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**Value of a Clear Desk: Sequencing
Decisions when Decision Capacity is
Limited**

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The Value of a Clear Desk: Sequencing Decisions when Decision Capacity is Limited*

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Abstract

Postponing a decision may allow a better decision later. However, depending on what else comes up, there may never be a moment when it makes sense to come back to a postponed decision, leaving potential gains unrealized. We develop a model of an agent with limited decision-making capacity who faces a sequence of decisions that vary stochastically in their importance and improvability. In each period he can either act on the newly-arrived opportunity or return to one carried from earlier. The prospect of future congestion in decisions generates an incentive to make prompt decisions, to “keep a clear desk”. The strength of that imperative is: (1) increasing in the expected importance of future opportunities but decreasing in the dispersion of their importance; (2) decreasing in the expected improvability of future opportunities, but ambiguously influenced by the dispersion of that improvability. The analysis illuminates some decision practices that would otherwise be hard to rationalize. Multiple equilibria in some cases rationalize persistently different behavior by two agents facing (almost) identical sequences of choices. The setting allows for a generalization of the concept of option value to congested decision environments and, by accounting for a plausible cost to postponement of action, offers a counter-force to the precautionary principle.

Keywords: Dynamic decision-making, limited attention, organizational bandwidth; committees, precautionary principle, option value

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1 Introduction

“Decisions are the prevailing fabric of our daily design” according to the Hall of Fame American Sports Journalist Don Yaeger. Both individuals and organizations face a multiplicity of situations that invite choices. Indeed some organisms, like committees and political bodies, exist only to make decisions on issues that appear before them.

In a business setting the serial entrepreneur and management ‘guru’ Ram Charan stresses the importance of the timing of decisions:

CEOs face countless decisions. The best executives understand which ones they need to focus on. They also know *when* to make a decision. And they’ve debated the risk of *not* doing it Any change in the landscape creates opportunities ... but their shelf life can be short. (From *You Can’t be a Wimp - Make the Tough Calls*, an interview with Ram Charan, Harvard Business Review (2013), emphasis in original).

When decisions once made are hard to reverse it is sometimes tempting to defer them. It is common to hear people and organizations saying that they will “return to this issue later”. An efficiency rationale might be that a better decision can be made later – that the decision is ‘improvable’. The gains from postponing irreversible decisions in a setting in which choice-relevant information is anticipated were first formalized by Arrow and Fisher (1974) and are captured by the notions of option and quasi-option value (Crabbe 1987). That the option value is positive provides the intellectual basis for the precautionary principle applied in policy evaluation (Atkinson et al 2006).¹

But postponement can have costs. For some decisions the benefits do not start to flow until a choice has been made. If a household without a dishwasher struggles to decide between brand A and brand B and so opts to postpone the choice, then the benefits of owning the new appliance start flowing later. Note

¹In these papers choice-relevant information arrives with passage of time. An alternative strand of research treats decision-makers as active gatherers of information. See for example Che and Mierendorff (2018) and references therein.

that this is not simply a question of discounting: the flow of benefits itself is truncated. Similar costs can apply to delays in investment decisions by firms (“should we locate the new production plant in city X or city Y?”) and in public policy choices. In other cases an investment or other opportunity may have a “shelf life” – consider a firm having the chance to be first entrant into a new market – such that doing it later may mean that the opportunity degrades or even expires altogether.

Our focus here is on a different cost of postponement, one that arises from constraints in future decision-making capacity. In essence, once a choice is postponed, (a) the decision maker might never find a time when it makes sense to go back to it, so that the benefits from action are never harvested; or (b) he may opt to revisit it in the future, but at the expense of diverting attention from some future choice that, considered in isolation, would have merited prompt action.

The existence of a finite limit on the capacity to make decisions is a common feature of many (perhaps almost all) decision environments. It is not difficult to think of rationales at a variety of levels.

Individuals. Human beings face time constraints that limit the number of issues to which they can productively attend (Heyes et al. 2018). Even ignoring such constraint the concept of ego depletion and limited ‘mental bandwidth’ (Schilback et al. 2016, Mullainathan and Shafir 2013) imply decision capacity is congestible. Many studies have shown that repeated decision-making causes decision fatigue (some cited in Amir 2008). This can substantially reduce decision quality, for example, among judges (Danziger 2011), financial professionals (Hirshleifer et al. 2018) and business leaders (Loewenstein et al. 2003). Various mechanisms may underpin such fatigue, including the glucose-depleting effect of mental exertion (Baumeister 2003). Similar effects have been found in other animals. Risk of decision fatigue leads some to develop strategies of ‘decision avoidance’ (Anderson 2003) and in the formal approach to decision quality management, specific techniques have been devised to help managers cope with decision fatigue including ensuring sufficient gaps between decisions (Saxena 2009).

Firms. The constrained decision-making capacity of a firm can be driven

by a number of factors. The most obvious is that key decision makers, from the CEO down, are themselves human and therefore subject to the physiological limitations outlined above. In short, the chief executive can only do so many things at once (Geanakoplos and Milgrom 1991).² “In the pursuit of value creation, people are limited by the capacity of their attention resources. When trying to achieve a goal (create value), one person can assimilate and understand only so much, reason so much, and take only so many actions in a period of time” (page 4, Nunamaker et al. 2001). But features of an organization, say how information flows within a firm, may *themselves* determine decision capacity, and the broader concept of *organizational* bandwidth is increasingly prominent. An early sketch of what it means, the limits upon it and how those limits are sensitive to organizational design and context can be found in Nunamaker et al. (2001) and the associated journal special issue *Enhancing Organizations’ Intellectual Bandwidth* to which that is an introduction.

Public and political organizations. Perhaps the most obvious institutional limits on decision capacity are to be found in some public and political bodies. Committees within organizations such as universities and school boards are often subject to rules about the frequency and timings of meetings and the length of their agenda (Hammond 1986). If agendas are set optimally in a forward-looking way then, depending upon what else comes along, something not resolved in one cycle will not necessarily command a place on the agenda in the next. Similarly legislatures, parliamentary committees, etc. convene for a limited number of days or hours per term and competition for space on the docket can be stiff (Brauniger and Debus 2011, Alesina and Perotti 1996) implying that only a limited number of issues can be subject to decision in any particular period.

When decision-making capacity is constrained, a decision maker facing a series of decisions that arrive in sequence faces a potentially complex problem.

²Geanakoplos and Milgrom (1991) preface their paper with a pertinent quotation from Herbert Simon: “The scarce resource is not information; it is processing capacity to attend to information. Attention is the chief bottleneck in organizational activity, and the bottleneck becomes narrower and narrower as we move to the tops of organizations, where parallel processing capacity becomes less easy to provide without damaging the coordinating function that is a prime responsibility of these levels (Simon 1976).

The constraint serves to connect decisions that could be treated independently and separately in an unconstrained environment. When a new decision arrives it has to compete for attention with unresolved prior decisions carried over from earlier. Relatedly, in opting to postpone a decision, he will recognize that so doing may impact what gets done in subsequent periods.

The rest of the paper is constructed as follows. In Section 2 we develop a stylized model of a capacity-constrained decision maker facing a sequence of decisions that vary in importance and in improvability/urgency. In the first instance we characterize the solution to the decision maker’s problem in a three-period setting which provides our basic results and makes transparent the mechanics. Section 3 considers a more general setting and Section 4 concludes.

2 Model

An agent faces a sequence of decision opportunities which we will refer to as **files**.³ These arrive one per period with D_t denoting the one that arrives in period t . Opportunities are time-limited – they have a ‘shelf-life’ – and expire after two periods. In other words, file D_t becomes worthless at $t + 2$. Most of the insight from the model can be gleaned from the case in which there are three periods (and two files) in the sequence: we treat that case in detail before generalizing.

A file is described by two attributes. For file D_t , we will denote by $\theta_t \geq 0$ the expected payoff if the decision is made on that file promptly at time t (in other words, immediately upon its arrival). This variable θ_t is a measure of the scale or **importance** of the opportunity. If the decision on that file is postponed to period $t + 1$ it delivers payoff $\alpha_t \theta_t$. For $\alpha_t > 1$ the decision is improved through postponement; if $\alpha_t < 1$, the value of the opportunity decays over time. We will refer to $\alpha_t > 0$ as the **improvability** of the period- t file. Conversely α_t^{-1} can be regarded as a measure of its **urgency**. The attributes θ_t and α_t are consistent with a variety of plausible micro-foundations. For our purposes they are sufficient to formulate the decision-maker’s problem and characterize its

³We will refer to the decision-maker as a single agent (‘he’) but it could equally well refer to other categories, including a committee or other collective.

solution.

The opportunities D_t over time are independent of each other. The decisions might involve completely distinct issues and there is, for example, no a priori complementarity between actions across decisions. We assume that θ_t and α_t are observable by the decision-maker at time t , in other words as soon as the file arrives, but that the characteristics of future files are uncertain.⁴

2.1 Optimal Timing with Unlimited Decision Capacity

To provide a benchmark we first consider the optimal timing of decisions when there is no limitation on decision-making capacity.

For this purpose we can consider file D_t in isolation, irrespective of the number of periods under consideration and whether or not the decision-maker arrives at period t carrying a previously postponed file.

The opportunity described in the file lasts two periods, so D_t calls for a decision either in period t or $t + 1$. That θ_t is non-negative implies that it is never optimal to allow the file to expire. A prompt decision has payoff θ_t . Delaying the decision for one period has payoff $\alpha_t\theta_t$. The optimal choice depends simply on the relative size of these payoffs (for the moment we abstract from any time preference).

Result 1 *With unlimited decision-making capacity it is optimal to make a prompt decision (i.e., at time t) on file $D_t = (\theta_t, \alpha_t)$ if and only if*

$$\theta_t \geq \alpha_t\theta_t. \tag{1}$$

Note that this result can be stated more simply: a prompt decision is optimal if and only if $\alpha_t \leq 1$, or that the project is not improved by waiting.

⁴At the cost of extra complexity the model could be extended to allow for (a) multiple files to arrive per period and/or, (b) files to have a shelf-life longer than two periods. An important implication of our approach is that in any period the number of unexpired files carried from earlier can be no more than one. This implies that we do not have to worry about dynamics that might arise if a larger portfolio of carried-over files were permitted. The current set-up allows us to develop the key arguments without excessive notation.

Importantly, in this unconstrained setting the optimal timing of decisions does not vary with θ_t , the importance of the file.

2.2 Optimal Timing with Limited Decision Capacity

When decision capacity is limited, even otherwise unrelated files become linked. In particular the way in which it is optimal to deal with file t becomes sensitive to what has happened in the past (in particular if decision on the prior file was postponed) and expectations about the characteristics of files likely to arrive in the future (in period $t + 1$ and beyond).

In our three-period setting we limit decision capacity starkly, assuming that the decision-maker can make at most one decision in any period.

Consider three alternative evolutions of decision points in this three-period setting. In period 1 the decision-maker receives D_1 . He can either make a prompt decision on that file or postpone it. A prompt decision on D_1 leaves him with a ‘clear desk’ for the arrival of file D_2 in the next period. In that case, in period 2 he may act promptly on D_2 or postpone it to period 3. In contrast, if a decision on the initial file D_1 is postponed, the decision maker is left with a cluttered desk and a more complex decision in period 2: he can either revert to the postponed file D_1 before it expires (in turn postponing D_2 to period 3), or can act on D_2 promptly leaving D_1 to expire. Depending on what he expects future files to look like, postponing D_1 , failing to leave a clear desk, can therefore be costly.

We evaluate the possible decision sequences backwards.

The choice in the final period 3 is straightforward: if holding an unexpired file (D_2), the agent makes a decision on it, receiving $\alpha_2\theta_2$.

In period 2 his choices depend on the state of (in)decision inherited from period 1 and on the realized characteristics of the period 2 file, (θ_2, α_2) .

First, consider the case in which decision on D_1 was postponed to period 2. If the new opportunity (θ_2, α_2) is revealed to be relatively unimportant and/or non-urgent he might revert to D_1 , with payoff $\alpha_1\theta_1$, rather than let it expire. This implies the postponement of D_2 to period 3, at which point he anticipates

it will be determined with payoff $\alpha_2\theta_2$. The total payoff in periods 2 and 3 along this trajectory is $\alpha_1\theta_1 + \alpha_2\theta_2$ (recall that we are ignoring any discounting).

On the other hand, if the new file D_2 is revealed to be sufficiently important and/or urgent then he will choose to attend to it promptly. This will deliver payoff θ_2 but imply the expiry of D_1 (and leave him with no decision to make in period 3).

To formalize this, when carrying unexpired file D_1 in period 2, the payoff to the optimal allocation of decision capacity, contingent on realization (θ_2, α_2) is

$$v_2(D_1; \theta_2, \alpha_2) = \max[\theta_2, \alpha_1\theta_1 + \alpha_2\theta_2] \quad (2)$$

From the perspective of period 1, before the attributes of D_2 are known, we can define the expectation over the distribution of $(\tilde{\theta}_2, \tilde{\alpha}_2)$ to compute

$$V_2(D_1) = \mathbb{E} [v_2(D_1; \tilde{\theta}_2, \tilde{\alpha}_2)].$$

$V_2(D_1)$ is then the period-2 ‘continuation value’ from postponement of decision on file D_1 .

Alternatively, consider a prompt decision on file D_1 , which gives payoff θ_1 and leaves a clear desk. The period-2 problem is then unencumbered, and leaves a choice between prompt decision on D_2 (yielding θ_2) or deferral to period 3 (yielding $\alpha_2\theta_2$). The optimized payoff is therefore

$$v_2(\emptyset; \theta_2, \alpha_2) = \max[\theta_2, \alpha_2\theta_2]. \quad (3)$$

Let $V_2(\emptyset)$ be the ex ante expectation of $v_2(\emptyset; \theta_2, \alpha_2)$: this is the continuation value of a clear desk.

We can now address the period 1 problem, to determine how the decision-capacity constraint affects the choice in the initial period. A prompt decision on D_1 pays θ_1 along with continuation value $V_2(\emptyset)$. Postponement delivers the continuation value $V_2(D_1)$. A prompt decision on D_1 is therefore optimal when $\theta_1 + V_2(\emptyset) \geq V_2(D_1)$.

To allow comparison with the unconstrained benchmark, define the differ-

ence between continuation values as

$$\Delta_2(D_1) = V_2(D_1) - V_2(\emptyset). \quad (4)$$

The optimal timing of the initial decision can then be summarized:

Result 2 *With limited decision capacity, a prompt decision on file D_1 is optimal if and only if*

$$\theta_1 \geq \Delta_2(D_1). \quad (5)$$

A comparison of Results 1 and 2 reveals how limiting decision capacity alters the threshold that D_1 must satisfy to merit prompt decision, from $\alpha_1\theta_1$ in the unconstrained case to $\Delta_2(D_1)$ in the capacity-constrained case. For further discussion it is useful to define the difference between the two thresholds as follows.

Definition (Value of a clear desk)

$$P(D_1, D_2) = \alpha_1\theta_1 - \Delta_2(D_1). \quad (6)$$

A positive value for $P(D_1, D_2)$ means that future decision-capacity constraints set a lower hurdle rate for prompt decision in period 1, namely $\Delta_2(D_1)$ rather than $\alpha_1\theta_1$ threshold when decision capacity is unlimited. The higher the magnitude of $P(D_1, D_2)$ the greater is the extent to which the constraint on decision capacity implies a propensity towards early decisions.⁵

The value $P(D_1, D_2)$ depends on the (known) attributes (θ_1, α_1) of the current file D_1 and the probability distribution of the attributes $(\tilde{\theta}_2, \tilde{\alpha}_2)$ of the future file D_2 . We can establish that $P(D_1, D_2)$ is non-negative, leading to the following proposition.

Proposition 1 *Limited decision-making capacity implies a non-negative premium to maintaining a clear desk, i.e. $P(D_1, D_2) \geq 0$, and (weakly) increases the likelihood of a prompt decision on the initial opportunity.*

⁵The relative sizes of the thresholds, $\Delta_2(D_1)/\alpha_1\theta_1$, measures the proportional reduction in the hurdle rate, an alternative measure that some readers might prefer. We adopt $P(D_1, D_2)$ because of its intuitive interpretation as the value of a clear desk.

Proof: Using definitions (4) and (6), we can write $P(D_1, D_2) = \mathbb{E}[p(D_1; \tilde{\theta}_2, \tilde{\alpha}_2)]$ where

$$p(D_1; \theta_2, \alpha_2) \equiv \alpha_1 \theta_1 - [(v_2(D_1; \theta_2, \alpha_2) - v_2(\emptyset; \theta_2, \alpha_2))].$$

Comparing (2) and (3) over various ranges of θ_2 and α_2 we find

$$p(D_1; \theta_2, \alpha_2) = \begin{cases} 0 & \text{if } \theta_2 \leq \alpha_2 \theta_2 \\ (1 - \alpha_2) \theta_2 & \text{if } \alpha_2 \theta_2 < \theta_2 \leq \alpha_1 \theta_1 + \alpha_2 \theta_2 \\ \alpha_1 \theta_1 & \text{otherwise.} \end{cases} \quad (7)$$

In each case $p(D_1; \theta_2, \alpha_2)$ lies in the interval $[0, \alpha_1 \theta_1]$: this is obvious in the first and third cases; for the second this follows from the restriction on the range. It follows that its expected value $P(D_1, D_2)$ must also lie in the interval $[0, \alpha_1 \theta_1]$. If so, $P(D_1, D_2) \geq 0$. ■

This is intuitive. Having a clear desk is never in itself costly. A cluttered one may or may not prove costly later, depending on the characteristics of D_2 realized subsequently, and the three cases delineated in the proof are illuminating. (1) In the first case, where $\alpha_2 > 1$, file D_2 turns out to be improvable/non-urgent to such an extent that postponement of a decision on that file to period 3 is warranted even if considered in isolation. In this case, a cluttered desk does not impose any cost (or, equivalently, there is no particular value to having kept the desk clear). (2) In the second case, decision 2 is not improvable, so that the attributes of D_2 do not, considered in isolation, warrant postponement. However, the cost of postponement is sufficiently small that – given the constraint on decision capacity – it makes sense to postpone it and go back to D_1 . (3) In the third case, D_2 is sufficiently significant and/or urgent that the constrained decision-maker chooses to abandon D_1 altogether. In the latter case, while allowing D_1 to expire is optimal ex post the decision-maker will regret not having processed it earlier.

The opportunity costs of postponement lie in the second case (where limited decision capacity induces inefficient postponement of future opportunities) and third case (where it leads to the abandonment of the initial opportunity). Case (3) corresponds to the concern that “... there may never be a moment when it

makes sense to come back to it” referred to in the abstract.

The set-up makes clear how the preferred handling of D_1 is highly dependent upon what future files are expected to look like. Recall that the magnitude of $P(D_1, D_2)$ measures the extent to which limitations of decision-making capacity bias the choice towards prompt decisions. The extent of this bias depends on the attributes of the current opportunity and the expected attributes of the future opportunity. We characterize this dependence through a sequence of results that assess how $P(D_1, D_2)$ varies with the probability distributions from which the parameters of future files will be drawn.

Result 3 *Other things being equal, the greater the importance of the current opportunity in file D_1 , the stronger the case for prompt decision. $P(D_1, D_2)$ is increasing in θ_1 .*

While this result seems intuitive, the specification in (7) is revealing. Here the case for a prompt decision of the initial file D_1 rests on the fear that postponement could result in abandonment of the file.

Result 4 *An increase in the expected importance of future decision opportunities – a shift in the distribution of θ_2 in the sense of first-order stochastic dominance – increases the value of a clear desk $P(D_1, D_2)$ and so makes a prompt decision on D_1 more likely.*

Again, the specification (7) is instructive. It shows that $p(\cdot)$ is strictly increasing in θ_2 in the second range and (and invariant to θ_2 in the first and third ranges). Here the value of a clear desk lies in avoiding sub-optimal postponement of future files. The expected value $P(D_1, D_2)$ is larger for a distribution that outranks another in the sense of first-order stochastic dominance over θ_2 . If future opportunities are likely to be more valuable then it is more likely that a decision-maker who does not decide file D_1 promptly will end up postponing future decisions that were relatively urgent. More simply, a clear desk is more valuable.

The decision environment in which an agent operates can vary in terms of how ‘choppy’ they are. In other words, to what extent are the files that appear

similar in importance. What effect does the variability of the importance of future projects have on the timing of decisions? Perhaps unexpectedly, we have the following.

Result 5 *An increase in the variability of the importance of future decision opportunities – a mean-preserving increase in the spread of the distribution of θ_2 – decreases the value of a clear desk $P(D_1, D_2)$ and makes a prompt decision on D_1 less likely.*

Once again, (7) is helpful. As $p(\cdot)$ is concave in θ_2 in the relevant range it follows, from Jensen's inequality, that a mean-preserving spread of the distribution of θ_2 lowers the expected value $P(D_1, D_2)$. Intuitively, if likely future opportunities are more dispersed in value then postponement of decision on D_1 is a little more desirable as it is more likely to be returned to for action later. In a sense the ability to revert to a past decision provides better insurance against low realizations of θ_2 .

To study the impact of varying the distribution of α_2 , the improvability of future opportunities, it helps to rewrite (7) to express $p(\cdot)$ over ascending ranges of α_2 .

$$p(\cdot) = \begin{cases} \alpha_1 \theta_1 & \text{if } \alpha_2 \leq 1 - (\alpha_1 \theta_1 / \theta_2) \\ \theta_2 - \theta_2 \alpha_2 & \text{if } 1 - (\alpha_1 \theta_1 / \theta_2) < \alpha_2 \leq 1 \\ 0 & \text{if } \alpha_2 > 1. \end{cases} \quad (8)$$

Across the three ranges, $p(\alpha_2, \cdot)$ is decreasing in α_2 (it is weakly decreasing in the first and the third range and strictly decreasing in the intermediate range). This leads to:

Result 6 *An increase in the expected improvability of future decision opportunities – a shift in the distribution of α_2 in the sense of first-order stochastic dominance – decreases the value of a clear desk $P(D_1, D_2)$ and makes a prompt decision on D_1 less likely.*

The intuition here is subtle. The greater the improvability of second-period

opportunities, the greater the likelihood that these will be postponed to period 3. This dilutes the advantage to entering period 2 with a clear desk.

Finally, to complete the analysis we consider the impact of variability in the improvability of future projects.

Result 7 *An increase in the variability of the improvability of future decision opportunities – a mean-preserving increase in the spread of the distribution of α_2 – may increase or decrease the value of a clear desk $P(D_1, D_2)$ and may make a prompt decision on D_1 more or less likely.*

The ambiguity follows from $p(\cdot)$ being neither convex nor concave in α_2 : its graph has a convex section in one range, concave in the other. As such it is not possible to make categorical statements without additional restriction. The worked example below will deliver a more precise condition for a particular specification.

2.3 An Example with Uniform Distributions

To sharpen our understanding of the magnitude of these effects, we can obtain a closed-form solution for a particular family of distributions.

We consider the special case where the two attributes of D_2 are distributed independently, and each distribution is uniform over a non-negative interval.

Assumption 1 *The attributes of decision opportunity D_2 are uniformly and independently distributed.*

(a) $\tilde{\theta}_2 \sim U[q, Q]$, where $0 \leq q < Q$;

(b) $\tilde{\alpha}_2 \sim U[a, A]$, where $0 \leq a < 1 < A$.

The restrictions on the parameter range are minimal for our purpose. That the lower limits of these distributions – q for the payoff and a for the improvability parameter – are both non-negative ensures that the payoff to the opportunity is non-negative, even if a decision on it is postponed. That the

upper limit for improvability $A > 1$ retains some richness to the setting by ensuring that future opportunities may be, in at least some realizations, sufficiently improvable to merit postponement. The choice of uniform distributions allows us to compute closed-form solutions.

The Appendix details how $P(D_1, D_2)$ can be evaluated for the distributions specified in Assumption 1. To save notation, we define $I \equiv \alpha_1 \theta_1$.

Result 8 *Under Assumption 1, we have*

$$P(\cdot) = \begin{cases} \frac{1}{(A-a)(Q-q)} \left[(1-a)(Q-q)I - \frac{1}{2}I^2 \ln\left(\frac{Q}{q}\right) \right] & \text{if } \frac{I}{1-a} \leq q, \\ \frac{1}{(A-a)(Q-q)} \left[I(1-a)Q - \frac{3}{4}I^2 - \frac{1}{4}(q(1-a))^2 - \frac{1}{2}I^2 \ln\left(\frac{Q(1-a)}{I}\right) \right] & \text{if } q < \frac{I}{1-a} \leq Q \\ \frac{1}{(A-a)(Q-q)} \left[\frac{1}{4}(1-a)^2(Q^2 - q^2) \right] & \text{otherwise.} \end{cases}$$

It is straightforward to verify that $P(D_1, D_2)$ is non-negative for each of the ranges above and that it is weakly increasing in θ_1 , the value of the initial opportunity.

The closed-form solution also allows us to confirm the impact of variations in the distribution of future opportunities, along the lines of our general Results 4 to 7. We consider small perturbations in the range of the distributions specified in Assumption 1. In each case $\epsilon > 0$ is a small change that captures a perturbation in the underlying distribution, broadly in the sense of first- or second-order stochastic dominance. The Appendix provides a proof of the following.

Result 9 *Impact of distributional changes on $P(D_1, D_2)$. Consider $\epsilon > 0$.*

1. *A mean-increasing shift in the distribution of θ_2 , from $U[q, Q]$ to $U[q + \epsilon, Q + \epsilon]$ increases $P(D_1, D_2)$.*
2. *A mean-preserving spread of the distribution of θ_2 , from $U[q, Q]$ to $U[q - \epsilon, Q + \epsilon]$ lowers $P(D_1, D_2)$.*
3. *A mean-increasing shift in the distribution of improvability parameter α_2 , from $U[a, A]$ to $U[a + \epsilon, A + \epsilon]$ lowers $P(D_1, D_2)$,*

4. A mean-preserving spread of the distribution of α_2 , from $U[a, A]$ to $U[a - \epsilon, A + \epsilon]$, increases $P(D_1, D_2)$ as long as the expected value of α_2 is not less than 1.

These results mirror our earlier statements for the general case. In the last case, where we had not been able to provide a categorical statement in the general case, our special case with uniform distributions is more helpful. As long as $\frac{a+A}{2} \geq 1$ – that is the future opportunity is *on average* improvable – increased dispersion of the improvability parameter reinforces the case for prompt decisions.

How large is the effect of limited decision capacity, in quantitative terms? Recall that we can think of $\Delta_2(D_1)/I$ as a measure of the relative diminution of the hurdle rate from I (with no decision capacity constraint) to $\Delta_2(D_1)$ (with constraint). We use Result 8 to compute numerical values for this ratio for a particular set of distributions. We set $\theta_2 \sim U[1, 100]$ to allow future opportunities to vary considerably in their importance. We set $\alpha_2 \sim U[0.6, 1.8]$, which implies that the future opportunity is positively improvable, on average by 20%, with significant variation around this average. For these distributions, we find that $\Delta_2(D_1)/I$ ranges from around 0.6 to 0.9, depending on I , the value of the initial opportunity. Put simply, the restriction that only one decision can be made in any period creates a relatively large bias towards prompt decisions, with 10 to 40% reduction in the ‘target return’. These values are not overly sensitive to our particular choice of parameters. In other words, a clear desk might be quite valuable.

3 Generalizing to Many Periods

We can generalize the insights developed above into decision sequences that last arbitrarily many periods. Given the potentially infinite decision horizon, we now allow for time preference, with $\beta < 1$ as the per-period discount factor.

Consider a setting with an arbitrary number of future files, D_{t+1}, D_{t+2}, \dots . As before, each file is characterized by two variables, $\theta_t \geq 0$ (its importance or size) and $\alpha_t \geq 0$ (its improvability). The attributes of current files are

known; those of future files are uncertain but the probability distributions over $(\theta_{t+i}, \alpha_{t+i})$ are known at time t .

Again it is helpful to identify the benchmark for the optimal timing of decisions in the absence of limitations in decision-making capacity.

For an isolated opportunity D_t , a prompt decision in period t has payoff θ_t . Postponing the decision to period $t + 1$ has payoff $\alpha_t \theta_t$, with present value $\beta \alpha_t \theta_t$. In line with Result 1 we have:

Result 10 *Consider an agent with unlimited decision-making capacity. It is optimal to make a decision on file $D_t = (\theta_t, \alpha_t)$ promptly (i.e. at time t) iff*

$$\theta_t \geq \beta \alpha_t \theta_t. \tag{9}$$

This is intuitive. Decisions that are sufficiently improvable (large enough α) are postponed and decided one period later. Those sufficiently ‘urgent’ relative to the discount factor – i.e. where $1/\alpha_t \geq \beta$ – are decided promptly.

This decision rule is optimal for any sequence of opportunities, $\dots D_{t-1}, D_t, D_{t+1} \dots$, as long as decision capacity is not limited.

3.1 Optimal Timing with Limited Decision Capacity

We now turn to the case with limited decision capacity. Once again we focus on a stark version of limited decision capacity by assuming that our decision maker can make at most one decision in any period.

Here the optimal decision at any time t may depend not only on the attributes of the current opportunity D_t , but also those of any file carried from the period before, and the decision maker’s conjectures about the attributes of all future opportunities, D_{t+1}, D_{t+2}, \dots .

For presentation we distinguish between two cases, based on whether the decision maker enters period t with a clear desk (no files postponed from previous periods) or a ‘cluttered desk’ (with a file postponed from the previous period).

Starting with a clear desk

Consider a state in which file $D_t = (\theta_t, \alpha_t)$ arrives in period t onto an empty desk (meaning that opportunity D_{t-1} was decided promptly in the previous period).

If the decision-maker decides at time t to address the newly-arrived opportunity, he obtains an immediate payoff θ_t and moves on to the next period with a clear desk. Let $V_{t+1}(\emptyset)$ be the continuation value of being in that future state with \emptyset indicating, as before, a clear desk. Discounting future values, the total payoff to a prompt decision on file D_t is $\theta_t + \beta V_{t+1}(\emptyset)$.

If the decision-maker chooses at time t to postpone the decision on file D_t , his immediate payoff is zero and the continuation value, denoted as $V_{t+1}(D_t)$, incorporates the option of returning to D_t in the next period.

In any period t that starts with a clear desk, a prompt decision on file D_t is optimal if and only if

$$\theta_t + \beta V_{t+1}(\emptyset) \geq \beta V_{t+1}(D_t). \quad (10)$$

As before, we save notation by defining

$$\Delta_{t+1}(D_t) \equiv V_{t+1}(D_t) - V_{t+1}(\emptyset). \quad (11)$$

Relations (10) and (11) allow the following characterization of the optimal timing of decisions:

Result 11 *Consider an agent with limited decision-making capacity. Starting any period t with a clear desk it is optimal to make a prompt decision (i.e., at time t) on file $D_t = (\theta_t, \alpha_t)$ iff*

$$\theta_t \geq \beta \Delta_{t+1}(D_t). \quad (12)$$

To assess how future limitations of decision-making capacity affect the propensity towards prompt decisions for current opportunities, we generalize our three-period model. From Result 10, absent decision capacity constraints,

the required threshold for a prompt decision was $\beta\alpha_t\theta_t$. In the presence of a decision capacity constraint, Result 11 tells us that the threshold is $\beta\Delta_{t+1}(D_t)$.

Denote the difference between these thresholds – the value of a clear desk when facing a future sequence $\{D_{t+i}\}$ of files – as

$$P(D_t, \{D_{t+i}\}) = \beta[\alpha_t\theta_t - \Delta_{t+1}(D_t)]. \quad (13)$$

It follows that decision capacity constraints will create a propensity towards prompt decisions whenever $P(D_t, \{D_{t+i}\}) \geq 0$. As Proposition 2 shows, this is indeed the case.

Proposition 2 *For an agent with a clear desk, limiting decision-making capacity (weakly) increases the propensity to make a prompt decision on the file that arrives.*

The proof of this Proposition, a generalization of the proof of Proposition 1, is relegated to the Appendix. In principle, the evaluation of $P(D_t, \{D_{t+i}\}) \geq 0$ is potentially complex because the continuation values $V_{t+1}(D_t)$ and $V_{t+1}(\emptyset)$ depend on the entire sequence of future decision opportunities. However, as the proof shows, the difference $\Delta_{t+1}(D_t)$ between the continuation values is bounded above by $\alpha_t\theta_t$, which allows us to establish that $P(D_t, \{D_{t+i}\})$ is non-negative.

To gain some intuition on why $\Delta_{t+1}(D_t)$ is bounded above by $\alpha_t\theta_t$, we must analyze the implications of postponing a decision on today's file, in terms of constraints on future choices. There are three scenarios. One, if tomorrow's file turns out to be non-urgent, it can be postponed to the day after, so that today's clutter imposes no cost at all. Or, it could be that tomorrow's file might ideally call for prompt action, but the cost of delay is less than the benefit of clearing the inherited backlog. Finally, in the worst case scenario, tomorrow's file might turn out to be so urgent and so important that it leads to the abandonment of the current file, losing $\alpha_t\theta_t$. In each case, tomorrow's cluttered desk (as a result of today's postponement) causes a loss of at most $\alpha_t\theta_t$. Hence the expected cost $\Delta_{t+1}(D_t) < \alpha_t\theta_t$. As long as the support of the distribution of future scenarios includes the second or third case above, the

value of $\Delta_{t+1}(D_t)$ will be strictly less than $\alpha_t\theta_t$.

There is another sense in which it is intuitively plausible that $\Delta_{t+1}(D_t) \leq \alpha_t\theta_t$. A decision-maker who find himself weighed down by past indecision can voluntarily abandon the previous file, whose value is $\alpha_t\theta_t$.⁶ The difference between the two future valuations $V_{t+1}(D_t)$ and $V_{t+1}(\emptyset)$ cannot exceed this value.

Note that the results mirror very closely the arguments that we had uncovered in the three-period model in the previous Section. This is not surprising: given our assumption that all files expire after two periods, there can be at most two files on the desk in any period, so the three period story suffices. However, there is one important aspect in which this general setting allows us to go beyond the previous model. In a three-period model, the decision-maker necessarily starts the initial period $t = 1$ with a clear desk. In contrast, when analyzing a three-period interval in a sequence of arbitrary length, we can allow the possibility that the decision maker finds himself with a cluttered desk at time t , due to indecision in the previous period $t - 1$. We turn to this next.

Starting with a cluttered desk: Backlog of past indecision

Consider the case where the decision-maker receives file D_t in period t but with file D_{t-1} still on his desk, having been postponed from the previous period. If he returns to that file he cannot make a decision on the newly arrived one; if he does not return to it the opportunity described in that file expires and is lost.

Consider the payoffs to the choices. If he proceed to a decision on the newly-arrived file D_t the payoff is $\theta_t + \beta V_{t+1}(\emptyset)$. If he reverts to D_{t-1} , the file inherited from the past, the payoff is $\alpha_{t-1}\theta_{t-1} + \beta V_{t+1}(D_t)$. A comparison of these payoffs leads to the following result.

Result 12 *Consider an agent with limited decision-making capacity. Starting any period t with a cluttered desk (carrying unexpired file D_{t-1}) it is optimal*

⁶It is a moot point that individuals are often attached to, even weighed down by, past issues, over and above the realizable value of lingering decisions.

to make a decision on the newly-arrived file D_t iff

$$\theta_t \geq \beta\Delta_{t+1}(D_t) + \alpha_{t-1}\theta_{t-1}. \quad (14)$$

A comparison of Results 11 and 12 is revealing. One, as we would expect, past indecision makes it less likely that new files will be decided upon promptly. Prompt action towards D_t requires that θ_t must exceed not only the future advantage from postponement – namely $\beta\Delta_{t+1}(D_t)$ – but also the payoff $\alpha_{t-1}\theta_{t-1}$ to the expiring file D_{t-1} . In fact, given that Δ_{t+1} is non-negative, it will never make sense to address a newly-arrived file promptly if its payoff is less than that of the inherited file.

Once again, we can examine how limitations of decision capacity affect the propensity towards prompt decisions. Absent limitations in decision capacity, the optimal timing of action on file D_t is given by Result 10, with for prompt action only if θ_t exceeds $\beta\alpha_t\theta_t$. With limited decision capacity the threshold is given by Result 12. When starting from a cluttered desk, the difference between these thresholds is

$$\hat{P}(D_t, \{D_{t+i}\}; D_{t-1}) = \beta[\alpha_t\theta_t - \Delta_{t+1}(D_t)] - \alpha_{t-1}\theta_{t-1}. \quad (15)$$

Depending on the value $\alpha_{t-1}\theta_{t-1}$, the expression $\hat{P}(D_t, \{D_{t+i}\})$ could be positive or negative. Our previous Proposition showed that with limited decision capacity, a decision maker with a clear desk is inclined to be more decisive towards current opportunities. The next Proposition, which follows directly from previous results, spells out the obverse of this, that a decision maker with a cluttered desk may end up being less attentive to current opportunities.

Proposition 3 *For a decision-maker who finds himself at a cluttered desk, limitations in decision-making capacity can decrease the propensity to react promptly to file that has just arrived.*

3.2 Persistent procrastination

Some individuals and organizations have a reputation for indecision, always postponing decisions on issues that arise. At the same time another decision-maker facing seemingly identical stream of decisions deals with them all promptly, always keeping a clear desk. It is tempting to ascribe the adoption of one decision practice over the other to differences in personality.

However our model can explain how these differences might be the outcome of rational calculation, and allow us to characterize the coexistence of such behaviors as multiple equilibria that differ in their initial conditions.

Consider a sequence of files $\{D_{t+i}\}$ that the decision-maker knows to be all of equal importance (that is, $\theta_{t+i} = \theta$ for all i), and all opportunities are positively improvable to the same extent ($\alpha_{t+i} = \alpha$ for all i). In essence the attributes of future opportunities are drawn from a common, degenerate distribution.

Assume that $\alpha \in (1, \frac{1}{\beta})$. In the absence of any limitations of decision capacity, in accordance with Result 10, it is optimal to act promptly on each file as it arrives, given the restriction on α . The same decision routine – to decide promptly in each period – is also optimal with limited decision capacity, *as long as the decision maker starts with an uncluttered desk*. To see why, it is easy to check that that $\Delta(D_t) = \frac{(\alpha-1)}{1-\beta}\theta$ in this setting. If so, the condition specified for prompt action in Result 11 – that $\theta_t > \beta\Delta(D_t)$ – is readily satisfied with the assumed restriction that $\alpha < 1/\beta$.

At the same time, for a decision maker who starts with a cluttered desk, the threshold for prompt decisions specified by Result 12 is never met for the parameter values. In particular, the requirement that $\theta_t > \beta\Delta(D_t) + \alpha\theta$ is never met for $\alpha > 1$.

Put simply, a decision-maker who enters any period with a clear desk would find it optimal to make an immediate decision on a newly-arrived file, maintaining such behavior in each subsequent period. In contrast, an individual who finds himself with a cluttered desk would find it optimal to address the postponed file and leave the newly-arrived file to the next period, when such behavior would be repeated.

Note that with a rigid rate of arrival of files (precisely one per period) and decision capacity (no more than one per period) the equilibrium behavior for our decision-maker could either be one of consistently prompt decision or of consistent delay, depending upon the initial state. In a slightly perturbed version of the setting, consider a decision-maker with a clear desk who receives in some period *two* files, or the decision-maker fails to make a decision in one period for whatever reason, the implied delay in processing one file would imply postponement of all subsequent files. Conversely, a decision-maker with a cluttered desk who is able to clear the backlog by making two decisions in one period (say, by working a double-shift) would then find it optimal to decide promptly on all subsequent files.

We summarize this as follows:

Result 13 *For mid-range values of improvability, in particular for $\alpha \in (1, \frac{1}{\beta})$, timing of decisions is history-dependent. Delay in decision making is persistent as is promptness.*

4 Conclusions

We have investigated the optimal behavior of a capacity-constrained decision-maker who faces a sequence of decisions.

Absent a limit on decision capacity each decision can be thought about in isolation. The decision-maker will choose to postpone a decision only if the improvement in the quality of the postponed decision enhances payoff more than the cost of delay due to discounting. The timing of decision-making on each file is independent of all other past and future files.

When a decision-maker is limited in the number of decisions he can make in a particular period, his problem is more complex. If he chooses to make progress on a newly-arrived file it might preclude his attending to a file carried over from a previous period. Relatedly, if he postpones a decision on a newly-arrived file he may not in the future have a time when it makes sense to come back to it, in which case the payoff opportunity embedded in that file may never be harvested. Alternatively, he may subsequently come back to it, but

in so doing distort the timing of decision-making on future files in undesirable ways. As such both past decision patterns and expected properties of *the whole sequence* of future files need to be accounted for in determining the optimal timing of decisions.

Our general finding is the recognition that future decision capacity is congestible creates a bias towards prompt action towards current decisions. That such a bias should exist seems self evident but we offer a method to understand the different future scenarios that lead to this bias and to the quantify the size of the bias in specific settings. We also study how the optimal timing of decisions depends, in less obvious ways, on the probability distribution of attributes associated with future opportunities. Making a prompt decision on a particular file becomes more attractive when future decisions are expected to be more important, since that makes it more likely that a file postponed today might never be revisited. But that may matter less if future files are expected to be more improvable, since even if an important file arrives next period it is itself likely to be postponed. Interestingly, the incentive to postpone a decision on a current file is increasing in how variable future files are expected to be in terms of importance, while the effect of increasing the variability in improvability (urgency) is in general ambiguous.

The framework also relates to the literature on irreversible choices in environments in which choice-relevant information is arriving (Arrow and Fisher 1974, Dixit and Pindyck 1994). By embedding the expected opportunity costs to delayed action when decision capacity is limited the current framework provides a counter-force to the option (and quasi-option) values associated with waiting.

The model we have presented has been stylized. It has allowed us to highlight the key inter-temporal inter-dependencies that are thrown up when a sequence of decisions confronts a capacity-constrained decision-maker. A richer version could, (1) allow for files that vary in shelf-life (or may not expire at all); (2) model a decision body that could make more than one decision period but subject to a cost either pecuniary or in terms of decision quality; (3) allow the possibility of a backlog greater than one, allowing for varying degrees of ‘decision clutter’; (4) permit some degree of interdependence (complementarity or

substitutability) between files through channels other than via the constraint on decision capacity. The precise extensions would depend on the specifics of the problems to be addressed, but we believe that our model provides a useful way to organize such analyses.

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Appendix

Proof of Result 8

By definition $P(D_1, D_2) = I - \Delta_2(D_1)$ where $I \equiv \alpha_1 \theta_1$ and $\Delta_2(D_1)$ is the expected value of

$$\delta_2(D_1; \tilde{\theta}_2, \tilde{\alpha}_2) = \begin{cases} I & \text{if } \tilde{\theta}_2 \leq \tilde{\alpha}_2 \tilde{\theta}_2, \\ I - (1 - \tilde{\alpha}_2) \tilde{\theta}_2 & \text{if } \tilde{\alpha}_2 \tilde{\theta}_2 < \tilde{\theta}_2 \leq I + \tilde{\alpha}_2 \tilde{\theta}_2 \\ 0 & \text{otherwise.} \end{cases}$$

We compute $\mathbb{E}(\delta_2)$ for the uniform distributions in Assumption 1 to evaluate $P(D_1, D_2)$. We have three cases, depending on the value of I relative to the parameters of the distributions.

Case 1: $I > Q(1 - a)$ (ie., when the unexpired opportunity is relatively large.)

For $\tilde{\alpha}_2 \geq 1$ we have $\delta_2(D_1) = I$, so $\int_q^Q \int_1^A \delta_2(\cdot) d\alpha_2 d\theta_2 = \left(\frac{A-1}{A-a}\right) I$. For $\tilde{\alpha}_2 < 1$, we have $\delta_2(D_1) = I - (1 - \tilde{\alpha}_2) \tilde{\theta}_2$, so

$$\int_q^Q \int_a^1 \delta_2(\cdot) d\alpha_2 d\theta_2 = \frac{1}{A-a} \left[(1-a)I - \frac{(1-a)^2 q + Q}{2} \right].$$

Adding across the two ranges, and simplifying we get

$$P(D_1, D_2) = \frac{1}{(A-a)(Q-q)} \left[\frac{1}{4} (1-a)^2 (Q^2 - q^2) \right].$$

Case 2: $q(1 - a) < I < Q(1 - a)$.

Once again, for $\tilde{\alpha}_2 \geq 1$ we have $\int_q^Q \int_1^A \delta_2(\cdot) d\alpha_2 d\theta_2 = \left(\frac{A-1}{A-a}\right) I$. For $\tilde{\alpha}_2 < 1$, the integral $\int_q^Q \int_a^1 \delta_2(\cdot) d\alpha_2 d\theta_2$ equals

$$\frac{1}{A-a} \frac{1}{Q-q} \left[\frac{1}{4} (I - q(1-a))(3I - q(1-a)) + \frac{1}{2} I^2 \ln \left(\frac{Q(1-a)}{I} \right) \right].$$

After straightforward manipulation,

$$P(D_1, D_2) = \frac{1}{(A-a)(Q-q)} \left[I(1-a)Q - \frac{3}{4} I^2 - \frac{1}{4} (q(1-a))^2 - \frac{1}{2} I^2 \ln \left(\frac{Q(1-a)}{I} \right) \right].$$

Case 3: $I < q(1 - a)$.

Again, for $\tilde{\alpha}_2 \geq 1$, we have $\int_q^Q \int_1^A \delta_2(\cdot) d\alpha_2 d\theta_2 = \left(\frac{A-1}{A-a}\right) I$. For $\tilde{\alpha}_2 < 1$, we have

$$\int_q^Q \int_a^1 \delta_2(\cdot) d\alpha_2 d\theta_2 = \frac{1}{2} \frac{1}{(A-a)(Q-q)} \left[I^2 \ln \left(\frac{Q}{q} \right) \right].$$

Hence

$$P(D_1, D_2) = \frac{1}{(A-a)(Q-q)} \left[(1-a)(Q-q)I - \frac{1}{2} I^2 \ln \left(\frac{Q}{q} \right) \right].$$

Together we write

$$P = \begin{cases} \frac{1}{(A-a)(Q-q)} \left[(1-a)(Q-q)I - \frac{1}{2} I^2 \ln \left(\frac{Q}{q} \right) \right] & \text{if } \frac{I}{1-a} \leq q, \\ \frac{1}{(A-a)(Q-q)} \left[I(1-a)Q - \frac{3}{4} I^2 - \frac{1}{4} (q(1-a))^2 - \frac{1}{2} I^2 \ln \left(\frac{Q(1-a)}{I} \right) \right] & \text{if } q < \frac{I}{1-a} \leq Q \\ \frac{1}{(A-a)(Q-q)} \left[\frac{1}{4} (1-a)^2 (Q^2 - q^2) \right] & \text{otherwise.} \end{cases}$$

It is tedious but straightforward to check that $P(D_1, D_2)$ is positive in each range.

It is easy to verify that $P(D_1, D_2)$ is (weakly) increasing in I . In the first case, it is sufficient to check that $(1-a)(Q-q) > I \ln \left(\frac{Q}{q} \right)$, or equivalently that $\frac{Q}{q} - 1 - \frac{I}{(1-a)q} \ln \left(\frac{Q}{q} \right)$ is positive. This is of the form $x - 1 - \gamma \ln x$ where $x > 1$ and $\gamma = \frac{I}{(1-a)q} \leq 1$, which is always positive. A similar argument can be made in the second case. In the third case, $P(D_1, D_2)$ does not vary with I . ■

Proof of Result 9

In each case we are looking at a perturbation of the uniform probability distribution, either of θ_2 or α_2 , where the perturbation affects the range of the distribution. We can capture these perturbations as a function $P(D_1, D_2; \epsilon)$ and then investigate if this function is increasing or decreasing in ϵ . The function is continuous over the ranges of I so it suffices to check that the comparative static result holds over each range.

1. A mean-enhancing shift of the distribution $U[q, Q]$ to $U[q + \epsilon, Q + \epsilon]$

For expositional simplicity, we write $P(\epsilon)$ to capture only the terms in $P(D_1, D_2; \epsilon)$ that vary with ϵ associated with this perturbation.

$$P(\epsilon) \sim \begin{cases} -\ln \left(\frac{Q+\epsilon}{q+\epsilon} \right) & \text{if } \frac{I}{1-a} \leq q, \\ I(1-a)(Q+\epsilon) - \frac{1}{4} (q+\epsilon)^2 (1-a)^2 - \frac{1}{2} I^2 \ln \left(\frac{(Q+\epsilon)(1-a)}{I} \right) & \text{if } q < \frac{I}{1-a} \leq Q \\ (Q + q + 2\epsilon) & \text{otherwise.} \end{cases}$$

For the first case

$$\text{sign} \left[\frac{dP}{d\epsilon} \right] = \text{sign} \left[\frac{Q - q}{(Q + \epsilon)(q + \epsilon)} \right] > 0.$$

For the second case, where $(1-a)q < I < (1-a)Q$,

$$\begin{aligned} \text{sign} \left[\frac{dP}{d\epsilon} \right] &= \text{sign} \left[I(1-a) - \frac{1}{2}(1-a)^2(q+\epsilon) - \frac{1}{2} \frac{I^2}{Q+\epsilon} \right] \\ &= \text{sign} \frac{1}{2} \left[(1-a) [I - (1-a)(q+\epsilon)] + I \left[(1-a) - \frac{I}{Q+\epsilon} \right] \right] \end{aligned}$$

For ϵ small enough, in the specified range the expression is positive.

For the third case, $P(D_1, D_2)$ is obviously increasing in ϵ .

In sum, across all three ranges, a mean-enhancing shift in the distribution of θ_2 , from $U[q, Q]$ to $U[q + \epsilon, Q + \epsilon]$, causes $P(D_1, D_2; \epsilon)$ to weakly increase. \square

2. A mean-preserving spread of the distribution of θ_2 , from $U[q, Q]$ to $U[q-\epsilon, Q+\epsilon]$.

Once again, $P(\epsilon)$ captures only the terms in $P(D_1, D_2; \epsilon)$ that vary with ϵ associated with this perturbation.

$$P \sim \begin{cases} \frac{1}{Q-q+2\epsilon} \left[(1-a)(Q-q)I - \frac{1}{2}I^2 \ln \left(\frac{Q}{q} \right) \right] & \text{if } \frac{I}{(1-a)} \leq q, \\ \frac{1}{Q-q+2\epsilon} \left[I(1-a)(Q+\epsilon) - \frac{3}{4}I^2 - \frac{1}{4}(q-\epsilon)^2(1-a)^2 - \frac{1}{2}I^2 \ln \left(\frac{(Q+\epsilon)(1-a)}{I} \right) \right] & \text{if } q < \frac{I}{1-a} \leq Q \\ \text{constant} & \text{otherwise.} \end{cases}$$

For the first case, differentiating $P(\epsilon)$ with respect to ϵ we have

$$\text{sign} \left[\frac{dP(\cdot)}{d\epsilon} \right] = \text{sign} \left[-x + \frac{1}{x} - 2 \ln x \right]$$

where $x(\epsilon) \equiv \frac{Q+\epsilon}{q-\epsilon}$. It is easy to verify that expression in square brackets in the right is negative for $x > 1$.

For the second case, where $(1-a)q < I < (1-a)Q$, after some manipulation, we find

$$\text{sign} \left[\frac{dP(\cdot)}{d\epsilon} \right] = \text{sign} \left[1 - m + \ln m + \frac{1}{2} \frac{q-\epsilon}{Q+\epsilon} (1-m)^2 \right]$$

where $m \equiv \frac{(1-a)(Q+\epsilon)}{I}$ must exceed 1 in this range. Once again, the expression in square brackets is negative. To see why, note that the expression is zero for $m = 1$, and its derivative equals $\frac{1-m}{(1-a)(Q+\epsilon)} [I - (1-a)(q+\epsilon)]$ which is negative for $m > 1$.

For the third case, $P(D_1, D_2)$ is independent of ϵ .

In sum, across the three ranges, a mean-preserving spread of the distribution of θ_2 , from $U[q, Q]$ to $U[q - \epsilon, Q + \epsilon]$ causes $P(D_1, D_2; \epsilon)$ to (weakly) decrease.

□

3. A mean-enhancing shift of the distribution of improvability parameter α_2 , from $U[a, A]$ to $U[a + \epsilon, A + \epsilon]$.

Once again, $P(\epsilon)$ captures only the terms in $P(D_1, D_2; \epsilon)$ that vary with ϵ associated with this perturbation.

$$P(\epsilon) \sim \begin{cases} (1-a-\epsilon)(Q-q)I & \text{if } \frac{I}{(1-a)} \leq q, \\ I(1-a-\epsilon)Q - \frac{1}{4}[q(1-a-\epsilon)]^2 - \frac{1}{2}I^2 \ln(1-a-\epsilon) & \text{if } q < \frac{I}{1-a} \leq Q \\ (1-a-\epsilon)^2(Q^2 - q^2) & \text{otherwise.} \end{cases}$$

For the first and the third cases, it is straightforward to check that $P(\cdot)$ is decreasing in ϵ . For the second case

$$\text{sign} \left[\frac{dP(\cdot)}{d\epsilon} \right] = \text{sign} \left[I \left(\frac{I}{1-a-\epsilon} - Q \right) + (q^2(1-a+\epsilon) - IQ) \right] < 0.$$

In sum, a mean-enhancing shift of the distribution of improvability parameter α_2 , from $U[a, A]$ to $U[a + \epsilon, A + \epsilon]$ causes $P(D_1, D_2; \epsilon)$ to decrease. □

4. A mean-preserving spread of the distribution of α_2 from $U[a, A]$ to $U[a - \epsilon, A + \epsilon]$.

Here

$$P = \begin{cases} \frac{1}{A-a+2\epsilon} \left[(1-a+\epsilon)(Q-q)I - \frac{1}{2}I^2 \ln \left(\frac{Q}{q} \right) \right] & \text{if } \frac{I}{(1-a)} \leq q, \\ \frac{1}{A-a+2\epsilon} \left[I(1-a+\epsilon)Q - \frac{3}{4}I^2 - \frac{1}{4}(q(1-a+\epsilon))^2 - \frac{1}{2}I^2 \ln \left(\frac{Q(1-a+\epsilon)}{I} \right) \right] & \text{if } q < \frac{I}{1-a} \leq Q \\ \frac{1}{A-a+2\epsilon} \left[\frac{1}{4}(1-a+\epsilon)^2(Q^2 - q^2) \right] & \text{otherwise.} \end{cases}$$

For the first case

$$\text{sign} \left[\frac{dP(\cdot)}{d\epsilon} \right] = \text{sign} \left[I(Q - q)(A + a - 2) + I^2 \ln(Q/q) \right]$$

which is positive if $\frac{1}{2}(A + a) > 1$. For the second case the argument is slightly more tedious but straightforward whenever $\frac{1}{2}(A + a) > 1$. For the third case, the argument is simple.

In sum, a mean-preserving spread of the distribution of α_2 from $U[a, A]$ to $U[a - \epsilon, A + \epsilon]$ increases $P(D_1, D_2; \epsilon)$ as long as the mean of the distribution $0.5(a + A) > 1$. ■

Proof of Proposition 2

The key step is to establish that $0 \leq \Delta_{t+1}(D_t) \leq \alpha_t \theta_t$.

We can write $V_{t+1}(D_t) = \mathbb{E} [v_{t+1}(D_t; \tilde{\theta}_{t+1}, \tilde{\alpha}_{t+1})]$, where $v_{t+1}(D_t; \tilde{\theta}_{t+1}, \tilde{\alpha}_{t+1})$ is the optimized value of a future decision sequence starting at $t + 1$, conditional on inheriting file D_t postponed from period t . Similarly, for the case where the decision maker enters next period with a clear desk we have $V_{t+1}(\emptyset) = \mathbb{E} [v_{t+1}(\emptyset; \tilde{\theta}_{t+1}, \tilde{\alpha}_{t+1})]$. Then $\Delta_{t+1}(D_t) = \mathbb{E} \delta_{t+1}(D_t; \cdot)$, where

$$\delta_{t+1}(D_t; \theta_{t+1}, \alpha_{t+1}) \equiv [v_{t+1}(D_t; \theta_{t+1}, \alpha_{t+1}) - v_{t+1}(\emptyset; \theta_{t+1}, \alpha_{t+1})].$$

To evaluate $v_{t+1}(D_t; \cdot)$, when the decision-maker carries the backlog of D_t , consider two scenarios. If the new opportunity D_{t+1} turns out to be unimportant nor non-urgent, the decision-maker reverts to D_t in period $t + 1$; given the constraint on decision-making capacity, this necessitates postponing D_{t+1} to period $t + 2$. The total payoff in this case is $\alpha_t \theta_t + \beta V_{t+2}(D_{t+1})$. On the other hand, if D_{t+1} turns out to be sufficiently important and urgent, it would displace D_t altogether: the individual will address D_{t+1} in period $t + 1$, and enter period $t + 2$ without any backlog. The total payoff is $\theta_{t+1} + \beta V_{t+2}(\emptyset)$ for this case. Choosing optimally between these two

$$v_{t+1}(D_t; \theta_{t+1}, \alpha_{t+1}) = \max[\alpha_t \theta_t + \beta V_{t+2}(D_{t+1}), \theta_{t+1} + \beta V_{t+2}(\emptyset)].$$

A similar analysis for the case where the decision-maker enters period $t + 1$ with a clear desk

$$v_{t+1}(\emptyset; \theta_{t+1}, \alpha_{t+1}) = \max[\beta V_{t+2}(D_{t+1}), \theta_{t+1} + \beta V_{t+2}(\emptyset)].$$

The difference $\delta_{t+1}(D_t; \cdot)$ between these two state-contingent values depends on θ_{t+1} relative to $\Delta_{t+2}(D_{t+1})$, as follows.

$$\delta_{t+1}(D_t; \cdot) = \begin{cases} \alpha_t \theta_t & \text{if } \theta_{t+1} \leq \beta \Delta_{t+2}, \\ \alpha_t \theta_t + \beta \Delta_{t+2} - \theta_{t+1} & \text{if } \beta \Delta_{t+2} < \theta_{t+1} < \beta \Delta_{t+2} + \alpha_t \theta_t \\ 0 & \text{otherwise.} \end{cases}$$

Note that in each of these three cases the value of $\delta_{t+1}(D_t; \cdot)$ is non-negative and bounded above by $\alpha_t \theta_t$. In the first case, opportunity D_{t+1} is such that a decision on it is best postponed to period $t + 2$. In the second case, the attributes of D_{t+1} by themselves do not warrant postponement to period $t + 2$, but as that postponement releases decision making-capacity to deal with D_t , it becomes worthwhile. Given the restriction on θ_{t+1} , it is easy to check that $\delta_{t+1}(D_t; \cdot)$ lies strictly between 0 and $\alpha_t \theta_t$ for this case. The third case is simply the one where next period's opportunity is so large that it is optimal to abandon D_t altogether: here the difference in future payoffs is zero.

If $\delta_{t+1}(D_t; \cdot) \leq \alpha_t \theta_t$ in each of the three cases, so must its expected value

$\Delta_{t+1}(D_t) = \mathbb{E} \delta_{t+1}(D_t; \cdot)$ If so, $P(D_t, \{D_{t+1}\}) = \beta[\alpha_t \theta_t - \Delta_{t+1}(D_t)] \geq 0$. ■