamics and learning, we ignore the incentive effects of taxation (see Piketty (1995)), but do account for the role of dead weight loss.

<sup>4</sup>Tucker (2006) shows that voting in the post-Soviet bloc reflects economic experiences: areas with poor outcomes support "Old Regime" parties while good outcomes provide support for liberal "New Regime" parties.

We argue it is the assumption that voters have full information about their economy’s income distribution dynamics that is most problematic, especially in transition economies where the electorates have had little prior experience with liberalized market economies (e.g. Przeworski, 1991).<sup>5</sup> In such circumstances, voters have little choice but to fall back on priors about how such an economy might work.<sup>6</sup> Edwards (1995), for example, largely credits the origins of the switch to liberal economic policies within Latin America to the failure of all other alternatives, although he notes that multilateral institutions influenced the “convergence of doctrinal views” through research, analysis, lending practices and conditionalities.

In Latin America, the shift to the liberal economic model was put forward on the grounds that it would boost incomes and well being for all, including the lower half of the income distribution.<sup>7</sup> Assuming that voters begin with this “POUM prior,” we go on to model voters as Bayesian learners who experientially update their expectations based on their own stochastic income experience. Leveraging the POUM and No-POUM distinction, we characterize “right” vs “left” Bayesian beliefs about income dynamics. We show that this model of forward-looking, Bayesian voters offers an empirically tenable explanation of the recent right to left political evolution in Latin America. We further show—under more restrictive assumptions—that taking into consideration the dead weight losses of redistribution amplifies this result and implies even stronger political volatility. While increased dead weight loss reduces support for redistribution for both right and left voters, the effect is proportionately stronger for right voters. This asymmetry then amplifies political volatility in which learning is moving some fraction of the electorate left.

The general tenor of this explanation is corroborated by public opinion survey data from Peru, a country whose politics mirror the right to left electoral pattern that motivates this paper. The Latinobarómetro Corporation began annual surveys of the Peruvian electorate in 1995, the year in which liberal candidate Alberto Fujimori was elected with a strong two-thirds majority.<sup>8</sup> The 1995 data reveal that 55% of Peruvian expressed prospects of upward mobility, expecting their personal economic circumstances to improve in the next 12 months.<sup>9</sup> At the same time, only 11% of survey respondents positioned them-

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<sup>5</sup>Fernandez and Rodrik (1991) point out the importance of uncertainty in policy reform, although they explain why policies might increase in popularity after implementation, rather than decrease as in Latin America. Van Wijnbergen and Willems (2012) extend their approach with learning dynamics to explain why policies may decrease in popularity.

<sup>6</sup>Roemer (1994) models this process in a Downsian framework. In contrast, this paper conceptualizes beliefs about empirically based income dynamics as an ‘ideology space’ which voters learn about through personal experience.

<sup>7</sup>See Williamson (1990) for a classic statement of the so-called Washington Consensus about the desirability of liberal economic policies for Latin America.

<sup>8</sup>Fujimori was first elected in 1990 as a progressive alternative to an economically more liberal candidate. Once elected, Fujimori moved sharply right, dissolved his own constitutional presidency and installed himself as *de facto* president through his “*auto-golpe*” in 1992, and was then re-elected in 1995 as the liberal candidate.

<sup>9</sup>The Latinobarómetro specifically questions respondents about perceived past and ex-

selves as left of center on a political scale, while another 23% said that they were either uncertain of their political position, or refused to reveal it. In the ensuing 5 years, the fraction of the electorate expressing positive prospects of upward mobility steadily declined from 55%, reaching a low of 27% in 1998, before rebounding to 36% in the months before the 2000 presidential election. Over this same time period, the fraction of the electorate who self-identified as left of center doubled to 22%, a trend that continued another 5 years when the self-identified left reached 33% of the electorate. Fujimori won the 2000 election, which was widely criticized as fraudulent, and then exiled himself from Peru later that year, to be replaced in 2001 by the more progressive candidate he had ostensibly defeated in the 2000 election.

While Peru fits the pattern whereby diminished expectations of upward mobility accompany a leftward shift in the electorate, observers of transition economy politics have noted similar patterns elsewhere. Weyland (2002), for example, notes that support for the liberal economic policies introduced in Latin America the late 1980s and early 1990s waned over time, especially as individuals began to learn and reassess their future prospects.<sup>10</sup> Similarly, Graham and Pettinato's (2002) analysis of Peru and Russia identifies a numerically significant group of "frustrated achievers," who benefited initially from liberal reforms, but came to see little prospect for further advance. These authors go on to note that this frustrated group showed waning support for market-oriented policies and speculate that their political behavior will likely change accordingly.<sup>11</sup>

Finally, evidence for the role of upward mobility in voting behavior has been mixed, and the reach of the income based approach can be extended by modeling more general income dynamics. Fong (2001) finds that variables reflecting personal benefit from redistribution are insignificant in predicting redistributive preferences in the US. On the other hand, Checchi and Filippin (2004) find experimental support that the POUM reduces chosen taxation rates and that

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pected future changes in their "economic situation." Whether individuals report changes based on their absolute or relative living standards is not clear from the wording of this question. Graham and Pettinato (2002) find that with the exception of the poorest cohort of respondents, individuals' perceptions of changes in their economic circumstances are influenced by their relative position in the income distribution, not absolute changes in their living standards.

<sup>10</sup>While documenting the same shift in policy and electoral preferences that motivate our work, Weyland (2002) offers a rather different explanation rooted in cumulative prospect theory. Whereas we emphasize learning about income distribution dynamics under the new economic regime, Weyland suggests that voters' subjective preferences changed as they become more risk averse—and safety net-loving—after an initial period in which they were willing to take the risk of neo-liberal shock therapy.

<sup>11</sup>Another possible explanation is that voters were disappointed in the aggregate growth performance achieved under the new regime and hence shifted to the other party. It is also of course possible that people are fooled, or fool themselves, about the nature of income dynamics and vote against their true economic interests. Survey research which assesses voter's subjective expectations about prospects has found "POUM captures hopes and expectations as well as realistic socioeconomic assessments" Graham and Pettinato (2002). Additional possibilities are considered by Putterman (1996). Herrera (2005) carefully studies how economic information was mediated by larger sets of social relations in post-USSR Russia.

longer time horizons tend to decrease chosen rates under POUM. Beckman and Zheng (2007) find tentative support for the POUM hypothesis using undergraduate surveys. At the international level, Wong (2004) examines the GSS and World Values Survey for redistributive preferences and finds the expected signs across incomes, but no evidence of the “tipping behavior” implied by median voter or POUM models.

There may of course be alternative explanations of the leftward shift of electorates and policy in transition economies. Callander (2011), for example, puts forward a model of pragmatic policy experimentation in which a single leader, with unchanging preferences, may alter a policy parameter as she learns about a complex mapping from the parameter to a univariate outcome measure (*e.g.*, environmental quality). While one can indeed observe such policy tinkering within regimes in transition economies, the policy experimentation approach does not accord well with the regime changes and the vitriolic electoral processes that accompanied the left-right shift in Latin America circa 2000.<sup>12</sup> Broadly similar to Callander (2011), Buera et al. (2011) examine policymakers as passive statistical learners who learn from policy mutations in neighboring countries. In contrast, our model provides a microfoundation for the electoral discontent that is a first or structural mover of fundamental policy change.

The remainder of this paper is organized as follows. Section 2 develops a basic framework for individual and aggregate income dynamics in the presence of transient shocks, and models political support for redistributive policies by both myopic and forward-looking voters who enjoy full information about the income dynamic process. Section 3 then introduces both concave (POUM) and poverty trap (No-POUM) dynamics, and derives results on the political preferences of forward-looking voters who may be fully informed or instead must learn about extant income dynamics through experience. The analysis of Section 3 is applied to Latin American income dynamics in Section 4. Section 5 shows this model of forward-looking Bayesian voters who confront a No-POUM world can give rise to the political polarization and the sudden political shifts that have been observed in twenty first century Latin America. Section 6 concludes.

## 2. Forward-looking Voters and the Demand for Redistribution

This section lays out machinery needed to discuss changing patterns in voting when the electorate can choose among income redistribution schemes. We assume a continuum of voters whose incomes evolve over time and fluctuate with idiosyncratic shocks each period. Voters care only about maximizing the present discounted value of income from all sources, whether public or private, and are thus “pocketbook voters.” We define the demand for redistribution as

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<sup>12</sup>In contrast to the Callander (2011) focus on policy complexity, we focus on voters who care about their individual incomes and are trying to figure out the *laissez faire* counterfactual for themselves (“will I be better off in 4 years?” to the quote the challenge that US president Reagan put to voters).

the fraction of voters who rationally prefer redistribution from a pocketbook perspective.

In order to evaluate a particular redistributive policy, each voter considers three things: their individual income path, the aggregate income path of the economy and the longevity of the policy. Changes in the economy over time can thereby induce changes in voting patterns, and support for a given policy is dependent on expected economic conditions. To help unpack these relationships, this section defines income transitions, redistributive schemes and forward-looking demand for redistribution. After developing baseline analytical results, this framework will be extended in the next section to consider the role of beliefs when income transitions are uncertain.

### 2.1. Income Transitions

To model the income transition process, we follow the lead of the rich theoretical literature on asset accumulation and income distribution dynamics and assume that at any time period  $t$ , individual  $i$ 's income is anchored to a structural component,  $y_{it}^s$ , that depends on accumulated human capital and other assets. Realized income is the product of this structural component with a transitory income shock:

$$y_{it} = y_{it}^s \varepsilon_{it},$$

where we assume that  $\varepsilon_{it}$  is distributed iid with  $E[\varepsilon_{it}] = 1$ .

Income transitions are driven by the evolution of the structural component of income, which we assume evolves according to the transition function<sup>13</sup>  $f$  such that structural component of income in period  $t + 1$  can be written as:

$$y_{it+1}^s = f(y_{it}^s),$$

implying that:

$$y_{it+1} = f(y_{it}^s) \varepsilon_{it+1}.$$

This specification is consistent with Deaton's (1991) canonical model of optimal savings and accumulation. The Deaton model is essentially a wage labor model, with expected earnings set by a pre-determined level of human capital. The only accumulable asset in this model is a buffer stock, which impatient, credit-constrained agents accumulate in order to smooth consumption in the face of negative income shocks. In the Deaton model, income shocks would not change future expected income, as in the transition specification above. Empirically, this notion of a structural anchor to the income distribution process is supported by micro studies of both middle and high income economies, which find that within one to two years, households return to their pre-shock level of earnings. For example, see the study on poverty spells and income shocks in the US by Gottschalk (1997) and Card and Blank (2008). Lokshin and Ravallion (2004)

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<sup>13</sup>Formally, by income transition we refer to any positive, strictly increasing and continuous function defined on a bounded interval of the form  $[0, B]$ .



similarly find that income shocks dissipate quickly in Hungary and Russia, as do Antman and McKenzie (2007) in the case of Mexico.<sup>14</sup>

Given this specification of the income transition process, we consider now an economy in which individuals are indexed by  $i$  and each person enjoys an initial non-stochastic income,  $y_{i0} = y_{i0}^s$ . The initial income distribution  $F_0$  is assumed to be bounded and absolutely continuous with positive support. These assumptions imply that the expected income path for any individual is deterministic, since

$$E_{\epsilon_{it}}[y_{it}] = f(E_{\epsilon_{it-1}}[y_{it-1}]) = f(f(E_{\epsilon_{it-2}}[y_{it-2}])) = f^{(t)}(E_{\epsilon_{i0}}[y_{i0}]) = f^{(t)}(y_{i0}),$$

while realized income can be written as  $y_{it} = f^{(t)}(y_{i0}) \cdot \epsilon_{it}$ .<sup>15</sup>

When making decisions about the future, we assume individuals care only about present discounted income, discounted at rate  $\delta$  each period. We can therefore write each individual's discounted income stream over  $T$  periods as:

$$\sum_{t=0}^T \delta^t y_{it} = \sum_{t=0}^T \delta^t f^{(t)}(y_{i0}) \cdot \epsilon_{it}. \quad (\text{Discounted Income Stream}) \quad (1)$$

Since  $E[\epsilon_{it}] = 1$ , expected present discounted income can be seen from (1) to be  $\sum_{t=0}^T \delta^t f^{(t)}(y_{i0})$ . We now turn to policies which might redistribute this income.

## 2.2. Myopic Demand for Redistribution

Consider the political preferences of myopic, pocketbook voters whose incomes evolve according to a known income transition function  $f$  as above. Pocketbook voters choose policies which maximize their income, and for simplicity we assume voters are risk neutral. Following the convention in much of the political economy literature (e.g. Persson and Tabellini (2000), Roemer (2001)), we define redistribution schemes composed of a flat tax  $\tau$  and a lump sum transfer to all voters. Thus if a tax  $\tau$  is enacted in period  $t$ , each voter  $i$  receives income:

$$(1 - \tau) \cdot y_{it} + \tau \cdot (1 - D) \mu_t \quad (2)$$

<sup>14</sup>In lower income economies, where substantial numbers of households do not earn wages but depend on agricultural and other forms of self-employment where earnings are a function of productive assets, it is not immediately obvious that the income transition process is unaffected by shocks. For examples, in models where higher earning entrepreneurial forms of self-employment require a minimum level of productive assets (e.g., Banerjee and Newman (1993) and Buera (2009)), too large a shock could push households below that minimum threshold and threaten to reduce their long-term expected income level. However, it is precisely in this class of models where households will optimally respond to shocks by cutting consumption and holding on to assets in order to avoid damaging their long-term income process (i.e. Sen and Dreze (1989), Zimmerman and Carter (2003) and Carter and Lybbert (2012)). It is beyond the scope of this paper to thoroughly investigate the microfoundations that underlie the income transition function, but our assumptions do accord with the insights from a number of key dynamic models of optimal savings and accumulation.

<sup>15</sup>While the individual's history of realized incomes does not matter for expected future income, this history will matter when the individual does not know the true nature of the transition process and must deduce it from his or her own lived experience. The next section considers ramifications.

Here  $\mu_t = E_0 [f^{(t)}(y)]$  denotes the mean income of the population at time  $t$ ,<sup>16</sup> and  $D \in [0, 1]$  denotes any dead weight loss under the redistributive scheme.<sup>17</sup> A myopic voter's most preferred policy  $\tau^*$  must maximize expected income. Since at any period  $t' < t$  we have

$$E_{t'} [(1 - \tau) \cdot y_{it} + \tau \cdot (1 - D) \mu_t] = (1 - \tau) \cdot f^{(t)}(y_{i0}) + \tau \cdot (1 - D) \mu_t,$$

either  $\tau^* = 1$  (complete redistribution) or  $\tau^* = 0$  (laissez-faire).

Now consider a majoritarian vote taken between  $\tau = 1$  and  $\tau = 0$  at the beginning of period  $t$  before idiosyncratic shocks are realized. A myopic voter  $i$  prefers  $\tau = 1$  to  $\tau = 0$  exactly when  $E[y_{it}] = f^{(t)}(y_{i0}) \leq (1 - D)\mu_t$ , which means they expect to be below average income, less any dead weight loss. Since  $f$  is strictly increasing,  $f$  can be inverted, and all voters with initial incomes  $y_{i0} \leq f^{(-t)}((1 - D)\mu_t)$  prefer  $\tau = 1$  to  $\tau = 0$ . The fraction of such voters in the population is determined by the initial distribution of income  $F_0$  to arrive at

$$\Pr(\text{Voter prefers } \tau = 1) = F_0 \left( f^{(-t)}((1 - D)\mu_t) \right) \quad (\text{Myopic Demand for Redistribution}).$$

### 2.3. Forward-looking Demand for Redistribution

Here we follow Benabou and Ok's framework of forward-looking voters who consider redistributive policies that last from period 0 through period  $T$ . Over this time frame, define a voter's discounted income stream under laissez-faire ( $\tau = 0$ ) as  $g^T(y_{i0})$ . From Equation (1),  $g^T(y_{i0}) = \sum_{t=0}^T \delta^t f^{(t)}(y_{i0})$  and the average of all voters' discounted income streams is therefore  $\mu^T \equiv \sum_{t=0}^T \delta^t \mu_t$ . Complete redistribution ( $\tau = 1$ ) over this period would pay out  $\mu^T$ , less any dead weight loss, giving a discounted income of  $(1 - D)\mu^T$ . Consequently, a voter prefers  $\tau = 1$  to  $\tau = 0$  from periods 0 through  $T$  if and only if  $g^T(y_{i0}) \leq (1 - D)\mu^T$ . Akin to the myopic case, the share of voters demanding redistribution is

$$\Pr(\text{Voter prefers } \tau = 1) = F_0 \left( [g^T]^{-1}((1 - D)\mu^T) \right) \quad (\text{Forward Demand for Redistribution}).$$

This equation shows that the fraction of the population who wants redistribution takes into account discounting and the evolution of income during the policy.  $[g^T]^{-1}((1 - D)\mu^T)$  is the forward-looking generalization of the term  $f^{(-t)}((1 - D)\mu_t)$  that determines the demand for redistribution in the myopic voter case. Note that a voter who looks forward only one period (or who considers a policy that will last only one year) has the same preferences as a myopic voter. The next section develops a method to explore voter dynamics under any

<sup>16</sup>Hereafter,  $E_0[\cdot]$  denotes the expectation at time 0 over initial incomes distributed  $F_0$  and all  $\{\epsilon_{it}\}_{i,t>0}$ .

<sup>17</sup>Under perfect information, dead weight loss serves no dynamic role, but will allow us to quantify the appropriate level of loss that would provide majoritarian support for laissez faire in highly unequal economics. Under incomplete information, dead weight loss has surprising implications for volatility, as we discuss in the last section.

income transition function, a family that is broad enough to encapsulate the processes implied by theories of both convergent and divergent income distribution dynamics.

### 3. Political Dynamics under Full and Imperfect Information

Initially assuming that voters have full information on the structure of income dynamics, this section first recaps the political implications of concave income dynamics, which exhibit upward mobility and have been studied in detail by Benabou and Ok (2001). We then consider the political implications of more complex income dynamics that do not exhibit such convergence (e.g. Banerjee and Newman (1994)), and fail to be concave or convex. Such NoPOUM dynamics may fail to offer upward prospects for segments of the population.

In reality, full information is unlikely as income dynamics and the prospects for mobility are complex and hard to understand, especially in economies which have fundamentally altered their economic model.<sup>18</sup> Later in this section, we thus relax the full information assumption and consider the behavior of voters who must learn about the true income dynamics from their own experience, and cast votes over future policies based on this learning in combination with ideological priors.<sup>19</sup>

#### 3.1. Political Dynamics under Prospects of Upward Mobility

The Solow model of neoclassical economic growth relies on an assumption of diminishing capital returns and implies that poorer nations will tend to catch up over time, or converge, with the incomes of richer nations.<sup>20</sup> When transported to the individual or microeconomic level, the Solow assumptions imply a process of convergence among the population of a single country.

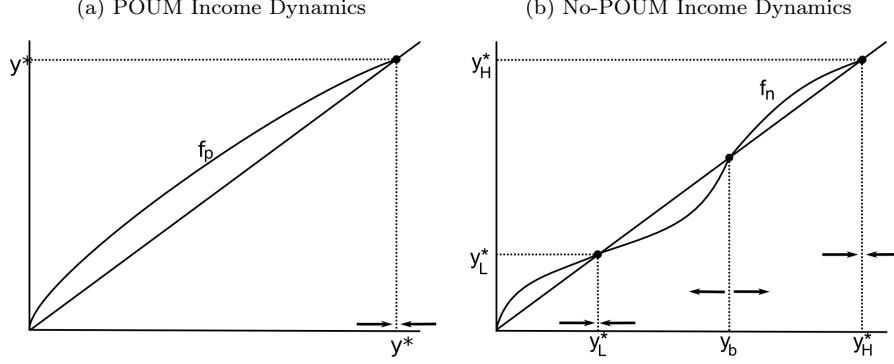
Figure 1(a) illustrates a typical income dynamic implied by accumulation under diminishing returns. Note that this concave transition process, maps incomes in period  $t$  into incomes in period  $t + 1$ , implies a unique long term or steady state income level,  $y^*$ , at the point where  $f_p(y)$  crosses the 45-degree line. Under this transition process, individuals who begin with incomes below the steady state level will converge towards it, while those who begin above the steady state level will drop back towards it. Note that this sort of concave

<sup>18</sup>This approach to modeling ideology as an idiosyncratic evolving process echoes Bates et al. (1998) as a means to complement cultural and ideological political theories with rational choice.

<sup>19</sup>This specification naturally incorporates the possibility that repression or fear constricted the political space, leading people to vote differently. In this case, the ideological space can be modeled as constrained to an ideological spectrum which expands with “political thawing” and faith in democratic institutions over time. This gradual expansion of publicly admissible views might also help explain large shifts.

<sup>20</sup>For an early review of both the theoretical and empirical controversies, see Romer (1994). A more recent review with a theoretical emphasis is Azariadis and Stachurski (2005). While the concavity of income transition processes are not necessary for convergence, they are ubiquitous in the theoretical and empirical literature.

Figure 1: POUM and No-POUM Income Transitions



income process offers prospects of upward mobility (POUM) to voters whose initial income levels are less than the steady state income level.

As shown in Section 2, the fraction of myopic voters who demand redistribution at time  $t$  is  $F_0(f^{(-t)}((1-D)\mu_t))$ . Whether this increases or decreases over time depends on  $f^{(-t)}((1-D)\mu_t)$ . In a POUM world, the global concavity of  $f$  implies (through Jensen's inequality) the demand for redistribution is always decreasing over time. Similarly, if voters are forward-looking, the fraction of the population that wants redistribution monotonically decreases as the duration of a policy increases. Therefore in a POUM world with forward-looking voters, the demand for redistribution decreases with time in two senses: as evaluations each single period and as policy longevity increases. This is the type of behavior that Benabou and Ok (2001) deduce and discuss, and these two aspects of POUM redistributive dynamics can be summarized as:

**PROPOSITION (POUM DYNAMICS).** Suppose  $f$  is concave. Then:

1. The demand for a single period of redistribution decreases over time.
2. The demand for redistribution over a  $T$  period horizon decreases in  $T$ .

The obvious twin result which follows from Benabou and Ok is that if income dynamics are convex, the demand for redistribution increases both over time and over longer policy periods. However, empirical income dynamics are seldom so ideal as to satisfy pure concavity or convexity.

### 3.2. Political Dynamics under (No) Prospects of Upward Mobility

In contrast to Figure 1(a), individuals need not face uniformly decreasing returns in asset accumulation. The increasingly well developed theory of poverty traps suggests a number of mechanisms that can trap households at low living standards (see the reviews in Azariadis and Stachurski (2004), Carter and Barrett (2006) and Barrett and Carter (2013)). Central to all of these theories

of poverty traps is exclusion from financial markets. Put differently, if households have access to loan markets and insurance instruments, then even when confronted by locally increasing returns to scale and risk, they can successfully engineer a strategy to obtain the assets needed to jump to a high level equilibrium. But absent access to those financial markets, households below a critical initial asset level may remain stuck in a low level, poverty trap equilibrium.

The result of such poverty trap models is Figure 1(b) which illustrates income transition dynamics with multiple steady states.<sup>21</sup> The non-concave income transition function,  $f_n(y)$ , has multiple crossings of the 45-degree line and admits multiple equilibria:  $y_H^*$  is the high income steady state;  $y_L^*$  is the low income steady state. Bifurcation occurs at the unstable equilibrium income level,  $y_b$ . Households with incomes in excess of  $y_b$  will tend toward the high level equilibrium while those that begin below this critical threshold will head towards the low level, poverty trap equilibrium,  $y_L^*$ . This implies no prospect of upward mobility (No-POUM) for voters below  $y_b$ . In contrast to an economy with a concave income transition function, economic polarization will occur and inequality can deepen when income transitions are governed by a non-concave function like  $f_n(y)$ .<sup>22</sup>

The remainder of this section considers any income transition function, allowing for both  $f_p$  and  $f_n$  types of income transitions and then derives a general set of results with political implications. We show that relaxing the assumption of concavity can generate rich patterns in the demand for redistribution. We then provide a theorem showing how these new income transitions create both increases and decreases in the demand for redistribution, even when the transition function is neither globally concave nor convex.

### 3.3. Demand for Redistribution: What's in the Envelope?

While voter dynamics under POUM are relatively straightforward, non-convexities under general income transitions lead to more complex political dynamics. To better describe this complex process, we connect the changing demand for redistribution to the upper and lower envelopes of an income dynamic. We first define the upper envelope of  $f$ ,  $\bar{f}$ , as the smallest concave function everywhere above  $f$ . The lower envelope  $\underline{f}$  is defined as the largest convex function everywhere below  $f$ . Both types of envelopes are illustrated in Figure 2 and defined in Equation (3).<sup>23</sup>

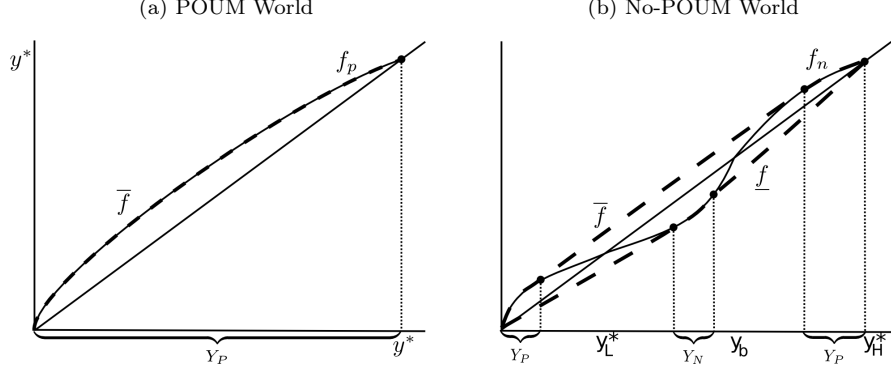
$$\bar{f}(x) \equiv \inf\{h(x) : h \text{ is concave, } h \geq f\}, \quad \underline{f}(x) \equiv \sup\{h(x) : h \text{ is convex, } f \geq h\}. \quad (3)$$

<sup>21</sup>For empirical examples see Lybbert et al. (2004) and Adato et al. (2006). Banerjee and Newman (2000) construct a model of “dynamic institutional change” which implies non-concave income dynamics and exhibits path dependence.

<sup>22</sup>Strictly speaking, this non-concave income transition function implies increasing polarization, not necessarily increasing inequality, as Esteban and Ray (1994) discuss.

<sup>23</sup>This is equivalent to finding the envelope created by tracing all lines which are above, but do not cross  $f$ . In practice, there are several efficient algorithms to construct such envelopes numerically, e.g. Jarvis (1973).

Figure 2: Upper and Lower Envelopes



Clearly for each  $y$  we have  $\bar{f}(y) \geq f(y) \geq \underline{f}(y)$  and necessarily  $\bar{f}$  is concave and  $\underline{f}$  is convex. Two special cases stand out. When  $f$  is concave,  $\bar{f}$  and  $f$  coincide. When  $f$  is convex,  $\underline{f}$  and  $f$  coincide. Therefore in a POUM world,  $f = \bar{f}$ . We define the sets of incomes where  $f$  and  $\bar{f}$  exactly coincide as  $Y_P$  (as this is the domain of upward mobility). Similarly define the domain of downward mobility,  $Y_N$ , as incomes where  $f$  and  $\underline{f}$  coincide. The relationship of  $Y_P$  and  $Y_N$  to the path of redistributive preferences is Proposition 1:

**Proposition 1.** If  $\mu_t \in Y_P$  then the demand for redistribution decreases in period  $t$  relative to period  $t - 1$ . Conversely, if  $\mu_t \in Y_N$  then the demand for redistribution increases. We consider  $\mu_t \in Y_P$  as the other case is similar. We want to show that  $f^{(-t)}(\mu_t) \geq f^{(-(t+1))}(\mu_{t+1})$ . This holds iff  $\mu_{t+1} \leq f(\mu_t)$  and by assumption  $f(\mu_t) = \bar{f}(\mu_t)$ . Therefore we are done if we can show  $\mu_{t+1} = \int f^{(t+1)} dF_0 \leq \bar{f}(\mu_t)$ . Since  $f \leq \bar{f}$  we know that  $\int f^{(t+1)} dF_0 \leq \int \bar{f} \circ f^{(t)} dF_0$  so we need to show  $\int \bar{f} \circ f^{(t)} dF_0 \leq \bar{f}(\mu_t)$ . In fact, this last inequality holds by Jensen's inequality since by construction,  $\bar{f}$  is concave.

Proposition 1 says that if the mean income next period  $\mu_t$  lies in  $Y_P$ , the demand for redistribution *decreases*. Conversely, if  $\mu_t$  lies in  $Y_N$ , the demand for redistribution *increases*. In this sense, the upper and lower envelopes of  $f$  are natural definitions of Right and Left income transitions based on  $f$  (where Right means a transition function—or belief in a transition function—which implies decreasing demand for redistribution over time and Left means the opposite). This proposition also highlights the differences between POUM and No-POUM income dynamics. In a POUM world,  $f$  is concave and equals  $\bar{f}$  so all incomes (including  $\mu_t$ ) are in  $Y_P$ . Therefore the demand for redistribution is always decreasing (Figure 2a). In contrast, a No-POUM income dynamic has both  $Y_P$  and  $Y_N$  regions. Depending on where  $\mu_t$  lies the next period, the demand for redistribution can either increase or decrease (Figure 2b).

In a No-POUM world, the determination of whether the demand for re-

distribution is increasing or decreasing depends simultaneously on the current period, the expected income transition and the initial distribution of income. Although the demand for redistribution may be directly computed, in general it is hard to derive a particular path analytically due to its dependence on the range of possible income distributions. In Appendix AppendixB, we illustrate this point with a concrete example where for a large class of income dynamics, small changes in  $F_0$  cause qualitative changes in the demand for redistribution over time.

In a POUM world, the demand for redistribution decreases with policy length, as discussed above. However, in a No-POUM world, longer policy horizons may include relatively precipitous drops in present discounted income for some segments of the population. In such cases, policy longevity inspires increased demand for redistribution. The following proposition formalizes this argument, the key condition being that mean incomes lie in the region of downward mobility,  $Y_N$ , for successive periods.

**Proposition 2.** Suppose for an income transition  $f$ ,  $\mu_t \in Y_N$  for all periods  $t$ . Then the demand for redistribution increases in policy longevity. Conversely, if  $\mu_t \in Y_P$  for all periods  $t$  then demand for redistribution decreases.

PROOF. By definition,  $g^T = g^{T-1} + \delta^T f^{(T)}$ , so consider the intuitive result (see appendix):

**Lemma 1.** Suppose  $f$  and  $g$  are income transition functions. Then the myopic demand for redistribution implied by the income transition  $f + g$  is between that implied by  $f$  and  $g$ .

It is therefore sufficient to show that  $f^{(-T)}(\mu_T) \geq (g^{T-1})^{-1}(E[g^{T-1}])$ , or equivalently

$$g^{T-1} \circ f^{(-T)}(\mu_T) \geq E[g^{T-1}]. \quad (4)$$

Now consider that  $\mu_t \in Y_N$  for all  $t$  implies  $f^{-1}(\mu_t) \geq \mu_{t-1}$  so  $\mu_t \geq f(\mu_{t-1})$ . Recursing this equation  $k$  times shows

$$\mu_t \geq f^{(k)}(\mu_{t-k}) \quad \text{for all } t, k \geq 0. \quad (5)$$

Expanding  $g^{T-1} \circ f^{(-T)}(\mu_T)$  and substituting in Equation (5) shows

$$g^{T-1} \circ f^{(-T)}(\mu_T) = \sum_{t=0}^{T-1} \delta^t f^{(t-T)}(\mu_T) \geq \sum_{t=0}^{T-1} \delta^t f^{(t-T)} \circ f^{(T-t)}(\mu_{T-(T-t)}) = E[g^{T-1}].$$

Which is precisely Equation (4), so demand for redistribution increases.  $\square$

### 3.4. Generalization to stochastic income transitions

While these results are based on a deterministic income transition process, they generalize to the case in which the income transition function is stochastic

and drawn from a family of income transition functions  $\{f_\theta(y)\}_{\theta \in \Theta}$ , where an idiosyncratic shock,  $\theta_{it}$  determines the income transition  $f_{\theta_{it}}$  faced each period, akin to the setting of Proposition 4 of Benabou and Ok (2001).<sup>24</sup> The generalization depends on domains of upward mobility and downward mobility of the *transitions between* the expected income transition function for each period,  $\tilde{f}^{(t)}(y) \equiv E_{\Theta_t, \dots, \Theta_1} [f_{\theta_{it}} \circ \dots \circ f_{\theta_{i1}}(y)]$  (with  $\tilde{f}^{(0)}(y) \equiv y$ ). Define the transition between period  $t$  and period  $t+1$  as  $\eta^t(y) \equiv \tilde{f}^{(t+1)} \circ \tilde{f}^{(-t)}(y)$ , i.e. for an individual with initial income  $y$ , the change in expected incomes between period  $t$  and  $t+1$ . Label the upwardly mobile domains  $\tilde{Y}_P^t$  as where  $\eta^t$  coincides with its upper envelope and the downwardly mobile domains  $\tilde{Y}_N^t$  as where  $\eta^t$  coincides with its lower envelope. Then the demand for redistribution changes with policy longevity as in the following Proposition:

**Proposition 3.** Under stochastic income transitions, the demand for redistribution will increase in policy longevity when each mean income  $\mu_t$  lies in the domain of downward mobility  $\tilde{Y}_N^t$  for the transition between expected incomes,  $\eta^t$ . Similarly, the demand for redistribution decreases if each mean income lies in the domain of upward mobility  $\tilde{Y}_P^t$  of  $\eta^t$ .

PROOF. Akin to the case of income transitions that are deterministic in expectation above, we define multiperiod expected income streams as  $\tilde{g}^T(y) \equiv \sum_{t=0}^T \delta^t \tilde{f}^{(t)}(y)$ . By the Lemma in the last proof,  $\tilde{g}^{T+1}$  generates (weakly) higher demand for redistribution than  $\tilde{g}^T$  so long as

$$\tilde{f}^{-(T+1)} \left( E \left[ \tilde{f}^{(T+1)}(y) \right] \right) \geq \tilde{f}^{-(T)} \left( E \left[ \tilde{f}^{(T)}(y) \right] \right),$$

which is to say exactly when  $E \left[ \tilde{f}^{(T+1)}(y) \right] \geq \eta^T \left( E \left[ \tilde{f}^{(T)}(y) \right] \right)$ , which we will show holds. Defining the lower envelope of  $\eta^T$  as  $\bar{\eta}^T$ , since  $\mu^T = E \left[ \tilde{f}^{(T)}(y) \right]$ , by assumption  $\eta^T \left( E \left[ \tilde{f}^{(T)}(y) \right] \right) = \bar{\eta}^T \left( E \left[ \tilde{f}^{(T)}(y) \right] \right)$ .  $\bar{\eta}^T$  is convex by construction so by Jensen's inequality,

$$\bar{\eta}^T \left( E \left[ \tilde{f}^{(T)}(y) \right] \right) \leq E \left[ \bar{\eta}^T \circ \tilde{f}^{(T)}(y) \right] = E \left[ \tilde{f}^{(T+1)}(y) \right].$$

Putting these equations together,  $E \left[ \tilde{f}^{(T+1)}(y) \right] \geq \eta^T \left( E \left[ \tilde{f}^{(T)}(y) \right] \right)$ , so by the above condition, each increase in policy longevity increases the demand for redistribution. The other claim follows similarly.  $\square$

### 3.5. Income Dynamics under Imperfect Information

Our analysis so far has assumed that voters know the true income transition function and use this knowledge to construct their forward looking income

<sup>24</sup>We additionally assume standard measurability and boundedness assumptions, e.g.  $\sup_{\theta, t} E_y [f^{(t)}(y, \theta)] < \infty$ , so that the definitions of the discussion above are well defined in this setting.



forecasts and vote accordingly. We now relax this assumption by considering the role of voters' beliefs, which though informed by experience, may reflect ideological priors rather than incumbent economic conditions.

To keep this problem manageable, we assume voters face a known family of possible expected income transition functions  $\{f_\lambda(y)\}_{\lambda \in [0,1]}$  indexed by the parameter  $\lambda$ . The family of income transitions is assumed to be bracketed by two extreme specifications, one representing a right perspective or vision of how the economy operates  $f_R$  (at  $\lambda = 1$ ) and the other a left perspective  $f_L$  ( $\lambda = 0$ ). Specifically, the right perspective is that the laissez faire economy offers substantial prospects of upward mobility such that voters need not support redistributive policies. In contrast, the left perspective is that the economy intrinsically offers few prospects for upward mobility, requiring redistributive policies if significant fractions of the electorate are to get ahead economically. We refer to these specifications as "ideologies," using this word to denote a model or understanding of how the world works.

At any point in time  $t$ , the individual's understanding of the economy can be represented by a probability density  $\pi_{it}(\lambda)$  over possible values of  $\lambda$  while the true value of  $\lambda$ , labeled  $\lambda_0$ , is unknown to the individual. Note that this specification naturally describes someone with a left view of the world as placing a large probability weight on low or left values of  $\lambda$ , whereas a right view of the world would have probability weight near the right side of the spectrum or 1. We normalize the true value of state of the world  $\lambda_0$  to be  $1/2$ . This specification of how voters predict their future income under incomplete information will be incorporated into our model of forward-looking voters. However, we first consider how the critical new element, the voter's probability distribution  $\pi_{it}(\lambda)$ , is formed and evolves over time.

Each voter  $i$  begins with a prior distribution  $\pi_{i0}(\lambda)$  over possible values of  $\lambda$ . We also assume that voters keep track of their idiosyncratic income histories  $H_{it} \equiv \{y_{i0}, \dots, y_{it}\}$ . The history  $H_{it}$  is used to update beliefs each period to a posterior belief  $\pi_{it}(\lambda|H_{it})$  according to Bayes rule. In our context, we can think of  $\pi_{i0}(\lambda)$  as the initial ideological beliefs a voter has about the income transitions they face, while  $\pi_{it}(\lambda|H_{it})$  are the voter's new ideological beliefs after  $t$  periods of learning about the true income dynamic. In order to make this learning process concrete, we will analyze it assuming an explicit structure of the transient income shocks specified in Assumption 1:

**Assumption 1.** *The income dynamic each voter faces satisfies the following:*

1. *The shock  $\epsilon_{it}$  is distributed Uniform( $1 - \sigma, 1 + \sigma$ ) for some  $\sigma \in (0, 1)$ .<sup>25</sup>*
2. *Voters know the value of  $\sigma$ .*

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<sup>25</sup>The distributional form of  $\epsilon_{it}$  is not crucial, since the driving force for voting is convergence through learning to a tight posterior around the true income transition state. Other distributions give similar results, but induce updating rules that include weights from  $\epsilon_{it}$  for each voter history that increase computational dimensionality.

Recalling the true state of the world is  $\lambda_0 = 1/2$ , actual incomes are  $y_{it} = f_{1/2}^{(t)}(y_{i0}) \cdot \epsilon_{it}$ , so voters receive some random fraction,  $\epsilon_{it}$ , of their true expected income. Under Assumption 1, the magnitude of  $\sigma$  determines whether fluctuations around the expected value are large or small, with a larger  $\sigma$  obscuring the true income dynamic from voters.

Now consider how voters update their beliefs under Assumption 1. Since for any true state of the world  $\lambda$ ,  $\epsilon_{it} = y_{it}/f_{\lambda}^{(t)}(y_{i0})$  and each voter knows that  $|\epsilon_{it} - 1| \leq \sigma$ , voters know

$$\left| y_{it}/f_{\lambda}^{(t)}(y_{i0}) - 1 \right| \leq \sigma. \quad (6)$$

Equation (6) encapsulates the fact that a voter knows that realized income  $y_{it}$  must be within the fraction  $\sigma$  of expected income  $f_{\lambda}^{(t)}(y_{i0})$ . Therefore any state  $\lambda$  for which Equation (6) fails to hold cannot correspond to the true income dynamic. Eliminating these impossible states is exactly what Bayes rule dictates as the updating rule. Accordingly,  $\pi_{it}(\lambda|H_{it})$  is exactly  $\pi_{i0}(\lambda)$  restricted to all values of  $\lambda$  that satisfy (6) for the voter's history  $H_{it}$ , normalized to integrate to one.

Appendix AppendixC develops the mechanics of learning dynamics in more detail. Under imperfect information, voting behavior is determined by each voter's beliefs given their income history and the expected redistributive transfer for each state of the world  $\lambda$ . A myopic voter (looking forward only one period) prefers redistribution in period  $t$  when he *believes* expected transfers are positive:

$$\int_0^1 \underbrace{\left[ (1-D) \int f_{\lambda}^{(t+1)}(y) dF_0(y) - f_{\lambda}^{(t+1)}(y_{i0}) \right]}_{\text{Expected Transfer}|\lambda, y_{i0}} \cdot \underbrace{\left[ \pi_{it}(\lambda|H_{it}) \right]}_{\text{Beliefs}|\lambda, H_{it}} d\lambda \geq 0.$$

This expression naturally generalizes to the case when policies persist and voters look forward by more than one period. As this expression makes clear, evolving voter beliefs inserts another dynamic element into the determination of political preferences.

### 3.6. Dead Weight Loss and the Amplification of Political Volatility

In the model of Bayesian voters faced with imperfect information just discussed, dead weight loss ( $D$ ) can have a surprising role in generating potentially radical political swings. At first glance, one might think that dead weight loss would uniformly depress the demand for redistribution, but would have no effect on its volatility. But under modest additional assumptions, this section shows that increases in  $D$  can amplify political volatility. As this section will show, this systematic effect results from the asymmetric effect that an increase in  $D$  has on a Right partisan with a strong belief in  $f_R$  in comparison to a Left partisan with a strong belief in  $f_L$ . Increases in  $D$  attrit support for redistribution much faster for a Right partisan than for a Left partisan, creating a wider gulf to cross as voters learn. As individuals learn and their beliefs move away from

$f_R$ , their sensitivity to dead weight losses evaporates, powering a rapid shift in the population's support for redistributive policies. Dead weight loss can thus amplify our core result on left-right political volatility when voters are learning about a NoPOUM income dynamic.

In order to formalize this idea, let  $\tilde{y}_R = f_R^{-1}((1-D)E_0[f_R])$  denote the income level of a voter who is indifferent about a single period of redistribution under  $f_R$ , and similarly let  $\tilde{y}_L = f_L^{-1}((1-D)E_0[f_L])$  denote the income of a voter who is indifferent under  $f_L$ . By definition, the fraction of voters preferring redistribution under  $f_L$  is  $F_0(\tilde{y}_L)$  while under  $f_R$  the fraction is  $F_0(\tilde{y}_R)$ . The gap between these two fractions,  $F_0(\tilde{y}_L) - F_0(\tilde{y}_R)$ , is completely accounted for by the range of possible beliefs held by voters, and quantifies potential political volatility. Under plausible assumptions, we will show that  $\partial[F_0(\tilde{y}_L) - F_0(\tilde{y}_R)]/\partial D > 0$ . Direct manipulation shows this inequality is equivalent to Equation (7):

$$(F'_0(\tilde{y}_L)/F'_0(\tilde{y}_R)) \cdot (E_0[f_L]/E_0[f_R]) < (f'_L(\tilde{y}_L)/f'_R(\tilde{y}_R)) \quad (7)$$

We will verify this inequality as we detail the assumptions under which it holds.

The first additional assumption required for the dead weight loss result concerns the income distribution. It is a stylized fact of real world income distributions that the median is below the mean. Similarly, most real world income distributions are unimodal, and the mode typically occurs below the mean. It follows that voters at or below this unique mode would likely vote for redistribution, even allowing for substantial dead weight loss of redistribution. This is Assumption 2:

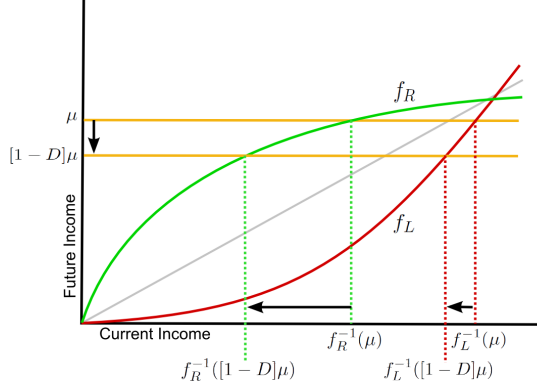
**Assumption 2.** *Voters at the unique mode of the income distribution prefer redistribution under both  $f_L$  and  $f_R$ .*

Our second additional assumption is that income transitions deserve the labels of Right and Left, in that  $f_L$  implies greater demand for redistribution than  $f_R$ , i.e.  $F_0(\tilde{y}_L) > F_0(\tilde{y}_R)$ . This assumption may be satisfied in many ways, not least by constructing a continuum of Right-Left income transitions via the “more concave than” ordering of Benabou and Ok (2001). We therefore assume that  $f_R$  is no more pessimistic about average growth than  $f_L$ , in that  $E_0[f_R] \geq E_0[f_L]$ . With reference to our particular construction of Right and Left,  $E_0[f_R] = E_0[f_L]$ . This is Assumption 3:

**Assumption 3.** *Left voters prefer more redistribution than Right voters. In addition, average growth under the Right transition is at least as high as under the Left transition.*

So far, Assumptions ?? and ?? guarantee the left hand side of Equation (7) is less than one. Assumption 3 directly implies  $E_0[f_L] \leq E_0[f_R]$ , and also that  $\tilde{y}_L > \tilde{y}_R$ . Since the income distribution is unimodal, the density of the distribution  $F'_0$  is decreasing for all incomes above the mode. Since by Assumption 2 the incomes  $\tilde{y}_L$  and  $\tilde{y}_R$  are above the mode, it follows that  $F'_0(\tilde{y}_L) < F'_0(\tilde{y}_R)$ .

Figure 3: Changes in Right vs Left Income as Dead Weight Loss Increases



Putting these together, we see (7) holds so long as  $f'_R(\tilde{y}_R) \leq f'_L(\tilde{y}_L)$ . Figure 3 makes this intuitive through a graphical analysis, which is a natural consequence of modeling Right and Left transitions in terms of curvature. This is Assumption 4:

**Assumption 4.** *At the income levels where Right and Left voters are (respectively) indifferent about redistribution, Left income is increasing faster than Right income.*

In order to explain Assumption 4, we depict idealized transitions  $f_R$  and  $f_L$  in Figure 3. This figure supposes  $f_R$  is concave while  $f_L$  is convex, which is approximately true when  $f_R$  and  $f_L$  are constructed as described above. Fix any future mean income  $\mu$  above median income, and consider the level of support for redistribution next period as dead weight loss  $D$  increases, depicted in Figure 3 as a shift in the horizontal line  $\mu$  to  $(1-D)\mu$ . As dead weight loss increases, the fraction of the population supporting redistribution decreases under both  $f_R$  and  $f_L$ . Under  $f_R$ , this decrease is from  $F_0(f_R^{-1}(\mu))$  to  $F_0(f_R^{-1}((1-D)\mu))$  which in Figure 3 is larger than the drop in support under  $f_L$ , from  $F_0(f_L^{-1}(\mu))$  to  $F_0(f_L^{-1}((1-D)\mu))$ . This asymmetric effect of dead weight loss holds because in the illustrated range, the concavity of  $f_R$  implies  $f_R$  is much flatter than  $f_L$ , which is convex. A local characterization that  $f_R$  is flatter than  $f_L$  is  $f'_R(\tilde{y}_R) \leq f'_L(\tilde{y}_L)$ , which is precisely Assumption 4 and ensures that Equation (7) holds.<sup>26</sup> Formally:

**Proposition 4.** Under Assumptions 2-4, the response of Right voters to dead weight loss is larger than the response of Left voters.

<sup>26</sup>This assumption is stronger than needed to achieve Equation (7). It can be relaxed to the extent that average growth is higher under  $f_R$  than  $f_L$  or that incomes are more concentrated at lower incomes, i.e. around  $\tilde{y}_R$  compared to  $\tilde{y}_L$ .

Proposition 4 summarizes that when ideologies are modeled as income dynamics, Right voters intrinsically have more aversion to dead weight loss than Left voters. Dead weight loss further polarizes support for redistribution between Right and Left, and when voters update their beliefs away from extreme priors, the effect will be to accelerate right-left (or left-right) political swings beyond what would happen in the absence of dead weight loss.

The next two sections quantify the political implications of this model based on estimates of actual income dynamics. We explore whether the forward-looking voter model, possibly augmented with Bayesian learning effects, can explain the Latin American political dynamics discussed in Section 1 above. We also explore whether dead weight loss amplifies these dynamics as hypothesized here.

#### 4. The Right-Left Political Shift in Latin America under Full Information

The prior section has shown that political dynamics for forward looking voters will depend on both the income transition and the initial distribution of income. This section asks if these two considerations can help us understand recent electoral dynamics in Latin America. Building on the method of Shorrocks and Wan (2008),<sup>27</sup> we first recover income distributions for several periods, and then use these to calibrate income transition functions as the basis for the analysis of political dynamics in Chile and Peru.

##### 4.1. Income Dynamics in Chile and Peru

The analysis here relies on income decile data from SEDLAC (CEDLAS and The World Bank, 2011). We use this data to construct approximate income distributions  $\{\hat{F}_t\}$  for each period by fitting a monotone spline to recover each distribution.<sup>28</sup> In order to recover income dynamics  $\hat{f}(y)$ , we consider all functions which are composed of line segments spanning each income decile. Letting  $\beta$  denote a vector of ten line slopes (one for each decile), we can write every admissible income dynamic  $\hat{f}(y)$  as  $f_\beta(y)$  for some  $\beta$ . To calibrate  $f_\beta(y)$ , we make use of the identity that if  $F_t(y)$  is the distribution of expected incomes  $E[y_{it}]$ , then

$$f^{(t)}(y) = F_t^{-1} \circ F_0(y) \quad (8)$$

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<sup>27</sup>These authors use a parametric approach to back out income distributions from income decile data. Synthetic income distributions generated by their method have surprising accuracy compared with known distributions.

<sup>28</sup>The SEDLAC income measures include monetary, non-monetary and transfer income in addition to imputed rent, and we use the following country-year pairs: Chile (1994, 1996, 1998, 2000, 2003, 2006) and Peru (1997-2006).

which fixes the relationship between the annual distribution of income  $F_t(y)$  and the true income dynamic  $f(y)$ .<sup>29</sup> Equation (8) combined with estimates of the income distribution each period, say  $\{\hat{F}_t\}$ , provides a basis to calibrate  $f_\beta(y)$  since Equation (8) implies  $f_\beta^{(t)}(y) \approx \hat{F}_t^{-1} \circ \hat{F}_0(y)$  for each observed period  $t$ .

Using these relationships, we then fit  $f_\beta(y)$  to best explain the recovered income distributions for each observed year,  $\{\hat{F}_t\}$ . To do this, we assume  $\hat{F}_t^{-1} \circ \hat{F}_0(y) = f_\beta^{(t)}(y) \cdot \epsilon(y)$  with the error term  $\epsilon(y)$  distributed lognormal  $(0, \sigma)$ . This implies that for each  $y$ ,  $\hat{F}_t^{-1} \circ \hat{F}_0(y)$  is distributed lognormal  $(f_\beta^{(t)}(y), \sigma)$ . Taking our cue from maximum likelihood estimation, let  $\phi(y, \mu, \sigma)$  denote the log normal likelihood for an observation  $y$ . We then maximize the log likelihood summed across all years and incomes by finding  $\beta$  and  $\sigma$  to solve (9), with further details in the Appendix:

$$\max_{\beta, \sigma} \sum_{t \text{ observed}} \int \ln \phi(y, f_\beta^{(t)}(y), \sigma) dF_0(y). \quad (9)$$

Having calibrated  $f_\beta(y)$  to recover income dynamics  $\hat{f}(y)$  for Chile and Peru, we present the results graphically in Figure 4. A benchmark of ten years is illustrated as this roughly corresponds to two presidential election cycles in Latin American countries. Thus Figure 4 shows the calibrated income dynamic over ten years ( $\hat{f}^{(10)}$ ) for each country, with a 95% confidence band illustrated in dashed lines. For both countries, the interval estimates are wide, signaling that it is difficult to precisely recover income dynamics, both for us as econometricians and presumably also for those individuals who were living that experience. In Section 5, we will explicitly model how noise in the income distribution process affects voters' ability to learn about income dynamics. For the remainder of this section, we will treat the estimated dynamic patterns as known and use these patterns to draw out the implications of the forward-looking voting model.

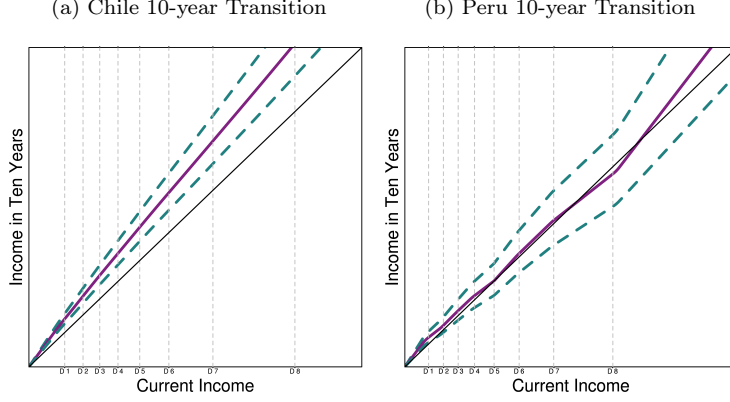
As can be seen in Figure 4, the expected income dynamics for Peru show areas of convexity for much of the income distribution and therefore exhibit NO-POUM dynamics. In particular, note that those who begin in the lowest five deciles are predicted to converge towards the initial median income level. Those who begin from about the sixtieth to the eighty-fifth percentiles converge to an intermediate income position equivalent to the starting level of the seventy-fifth percentile, while those who begin above the eighty-fifth percentile grow rapidly towards ever higher income levels. In contrast, the income dynamics for Chile show prospects for absolute, if not relative, mobility for all deciles of the income

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<sup>29</sup>To see this identity, note that inverting both sides of  $f^{(t)}(y) = F_t^{-1} \circ F_0(y)$  shows  $F_0^{-1} \circ F_t(y) = f^{(-t)}(y)$ , and

$$F_t(y) = \Pr(y_{it} \leq y) = \Pr(f^{(t)}(y_{i0}) \leq y) = \Pr(y_{i0} \leq f^{(-t)}(y)) = F_0(f^{(-t)}(y)).$$

Figure 4: Calibrated Income Transition Functions



distribution.

#### 4.2. Implied Political Dynamics Under Full Information

Using the recovered income transition functions for Peru and Chile, we now derive the electoral dynamics implied by our model of voters who possess full information on the underlying income transition process. We consider the historical time period covered by our income decile data (mid-1990s to the mid-2000s) and consider policy longevities (degrees of forward-lookingness) of 1 to 10 years. Computing the demand for redistribution across time and for varying policy lengths then follows the development above. At any point in historical time,  $H$ , and for any degree of policy longevity,  $T$ , we calculate the fraction of the electorate supporting different policies as implied by our model:

$$\Pr(\text{Voter prefers } \tau = 1) = \tilde{F}_H \left( [\hat{g}^T]^{-1} (\hat{\mu}^T) \right) \quad (\text{Forward Demand for Redistribution})$$

where  $\hat{g}^T(y_{i0}) \equiv \sum_{t=0}^T \delta^t \hat{f}^{(t)}(y_{i0})$ ,  $\hat{\mu}^T \equiv E_0 [\hat{g}^T(y_{i0})]$  and  $\tilde{F}_H(y) \equiv \hat{F}_0 \circ \hat{f}^{(-H)}(y)$ .

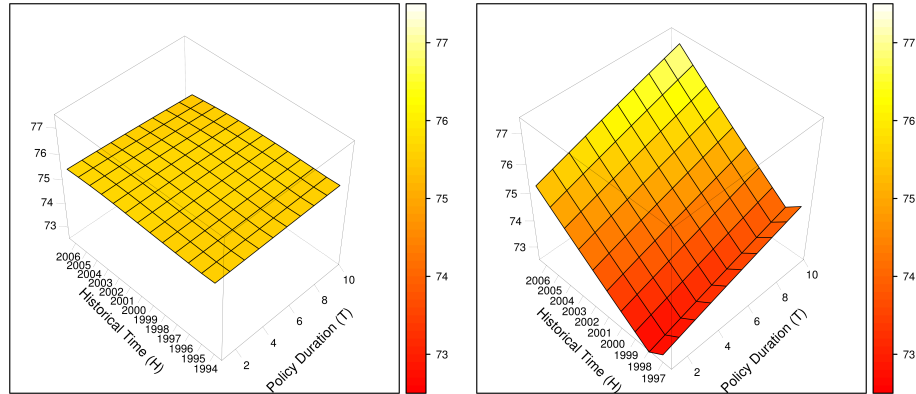
Figure 5 graphs the results of these calculations for Chile and Peru under the assumptions that redistribution incurs no dead weight loss ( $D = 0$ ) and that  $\delta = .95$ . First, consider the myopic ( $T = 1$ ) demand for redistribution in each country. Over time, Chile shows a fairly linear pattern in Figure 4(a), which implies fairly flat redistributive preferences over the period as calculated in Figure 5(a). In contrast, reflecting the non-concavities in its calibrated income dynamics, Peru shows a pattern in which the myopic demand for redistribution increases over time.

Figure 5 also allows us to see what happens over time when voters are forward-looking (and as policy longevity increases). In the case of Chile, more forward-looking voters and longer-lasting policies barely perturbs the demand for redistribution at any point in time. Peru again presents an interesting picture

Figure 5: Evolving Demand for Redistribution

(a) % Demanding Redistribution: Chile

(b) % Demanding Redistribution: Peru



as more forward-looking voters support redistribution more strongly than do myopic voters, increasingly so over historical time.

While the contrast between Chile and Peru illustrates the importance of income transition dynamics for political dynamics, the calculated level of support for redistribution is remarkable high for both countries, in all time periods and under any degree of forward-lookingness. Put differently, the full information voter model predicts that there would have been strong support for redistributive policies long before such support actually emerged. While there are many possible explanations for the tardy arrival of support for more aggressively redistributive policies, one is that voters perceived significant dead weight losses to redistribution. To explore this idea, we calculate the level of dead weight taxation loss that would have been necessary to provide majoritarian support for *laissez faire* policies in Chile and Peru under the assumptions used to generate Figure 5. These levels are 45-48% in Chile and 43-47% in Peru, and are exceedingly high in comparison to existing estimates of dead weight loss (e.g. Olken (2006)), making it unlikely that dead weight losses explain the mismatch between model prediction and reality.

## 5. The Right-Left Political Shift in Latin America Under Uncertainty

While the analysis so far is consistent with the left turn that took place in Latin America politics, it cannot account for the timing of that shift, throwing into sharp relief the question as to why so many voted for largely *laissez faire* policies prior to the early part of this century. The answer cannot be found in the prospect of upward mobility as the recovered income transitions suggest that there were not prospects of upward mobility for important segments of the



electorate. Indeed, forward-looking pocketbook voters with perfect information on the nature of income dynamics would have supported redistributive policies sooner and more forcefully than they actually did.

This section employs the imperfect information model of forward-looking, Bayesian voters to analyze the right to left political shift observed across contemporary Latin America. To do this, we first provide an empirically grounded approach for representing left and right political ideologies. Second, we argue that economic crises of the 1980s put the left in disarray, and at the time of reforms, voters adopted a POUM prior, since prior crises left no credible alternative to the emergent neoliberal model. Applying these assumptions to Peru, we show that over the course of a dozen years, voter learning would generate a 22 percentage point increase in the fraction of the electorate preferring redistribution, assuming that there is a zero dead weight loss associated with redistribution. The electoral shift jumps to 27 percentage points when redistribution carries a 10% dead weight loss. In both cases, those preferring redistribution move from a minority to a majority of the population. While the exact shift in the electorate will vary with how learning is modeled, the magnitude we find highlights this mechanism as viable explanation of observed shifts in contrast to a model of fully informed voters.

### 5.1. Empirical Approximation of Left and Right Ideologies

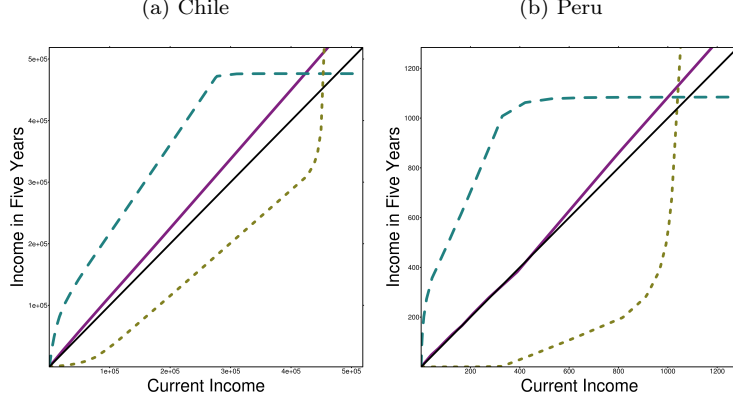
In order to arrive at plausible left and right ideological models of income dynamics, we construct two functions ( $f_R$  and  $f_L$ ) that surround and exaggerate the true empirical income transition function,  $\hat{f}(y)$ . We begin by characterizing the right income transition model as one that offers greater prospects for upward mobility and implies less demand for redistribution than does  $\hat{f}$ . For a given  $f_R$ , we then residually construct  $f_L$  so that the true function can be expressed as a linear combination of the left and right ideologies as specified in Equation (10):

$$f_\lambda(y) = (1 - \lambda)f_L(y) + \lambda f_R(y). \quad (10)$$

We now describe the construction of  $f_R$  and  $f_L$  from the empirical income transition  $\hat{f}(y)$ . First define  $\bar{f}(y)$ , the upper envelope of  $\hat{f}(y)$  (which is necessarily concave, thereby inducing POUM dynamics). Now consider the income transition  $C(y) \equiv \bar{f}(y) - \kappa \cdot y$  where  $\kappa \equiv E[\bar{f}(y)]/E[y]$ .  $C(y)$  has the same curvature as  $\bar{f}(y)$  since  $C''(y) = \bar{f}''(y)$ , yet implies no change in mean income as  $E[C(y)] = E[\bar{f}(y)] - E[\kappa \cdot y] = 0$ . We conceptualize the “Right Ideology”  $f_R(y)$  by adding a multiple of  $C(y)$  to the empirical income transition  $\hat{f}(y)$  and subtracting the same multiple from  $\hat{f}(y)$  to arrive at the “Left Ideology”  $f_L(y)$ . We denote the constant that multiplies  $C(y)$  by  $\rho$ , giving the following expressions for  $f_R(y)$  and  $f_L(y)$ :

$$f_R(y) \equiv \hat{f}(y) + \rho [\bar{f}(y) - \kappa \cdot y], \quad f_L(y) \equiv \hat{f}(y) - \rho [\bar{f}(y) - \kappa \cdot y]. \quad (11)$$

Figure 6: Stylized Right and Left Income Ideologies



Therefore, combining Equations (10) and (11), we have

$$f_\lambda(y) = \hat{f}(y) + (2\lambda - 1) \rho [\bar{f}(y) - \kappa \cdot y] .$$

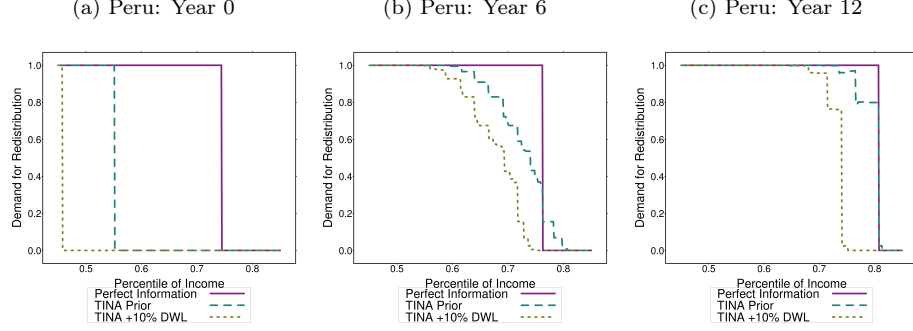
Increases in the constant  $\rho$  decrease the demand for redistribution under  $f_R(y)$ , in line with the intuition that as  $\rho$  increases, more of the curvature from  $\bar{f}(y)$  is present in  $f_R$ .

Figure 6 illustrates the application of this approach of constructing  $f_R(y)$  and  $f_L(y)$  in Chile and Peru. The uppermost curve is  $f_R(y)$ , representing the effect of adding the curvature term  $\rho \cdot C(y)$  to  $f(y)$ , while the bottom curve is  $f_L(y)$  with the same term  $\rho \cdot C(y)$  subtracted from  $f(y)$ . Referring back to Equation (11), the parameter  $\rho$  determines the spectrum of possible income transitions between  $f_R(y)$  and  $f_L(y)$  by making both of these bounding transitions more extreme. Using the empirical income transition functions of Figure 4, we fix the constant  $\rho$  to be the large as possible to capture the widest range of possibilities, subject to the constraint that both  $f_R(y)$  and  $f_L(y)$  are increasing. These maximal values of  $\rho$  are 28.4 (Chile) and 43.2 (Peru). While these values for  $\rho$  may seem large, any values used should allow for a large degree of ignorance about actual income dynamics. In any case, we next introduce Bayesian learning dynamics, so that beliefs in these maximal values of  $\rho$  will be adjusted in line with voters' observed income dynamics.

### 5.2. Learning and Political Dynamics under a POUM Prior

The final element needed is a specification of voters' initial beliefs about income dynamics at the beginning of the 1990s. To illustrate the implications of our model, we take seriously the then common observation that there was an exhaustion of credible political alternatives to a liberal economic regime. As Margaret Thatcher famously intoned: "TINA—There Is No Alternative" to free markets. Thatcher's statements motivate what we call the TINA or POUM prior,

Figure 7: Demand for a 10 Year Redistributive Policy by Income Percentile



meaning an initial set of beliefs,  $\pi_{i0}(\lambda)$ , that heavily weight the right perspective on the income process and its promise of upward mobility. In the numerical analysis that follows, we use a simple prior form which places exponential weight on Right beliefs for voters in the initial period, namely  $\pi_{i0}(\lambda) = 10e^{10\lambda} / (e^{10} - 1)$ . Although this is one particular prior, the results are fairly robust to any prior highly weighted to the Right ideology.

With the POUM prior in hand, and the empirically grounded representations of Left and Right ideologies in Figure 6, we now numerically simulate political dynamics in Chile and Peru, assuming an idiosyncratic income shock parameter  $\sigma = 1/2$ , and that voters look forward ten years and have a discount rate of 95%.<sup>30</sup> Figure 7 shows the simulated evolution of political preferences for Peru in the initial period, six years later and twelve years later. The  $y$ -axis shows the percentage of the electorate preferring redistribution by income percentile. The solid line in each figure is the full information benchmark, showing what political preferences would have looked like had voters had perfect information on the true income dynamic. The dashed line shows political preferences by decile under imperfect information (and when voters begin with the POUM prior) and when there are no dead weight losses associated with redistribution. The dotted line shows the same imperfect information scenario but assumes that redistribution is associated with a 10% dead weight loss.<sup>31</sup>

As can be seen, the preferences of fully and imperfectly informed voters are quite different, although absent any dead weight loss the median voter would have preferred redistribution from the outset. However, with a 10% dead weight

<sup>30</sup>Necessarily, the variance recovered from our fitting of income transitions to country decile data is a lower bound for the variance of idiosyncratic income shocks, since the decile data has been aggregated across individuals.  $\sigma = 1/2$  satisfies this lower bound and is close to known estimates: see Musgrove (1979).

<sup>31</sup>Given the new insights above regarding the asymmetric effect of dead weight loss on Right versus Left voters, modeling uncertainty about dead weight loss (e.g. Roemer (1994)) could produce even larger swings in the electorate.

loss, the median, forward-looking voter would have initially voted against redistribution under the TINA prior.<sup>32</sup> However, after six years of living and learning from the actual income distribution process, the median voter, and most voters in the lowest seven deciles of the income distribution would have favored redistributive policies. After a dozen years, the preferences of most voters approach those that would hold under full information, implying a major political shift from minority to majority support for redistribution.

Figure 8 provides another look at the political dynamics implied by our model. The vertical axis now displays the fraction of the electorate at each point in time that is expected to vote for redistribution. Absent dead weight losses, over the 1997 to 2009 simulation period in Peru, the fraction voting for redistribution rises by some 22 percentage points, again approaching the levels that would be expected under full information by 2010. With dead weight losses, politics become even more volatile with a 27% shift over this 12-year period.<sup>33</sup>

These sharp swings in policy preferences are largely driven by swing voters' reevaluation of their prospects for upward mobility as they learn from the actual operation of the Peruvian economy. An interesting contrast to these results is provided by undertaking a similar exercise for the Chilean economy. The estimated Chilean income transition function of Figure 6(a) is one that shows absolute upward income mobility for all classes, though not much relative improvement for the initially lower income deciles. While simulated preferences for redistribution in the Chilean case are strong, they remain quite stable over time, offering a vision of much more stable politics in Chile than in a country with a polarizing income distribution process.

## 6. Conclusion

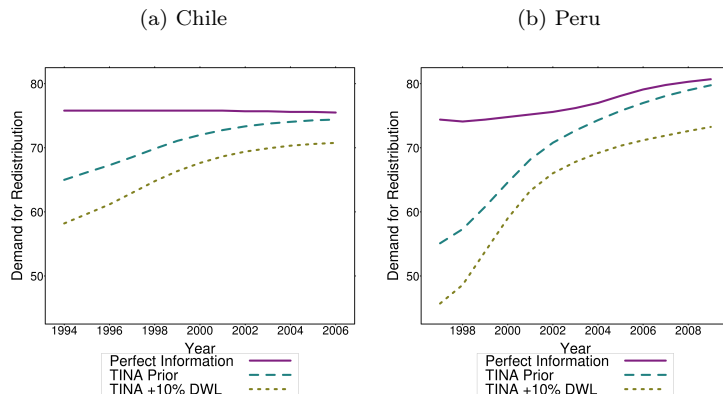
Adopting the perspective that voters are forward-looking and pay attention to income dynamics, not just to their place in the contemporaneous income distribution, this paper has explored the right to left shift in the politics of Latin American countries over the last twenty-five years. Two analytical innovations are key to this exploration. The first is a generalization of earlier work on forward-looking voters. We here model political preferences under families of income distribution dynamics beyond concave dynamics that offer prospects of upward mobility. This generalization, motivated by empirical evidence of polarizing, non-concave dynamics that offer no prospects of upward mobility for segments of the population, shows that preferences for redistributive policies may increase, not decrease over time when voters are forward-looking. The key

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<sup>32</sup>It is difficult to ascertain what the level of dead weight loss is for these countries for this period. As a benchmark, Olken (2006) estimates a dead weight loss of redistribution of 18% in Indonesia. As higher levels of dead weight loss amplify our results (as shown theoretically above), we have chosen a conservative assumption of 10%.

<sup>33</sup>This perhaps counterintuitive result is consistent with Proposition ?? above, although we have not specifically calibrated this section's simulation analysis to match the assumptions under which we have shown this proposition to be true.

Figure 8: Aggregate Demand for a 10 Year Redistributive Policy



message is that unlike a world which offers upward mobility to low income voters, the dynamics of demand for redistribution are not a foregone conclusion and may manifest in volatile political patterns. This points to evaluating the relationship between income dynamics and political choices in light of the conditions voters face on a country-specific basis.

However, detailed analysis of the case of Peru suggests that there would have been initially strong support for redistribution had voters been fully informed about the nature of the income distribution dynamics, making it extremely hard to account for the elections in Peru and elsewhere in Latin America in the 1990s that brought conservative candidates to power. Indeed, the Latinobarómetro public opinion data discussed in Section 1 shows that the high point of support for conservative policy in the mid-1990s was marked by extremely high prospects of self-reported upward mobility.

These observations motivate this paper's second innovation, namely its modeling of voters as Bayesian learners who update their understanding of income distribution dynamics based on their own lived experience. Given that most voters in Peru (and other countries which saw a transition to a market economy in the late 1980s and early 1990s) had little prior experience with the new economic model, we assume that they initially adopted a prior probability distribution that put substantial weight on an ideological position that attached strong prospects for upward mobility to the region's new economic model. While this assumption is admittedly strong, we find that learning rather quickly erases ideological errors. Numerical simulation of political preferences as voters received noisy draws from the true (calibrated) income distribution process shows that a substantial shift from strong right political majority to a strong left political majority over the course of about a dozen years. Somewhat surprisingly, political volatility is actually increased when the electorate believes that redistributive policies carry dead weight losses. We show that the additional political volatility created by dead weight loss is to be expected under assumptions plausibly

satisfied by real world data.

While there are of course competing explanations of the kinds of policy shifts we explain with our voter learning model (see especially the work of Callander (2011), Buera et al. (2011) and Weyland (2002) discussed in the Introduction above), our theoretical model suggests a number of empirically testable implications about the nature of political change that may help to distinguish between these alternative explanations. As is clear from our theoretical framework, a key parameter that will influence the speed and acceleration of political dynamics is the amount of noise in the income distribution process ( $\sigma$ , the variance of  $\varepsilon_{it}$  in our notation). Exploration of these dynamics is beyond the scope of this paper, but we hope that our theoretical framing motivates an agenda of empirical work in this area.

Latin America of the 1990s is not only the region to have transitioned to a market economy. While there can certainly be no claim that the precise voting dynamics derived here for Peru apply to other countries, the information deficit and voting dilemma confronted by the Peruvian electorate has had its reflection in a much larger number of countries that have transitioned to political democracy and market economies. Modeling the evolving political preferences of voters in these regions as forward-looking, Bayesian learners offers insights into the complex and often unstable voting patterns observed in these other regions.

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Table AppendixA.1: Income Transition Parameters

	Slope of Income Transition in each Decile									
	1	2	3	4	5	6	7	8	9	10
Chile	1.035	1.025	1.028	1.024	1.022	1.023	1.020	1.021	1.024	1.025
Peru	1.032	0.979	1.004	0.998	0.976	1.014	1.000	0.980	1.031	1.023

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## AppendixA. Recovery of Simple Income Dynamics

As developed above, we fit a class of income transitions  $f_\beta(y)$  defined by piecewise line segments spanning each income decile. Specifically, these line segments span the midpoints between each income decile (with endpoints defined by incomes at zero and twice the tenth income decile). The years used to determine the midpoints are roughly in the middle of our data sample years: 2001 in Chile and 1998 in Peru. The values of  $\beta$  which maximize Equation (9) for each country are reported in Table AppendixA.1, while the 95% confidence band for incomes graphed in Figure 4 are based on the likelihood maximizing values  $\sigma_{\text{Peru}} = 0.0943$  and  $\sigma_{\text{Chile}} = 0.0540$ . The parameter values of  $\rho$  and  $\bar{g}$  which determine Right and Left income transitions are  $\rho_{\text{Chile}} = 28.42$ ,  $\rho_{\text{Peru}} = 43.16$  and  $\kappa_{\text{Chile}} = 1.02$ ,  $\kappa_{\text{Peru}} = 1.009$ .

## AppendixB. Dependence of redistributive demand on initial income

### AppendixB.1. Fragility of Redistributive Dynamics

Consider an income distribution  $F_0$  composed of three equally sized groups with incomes  $Y_1, Y_2, Y_3$  where  $0 < Y_1 < Y_2 < Y_3 < \bar{Y}$ . Then  $\mu_t = \sum (1/3) \cdot f^{(t)}(Y_i)$  and so clearly depends on the initial distribution of income  $F_0$  interacting with the evolution of group incomes  $f^{(t)}(Y_i)$ . Now also suppose  $f$  has three fixed points  $Y_{\text{Trap}} < Y_{\text{Escape}} < \bar{Y}$  where for  $y < Y_{\text{Escape}}$ ,  $f^{(t)}(y) \rightarrow Y_{\text{Trap}}$  and for  $y > Y_{\text{Escape}}$ ,  $f^{(t)}(y) \rightarrow \bar{Y}$ . In this case all groups are going to either  $Y_{\text{Trap}}$  or  $\bar{Y}$ . To fix ideas, assume  $Y_1 < Y_{\text{Escape}} < Y_3$  so the  $Y_1$  group converges to  $Y_{\text{Trap}}$  while  $Y_3$  converges to  $\bar{Y}$ . Clearly  $Y_1$  prefers the complete redistribution scheme  $r_1$  while  $Y_3$  prefers  $r_0$ . This leaves open the middle class of “swing voters”  $Y_2$ . If  $Y_2 > Y_{\text{Escape}}$  then the middle class eventually climbs the income ladder to  $\bar{Y}$  and joins the (now majoritarian) voting block of  $Y_3$ . Otherwise, if  $Y_2 < Y_{\text{Escape}}$  there is a thinning of the middle class and swing voters eventually join with  $Y_1$ , implying the median voter prefers redistribution. Note the fragility of the eventual voting outcomes: a small income difference  $\delta$  in  $Y_2$  can push  $Y_2 + \delta$  to be greater or less than  $Y_{\text{Escape}}$ . This eventually results in a large fraction  $p_2$  of swing voters to switch their vote as income evolves. Similar consequences can arise if voters lack perfect information about  $F_0$  or  $f$  so that small changes in beliefs can give rise to large changes in redistributive preferences.

### AppendixB.2. The Same Income Transition Often Can Imply Both Increase and Decreasing Redistributive Demand

A broad class of dynamics can exhibit either increasing or decreasing demand for redistribution. The deciding factor for redistributive dynamics, even for a fixed dynamic, is the initial distribution of income. This emphasizes the interrelationship between “Upward/No Mobility” in the dynamic role of income transitions and the “existing order” in the role of the income distribution: political implications cannot be drawn without considering both:

**PROPOSITION.** Suppose  $f$  is bounded and define  $f^{(\infty)}(y) \equiv \lim_{t \rightarrow \infty} f^{(t)}(y)$  as an individual’s income after an arbitrarily long period of time. Let  $\underline{\mu}_\infty$  be the least possible per capita income under  $f^{(\infty)}$  and let  $\bar{\mu}_\infty$  be the highest possible per capita income under  $f^{(\infty)}$ .

1. (POUM Forever) If  $[\underline{\mu}_\infty, \bar{\mu}_\infty]$  intersects the interior of  $Y_P$ , there is an initial distribution of income where the demand for redistribution always decreases.
2. (No-POUM Forever) If  $[\underline{\mu}_\infty, \bar{\mu}_\infty]$  intersects the interior of  $Y_N$  there is an initial distribution of income where the demand for redistribution always increases.

PROOF. Formally, we have  $\underline{\mu}_\infty \equiv \inf_{F_0} \int f^{(\infty)} dF_0$  and  $\bar{\mu}_\infty \equiv \sup_{F_0} \int f^{(\infty)} dF_0$  where the infimum and supremum are taken over bounded, absolutely continuous initial income distributions.  $\underline{\mu}_\infty$  and  $\bar{\mu}_\infty$  are both in  $[0, \infty)$  as for any fixed  $F_0$ ,  $f$  is bounded so  $\int f^{(\infty)} dF_0 = \lim_{t \rightarrow \infty} \int f^{(t)} dF_0$  by bounded convergence, and clearly each  $\int f^{(t)} dF_0 \in [0, \sup f]$ . We will consider the POUM case as the other case is similar. By hypothesis, there is some  $\mu^* \in [\underline{\mu}_\infty, \bar{\mu}_\infty]$  and an open set  $V$  such that  $\mu^* \in V \subset Y_P$ . Fix  $\epsilon > 0$  such that  $\mu^* \pm \epsilon \in V$  and fix  $\lambda$  such that  $\mu^* = \lambda \underline{\mu}_\infty + (1 - \lambda) \bar{\mu}_\infty$  (note this forces  $\lambda \in [0, 1]$ ). Now define, for any initial income distribution  $G$ ,  $\mu_t^G \equiv \int f^{(t)} dG$ . By definition of  $\underline{\mu}_\infty$  and  $\bar{\mu}_\infty$  we may choose initial income distributions  $\underline{F}$  and  $\bar{F}$  where

$$\lim_{t \rightarrow \infty} \mu_t^{\underline{F}} < \underline{\mu}_\infty + \epsilon/2 \quad \text{and} \quad \lim_{t \rightarrow \infty} \mu_t^{\bar{F}} > \bar{\mu}_\infty - \epsilon/2.$$

It follows that there is a  $T$  such that for all  $t \geq T$ ,  $\mu_t^{\underline{F}} < \underline{\mu}_\infty + \epsilon$  and  $\mu_t^{\bar{F}} > \bar{\mu}_\infty - \epsilon$ . This implies  $\left| \lambda \mu_t^{\underline{F}} + (1 - \lambda) \mu_t^{\bar{F}} - \left[ \lambda \underline{\mu}_\infty + (1 - \lambda) \bar{\mu}_\infty \right] \right| < \epsilon$  for all  $t \geq T$ . Since  $\lambda \underline{\mu}_\infty + (1 - \lambda) \bar{\mu}_\infty = \mu^*$  and  $\tilde{F} \equiv \lambda \underline{F} + (1 - \lambda) \bar{F}$  is an admissible initial income distribution, we see  $\left| \mu_t^{\tilde{F}} - \mu^* \right| < \epsilon$  for all  $t \geq T$ . By construction, this implies  $\mu_t^{\tilde{F}} \in V \subset Y_P$ , so by Proposition 1, the demand for redistribution is always decreasing when the initial distribution of income is  $F_0(y) \equiv \tilde{F}(f^{(T)}(y))$ .  $\square$

### Appendix C. Demand for Redistribution under Imperfect Information

In general, neither  $f_L$  nor  $f_R$  need reflect reality but rather idealized versions of what Left and Right ideologues might represent in a manifesto. Clearly the relative strength of a voter's beliefs in these world views will influence voting behavior. In order to emphasize the role of beliefs  $\pi_{it}(\lambda|H_{it})$  in deriving a voter's expected income, we now illustrate the decisions of a pocketbook voter who is also a Bayesian learner.

If a voter knows the true value of  $\lambda$ , namely  $\lambda_0$ , then beliefs  $\pi_{i0}(\lambda)$  put a point mass of 1 on  $\lambda_0$ . This is the perfect information case, and as above expected income in period 1 would be given by  $E[y_{i1}|H_{i0}, \pi_{i0}] = f_{\lambda_0}(y_{i0})$ . However, each voter does not know  $\lambda_0$  with certainty but has a non-degenerate prior density  $\pi_{i0}(\lambda)$  over possible values of  $\lambda$ . Now consider a voter in period 0 with income  $y_{i0}$ . The voter's expected income in period 1 is a weighted average of expected income over plausible values of  $\lambda$ , namely  $E[y_{i1}|H_{i0}, \lambda = \lambda_0] = f_\lambda(y_{i0})$  weighted by  $\pi_{i0}(\lambda)$ . Specifically,

$$E[y_{i1}|H_{i0}, \pi_{i0}] = \int_0^1 E[y_{i1}|H_{i0}, \lambda] \pi_{i0}(\lambda) d\lambda = \int_0^1 f_\lambda(y_{i0}) \pi_{i0}(\lambda) d\lambda.$$

More generally, at the end of periods 1 to  $t$ , a voter updates his prior  $\pi_{i0}(\lambda)$  to a posterior  $\pi_{it}(\lambda)$  using their history  $H_{it} = \{y_{i0}, \dots, y_{it}\}$ . Therefore expected

income in period  $t + 1$  is

$$E[y_{it+1}|H_{it}, \pi_{i0}] = \int \underbrace{f_{\lambda}^{(t+1)}(y_{i0})}_{\text{Expected Income}|\lambda} \cdot \underbrace{\pi_{it}(\lambda|H_{it})}_{\text{History Dependent Beliefs Over } \lambda} d\lambda. \quad (\text{C.1})$$

Equation (C.1) highlights the two dynamic factors which influence a voter's beliefs about expected income. The first element, expected income given  $\lambda$  is the true state of the world, is deterministic as under perfect information. The second element, a voter's ideological beliefs, evolve as information is collected in the form of the idiosyncratic income history  $H_{it}$ .

Since each possible value of  $\lambda$  corresponds to an economy wide income dynamic, a voter's beliefs also impact expectations about mean income next period. Given that  $f_{\lambda}$  is the true income dynamic, mean income in period  $t + 1$  is  $\mu_{t+1}|\lambda \equiv \int f_{\lambda}^{(t+1)}(y)dF(y)$ . Thus, a voter with an income history  $H_{it}$  and prior  $\pi_{i0}$  believes mean income in period  $t + 1$  is

$$\begin{aligned} E[\mu_{t+1}|H_{it}, \pi_{i0}] &= \int_0^1 [\mu_{t+1}|\lambda] \cdot \pi_{it}(\lambda|H_{it})d\lambda \\ &= \int_0^1 \left[ \int f_{\lambda}^{(t+1)}(y)dF(y) \right] \pi_{it}(\lambda|H_{it})d\lambda. \end{aligned} \quad (\text{C.2})$$

After accounting for any dead weight loss  $D$ , a voter will prefer redistribution if and only if  $E[(1 - D)\mu_{t+1}|H_{it}, \pi_{i0}] \geq E[y_{it+1}|H_{it}, \pi_{i0}]$ . From Equations (C.1) and (C.2) this means voters prefer redistribution when

$$\int_0^1 \underbrace{\left[ (1 - D) \int f_{\lambda}^{(t+1)}(y)dF(y) - f_{\lambda}^{(t+1)}(y_{i0}) \right]}_{\text{Expected Transfer}|\lambda} \cdot \pi_{it}(\lambda|H_{it})d\lambda \geq 0. \quad (\text{C.3})$$

Equation (C.3) shows that a voter prefers redistribution when, given their history  $H_{it}$ , they believe the expected transfer from redistribution will be positive. Note that two voters with the same initial incomes need not have the same redistributive preferences: whether Equation (C.3) holds depends on each voter's income history through their beliefs  $\pi_{it}(\lambda|H_{it})$ . This implies the popularity of redistributive policies varies in a nontrivial way across initial incomes. Preferred policies for each voter over redistribution in period  $t + 1$ , conditional on their history  $H_{it}$ , are summarized as

$$\begin{aligned} \text{Prefer Redistribution: } & \int \left[ (1 - D)\mu_{t+1}|\lambda - f_{\lambda}^{(t+1)}(y_{i0}) \right] \pi_{it}(\lambda|H_{it})d\lambda \geq 0, \\ \text{Prefer Laissez Faire: } & \int \left[ (1 - D)\mu_{t+1}|\lambda - f_{\lambda}^{(t+1)}(y_{i0}) \right] \pi_{it}(\lambda|H_{it})d\lambda \leq 0. \end{aligned}$$

A clear connection from ideological beliefs to demand for redistribution is given by:

**Assumption 5.** *Increases in  $\lambda$  imply expected income improves relative to transfers*

$(df_{\lambda}^{(t)}(y_{i0})/d\lambda \geq d(1-D)\mu_t|\lambda/d\lambda)$  for all swing voters defined as  $y_{i0}$  where

$$y_{i0} \in \left[ f_R^{(-t)}((1-D)\mu_t|\lambda=1), f_L^{(-t)}((1-D)\mu_t|\lambda=0) \right] \quad (\text{Swing Voters})$$

This Assumption says that as  $\lambda$  increases (moves to the Right), each voter believes his expected income  $f_{\lambda}^{(t)}(y_{i0})$  increases *relatively more* than expected transfers  $(1-D)\mu_t|\lambda$ . Furthermore, we only require this to hold for voters who might potentially change their vote: the votes of both destitute ( $f_{\lambda}^{(t)}(y_{i0}) < (1-D)\mu_t|\lambda$  for all  $\lambda$ ) and well-to-do ( $f_{\lambda}^{(t)}(y_{i0}) > (1-D)\mu_t|\lambda$  for all  $\lambda$ ) are unaffected by belief. Crucially, the expected transfer  $(1-D)\mu_t|\lambda - f_{\lambda}^{(t)}(y_{i0})$  is decreasing in  $\lambda$ . It follows that voter  $j$  tends to prefer less redistribution than voter  $i$  when voter  $j$ 's beliefs  $\pi_{jt}$  are “to the Right” of a voter  $i$ 's beliefs  $\pi_{it}$ . To make this precise, assume that  $\pi_j$  stochastically dominates  $\pi_i$  and  $i$  and  $j$  have the same initial incomes. Since for each fixed  $\lambda$ ,  $(1-D)\mu_t|\lambda - f_{\lambda}^{(t)}(y_{i0}) = (1-D)\mu_t|\lambda - f_{\lambda}^{(t)}(y_{j0})$  and this equation is decreasing in  $\lambda$ , the dominance of  $\pi_j$  over  $\pi_i$  implies

$$\int \underbrace{\left[ (1-D)\mu_t|\lambda - f_{\lambda}^{(t)}(y_{j0}) \right]}_{\text{Expected Transfer}|\lambda} \pi_{jt}(\lambda) d\lambda \leq \int \underbrace{\left[ (1-D)\mu_t|\lambda - f_{\lambda}^{(t)}(y_{i0}) \right]}_{\text{Expected Transfer}|\lambda} \pi_{it}(\lambda) d\lambda \quad (\text{C.4})$$

(C.4) shows that the “Right” voter  $j$  believes they will receive a lower net transfer from redistribution than the “Left” voter  $i$ . Therefore voter  $j$  tends to prefer less redistribution than voter  $i$ . This result is summarized as Proposition 5:

**Proposition 5.** Suppose the Assumption above holds. If voters  $i$  and  $j$  are identical except voter  $j$ 's beliefs  $\pi_{jt}$  stochastically dominate voter  $i$ 's beliefs  $\pi_{it}$ , then  $j$  prefers less redistribution than  $i$ .

Proposition 5 shows that the further to the ideological Right a voter is, the less redistribution they prefer. In this framework, one would expect that the speed of learning would be related to both the variability of income signals and the gap between left and right predictions for an individual's future income position.<sup>34</sup> These expectations imply a rich set of testable implications about the evolution of political preferences and voting.

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<sup>34</sup>Voters may of course learn about additional aspects of the economy besides income mobility, which could influence political preferences. In particular, we have not modeled the effect of learning the level of dead weight loss present in any prospective redistributive policy, although Proposition 4 suggests the impact this added dimension would have.

## Appendix D. Proofs

**Lemma 2.** *Suppose  $f$  and  $g$  are income transition functions. Then the myopic demand for redistribution implied by the income transition  $f + g$  is between that implied by  $f$  and  $g$ .*

PROOF. Formally, we wish to show that

$$(f + g)^{-1}(E[f + g]) \in [f^{-1}(E[f]), g^{-1}(E[g])]. \quad (\text{D.1})$$

Since  $f$  and  $g$  are strictly increasing, composing each side with  $f + g$  shows (D.1) holds iff

$$E[f] + E[g] \in [E[f] + g \circ f^{-1}(E[f]), E[g] + f \circ g^{-1}(E[g])]. \quad (\text{D.2})$$

Subtracting  $E[f] + E[g]$  from both sides of Equation (D.2) shows the equation holds iff

$$0 \in [g \circ f^{-1}(E[f]) - E[g], f \circ g^{-1}(E[g]) - E[f]]. \quad (\text{D.3})$$

This always holds, since considering the two terms on the RHS of (D.2) have different signs, since

$$\begin{aligned} \text{sign}\{g \circ f^{-1}(E[f]) - E[g]\} &= \text{sign}\{f^{-1}(E[f]) - g^{-1}(E[g])\}, \\ \text{sign}\{f \circ g^{-1}(E[g]) - E[f]\} &= \text{sign}\{g^{-1}(E[g]) - f^{-1}(E[f])\}. \end{aligned}$$

□