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Abstract

This paper considers tests of the effectiveness of a policy intervention, defined as a change in the parameters of a policy rule, in the context of a macroeconometric dynamic stochastic general equilibrium (DSGE) model. We consider two types of intervention, first the standard case of a parameter change that does not alter the steady state, and second one that does alter the steady state, e.g. the target rate of inflation. We consider two types of test, one a multi-horizon test, where the post-intervention policy horizon, H, is small and fixed, and a mean policy effect test where H is allowed to increase without bounds. The multi-horizon test requires Gaussian errors, but the mean policy effect test does not. It is shown that neither of these two tests are consistent, in the sense that the the power of the tests does not tend to unity as $H \to \infty$, unless the intervention alters the steady state. This follows directly from the fact that DSGE variables are measured as deviations from the steady state, and the effects of policy change on target variables decay exponentially fast. We investigate the size and power of the proposed mean effect test by simulating a standard three equation New Keynesian DSGE model. The simulation results are in line with our theoretical findings and show that in all applications the tests have the correct size; but unless the intervention alters the steady state, their power does not go to unity with H.

Keywords: Counterfactuals, policy analysis, policy ineffectiveness test, macroeconomics
JEL classification: C18, C54, E65,

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1 Introduction

This paper considers testing the effectiveness of a policy intervention given time-series data on outcome variables, both before and after the policy change. The policy effect is measured as the difference between the policy outcome, the post-intervention realized values, and a counterfactual. The counterfactual is constructed assuming no policy intervention, using parameters estimated on the pre-intervention sample.[1] While there are many ways that one could construct such a counterfactual, this paper considers the case where it is obtained from a dynamic stochastic general equilibrium (DSGE) model whose variables are measured as deviations from the steady state.[2] The realized policy outcomes will reflect both a deterministic component, the effect of the intervention, and a stochastic component, the post-intervention disturbances or shocks.

In the DSGE literature a typical policy intervention is a monetary policy shock, calculated as a one standard error displacement of the structural disturbance of a policy equation, such as a Taylor rule. The impulse response function (IRF) is the time profile of the deterministic component of the effect of such a displacement, and as discussed in Section 2.1, yields *ex ante* information about the way the model responds to such a displacement, not an *ex post* evaluation of the effectiveness of an actual policy intervention. As we shall see, IRFs *ignore* the cumulative uncertainty associated with the stochastic component, the post-intervention disturbances. While one can construct tests for such displacements, we focus on interventions that change policy parameters. The first type of intervention, such as changing parameters of the Taylor Rule, does not alter the steady state. The second type, such as changing the target rate of inflation, does alter the steady state. We show that if the intervention does not alter the steady state, the power of the tests will not go to unity as the post-intervention horizon, H, gets large. This is an inherent consequence of the fact that DSGE models use variables measured as deviations from the steady state and the effects of policy changes decay exponentially fast. Thus unless the intervention alters the steady state we cannot be sure that it has had an effect.

Tests based on the differences between realizations and counterfactuals are standard in the statistical literature and have been used to examine a range of macroeconometric questions. Abadie and Gardeazabal (2003) examine the effect of terrorism on the Basque country using a "synthetic control region" as a counterfactual. Hsiao, Ching and Wan (2012) examine the effect on growth in Hong Kong of political and economic integration with mainland China, using a panel data approach to construct a counterfactual using predictions from similar economies. Synthetic control and panel data counterfactuals are compared by Gardeazabal and Vega-Bayo (2016). Pesaran, Smith and Smith (2007) examine what would have happened to the economies of the UK and the eurozone had the UK joined the euro in 1999, using "euro" restrictions on a GVAR

[1] The Lucas Critique does not apply since counterfactuals are estimated using the pre-intervention sample, and the policy-induced parameter change gets reflected in the realized post-intervention outcomes, which embody the effect of the change in the policy parameters and any consequent changes to expectations.

[2] Policy ineffectiveness tests where the counterfactuals are obtained from reduced or final form specifications are considered in Pesaran and Smith (2016).

model to construct a counterfactual. Fagan, Lothian and McNelis (2013) examine whether the Gold Standard was in fact destabilising, constructing the counterfactual by replacing the pre-1914 US money supply process with a Taylor rule in a DSGE model.

Rather than comparing actuals with counterfactuals, the mainstream macro-economic literature has tended to emphasise estimation issues in the context of structural vector autoregressions (VARs) and DSGEs. For instance, in the case of the Volcker disinflation which marked the transition from an era of macroeconomic turbulence and high inflation to an era of "Great Moderation" and low inflation, Primiceri (2006) provides an explanation of changes in policy in terms of learning about the parameters of the Phillips curve. Sims and Zha (2006) estimate regime switching structural VARs and find that the best fit allows time variation in the error variances only. Boivin and Giannoni (2006) argue that by responding more strongly to inflation expectations, monetary policy stabilised the economy more effectively after 1980. Inoue and Rossi (2011) investigate the sources of the Great Moderation and, using a representative New Keynesian and structural VAR models, show that the substantial decrease in output growth volatility during mid-1980s was due to changes in monetary policy parameters, as well as to other parameters rather than just good luck (reduced shock volatilities).

In contrast, the focus of the present paper is on formal statistical tests of the effects of a policy intervention. The null hypothesis is policy ineffectiveness: no change in policy parameters.[3] This null hypothesis is tested in conjunction with the maintained assumption that non-policy parameters remain stable.[4] We consider two types of test. The first is a multi-horizon test, where the post-intervention policy horizon, H, is small and fixed. This has a chi-squared distribution. The second is a test of the mean policy effect where H tends to infinity, which has a standard normal distribution. The multi-horizon test will have more power but requires Gaussian errors, whereas the mean effect test does not require Gaussianity. There is thus a trade-off between power and robustness to the failure of the Gaussian assumption. In both cases the power of the test will only go to one as H goes to infinity if the intervention changes the steady state. We assume a best case scenario for the tests namely that the DSGE model is well specified and identified, such that we can obtain $\sqrt{T}$ consistent estimates of its parameters.

The power of the test increases with the degree of persistence of the model. The higher the degree of persistence, the longer the effects of the intervention last, the easier it is to detect them. The power is also increased by the distance the economy is from its steady state at the time of intervention. A parameter change when the economy is in steady state cannot be detected. In practice, major policy interventions tend to take place at times when the economy is far from its steady state, increasing power.

We investigate the size and power of the proposed mean policy effect test by simulating a

[3] Note that we are concerned with the empirical issue of whether the effect of a policy change can be detected not the theoretical policy ineffectiveness proposition of Sargent and Wallace (1975).

[4] In practice, it is difficult to isolate the effects of policy change from other non-policy related parameter changes. All statistical tests, in one form or another, are joint tests of model specification and the null hypothesis of interest. As a result, the test outcomes must be interpreted with care.

standard three equation New Keynesian DSGE model. The model exhibits the usual impulse responses when subject to demand, supply and monetary policy shocks. To illustrate the factors influencing power of the policy ineffectiveness test, we also present policy impulse response functions that show the time profile of the effects of a change in policy parameters. The simulation results are in line with our theoretical findings and show that in all applications the tests have the correct size; but unless the intervention changes the steady state, their power can be low. When the policy intervention changes the inflation target, the test has power for the effects on inflation and interest rates, but not on output due to model's long run neutrality properties.

The rest of the paper is organized as follows: Section 2 specifies the DSGE model. Section 3 describes the types of policy interventions to be considered. Section 4 sets out the policy intervention tests and their underlying assumptions, derives their asymptotic distributions, and discusses their power properties. Tests of policy interventions that do not change the steady state are considered in sub-section 4.1, where the properties of the multi-horizon and the mean effect tests are considered. Sub-section 4.2 discusses the properties of the two types of tests in the case of interventions that only change the steady state. Section 5 provides a simulated policy analysis of the New Keynesian model and the performance of the mean effect test, including tests for interventions which change both the parameters of the DSGE model and of the steady state. Section 6 provides some concluding remarks. The more technical derivations are given in an online supplement.

2 Specification of the DSGE model

Consider a standard rational expectations (RE) DSGE model, where all the variables are endogenous.[5] Denote the set of endogenous variables by the $(k_z + 1) \times 1$ vector $\mathbf{q}_t = (y_t, \mathbf{z}_t')'$, where y_t is the target variable, and $\mathbf{q}_t$, contains policy and non policy variables. The deviations from steady state are $\widetilde{\mathbf{q}}_t = \mathbf{q}_t - \mathbf{q}_t^*(\boldsymbol{\alpha})$, where $\boldsymbol{\alpha}$ is a vector of parameters that affect the steady state, including policy variables like the target rate of inflation or the target rate of growth of money supply. The DSGE model determining deviations from steady state is given by

$$\mathbf{A}_0\widetilde{\mathbf{q}}_t = \mathbf{A}_1 E_t(\widetilde{\mathbf{q}}_{t+1}) + \mathbf{A}_2\widetilde{\mathbf{q}}_{t-1} + \mathbf{u}_t, \tag{1}$$

where the structural disturbances, $\mathbf{u}_t$, have mean zero, $E(\mathbf{u}_t) = 0$, are serially uncorrelated and have a constant, typically diagonal, variance matrix, $E(\mathbf{u}_t\mathbf{u}_t') = \boldsymbol{\Sigma}_u$. $E_t(\widetilde{\mathbf{q}}_{t+1}) = E(\widetilde{\mathbf{q}}_{t+1} \mid \mathfrak{I}_t)$, where $\mathfrak{I}_t$ is the information set that includes $\mathbf{u}_t$, and the lagged values of the variables, $\widetilde{\mathbf{q}}_t$. By construction it is assumed that $E\left(\widetilde{\mathbf{q}}_t\right) = \mathbf{0}$.

Initially we abstract from parameter estimation uncertainty and denote the vector of structural and policy parameters of the DSGE model by $\boldsymbol{\theta}$, and assume that $\boldsymbol{\Sigma}_u$ remains invariant to any policy change.[6] Thus $\widetilde{\mathbf{q}}_t = \widetilde{\mathbf{q}}_t(\boldsymbol{\alpha}, \boldsymbol{\theta})$ and the deviations from steady state are a function of both

[5] The model can be readily extended to allow for exogenous variables, such as oil prices or foreign variables.

[6] $\boldsymbol{\theta}$ is often referred to as a vector of deep parameters and elements of matrices, $\mathbf{A}_i$, $i = 0, 1, 2$ are defined as functions of $\boldsymbol{\theta}$.

the parameters that determine the steady state, $\boldsymbol{\alpha}$, and those that appear in the DSGE model, $\boldsymbol{\theta}$. Unless necessary for clarity, we will suppress the dependence of $\widetilde{\mathbf{q}}_t$ on $\boldsymbol{\alpha}$, and $\boldsymbol{\theta}$.

An example of such a system is the three equation New Keynesian DSGE model, which we simulate in Section 5. In this model, the DSGE policy parameters are the parameters of the Taylor rule, the steady state policy parameter is the target rate of inflation.

Under the above set up, the RE model (1) has the unique solution[7]

$$\widetilde{\mathbf{q}}_t = \mathbf{\Phi}(\boldsymbol{\theta})\widetilde{\mathbf{q}}_{t-1} + \mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{u}_t, \tag{2}$$

if the quadratic matrix equation

$$\mathbf{A}_1\mathbf{\Phi}^2 - \mathbf{A}_0\mathbf{\Phi} + \mathbf{A}_2 = \mathbf{0}, \tag{3}$$

has a solution, $\mathbf{\Phi}$, with all its eigenvalues inside the unit circle, and $\mathbf{\Gamma}(\boldsymbol{\theta}) = (\mathbf{A_0} - \mathbf{A_1}\mathbf{\Phi})^{-\mathbf{1}}$. Below we shall also use the reduced form disturbances, $\boldsymbol{\varepsilon}_t = \mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{u}_t$, and note that

$$\mathbf{\Sigma}_\varepsilon(\boldsymbol{\theta}) = E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = (\mathbf{A}_0 - \mathbf{A}_1\mathbf{\Phi})^{-1}\mathbf{\Sigma}_u(\mathbf{A}_0 - \mathbf{A}_1\mathbf{\Phi})^{'-1}. \tag{4}$$

The unique solution of the RE model in (2) is a vector autoregression, and corresponds to the reduced form of a standard simultaneous equations model where there are no exogenous variables.

The DSGE parameter vector, $\boldsymbol{\theta}$, is composed of a set of policy parameters, $\boldsymbol{\theta}_p$, and a set of structural parameters, $\boldsymbol{\theta}_s$, that are invariant to changes in $\boldsymbol{\theta}_p$. Similarly, the steady state parameters can be divided into $\boldsymbol{\alpha}_p$ and $\boldsymbol{\alpha}_s$. A policy intervention is defined in terms of a change in one or more elements of $\boldsymbol{\theta}_p$ or $\boldsymbol{\alpha}_p$. It is assumed that the policy parameters are under the control of the policy maker. The null hypothesis of our test is policy ineffectiveness, defined as no change in the parameters. We assume that the model is known by economic agents, the announcement and implementation of the intervention are credible, and no further changes are expected.[8] We suppose that the policy intervention occurs at the end of time $t = T_0$, and we have a pre-intervention sample, $t = M, M+1, ..., T_0$, and a post-intervention sample, $t = T_0+1, T_0+2, .., T_0+H$. Therefore, the post-intervention evaluation horizon is H and the sample size for estimation of the pre-intervention parameters is $T = T_0 - M + 1$. This notation allows us to increase the sample size T (by letting $M \to -\infty$), while keeping the date of the intervention fixed at T_0.

2.1 Policy effects and impulse responses

In the literature the effects of policy interventions are usually investigated using the impulse response function. Consider a policy intervention at time $T_0 + 1$ defined by a one standard error, σ_i, displacement of the disturbance of the i^{th} equation of the model (taken to be a policy equation). Since the structural disturbances are assumed to be orthogonal, then $\mathbf{u}_{T_0+1} = \sigma_i\mathbf{e}_i$, where $\mathbf{e}_i$ is a vector of zeros, except for its i^{th} element which is set to unity. It is assumed

[7] For a discussion of alternative solutions of RE models see Chapter 20 of Pesaran (2015).

[8] Kulish and Pagan (2017) consider solutions of forward looking models in the case of imperfect credibility where policy announcements are not necessarily incorporated into expectations.

that the structural parameters, $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$, are invariant to such an intervention. Therefore, the counterfactual associated with the event $\mathbf{u}_{T_0+1} = \sigma_i \mathbf{e}_i$ is given by $\widetilde{\mathbf{q}}_{T_0+h}(\sigma_i) = E(\widetilde{\mathbf{q}}_{T_0+h} \mid \mathbf{u}_{T_0+1} = \sigma_i \mathbf{e}_i, \mathfrak{I}_{T_0})$, where iterating (2) forward from T_0 yields

$$\widetilde{\mathbf{q}}_{T_0+h} = \mathbf{\Phi}^h(\boldsymbol{\theta})\widetilde{\mathbf{q}}_{T_0} + \sum_{j=0}^{h-1} \mathbf{\Phi}^j(\boldsymbol{\theta})\,\mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{u}_{T_0+h-j},\ h = 1, 2, ..., H. \tag{5}$$

Hence

$$\widetilde{\mathbf{q}}_{T_0+h}(\sigma_i) = \mathbf{\Phi}^h(\boldsymbol{\theta})\,\widetilde{\mathbf{q}}_{T_0} + \sigma_i \mathbf{\Phi}^{h-1}(\boldsymbol{\theta})\,\mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{e}_i. \tag{6}$$

The realized policy effect of this displacement is

$$\mathbf{d}_{T_0+h}(\sigma_i) = \widetilde{\mathbf{q}}_{T_0+h} - \widetilde{\mathbf{q}}_{T_0+h}(\sigma_i), \tag{7}$$

which upon using (5) and (6) gives

$$\mathbf{d}_{T_0+h}(\sigma_i) = -\sigma_i \mathbf{\Phi}^{h-1}(\boldsymbol{\theta})\,\mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{e}_i + \mathbf{V}_{T_0,h}(\boldsymbol{\theta}), \tag{8}$$

where

$$\mathbf{V}_{T_0,h}(\boldsymbol{\theta}) = \sum_{j=0}^{h-1} \mathbf{\Phi}^j(\boldsymbol{\theta})\,\mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{u}_{T_0+h-j}. \tag{9}$$

The IRF for this shock scenario is given by,

$$IRF(h, \sigma_i, \boldsymbol{\theta}) = \sigma_i \mathbf{\Phi}^{h-1}(\boldsymbol{\theta})\,\mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{e}_i, \tag{10}$$

which refers only to the deterministic component of the realized effect of the policy, $\mathbf{d}_{T_0+h}(\sigma_i)$, and ignores the random component $\mathbf{V}_{T_0,h}(\boldsymbol{\theta})$, which could be quite important, particularly considering the cumulative nature of the future shocks in (9). Note that

$$Var\left[\mathbf{V}_{T_0,h}(\boldsymbol{\theta})\right] = \sum_{j=0}^{h-1} \mathbf{\Phi}^j(\boldsymbol{\theta})\,\mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{\Sigma}_u \mathbf{\Gamma}'(\boldsymbol{\theta})\mathbf{\Phi}'^j(\boldsymbol{\theta}), \tag{11}$$

which is increasing in h and can be quite sizeable as compared to the $IRF(h, \sigma_i, \boldsymbol{\theta})$ component of $\mathbf{d}_{T_0+h}(\sigma_i)$. Any attempt at *ex post* policy evaluation must also take account of the possible effects of the future disturbances, $\mathbf{u}_{T_0+h}$ for $h = 1, 2, ..., H$, on the outcomes. In fact, Benati and Surico (2009) show how structural VAR based counterfactuals and impulse response functions, which treat policies as being about shock impulse response functions, may be misleading in not revealing changes in policy parameters, which is the focus of concern here.

In what follows we focus on policy interventions that involve changes in $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$; introduce the concept of policy impulse response function (PIRF) to be contrasted with the IRF; and propose *ex post* tests of policy effectiveness.

3 Types of policy interventions

3.1 Policy interventions that do not change the steady state

First consider a policy intervention that changes one or more elements of $\boldsymbol{\theta}_p$, but leaves the steady state unchanged. This will affect the mean outcomes through changes in $\boldsymbol{\Phi}(\boldsymbol{\theta})$, and the variance of the outcomes through changes in $\boldsymbol{\Gamma}(\boldsymbol{\theta})$. Denote the pre-intervention parameters by $\boldsymbol{\theta}^0 = (\boldsymbol{\theta}_p^{0\prime}, \boldsymbol{\theta}_s^{\prime})^{\prime}$, and the post-intervention parameters by $\boldsymbol{\theta}^1 = (\boldsymbol{\theta}_p^{1\prime}, \boldsymbol{\theta}_s^{\prime})^{\prime}$, where one or more elements of $\boldsymbol{\theta}_p$ are changed. If the intervention at $t = T_0$ is fully communicated, and seen to be credible, with expectations adjusting immediately, the process switches from

$$\widetilde{\mathbf{q}}_t = \boldsymbol{\Phi}(\boldsymbol{\theta}^0)\widetilde{\mathbf{q}}_{t-1} + \boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{u}_t,\ t = M, M+1, M+2, ..., T_0,$$

to

$$\widetilde{\mathbf{q}}_t = \boldsymbol{\Phi}(\boldsymbol{\theta}^1)\widetilde{\mathbf{q}}_{t-1} + \boldsymbol{\Gamma}(\boldsymbol{\theta}^1)\mathbf{u}_t,\ t = T_0+1, T_0+2, ..., T_0+H.$$

Suppose now that we are interested in the effects of the intervention on the target variable $\tilde{y}_t = \mathfrak{s}'\widetilde{\mathbf{q}}_t$, where $\mathfrak{s}$ is a the $(k_z+1)\times 1$ selection vector with all its elements zero except for its first element which is set to unity.[9] The counterfactual values of the target variable, $\widetilde{y}_{T_0+h}$, under the null hypothesis of no policy change is given by

$$\widetilde{y}^0_{T_0+h} = \mathfrak{s}'\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\widetilde{\mathbf{q}}_{T_0},\ \text{for } h = 1, 2, ..., H, \tag{12}$$

and only requires estimation of $\boldsymbol{\theta}^0$. The policy effect of the intervention on the target variable is then defined as the difference between the realized value, y_{T_0+h}, and the associated counterfactual, $y^0_{T_0+h}$, namely

$$d_{T_0+h}(\boldsymbol{\theta}^0) = y_{T_0+h} - y^0_{T_0+h},\ h = 1, 2, ..., H. \tag{13}$$

In the present case, where the intervention does not alter the steady state, then (13) can also be written as

$$d_{T_0+h}(\boldsymbol{\theta}^0) = \widetilde{y}_{T_0+h} - \widetilde{y}^0_{T_0+h},\ h = 1, 2, ..., H. \tag{14}$$

Now using (5) and (12), the realized policy effects can be written as

$$d_{T_0+h}(\boldsymbol{\theta}^1, \boldsymbol{\theta}^0) = \mathfrak{s}'\left[\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right]\widetilde{\mathbf{q}}_{T_0} + \mathbf{V}_{T_0,h}\left(\boldsymbol{\theta}^1\right),$$

where $\mathbf{V}_{T_0,h}\left(\boldsymbol{\theta}^1\right)$ is defined by (9) and evaluated at $\boldsymbol{\theta} = \boldsymbol{\theta}^1$. As in the case of policy shocks discussed in sub-section 2.1, the outcome of the policy change has a deterministic component, which we refer to as the policy impulse response function (PIRF) given by

$$PIRF_y(h, \boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0}) = \mathfrak{s}'\left[\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right]\widetilde{\mathbf{q}}_{T_0}. \tag{15}$$

PIRF is the expected difference in the outcomes associated with the pre- and post-policy parameters. Also, unlike the impulse response function, (10), the PIRF depends on the state of the

[9] If we are interested in more than one target variable the selection vector can be replaced by a selection matrix.

economy at the time of the intervention, $\widetilde{\mathbf{q}}_{T_0}$. But like the IRF, due to the stationary nature of the underlying DSGE model, the PIRF also tends to zero exponentially, which turns out to be important for the power of the policy effectiveness tests discussed below.

The stochastic component of the policy effect $d_{T_0+h}(\boldsymbol{\theta}^1, \boldsymbol{\theta}^0)$, namely $\mathbf{V}_{T_0,h}\left(\boldsymbol{\theta}^1\right)$, increases in importance with h, and also plays an important role in the policy ineffectiveness tests.

3.2 Policy interventions that change the steady state

Suppose that one or more parameters that determine the steady state and which are under the control of the policy maker, denoted by $\boldsymbol{\alpha}_p$, are changed from $\boldsymbol{\alpha}_p^0$ at the end of period T_0, to $\boldsymbol{\alpha}_p^1$, over the period $t = T_0 + 1, T_0 + 2, ...T_0 + H$. The policy effects in this case are measured in terms of the original variables y_{T_0+h}, not deviations from the steady state, and are given by

$$d_{T_0+h}(\boldsymbol{\theta}, \boldsymbol{\alpha}^0) = y_{T_0+h} - y^0_{T_0+h},\ h = 1, 2, ..., H, \tag{16}$$

where y_{T_0+h} is the realized value of the target variable, y_t, post-intervention and the counterfactual is given by

$$y^0_{T_0+h} = y^*_{T_0+h}\left(\boldsymbol{\alpha}^0\right) + \mathfrak{s}'\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}\right)\left[\mathbf{q}_{T_0} - \mathbf{q}^*_{T_0}\left(\boldsymbol{\alpha}^0\right)\right],\ \text{for } h = 1, 2, ..., H. \tag{17}$$

Note that in this case $\boldsymbol{\theta}$ has the same value before and after the policy change.[10]

3.3 Policy interventions that change volatilities

So far we have considered how the policy intervention changes the levels of the variables. One can also consider how policy interventions change volatilities. Using (11), we can see that the effect of an intervention on volatility, the difference between the variance with and without the policy change, is given by

$$\begin{aligned}
&Var\left(\widetilde{\mathbf{q}}_{T_0+h} \mid \Im_{T_0}, \boldsymbol{\theta}^1\right) - Var\left(\widetilde{\mathbf{q}}_{T_0+h} \mid \Im_{T_0}, \boldsymbol{\theta}^0\right) \\
&= \sum_{j=0}^{h-1}\boldsymbol{\Phi}^j\left(\boldsymbol{\theta}^1\right)\boldsymbol{\Gamma}(\boldsymbol{\theta}^1)\boldsymbol{\Sigma}_u\boldsymbol{\Gamma}'(\boldsymbol{\theta}^1)\boldsymbol{\Phi}'^j\left(\boldsymbol{\theta}^1\right) - \sum_{j=0}^{h-1}\boldsymbol{\Phi}^j\left(\boldsymbol{\theta}^0\right)\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\boldsymbol{\Sigma}_u\boldsymbol{\Gamma}'(\boldsymbol{\theta}^0)\boldsymbol{\Phi}'^j\left(\boldsymbol{\theta}^0\right).
\end{aligned}$$

Conditional on the structural error variances, $\boldsymbol{\Sigma}_u$, remaining constant, one could, in principle, derive a test statistic, for a policy induced volatility change. This is beyond the scope of this paper and is likely to be more challenging than the case discussed below of deriving a test for a policy induced level change in $\widetilde{\mathbf{q}}_{T_0+h}$. To test for the effect of a policy induced level change one only needs to estimate $\boldsymbol{\theta}$ on the pre-intervention sample, but to test for a policy induced volatility change one would also need estimates of $\boldsymbol{\theta}$ for the post-intervention sample.

While much of the theoretical literature is concerned with the issue of policy induced reduction in volatilities, in practice policy objectives are nearly always expressed in terms of their intended

[10] It is also possible to consider a policy intervention that involves changes to both $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$, and this case will be considered in the simulations.

level effects; reducing inflation, raising output, or reducing unemployment. Examples include the Volcker commitment to reduce the level of inflation, or the Japanese Government commitments to increase inflation/stop deflation.

Finally, a mean shift is relatively easy to interpret, few would disagree that the reduction of inflation following the Volcker disinflation (what we would regard as a change in the steady state) was the result of policy. A variance shift is more difficult to interpret. In the case of the Great Moderation, there is considerable dispute about whether the reduction in the variance of output growth was due to good policy (changes in policy parameters, $\boldsymbol{\theta}_p$) or good luck (reductions in $\|\boldsymbol{\Sigma}_u\|$).

4 Derivation of policy ineffectiveness tests

We derive tests for policy interventions that (i) change parameters of the DSGE model, $\boldsymbol{\theta}_p$, without changing the steady state, and (ii) policy interventions that change $\boldsymbol{\alpha}_p$, the policy parameter of the steady state and do not change $\boldsymbol{\theta}$.[11] Similar tests can also be developed for the policy shocks discussed in sub-section 2.1.

We consider two types of closely related tests: a multi-horizon test and a mean effect test. The former takes the evaluation horizon, H, as fixed, but assumes that the post policy intervention errors, u_{T_0+h}, for $h = 1, 2, ..., H$, are Gaussian. The latter aims at reducing the dependence of the test on the Gaussian assumption by considering policy effects averaged over relatively long evaluation horizon. Note, however, that neither tests require pre-policy intervention errors to be Gaussian for sufficiently large T.

To simplify the exposition we continue with the case of a scalar target variable y_t, being the first element of the $(k_z + 1) \times 1$ vector of endogenous variables, $\mathbf{q}_t = (y_t, \mathbf{z}_t')'$. But the proposed tests readily generalise to the case where there are more target variables. For notational convenience we also set $m = k_z + 1$.

To develop tests of policy effects and derive their distribution we shall adopt the following assumptions.

Assumption 1: The RE model defined by (1) has a unique solution given by (2), and the structural parameters, $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, are identified at $\boldsymbol{\theta}^0$ and $\boldsymbol{\theta}^1$ (the pre and post-intervention parameters). The structural errors, $\mathbf{u}_t$, are serially uncorrelated with zero means and a constant covariance matrix, $\boldsymbol{\Sigma}_u$. In particular, $\boldsymbol{\Sigma}_u$ is invariant to the policy change.

Assumption 2a: The spectral radius of $\boldsymbol{\Phi}(\boldsymbol{\theta})$, defined by $|\lambda_{\max}[\boldsymbol{\Phi}(\boldsymbol{\theta})]|$, is strictly less than unity for values of $\boldsymbol{\theta} = \boldsymbol{\theta}^0$ and $\boldsymbol{\theta}^1 \in \boldsymbol{\Theta}$.[12]

Assumption 2b: There exists a matrix norm of $\boldsymbol{\Phi}(\boldsymbol{\theta})$, denoted by $\|\boldsymbol{\Phi}(\boldsymbol{\theta})\|$, such that $\|\boldsymbol{\Phi}(\boldsymbol{\theta})\| < 1$, for values of $\boldsymbol{\theta} = \boldsymbol{\theta}^0$ and $\boldsymbol{\theta}^1 \in \boldsymbol{\Theta}$.

Assumption 3: Standard regularity assumptions on the structural errors, $\mathbf{u}_t$, and the

[11] See also sub-sections 3.1 and 3.2.

[12] $\lambda_{\max}(\mathbf{A})$ stands for the largest eigenvalue of matrix $\mathbf{A}$.

processes generating the exogenous variables (if any) apply such that θ^0 can be consistently estimated by $\hat{\boldsymbol{\theta}}_T^0$ based on the pre-intervention sample, $t = M, M+1, M+2, ..., T_0$, where $T = T_0 - M + 1$, and $\hat{\boldsymbol{\theta}}_T^0 = \boldsymbol{\theta}^0 + O_p\left(T^{-1/2}\right)$. In particular

$$\sqrt{T}\left(\hat{\boldsymbol{\theta}}_T^0 - \boldsymbol{\theta}^0\right) \overset{a}{\sim} N(\mathbf{0}, \boldsymbol{\Sigma}_{\theta^0}), \text{ and } E\left\|\hat{\boldsymbol{\theta}}_T^0 - \boldsymbol{\theta}^0\right\| = O(T^{-1/2}), \tag{18}$$

where $\boldsymbol{\Sigma}_{\theta^0}$ is a symmetric positive definite matrix.

Assumption 4: $\boldsymbol{\Phi}(\boldsymbol{\theta}) = (\phi_{ij}(\boldsymbol{\theta}))$, is bounded and continuously differentiable in $\boldsymbol{\theta}$, such that $\left\|\partial\phi_{ij}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}'\right\|$, for all i and j exist and are bounded.

Assumption 5: The initial values, $\widetilde{\mathbf{q}}_{T_0}$, are bounded, namely $\|\widetilde{\mathbf{q}}_{T_0}\| < K$, where K is a fixed positive constant.

Assumptions 1, 2a, 3 and 4 are standard in the literature on the econometric analysis of DSGE models. The conditions for identification in Assumption 1 are discussed in Koop, Pesaran and Smith (2013). Assumption 2a ensures that $\|\boldsymbol{\Phi}(\boldsymbol{\theta})\| < \lambda$, where λ is a finite positive constant.[13] Assumption 2b is stronger than 2a and further requires that $\lambda < 1$. This latter restriction allows us to simplify the proofs considerably and obtain the main theoretical results without requiring high order differentiability of $\boldsymbol{\Phi}(\theta)$ which will be needed in the absence of Assumption 2b.

In cases where both H and T go to infinity we shall also consider the following joint asymptotic condition:

Condition 1 *The post-intervention sample size, H, rises with the pre-intervention sample size, T, such that $H = \kappa T^{\epsilon}$, where κ is a fixed positive constant, and $\epsilon \leq 1/2$.*

4.1 Tests of policy interventions that do not change the steady state

Using (12), estimates of the counterfactuals in the absence of a policy change are given by

$$\widehat{\widetilde{y}}_{T_0+h}^0 = \mathfrak{s}'\boldsymbol{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right)\widetilde{\mathbf{q}}_{T_0}, \ h = 1, 2, ..., H, \tag{19}$$

where under Assumption 3, $\hat{\boldsymbol{\theta}}_T^0$ is a $\sqrt{T}$ consistent estimator of $\boldsymbol{\theta}$ based on the pre-intervention sample. Therefore, the estimated policy effects are given by

$$\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0) = \mathfrak{s}'\widetilde{\mathbf{q}}_{T_0+h} - \mathfrak{s}'\boldsymbol{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right)\widetilde{\mathbf{q}}_{T_0}, \ h = 1, 2, ..., H. \tag{20}$$

The sampling distribution of $\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0)$, depends on post-intervention parameters only under the alternative that the policy is effective, but not under the null hypothesis of no policy effect as defined by

$$H_0 : \boldsymbol{\theta}^1 = \boldsymbol{\theta}^0. \tag{21}$$

To derive the distribution of the policy effects, $\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0)$, (for $h = 1, 2, ..., H$), first recall that post-intervention realized values, $\widetilde{\mathbf{q}}_{T_0+h}$, are given by

[13] Note that there exists a matrix norm, $\|\mathbf{A}\|$, such that $|\lambda_{\max}(A)| \leq \|\mathbf{A}\| \leq |\lambda_{\max}(A)| + \epsilon$, where ϵ is a positive constant. See, for example, Lemma 5.10.10 in Horn and Johnson (1985).

$$\widetilde{\mathbf{q}}_{T_0+h} = \mathbf{\Phi}^h\left(\boldsymbol{\theta}^1\right)\widetilde{\mathbf{q}}_{T_0} + \sum_{j=0}^{h-1}\mathbf{\Phi}^j\left(\boldsymbol{\theta}^1\right)\mathbf{\Gamma}(\boldsymbol{\theta}^1)\mathbf{u}_{T_0+h-j}. \tag{22}$$

Using (22) and substituting the results for $\widetilde{\mathbf{q}}_{T_0+h}$ in (20) we have

$$\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0) = \hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) + v_{T_0,h}\left(\boldsymbol{\theta}^1\right), \tag{23}$$

where

$$\hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) = -\mathfrak{s}'\left[\mathbf{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right) - \mathbf{\Phi}^h\left(\boldsymbol{\theta}^1\right)\right]\widetilde{\mathbf{q}}_{T_0}, \tag{24}$$

$$v_{T_0,h}\left(\boldsymbol{\theta}^1\right) = \mathfrak{s}'\mathbf{V}_{T_0,h}\left(\boldsymbol{\theta}^1\right) = \sum_{j=0}^{h-1}\mathfrak{s}'\mathbf{\Phi}^j\left(\boldsymbol{\theta}^1\right)\mathbf{\Gamma}(\boldsymbol{\theta}^1)\mathbf{u}_{T_0+h-j}. \tag{25}$$

In (23) the estimated policy effect, $\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0)$, has a systematic component, $\hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0)$, and a stochastic component, $v_{T_0,h}\left(\boldsymbol{\theta}^1\right)$, which is a weighted linear combination of serially uncorrelated disturbances, $\mathbf{u}_t$, with the weights decaying exponentially under Assumption 2a.

4.1.1 A multi-horizon policy ineffectiveness test

A multi-horizon policy ineffectiveness test of H_0 can now be based on the policy effects, $\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0)$, $h = 1, 2, ..., H$, jointly. When H is fixed and T sufficiently large, such a policy ineffectiveness test can be developed if we are prepared to make distributional assumptions regarding the post policy shocks, $\mathbf{u}_{T_0+h}$ for $h = 1, 2, ..., H$. For example, assuming that post policy shocks are Gaussian it readily follows that, under the null of no policy effects, (21), and with T sufficiently large, $\hat{\mathbf{d}}_H = \left(\hat{d}_{T_0+1}(\hat{\boldsymbol{\theta}}_T^0), \hat{d}_{T_0+2}(\hat{\boldsymbol{\theta}}_T^0), ..., \hat{d}_{T_0+H}(\hat{\boldsymbol{\theta}}_T^0)\right)'$, has a multivariate normal distribution with zero mean. This follows using (24) in (23), and noting that by Assumption 3 and under $H_0 : \boldsymbol{\theta}^1 = \boldsymbol{\theta}^0$ we have

$$\hat{\mathbf{d}}_H = \left(\mathfrak{s}' \otimes \mathbf{I}_H\right)\mathbf{v}_H\left(\boldsymbol{\theta}^0\right) + O_p\left(T^{-1/2}\right), \tag{26}$$

where $\mathbf{v}_H\left(\boldsymbol{\theta}^0\right) = \left(v_{T_0,1}\left(\boldsymbol{\theta}^0\right), v_{T_0,2}\left(\boldsymbol{\theta}^0\right), ..., v_{T_0,H}\left(\boldsymbol{\theta}^0\right)\right)'$. Furthermore, using (25) and recalling that $m = k_z + 1$ we obtain

$$\begin{aligned}\mathbf{v}_H\left(\boldsymbol{\theta}^0\right) &= \begin{pmatrix} \mathbf{I}_m & \mathbf{0}_m & \dots & \mathbf{0}_m \\ \mathbf{\Phi}\left(\boldsymbol{\theta}^0\right) & \mathbf{I}_m & \dots & \mathbf{0}_m \\ \vdots & \vdots & \dots & \vdots \\ \mathbf{\Phi}^{h-1}\left(\boldsymbol{\theta}^0\right) & \mathbf{\Phi}^{h-2}\left(\boldsymbol{\theta}^0\right) & \dots & \mathbf{I}_m \end{pmatrix} \begin{pmatrix} \mathbf{\Gamma}(\boldsymbol{\theta}^0) & \mathbf{0}_m & \dots & \mathbf{0}_m \\ \mathbf{0}_m & \mathbf{\Gamma}(\boldsymbol{\theta}^0) & \dots & \mathbf{0}_m \\ \vdots & \vdots & \dots & \vdots \\ \mathbf{0}_m & \mathbf{0}_m & \dots & \mathbf{\Gamma}(\boldsymbol{\theta}^0) \end{pmatrix} \begin{pmatrix} \mathbf{u}_{T_0+1} \\ \mathbf{u}_{T_0+2} \\ \vdots \\ \mathbf{u}_{T_0+H} \end{pmatrix} \\ &= \mathbf{\Psi}\left(\boldsymbol{\theta}^0\right)\left(\mathbf{\Gamma}(\boldsymbol{\theta}^0) \otimes \mathbf{I}_H\right)\mathbf{u}_H. \end{aligned} \tag{27}$$

Also, under Assumption 1, $Var\left(\mathbf{u}_H\right) = \left(\mathbf{\Sigma}_u \otimes \mathbf{I}_H\right)$, and hence

$$Var\left[\mathbf{v}_H\left(\boldsymbol{\theta}^0\right)\right] = \mathbf{\Psi}\left(\boldsymbol{\theta}^0\right)\left[\mathbf{\Gamma}(\boldsymbol{\theta}^0)\mathbf{\Sigma}_u\mathbf{\Gamma}'(\boldsymbol{\theta}^0) \otimes \mathbf{I}_H\right]\mathbf{\Psi}'\left(\boldsymbol{\theta}^0\right),$$

which is an $mH \times mH$ matrix. Hence, for T sufficiently large we have

$$Var\left(\hat{\mathbf{d}}_H\right) = \left(\mathfrak{s}' \otimes \mathbf{I}_H\right)\mathbf{\Psi}\left(\boldsymbol{\theta}^0\right)\left[\mathbf{\Gamma}(\boldsymbol{\theta}^0)\mathbf{\Sigma}_u\mathbf{\Gamma}'(\boldsymbol{\theta}^0) \otimes \mathbf{I}_H\right]\mathbf{\Psi}'\left(\boldsymbol{\theta}^0\right)\left(\mathfrak{s} \otimes \mathbf{I}_H\right). \tag{28}$$

The following test statistics can now be considered

$$\mathcal{T}_H = \hat{\mathbf{d}}_H' \left[\widehat{Var\left(\hat{\mathbf{d}}_H\right)} \right]^{-1} \hat{\mathbf{d}}_H, \tag{29}$$

where

$$\widehat{Var\left(\hat{\mathbf{d}}_H\right)} = \left(\mathfrak{s}' \otimes \mathbf{I}_H\right) \boldsymbol{\Psi}\left(\hat{\boldsymbol{\theta}}_T^0\right) \left[\boldsymbol{\Gamma}(\hat{\boldsymbol{\theta}}_T^0)\hat{\boldsymbol{\Sigma}}_u \boldsymbol{\Gamma}'(\hat{\boldsymbol{\theta}}_T^0) \otimes \mathbf{I}_H\right] \boldsymbol{\Psi}'\left(\hat{\boldsymbol{\theta}}_T^0\right) \left(\mathfrak{s} \otimes \mathbf{I}_H\right). \tag{30}$$

The estimators, $\hat{\boldsymbol{\Sigma}}_u$ and $\hat{\boldsymbol{\theta}}_T^0$, can be computed using the pre-intervention sample. Then under Assumptions 1, 2a, 3, 4 and 5, it follows that $\mathcal{T}_H \to \chi^2_H$, for a fixed H and as $T \to \infty$. The test can be extended readily to more than one target variable by replacing the selection vector $\mathfrak{s}$ with an appropriate selection matrix.

The power of the $\mathcal{T}_H$ test Using (23), under the alternative hypothesis, $H_1 : \boldsymbol{\theta}^1 \neq \boldsymbol{\theta}^1$, we have

$$\hat{\mathbf{d}}_H = \left(\mathfrak{s}' \otimes \mathbf{I}_H\right) \boldsymbol{\mu}_H + \left(\mathfrak{s}' \otimes \mathbf{I}_H\right) \mathbf{v}_H\left(\boldsymbol{\theta}^1\right) + O_p\left(T^{-1/2}\right),$$

where $\boldsymbol{\mu}_H = \left[\mu_1(\boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0}), \mu_2(\boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0}), ..., \mu_H(\boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0})\right]'$, and $\mu_h(\boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0})$ is the probability limit of $\hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0)$, under H_1. Using (24), we have

$$\hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) \to_p \mu_h(\boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0}) = \mathfrak{s}' \left[\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right] \widetilde{\mathbf{q}}_{T_0}.$$

It is now readily follows that under H_1, the multi-horizon test statistic, $\mathcal{T}_H$, defined by (29) is distributed as a non-central $\chi^2_H(C_H)$, with the non-centrality parameter $C_H = \boldsymbol{\mu}_H' \left[Var\left(\hat{\mathbf{d}}_H\right)\right]^{-1} \boldsymbol{\mu}_H > 0$, where $Var\left(\hat{\mathbf{d}}_H\right)$ is given by (28). The power is increasing in C_H which in turn depends on the norm of $\boldsymbol{\mu}_H$. Specifically, $C_H \leq \lambda_{\max}\left(\left[Var\left(\hat{\mathbf{d}}_H\right)\right]^{-1}\right) \boldsymbol{\mu}_H'\boldsymbol{\mu}_H$, where $\lambda_{\max}\left(\left[Var\left(\hat{\mathbf{d}}_H\right)\right]^{-1}\right) = \lambda_{\min}\left(\left[Var\left(\hat{\mathbf{d}}_H\right)\right]\right)$ is strictly positive abd bounded in H. Furthermore, so long as $\widetilde{\mathbf{q}}_{T_0} \neq 0$, then $\boldsymbol{\mu}_H'\boldsymbol{\mu}_H = \sum_{h=1}^H \mu_h^2(\boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0}) > 0$, and the test has power for any H. In this case he power of the test rises with $\left\|\boldsymbol{\theta}^1 - \boldsymbol{\theta}^0\right\|$ and $\|\widetilde{\mathbf{q}}_{T_0}\|$, and falls with $\left\|Var\left(\hat{\mathbf{d}}_H\right)\right\|$. However, due to the stationary nature of the DSGE model, as formalized by Assumption 2, $\mu_h(\boldsymbol{\theta}^0, \boldsymbol{\theta}^1, \widetilde{\mathbf{q}}_{T_0}) \to 0$, as $h \to \infty$, at the exponential rate of $[\lambda_{\max}(\boldsymbol{\Phi}(\boldsymbol{\theta}))]^h$, which ensures that $\boldsymbol{\mu}_H'\boldsymbol{\mu}_H$ is bounded in H. This in turn means that the test need not be consistent, in the sense that the power of the test need not tend to unity as $H \to \infty$.

4.1.2 Mean policy effects test without Gaussianity

We can minimize the role of the Gaussianity assumption by basing the policy ineffectiveness test on a "mean policy effect", computed over the post-intervention horizon $T_0 + h$, for $h = 1, 2, ..., H$, namely

$$\bar{\hat{d}}_H(\hat{\boldsymbol{\theta}}_T^0) = \frac{1}{H}\sum_{h=1}^{H} \hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0). \tag{31}$$

For a fixed H, the implicit null hypothesis of no policy effects can now be specified as

$$H_0' : p \lim_{T\to\infty} \left(H^{-1/2} \sum_{h=1}^{H} \hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) \right) = 0. \tag{32}$$

As we shall see, this condition is met under Assumptions 1, 2a, 3 and 4 when H is fixed and as $T \to \infty$.

Interestingly enough, H_0' continues to hold even if $H \to \infty$, so long as Assumption 2b holds and the rate of increase of H in relation to T is governed by the joint asymptotic condition 1. If the underlying RE model is correctly specified, then under the null of no policy change, H_0, we have

$$H^{-1/2} \sum_{h=1}^{H} \hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) = -\mathfrak{s}' \left\{ H^{-1/2} \sum_{h=1}^{H} \left[\boldsymbol{\Phi}^h \left(\hat{\boldsymbol{\theta}}_T^0 \right) - \boldsymbol{\Phi}^h \left(\boldsymbol{\theta}^0 \right) \right] \right\} \widetilde{\mathbf{q}}_{T_0}. \tag{33}$$

Now using results in Lemmas S2 and S3, given in the online supplement, we have

$$\begin{aligned} \left\| H^{-1/2} \sum_{h=1}^{H} \hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) \right\| &\leq \left\| \mathfrak{s}' \right\| \left\| \widetilde{\mathbf{q}}_{T_0} \right\| H^{-1/2} \left\| \sum_{h=1}^{H} \left[\boldsymbol{\Phi}^h \left(\hat{\boldsymbol{\theta}}_T^0 \right) - \boldsymbol{\Phi}^h \left(\boldsymbol{\theta}^0 \right) \right] \right\| \\ &\leq K \left\| \mathfrak{s}' \right\| \left\| \widetilde{\mathbf{q}}_{T_0} \right\| H^{-1/2} \left(\sum_{h=1}^{H} h \lambda_T^{h-1} \right) \left\| \hat{\boldsymbol{\theta}}_T^0 - \boldsymbol{\theta}^0 \right\|, \end{aligned} \tag{34}$$

where K is a fixed constant. Using (A.3) in Lemma S3, we have

$$\left\| \boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0) \right\| \leq \left\| \boldsymbol{\Phi}(\boldsymbol{\theta}^0) \right\| + a_T \left\| \hat{\boldsymbol{\theta}}_T^0 - \boldsymbol{\theta}^0 \right\|,$$

where $a_T = \left\| \partial \boldsymbol{\Phi} \left(\bar{\boldsymbol{\theta}}_T^0 \right) / \partial \boldsymbol{\theta}' \right\|$, and elements of $\bar{\boldsymbol{\theta}}_T^0$ lie on the line segment joining $\boldsymbol{\theta}^0$ and $\hat{\boldsymbol{\theta}}_T^0$. Considering that $\bar{\boldsymbol{\theta}}_T^0 \to_p \boldsymbol{\theta}^0$, and by Assumption 4, $\left\| \partial \phi_{ij}(\boldsymbol{\theta}) / \partial \boldsymbol{\theta}' \right\|$ for all i and j exist and are bounded, then it must also follow that a_T is bounded in T. Hence, recalling that under Assumption 3, $\sqrt{T} \left\| \hat{\boldsymbol{\theta}}_T^0 - \boldsymbol{\theta}^0 \right\| = O_p(1)$, then $\lambda_T \leq \lambda + a_T T^{-1/2}$, where $\left\| \boldsymbol{\Phi} \left(\boldsymbol{\theta}^0 \right) \right\| \leq \lambda$, and a_T is bounded in T. In the case where H is fixed and $T \to \infty$,

$$\left| H^{-1/2} \left(\sum_{h=1}^{H} h \lambda_T^{h-1} \right) \right| \leq H^{-1/2} \sum_{h=1}^{H} h \left(\lambda + a_T T^{-1/2} \right)^{h-1} \to H^{-1/2} \sum_{h=1}^{H} h \lambda^{h-1} < K, \text{ as } T \to \infty.$$

Using this result in (34) and noting that under Assumptions 3 and 5, $\left\| \widetilde{\mathbf{q}}_{T_0} \right\|$ is bounded in T, and $\left\| \hat{\boldsymbol{\theta}}_T^0 - \boldsymbol{\theta}^0 \right\| = O_p\left(T^{-1/2} \right)$, then under the null of no policy change, H_0, for a fixed H and as $T \to \infty$, we have $\left\| H^{-1/2} \sum_{h=1}^{H} \hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) \right\| \to_p 0$, as required.

Consider now the case where H rises with T and the rate of increase of H in relation to T is governed by the joint asymptotic condition 1. Note also that under Assumption 2b, $\lambda < 1$. Then using (A.4) and (A.5) in Lemma S4 we have

$$\sum_{h=1}^{H} h \lambda_T^{h-1} = \frac{1}{(1-\lambda)^2} + O_p\left(T^{-1/2} \right) + O_p\left(H \lambda^H \right), \tag{35}$$

$$\sum_{h=1}^{H}\sum_{j=0}^{h-1} j\lambda_T^{j-1} = \frac{1}{(1-\lambda)^2}\left(H - \frac{1+\lambda}{1-\lambda}\right) + O_p\left(T^{-1/2}\right) + O_p\left(H\lambda^H\right). \tag{36}$$

Using (35) in (34), and (36) we obtain

$$H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0) = O_p\left(H^{-1/2}T^{-1/2}\right) + O_p\left(\frac{H^{-1/2}\lambda^H}{T^{-1/2}}\right), \text{ under } H_0 \tag{37}$$

Therefore, under H_0, $H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h}(\hat{\boldsymbol{\theta}}_T^0)$ tends to zero in probability if $H = \kappa T^{\epsilon}$, for $\epsilon \leq 1/2$, as H and $T \to \infty$ (the joint asymptotic condition 1).

To derive the distribution of $\bar{\hat{d}}_H(\hat{\boldsymbol{\theta}}_T^0)$, using Lemma S1, in the online supplement, we first note that

$$\frac{1}{H}\sum_{h=1}^{H} \upsilon_{T_0,h} = \frac{1}{H}\sum_{h=1}^{H}\sum_{j=0}^{h-1}\mathfrak{s}'\boldsymbol{\Phi}^j(\boldsymbol{\theta}^1)\boldsymbol{\Gamma}(\boldsymbol{\theta}^1)\mathbf{u}_{T_0+h-j} = \frac{1}{H}\sum_{j=1}^{H}\mathfrak{s}'\mathcal{A}_{H-j}\left(\boldsymbol{\Phi}_1\right)\boldsymbol{\Gamma}(\boldsymbol{\theta}^1)\mathbf{u}_{T_0+h-j}, \tag{38}$$

where

$$\mathcal{A}_{H-j}\left(\boldsymbol{\Phi}_1\right) = \mathbf{I}_{k_z+1} + \boldsymbol{\Phi}_1 + \boldsymbol{\Phi}_1^2 + ... + \boldsymbol{\Phi}_1^{H-j} = \left(\mathbf{I}_{k_z+1} - \boldsymbol{\Phi}_1\right)^{-1}\left(\mathbf{I}_{k_z+1} - \boldsymbol{\Phi}_1^{H-j+1}\right). \tag{39}$$

To simplify notations we have used $\boldsymbol{\Phi}_1$ for $\boldsymbol{\Phi}(\boldsymbol{\theta}^1)$. Considering that under H_0, $\bar{\hat{\mu}}_{T_0,H}(\hat{\boldsymbol{\theta}}_T^0) = O_p(T^{-1/2})$, we have

$$Var\left(\sqrt{H}\bar{\hat{d}}_H(\hat{\boldsymbol{\theta}}_T^0)\right) = \omega_{0q}^2 + o(1),$$

where

$$\omega_{0q}^2 = \mathfrak{s}'\left[H^{-1}\sum_{j=1}^{H}\mathcal{A}_{H-j}\left(\boldsymbol{\Phi}_1\right)\boldsymbol{\Sigma}_{\varepsilon}(\boldsymbol{\theta}^1)\mathcal{A}'_{H-j}\left(\boldsymbol{\Phi}_1\right)\right]\mathfrak{s},$$

and $\boldsymbol{\Sigma}_{\varepsilon}(\boldsymbol{\theta}^1) = E(\boldsymbol{\varepsilon}_{T+j}\boldsymbol{\varepsilon}'_{T+j}) = \boldsymbol{\Gamma}(\boldsymbol{\theta}^1)\boldsymbol{\Sigma}_u\boldsymbol{\Gamma}(\boldsymbol{\theta}^1)'$. Therefore, the mean policy effect test statistic can be written as

$$\bar{\mathcal{T}}_H = \frac{\sqrt{H}\bar{\hat{d}}_H(\hat{\boldsymbol{\theta}}_T^0)}{\sqrt{\hat{\omega}_{0q}^2}}, \tag{40}$$

where ω_{0q}^2 can be estimated using pre-intervention sample as:

$$\hat{\omega}_{0q}^2 = \mathfrak{s}'\left\{H^{-1}\sum_{j=1}^{H}\mathcal{A}_{H-j}\left(\boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0)\right)\boldsymbol{\Sigma}_{\varepsilon}\left(\hat{\boldsymbol{\theta}}_T^0\right)\mathcal{A}'_{H-j}\left(\boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0)\right)\right\}\mathfrak{s}, \tag{41}$$

where

$$\mathcal{A}_{H-j}\left(\boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0)\right) = \mathbf{I}_{k_z+1} + \boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0) + \left[\boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0)\right]^2 + ... + \left[\boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0)\right]^{H-j} \tag{42}$$

$$\boldsymbol{\Sigma}_{\varepsilon}\left(\hat{\boldsymbol{\theta}}_T^0\right) = T^{-1}\sum_{t=M}^{T_0}\left[\tilde{\mathbf{q}}_t - \boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0)\tilde{\mathbf{q}}_{t-1}\right]\left[\tilde{\mathbf{q}}_t - \boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_T^0)\tilde{\mathbf{q}}_{t-1}\right]', \tag{43}$$

Under the null hypothesis of policy ineffectiveness, and assuming that the underlying RE model is correctly specified and the innovations $\boldsymbol{\varepsilon}_{T_0+h} = \boldsymbol{\Gamma}(\boldsymbol{\theta})\mathbf{u}_{T_0+h}$ for $h = 1, 2, ..., H$ are normally distributed, then for a fixed H and as $T \to \infty$, we have $\bar{\mathcal{T}}_H \to_d N(0,1)$. For moderate values of H, small departures from normality of the innovations over the post-intervention sample might not be that serious for the validity of the test.

The power of $\bar{\mathcal{T}}_H$ test The power of the $\bar{\mathcal{T}}_H$ test, defined by (40), depends on the probability limit of $\bar{\mathcal{T}}_H$ under the alternative hypothesis that $\boldsymbol{\theta}^1 \neq \boldsymbol{\theta}^0$. Using (23) and suppressing the dependence on $(\hat{\boldsymbol{\theta}}_T^0)$ for simplicity, we have

$$\sqrt{H}\bar{\hat{d}}_H = H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h} + H^{-1/2}\sum_{h=1}^{H}\upsilon_{T_0,h}. \tag{44}$$

The random component, $H^{-1/2}\sum_{h=1}^{H}\upsilon_{T_0,h}$, has a limiting distribution with mean zero and a finite variance both under the null and the alternative hypotheses. Therefore, for the test to be consistent $H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h}$ must diverge to infinity with H. Under $H_1 : \boldsymbol{\theta}^1 \neq \boldsymbol{\theta}^0$, we have

$$\begin{aligned} H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h} &= -\mathfrak{s}'\left\{H^{-1/2}\sum_{h=1}^{H}\left[\boldsymbol{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right)\right]\right\}\widetilde{\mathbf{q}}_{T_0} \\ &= \mathfrak{s}'\left\{H^{-1/2}\sum_{h=1}^{H}\left[\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right]\right\}\mathbf{q}_{T_0} - \mathfrak{s}'\left\{H^{-1/2}\sum_{h=1}^{H}\left[\boldsymbol{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right]\right\}\widetilde{\mathbf{q}}_{T_0}. \end{aligned} \tag{45}$$

But it has been already established that (see (37))

$$\mathfrak{s}'\left\{H^{-1/2}\sum_{h=1}^{H}\left[\boldsymbol{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right]\right\}\widetilde{\mathbf{q}}_{T_0} = O_p\left(H^{-1/2}T^{-1/2}\right) + O_p\left(\frac{H^{-1/2}\lambda^H}{T^{-1/2}}\right).$$

Hence, under H_1

$$H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h} = \mathfrak{s}'\left\{H^{-1/2}\sum_{h=1}^{H}\left[\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right]\right\}\widetilde{\mathbf{q}}_{T_0} + O_p\left(H^{-1/2}T^{-1/2}\right) + O_p\left(\frac{H^{-1/2}\lambda^H}{T^{-1/2}}\right).$$

Now set $\boldsymbol{\Phi}_1 = \boldsymbol{\Phi}\left(\boldsymbol{\theta}^1\right)$ and $\boldsymbol{\Phi}_0 = \boldsymbol{\Phi}\left(\boldsymbol{\theta}^0\right)$, and note that $\sum_{h=1}^{H}\boldsymbol{\Phi}_1^h = \boldsymbol{\Phi}_1(\mathbf{I}_{k_z+1} - \boldsymbol{\Phi}_1^H)(\mathbf{I}_{k_z+1} - \boldsymbol{\Phi}_1)^{-1}$. Under Assumption 2, $(\mathbf{I}_{k_z+1} - \boldsymbol{\Phi}_1)^{-1}$ exists and is finite and $\boldsymbol{\Phi}_1^H \to \mathbf{0}$ as $H \to \infty$. Hence,

$$H^{-1/2}\sum_{h=1}^{H}\boldsymbol{\Phi}^h\left(\theta^1\right) = H^{-1/2}\boldsymbol{\Phi}_1(\mathbf{I}_{k_z+1} - \boldsymbol{\Phi}_1^H)(\mathbf{I}_{k_z+1} - \boldsymbol{\Phi}_1)^{-1} \to \mathbf{0}, \text{ as } H \to \infty.$$

Similarly, $H^{-1/2}\sum_{h=1}^{H}\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right) \to \mathbf{0}$, with H, and $H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h} = o_p(1)$, under the alternative hypothesis. Hence, $H^{-1/2}\sum_{h=1}^{H}\hat{\mu}_{T_0,h} \to_p 0$ under both the null and the alternative hypotheses as T and $H \to \infty$, subject to the joint asymptotic condition 1. Therefore, the internal dynamics of the RE model do not contribute to the power of the policy ineffectiveness test for T and H large. Thus tests based on the average policy effects, $\bar{\hat{d}}_H$, will not be consistent in the case of stationary DSGE models. In such cases, the best that can be hoped for is to base the test of the policy ineffectiveness on a short post-intervention sample and accept that the test is likely to be sensitive to the specifications of the post-intervention disturbances, $\boldsymbol{u}_{T_0+h}$, $h = 1, 2, ..., H$.

4.2 Tests of policy interventions that change the steady state

In this case the counterfactual values of y_{T_0+h} are defined by (17) and the associated policy effects by (16). Hence, the estimated values of policy effects can be computed as

$$\begin{aligned}\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0, \boldsymbol{\alpha}^0) &= y_{T_0+h} - \hat{y}^0_{T_0+h} \\ &= y_{T_0+h} - y^*_{T_0+h}\left(\boldsymbol{\alpha}^0\right) - \mathfrak{s}'\boldsymbol{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right)\left[\mathbf{q}_{T_0} - \mathbf{q}^*_{T_0}\left(\boldsymbol{\alpha}^0\right)\right], \text{ for } h = 1, 2, ..., H, \qquad (46)\end{aligned}$$

where $\boldsymbol{\alpha}^0$ is the pre-intervention parameter vector of the steady state.[14] Furthermore, since it is only the steady state parameters that change, $\boldsymbol{\theta}^1 = \boldsymbol{\theta}^0$, and post-intervention realized values of y_{T_0+h} are given by

$$y_{T_0+h} = y^*_{T_0+h}\left(\boldsymbol{\alpha}^1\right) + \mathfrak{s}'\boldsymbol{\Phi}^h(\boldsymbol{\theta}^0)\left[\mathbf{q}_{T_0} - \mathbf{q}^*_{T_0}\left(\boldsymbol{\alpha}^0\right)\right] + \sum_{j=0}^{h-1}\boldsymbol{\Phi}^j\left(\boldsymbol{\theta}^0\right)\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{u}_{T_0+h-j}, \text{for } h = 1, 2, ..., H,$$

which if used in (46) yields

$$\begin{aligned}\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0, \boldsymbol{\alpha}^0) = &\left[y^*_{T_0+h}\left(\boldsymbol{\alpha}^1\right) - y^*_{T_0+h}\left(\boldsymbol{\alpha}^0\right)\right] - \mathfrak{s}'\left[\boldsymbol{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right) - \boldsymbol{\Phi}^h(\boldsymbol{\theta}^0)\right]\left[\mathbf{q}_{T_0} - \mathbf{q}^*_{T_0}\left(\boldsymbol{\alpha}^0\right)\right] \\ &+ \sum_{j=0}^{h-1}\boldsymbol{\Phi}^j\left(\boldsymbol{\theta}^0\right)\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{u}_{T_0+h-j}.\end{aligned}$$

It is clear that under the null of no policy change, $H_0^\alpha : \boldsymbol{\alpha}^1 = \boldsymbol{\alpha}^0$, $\hat{d}_{T_0+h}(\hat{\boldsymbol{\theta}}_T^0, \boldsymbol{\alpha}^0)$ has the same asymptotic distribution as the one obtained in sub-sections 4.1.1 and 4.1.2 for the case when the policy intervention applied to $\boldsymbol{\theta}$. The main difference between the two types of policy change relates to the their power under the alternative hypothesis. In the present case the power of the test depends on $\delta_{T_0+h} = y^*_{T_0+h}\left(\boldsymbol{\alpha}^1\right) - y^*_{T_0+h}\left(\boldsymbol{\alpha}^0\right)$ for $h = 1, 2, ..., H$, and rises with H, so long as the policy change is permanent, namely $|\delta_{T_0+h}|$ remains bounded away from zero for all h. This contrasts to the power of the test when the policy change only affects deviations from the steady state.

5 Simulated policy analysis using a new-Keynesian model

5.1 The model

In this section we illustrate the performance of policy tests based on mean effects using simulations from a standard three equation New Keynesian DSGE model. Similar results can be obtained for the multi-horizon version of the test.

The new-Keynesian (NK) model is calibrated using parameter estimates from the literature and we assume that there is no parameter or specification uncertainty. The variables, which are measured in deviations from their steady states, are $\widetilde{R}_t = R_t - R_t^*$, the interest rate deviation,

[14] Here we are assuming that policy parameters affecting the steady state are known, which is the case when we consider changes to the inflation target. The analysis can be readily modified to allow for possible uncertainty in the policy parameters of the steady state.

$\widetilde{y}_t = y_t - y_t^*$, log real output gap, and $\widetilde{\pi}_t = \pi_t - \pi_t^*$, the deviation of inflation rate from target. As above, the policy intervention takes place at time T_0, with a post-intervention sample, $T_0+1, T_0+2, ..., T_0+H$. After setting out the model, we first consider an intervention which changes the DSGE parameters of the Taylor rule, but not the steady state, and examine the size and power of the test. Then we consider an intervention where the steady state inflation target is changed as well as the parameters of the Taylor rule. The model, in terms of deviations from steady state is

$$\widetilde{R}_t = \delta_R \widetilde{R}_{t-1} + (1-\delta_R)(\psi_\pi \widetilde{\pi}_t + \psi_y \widetilde{y}_t) + u_{Rt}, \tag{47a}$$

$$\widetilde{y}_t = \delta_y \widetilde{y}_{t-1} + \kappa E(\widetilde{y}_{t+1} \,|\mathfrak{I}_t) - \sigma \left[\widetilde{R}_t - E(\widetilde{\pi}_{t+1} \,|\mathfrak{I}_t) \right] + u_{yt}, \tag{47b}$$

$$\widetilde{\pi}_t = \delta_\pi \widetilde{\pi}_{t-1} + \beta E(\widetilde{\pi}_{t+1} \,|\mathfrak{I}_t) + \gamma \widetilde{y}_t + u_{\pi t}, \tag{47c}$$

which is of the form (1) where $\widetilde{\mathbf{q}}_t = (\widetilde{R}_t, \widetilde{y}_t, \widetilde{\pi}_t)'$, $\mathbf{u}_t = (u_{Rt}, u_{yt}, u_{\pi t})'$,

$$\mathbf{A}_0 = \begin{pmatrix} 1 & -(1-\delta_r)\psi_y & -(1-\delta_r)\psi_\pi \\ \sigma & 1 & 0 \\ 0 & -\gamma & 1 \end{pmatrix}, \ \mathbf{A}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \kappa & \sigma \\ 0 & 0 & \beta \end{pmatrix}, \ \mathbf{A}_2 = \begin{pmatrix} \delta_R & 0 & 0 \\ 0 & \delta_y & 0 \\ 0 & 0 & \delta_\pi \end{pmatrix}, \tag{48}$$

and $Var(\mathbf{u}_t) = diag(\sigma^2_{uR}, \sigma^2_{uy}, \sigma^2_{u\pi})$.

The parameters are calibrated using estimates from the DSGE literature. Parameters of (47c) are calibrated based on average estimates from eight major economies as summarized in Table 5 of Dees et al (2009). The parameters of (47b) and the long run parameters of the Taylor rule, (47a), are calibrated using the results in Dennis (2009). The calibrated values of $\boldsymbol{\theta}^0$ are summarized in Table 1 below. The standard deviations of the errors were all set equal to 0.005, or half a percent per quarter, which is similar to the US values found in Dees et al. (2009).

Table 1. Pre-intervention parameter values, $\boldsymbol{\theta}^0$, used in the Monte Carlo Analysis

$\sigma = 0.065$	$\kappa = 0.57$	$\beta = 0.65$	$\gamma = 0.045$	$\psi_\pi = 1.5$	$\psi_y = 0.5$
$\delta_y = 0.42$	$\delta_\pi = 0.34$	$\delta_R = 0.7$	$\sigma_{u\pi} = 0.005$	$\sigma_{uy} = 0.005$	$\sigma_{uR} = 0.005$

The solution for these calibrated parameters is given by[15]

$$\widetilde{\mathbf{q}}_t = \boldsymbol{\Phi}(\boldsymbol{\theta}^0)\widetilde{\mathbf{q}}_{t-1} + \boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{u}_t, \tag{49}$$

$$\boldsymbol{\Phi}(\boldsymbol{\theta}^0) = \begin{pmatrix} 0.65 & 0.13 & 0.20 \\ -0.17 & 0.62 & -0.05 \\ -0.06 & 0.08 & 0.47 \end{pmatrix}, \ \boldsymbol{\Gamma}(\boldsymbol{\theta}^0) = \begin{pmatrix} 0.93 & 0.31 & 0.60 \\ -0.24 & 1.49 & -0.15 \\ -0.08 & 0.19 & 1.39 \end{pmatrix}. \tag{50}$$

The dynamic responses of the model to the effects of monetary policy, demand and supply shocks are documented in the online supplement, where we also provide policy impulse response functions.

[15] This solution is obtained using the iterative back-substitution procedure advanced in Binder and Pesaran (1995). See the online supplement for further details.

The results are as to be expected and to save space will not be discussed here. Instead we focus on the small sample properties of the policy ineffectiveness test proposed in the paper.

We consider four separate policy interventions, in which each of the parameters of the Taylor rule are changed one at a time, leaving the other parameters unchanged. Intervention 1_A increases the interest rate persistence in the Taylor Rule, δ_R, from 0.7 to 0.9. Intervention 1_B reduces δ_R from 0.7 to 0.25. Intervention 1_C increases the inflation coefficient in the Taylor rule, ψ_π, from 1.5 to 2.5. Intervention 1_D increases the output coefficient in the Taylor rule, ψ_y, from 0.5 to 1. The values of θ^1 that are changed under alternative policy interventions are given in Table 2.

Table 2: Policy interventions

Interventions*	$\boldsymbol{\theta}^0$	$\boldsymbol{\theta}^1$
1_A	$\delta_R = 0.7$	$\delta_R = 0.9$
1_B	$\delta_R = 0.7$	$\delta_R = 0.25$
1_C	$\psi_\pi = 1.5$	$\psi_\pi = 2.5$
1_D	$\psi_y = 0.5$	$\psi_y = 1.0$

* The other elements of $\boldsymbol{\theta}^1$ are kept at their pre-intervention values.

The initial values, $\widetilde{\mathbf{q}}_{T_0}$, play an important role and should reflect a sensible combination of values of interest rate, inflation and output. One possible approach is to set $\widetilde{\mathbf{q}}_{T_0}$ equal to the impact effects of IRFs. For example, one could set $\widetilde{\mathbf{q}}_{T_0}$ to $\widetilde{\mathbf{q}}_{R,T_0} = \sigma_{uR}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_R$, which is the impact effect of a monetary policy shock. Similarly, for the demand and supply shocks $\mathbf{q}_{T_0}$ can be set to $\widetilde{\mathbf{q}}_{y,T_0} = \sigma_{u\,y}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_y$ and $\widetilde{\mathbf{q}}_{\pi,T_0} = \sigma_{u\pi}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_\pi$, respectively, where $\mathbf{e}_y = (0,1,0)'$ and $\mathbf{e}_\pi = (0,0,1)'$. These values are given by the columns of $\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)$ defined by (50). Multiples of the effects of such shocks represent different degrees of deviations from equilibrium. The power of the policy ineffectiveness test will then be an increasing function of the extent to which, at the time of the policy change, the economy has deviated from steady state.

5.2 Tests for interventions that do not change the steady state

We computed size and power of the policy ineffectiveness tests using the calibrated values of $\boldsymbol{\theta}^0$, for different initial states, $\widetilde{\mathbf{q}}_{T_0}$. We generated values of $\widetilde{\mathbf{q}}_{T_0+h}$, $h = 1, 2, ..., H$, for horizons $H = 8$, and $H = 24$ from (49) assuming $\mathbf{u}_t^{(b)} \sim IIDN(\mathbf{0}, \boldsymbol{\Sigma}_u)$,for $b = 1, 2, .., 2000$, replications, where $\boldsymbol{\Sigma}_u = diag(\sigma_{uR}^2, \sigma_{uy}^2, \sigma_{u\pi}^2)$.[16] For replication b the policy effects are simulated as

$$\hat{\mathbf{d}}_{T_0+h}^{(b)} = \widetilde{\mathbf{q}}_{T_0+h}^{(b)} - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\widetilde{\mathbf{q}}_{T_0}, \tag{51}$$

for $h = 1, 2, ..., H$. The policy mean effect is calculated as $\overline{\hat{\mathbf{d}}}_H^{(b)} = H^{-1}\sum_{h=1}^{H}\hat{\mathbf{d}}_{T_0+h}^{(b)}$, and the test statistic as $\mathcal{T}_{d,H}^{(b)} = \sqrt{H}\overline{\hat{\mathbf{d}}}_H^{(b)}/\hat{\omega}_{0q}$, where

$$\hat{\omega}_{0q}^2 = \left\{H^{-1}\sum_{j=1}^{H}\mathcal{A}_{H-j}\left(\boldsymbol{\Phi}(\boldsymbol{\theta}^0)\right)\boldsymbol{\Sigma}_\varepsilon\left(\boldsymbol{\theta}^0\right)\mathcal{A}'_{H-j}\left(\boldsymbol{\Phi}(\boldsymbol{\theta}^0)\right)\right\},$$

[16] More specifically, $\widetilde{\mathbf{q}}_{T_0+h}^{(b)} = \boldsymbol{\Phi}(\boldsymbol{\theta})\widetilde{\mathbf{q}}_{T_0+h-1}^{(b)} + \boldsymbol{\Gamma}(\boldsymbol{\theta})\mathbf{u}_{T_0+h}^{(b)}$, for $h = 1, 2, ..., H$, with $\widetilde{\mathbf{q}}_{T_0}^{(b)} = \widetilde{\mathbf{q}}_{T_0}$.

and

$$\mathcal{A}_{H-j}\left(\boldsymbol{\Phi}(\boldsymbol{\theta}^0)\right) = \mathbf{I}_{k_z+1} + \boldsymbol{\Phi}(\boldsymbol{\theta}^0) + \boldsymbol{\Phi}^2(\boldsymbol{\theta}^0) + ... + \ \boldsymbol{\Phi}^{H-j}(\boldsymbol{\theta}^0).$$

Table 3 shows the size and power of the policy ineffectiveness tests against four alternative policy interventions, two evaluation horizons and three initial states. The size of the test was calculated with $\widetilde{\mathbf{q}}^{(b)}_{T_0+h}$ generated using $\boldsymbol{\theta}^0$, and the power was obtained with $\widetilde{\mathbf{q}}^{(b)}_{T_0+h}$ generated using one of the four alternative policy interventions which change $\boldsymbol{\theta}^0$ to $\boldsymbol{\theta}^{1A}, ..., \boldsymbol{\theta}^{1D}$, as set out in Table 2. The initial states are given in different rows of the Table. The rows labelled $\widetilde{\mathbf{q}}_{R,T_0}$ give the rejection frequencies for the initial state corresponding to the effects of a one standard deviation monetary policy shock; the rows labelled $\widetilde{\mathbf{q}}_{y,T_0}$ refer to a demand shock and the rows labelled $\widetilde{\mathbf{q}}_{\pi,T_0}$ refer to a supply shock.

Table 3: Size, $\boldsymbol{\theta}^0$, and power of policy ineffectiveness tests against 4 alternatives $\boldsymbol{\theta}^{1A}, \boldsymbol{\theta}^{1B}, \boldsymbol{\theta}^{1C}, \boldsymbol{\theta}^{1D}$; horizons $H = 8, 24$; 3 initial states

	Size ($\boldsymbol{\theta}^0$)			Power ($\boldsymbol{\theta}^{1A}$)			Power ($\boldsymbol{\theta}^{1B}$)			Power ($\boldsymbol{\theta}^{1C}$)			Power ($\boldsymbol{\theta}^{1D}$)		
	$\widetilde{R}$	$\widetilde{y}$	$\widetilde{\pi}$	$\widetilde{R}$	$\widetilde{y}$	$\widetilde{\pi}$	$\widetilde{R}$	$\widetilde{y}$	$\widetilde{\pi}$	$\widetilde{R}$	$\widetilde{y}$	$\widetilde{\pi}$	$\widetilde{R}$	$\widetilde{y}$	$\widetilde{\pi}$
	$H = 8$														
$\widetilde{\mathbf{q}}_{R,T_0}$	0.05	0.05	0.05	0.03	0.20	0.13	0.13	0.04	0.08	0.11	0.06	0.03	0.07	0.02	0.07
$\widetilde{\mathbf{q}}_{y,T_0}$	0.04	0.05	0.05	0.03	0.18	0.12	0.11	0.04	0.07	0.10	0.06	0.03	0.07	0.01	0.06
$\widetilde{\mathbf{q}}_{\pi,T_0}$	0.05	0.04	0.05	0.04	0.20	0.12	0.12	0.04	0.08	0.12	0.05	0.03	0.07	0.02	0.06
	$H = 24$														
$\widetilde{\mathbf{q}}_{R,T_0}$	0.05	0.05	0.05	0.04	0.25	0.17	0.11	0.04	0.09	0.10	0.06	0.02	0.07	0.02	0.07
$\widetilde{\mathbf{q}}_{y,T_0}$	0.05	0.06	0.05	0.04	0.25	0.16	0.11	0.03	0.09	0.10	0.05	0.02	0.07	0.01	0.06
$\widetilde{\mathbf{q}}_{\pi,T_0}$	0.05	0.04	0.05	0.04	0.24	0.18	0.12	0.04	0.09	0.10	0.07	0.02	0.07	0.02	0.06

Notes: The rows labelled $\widetilde{\mathbf{q}}_{R,T_0}$ set the initial state $\widetilde{\mathbf{q}}_{T_0} = \sigma_{uR}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_R$. Similarly for $\widetilde{\mathbf{q}}_{y,T_0} = \sigma_{uy}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_y$, and $\widetilde{\mathbf{q}}_{\pi,T_0} = \sigma_{u\pi}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_\pi$. $\mathbf{e}_R = (1, 0, 0)$, $\mathbf{e}_y = (0, 1, 0)$, $\mathbf{e}_\pi = (0, 0, 1)$. The alternative hypotheses are set out in Table 2.

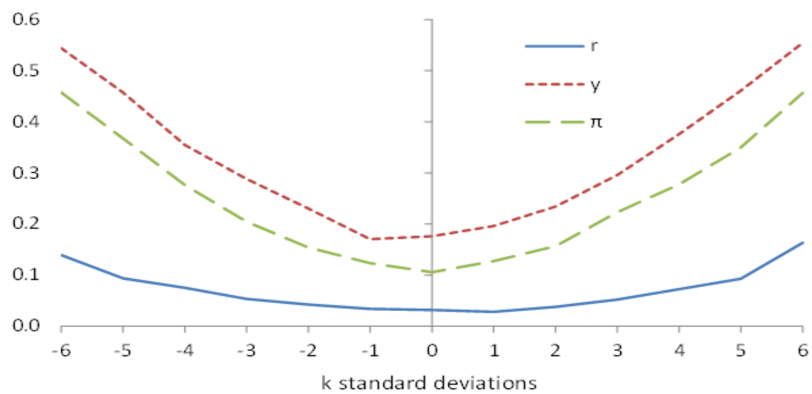
The test sizes are close to the nominal value of 5%. The power is highest for intervention, 1_A, where the degree of persistence of the Taylor rule increases from $\delta_R = 0.7$, to $\delta_R = 0.9$. Even in case 1_A the power is not high. At $H = 8$ the highest power is 20% for testing the effect on y_t and using the initial state, $\widetilde{\mathbf{q}}_{R,T_0}$ or $\widetilde{\mathbf{q}}_{\pi,T_0}$. At $H = 24$ the highest power is 25% for testing the effect on y_t. The test has little power against the other three types of interventions.[17] Whereas the test has power against the increase in persistence of the Taylor rule it has less power against the reduction in the persistence of the Taylor rule for output and inflation because the variables return to zero quickly. The test also has little power against changes in the coefficients of inflation and output in the Taylor rule because they have relatively little effect on the other variables on impact.

Figure 1 shows the rejection frequency for intervention 1_A (increasing the degree of interest rate smoothing) as a function of the initial deviation from steady state, measured in standard

[17] Similar outcomes are also reported by Rudebusch (2005) who, in the context of the Lucas Critique, shows that the apparent policy invariance of reduced forms is consistent with the magnitude of historical policy shifts and the relative insensitivity of the reduced forms of plausible forward looking macroeconomic specifications to policy shifts. However, here we use formal tests based on structural models.

deviations of monetary policy shock, $\widetilde{\mathbf{q}}_{R,T_0}$. The rejection frequencies increase with the deviation of the initial value from zero, and are roughly symmetric for positive and negative values. The rejection frequencies are highest for output, intermediate for inflation and lowest for interest rates. The results are similar, but with lower rejection frequencies, when the initial states are set to multiples of demand and supply shocks.

Figure 1. Rejection frequencies for intervention 1_A (increasing δ_R from 0.7 to 0.9) with the initial states at k standard deviations of $\widetilde{\mathbf{q}}_{R,T_0}$, and $H = 8$ quarters



These simulations confirm the theoretical results. The size of the test is correct. The effect of the policy intervention depends on the dynamics, reductions in the degree of persistence reduce the effect of changing the policy parameters. The power of the test depends on the state of the economy at the time of the policy intervention. In our example, the test has some power against increases in the persistence of the Taylor rule, but not against the other policy changes considered. However, the effects of all these policy changes are transitory, none have any effect on the steady states.

5.3 Tests for interventions that change the steady state inflation target

Consider now interventions that change the steady state. As an example, suppose the policy maker changes the target rate of inflation which we assume constant and denote by π_*. We assume the announcement of the change in the inflation target is credible and fully understood.[18] To represent this intervention in the New Keynesian example, where the variables are measured as deviations from steady state, we need to re-write the inflation and interest rate deviations in terms of their realized values which we denote by π_t and R_t. Note that $\pi_t = \widetilde{\pi}_t + \pi_*$ and $R_t = \widetilde{R}_t + (r_* + \pi_*)$, where π_* is the target rate of inflation, and r_* denotes the steady state value of the real interest rate. In terms of the realized values of inflation and interest rates (π_t and R_t) and deviations for the output gap ($\widetilde{y}_t$), we have

[18] Kulish and Pagan (2017) consider a change in inflation target when there is both perfect and imperfect credibility.

$$R_t = (1-\delta_R)\left[r_* + (1-\psi_\pi)\pi_*\right] + \delta_R R_{t-1} + (1-\delta_R)(\psi_\pi \pi_t + \psi_y \widetilde{y}_t) + u_{Rt}$$
$$\widetilde{y}_t = -\sigma r_* + \delta_y \widetilde{y}_{t-1} + \kappa E(\widetilde{y}_{t+1} \,|\mathfrak{I}_t\,) - \sigma\left[R_t - E(\pi_{t+1} \,|\mathfrak{I}_t\,)\right] + u_{yt}$$
$$\pi_t = (1-\delta_\pi - \beta)\,\pi_* + \delta_\pi \pi_{t-1} + \beta E(\pi_{t+1} \,|\mathfrak{I}_t\,) + \gamma \widetilde{y}_t + u_{\pi t}.$$

and setting $\mathring{\mathbf{q}}_t = (R_t, \widetilde{y}_t, \pi_t)'$, we obtain

$$\mathbf{A}_0 \mathring{\mathbf{q}}_t = \mathbf{A}_1 E_t(\mathring{\mathbf{q}}_{t+1}) + \mathbf{A}_2 \mathring{\mathbf{q}}_{t-1} + \mathbf{A}_3 + \mathbf{u}_t,$$

where

$$\mathbf{A}_3 = \begin{pmatrix} (1-\delta_R)\left[r_* + (1-\psi_\pi)\pi_*\right] \\ -\sigma r_* \\ (1-\delta_\pi - \beta)\,\pi_* \end{pmatrix}.$$

The other matrices, $\mathbf{A}_0$, $\mathbf{A}_1$, and $\mathbf{A}_2$, are given as before by (48). The solution in terms of $\mathring{\mathbf{q}}_t$ is given by

$$\mathring{\mathbf{q}}_t = \left[\mathbf{I}_3 - \mathbf{\Phi}(\boldsymbol{\theta})\right] \mathring{\mathbf{q}}_* + \mathbf{\Phi}(\boldsymbol{\theta}) \mathring{\mathbf{q}}_{t-1} + \mathbf{\Gamma}(\boldsymbol{\theta}) \mathbf{u}_t,$$

where $\mathring{\mathbf{q}}_* = (r_* + \pi_*, 0, \pi_*)'$, and $\mathbf{\Phi}(\boldsymbol{\theta})$ and $\mathbf{\Gamma}(\boldsymbol{\theta})$ are defined as before.

Suppose now that the policy intervention at time T_0 took the form of changing the inflation target from π_*^0 to π_*^1. In this case the policy effects are given by

$$\hat{d}_{T_0+h} = \mathfrak{s}' \mathring{\mathbf{q}}_{T_0+h} - \mathfrak{s}' \mathbf{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right) \mathring{\mathbf{q}}_{T_0} - \mathfrak{s}' \sum_{j=0}^{h-1} \mathbf{\Phi}^j\left(\hat{\boldsymbol{\theta}}_T^0\right)\left[\mathbf{I}_3 - \mathbf{\Phi}\left(\hat{\boldsymbol{\theta}}_T^0\right)\right] \mathring{\mathbf{q}}_*^0,$$

where $\mathring{\mathbf{q}}_*^0 = (r + \pi_*^0, 0, \pi_*^0)'$

$$\hat{d}_{T_0+h} = \mathfrak{s}' \mathring{\mathbf{q}}_{T_0+h} - \mathfrak{s}' \mathbf{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right) \mathring{\mathbf{q}}_{T_0} - \mathfrak{s}' \left\{\mathbf{I}_3 - \mathbf{\Phi}^h\left(\hat{\boldsymbol{\theta}}_T^0\right)\right\} \mathring{\mathbf{q}}_*^0, \tag{52}$$

Where only the inflation target is changed the power of the test rises with $\sqrt{H}\mathfrak{s}'\left(\mathring{\mathbf{q}}_*^1 - \mathring{\mathbf{q}}_*^0\right) = \sqrt{H}\left(\pi_*^1 - \pi_*^0\right)(1,0,1)'\mathfrak{s}$, and tends to unity in the case of inflation and the nominal interest rate, as to be expected, and has no power as $H \to \infty$, for real output deviations, $\widetilde{y}_t$. Nevertheless, the change in the inflation target does have short run effects on real output. This is reflected in the policy impulse response function and the test outcomes. The policy impulse response function when only the inflation target is changed is given by

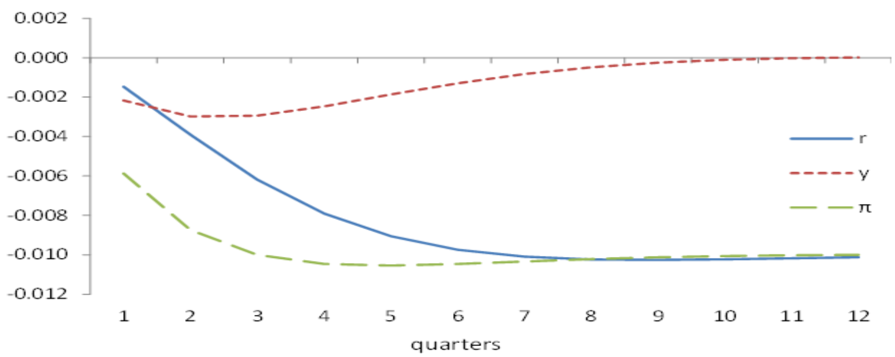
$$PIRF(h, \pi_*^1 - \pi_*^0, \theta) = \left(\pi_*^1 - \pi_*^0\right)\left\{\mathbf{I}_3 - \mathbf{\Phi}^h\left(\theta\right)\right\} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \text{ for } h = 1, 2, ..., H. \tag{53}$$

It is clear that in the limit as $H \to \infty$, the PIRF tends to $\left(\pi_*^1 - \pi_*^0\right)(1,0,1)'$, which also confirms that in the NK model only nominal values are affected in the long run by changes in the inflation target.

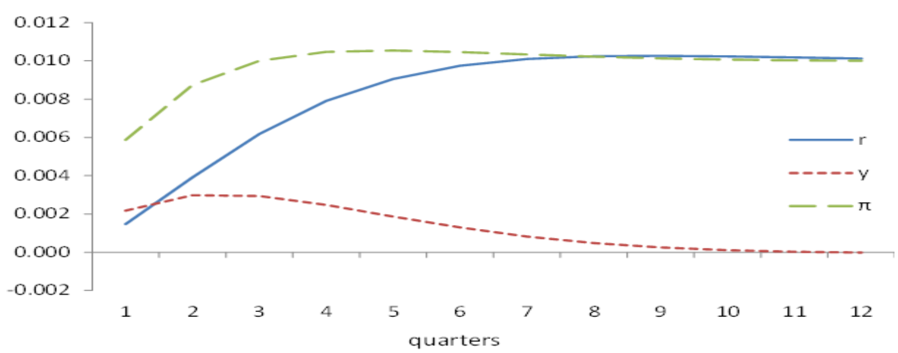
The short run impacts of changes in the inflation target can be illustrated using the parameterization given above. For this purpose we consider two scenarios, a reduction of π_*^0 from 2% to

1% per quarter and an increase of π_*^0 from 1% to 2% per quarter. Initially we do not change any of the other policy parameters, which are kept at the baseline values listed in Table 1. Figure 2a gives the responses to the reduction and 2b to the increase in the inflation target. In the case of a reduction, inflation falls more than the interest rate, raising the real interest rate on impact to 0.44% and thus depressing output. The real interest rate and output return to zero, leaving the nominal interest rate and inflation rate at the new target 1% lower after about seven quarters. When the target rate of inflation is increased the effects are reversed: inflation jumps more than interest rates, the real interest rate falls on impact to -0.44%, temporarily raising output.

Figure 2: Policy impulse response functions for changes in target rates of inflation



2a. Reduction of $\pi_*^0 = 2\%$ to $\pi_*^1 = 1\%$ per quarter



2b. Increase of $\pi_*^0 = 1\%$ to $\pi_*^1 = 2\%$ per quarter

In the case where there is both a change in the steady state and a change in the policy rule parameters, the policy impulse response functions are given by

$$PIRF(h, \pi_*^1, \boldsymbol{\theta}^1, \pi_*^0, \boldsymbol{\theta}^0) = \left[\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right]\mathring{\mathbf{q}}_{T_0} + \left\{\mathbf{I}_3 - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right)\right\}\mathring{\mathbf{q}}_*^1 - \left\{\mathbf{I}_3 - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right\}\mathring{\mathbf{q}}_*^0,$$
$$= \left\{\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right\}\left(\mathring{\mathbf{q}}_{T_0} - \mathring{\mathbf{q}}_*^0\right) + \left[\mathbf{I}_3 - \boldsymbol{\Phi}\left(\boldsymbol{\theta}^1\right)\right]^h\left(\mathring{\mathbf{q}}_*^1 - \mathring{\mathbf{q}}_*^0\right)$$

where

$$\mathring{\mathbf{q}}_*^0 = \begin{pmatrix} r + \pi_*^0 \\ 0 \\ \pi_*^0 \end{pmatrix}, \mathring{\mathbf{q}}_*^1 - \mathring{\mathbf{q}}_*^0 = \left(\pi_*^1 - \pi_*^0\right)\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

More specifically, for a unit monetary policy shock at the point of intervention, we set $\mathring{\mathbf{q}}_{T_0} = \mathbf{q}_*^0 + \sigma_{uR}\Gamma(\hat{\theta}_T^0)\mathbf{e}_R$, and hence

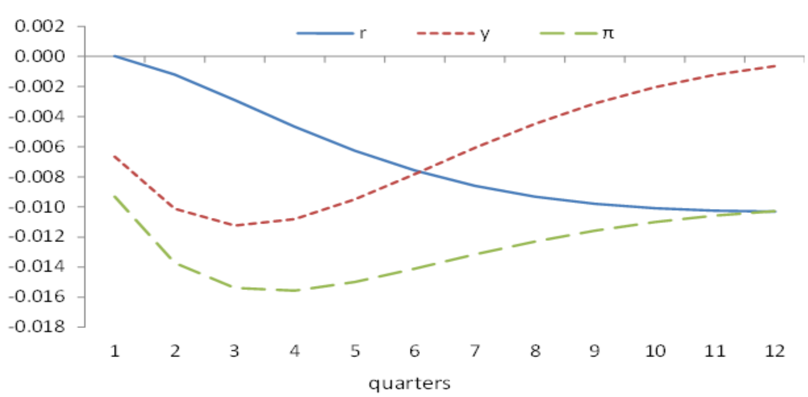
$$PIRF(h, \pi_*^1, \boldsymbol{\theta}^1, \pi_*^0, \boldsymbol{\theta}^0) = \sigma_{uR}\left\{\boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}^h\left(\boldsymbol{\theta}^0\right)\right\}\Gamma(\boldsymbol{\theta}^0)\mathbf{e}_R + \left(\pi_*^1 - \pi_*^0\right)\left[\mathbf{I}_3 - \boldsymbol{\Phi}\left(\boldsymbol{\theta}^1\right)\right]^h\begin{pmatrix}1\\0\\1\end{pmatrix}. \tag{54}$$

which reduces to (53) when only the inflation target is changed. Similar expressions can be obtained when the initial state is set to values of $\mathring{\mathbf{q}}$ that arise on impact from demand or supply shocks.

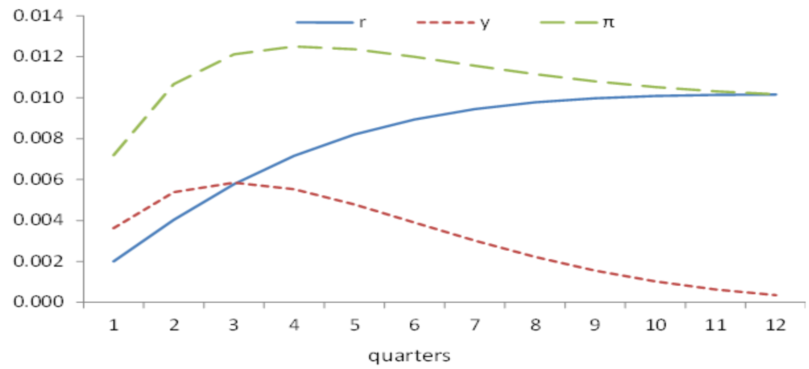
We now consider combining the change in the inflation target with changes in the degree of inflation smoothing. Figure 3a presents the effects of simultaneously reducing the inflation target from 2% to 1% and increasing the inflation smoothing parameter, δ_R, from 0.7 to 0.9**,** intervention 1_A above, with the initial state set to $\mathring{\mathbf{q}}_{R,T_0}$. This intervention causes inflation to drop sharply, overshooting its steady state of 1%, hitting 1.55% after about 4 quarters. The real interest rate rises to 1.25%, depressing output, before the variables return to their steady state. Figure 3b shows that increasing the target rate of inflation has similar but the opposite effects. Comparing the reduction in the target rate of inflation in Figure 3a with that in Figure 2a, the increased interest rate smoothing has resulted in a much larger loss of output. Whereas in Figure 4a the maximum loss of output was 0.3% per quarter, in figure 5a the maximum loss was 1.1%, in both cases around quarter 3.

Figure 3: Policy impulse response functions for changes in target rates of inflation plus increased interest rate smoothing

Intervention 1_A : δ_R from 0.7 to 0.9, initial state $\mathring{\mathbf{q}}_{R,T_0}$



3a. Reduction of $\pi_*^0 = 2\%$ to $\pi_*^1 = 1\%$ per quarter

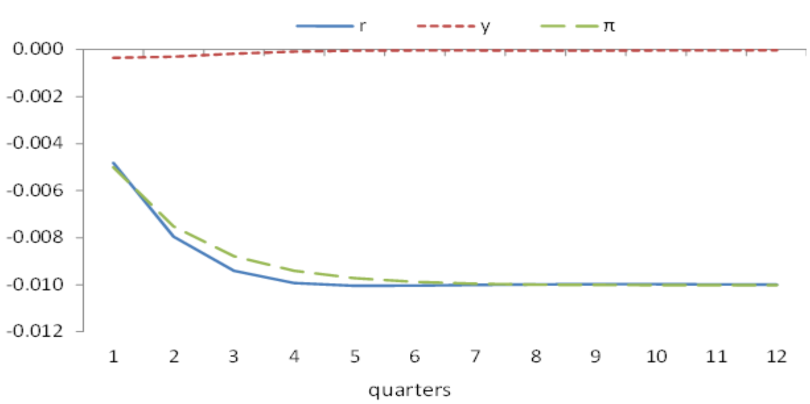


3b. Increase of $\pi_*^0 = 1\%$ to $\pi_*^1 = 2\%$ per quarter

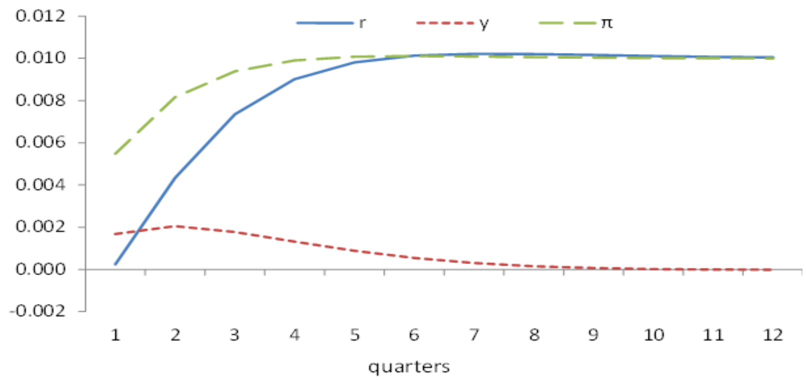
Figure 4 shows the results when the change in inflation target is combined with reduced interest rate smoothing. For a credible reduction in the inflation target and very little interest rate smoothing, the interest rate and the inflation rate reduce by almost exactly the same amount and output hardly falls. With an increase in the inflation target and reduced interest rate smoothing, inflation increases more than interest rates and the lower real interest rates provides a boost to output. While the results are specific to this parameterisation and the assumption of credibility, it seems likely that less interest rate smoothing is appropriate when reducing the target rate of inflation, as in Figure 4a, since this causes less output loss, and more interest rate smoothing seems more appropriate when increasing the target rate of inflation, since this provides a bigger boost to output.

Figure 4: Policy impulse response functions for changes in target rates of inflation plus reduced interest rate smoothing

Intervention 1_B : δ_R from 0.7 to 0.25, initial state $\mathring{\mathbf{q}}_{R,T_0}$



4a. Reduction of $\pi_*^0 = 2\%$ to $\pi_*^1 = 1\%$ per quarter



4b. Increase of $\pi_*^0 = 1\%$ to $\pi_*^1 = 2\%$ per quarter

We now consider the effect on size and power of the policy ineffectiveness test in detecting the effects of changes in the target rate of inflation on inflation, output deviations and the interest rate. We only consider the case where the inflation target is reduced from 2% to 1% per quarter, the results for an increase were almost identical. We consider two interventions. In the first, called θ^{1E}, the interest rate smoothing parameter is left unchanged at $\delta_R = 0.7$, in the second, called θ^{1F}, δ_R is increased to 0.9 at the same time as the reduction in the inflation target is announced. If the target rate is reduced without any other policy changes, the power of the tests based on the nominal interest rate and the inflation rate are quite high and rise substantially as the horizon of the test is increased from $H = 8$ to 24 quarters. In contrast, and as to be expected noting the PIRFs depicted in Figure 2, the test has little power for output, since the effect of a change in the inflation target on the real output is small and transitory. Under intervention θ^{1F}, when there is both a change in the inflation target and an increase in interest rate smoothing, the power of the test based on inflation outcomes is increased, but for interest rates the power is reduced relative to the case θ^{1E}, since the increased smoothing means that interest rates do not change as much. The increased smoothing causes a larger movement in real interest rates as noted above and this causes a greater effect on output hence a higher power in detecting the effects of the policy change on realized values of output deviations. Whereas for interest rates and inflation, the power increases as the horizon is extended, for output deviations, which is moving back to its steady state value of zero, the power falls as the horizon is extended.

Table 4: Size and power of policy ineffectiveness tests against reducing the inflation target only ($\boldsymbol{\theta}^{1E}$) and when inflation target reduction is accompanied by a rise in interest rate smoothing ($\boldsymbol{\theta}^{1F}$)- Horizons $H = 8, 24$; 3 initial states ($\mathring{\mathbf{q}}_{T_0}$)

	Size ($\boldsymbol{\theta}^0$)			Power ($\boldsymbol{\theta}^{1E}$)			Power ($\boldsymbol{\theta}^{1F}$)		
	R	$\widetilde{y}$	π	R	$\widetilde{y}$	π	R	$\widetilde{y}$	π
Initial states	$H = 8$								
$\mathring{\mathbf{q}}_{R,T_0}$	0.05	0.05	0.05	0.29	0.07	0.72	0.13	0.39	0.90
$\mathring{\mathbf{q}}_{y,T_0}$	0.06	0.05	0.06	0.26	0.07	0.68	0.17	0.33	0.86
$\mathring{\mathbf{q}}_{\pi,T_0}$	0.05	0.06	0.06	0.28	0.06	0.70	0.16	0.35	0.88
	$H = 24$								
$\mathring{\mathbf{q}}_{R,T_0}$	0.06	0.04	0.05	0.73	0.07	0.99	0.65	0.30	0.98
$\mathring{\mathbf{q}}_{y,T_0}$	0.05	0.06	0.05	0.73	0.05	0.99	0.70	0.28	0.98
$\mathring{\mathbf{q}}_{\pi,T_0}$	0.05	0.05	0.05	0.71	0.05	0.99	0.68	0.29	0.99

Notes: See notes to Table 3. Alternative hypothesis $\boldsymbol{\theta}^{1E}$ assumes that the inflation target is reduced from $\pi_*^0 = 2\%$ to $\pi_*^1 = 1\%$ per quarter. Alternative hypothesis $\boldsymbol{\theta}^{1F}$ combines the reduction of the inflation target from $\pi_*^0 = 2\%$ to $\pi_*^1 = 1\%$ per quarter with a higher degree of interest rate smoothing, raising δ_R from 0.7 to 0.9.

6 Concluding remarks

In this paper we propose formal tests for two types of policy intervention in the context of DSGE models. One involves a change in a policy parameter that does not alter the steady state, as is standard in the literature, the other a change in a policy parameter that only alters the steady state. Two versions of the policy ineffectiveness tests are considered, a multi-horizon version and a mean effect version.

The tests are based on the differences, over a given policy evaluation horizon, between the post-intervention realizations of the target variable and the associated counterfactual outcomes based on the parameters estimated using data before the policy intervention. The power of the policy ineffectiveness tests depends on the degree of persistence of the model and the deviation from steady state at the time of the intervention, but the power will not go to unity as the evaluation horizon increases, that is the tests will not be consistent, unless the policy intervention changes the steady state.

The formal tests considered in the paper are important for policy analysis as they highlight the importance of allowing for future shocks in policy evaluations. A policy impulse response function is also proposed which is more relevant when the policy intervention is formulated in terms of a change in a policy parameter, as compared to standard impulse response function that considers the deterministic effects of a policy shock, defined as a one standard deviation change in a structural disturbance of interest. The policy ineffectiveness tests that we have developed take account of deterministic as well as random components of policy outcomes and are likely to be more relevant for *ex post* policy analyses.

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Online Supplement
Tests of Policy Interventions in DSGE Models
by
M. Hashem Pesaran and R.P. Smith

S1 Statement and proof of lemmas

Lemma S1 *Let* $\mathbf{A}$ *be a* $k \times k$ *matrix and* $\mathbf{x}_{T+h-j}$ *a* $k \times 1$ *vector, and suppose that* $\mathbf{I}_k - \mathbf{A}$ *is invertible, then*

$$\begin{aligned}
H^{-1}\sum_{h=1}^{H}\sum_{j=0}^{h-1}\mathbf{A}^j\mathbf{x}_{T+h-j} &= H^{-1}\sum_{j=1}^{H}\left(\mathbf{I}_k+\mathbf{A}+...+\mathbf{A}^{H-j}\right)\mathbf{x}_{T+j} \\
&= H^{-1}\left(\mathbf{I}_k-\mathbf{A}\right)^{-1}\sum_{j=1}^{H}\left(\mathbf{I}_k-\mathbf{A}^{H-j+1}\right)\mathbf{x}_{T+j} \\
&= \left(\mathbf{I}_k-\mathbf{A}\right)^{-1}\left(H^{-1}\sum_{j=1}^{H}\mathbf{x}_{T+j}\right)-\left(\mathbf{I}_k-\mathbf{A}\right)^{-1}\left(H^{-1}\sum_{j=1}^{H}\mathbf{A}^{H-j+1}\mathbf{x}_{T+j}\right).
\end{aligned}$$

Proof. The result follows by direct manipulation of the terms. ■

Lemma S2 *Suppose that the* $k \times k$ *matrices* $\mathbf{A}$ *and* $\mathbf{B}$ *have bounded spectral norms* $\|\mathbf{A}\| \leq \lambda$ *and* $\|\mathbf{B}\| \leq \lambda$*, for some fixed positive constant* λ*. Then*

$$\left\|\mathbf{A}^h-\mathbf{B}^h\right\| \leq h\lambda^{h-1}\left\|\mathbf{A}-\mathbf{B}\right\|, \text{ for } h=1,2,.... \tag{A.1}$$

Proof. We establish this result by backward induction. It is clear that it holds for $h = 1$. For $h = 2$, using the identity

$$\mathbf{A}^2-\mathbf{B}^2=\mathbf{A}(\mathbf{A}-\mathbf{B})+(\mathbf{A}-\mathbf{B})\mathbf{B},$$

the result for $h = 2$ follows

$$\left\|\mathbf{A}^2-\mathbf{B}^2\right\| \leq \left(\|\mathbf{A}\|+\|\mathbf{B}\|\right)\|\mathbf{A}-\mathbf{B}\| = 2\lambda\|\mathbf{A}-\mathbf{B}\|.$$

More generally, we have the identity

$$\mathbf{A}^h-\mathbf{B}^h=\mathbf{A}^h(\mathbf{A}-\mathbf{B})+(\mathbf{A}-\mathbf{B})\mathbf{B}^h+\mathbf{A}(\mathbf{A}^{h-2}-\mathbf{B}^{h-2})\mathbf{B}.$$

Now suppose now that (A.1) holds for $h-2$, and using the above note that

$$\begin{aligned}
\left\|\mathbf{A}^h-\mathbf{B}^h\right\| &\leq \left\|\mathbf{A}^{h-1}\right\|\|\mathbf{A}-\mathbf{B}\|+\|\mathbf{A}-\mathbf{B}\|\left\|\mathbf{B}^{h-1}\right\|+\|\mathbf{A}\|\left\|\mathbf{A}^{h-2}-\mathbf{B}^{h-2}\right\|\|\mathbf{B}\| \\
&\leq \|\mathbf{A}\|^{h-1}\|\mathbf{A}-\mathbf{B}\|+\|\mathbf{A}-\mathbf{B}\|\|\mathbf{B}\|^{h-1}+\|\mathbf{A}\|\left\|\mathbf{A}^{h-2}-\mathbf{B}^{h-2}\right\|\|\mathbf{B}\| \\
&\leq 2\lambda^{h-1}\|\mathbf{A}-\mathbf{B}\|+\lambda^2\left\|\mathbf{A}^{h-2}-\mathbf{B}^{h-2}\right\| \\
&\leq 2\lambda^{h-1}\|\mathbf{A}-\mathbf{B}\|+\lambda^2\left[(h-2)\lambda^{h-3}\|\mathbf{A}-\mathbf{B}\|\right] \\
&\leq h\lambda^{h-1}\|\mathbf{A}-\mathbf{B}\|.
\end{aligned}$$

Hence, if (A.1) holds for $h-2$, then it must also hold for h. But since we have established that (A.1) holds for $h=1$ and $h=2$, then it must hold for any h. ■

Lemma S3 *Consider the $k \times k$ matrix $\mathbf{A}(\boldsymbol{\theta}) = (a_{ij}(\boldsymbol{\theta}))$, where k is a finite integer and $a_{ij}(\boldsymbol{\theta})$, for all $i, j = 1, 2, .., k$, are continuously differentiable real-valued functions of the $p \times 1$ vector of parameters, $\theta \in \boldsymbol{\Theta}$. Suppose that $a_{ij}(\theta)$ has finite first order derivatives at all points in $\boldsymbol{\Theta}$, and assume that $\hat{\boldsymbol{\theta}}_T$ is a $\sqrt{T}$ consistent estimator of $\boldsymbol{\theta}^0$. Then*

$$\left\| \mathbf{A}(\hat{\boldsymbol{\theta}}_T) - \mathbf{A}(\boldsymbol{\theta}^0) \right\| \leq a_T \left\| \hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}^0 \right\|, \tag{A.2}$$

$$\left\| \mathbf{A}(\hat{\boldsymbol{\theta}}_T) \right\| \leq \left\| \mathbf{A}(\boldsymbol{\theta}^0) \right\| + a_T \left\| \hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}^0 \right\|, \tag{A.3}$$

where $a_T = \left\| \partial \mathbf{A}\left(\bar{\boldsymbol{\theta}}_T\right) / \partial \boldsymbol{\theta}' \right\|$ is bounded in T, and elements of $\bar{\boldsymbol{\theta}}_T \in \boldsymbol{\Theta}$ lie on the line segment joining $\boldsymbol{\theta}^0$ and $\hat{\boldsymbol{\theta}}_T$

Proof. Consider the mean-value expansions

$$a_{ij}\left(\hat{\theta}_T\right) - a_{ij}\left(\theta^0\right) = \frac{\partial a_{ij}\left(\bar{\theta}_T\right)}{\partial \theta'} \left(\hat{\theta}_T - \theta^0\right), \text{ for } i, j = 1, 2, ..., k,$$

where elements of $\bar{\theta}_T$ lie on the line segment joining θ^0 and $\hat{\theta}_T$. Given that $\hat{\theta}_T$ is consistent for θ^0, it must also be that $\bar{\theta}_T \to_p \theta^0$, as $T \to \infty$. Collecting all the k^2 terms we have

$$\mathbf{A}(\hat{\boldsymbol{\theta}}_T) - \mathbf{A}(\boldsymbol{\theta}^0) = \left(\frac{\partial \mathbf{A}\left(\bar{\boldsymbol{\theta}}_T\right)}{\partial \boldsymbol{\theta}'} \right) \left[\mathbf{I}_k \otimes \left(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}^0 \right) \right],$$

where $\otimes$ denotes the Kronecker matrix product. Hence

$$\left\| \mathbf{A}(\hat{\boldsymbol{\theta}}_T) - \mathbf{A}(\boldsymbol{\theta}^0) \right\| \leq \left\| \frac{\partial \mathbf{A}\left(\bar{\boldsymbol{\theta}}_T\right)}{\partial \boldsymbol{\theta}'} \right\| \left\| \hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}^0 \right\|,$$

$$\left\| \mathbf{A}(\hat{\boldsymbol{\theta}}_T) \right\| = \left\| \mathbf{A}(\boldsymbol{\theta}^0) + \left(\frac{\partial \mathbf{A}\left(\bar{\boldsymbol{\theta}}_T\right)}{\partial \boldsymbol{\theta}'} \right) \left[\mathbf{I}_k \otimes \left(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}^0 \right) \right] \right\| \leq \left\| \mathbf{A}(\boldsymbol{\theta}^0) \right\| + \left\| \frac{\partial \mathbf{A}\left(\bar{\boldsymbol{\theta}}_T\right)}{\partial \boldsymbol{\theta}'} \right\| \left\| \hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}^0 \right\|.$$

The results (A.2) and (A.3) now follow since $\bar{\boldsymbol{\theta}}_T \to_p \boldsymbol{\theta}^0$, and by assumption the derivatives $\partial a_{ij}\left(\boldsymbol{\theta}^0\right) / \partial \boldsymbol{\theta}'$ exist and are bounded in T. ■

Lemma S4 *Suppose that $\lambda_T = \lambda + T^{-1/2} a_T$, $a_T > 0$ and bounded in T, $\lambda_T \neq 1$, $H = \kappa T^{\epsilon}$, where $\epsilon \leq 1/2$, $0 < \lambda < 1$, and κ is a positive fixed constant. Then*

$$\sum_{h=1}^{H} h \lambda_T^{h-1} = \frac{1}{(1-\lambda)^2} + O_p\left(T^{-1/2}\right) + O_p\left(H \lambda^H\right), \tag{A.4}$$

and

$$\sum_{h=1}^{H} \sum_{j=0}^{h-1} j \lambda_T^{j-1} = \frac{1}{(1-\lambda)^2} \left(H - \frac{1+\lambda}{1-\lambda} \right) + O_p\left(T^{-1/2}\right) + O_p\left(H \lambda^H\right). \tag{A.5}$$

Proof. We first note that

$$\sum_{h=1}^{H} h\lambda_T^{h-1} = \frac{\partial}{\partial \lambda_T}\left(\sum_{h=1}^{H}\lambda_T^h\right)$$
$$= \frac{1-\lambda_T^H}{(1-\lambda_T)^2} - \frac{H\lambda_T^H}{(1-\lambda_T)}, \tag{A.6}$$

Also since $\lambda_T = \lambda + O_p\left(T^{-1/2}\right)$

$$\sum_{h=1}^{H} h\lambda_T^{h-1} = \frac{1}{(1-\lambda)^2} + O_p\left(T^{-1/2}\right) + O_p\left(H\lambda_T^H\right). \tag{A.7}$$

But,

$$\lambda_T^H = \left(\lambda + T^{-1/2}a_T\right)^H = \lambda^H\left(1 + \frac{T^{-1/2}a_T}{\lambda}\right)^H = O_p\left(\lambda^H e^{d_T H/\sqrt{T}}\right), \tag{A.8}$$

where $d_T = a_T/\lambda$, which is also bounded in T. Finally, $H/\sqrt{T} = T^{1-\epsilon/2}$ and for $\epsilon \leq 1/2$, $e^{d_T H/\sqrt{T}}$ will be bounded in T. Using this result in (A.7) yields (A.4), as desired. Similarly,

$$\sum_{h=1}^{H}\sum_{j=0}^{h-1} j\lambda_T^{j-1} = \sum_{h=1}^{H}\left[\frac{(1-\lambda_T^h) - h(1-\lambda_T)\lambda_T^{h-1}}{(1-\lambda_T)^2}\right]$$
$$= \frac{1}{(1-\lambda_T)^2}\left[\sum_{h=1}^{H}\left[\left(1-\lambda_T^h\right) - h(1-\lambda_T)\lambda_T^{h-1}\right]\right]$$
$$= \frac{1}{(1-\lambda_T)^2}\left[H - \sum_{h=1}^{H}\lambda_T^h - (1-\lambda_T)\sum_{h=1}^{H}h\lambda_T^{h-1}\right].$$

Using (A.6) we have

$$\sum_{h=1}^{H}\sum_{j=0}^{h-1} j\lambda_T^{j-1} = \frac{1}{(1-\lambda_T)^2}\left\{H - \frac{\lambda_T - \lambda_T^{H+1}}{1-\lambda_T} - (1-\lambda_T)\left[\frac{1-\lambda_T^H}{(1-\lambda_T)^2} - \frac{H\lambda_T^H}{(1-\lambda_T)}\right]\right\}.$$

Now using (A.8) and recalling that $\lambda_T = \lambda + O_p\left(T^{-1/2}\right)$, we obtain (A.5). ■

S2 The numerical solution of the DGSE model used in Section 5

The unique solution of the New Keynesian model is given by (see also equation (2) in the paper):

$$\widetilde{\mathbf{q}}_t = \mathbf{\Phi}(\boldsymbol{\theta})\widetilde{\mathbf{q}}_{t-1} + \mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{u}_t,$$

where $\mathbf{\Phi}(\boldsymbol{\theta})$ solves the quadratic matrix equation $\mathbf{A}_1\mathbf{\Phi}^2(\boldsymbol{\theta} - \mathbf{A}_0\mathbf{\Phi}(\boldsymbol{\theta}) + \mathbf{A}_2 = \mathbf{0}$, and $\mathbf{\Gamma} = [\mathbf{A}_0 - \mathbf{A}_1\mathbf{\Phi}(\boldsymbol{\theta})]^{-1}$. $\mathbf{\Phi}(\boldsymbol{\theta})$ can be solved numerically by iterative back-substitution procedure which involves iterating on an initial arbitrary choice of $\mathbf{\Phi}(\boldsymbol{\theta})$ say $\mathbf{\Phi}(\boldsymbol{\theta}_{(0)}) = \mathbf{\Phi}_{(0)}$ using the recursive relation

$$\mathbf{\Phi}_{(r)} = [\mathbf{I}_k - (\mathbf{A}_0^{-1}\mathbf{A}_1)\mathbf{\Phi}_{(r-1)}]^{-1}(\mathbf{A}_0^{-1}\mathbf{A}_2).$$

See Binder and Pesaran (1995) for further details. The iterative procedure is continued until convergence using the criteria $\| \mathbf{\Phi}_{(r)} - \mathbf{\Phi}_{(r-1)} \|_{\max} \leq 10^{-6}$.

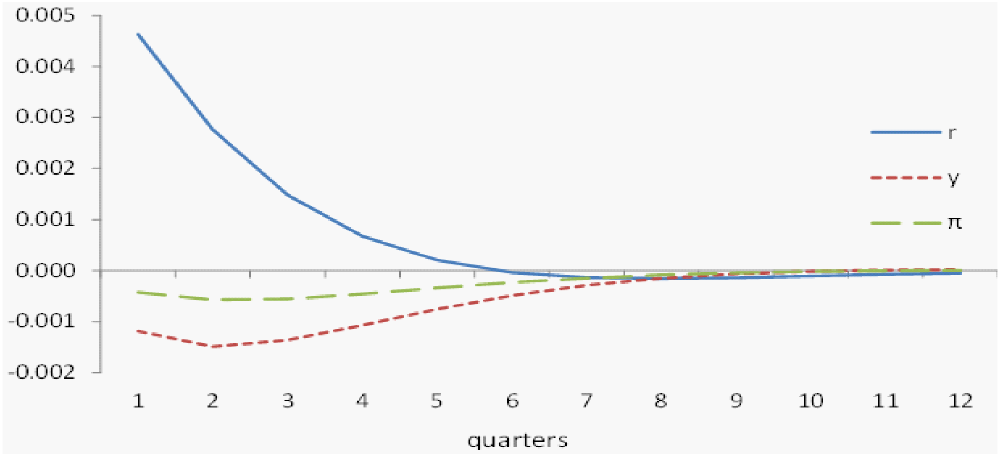
S3 Standard and Policy Impulse Response Functions for the new-Keynesian model

Here we first provide impulse response functions, IRFs for the effects of monetary policy, demand and supply shocks in the new-Keynesian model. As Figure S1 shows a contractionary monetary policy shock raises interest rates and reduces output and inflation, with output falling by more than inflation. A positive demand shock increases all three variables; output by the most, then interest rates, and then inflation. A negative supply shock, increases inflation, the interest rate rises to offset the higher inflation, but not by as much as inflation and output falls. The impact effects of the monetary policy shock are given by the first column of $\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)$ defined by equation (50) of the paper, while the impact effects of the demand and supply shocks are given by its second and third columns. It is clear that in terms of IRFs the behaviour of the model is as expected.

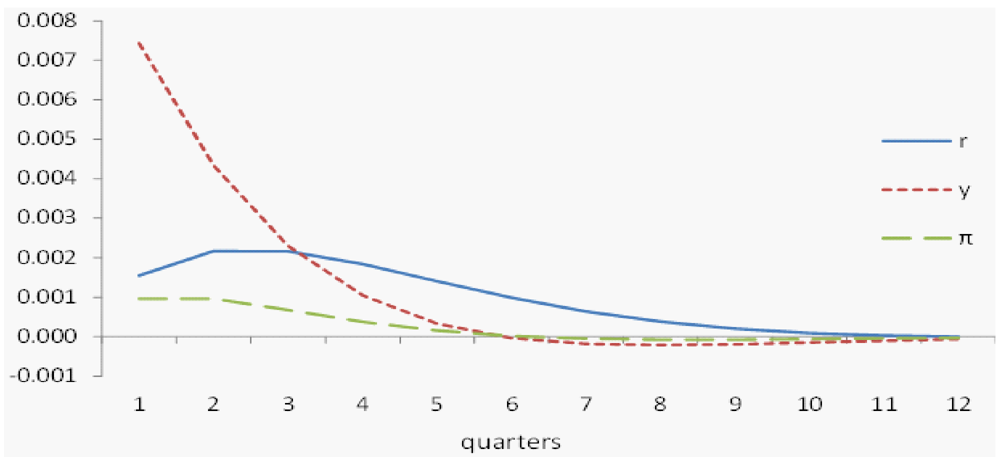
Turning to the policy impulse response function, PIRF, discussed in Section 3.1 of the paper, as noted in the text it is important that the choice of $\widetilde{\mathbf{q}}_{T_0}$ reflects a sensible combination of values of interest rate, inflation and output. One possible approach is to set $\widetilde{\mathbf{q}}_{T_0}$ equal to the impact effects of IRFs. For example, one could set $\widetilde{\mathbf{q}}_{T_0}$ to $\widetilde{\mathbf{q}}_{R,T_0} = \sigma_{uR}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_R$, which is the impact effect of a monetary policy shock. Similarly, for the demand and supply shocks $\mathbf{q}_{T_0}$ can be set to $\widetilde{\mathbf{q}}_{y,T_0} = \sigma_{u\ y}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_y$ and $\widetilde{\mathbf{q}}_{\pi,T_0} = \sigma_{u\pi}\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_\pi$, respectively, where $\mathbf{e}_y = (0,1,0)'$ and $\mathbf{e}_\pi = (0,0,1)'$. These values are given by the columns of $\boldsymbol{\Gamma}(\boldsymbol{\theta}^0)$ defined by equation (50) of the paper. We will also consider multiples of the effects of such shocks as representing different degrees of deviations from equilibrium. The power of the policy ineffectiveness test will then be an increasing function of the extent to which, at the time of the policy change, the economy has deviated from steady state.

Figure S2 shows PIRFs for the effects of changing the degree of persistence (or the interest rate smoothing) associated with the Taylor rule, Figure S2a shows the effect of intervention 1_A and Figure S2b of 1_B. These are the only policy changes which have much effect. This is consistent with the theoretical results that it is the dynamics that are central to policy having mean effects. Intervention 1_A increases the degree of persistence from $\delta_R = 0.7$, to $\delta_R = 0.9$. This causes the interest rate to rise and output and inflation to fall initially, with a maximum effect after about three periods before returning to zero. Intervention 1_B reduces the degree of persistence from $\delta_R = 0.7$, to $\delta_R = 0.25$. This has the opposite effect causing the interest rate to fall, by more than it rose in case 1_A, and output and inflation to rise by rather less than they fell under case 1_A. The initial effects are the same as the values of $\left[\boldsymbol{\Phi}\left(\boldsymbol{\theta}^1\right) - \boldsymbol{\Phi}\left(\boldsymbol{\theta}^0\right)\right]$ for the two cases. When the degree of persistence is low as in intervention 1_B, the variables return to zero much faster, making the mean effect of policy much smaller. This is reflected in the power of the policy ineffectiveness tests discussed in the text.

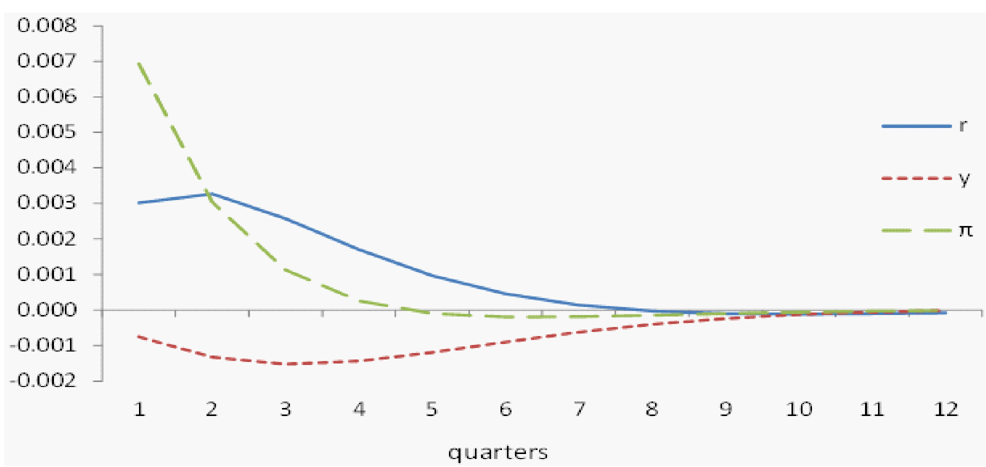
Figure S1: Impulse response functions for interest rates, $\widetilde{R}_t$, output, $\widetilde{y}_t$, and inflation $\widetilde{\pi}_t$ deviations



S1a. Monetary Policy Shock

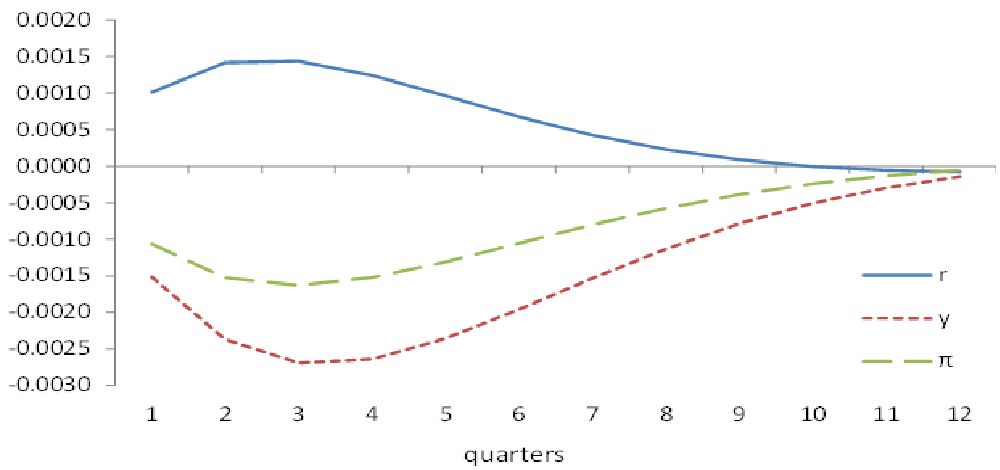


S1b. Demand Shock

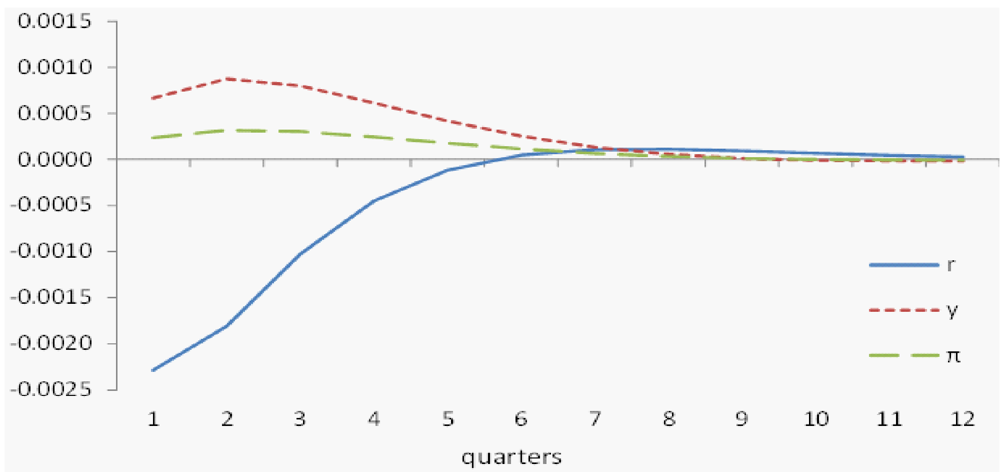


S1c. Supply Shock

Figure S2: Policy impulse response functions: $\widetilde{\mathbf{q}}_{R,T_0} = \sigma_{uR}\mathbf{\Gamma}(\boldsymbol{\theta}^0)\mathbf{e}_R$.



S2a. Intervention $1_A : \delta_R = 0.7$, to $\delta_R = 0.9$



S2b. Intervention $1_B : \delta_R = 0.7$, to $\delta_R = 0.25$