Indifference Curve Analysis of Banks' Risk-taking and CoCo Covenants*

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Abstract

This paper investigates two repercussions of the contingent convertible (CoCo) bond bail-in framework: the agency costs and the resulting monitoring costs. For the first, the equityholders' behaviour is analysed as a trade-off between the value of the bank and the risk taken by using an indifference curve model. While the first-best optimal risk maximises the value of the bank, the equityholders select sub-optimally high risk level under bail-in structures. This leads to both wealth transfer and value destruction agency costs. For the second, the increased required rate of return by bondholders that reflects the cost of monitoring is shown to act as a "Pigouvian tax" on the equityholders' behaviour. Utilising this, we propose different types of covenants within CoCo bonds indenture as a solution to the sub-optimal risk-taking behaviour.

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1 Introduction

The new financial regulation aims to impose losses on bondholders on a going-concern basis by the use of contingent convertible (CoCo) bonds. These are bonds that either convert to equity or are written-down/off, when a bank's capital ratio hits a pre-specified trigger ratio. This formalises the deviation from absolute priority rule (DAPR), where under the absolute priority rule (APR) bondholders do not bear losses until equityholders have been wiped out. This paper focusses on the two of the consequences of the DAPR: the agency costs of the bail-in structures, and the effects of the resulting monitoring costs of the bondholders on the equityholders' behaviour.

The introduction of DAPR increases equityholders' incentive to "loot", where the bank's wealth is extracted for the benefit of a group of stakeholders (in the equityholders' case, for example by unreasonably high dividend payouts), or to "gamble-for-resurrection", where as a last resort to revive a bank's fortune, a high risk strategy is undertaken in the knowledge that benefits would accrue to the equityholders while the losses are mostly borne by the bondholders. This is because the equityholders know their maximum loss is limited by the CoCo trigger (if the bond is a write-down/off type), or with further dilution (if the CoCo is an equity-conversion type).[1] These are behaviours recognised in the literature as agency costs, for which there are broadly two types, the wealth transfer and the value destruction.[2] These are typically identified by the positive vega of the equityholders' position[3] (e.g. Berg and Kaserer (2011), Hori and Martin Cerón (2017)). Here we develop a different approach using equityholders' indifference curves. This enables us to investigate the risk-taking behaviours of banks, both the first-best and the sub-optimal, and allows us to propose solutions for alleviating these agency costs.

[1] Note the equity-conversion CoCo bond is a non-admissible debt to equity swap (NADES), and thus the dilution of shareholders is lower than it would be if they had to issue private equity at distressed share prices. For an analysis of CoCo bond bail-in as a form of DES, see Hori and Martin Cerón (2017).

[2] See, for example, Berg and Kaserer (2011) (for wealth transfer) and Eberhart and Senbet (1993) (for value destruction).

[3] *Vega* is the sensitivity of the value of an option, C, with respect to the volatility of the underlying asset price, σ, $Vega = \frac{\partial C}{\partial \sigma}$.

More specifically, a bail-out or a bail-in structure can be shown to be construed as a sale of an option structure from the bondholders to the equityholders, where the buyer of the option, the equityholders, have the right to choose its volatility. The question that can then be asked is what the chosen level of volatility is, and how it is affected by the new bail-in structures. The vast theory of firm literature does not tell us how a firm selects its optimal level of risk. As well established in the literature,[4] under the APR the equityholders have a payoff structure that is a call option (their loss is limited to their current position while the potential income upside is unbounded). The positive vega position of an option means that the value of the equityholders' position increases as the volatility increases. There is, however, no solution to the equityholders' optimal risk selection problem. In this paper, we develop a model of firm where a firm's activities are a portfolio of correlated projects, with uncertain future values and independent project-specific risks. Analogous to Markowitz's (1952) portfolio theory, there is a concave risk-future value "portfolio frontier" (here termed "project plans"). The future values are discounted at the required rate of return for the associated risk, leading to better diversified project plans to be discounted at a lower rate. This results in a trade-off between the present value of the firm, V_0, and its associated risk, σ. The first-best optimal choice of risk maximises the value of the firm. The equityholders' choice of risk is determined by their indifference curves that is a loci of the pairs (V_0, σ) which yields the same present value of their position. They choose the highest indifference curve that is feasible with the firm's possible project plans, given by the one that is tangent to the project plans curve. Different bail-out / bail-in schemes imply different set of indifference curves, with steeper indifference curves yielding higher optimal risk choices and lower present values. Our analysis shows that higher leverage means higher risk choice, and for reasonable levels of leverage, the equityholders choose risks in the ascending order for the cases of no bail-in / bail-out (i.e. APR), bondholder bail-in with equity-conversion CoCo bond and bail-in with write-off CoCo bond. The choice of higher volatility means a transfer of wealth from bondholders to equityholders (from the seller of the option to the buyer), while non-value maximising choice means value destruction.

Given this result, the next question asked is whether there is a way of alleviating the equityholders' incentive for high risk-taking. Using the above model, we are able to show that by

[4] E.g. Jensen and Meckling (1976).

imposing higher required rate of return for higher risk-taking, the equityholders will rationally reduce the risk-level chosen. In policy terms this is analogous to Pigouvian tax. The agency costs are the negative externality of equityholders' actions on the remaining stakeholders of the bank. By implementing higher costs on their action, the regulators can force the equityholders to select less risky choice. Alternatively, leaving it to the bondholders, this is akin to them demanding higher cost of debt, compensating them for their cost of needing to closely monitor the credit quality of the firm.

We utilise this result to propose practical solutions to alleviate agency costs of CoCo bond bail-in. Mirroring the practice in the corporate bond market, we suggest covenants as an efficient way for bondholders to monitor the credit quality and risk-taking profile of a bank. Specifically, we propose different types of covenants within the CoCo bond indentures. When the bank's solvency is reasonably high, we propose a "ratchet" coupon financial accounting covenant. With a rise in the leverage ratio (due to depleting equity capital) the ratchet is triggered, automatically increasing the cost of debt and reducing the return on equity (ROE) of the firm. More precisely, at each ratchet trigger point there is a step down in the ROE. This introduces concavity in equityholders' return on equity (as opposed to convexity at the CoCo trigger point), and this and the higher cost of capital once triggered (acting as Pigouvian tax) discourage equityholders from taking higher risk. This type of covenants are argued to be effective when the firm's solvency is high. In a falling solvency scenario we propose a different type of covenants, specifically asset sweep and debt sweep covenants. In the former the asset is partially sold off to pay down some of the debt, while in the latter newly issued debt is used to repay existing debts. Both discourage (or prevent, in the case of debt sweep) equityholders from piling on more debt to attempt "gamble-for-ressurection". The mechanisms of these covenants are outlined in detail in the text.

So far the literature on agency costs associated with CoCo bond bail-in has focused mainly on highlighting the over-investment problem (for example Berg and Kaserer (2011), Koziol and Lawrenz (2012), Pennacchi, Vermaelen and Wolf (2014), Hilscher and Raviv (2014) and Hori and Martin Cerón (2017)). None of these consider bondholders' monitoring cost, ultimately borne by equityholders via higher cost of capital, as a mean to alleviate the agency problem. Jensen and Meckling (1976) investigates the role of monitoring cost in the context of a trade-off between firm value and non-pecuniary benefit for the managers of a firm. Their indifference curve analysis is akin to the one developed here. In a wider sense, our model contributes to the broader literature of theory of firm. Whereas previous works point out the positive vega of the equityholders' position as the root of a firm's over-investment problem (e.g. Eberhart and Senbet (1993), Berg and Kaserer (2011), Hori and Martin Cerón (2017)), which leaves the question of "Then why do firms not keep increasing its risk-taking?", our optimal risk model is able to answer this question specifically which leads to concrete policy suggestions.

The paper is structured as follows. Section 2 briefly reviews the related literature, and describes different bail-out / bail-in scenarios with their payoffs and valuations. Section 3 develops the optimal risk model using indifference curves and derives the main theoretical results. In Section 4, we suggest a practical way of implementing these results by proposing financial and non-financial covenants in the CoCo bond indenture. Finally, Section 5 gives concluding remarks.

2 CoCo Bonds

Contingent convertible bonds, or CoCo bonds, are bonds that either convert to equity or are written-down/off when a bank's capital ratio hits a trigger ratio. The bond is designed to establish "bail-in" by the creditors, replacing "bail-out" by the government. This formalises deviation from absolute priority rule (DAPR), where under the absolute priority rule (APR) bondholders do not bear losses until equityholders have been wiped out.

CoCo bonds, initially termed "reverse convertible debentures" (RCDs), were first recommended by Flannery (2005). The idea was to counter a firm's incentive to use tax-advantaged debt rather than equity, that also reduces the firm's ability to take losses. Flannery argued that the issuance of RCDs would still maintain the tax advantage whilst reducing the latter risk. In more recent terminology the suggested structure was an equity-conversion CoCo bond with a market value trigger (explained below). In terms of post-trigger treatments there are two types of CoCo bonds: equity-conversion, and write-down or write-off bonds. In the former, upon trigger CoCo bonds are converted into common equity,[5] whilst in the latter, bonds are

[5] Coffee (2010) suggests a conversion into preference shares with cumulative dividends and voting rights, for

either partially written down or wholly written off to cover the incurred loss. For the trigger mechanism, broadly two types are suggested in the literature: an accounting ratio trigger and a market value trigger. Himmelberg and Tsyplakov (2011), Berg and Kaserer (2011) and Hilscher and Raviv (2014) are examples of the former. However Flannery (2014), amongst others, argues that "accounting measures trail economic developments when a firm encounters difficulties, and managers can manipulate accounting statements" (p235). Pennacchi (2010), Prescott (2011), Glasserman and Nouri (2012), Koziol and Lawrenz (2012) and Albul, Jaffee and Tchistyi (2013) are examples that adopt the latter. In this case Sundaresan and Wang (2014) point out that a market trigger bail-in does not lead to a unique competitive equilibrium. This problem arises from the fact that the share price reflects both the current value of the firm (say below the CoCo trigger value) and the post-bail-in value of shares (which would then be above the trigger value). Many have sought solutions to this: Pennacchi (2010) by including CoCo bond values in the capital ratio's numerator; Prescott (2011) by introducing a "sliding conversion rule"; Glasserman and Nouri (2012) argue that the multiple equilibria problem is a feature of discrete-time models; Albul, Jaffee and Tchistyi (2013) achieve unique equilibrium by placing the trigger directly on the asset value. However market value trigger also suffers from the possibility of price manipulation; as suggested by Pennacchi, Vermaelen and Wolff (2014), "the financial industry justifies its objection to CoCos with market based triggers on the basis of... manipulation/death spiral fears." (p550-1).[6] In this paper we follow the common market practice and focus on accounting capital ratio trigger CoCos.[7]

In this paper, we are interested in the equityholders' optimal choice of project given their payoffs under different scenarios of bail-out / bail-in. The scenarios that we focus are: no bail-out/in, government bail-out, bail-in with equity-conversion CoCo bonds, and bail-in with write-off CoCo bonds. We make use of the payoffs derived and analysed in detail in Hori and Martin Cerón (2017) for each of these cases in a simple firm financed by common equity capital and discount bonds (vanilla or CoCo). The summary of these are now reproduced below.

The set-up is as follows. The total face value of the bonds is F, which may include equity-

risk incentive reasons.

[6] See for example, "'Coco' trigger plan draws wary response", *The Financial Times*, April 4, 2011. Duffie (2010) suggests using multiday average as a solution to this.

[7] McDonald (2011) suggests a dual price trigger that depends on both the bank's share price and the value of a market stock index.

conversion CoCo bond (face value F_C) or write-off CoCo bond (face value F_W). The face value of the plain vanilla bond is F_B. Therefore the firms have either $F = F_B$ (no bail-out/in or government bail-out), $F = F_B + F_C$ (equity-conversion CoCo bond bail-in) or $F = F_B + F_W$ (write-off CoCo bond bail-in). The equity value at time 0 is E_0. The total asset value at the maturity of the bonds T is V_T. All bail-outs / bail-ins trigger at a trigger capital ratio τ. There exists a minimum capital ratio $\underline{E}$ set by the regulator, where $\underline{E} > \tau$. In all cases, where possible, when bailed-out/in the equity is boosted to this minimum capital ratio $\underline{E}$.

In the case that there is *no bail-out / bail-in*, the payoffs to bondholders and equityholders follow the absolute priority rule (APR), where the equityholders are first wiped out before bondholders' positions are affected. As well established in the literature, the equityholders' payoff at T is that of a call option with strike price F, while the bondholders receive the bond face value F unless they become the residual claimants when $V_T < F$. These payoffs can be summarised as,

$$\begin{aligned} E^N &= \max\left[V_T - F, 0\right] \\ D^N &= \min\left[V_T, F\right]. \end{aligned} \tag{1}$$

The Black-Scholes-Merton valuation of the debt and equity holdings at time $t = 0$ are (see for example Merton (1974)),

$$\begin{aligned} V_E^N &= C\left(F\right) \\ V_D^N &= Fe^{-rT} - P\left(F\right) \end{aligned} \tag{2}$$

where $C\left(K\right)$ and $P\left(K\right)$ are call and put option values with strike K given by

$$\begin{aligned} C\left(K\right) &= V_0 N\left(d_1\left(K\right)\right) - Ke^{-rT} N\left(d_2\left(K\right)\right) \\ P\left(K\right) &= -V_0 N\left(-d_1\left(K\right)\right) + Ke^{-rT} N\left(-d_2\left(K\right)\right) \\ \text{with } d_1\left(K\right) &= \frac{\ln\left(\frac{V_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\text{ , } d_2\left(K\right) = d_1 - \sigma\sqrt{T}, \end{aligned} \tag{3}$$

and r is the risk-free rate, T is the options' time to maturity and σ is the asset volatility. Note that by the use of the put-call parity[8] (see for example Hull (2017)), the equityholders'

[8] The *put-call parity* states that for a non-dividend paying asset the following parity holds at time $t \leq T$,

$$P_t\left(K\right) + S_t = C_t\left(K\right) + Ke^{-r(T-t)},$$

where $P_t\left(K\right)$ and $C_t\left(K\right)$ are the prices of put and call options with strike price K and maturity T, S_t is the position is equivalent to,

$$V_E^N = V_T - Fe^{-rT} + P(F). \tag{4}$$

In other words their position is protected by the put option $P(F)$ in the case that $V_T < F$, provided by the bondholders.

In the case of a *government bail-out*, the APR is still followed, but once the capital ratio τ is breached the government injects common equity E_G to ensure that the minimum common capital ratio is maintained at $\underline{E}$.[9] The bondholders are fully protected at their face value F, and thus the balance sheet is restored to $\frac{F}{1-\underline{E}}$. The equityholders' and bondholders' payoffs are,

$$\begin{aligned} E^{BO} &= \max[V_T - F, 0] \\ D^{BO} &= F. \end{aligned} \tag{5}$$

Their valuations at $t = 0$ are,

$$\begin{aligned} V_E^{BO} &= C(F) \\ V_D^{BO} &= Fe^{-rT}. \end{aligned} \tag{6}$$

Comparing Eqn (6) with Eqn (2) suggests that, in government bail-out, the government replaces the bondholders as the provider of the put option hedge $P(F)$ to the equityholders. However the equityholders are no different with the government bail-out as they were with no bail-out/in. This is so, as we are considering payoffs at the bond maturity T. Hori and Martin Cerón (2017) relaxes this assumption and considers the case where the firm's solvency is reviewed at $t < T$, in which case the equityholders also benefit from the bail-out in cases where the bank is otherwise insolvent.

With the *equity-conversion CoCo bond bail-in*, the CoCo bail-in is triggered when the pre-conversion capital ratio falls below τ. Where possible, the capital ratio is restored to $\underline{E}$ with the converted equity. As opposed to the government bail-out case there is no external capital injection, and therefore the balance sheet remains depleted. In this paper we assume that, in the extreme case that the CoCo bond is not enough to cover the whole of the loss, the regulator will exercise its bail-in power to convert the necessary plain vanilla debt into

price of the underlying asset, and $Ke^{-r(T-t)}$ is the price of a zero coupon bond with face value K and r is the risk-free rate.

[9] Hori and Martin Cerón (2017) also considers the case of a government bail-out with preferece shares.

ordinary shares to restore solvency.[10] The equityholders are guaranteed the minimum of τV_T, at which point the CoCo conversion is triggered. The bondholders (vanilla and CoCo) hold the remaining $(1-\tau)V_T$ in the forms of either plain vanilla bond, unconverted CoCo bond or CoCo-converted equity. The payoffs at T are,

$$\begin{aligned} E^C &= \max\left[V_T - F, \tau V_T\right] \\ D^C &= \min\left[F, (1-\tau)V_T\right]. \end{aligned} \tag{7}$$

The valuations at $t = 0$ are,

$$\begin{aligned} V_E^C &= C(F) + \left[(1-\tau)P\left(\tfrac{F}{1-\tau}\right) - P(F)\right] \\ V_D^C &= Fe^{-rT} - (1-\tau)P\left(\tfrac{F}{1-\tau}\right). \end{aligned} \tag{8}$$

By using the put-call parity again, V_E^C can be rewritten as,

$$V_E^C = V_T - Fe^{-rT} + (1-\tau)P\left(\frac{F}{1-\tau}\right). \tag{9}$$

Comparing this with Eqn (4) suggests that the equityholders are protected by an extra 'bear spread'-like position[11] provided by the CoCo bondholders, on top of the put option $P(F)$,

$$(1-\tau)P\left(\frac{F}{1-\tau}\right) - P(F). \tag{10}$$

Note that the expression in Eqn (10) is always positive.[12]

Finally, with the *write-off CoCo bond bail-in*, unlike the equity-conversion CoCo bonds case where the bonds are partially converted, here once triggered the whole of the CoCo bonds are immediately written off.[13] It is unclear what happens in reality to the remainder of the written-off bond when the write-off more than covers the bank's loss. Here it is assumed that

[10] In Hori and Martin Cerón (2017), this case is termed *equity-conversion CoCo bail-in-bail-in*. Two other cases of equity-conversion CoCo bail-in are also considered, namely *bail-in-no-bail-out/in*, where APR is restored once all of the CoCo bond is used up to cover the loss, and *bail-in-bail-out*, where the government steps in to inject common equity after all possible CoCo bail-in is exhausted.

[11] A *bear spread* is created by buying a put option at a higher strike price and selling a put option at a lower strike price. The holder of the structure gains from a fall in the underlying asset price.

[12] See the proof of Prop 3 below.

[13] CoCo bonds which are partially written-off are called the *write-down CoCo bonds*. For details see Hori and Martin Cerón (2017).

this net amount is added to the equityholders' position as a contingent capital reserve. Then there is a discontinuity in the payoffs of the equityholders and the bondholders (vanilla plus write-off CoCo bonds) at the trigger point which is represented by the indicator function below,

$$\begin{aligned} E^W &= \max\left[V_T - F, \tau V_T - F_W\right] + F_W \chi_{V_T \leq \frac{F}{1-\tau}} \\ D^W &= \min\left[F, (1-\tau) V_T + F_W\right] - F_W \chi_{V_T \leq \frac{F}{1-\tau}} \end{aligned} \tag{11}$$

where $\chi_{V_T \leq \frac{F}{1-\tau}}$ is the indicator function,

$$\chi_{V_T \leq \frac{F}{1-\tau}} = \begin{cases} 1 & \text{if} \quad V_T \leq \frac{F}{1-\tau} \\ 0 & \text{if} \quad V_T > \frac{F}{1-\tau} \end{cases} . \tag{12}$$

Note as with the equity-conversion case above, a forced bail-in of vanilla bond is assumed when whole of the write-off CoCo bond is not enough to cover the loss. The values of these at $t = 0$ are,

$$\begin{aligned} V_E^W &= C(F) + F_W B_P\left(\tfrac{F}{1-\tau}\right) - \left[P(F) - (1-\tau) P\left(\tfrac{F_B}{1-\tau}\right)\right] \\ V_D^W &= F e^{-rT} - F_W B_P\left(\tfrac{F}{1-\tau}\right) - (1-\tau) P\left(\tfrac{F_B}{1-\tau}\right), \end{aligned} \tag{13}$$

where $B_P(K)$ is the price of a binary put option with unit payout at strike K,

$$B_P(K) = e^{-rT} N(-d_2(K)). \tag{14}$$

In this case, the equityholders have an extra protection given by a 'condor'-like position[14], on top of the put option $P(F)$ that is inherently present in the no bail-out/in and government bail-out cases,

$$F_W B_P\left(\frac{F}{1-\tau}\right) - \left[P(F) - (1-\tau) P\left(\frac{F_B}{1-\tau}\right)\right]. \tag{15}$$

The equityholders' payoffs for all four restructuring scenarios are depicted in Fig 1.

[14] A *condor* is created by a combination of a bull put spread with a bear put spread, where a bull put spread is formed by buying a put option at a lower strike price and selling a put option at a higher strike price. A holder of such a structure gains if the underlying asset price remains within a range.

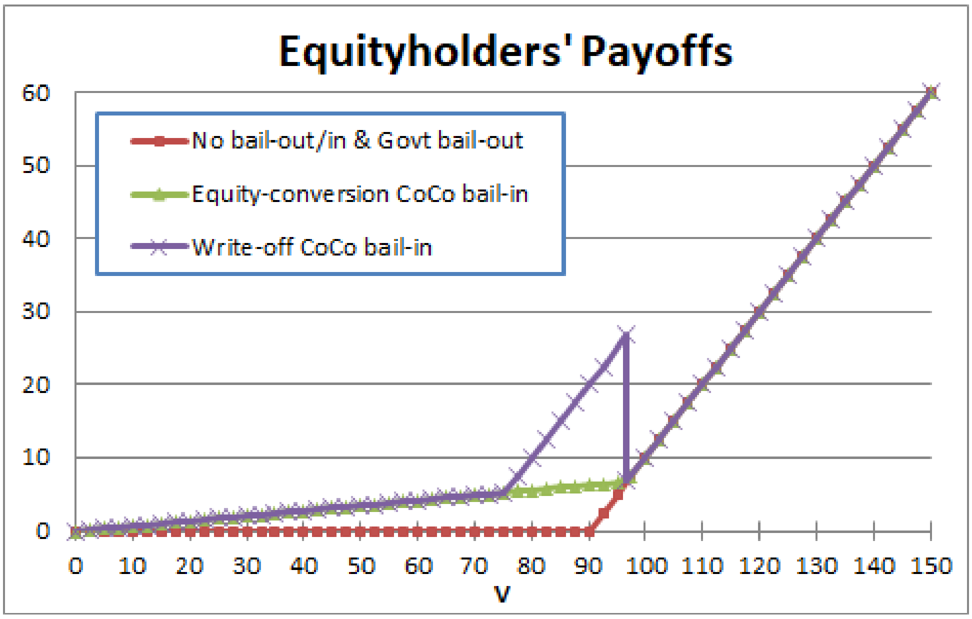

Figure 1: Equityholders' pay-offs at time T under different bail-out/in scenarios

3 Indifference Curve Analysis

Consider a bank that raises fund by equity and debt. The fund is invested in a portfolio of projects, whose outcomes are uncertain. Then for risk-averse investors, the present value of the bank's asset value is estimated using an appropriate risk-adjusted discount rate. The optimal portfolio decision is then dependent on the risk-return profiles of the feasible project mixes. In particular there may be a trade-off between higher risk-taking, leading to a possible higher expected outcome at maturity, and higher discounted rate. The current literature does not provide an answer to how the bank would select its optimal portfolio under such scenario.

Here we build a model of a simple bank with two possible investment projects with uncertain outcomes. Assuming sufficiently low correlation between the two outcomes, there is a risk-diversification effect in choosing a portfolio of the two projects. This results in a higher present value of the portfolio due to a lower risk-adjusted rate required for discounting. Under a simple condition then there exists an interior solution to the bank's maximum value, which is the bank's first-best choice of portfolio. However the decision of portfolio selection is taken by the equityholders, whose convex payoff structure (they gain fully from the bank's success, but their loss is limited) means that their optimisation behaviour of the value of their holdings would lead to a sub-optimal choice of portfolio selection. This can be analysed using an indifference curve model, similar to that adopted by Jensen and Meckling (1976) in their analysis of monitoring of the behaviour of the managers with non-pecuniary benefits. The indifference curves here describe the trade-off between risk (volatility) and value. The equityholders' choice of portfolio is then given by the tangent point between the bank's possible project portfolio frontier and their highest attainable indifference curve. We analyse these under different restructuring scenarios of no bail-out/in, government bail-out, equity-conversion CoCo bond bail-in and write-off CoCo bond bail-in.

The set-up differs from the traditional models in two ways. Firstly, it assumes the distribution of possible future outcomes of projects to be given, which is then discounted to today to estimate the present values. Thus the choice of projects affect the present value, not only through the chosen expected future value, but also via its effect on the required discount rate. This differs from the Merton (1974) set-up, where the future values of a security is given as a distribution of outcomes given today's value of the security.[15] Secondly the model contrasts with the CAPM set-up where the securities are priced assuming that all of idiosyncratic risk have been diversified away, which cannot be assumed for the limited number of possible projects available to a bank.

3.1 Bank's Project Plans

The bank has two possible projects, $i = 1, 2$, both of which mature at T. Their expected value and variance are given by $E\left[V_T^i\right]$ and σ_i^2, with correlation ρ. A "project plan" is given by the weights $(w, 1-w)$ of the two projects, and has the expected value and the variance,

$$\begin{aligned} E\left[V_T(w)\right] &= wE\left[V_T^1\right] + (1-w)E\left[V_T^2\right] \\ \sigma^2(w) &= w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\rho\sigma_1\sigma_2. \end{aligned} \tag{16}$$

[15] Jensen and Meckling (1976) make the same point when they state, "While we used the option pricing model above to motivate the discussion and provide some intuitive understanding of the incentives facing the equity holders, the option pricing solutions of Black and Scholes (1973) do not apply when incentive effects cause V to be a function of the debt/equity ratio as it is in general and in this example. Long (1974) points out this difficulty with respect to the usefullness of the model in the context of tax subsidies on interest and bankruptcy cost. The results of Merton (1974) and Galai and Masulis (1976) must be interpreted with care since the solutions are strictly incorrect in the contect of tax subsidies and/or agency costs."

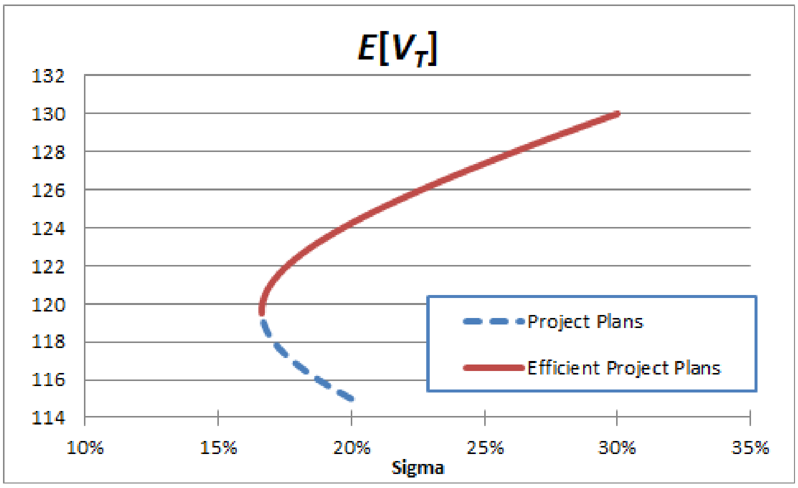

Figure 2: Expected bank value $E[V_T]$ for different values of w

Assume project 1 is riskier than project 2, i.e. $E\left[V_T^1\right] > E\left[V_T^2\right]$ and $\sigma_1 > \sigma_2$. Then in minimising $\sigma^2(w)$ with respect to $w \in (0,1)$, for low enough ρ, namely $\rho \in \left[-1, \frac{\sigma_2}{\sigma_1}\right)$, there exists a minimum-variance plan $w_{\min}$ with $\sigma_{\min} < \sigma_2$ given by,

$$\begin{aligned} w_{\min} &= \tfrac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2} \\ \sigma_{\min} &= \sigma\left(w_{\min}\right) = \tfrac{\left(1-\rho^2\right)\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2}. \end{aligned} \tag{17}$$

Project plans $w \in [w_{\min}, 1]$ then represent the set of efficient plans,[16] with $\sigma \in [\sigma_{\min}, \sigma_1]$. In this region then,

$$\frac{d\sigma}{dw} > 0 \text{ for } \sigma \in (\sigma_{\min}, \sigma_1] \tag{18}$$

where

$$\frac{d\sigma}{dw} = \frac{1}{\sigma}\left[w\sigma_1^2 - (1-w)\,\sigma_2^2 + (1-2w)\,\rho\sigma_1\sigma_2\right]. \tag{19}$$

Fig 2 depicts the graph of $E[V_T]$ for different values of $w \in [0,1]$, with its minimum-variance plan and the efficient plans on the upper branch.

The bank discounts its chosen project plan at the risk-adjusted rate $r(w, \sigma(w))$, where

[16] An efficient plan is one that attains the highest expected value $E[V_T(w)]$ for a given risk $\sigma(w)$, or equivalently, that attains the given expected value with the lowest risk.

$\frac{dr}{d\sigma} > 0$. Therefore the current market value of the bank is given by,

$$V_0(w) = e^{-r(w,\sigma(w))T} E[V_T(w)]. \tag{20}$$

3.2 The First-best Optimal Plan

The first-best optimal plan for the bank is w^* such that

$$w^* = \arg \max_{w \in [w_{\min}, 1]} V_0(w). \tag{21}$$

To find the optimal plan we compute and equate to zero the derivative,

$$\frac{dV_0}{dw}(w) = -T\frac{dr}{dw} e^{-r(w,\sigma(w))T} E[V_T(w)] + e^{-r(w,\sigma(w))T} \left(E\left[V_T^1\right] - E\left[V_T^2\right]\right). \tag{22}$$

To rule out corner solutions, we require $\frac{dV_0}{dw}(w_{\min}) > 0$ and $\frac{dV_0}{dw}(1) < 0$. In particular, for the case $r(w, \sigma(w)) \equiv r(\sigma(w)) \Rightarrow \frac{dr}{dw} = \frac{dr}{d\sigma}\frac{d\sigma}{dw}$, as $\frac{d\sigma}{dw} = 0$ at $w = w_{\min}$,

$$\frac{dV_0}{dw}(w_{\min}) = e^{-r(\sigma_{\min})T} \left(E\left[V_T^1\right] - E\left[V_T^2\right]\right) > 0. \tag{23}$$

As this is strictly positive the solution cannot be at this point, i.e. $w^* > w_{\min}$. At $w = 1$, as then $E[V_T(1)] = E\left[V_T^1\right]$, $\sigma(1) = \sigma_1$ and $\frac{d\sigma}{dw}(1) = \sigma_1 - \rho\sigma_2$,

$$\frac{dV_0}{dw}(1) = e^{-r(1,\sigma_1)T} \left[-T\frac{dr}{d\sigma}(\sigma_1 - \rho\sigma_2) E\left[V_T^1\right] + \left(E\left[V_T^1\right] - E\left[V_T^2\right]\right)\right]. \tag{24}$$

Then for an interior solution w^* that satisfies

$$\frac{dV_0}{dw}(w^*) = 0, \tag{25}$$

we require $\frac{dV_0}{dw}(1) < 0$, i.e.

$$\frac{E\left[V_T^1\right] - E\left[V_T^2\right]}{T(\sigma_1 - \rho\sigma_2) E\left[V_T^1\right]} < \frac{dr}{d\sigma}(1). \tag{26}$$

What condition (26) implies is that, for an internal solution, the project plan risk needs to be sufficiently costly (the right-hand side is large), or else the bank would simply choose $w^* = 1$

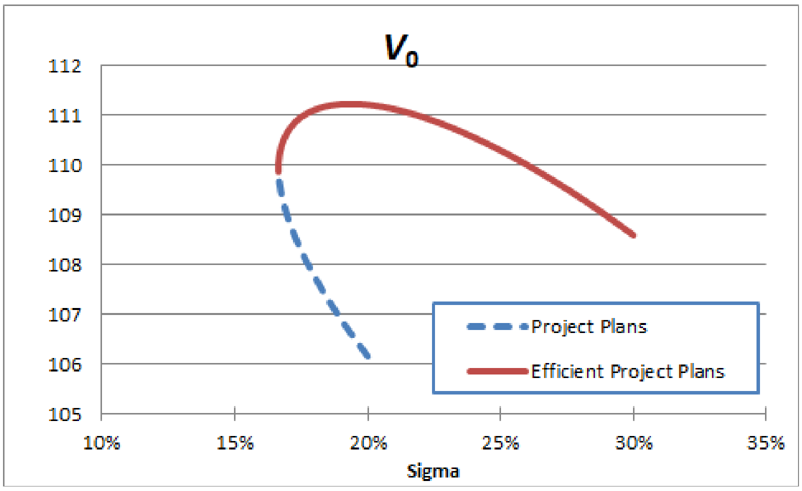

Figure 3: Bank's present value V_0 for different values of w

and only invest in the project with the higher expected value. Fig 3 depicts the graph of the present value V_0 in Eqn (20) given that condition (26) is satisfied. The optimal plan w^* is given by the curve's maximum point.[17]

Example 1 (Constant Market Price of Risk)

$$r\left(w,\sigma\left(w\right)\right)=r_f+\lambda\sigma\left(w\right). \tag{27}$$

where λ is the constant market price of risk and r_f is the market risk-free rate. Then $\frac{dr}{dw}=0$ and $\frac{dr}{d\sigma}=\lambda$, and so an internal solution exists for $\lambda>\frac{E\left[V_T^1\right]-E\left[V_T^2\right]}{T(\sigma_1-\rho\sigma_2)E\left[V_T^1\right]}$.

Example 2 (Linear Average Market Price of Risk)

$$r\left(w,\sigma\left(w\right)\right)=r_f+\lambda\left(w\right)\sigma\left(w\right). \tag{28}$$

where $\lambda\left(w\right)$ is given by the weighted average of the respective market prices of risk of the two projects, λ_1 and λ_2,

$$\lambda\left(w\right)=w\lambda_1+\left(1-w\right)\lambda_2. \tag{29}$$

For our numerical analyses in this section we use the linear average market price of risk given in Example 2.

[17] Fig 3 is simluated using the linear average market price of risk case given in Example 2 below.

3.2.1 Discussion: The CAPM

As opposed to the above examples, in the Capital Asset Pricing Model (CAPM) the risk-adjusted rate of return is linear in w,

$$r(w) = wE[r_1] + (1-w)E[r_2] = r_f + [w\beta_1 + (1-w)\beta_2]MRP \tag{30}$$

where MRP is the market risk premium, $MRP = E[r_M] - r_f$, and $E[r_M]$ is the expected market return rate. In this case then, $\frac{dr}{dw} = (\beta_1 - \beta_2)MRP$, and so,

$$\frac{dV_0}{dw}(w) = -T(\beta_1 - \beta_2)MRP\ e^{-r(w)T}E[V_T(w)] + e^{-r(w)T}\left(E\left[V_T^1\right] - E\left[V_T^2\right]\right), \tag{31}$$

for which,

$$\begin{aligned} \tfrac{dV_0}{dw}(1) &= -T(\beta_1 - \beta_2)MRP\ e^{-E[r_1]T}E\left[V_T^1\right] + e^{-E[r_1]T}\left(E\left[V_T^1\right] - E\left[V_T^2\right]\right) \\ &= [1 - T(\beta_1 - \beta_2)MRP]\,e^{-E[r_1]T}E\left[V_T^1\right] - e^{-E[r_1]T}E\left[V_T^2\right] \\ &\approx e^{-T(\beta_1-\beta_2)MRP}\left(e^{-E[r_1]T}E\left[V_T^1\right] - e^{T(\beta_1-\beta_2)MRP}e^{-E[r_1]T}E\left[V_T^2\right]\right) \\ &= e^{-T(\beta_1-\beta_2)MRP}\left(V_0^1 - V_0^2\right). \end{aligned} \tag{32}$$

This is positive assuming $V_0^1 > V_0^2$. Hence there is no internal solution for the optimal plan for the bank. Specifically, the bank would always simply choose the riskier project 1.

The reason for this is that, with the CAPM, the project-specific idiosyncratic risks are assumed to have been diversified away. In contrast, in the constant and linear average λ examples above, for ρ low enough there is enough risk-diversification effect such that a combination of the two projects would have a higher present value than the present values of the single projects, due to the lower risk-adjusted discount rate.

3.3 Equityholders' Choice of Risk

In the above section we established the bank's first-best choice of risk. However, it is the equityholders who choose the bank's project plan.[18] In this section we investigate their choice

[18] Here we ignore the principal-agent problem between the equityholders (principals) and the managers (agents).

under the different restructuring scenarios.

3.3.1 No Bail-out/in and Government Bail-out

As discussed in Section 2, the equityholders' payoff at T is the same under the cases of no bail-out/in and the government bail-out, where their payoff equals that of a call option with strike price F. The value of this is given by the familiar Black-Scholes option pricing formula in Eqns (2) and (6),

$$V_E^N = C(F) \tag{33}$$

where $C(F)$ is given by Eqn (3).

The equityholders' *indifference curves* (ICs) are defined as the loci of pairs (V_0, σ) that yields the same value of V_E^N, i.e. all values of V_0 and σ such that,

$$V_E^N(V_0, \sigma) = V \tag{34}$$

for a given value of V. We list some properties of the ICs:

Properties 1 (Equityholders' Indifference Curves) *The ICs have the following properties:*

1. *The ICs are downward-sloping.*
2. *The ICs are quasi-concave.*
3. *Given V_E^N, the IC steepens as F increases, pivoted at $\sigma = \infty$.*

Proof. From the definition of the ICs,

$$\begin{aligned} dV_E^N &= \frac{\partial V_E^N}{\partial V_0} dV_0 + \frac{\partial V_E^N}{\partial \sigma} d\sigma = 0 \\ \Leftrightarrow \frac{dV_0}{d\sigma} &= -\frac{\partial V_E^N / \partial \sigma}{\partial V_E^N / \partial V_0} = -\frac{vega^N}{\Delta^N} = MRS^N. \end{aligned} \tag{35}$$

MRS is the marginal rate of substitution between V_0 and σ along the IC, and,

$$\begin{aligned} \Delta^N &= N(d_1) \\ vega^N &= V_0 \sqrt{T} N'(d_1). \end{aligned} \tag{36}$$

Immediately, ICs are downward-sloping as both Δ^N and $vega^N$ are strictly positive,

$$MRS^N = -\frac{V_0\sqrt{T}N'(d_1)}{N(d_1)} < 0. \tag{37}$$

To check the curvature of the ICs, consider the limits of σ. First when it becomes large, using the limiting properties of call option values outlined in Properties B1 of Appendix B,

$$\lim_{\sigma\to\infty} V_E^N = V_0 \tag{38}$$

i.e. the equityholders' position approximates the asset value. Therefore at this point $\Delta = 1$ and $vega = 0$, and hence $MRS^N \to 0$ as σ becomes large. At the other limit when σ approaches zero, again from Property B1,

$$\lim_{\sigma\to 0} V_E^N = \max\left[V_0 - Fe^{-rT}, 0\right]. \tag{39}$$

On an IC where $V_E^N > 0$ then, V_0 must unambiguously be greater than Fe^{-rT} as $\sigma \to 0$. Therefore, at this point the equityholders' position approximates the value of the forward $V_0 - Fe^{-rT}$, and thus again $\Delta = 1$ and $vega = 0$, making $MRS^N \to 0$ as $\sigma \to 0$. Given that $MRS^N < 0$ for $\sigma \in (0, \infty)$, the curves must therefore be quasi-concave.[19] Finally, as F increases V_0 has to adjust in order to keep V_E^N constant for given σ,

$$\begin{aligned} dV_E^N &= \frac{\partial V_E^N}{\partial F}dF + \frac{\partial V_E^N}{\partial V_0}dV_0 = 0 \\ \Leftrightarrow \frac{dV_0}{dF} &= -\frac{\partial V_E^N/\partial F}{\partial V_E^N/\partial V_0} = \frac{e^{-rT}N(d_2)}{N(d_1)} > 0 \text{ for } \sigma < \infty. \end{aligned} \tag{41}$$

Thus the IC shifts up. To show that this shift is a steepening of the curve pivoted at $\sigma = \infty$,

[19] The curvature of the ICs is given by,

$$\frac{d^2V_0}{d\sigma^2} = -\frac{V_0\sqrt{T}}{\sigma}\frac{N'(d_1)}{N(d_1)}\left[\left(\frac{N'(d_1)}{N(d_1)} + d_2\right)^2 + d_2\sigma\sqrt{T}\right]. \tag{40}$$

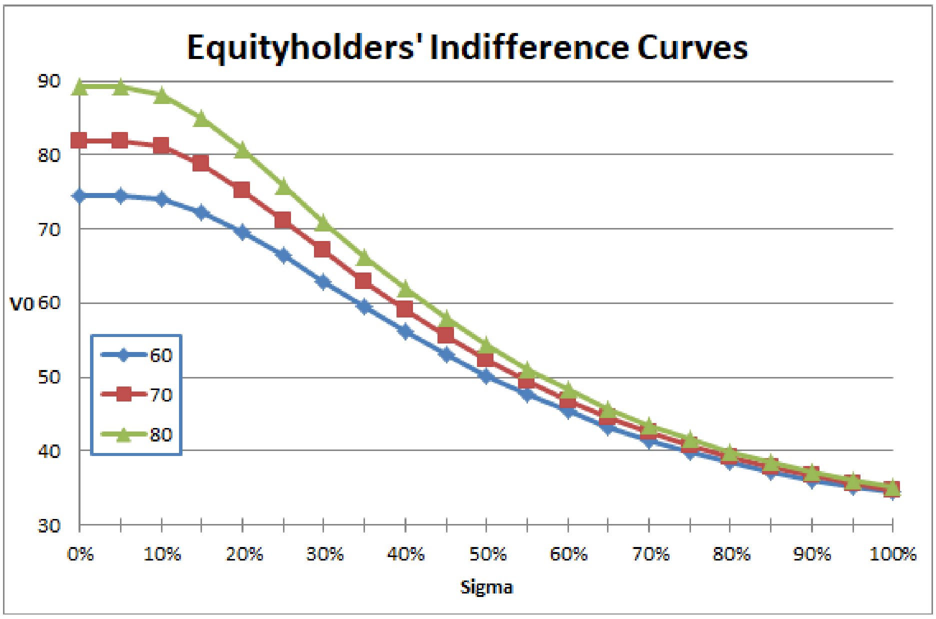

Figure 4: Equityholders' indifference curves for different debt levels F for constant V_E^N

note first that $\lim_{\sigma\to\infty} \frac{dV_0}{dF} = 0$. At any other values of σ,

$$\begin{aligned} \frac{dMRS^N}{dF} &= \frac{\partial MRS^N}{\partial F} + \frac{\partial MRS^N}{\partial V_0}\frac{dV_0}{dF} \\ &= \left[-\frac{V_0}{F\sigma}\frac{N'(d_1)}{N(d_1)}\left(d_1 + \frac{N'(d_1)}{N(d_1)}\right)\right] + \left[\frac{N'(d_1)}{\sigma N(d_1)}\left(d_2 + \frac{N'(d_1)}{N(d_1)}\right)\right]\frac{e^{-rT}N(d_2)}{N(d_1)} \\ &= -\frac{N'(d_1)}{\sigma N(d_1)}\left[\frac{V_E^N}{FN(d_1)}\left(d_1 + \frac{N'(d_1)}{N(d_1)}\right) + \sigma\sqrt{T}\frac{e^{-rT}N(d_2)}{N(d_1)}\right] < 0. \end{aligned} \tag{42}$$

The final line is negative $\forall\sigma < \infty$ as each term within it are positive except for d_1, and $xN(x) + N'(x) > 0\ \forall x$ which is shown in Property A2 of Appendix A. Thus the ICs steepens as F increases. ■

These properties are demonstrated in Fig 4. Intuitively, what Property 1 states is as follows. Firstly, for equityholders there is a trade-off between the present value of the bank's project plan, V_0, and its volatility, σ. In fact there is a continuum of pairs of (V_0, σ) for which the value of the equityholders' position $V_E^N(V_0, \sigma)$ is the same. This is the downward-sloping indifference curve. Secondly, the marginal value of σ is higher for medium values of σ than for extreme values. This means that the equityholders are willing to give up more of V_0 in exchange for higher σ, i.e. the ICs are steeper, for those values of σ. This makes the shape of

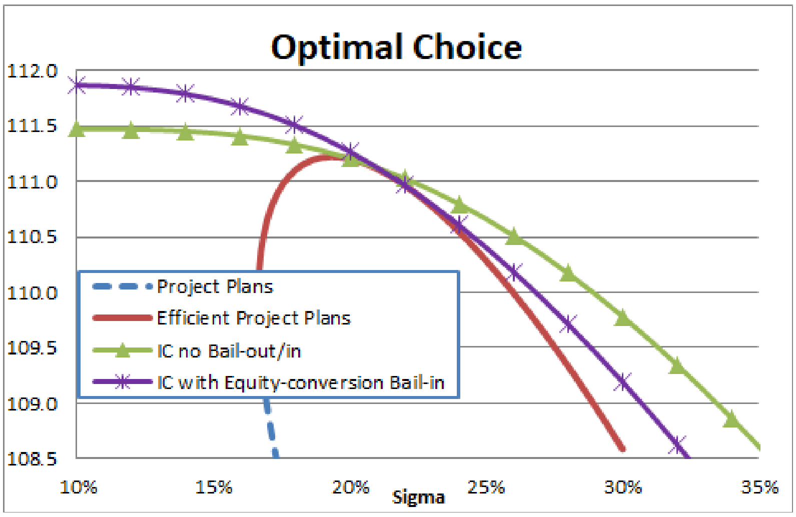

Figure 5: Equityholders' optimal choice with (i) no bail-out/in, and (ii) equity-conversion CoCo bail-in.

ICs quasi-concave. Finally, the higher the liability raised by debt, the lower the value of firm claimed by the equityholders, and therefore for a given level of risk, the higher the level of V_0 required for them to attain the same level of V_E^N. This is shown by the steepening ICs.

The equityholders' optimisation problem is the selection of the highest attainable indifference curve given project plans (20). Diagrammatically, the solution is given by the tangent point, as shown in Fig 5. The graph suggests that the equityholders would choose a higher risk project plan w^N than the bank's first-best choice w^*. More formally,

Proposition 1 $w^N > w^*$.

Proof. The equityholders' choice of the optimal project plan is determined by,

$$\max_{w\in[w_{\min},1]} V_E^N = C(F) \text{ subject to } V_0(w) = e^{-r(w,\sigma(w))T} E[V_T(w)]. \tag{43}$$

The solution w^N is the w that satisfies,

$$\frac{dV_E^N}{dw}(w) = \frac{\partial V_E^N}{\partial V_0}\frac{dV_0}{dw} + \frac{\partial V_E^N}{\partial \sigma}\frac{d\sigma}{dw} = N(d_1(F))\frac{dV_0}{dw}(w) + V_0\sqrt{T}N'(d_1(F))\frac{d\sigma}{dw}(w) = 0. \tag{44}$$

However at w^*, we know from Eqn (25) that $\frac{dV_0}{dw}(w^*) = 0$. On the other hand $\frac{d\sigma}{dw} > 0$ for $w \in [w_{\min}, 1]$, and hence $\frac{dV_E^N}{dw} > 0$ at w^*. Therefore $w^N > w^*$. ■

The indifference curve analysis is particularly useful as it demonstrates clearly the agency costs associated with having the equityholders as the decision maker of firm's risk-taking. As proved the equityholders select a higher risk project plan w^N compared with the firm value maximising w^*. Immediately then, by equityholders optimising the value of their holdings and not that of the firm, it results in *value destruction* of the firm. This is one type of agency costs. The other type is the *wealth transfer*. The equityholders' position is a long call option as given above, the value of which increases with the increase in the risk chosen. In contrast, for the no bail-out/in scenario the bondholders hold a short put option, as seen in Eqn (2),

$$V_D^N = Fe^{-rT} - P\left(F\right) \tag{45}$$

where $P\left(F\right)$ is given by Eqn (3). Put-call parity[20] implies that V_D^N decreases by the same amount as the increase in V_E^N with the increase in the risk. The equityholders' choice of w^N away from the first-best w^* therefore results in the wealth being transferred from the bondholders to the equityholders. On the other hand, in the government bail-out scenario, the bondholders are unaffected as they are guaranteed at F as was shown in Eqn (5),

$$V_D^{BO} = Fe^{-rT}. \tag{46}$$

In this case then the wealth transfer is from the taxpayers to the equityholders.

Note, here it is assumed that the government bail-out takes the form of common shares capital injection. Another possibility would be for the capital injection to be in the form of preference shares. This will have two opposing effects on the equityholders' position. The positive effect is that of smaller (or no) dilution. The negative effect is that of a reduced claim on the asset, due to higher ranking of preference shares. Hori and Martin Cerón (2017) shows that, in its set-up, the positive effect always outweighs the negative, and therefore the equityholders take higher risks with the preference shares bail-out. We expect the same in this paper's set-up.

[20] The *put-call parity* states that, for a non-dividend paying underlying asset, the value of a call option plus a bond equals the value of a put option plus the underlying asset, i.e. $C\left(K\right) + Ke^{-rT} = P\left(K\right) + S$. As values of the bond or the underlying asset do not depend on the volatility of the underlying asset price, an increase in the volatility therefore must induce the same increase in the values of the call and the put options.

We can also state the following for the no bail-out/in and common shares government bail-out schemes:

Proposition 2 *The risk-taking is higher, the higher the leverage.*

Proof. This follows immediately from the third property of Properties 1 - the steeper the IC, the further along to the right the tangent point is in Fig 5. ■

This implies higher agency costs for higher leveraged banks.

3.3.2 Equity-conversion CoCo Bond Bail-in

We apply the same analysis to equity-conversion CoCo bond bail-in. We established in Eqn (8) that the value of equityholders' position when the bank has issued equity-conversion CoCo bonds is,

$$V_E^C = C(F) + \left[(1-\tau) P\left(\frac{F}{1-\tau}\right) - P(F)\right]. \tag{47}$$

The equityholders' project plan choice w^C is then determined by,

$$\max_{w \in [w_{\min}, 1]} V_E^C \text{ subject to } V_0(w) = e^{-r(w,\sigma(w))T} E[V_T(w)]. \tag{48}$$

Firstly,

Proposition 3 *For $V_0 > \frac{F}{1-\tau}$, the IC for equity-conversion CoCo bond bail-in is below the IC for no bail-out/in with the same equityholders' value, $V_E^N = V_E^C$.*

Proof. Consider the ICs, IC^N and IC^C, with the same values for equityholders $V_E^N = V_E^C > 0$. First investigate what happens when $\sigma \to 0$. Applying Properties B1 in Appendix B, when $V_0 > \frac{F}{1-\tau}$,

$$\lim_{\sigma \to 0} V_E^N = \lim_{\sigma \to 0} V_E^C = V_0 - Fe^{-rT}. \tag{49}$$

Hence $V_0^N(0) = V_0^C(0)$ at $\sigma = 0$, where $V_0^N(\sigma)$ and $V_0^C(\sigma)$ are the values of V_0 required to attain a given equityholders' value V_E^N when the volatility is σ. Similarly for $\sigma \to \infty$, again using Properties B1,

$$\lim_{\sigma \to \infty} V_E^N = \lim_{\sigma \to \infty} V_E^C = V_0, \tag{50}$$

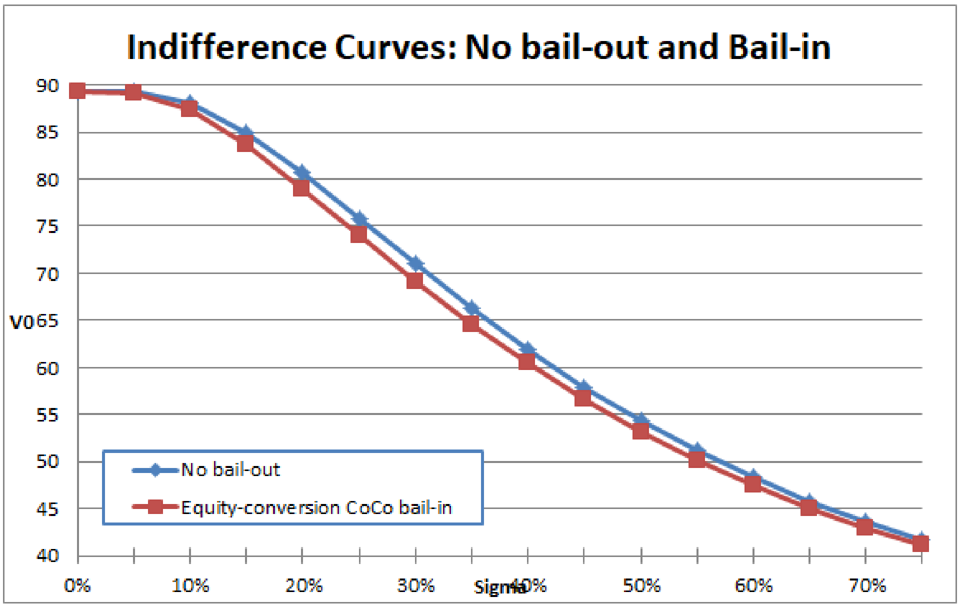

Figure 6: Equityholders' indifference curves for (i) no bail-out/in and (ii) bail-in cases

and thus $V_0^N(\infty) = V_0^C(\infty)$ at $\sigma = \infty$. Finally for $\sigma \in (0, \infty)$, note that

$$(1-\tau)\, P\left(\frac{F}{1-\tau}\right) - P(F) > 0 \;\forall \sigma \in (0,\infty)\,, \tag{51}$$

as $(1-\tau)\max\left[\frac{F}{1-\tau} - V_0, 0\right] = \max\left[F - (1-\tau)V_0, 0\right] > \max\left[F - V_0, 0\right]$ at all values of V_0. Thus to equate V_E^C with V_E^N for a given $\sigma \in (0,\infty)$, $V_0^C(\sigma)$ must be smaller than $V_0^N(\sigma)$. ■

The proposition states that in order to attain the same equityholders' value $V_E^N = V_E^C$, the equityholders are able to choose a lower $V_0(w)$ for a given σ under equity-conversion CoCo bond bail-in than under no bail-out/in. The condition $V_0 > \frac{F}{1-\tau}$ assures that the firm is not already in distress at time 0. Fig 6 demonstrates this result.

Secondly, Fig 5 suggests that the equityholders would choose a higher risk project plan w^C under equity-conversion CoCo bond bail-in than w^N under no bail-out/in:

Proposition 4 $w^C > w^N$.

Proof. From Eqn (44), the solution w^N satisfies,

$$\begin{aligned}\frac{dV_E^N}{dw}\left(w^N\right) &= \frac{\partial V_E^N}{\partial V_0}\frac{dV_0}{dw}\left(w^N\right)+\frac{\partial V_E^N}{\partial \sigma}\frac{d\sigma}{dw}\left(w^N\right)=0\\ \Leftrightarrow & \frac{dV_0}{dw}\left(w^N\right)=-\frac{vega_E^N}{\Delta_E^N}\frac{d\sigma}{dw}\left(w^N\right).\end{aligned} \tag{52}$$

Similarly, the solution w^C for Eqn (48) satisfies,

$$\begin{aligned}\frac{dV_E^C}{dw}\left(w^C\right) &= \frac{\partial V_E^C}{\partial V_0}\frac{dV_0}{dw}\left(w^C\right)+\frac{\partial V_E^C}{\partial \sigma}\frac{d\sigma}{dw}\left(w^C\right)\\ &= \Delta_E^C\times\frac{dV_0}{dw}\left(w^C\right)+vega_E^C\times\frac{d\sigma}{dw}\left(w^C\right)=0.\end{aligned} \tag{53}$$

Now at w^N, using Eqn (52),

$$\frac{dV_E^C}{dw}\left(w^N\right)=vega_E^N\left(-\frac{\Delta_E^C}{\Delta_E^N}+\frac{vega_E^C}{vega_E^N}\right)\frac{d\sigma}{dw}\left(w^N\right). \tag{54}$$

However, Appendix C shows that, when $V_0>\frac{F}{1-\tau}$, $\Delta_E^C<\Delta_E^N$ and $vega_E^C>vega_E^N$. Hence,

$$\frac{vega_E^C}{vega_E^N}>1>\frac{\Delta_E^C}{\Delta_E^N}\Rightarrow\frac{dV_E^C}{dw}\left(w^N\right)>0. \tag{55}$$

Thus at $w=w^N$ the equityholders will still be able to increase its value by increasing.w. Hence $w^C>w^N$.[21] ■

As demonstrated in Fig 5, this means that under equity-conversion CoCo bond bail-in, the equityholders would choose an efficient project plan with higher σ but lower V_0, implying an aggravation of both wealth transfer and value destruction agency costs compared with the case of no bail-out/in.

[21] An alternative intuitive proof is as follows. As already discussed the equityholders' position under equity-conversion CoCo bond bail-in is the no bail-out/in position $V_E^N=C(F)$ plus a long put bear spread-like structure $(1-\tau)P\left(\frac{F}{1-\tau}\right)-P(F)$, which represents the CoCo bail-in guarantee. For the range of values of V_0 and F that we are interested, the delta of this long put bear spread is negative. The vega of a put bear spread is always positive. This implies that $\Delta_E^C<\Delta_E^N$ and $vega_E^C>vega_E^N$. In Eqn (35) we derived that the slope of the IC is the MRS which is the negative of the ratio of vega to delta. Therefore MRS^C is more negative than MRS^N, implying a steeper IC under bail-in than under no bail-out/in. Thus $w^C>w^N$.

3.3.3 Write-off CoCo Bond Bail-in

Finally, we consider the write-off CoCo bond bail-in. In Eqn (8), we derived the value of equityholders' position (the total of the original capital and the post-trigger contingent capital reserve) as,

$$V_E^W = C(F) + F_W B_P\left(\frac{F}{1-\tau}\right) - \left[P(F) - (1-\tau)P\left(\frac{F_B}{1-\tau}\right)\right]. \tag{56}$$

The equityholders' project plan choice w^W is determined by,

$$\max_{w\in[w_{\min},1]} V_E^W = C(F) + F_W B_P\left(\frac{F}{1-\tau}\right) - \left[P(F) - (1-\tau)P\left(\frac{F_B}{1-\tau}\right)\right] \tag{57}$$
$$\text{subject to } V_0(w) = e^{-r(w,\sigma(w))T} E\left[V_T(w)\right].$$

Then,

Proposition 5 $w^W > w^N$.

Proof. Very similar to the proof of Proposition 4. ■

3.4 Simulated Results

To compare the outcomes between the different restructuring structures, we simulate the optimal risk chosen by the equityholders under the different structures for varying values of F. This is done by numerically solving the equityholders' maximisation problems for each scenario, namely Eqns (43), (48) and (57), using Newton-Raphson numerical estimation to solve for the values w such that $\frac{dV_E^X}{dw} = 0$ for $X \in \{N, C, W\}$. This is then applied to (16) to compute the optimal $\sigma^X(w)$. The values used for the simulation are: $E\left[V_T^1\right] = 130$, $E\left[V_T^2\right] = 115$, $\sigma_1 = 30\%$, $\sigma_2 = 20\%$, $\rho = 0$, $\lambda_1 = 0.5$, $\lambda_2 = 0.25$, $T = 1$ and $r_f = 3\%$. Where required, the level of CoCo bond is assumed to be 10% of the total debt level, with the CoCo trigger level $\tau = 7\%$. The minimum capital ratio is $\underline{E} = 10\%$. Given these volatilities and the market price of risk, the present values of the two projects are $V_0^1 = 108.6$ and $V_0^2 = 106.2$. The simulated results for the optimal $\sigma(w)$ are shown in Table 1. The graph of the results are plotted in Fig 7.

F	First-best	No bail-out/in	Equity-conversion CoCo	Write-off CoCo
30	19.36%	19.36%	19.36%	19.36%
35	19.36%	19.36%	19.36%	19.36%
40	19.36%	19.36%	19.36%	19.36%
45	19.36%	19.36%	19.36%	19.36%
50	19.36%	19.36%	19.36%	19.36%
55	19.36%	19.36%	19.37%	19.37%
60	19.36%	19.37%	19.40%	19.43%
65	19.36%	19.40%	19.49%	19.57%
70	19.36%	19.50%	19.74%	19.93%
75	19.36%	19.75%	20.34%	20.37%
80	19.36%	20.34%	21.86%	22.49%
85	19.36%	21.81%	26.12%	26.30%
90	19.36%	26.45%	33.63%	32.41%
95	19.36%	36.00%	39.95%	38.34%

Table 1: Simulated results for equityholders' risk choice for different leverage

The simulated results suggest the following,

- For low leverage, the risk chosen approaches the first-best.
- For higher leverage, the equityholders choose the risks in the ascending order of no bail-out/in, equity-conversion CoCo bond bail-in and write-off CoCo bond bail-in.
- For very high leverage, the equity-conversion CoCo bond bail-in induces higher risk-taking than the write-off CoCo bond bail-in.

The explanation for the last point is as follows. In the write-off CoCo bond bail-in, the jump up in the payoff for the equityholders at the strike price $\frac{F}{1-\tau}$ due to the binary put (as shown in Eqn (56) and Fig 1) implies that, close to the strike price, the equityholders would actually prefer not to have high volatility as it reduces the probability of ending up with a high payoff (within the triangle area in Fig 1). This reduces the incentive for high risk-taking in high-leverage cases for the write-off CoCo bond bail-in structure.

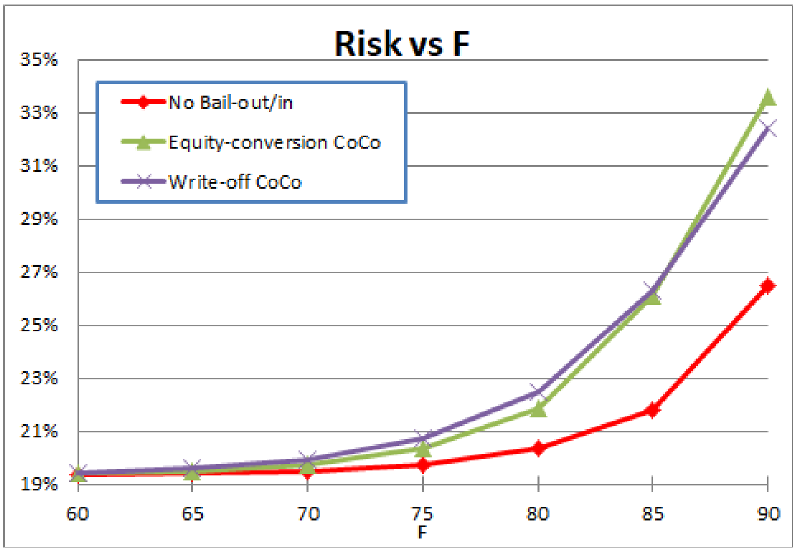

Figure 7: Equityholders' chosen risk for different leverage

3.5 Falling Capital Ratio

Up to this point, we have been investigating the equityholders' choice of risk given the set of possible projects, $E\left[V_T^i\right]$. In this section we investigate what happens under a falling capital ratio scenario. We do this by introducing a factor $\alpha \geq 0$, which represents an expansion of the asset value if $\alpha > 1$ and a contraction if $\alpha \in [0, 1)$. Then for the no bail-out/in or government bail-out cases, the equityholders' choice of the optimal project plan is determined by the following, which is Eqn (43) with the additional factor α,

$$\max_{w \in [w_{\min}, 1]} V_E^N = C(F) \text{ subject to } V_0(w) = e^{-r(w,\sigma(w))T} \alpha E\left[V_T(w)\right]. \tag{58}$$

From Eqn (44) the solution w satisfies,

$$\frac{dV_E^N}{dw} = N(d_1)\left[\frac{dV_0}{dw} + V_0\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right] = 0. \tag{59}$$

In other words, the solution is where the slope of the project plan curve, $\frac{dV_0}{dw}$, equals the slope of the IC, $-\frac{V_0\sqrt{T}N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}$,[22] in the $V_0 - \sigma$ plane. Then,

[22] See Eqn (37).

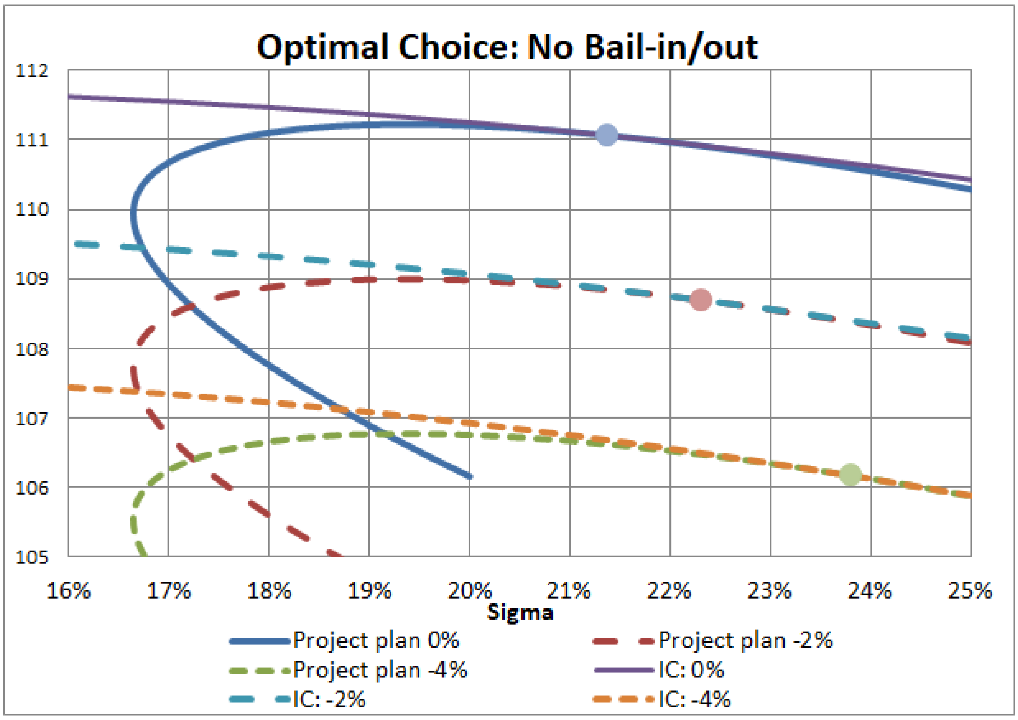

Figure 8: Equityholders' choice with falling capital ratio: no bail-out/in

Proposition 6 *Under the falling capital ratio scenario, the equityholders would increase their risk-taking behaviour, i.e.* $\frac{\partial w^N}{\partial \alpha} < 0$.

Proof. See Appendix D. ■

Note $\frac{\partial w^N}{\partial \alpha} < 0$ means that under contracting asset values, i.e. decreasing α value, w^N increases, and hence the equityholders' risk-taking increases. Figs 8 and 9 demonstrate this for no bail-out/in and equity-conversion CoCo bond bail-in cases. For example in Fig 9, the face values of the vanilla bonds and the CoCo bonds are fixed at $F_B = 76$ and $F_C = 4$, respectively. Initially the expected values of the two projects are $E\left[V_T^1\right] = 130$ and $E\left[V_T^2\right] = 115$. The falling capital ratio is simulated as 2% and 4% falls in these expected values, which shift the project plan curves down as shown. The equityholders' choices are again the tangent point between the project plan curves and the highest attainable ICs, shown by the dots. The simulated results demonstrate the equityholders' increasingly risky choice as the capital ratio falls: σ^C increases from 21.8% to 22.8% to 24.3%. Similar results are attained for write-off CoCo bond bail-in.

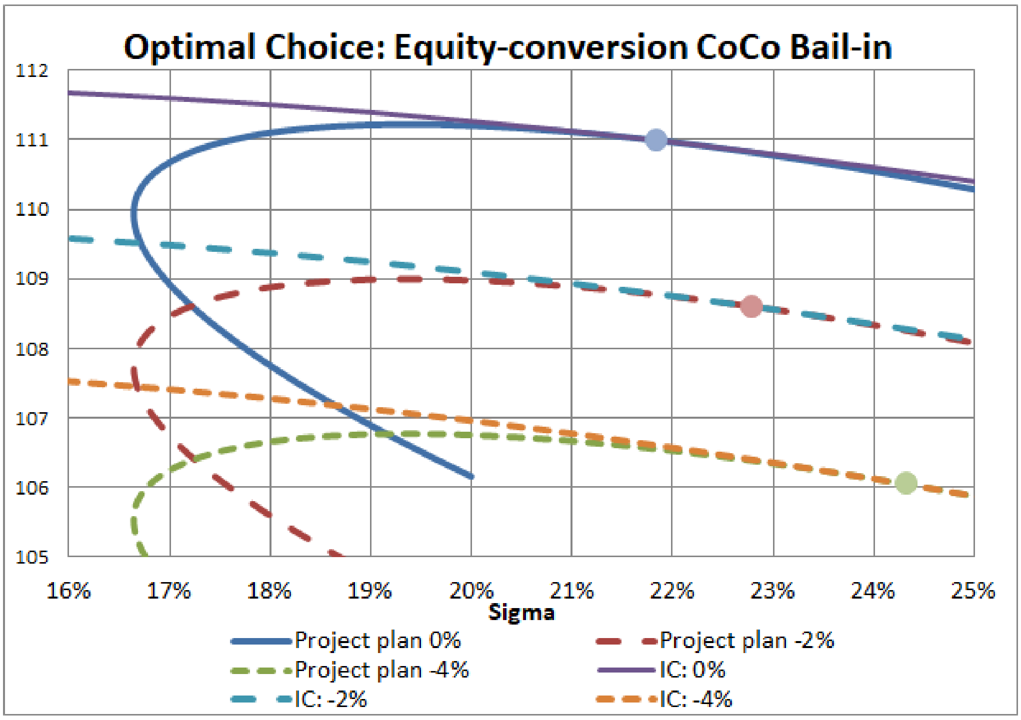

Figure 9: Equityholders' choice with falling capital ratio: equity-conversion CoCo bail-in

3.6 Monitoring Costs in WACC

The inevitable agency costs of bail-in should encourage bondholders to monitor equityholders' behaviour. This results in monitoring costs to the bondholders. For example, a passive asset manager who has been investing in financial bonds for its index tracking portfolio now needs fundamental analysts to monitor the credit quality of the banks. This is costly. Jensen and Meckling (1976) argues that these monitoring costs are ultimately borne by equityholders via higher weighted average cost of capital (WACC) demanded. The threat of the fall in the bank's value should in turn curb the bank's risk-taking behaviour and discipline equityholders. In this section we demonstrate this mechanism using the model developed above.

The rise in the return demanded by the investors, due to the monitoring costs, is here represented by an increase in the market price of risk for the higher risk project 1 from λ_1 to λ_1'. This is the higher WACC of choosing a higher proportion of the riskier project. As a result, the present value of the project plans are reduced for riskier choices. This is depicted in Fig 10 by the "squeezing" of the project plan curves. In again selecting the tangent point with

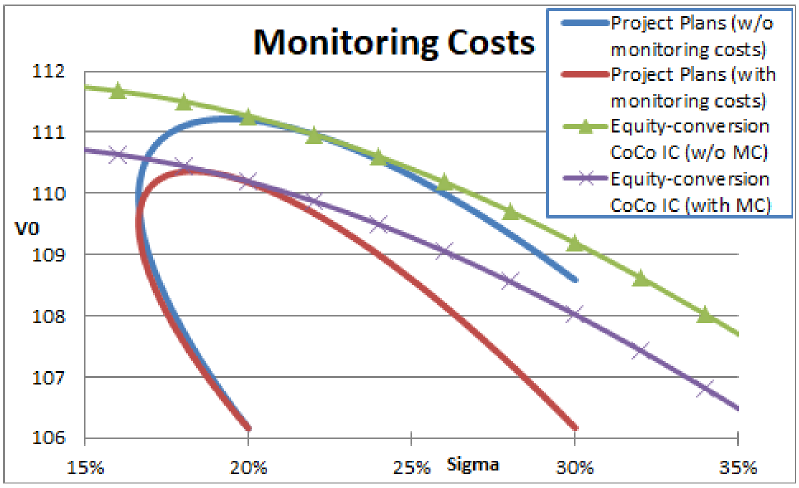

Figure 10: Effect of Monitoring Costs on Equityholders' Choice of Risk

their ICs (here with equity-conversion CoCo bond bail-in), the equityholders' optimal choice of project plan is now of a lower risk.

In policy terms this is analogous to Pigouvian tax. The agency costs are the negative externality of equityholders' actions on the remaining stakeholders of the bank. By implementing higher costs on their action, the regulators can force the equityholders to select less risky choice. As a demonstration, Fig 11 simulates the resulting behaviour of the equityholders with varying degrees of proportional increase in λ_1, $\Delta\lambda_1$, when $F = 80$. It shows that with a sufficient rise in the cost of choosing the riskier project, the equityholders can be made to choose the first-best risk level for all structures. For example with equity-conversion CoCo bond bail-in, for the values $\lambda_1 = 0.5$ and $\sigma_1 = 30\%$ the equityholders would choose the risk level $\sigma = 21.9\%$ (the y-axis intercept). The simulation then suggests that a proportional increase in λ of 15%, which translates to an increase in project 1's required rate of return by 2.3%, would induce the equityholders to choose the first-best risk level of $\sigma^* = 19.4\%$. There is, however, a social cost of the value of firm being lowered as a result, as shown in Fig 10.

More formally,

Proposition 7 *The higher the required rate of return of the riskier project* 1*, the lower the equityholders' choice of risk.*

Proof. To show this, consider two downward-sloping curves as a function of a variable x,

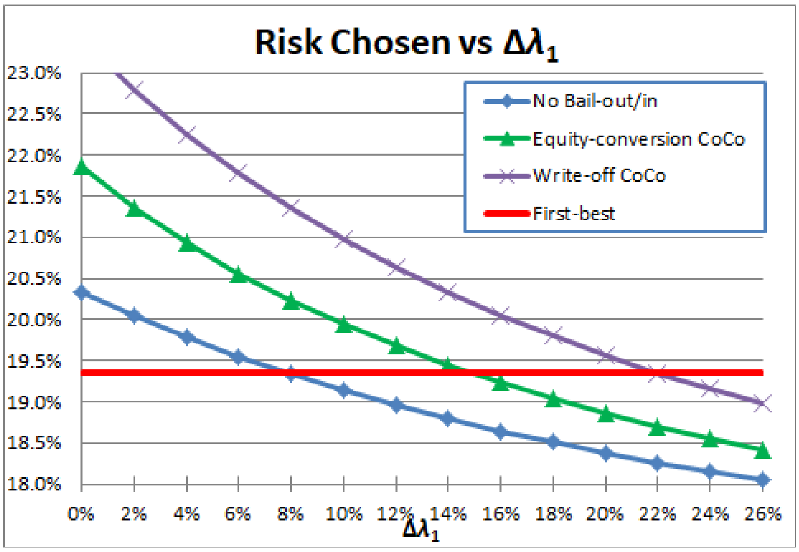

Figure 11: Risk chosen with monitoring costs

$f(x;\phi)$ and $g(x)$. ϕ is an exogenous parameter. Let the derivatives of the functions be such that $f_x < 0$, $g_x < 0$, $f_{xx} < g_{xx}$, $f_{xx} < 0$ and $f_{x\phi} < 0$. This implies that, (i) $f(x;\phi)$ is strictly concave in x; (ii) $g(x)$ is less concave than $f(x)$ and can even be linear or convex; and (iii) $f(x;\phi)$ becomes steeper (more downward-sloping) with an increase in ϕ. Let now x^* be the value of x where the two curves are tangent, i.e. $f_x(x^*;\phi) = g_x(x^*)$. Then as ϕ increases,

$$\frac{d}{d\phi} f_x(x^*;\phi) = f_{xx}\frac{dx^*}{d\phi} + f_{x\phi}. \tag{60}$$

For the two curves to be tangent again, this must equal $\frac{d}{d\phi} g_x(x^*) = g_{xx}\frac{dx^*}{d\phi}$. Thus,

$$\frac{dx^*}{d\phi} = \frac{f_{x\phi}}{g_{xx} - f_{xx}} < 0. \tag{61}$$

This is negative from the conditions on the derivatives. Our tangency analysis between the concave project plan curves ($f(x;\phi)$) and the quasi-concave ICs ($g(x)$) satisfy these conditions, where $x = \sigma$ and $\phi = \lambda_1$. Hence the choice of σ decreases with higher λ_1. ■

The rise in the WACC required to attain the first-best risk level depends on how far the bank's balance sheet is from the trigger level,

Proposition 8 *The rise in the WACC required to induce equityholders to choose the first-best*

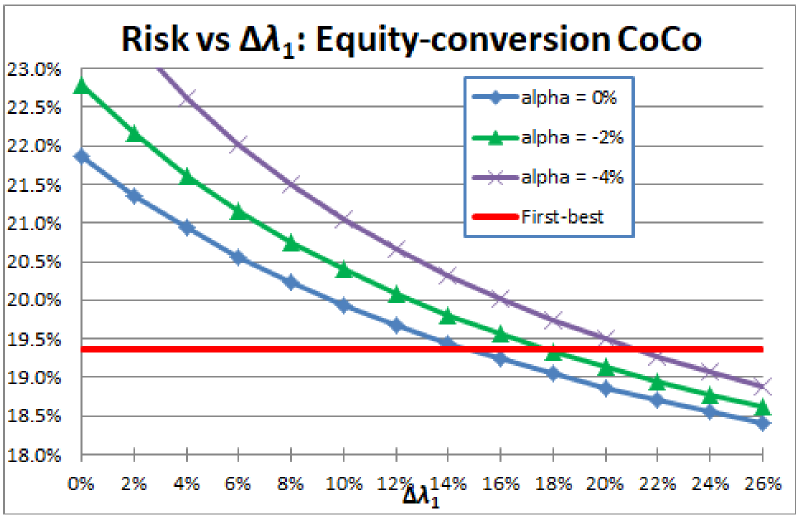

Figure 12: $\Delta\lambda_1$ required for different values of α: equity-conversion CoCo bond bail-in

risk level is smaller, the further away the bank is from the restructuring point.

Proof. This follows immediately from Proposition 6, where it was shown that the closer the bank is to the trigger restructuring point, the further away the chosen risk is from the first-best. Hence a larger WACC rise is required. ■

Fig 12 demonstrates this for equity-conversion CoCo bond bail-in. As was introduced in Section 3.5, $\alpha \geq 0$ is a factor which represents expansion / contraction of the value of the bank's asset. The figure shows that for $\alpha = -4\%$, $\Delta\lambda_1$ of 21% is required to attain the first-best risk level of 19.4%, as opposed to $\Delta\lambda_1 = 15\%$ when there is no depletion in the value of the asset.

Given these results, we can now propose financial and non-financial covenants as an effective way to articulate the monitoring effort.

4 Covenants

We have established above that the agency costs can, to a certain extent, be mitigated by rising required rate of return when riskier project plan is chosen. In practice this equates to the bondholders demanding higher cost of debt to compensate for their monitoring cost. In this section we propose covenants within the CoCo bond indentures to implement this idea.

Covenants are regularly inserted in corporate bonds (especially from high yield issuers) to monitor more effectively the investment and financial policies of the company. Corporate covenants have been argued to successfully attenuate investment distortions and risk-taking incentives in the corporate bond market, reducing agency costs between equityholders and bondholders (see for example, Gamba and Triantis (2014)[23]). In a similar manner, we propose covenants in the CoCo bond indentures to promote financial and investment discipline in banks to curb risk-taking appetite.

We suggest *financial accounting covenants with a "ratchet coupon" system* whereby the CoCo bond coupon rate gradually increases as the fundamentals of the bank debilitate and the covenants are breached. The ratchet system suggested in the empirical literature for corporate bond covenants (e.g. Bradley and Roberts (2004), Gamba and Triantis (2014)) tend to indicate that the conventional covenants are usually fixed at relatively low levels and fail to exert the necessary financial discipline on the company. Instead, introducing a ratchet with different covenant levels enables the bondholders to monitor and control the bank's risk-taking more effectively. With the rising risk-taking, bondholders are compensated through a coupon increase that mirrors the increasing risk premium. The regulator, on the other hand, can prevent equityholders from taking actions that would extract wealth from bondholders as the solvency ratio falls towards the CoCo trigger or the point of no viability (PONV). The covenant will exert discipline on managers and equityholders due to onerous coupon increases which will automatically dent equityholders' returns. Our result in Proposition 8 suggests that this type of covenant based-discipline is more effective during times of stable solvency, away from the point of restructuring. Later, for covenants close to restructuring point, we suggest different kind of covenants, namely *asset sweep* and *debt sweep covenants*.

4.1 Ratchet Coupon Financial Accounting Covenants

Following empirical evidence (e.g. Bradley and Roberts (2004), Billet, King and Mauer (2007), Chava and Roberts (2008), Roberts and Sufi (2009), Gamba and Triantis (2014)) of covenants on corporate bonds, we suggest three candidates for the index for the ratchet trigger in our

[23] They show how effective debt covenant restrictions can shift shareholders' financing and investments towards value maximisation.

proposed financial accounting covenants:

- Fully loaded Core Tier 1 Ratio: $\frac{CT1}{RWA}$
- Leverage Ratio: $\frac{Assets}{Equity+AT1}$, where AT1 is the Tier 1 CoCo bonds
- Interest Coverage / ADI[24] interest coverage: $\frac{ADI}{\text{CoCo coupon payment}}$

Financial accounting covenants are suggested, as it is difficult to use income statement based covenants due to the ongoing presence of exceptional and one-off items in the banks' profit and loss account. The choice of the CT1 ratio would be a natural one, if the CoCo trigger is also linked to the CT1 ratio, allowing bondholders to evaluate the degree of headroom against CoCo trigger and coupon suspension on AT1s. The leverage ratio covenant would allow bondholders to monitor bank's indebtness to restrict balance sheet expansion and constrain managers to constantly pursue equityholders-friendly investments.[25] The interest coverage covenant (to monitor the buffer on the coupon payments for AT1s) is a common covenant in corporate bonds to assess the ability of the company to service its coupon payments. These covenants would have to be calibrated to suit the nuances of each CoCo bond class. Specifically, CoCo T2s have a specific maturity and mandatory coupon payments, and hence the introduction of covenants and ratchets would be easier as these bonds are similar to unsecured corporate bonds. On the other hand, CoCo AT1s are perpetual and their coupons payments are not mandatory,[26] and so imposing a ratchet covenant structure on these may be more challenging.[27]

Here we describe the covenant's mechanism with a simple model using the leverage ratio-based ratchet. Assume that the value of the asset A_t at period $t \in \{0, 1\}$ is the sum of the equity E_t, the plain vanilla bond D_t and the T2 CoCo bond C_t,

$$A_t = E_t + D_t + C_t. \tag{62}$$

[24] Maximum amount of distributable items linked to the holding company or the distance to coupon suspension on the combined buffer (minimum CT1 including all additional buffers).

[25] For example the sovereign bond "carry trade", as sovereign bonds are zero risk weighted asset.

[26] AT1 CoCo bond coupons can be partially or totally suspended if there is not enough distributable amounts within the equity (set by the MDA - maximum distributable amount) or if the combined buffer (made up of the countercyclical, conservation and systemic equity buffers) is breached.

[27] Specifically, if the capital ratios fall below the Basel III combined buffer (the buffer above the minimum capital ratio), the coupon payments can be reduced or even suspended. Furthermore, the indenture does not contemplate any dividend stopper (CoCo bond coupons can be suspended whilst dividends are paid out) or pushers (dividends payments can be resumed whilst CoCo bond coupons can still be missed).

The leverage ratio L_t at period t is then,[28]

$$L_t = \frac{A_t}{E_t}. \tag{63}$$

Let the costs of debt of the plain vanilla bond and the CoCo bond be r_D and r_C, respectively, and the return on asset be $r_A > 0$. Then at the end of period 0, the firm produces the earnings before interest and tax (EBIT) of $r_A A_0$. Assuming zero tax rate, the bank's net income is then,

$$n_0 = r_A A_0 - r_D D_0 - r_C C_0. \tag{64}$$

The bank's return on equity (ROE) is,

$$r_E = \frac{n_0}{E_0}. \tag{65}$$

We assume that this net income is added to the bank's capital in whole as retained earnings. Period 1 asset and leverage ratio are then given by (62) and (63) above with no extra bond funding $D_1 = D_0$ and $C_1 = C_0$, and

$$E_1 = E_0 + n_0 \text{ and } A_1 = A_0 + n_0. \tag{66}$$

This implies an increasing leverage ratio $L_1 = \frac{A_1}{E_1} < \frac{A_0}{E_0} = L_0$ for a profit-making bank with $n_0 > 0$.[29]

Consider now a new risky investment I_0 at period 0 with an uncertain return $\widetilde{r}_I$. The investment is funded by an increase in the plain vanilla bond issuance, $I_0 = D_0' - D_0$. The funding cost is assumed unaffected at r_D despite the increase in the leverage ratio to,

$$L_0' = \frac{A_0'}{E_0} = \frac{A_0 + I}{E_0}. \tag{67}$$

[28] The Basel III leverage ratio allows for the AT1 CoCo bond (effectively a deeply perpetual subordinated Tier 1 bond) to be included.

[29] $E_0 < A_0 \Leftrightarrow A_0E_0 + n_0E_0 < A_0E_0 + n_0A_0 \Leftrightarrow \frac{A_0+n_0}{E_0+n_0} < \frac{A_0}{E_0}$.

The leverage ratio in period 1 is now,

$$L_1' = \frac{A_0 + I + n_0 + (\widetilde{r}_I - r_D)\, I}{E_0 + n_0 + (\widetilde{r}_I - r_D)\, I} = \frac{A_1 + I + (\widetilde{r}_I - r_D)\, I}{E_1 + (\widetilde{r}_I - r_D)\, I}. \tag{68}$$

The covenant specifies the ratchet trigger leverage ratios and the corresponding CoCo bond coupon levels $\{K^i, r_C^i\}$, $i = 1, 2, ...,$ with $L_0 < K^1 < K^2 < ...$ and $r_C < r_C^1 < r_C^2 < ...$. Consider the first trigger level K^1. This is breached when $L_1' \geq K^1$, or

$$r_I \leq r_D - \frac{E_1 K^1 - (A_1 + I)}{(K^1 - 1)\, I} = r_I^1, \tag{69}$$

where r_I is the realised rate of return of $\widetilde{r}_I$ and r_I^i is the rate of return of the risky investment corresponding to the ratchet trigger leverage ratio K^i, above which the ratchet trigger of the CoCo bond coupon occurs. Note where the right-hand side is positive, this implies that the ratchet would be triggered even when the risky investment returns a positive rate. Similarly, the second ratchet is triggered if leverage ratio L_1' breaches K^2 also,

$$L_1' = \frac{A_1 + I + (\widetilde{r}_I - r_D)\, I - \left(r_C^1 - r_C\right) C_0}{E_1 + (\widetilde{r}_I - r_D)\, I - \left(r_C^1 - r_C\right) C_0} \geq K^2, \tag{70}$$

which now reflects the extra coupon cost of the CoCo bond.[30] In terms of the realised rate of return, the CoCo coupon is ratcheted up to r_C^2 when,

$$r_I \leq r_D - \frac{E_1 K^2 - \left(r_C^1 - r_C\right) C_0 K^2 - \left[A_1 + I - \left(r_C^1 - r_C\right) C_0\right]}{(K^2 - 1)\, I} = r_I^2. \tag{71}$$

What we are interested in is the equityholders' behaviour. Their realised ROE depends on the ratchet triggers,

$$r_E' = \begin{cases} r_E + (r_I - r_D)\, \frac{I}{E_0}, & \text{if} \quad r_I^1 < r_I \\ r_E + (r_I - r_D)\, \frac{I}{E_0} - \left(r_C^1 - r_C\right) \frac{C_0}{E_0}, & \text{if} \quad r_I^2 < r_I \leq r_I^1 \\ r_E + (r_I - r_D)\, \frac{I}{E_0} - \left(r_C^2 - r_C\right) \frac{C_0}{E_0}, & \text{if} \quad r_I^3 < r_I \leq r_I^2 \\ \vdots & \quad \vdots \end{cases} . \tag{72}$$

[30] Here, for simplicity the higher coupon rate is applied to the whole period. The analysis is purely illustrative.

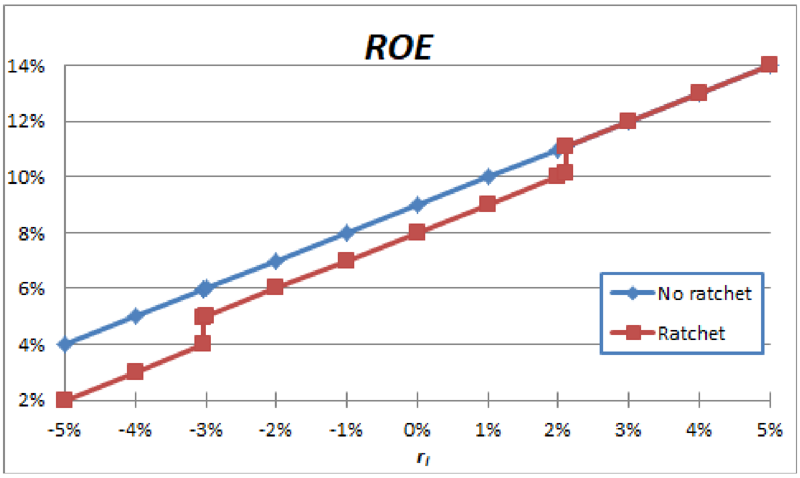

Figure 13: ROE vs r_I with Ratchet Coupon Covenant

Fig 13 depicts the effect of the ratchet coupon covenant on the equityholders' return. This financial accounting covenant encourages lower risk-taking through two channels. Firstly, the step-down nature of the ROE as $\widetilde{r}_I$ falls *introduces concavity* in the return profile of the equityholders. In much the same way as the convexity in the equityholders' payoff at bail-out / bail-in trigger points creates an incentive for higher risk-taking (due to a long vega position), the concavity at the ratchet trigger points (short vega position) discourages risk-taking. Secondly, once the ratchet is triggered, the higher CoCo coupon means *higher cost of debt* for the bank, which leads to lower risk-taking as the equityholders' optimal choice, as discussed in Proposition 7. In terms of externality, this is analogous to extra Pigouvian tax kicking in automatically.

However, both of these two channels become less effective as the bank's balance sheet nears the PONV or the CoCo trigger. For the first channel, this is because the concavity at the ratchet triggers are offset by the convexity at the CoCo trigger. For the second channel, we saw in Proposition 8 that the rise in the required rate of return required becomes increasingly large to induce equityholders to behave optimally. Intuitively, the appeal of "gamble-for-resurrection" or "looting" more than offsets the effectiveness of these covenants. We therefore suggest introducing these financial accounting covenants at the inception of CoCo bonds, when the solvency ratio of the bank is healthy.

4.2 Asset Sweep / Debt Sweep Covenants

In a falling solvency scenario where risk-taking incentives are high, we propose the following types of covenants instead:

- Asset Sweep Covenant
- Debt Sweep Covenant

Asset sweeps are common in private debt placements (see for example Bradley and Roberts (2004)). Once triggered, the bank is forced to sell assets to pay down debt, and thus decreasing the leverage ratio. This prevents equityholders from liquidating assets to receive a large dividend ("looting"), or take on new debt in order to finance a risky investment ("gamble-for-resurrection"), reducing the agency costs of bail-in near the CoCo trigger point / PONV.

Let now β be the proportion of the asset A' that has to be divested. We assume that the whole of this is used to pay down the debt, in which case the debt is reduced to $D_1' - \beta A_1'$.[31] The bank will seek to sell those assets in which it can materialise a capital gain, i.e. $b > 1$, where b is the ratio of the asset's market value and its nominal value. The profit from this divestiture is therefore $(b-1)\,\beta A_1'$, which is added to the bank's capital as retained earnings. As a result, the equity is increased to $E_1' + (b-1)\,\beta A_1'$, while the final value of the asset is $(1-\beta)\,A_1' + (b-1)\,\beta A_1' = [1+(b-2)\,\beta]\,A_1'$. Therefore the bank's leverage ratio becomes,

$$L_1' = \frac{[1+(b-2)\,\beta]\,A_1'}{E_1' + (b-1)\,\beta A_1'}. \tag{73}$$

This is smaller than $\frac{A_1'}{E_1'}$.[32] When $b < 2$, the leverage ratio is reduced by both a lower numerator (asset) and a higher denominator (equity). The trigger level $\frac{A_1'}{E_1'}$ and the divestiture ratio β are designed such that the leverage ratio is likely to be reduced to a maximum level desired by the regulator, although this will depend on the capital gain ratio b. However, in the case that the

[31] Going forward, all unsecured debt, including senior debt, will be loss-absorbing. However, the regulator is targeting a minimum loss absorbing capacity (TLAC). Since the regulator are unlikely to allow CoCo bonds or any loss absorbing debt that makes up the TLAC to be repaid, we argue that the asset sweep should work for any non-TLAC debt (both secured and unsecured).

[32] L_1' is decreased for all values of $b > 1$, as $\frac{[1+(b-2)\beta]A_1'}{E_1'+(b-1)\beta A_1'} < \frac{A_1'}{E_1'} \Leftrightarrow (b-2)\,E_1' < (b-1)\,A_1'$, which is true both by $b-2 < b-1$ and $E_1' < A_1'$.

asset sale fails to reduce the leverage ratio to the desired level, the regulator can impose further non-core asset sales and dividend / CoCo bond payouts cancellations[33] to bolster solvency and reduce the leverage ratio.

The *debt sweep* covenant resembles the asset sweep covenant, where any proceeds from newly issued debts are used to repay existing debts, if the leverage ratio has breached a trigger level. In our proposed covenant system, this can be the final ratchet trigger level of our ratchet coupon financial accounting covenant. This sets a ceiling to a bank's leverage ratio, curbing equityholders' appetite for leveraging up as the bank's solvency deteriorates.

We believe through a combination of these three types of CoCo covenants the agency costs can be contained. Financial covenants are useful to keep risk and leverage appetite down (especially in a low profitability environment) from the bond inception, enabling bondholders to monitor the banks and incorporate the increasing risk premium in the market price of the bonds as soon as the risk profile of the bank creeps up. We have proposed a ratchet coupon structure which we believe is an effective way of doing this. Both asset and debt sweep covenants contribute to the diminishing of the appetite for debt when risky investment opportunities become more appealing for the bank, as they alleviate the incentives of the banks to issue new debt to extract wealth from existing bondholders. These two covenants should be triggered when the solvency ratios are decreasing (but still high enough[34]) and the financial covenants are no longer effective near the PONV / CoCo trigger point.

5 Concluding Remarks

We have suggested that the new bail-in framework, achieved by the issuance of CoCo bonds, will prove not to solve the moral hazard problem that is inherent in the old bail-out framework. Instead, there are agency costs that lie behind the new relationship between equityholders and bondholders. The deviation from absolute priority rule (DAPR), formalised by the bail-in bonds, changes the profile of the trade-off between value and risk for equityholders. This was investigated using indifference curve analysis, which demonstrated the higher risk-taking

[33] The bond indenture should contemplate this event, especially in AT1 CoCo bonds.
[34] We suggest, for example, 9% CT1.

appetite for equityholders under equity-conversion CoCo bond and write-off CoCo bond bail-ins, relative to the no bail-in / bail-out and traditional government bail-out scenarios.

We also believe that covenants in CoCo bonds proposed in this paper are an effective way to curb risk-taking. When solvency is relatively high, contractual covenants that systematically incorporate the additional risk premium via upward coupon resetting exert discipline on banks and dent equityholder returns. The equityholders are not able to trade easily the bank value off against higher risk-taking for two reasons: the concave nature of the equityholders' return on equity at each ratchet trigger point, and the higher cost of capital once triggered. However as solvency deteriorates towards the point of no viability (PONV) or the CoCo trigger point, these become less effective mainly as the equityholders' incentive to "gamble-for-resurrection" negate the above effects. In these cases we propose a different type of covenants, namely asset and debt sweeps. Further investigations of other solutions to alleviate moral hazard and agency costs, especially near the PONV / CoCo trigger point, are forthcoming in future research.

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6 Appendix

A Properties of $N(x)$

$N(x)$ is the cumulative distribution function for a standard normally distributed random variable $X \sim N(0,1)$ such that $N(x) = \Pr(X \leq x)$. Then $N'(x)$ is the probability density function $N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}$ and $N''(x) = -xN'(x)$.

Property A1

$$\begin{aligned} \lim_{x\to\infty} \tfrac{N'(x)}{N(x)} &= 0 \\ \lim_{x\to-\infty} \tfrac{N'(x)}{N(x)} &= -x. \end{aligned} \tag{74}$$

Proof. The first limit is trivial as $\lim_{x\to\infty} N'(x) = 0$ and $\lim_{x\to\infty} N(x) = 1$. For the second, note first that $\lim_{x\to-\infty} N'(x) = 0$ and $\lim_{x\to-\infty} N(x) = 0$. The limit can therefore be found using L'Hôpital's rule,

$$\lim_{x\to-\infty} \frac{N'(x)}{N(x)} = \lim_{x\to-\infty} \frac{N''(x)}{N'(x)} = \lim_{x\to-\infty} \frac{-xN'(x)}{N'(x)} = -x. \tag{75}$$

■

Property A2

$$\frac{d}{dx}\left(\frac{N'(x)}{N(x)}\right) < 0 \ \forall x. \tag{76}$$

Proof. Expanding,

$$\frac{d}{dx}\left(\frac{N'(x)}{N(x)}\right) = -\frac{xN'(x)}{N(x)} - \left(\frac{N'(x)}{N(x)}\right)^2 = -\frac{N'(x)}{N(x)}\left(\frac{xN(x)+N'(x)}{N(x)}\right). \tag{77}$$

This is negative if and only if $xN(x) + N'(x) > 0$. This is obvious for $x \geq 0$. For $x < 0$, first check the limits,

$$\begin{aligned} \lim_{x\to 0} xN(x) + N'(x) &= N'(0) = \tfrac{1}{\sqrt{2\pi}} > 0 \\ \lim_{x\to-\infty} xN(x) + N'(x) &= \lim_{x\to-\infty} N(x)\left(x + \tfrac{N'(x)}{N(x)}\right) = 0 \end{aligned} \tag{78}$$

using Property A1. Now,

$$\frac{d}{dx}\left(xN\left(x\right)+N'\left(x\right)\right)=N\left(x\right)+xN'\left(x\right)-xN'\left(x\right)=N\left(x\right)>0\;\forall x \tag{79}$$

which proves that $xN\left(x\right)+N'\left(x\right)>0\;\forall x<0$. ■

B Limiting Properties of Call and Put Options

Properties B1 (Properties at the limits of call and put option values)

1. For large σ,

$$\left.\begin{array}{lll}\lim_{\sigma\to\infty}d_1=\infty & \Rightarrow & \lim_{\sigma\to\infty}N\left(d_1\right)=1\\ \lim_{\sigma\to\infty}d_2=-\infty & \Rightarrow & \lim_{\sigma\to\infty}N\left(d_2\right)=0\end{array}\right\}\Rightarrow\left\{\begin{array}{l}\lim_{\sigma\to\infty}C\left(K\right)=V_0\\ \lim_{\sigma\to\infty}P\left(K\right)=Ke^{-rT}\end{array}\right. . \tag{80}$$

   Therefore a call option behaves like the underlying asset while a put option behaves like a bond.

2. For $\sigma\to 0$,

$$\lim_{\sigma\to0}d_1=\lim_{\sigma\to0}d_2=\left\{\begin{array}{ccc}-\infty & \text{if} & V_0<Ke^{-rT}\\ 0 & \text{if} & V_0=Ke^{-rT}\\ \infty & \text{if} & V_0>Ke^{-rT}\end{array}\right.\Rightarrow\lim_{\sigma\to0}N\left(d_1\right)=\lim_{\sigma\to0}N\left(d_2\right)=\left\{\begin{array}{c}0\\ \frac{1}{2}\\ 1\end{array}\right. . \tag{81}$$

   Thus,

$$\begin{array}{l}\lim_{\sigma\to0}C\left(K\right)=\max\left[V_0-Ke^{-rT},0\right]\\ \lim_{\sigma\to0}P\left(K\right)=\max\left[Ke^{-rT}-V_0,0\right].\end{array} \tag{82}$$

   Intuitively the options approximate their intrinsic values.

C Comparing Delta and Vega

The value of equityholders' positions under no bail-out/in and equity-conversion CoCo bond bail-in are given in Eqns (1) and (8) as,

$$
\begin{aligned}
V_E^N &= C(F) \\
V_E^C &= C(F) + \left[(1-\tau) P\left(\tfrac{F}{1-\tau}\right) - P(F)\right].
\end{aligned}
\tag{83}
$$

The delta and vega of these positions are given by the derivatives with respect to V_0 and σ, respectively,

$$
\begin{aligned}
\Delta_E^N &= N(d_1(F)) \\
\Delta_E^C &= \tau + (1-\tau) N\left(d_1\left(\tfrac{F}{1-\tau}\right)\right),
\end{aligned}
\tag{84}
$$

$$
\begin{aligned}
vega_E^N &= V_0\sqrt{T} N'(d_1(F)) \\
vega_E^C &= (1-\tau) V_0\sqrt{T} N'\left(d_1\left(\tfrac{F}{1-\tau}\right)\right).
\end{aligned}
\tag{85}
$$

First, we show that $\Delta_E^N > \Delta_E^C$ for values $V_0 > \frac{F}{1-\tau}$. For this to be true, it must be that,

$$
\begin{aligned}
& N(d_1(F)) > \tau + (1-\tau) N\left(d_1\left(\frac{F}{1-\tau}\right)\right) \\
\Leftrightarrow\; & N(-d_1(F)) < (1-\tau) N\left(-d_1\left(\frac{F}{1-\tau}\right)\right).
\end{aligned}
\tag{86}
$$

To show this, consider the following derivative:

$$
\begin{aligned}
& \frac{\partial}{\partial V_0}\left[N(-d_1(F)) - (1-\tau) N\left(-d_1\left(\frac{F}{1-\tau}\right)\right)\right] \\
= \;& -\frac{1}{V_0\sigma\sqrt{T}}\left[N'(-d_1(F)) - (1-\tau) N'\left(-d_1\left(\frac{F}{1-\tau}\right)\right)\right].
\end{aligned}
\tag{87}
$$

As $N'(-d_1(.)) = N'(d_1(.))$, this is positive if $(1-\tau) N'\left(d_1\left(\frac{F}{1-\tau}\right)\right) > N'(d_1(F))$. For this to be true, we require,

$$
(1-\tau) e^{-\frac{1}{2}d_1^2\left(\frac{F}{1-\tau}\right)} > e^{-\frac{1}{2}d_1^2(F)}.
\tag{88}
$$

Now,

$$
d_1(F) = d_1\left(\frac{F}{1-\tau}\right) - \frac{1}{\sigma\sqrt{T}}\ln(1-\tau).
\tag{89}
$$

Hence,

$$e^{-\frac{1}{2}d_1^2(F)} = e^{-\frac{1}{2}d_1^2\left(\frac{F}{1-\tau}\right)}e^{-\frac{1}{2}\left\{-\frac{2}{\sigma\sqrt{T}}\ln(1-\tau)d_1\left(\frac{F}{1-\tau}\right)+\frac{1}{\sigma^2 T}[\ln(1-\tau)]^2\right\}}. \tag{90}$$

Thus for Eqn (88) to be true,

$$\begin{aligned}
1-\tau &> e^{-\frac{1}{2}\left\{-\frac{2}{\sigma\sqrt{T}}\ln(1-\tau)d_1\left(\frac{F}{1-\tau}\right)+\frac{1}{\sigma^2 T}[\ln(1-\tau)]^2\right\}} \\
\Leftrightarrow \ln(1-\tau) &> \frac{1}{\sigma\sqrt{T}}\ln(1-\tau)\,d_1\left(\frac{F}{1-\tau}\right) - \frac{1}{2\sigma^2 T}\left[\ln(1-\tau)\right]^2 \\
\Leftrightarrow 1 &< \frac{1}{\sigma\sqrt{T}}d_1\left(\frac{F}{1-\tau}\right) - \frac{1}{2\sigma^2 T}\ln(1-\tau) \\
\Leftrightarrow \frac{1}{2\sigma\sqrt{T}}\ln(1-\tau) &< d_1\left(\frac{F}{1-\tau}\right) - \sigma\sqrt{T} = d_2\left(\frac{F}{1-\tau}\right).
\end{aligned} \tag{91}$$

Noting that $\ln(1-\tau) < 0$ for $\tau > 1$, this is unambiguously satisfied when $d_2\left(\frac{F}{1-\tau}\right) > 0 \Leftrightarrow V_0 > \frac{F}{1-\tau}e^{-\left(r-\frac{\sigma^2}{2}\right)T}$, or definitely when V_0 is above the critical level $\frac{F}{1-\tau}$. Hence Eqn (87) is positive for these values of V_0. Also,

$$\lim_{V_o\to\infty}\left[N(-d_1(F)) - (1-\tau)N\left(-d_1\left(\frac{F}{1-\tau}\right)\right)\right] = 0 \tag{92}$$

as the limit for both terms are zero. This means that $N(-d_1(F)) - (1-\tau)N\left(-d_1\left(\frac{F}{1-\tau}\right)\right)$ approaches 0 from below as V_0 increases from $\frac{F}{1-\tau}$, proving that $\Delta_E^C < \Delta_E^N$ for $V_0 > \frac{F}{1-\tau}$.

Similarly, for $vega_E^C > vega_E^N$ when $V_0 > \frac{F}{1-\tau}$, we require,

$$(1-\tau)V_0\sqrt{T}N'\left(d_1\left(\frac{F}{1-\tau}\right)\right) > V_0\sqrt{T}N'(d_1(F)). \tag{93}$$

This is already proved above.

D Proof of Proposition 6

Consider the effect of an increase in α on the first-order condition Eqn (59),

$$\begin{aligned}
\frac{\partial}{\partial\alpha}\left(\frac{dV_E^N}{dw}\right) &= N'(d_1)\left[\frac{dV_0}{dw} + V_0\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right] + N(d_1)\frac{\partial}{\partial\alpha}\left(\frac{dV_0}{dw} + V_0\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right) \\
&= N(d_1)\left[\frac{\partial}{\partial\alpha}\left(\frac{dV_0}{dw}\right) + \frac{\partial V_0}{\partial\alpha}\left(\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right) + V_0\sqrt{T}\frac{\partial}{\partial\alpha}\left(\frac{N'(d_1)}{N(d_1)}\right)\frac{d\sigma}{dw}\right]
\end{aligned} \tag{94}$$

Note, the first term of the first line is zero from Eqn (59). Now from $V_0(w) = e^{-r(w,\sigma(w))T}\alpha E[V_T(w)]$, we have,

$$\frac{dV_0}{dw} = e^{-rT}\alpha\left\{-T\frac{dr}{dw}E[V_T(w)] + \left(E\left[V_T^1\right] - E\left[V_T^2\right]\right)\right\} \Rightarrow \frac{\partial}{\partial\alpha}\left(\frac{dV_0}{dw}\right) = \frac{1}{\alpha}\frac{dV_0}{dw} \tag{95}$$

and

$$\frac{\partial V_0}{\partial\alpha} = \frac{1}{\alpha}V_0 \Rightarrow \frac{\partial V_0}{\partial\alpha}\left(\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right) = \frac{1}{\alpha}V_0\left(\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right), \tag{96}$$

and therefore,

$$\frac{\partial}{\partial\alpha}\left(\frac{dV_0}{dw}\right) + \frac{\partial V_0}{\partial\alpha}\left(\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right) = \frac{1}{\alpha}\left(\frac{dV_0}{dw} + V_0\sqrt{T}\frac{N'(d_1)}{N(d_1)}\frac{d\sigma}{dw}\right) = 0, \tag{97}$$

again from Eqn (59). Therefore,

$$\frac{\partial}{\partial\alpha}\left(\frac{dV_E^N}{dw}\right) = N(d_1)V_0\sqrt{T}\frac{\partial}{\partial\alpha}\left(\frac{N'(d_1)}{N(d_1)}\right)\frac{d\sigma}{dw}. \tag{98}$$

However we know from Property A2 that $\frac{d}{dx}\left(\frac{N'(x)}{N(x)}\right) < 0\ \forall x$. With all other terms positive, this implies that $\frac{\partial}{\partial\alpha}\left(\frac{dV_E^N}{dw}\right) < 0$. Hence with an increasing α, w has to decrease to reattain $\frac{dV_E^N}{dw} = 0$, and thus $\frac{\partial w^N}{\partial\alpha} < 0$ as desired.