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Real options, intellectual property, R&D, geometric Brownian motion, Stackelberg games

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R&D appropriability and market structure in a preemption model.

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Abstract

Numerous studies have examined how market structure affects appropriability of R&D returns and, in turn, R&D investment and innovation speed. Less effort has been spent on the opposite relationship which is instead our focus. In a continuous time model, two firms compete in R&D, with the leading patent affecting the probability of success of a second patent (competing in the same product market); the size and the direction of this effect depends on the level of appropriability, which, unlike previous research, connects competition in R&D and competition in the product market. We find that low appropriability delays R&D investments and thus discovery, with the (future) benefit of a more competitive product market. Secondly, we show that the relation between concentration in R&D and concentration in product markets can be positive or negative depending on the probability of success of an innovation and its level of appropriability. Also, we find that an increase in the probability of success of innovation does not necessarily speed up investment in R&D.

JEL Classification: C7, D8, O3, K4.

Keywords: real options, intellectual property, R&D, geometric Brownian motion, Stackelberg games.

Introduction

The performance of knowledged-based industries is central to growth in modern economies. Understanding the role of innovation in their investment decisions and how these are affected by institutional and technological aspects protecting appropriability of R&D is, hence, a key topic for economic policymakers. This is particularly true for pharmaceutical, chemical and bio-tech industries where investment in R&D is exceptionally high compared to other industries.

This paper analyses the interaction between competing patents targeting the same Product Market (PM) within a real options framework. Firms compete in producing innovations via R&D and then compete in the PM. The firm that makes the first discovery patents the result and thereby the lead patent has an effect on the probability of innovation of its rival. Understanding this effect, its direction and size, and its implications on PM competition is the central element to our analysis as in turn

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this drives investment in R&D. For instance, the first patent might reduce (and not necessarily to zero) the probability of a second innovation thereby reducing competition in the PM; or despite the design efforts of a lead innovator, the lead patent might increase the probability of rivals' innovation thus intensifying competition in the PM. In turn the effect of appropriability of R&D on PM competition drives R&D investment.

Beside uncertainty in R&D with dependency between patents, which is the new methodological feature of the current study, a second form of uncertainty in our model, very common in the real-option preemption literature, is uncertainty in the PM.[1]

Unlike previous research, by looking at the probabilistic nature of innovations with its correlations to future discoveries, we can analyse a wide spectrum of cases -from innovations with fully blocking patents to those with large spillovers. Earlier related research on R&D races has focussed either on winner-takes-all patent races in which the successful innovator acquires a permanent monopoly in the PM (see, for example, Hsu and Lambrecht (2007), Weeds (2002), Lambrecht (1999)), or on PM competition with no uncertainty in R&D, such as Grenadier[2] (2002) or on independent innovations based on different technologies which can be separately patentable as in Miltersen and Schwartz (2004). Hence, apart from Miltersen and Schwartz, in these past studies, patents are modelled in extreme ways either as providing the holder with full IP protection (i.e., fully blocking competition for the duration of the patent) or as providing no protection at all. Therefore, by considering a varying degree of R&D appropriability one can examine the entire spectrum of cases between these two extremes.

The decision variable of our work is the timing of R&D investment, which we model as a Stackelberg-real-option game. However, this apparently simple decision variable ('when' rather than 'how much' to invest) can explain how apropriability affects the market structure and the speed of innovation.

We find that strong appropriability accelerates R&D investment, thus making discovery more likely to occur at an early stage. This socially desirable outcome must be weighted against the cost of a long lasting monopoly. On the contrary, low appropriability weakens competition in R&D, delaying R&D investment and thus discovery, with the (future) benefit of a more competitive PM though. In general terms, this result contradicts the belief that competition in R&D is positively related to competition in the product market[3].

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[1]In our paper, as in a large number of studies which stem from the seminal work of Smets (1991), PM demand is stochastic.

[2]This paper is a general strategic exercise game of the option to invest in additional capacity and increase output, which, as suggested in the paper, can be applied to model *R&D* investment decisions.

[3]Dasgupta and Stiglitz (1980) look at the effect of PM on R&D (thus, with reverse causality compared to our study) and find that "competition in R&D necessitates imperfect competition in product markets", hence in line with our result.

In other words, we identify a trade off between timing of innovation and degree of competition in the product market, and where a particular product or industry stands along this trade off depends on the level of appropriability of innovations. A welfare analysis goes beyond the scope of our study, however recognising the trade off between timing of innovation and market structure is a necessary step to analyse the welfare implication of IP protection.

An example can help clarifying. Imagine that two identical firms can undertake costly R&D to discover a drug whose final demand depends on the impact of a certain disease. If a wide patent can be obtained upon discovery, even if one of the two firms has already invested in R&D, the prospect of monopolistic revenues (due to the strength of the first patent) can still attract the rival to undergo R&D at an early stage, that is, even if the PM demand is not "booming". Therefore competition in R&D starts early (relatively to the state of the demand) and, with both firms undergoing R&D, discovery is more likely to occur. On the contrary, if the leading patent is weak, then the demand for the drug should be sufficiently high for lower, competitive profits to attract a competitor into R&D investment. Thus, competition in R&D occurs at a later stage and discovery is delayed. In other words, as appropriability narrows, competition in R&D is postponed and, with only a leading firm undergoing R&D for a longer period of time (until the demand is sufficiently high), the probability of discovery decreases.

Secondly, we find that the relation between competition in R&D and competition in the product market is affected by the interaction between the degree of appropriability and the technological challenges of R&D. Our study predicts that, when appropriability is high (thus leading to more persistent monopoly in PM), there are longer gaps between consecutive R&D investments in more challenging innovations and shorter gaps between R&D investments in less challenging innovations; with low appropriability the opposite situation results.

Our approach, based on a two-stage model of competition, seems particularly relevant for high-technology and intensive-knowledge based industries such as pharmaceutical, chemical, bio-tech. In these industries it is common to observe a few patented products of similarly perceived quality competing in the market. Typically, pharmaceutical R&D occurs along these lines, with a very small number of firms targeting a particular therapeutic class and competing in R&D.[4] Patents are normally designed to reduce the number of possible variants (i.e. alternative technologies) which enable production of substitute drugs by competitors. How wide the spectrum of alternative technologies is and how well a patent can carve out part of this spectrum is the main task of a patent agent, whose aim is eventually to reduce (ideally to zero) the probability of substitute discoveries by competitors.

The remainder of the paper is organised as follows. We present the related literature in Section I. Then in Section II, we give an overview of the model and introduce our assumptions. In Section III, we characterize a Stackelberg equilibrium and find

[4] Schweitzer (2007).

an equilibrium solution. Section IV describes the model implications and Section V clarifies some aspects of pharmaceutical innovations and patents, which are in line with the methodology used in our model. In the concluding section we suggest some directions to extend the current work.

I Related literature

There is a vast literature in growth theory on the interaction between PM and R&D competition (see Aghion and Howitt 2011 for a survey), however we limit this section to examine patent races within the real option literature which is methodologically in line with our study.

Our model is closely related to the seminal study of Smets (1991) where, in its simplified version discussed in Dixit and Pindyck (1994, pp. 309-14), two firms face the decision to enter a new market[5] with uncertainty on the demand side (hence, on the returns of the investment). Irreversibility of investment and uncertainty create an advantage to delay entry (the option value of waiting), however a first mover advantage (given by the opportunity of becoming a monopolist until the rival firm enters) creates fear of preemption which speeds up entry. This new dynamic in real option theory, which reduces the option value of waiting, has inspired a growing literature combining real options and preemption games.

Within the preemption literature, closer to our methodology is the study of Weeds (2002), who considers the interaction between two firms facing the opportunity to invest in *R&D* and compete in a winner-takes-all patent race (i.e. on winning the race, the patent holder gains a permanent monopoly). Beside uncertainty over the patent value (which, in line with Smets, evolves according to a geometric brownian motion), Weeds considers technological uncertainty so that, during R&D, discovery is a Poisson arrival[6]. The first innovator patents (without delay and at no cost) and fully preempts the rival, thus becoming a monopolist in the product market. Our study expands the IP setting of Weeds' model and while, after preempting the rival, the leader becomes a monopolist as in Weeds, the duration of the monopoly is given by the level of appropriability of the first innovation. For instance, the first innovation might reduce the competitor's probability of discovering a similar product but still leave opportunities to a competitor to come up with a substitute product and compete with the former leader; in this case the monopoly (however persistent given the negative externality from the first innovation to the second one) will terminate and be replaced by a duopolistic PM. Also, in our setting, appropriability can be very narrow, leading to large positive spillovers which will make a second discovery even more likely than the first one, thus significantly reducing the duration of the leader's

[5]See also Grenadier (1996) where two firms face an 'expansion/development decision' and both firms receive deterministic cash flows prior to expansion.

[6]This assumption is very common in the R&D literature despite the consideration that knowledge tends to cumulate and makes R&D output dependent on time, which particularly complicates the mathematical tractability of real option models.

monopoly.

In other words, compared to Weeds, our model uses a varying degree of IP appropriability rather than one fully blocking patent and by doing this we are able to study the implications of R&D on PM. In modeling PM, we formulate similar assumptions as in Smets regarding a stochastic demands.

Throughout our study we assume full information, however the literature on preemption games has incorporated a number of informational issues which expand the strategy space (as firms conjecture about competitors' entry levels) while explaining different phenomena such as sleeping patents (see the seminal work by Lambrecht, 1999, where patenting and commercialization are distinct choices), positive skewness and jumps in equity returns (as in Lambrecht and Perraudin, 2003, who analyse investment opportunities when competitors' investment costs are private information). The sensitivity of preemption to the information structure has prompted further research in asymmetric information structure such as Hsu and Lambrecht (2007) where fear of preemption of an informationally disadvantaged incumbent triggers inefficiently early investment.

II Model Overview and Assumptions

This section provides an overview of the preemption model set in this paper. We consider two identical firms, each facing the decision to invest in *R&D* by paying an upfront fixed cost. Both firms' R&D is targeted to develop a product in a given market. The first patent obtained by one firm, the lead patent, does not preclude the other firm from obtaining a patent which enables production of a perfectly substitute product. After investment and before discovery by either firm, the probability of making a discovery and obtaining a patent is the same for both firms. However, after discovery by one firm, the lead patent affects the probability of a subsequent innovation by the rival firm.[7] We assume that the patent fee is zero so that a discovery is always immediately patented.[8] When a firm innovates, the product is commercialized either in a monopoly market (if only one firm has innovated) or in a duopoly market (if one innovation is already on the market). Once on the market, each product generates an instantaneous cash flow which depends on the state of the market demand. As common with preemption models[9], the market demand includes a stochastic component which reflects changes in preferences.

We summarize our assumptions and provide additional explanations below.

A1. Two identical firm have a given amount K to invest in *R&D*. The *R&D* investment cost is sunk.

[7]This probability can increase (if there are spillovers from the lead patent) or decrease (if the first patent has ample scope and there are no significant gaps in the patent).

[8]In the remainder, we will use the terms "innovation" and "patent" interchangeably.

[9]See among others Smets (1999), Grenadier (1996, 1999, 2002), Lambrecht (2000), Weeds (2002), Lambrecht and Perraudin (2003), Hsu and Lambrecht (2007), Miltersen and Schwartz (2004).

A2. When both firms are engaged in R&D, the time of discovery of the leader (first firm to invest), T_l, and the follower[10] (second firm to invest), T_f, is distributed according to the Freund extension of the bivariate exponential distribution. Thus, the joint density of T_l and T_f is given by[11]:

$$f(T_l, T_f) = \begin{cases} h_1 h_2 e^{-h_2 T_f - (2h_1 - h_2) T_l} & if \quad 0 < T_l < T_f \\ h_1 h_2 e^{-h_2 T_l - (2h_1 - h_2) T_f} & if \quad 0 < T_f < T_l \end{cases} \tag{1}$$

with $h_1, h_2 > 0$. When instead only the leader is engaged in R&D then T_l has the density of a simple exponential with hazard rate h_1.

A3. The duration of the patent is infinity.

A4. The patenting fee and the production cost are zero.

A5. Patented products are perfect substitutes and share the same market.

A6. Agents are risk neutral; the risk free interest rate is constant and equal to r.

A7. A patented product generates a continuous stream of revenues, with instantaneous level given by

$$p_t D(Q_t) Q_{it}, \qquad \text{for} \quad i = 1, 2 \tag{2}$$

where Q_{it} is the market supply at time t by innovator i and Q_t is the total market supply at time t, i.e. $Q_t = Q_{1t}$ if only one firm has innovated by time t and $Q_t = Q_{1t} + Q_{2t}$ if both firms have innovated by time t. The term $p_t D(Q_t)$ is the inverse demand function with $D(\cdot)$ differentiable and $D'(\cdot) < 0$. p_t is a multiplicative demand shock which evolves according to a geometric Brownian motion,

$$dp_t = \mu p_t dt + \sigma p_t dW_t, \tag{3}$$

where μ and σ are the drift and the volatility of the process respectively and dW_t is a Wiener process. We also assume that $\mu > \sigma^2/2$ and $\mu < r$.

Assumption A7 can be simplified further. Because production costs are zero and the stochastic shock enters the demand in a multiplicative form, the optimal quantity Q_{it}, which maximize the instantaneous revenues, does not depend on p_t[12] as it solves the static problem of maximizing instantaneous revenues. Without loss of generality, we can treat the revenue level, net of shock, as a given parameter and use a more compact and conventional notation (as in Dixit and Pindyck, 1994).

[10]Notice that first firm to invest, the leader, is not necessarily the first firm to innovate.

[11]See Freund (1961).

[12]For instance if production costs were positive, then the optimal quantity produced would depend on the level of p_t and, in turn, on the model equilibrium. The problem would be more complex as shown by Grenadier (2002).

We summarize this consideration in the following lemma.

Lemma 1 *The revenues (net of the shock) $D(Q_t)Q_{it}$, take two values, independent of p_t: i) $D(Q_t)Q_{it} = D_1$ if only one innovator is on the market, or ii) $D(Q_t)Qi_t = D_2$ if both firms innovate and share the market. As demand is downward sloping, monopolistic revenues are larger than duopolistic revenues at the same level of the state variable, p_t, thus D_2 is strictly less than D_1.*

The implication of Lemma 1 is that the optimal revenue level net of shock, $D(Q_t)Qi_t$, is not endogenously determined and can be exogenously assigned after specifying the type of competition and the form of $D(Q_t)$.

Apart from assumption A2, the remaining assumptions are standard in the literature of preemption. For clarity, we discuss in more detail some of our assumptions below.

With assumption A2, when both firms are in the R&D phase, the time of discovery T_l and T_f are dependent in that the first discovery by either firm (leader or follower) changes the hazard rate of the distribution of the other firm from h_1 to h_2. The Freund extension of the bivariate exponential is a new feature proposed in the current paper[13], however the use of the exponential time is very common within the real option literature on R&D[14]. Although, our characterization of the discovery time belongs to the exponential family[15], the Freund extension allows us to capture the idea that the first patent affects the hazard rate of the rival's probability of innovation where h_2 can be greater than equal or less than h_1 and the direction and size of the inequality reflect the amount of positive or negative spillovers (thus our measure of appropriability). Although we are modeling identical firms, the Freund exponential can be used in its more general formulation so that one can accommodate asymmetries between the two firms [16]

Assumption A4 serves to simplify the analysis[17] and avoid introducing further strategic actions, such as when to patent after a discovery, how to reset output when

[13]This distribution is very common in engineering applications and it is often used to model situations where the lifetime of a component is exponential, however it depends on the working status of another component.

[14]See also Loury (1979), Lee and Wilde (1980), Reinganum (1982), Weeds (2002), Miltersen and Schwartz (2004).

[15]Memoryless patent races associated with the exponential distribution are very common in the R&D literature, mainly because time does not enter the model explicitly which generally simplifies the analysis. This is particularly true when also the state of the demand in the product market is stochastic and thus the lack of memory in R&D avoids dependency on the state of the demand, which allows to develop simple time homogeneous models. As an example (not in real-option theory though) of time dependent R&D, see Doraszelski (2003) who augments the hazard rate with a history dependent component so that past R&D learning cumulates in knowledge stock.

[16]For instance, the hazard rates could reflect exogenous differences amongst the two firms, so that one would replace h_1 with h_{1i} and/or h_2 with h_{2i} (with $i = 1, 2$).

[17]Similar assumptions can be found in Weeds (2002) and Miltersen and Schwartz (2004).

the level of p_t changes. In reality the cost of patenting is not zero. However, patent fees reflect the administrative cost of the patent office rather than a share of the patent value and they are generally negligible compared to the R&D investment cost.[18] As mentioned, introducing production costs (were these costs large relative to R&D cost) would add further complexity to the model without changing the qualitative result of our analysis.[19]

Assumption A7 follows Dixit and Pindyck[20] (1994) and serves to identify the advantage of being the first to discover and become a monopolist with larger revenues ($p_t D_1$) than a duopolist ($p_t D_2$).

III Leader-Follower Investment Decision

As standard in dynamic leader-follower models, we assume that one of the two firms (the leader) invests in $R\&D$ by paying a sunk cost K strictly before its rival (the follower). Therefore, as the leader has already invested in R&D, one can use backward induction and solve first the investment decision faced by the follower; then, given the follower's reaction, one can determine the leader's investment decision. In the section below, we briefly characterise a Stackelberg equilibrium and in the next section we use backward induction to solve for the model equilibrium.

A Characterisation and Existence of a Stackelberg Equilibrium

Intuitively one can imagine that an equilibrium is characterised by a set of threshold levels of the state variable, say $\{p_L, p_F\}$, which triggers sequential investment, that is, the leader invests when p_t crosses the level p_L, and the follower afterwards when p_t crosses the level p_F, provided that $p_L \leq p_F$.

However, before providing a formal definition of a dynamic Stackelberg equilibrium, we can examine how the follower's investment decision is affected by the occurrence of a discovery by the leader. As the leader has already invested, when the follower invests in R&D, either the leader has made a discovery or not and thus the follower investment decision at time t depends on the random variable T_l.

More formally, denote the two states, with and without the leader's innovation, by the subscript 1 and 0 respectively. Then, the follower's entry decision, p_F, consists of a pair of threshold levels of the state variable, say p_{F0} and p_{F1}, such that, if no innovation is in place yet (state 0), the follower invests when p_t crosses p_{F0} and, if the leader has already innovated (state 1), the follower invests when p_t crosses p_{F1}. In terms of stopping times the follower's entry decision is characterised as follows.

[18]When patent fees are not negligible relative to R&D cost, we refer the reader to Lambrecht (2000) who analyses the effect of patent fees on R&D investment decisions in a patent race model.

[19]Grenadier (2002) solves a stochastic optimal control problem with a linear cost for increasing output in a Cournot framework.

[20]In particular, see Smets (1991), Grenadier (1996).

Definition 1 *The follower's stopping rule consists of a pair of stopping times:*

$$\tau_F = \begin{cases} \tau_0 & = & \inf\{0 \leq t < T_l : p_t \geq p_{F0}\} \\ \tau_1 & = & \inf\{T_l \leq t : p_t \geq p_{F1}\} \end{cases} \tag{4}$$

This tells us that the stopping time τ_F switches on the occurrence of T_l, that is: if $t < T_l$ the follower invests at time τ_0, otherwise if $T_l \leq t$ the follower invests at time τ_1. This leads to the following proposition

Proposition 1 *The time τ_F is a stopping time if and only if $p_{F0} \leq p_{F1}$*

Proof: A random time τ is a stopping time relative to a Brownian filtration $(\mathcal{F}_t^W)_{t\geq 0}$ if by observing the Brownian trajectory $(W_s)_{0\leq s\leq t}$ up till t we know whether τ has occurred or not. Now, suppose that $p_{F1} < p_{F0}$. The state variable p_t might cross p_{F1} from below without triggering investment at p_{F1} if T_l does not occur before the first passage. This possibility contradicts the definition of τ_1 (that is, investment occurs at the first crossing of p_{F1}). In other words τ_1 is not a stopping time relative to the Brownian filtration. □

We can now characterise a dynamic Stackelberg equilibrium by summarising these considerations.

Definition 2 *A Stackelberg equilibrium consists of a pair of threshold levels of the state variable, $\{p_L, p_F\}$, with $p_F = \{p_{F0}, p_{F1}\}$ which triggers sequential investment, with the leader investing as soon as p_t crosses the level p_L and the follower invests: i) when p_t crosses p_{F0} if the leader has not discovered yet or ii) when p_t crosses p_{F1} if the leader has already made a discovery. The existence of an equilibrium requires: $p_L \leq p_F$ and $p_{F0} \leq p_{F1}$.*

B Follower's Decision

In order to determine the follower's entry thresholds, $\{p_{F0}, p_{F1}\}$, we make the following hypothesis[21].

Hypothesis 1 *There exists a pair of optimal investment triggers p_{F0}, p_{F1} such that $p_{F0} \leq p_{F1}$.*

Below, we derive the follower's investment values in the three regions $\{(0, p_{F0}), (p_{F0}, p_{F1}), (p_{F1}, \infty)\}$ and the thresholds $\{p_{F0}, p_{F1}\}$, which are selected in order to maximise the investment values.

B.1 Investment Values

In this section we determine the values of the investment by the follower in the three regions: $(0, p_{F0})$, $[p_{F0}, p_{F1})$ and $[p_{F1}, \infty)$. Let F_0 and F_1 denote the follower

[21]We show in Lemma 2 at the end of the next sub-section the range of parameters which satisfies hypothesis 1.

investment values in state 0 (where the leader has not discovered yet) and state 1 (where the leader has already discovered) respectively.

We start with the region $[p_{F1}, \infty)$. Because $p_{F0} \leq p_{F1}$ by Hypothesis 1, at any level of $p_t \in [p_{F1}, \infty)$, in either state 0 or 1, the follower invests at once. First consider the follower revenue, say $R(p_{T_f})$, at the time of innovation T_f. We can have either $T_f \leq T_l$ or $T_f > T_l$, thus the revenue equals

$$R(p_{T_f}) = \begin{cases} R_0(p_{T_f}) = \frac{D_1 p_{T_f}}{r-\mu} + \frac{(D_2-D_1)p_{T_f}}{r-\mu}\frac{h_2}{r+h_2-\mu} & if \quad T_f \leq T_l \\ R_1(p_{T_f}) = \frac{D_2 p_{T_f}}{r-\mu} & if \quad T_f > T_l \end{cases} \tag{5}$$

Therefore, the investment values F_0 and F_1 are simply given by[22]

$$F_0(p_t) = E[R_0(p_{T_f})e^{-r(T_f-t)} \mid T_l > t] - K \tag{6}$$
$$F_1(p_t) = E[R_1(p_{T_f})e^{-r(T_f-t)} \mid T_l \leq t] - K \tag{7}$$

We show the result of expectations (6) and (7) in the following proposition.

Proposition 1 *For $p_t \in [p_{F1}, \infty)$ the follower investment values $F_0(p_t)$ and $F_1(p_t)$ are given by:*

$$F_0(p_t) = \frac{p_t}{r-\mu}d_0 - K, \qquad with \;\; d_0 \equiv \frac{h_1}{r+2h_1-\mu}\frac{2D_2h_2 + D_1(r-\mu)}{r+h_2-\mu}, \tag{8}$$

$$F_1(p_t) = \frac{p_t}{r-\mu}d_1 - K, \qquad with \;\; d_1 \equiv \frac{h_2 D_2}{r+h_2-\mu} \tag{9}$$

.

Proof: See Appendix. □

Over the range $[p_{F0}, p_{F1})$, the follower invests at once if the leader has not yet innovated, thus F_0 is given by (8). If instead the leader has innovated, the follower waits to invest until p_t reaches p_{F1}. This is a simple entry decision and we know that F_1 must solve:

$$\frac{\sigma^2}{2}F_1''(p_t)p_t^2 + \mu F_1'(p_t)p_t = rF_1(p_t), \tag{10}$$

where F_1'' and F_1' denote the first and second derivative with respect to p_t.

Over the range $(0, p_{F0})$, F_1 must still solve (10). Therefore, F_1 can be easily found by solving (10) over the entire range $(0, p_{F1}]$. By setting standard boundary and

[22]For sake of brevity we omit the σ-algebra in the expectations. In general, all information on the Brownian motion and the exponential time up to time t should be described by the enlarged σ-algebra $\mathcal{G}_t = \sigma(\mathcal{F}_t \cup \mathcal{H}_t)$, generated by $\mathcal{F}_t$ and $\mathcal{H}_t$, where $\mathcal{H}_t = \sigma(\{T_l \leq s\} : s \leq t)$, and $\mathcal{F}_t$ is the Brownian filtration.

smooth-pasting conditions[23], thus,

$$F_1(p_t) = \left[\frac{p_{F1}}{r-\mu}d_1 - K\right]\left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} \tag{11}$$

with $\lambda_0 = \frac{-(\mu-\sigma^2/2)+\sqrt{(\mu-\sigma^2)^2+2\sigma^2 r}}{\sigma^2}$ and

$$p_{F1} = \frac{\lambda_0}{\lambda_0 - 1}\frac{K}{d_1}(r-\mu) \tag{12}$$

Last, we need to determine F_0 over $(0, p_{F0})$. This can be easily done by noticing that F_0 must solve[24]:

$$\frac{\sigma^2}{2}F_0''(p_t)p_t^2 + \mu F_0'(p_t)p_t + h_1(F_1(p_t) - F_0(p_t)) = rF_0(p_t). \tag{13}$$

This together with boundary and smooth-pasting conditions (see Appendix) leads to the result,

$$F_0(p_t) = F_0(p_{F0})\left(\frac{p_t}{p_{F0}}\right)^{\lambda_1} + F_1(p_{F1})\left[\left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} - \left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0}\left(\frac{p_t}{p_{F0}}\right)^{\lambda_1}\right] \tag{14}$$

where $\lambda_1 = \frac{-(\mu-\sigma^2/2)+\sqrt{(\mu-\sigma^2)^2+2\sigma^2(r+h_1)}}{\sigma^2}$ and p_{F0} is the implicit solution to:

$$\frac{\lambda_1}{\lambda_1 - 1}K = -\frac{\lambda_1 - \lambda_0}{\lambda_1 - 1}F_1(p_{F1})\left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0} + \frac{p_{F0}}{r-\mu}d_0. \tag{15}$$

We summarize the follower values, F_0 and F_1, in the following proposition.

Proposition 2 *The follower's R&D investment value is a piecewise function which takes the following values:*

$$F_0(p_t) = \begin{cases} F_0(p_{F0})\left(\frac{p_t}{p_{F0}}\right)^{\lambda_1} + F_1(p_{F1})\left[\left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} - \left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0}\left(\frac{p_t}{p_{F0}}\right)^{\lambda_1}\right] & if \quad p_t < p_{F0} \\ \frac{p_t}{r-\mu}d_0 - K & if \quad p_t \geq p_{F0} \end{cases}$$
$$F_1(p_t) = \begin{cases} F_1(p_{F1})\left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} & if \quad p_t < p_{F1} \\ \frac{p_t}{r-\mu}d_1 - K & if \quad p_t \geq p_{F1} \end{cases} \tag{16}$$

with p_{F0} and p_{F1} given by (15) and (12) respectively and d_0, d_1 defined in (8) and

[23] Boundaries at 0 and p_{F1} yield $\lim_{p_t\to 0} F_1(p_t) = 0$, $\lim_{p_t \to p_{F1}} F_1(p_t) = \frac{p_{F1}}{r-\mu}d_1 - K$. Smooth-pasting implies $\frac{\partial F_1(p_t)}{\partial p_t}\mid_{p_{F1}} = \frac{\partial p_{F1}d_1/(r-\mu)-K}{\partial p_t}\mid_{p_{F1}}$.

[24] The investment decision here resembles the one in Dixit and Pindyck, 1994, p.305 (an investment decision under policy uncertainty based on a simplified version of Metcalf and Hassett, 1993).

(9).

Notice that $F_0(p_t)$ for $p_t < p_{F0}$ is a weighted average of the expected discounted profits (evaluated at the time of investment in R&D, i.e. at p_{F0} and p_{F1}) conditional on the two states 0 and 1 and the weights correspond to the stochastic discount factors. $Ee^{-r(\tau_0-t)}1_{\{\tau_0<T_l\}}$ and $Ee^{-r(\tau_1-t)}1_{\{\tau_0>T_l\}}$ (see a proof at the end of Appendix).

Lemma 2 *The investment threshold p_{F0} is greater or equal to p_{F1} if and only if $h_2/h_1 \leq D_1/D_2$.*

Proof: The relation $p_{F0} \leq p_{F1}$ holds iff $F_0(p_t) \geq F_1(p_t)$ which occurs when $d_0 \geq d_1$, that is (with d_0 and d_1 given by (8) and (9)), when $h_1 D_1 \geq h_2 D_2$. □

The point of lemma (2) is intuitive as it simply shows that there is an advantage in being the first to discover if the temporary monopoly revenues, which occur with hazard h_1, are larger than the duopoly revenues occurring with hazard rate h_2. Also notice, the range of parameters which verifies Hypothesis 1 is sufficiently broad (given that $D_1/D_2 \geq 1$) to account for spillovers with $h_2 > h_1$. The follower value is depicted in Figure 1.

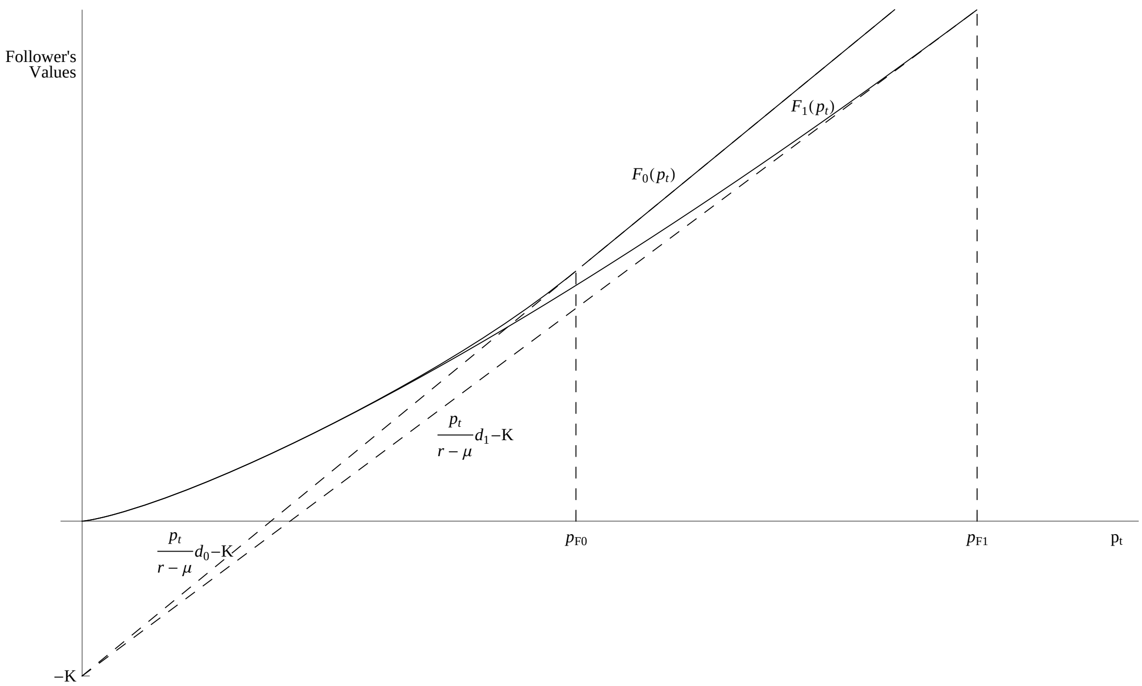

Figure 1: Before the leader discovers, the follower's expected discounted profits are equal to $F_0(p_t)$ with optimal entry trigger p_{F0}. If the leader innovates before the follower invests (that is, when $p_t \leq p_{F0}$), then the follower's value declines from $F_0(p_t)$ to $F_1(p_t)$, with new entry threshold p_{F1}.

C Leader's Decision

As standard in preemption games, suppose that neither firm has invested, and the first firm to invest, the leader, triggers the follower's investment decision analysed in the previous section. Given the follower's reaction, we can determine the leader's investment value, $L(p_t)$, over the two regions $(0, p_{F0})$ and $[p_{F0}, \infty)$. If the leader invests when p_t is in the region $[p_{F0}, \infty)$, the follower will invest at once. Thus, the leader's (revenues and therefore) investment value is equal to the follower's one $F_0(p_t)$.

Next consider the leader's investment region $(0, p_{F0})$. The leader's revenue is affected by the timing of innovation, that is, by whether T_l occurs before or after τ_0. In particular,

1. either $T_l > \tau_0$, i.e. the leader discovers after p_t crosses p_{F0}. In this case, the expected discounted revenues of the leader when p_t hits p_{F0} are the same as the follower's revenue in equation 8), thus we can write the leader's revenue as

$$F_0(p_{F0}) + K = \frac{p_{F0}}{r-\mu} d_0; \tag{17}$$

2. or $T_l \leq \tau_0$, i.e. the leader discovers before p_t hits the threshold p_{F0}. In this case we know that the follower delays his investment decision until p_t hits the new threshold level p_{F1}. By knowing the reaction of the follower, we can write the following lemma.

Lemma 3 *If $T_l \leq \tau_0$ then $T_f > \tau_1$ and the leader's expected revenues at the time of discovery T_l are equal to:*

$$\frac{D_1 p_{T_l}}{r-\mu} + \frac{(D_2 - D_1) p_{F1}}{r-\mu} \frac{h_2}{r + h_2 - \mu} \left(\frac{p_{T_l}}{p_{F1}} \right)^{\lambda_0} \tag{18}$$

Proof: One can compare this with the first of equations 5, the only difference here is that the leader loses its monopoly profits and starts earning duopoly ones at Tf which occurs only after the follower has entered at p_{F1}, therefore the term p_{Tf} in the first of equations 5 is replaced by $p_{F1}(p_{T_l}/p_{F1})^{\lambda_0}$. □

Having derived revenues in the two possible scenarios above, we can write the leader's expected discounted profits when investment in *R&D* takes place as

$$\begin{aligned} L(p_t) = E \Bigg\{ & \frac{p_{F0}}{r-\mu} d_0 e^{-r(\tau_0 - t)} 1_{\{T_l > \tau_0\}} \\ & + \left[\frac{D_1 p_{T_l}}{r-\mu} + \frac{(D_2 - D_1) p_{F1}}{r-\mu} \frac{h_2}{r + h_2 - \mu} \left(\frac{p_{T_l}}{p_{F1}} \right)^{\lambda_0} \right] e^{-r(T_l - t)} 1_{\{T_l \leq \tau_0\}} \Bigg\} - K \end{aligned} \tag{19}$$

We give the result of (19) in the following proposition.

Proposition 3 *The leader's R&D investment value over the region* $(0, p_{F0})$ *is given by*

$$L(p_t) = \frac{p_{F0}}{r-\mu} d_0 \left(\frac{p_t}{p_{F0}}\right)^{\lambda_1} + \frac{p_t}{r-\mu} \frac{D_1 h_1}{r+h_1-\mu} \left(1 - \left(\frac{p_t}{p_{F0}}\right)^{\lambda_1 - 1}\right) + \frac{(D_2 - D_1) p_{F1}}{r-\mu} \frac{h_2}{r+h_2-\mu} \left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} \left(1 - \left(\frac{p_t}{p_{F0}}\right)^{\lambda_1 - \lambda_0}\right) \quad (20)$$

D Leader-Follower Equilibrium

In order to solve for the leaders' investment trigger, p_L, the argument runs as in Dixit and Pindyck (1994). If there is any incentive to be a leader, that is, to invest when the state variable is below p_{F0}, it must be that the payoff from being the leader is greater or equal to the payoff of being the follower. Then, the optimal entry threshold for the leader must be such that at p_L, the leader's payoff is equal or marginally greater than the follower's payoff. This implies that p_L must be the root of the following equation

$$L(p_L) = F_0(p_L). \quad (21)$$

As common in preemption models, the leader's entry p_L has no closed form solution, however, one can easily see that p_L and p_{F0} are the equilibrium strategies of the Stackelberg game as long as:

$$\begin{cases} L(p_t) < F_0(p_t) & \text{for } p_t < p_L, \\ L(p_t) \geq F_0(p_t) & \text{for } p_L \leq p_t < p_{F0}, \\ L(p_t) = F_0(p_t) & \text{for } p_t \geq p_{F0}, \end{cases} \quad (22)$$

The former conditions simply tell us that, for any initial p_t below p_L, both firms prefer to wait as the value of waiting (i.e. being the follower) exceeds that from immediate investment (the value of being the leader); if the initial level of p_t is equal to or greater than p_{F0}, then investment occurs immediately as the value of being the leader is greater than that of being the follower. In this case one firm at random invests and becomes the leader while the other firm waits until p_t reaches either p_{F0} (if the leader does not discover) or p_{F1} (if the leader discovers). Therefore these conditions guarantee that p_L is the first entry threshold. Last, if the initial p_t starts above p_{F0}, the leader and the follower are in a symmetric race and invest both at once (the follower optimal entry trigger is p_{F0}).

Figure 2 shows the equilibrium values of the leader and the follower.

Last, one can notice that if h_2 goes to zero, our equilibrium entry thresholds p_L, p_{F0} approach the leader's and follower's entry triggers in Weeds' model[25] (2002). In this

[25] To be precise, there is a technical -but not substantial- difference between our entry levels and Weed's ones. This is because, unlike Weeds, where the value of the innovation follows a geometric brownian motion, in our model it is the cash flows from the innovation to follow a geaometric

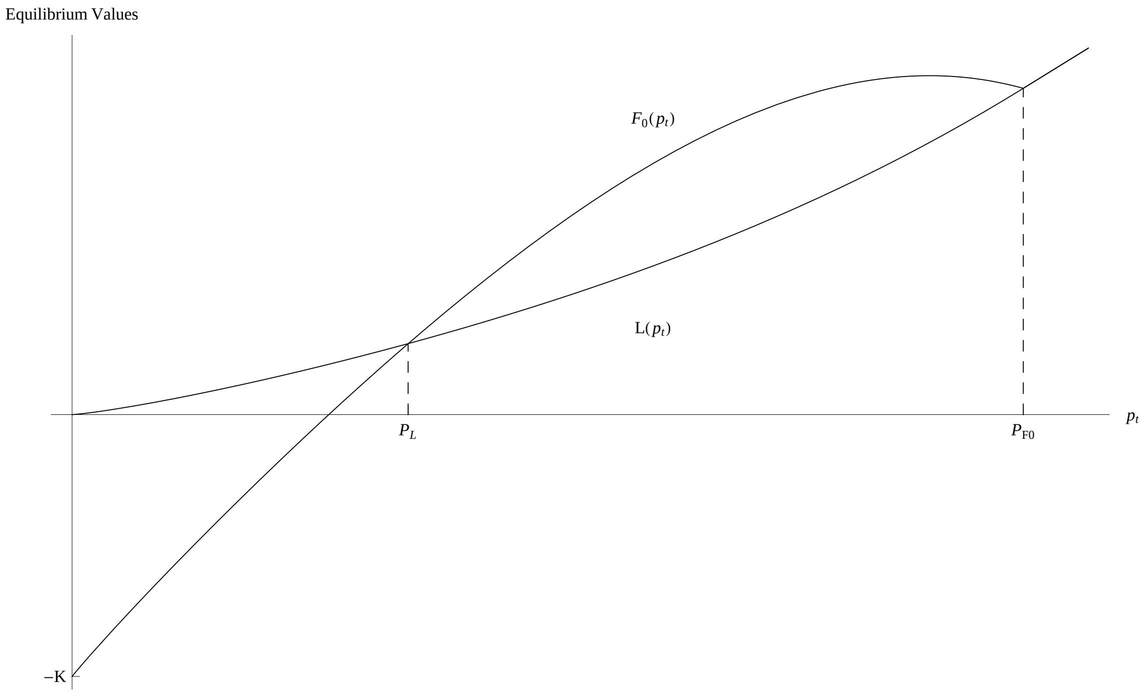

Figure 2: When $F(p_t) > L(p_t)$, both firms prefer to wait rather than investing. If p_t is such that $F(p_t) \leq L(p_t)$ both firms try to invest in $R\&D$. One at random is the first to invest and the other acts then as a follower and delays investment.

limiting case, it is obvious that the follower does not invest if the leader discovers before p_t reaches the level p_{F0} (that is, p_{F1} tends to infinity when h_2 tends to zero).

IV Results

In this section we examine the effect of our parameters on the equilibrium solution $\{p_L, \{p_{F0}, p_{F1}\}\}$ and in particular how appropriability affects the market structure and the speed of innovation. In order to assess the implication of appropriability, we should assess the effect of the first innovation on the rival's innovation by setting $h_2 = f(h_1, y)$ where y is a measure of spillovers from the first innovation. A simple form for f is[26] $h_2 = yh_1$, where the amount of spillovers is measured as a scale factor which reduces or expands the technological opportunities, so that, if $y < 1$, the first innovation reduces the spectrum of possibilities open to a second innovator (negative spillovers, e.g. due to ample scope granted to the first patent), if $y = 1$ the innovations are independent and if $y > 1$ the first innovation generates spill-overs which cannot

brownian motion; we have opted for a cash flow model as it allows us to characterise the interaction between the two firms in the PM.

[26]One can model this function in different ways, according to specific innovations, as long as h_2 increases with spillovers and technological opportunities, that is $\delta f/\delta h_1 > 0$ and $\delta f/\delta y > 0$

be internalized. Also as the existence of an equilibrium requires that $h_2/h_1 \leq D_1/D_2$ (see lemma (2)), thus we set $y \leq D_1/D_2$.

Figure 3 shows the effect of y on the equilibrium entry thresholds p_L, p_{F0} and p_{F1}. Both entry levels p_L and p_{F0} increase with y, while p_{F1} decreases and it equals p_{F0} when $y = D_1/D_2$, that is, the boundary of lemma (2) is met. The reason why p_L increases with y is the following. The first mover advantage consists of the possibility of discovering before the rival invests and earning monopoly revenues at least until the demand reaches the level p_{F1} (when the follower invests). If the leader's innovation spills over to the rival R&D (large y), a second discovery is more likely to occur and the duration of the leader's monopoly is shortened. Therefore low appropriability reduces the first mover advantage and delays the leader's entry threshold (as the preemption threat is weaker).

As to p_{F0}, the argument is more complex, though intuitive. There are two elements driving the value of the follower: 1) if the leader does not discover before p_{F0}, the follower will be competing with the leader on equal terms and the possibility of winning the (symmetric) race and be a temporary monopolist is the prize of the race, 2) if the leader does discover before p_{F0}, the profits of the follower are given by the possibility of producing a second innovation. With regard to point 1) above, a larger y has a detrimental effect as it reduces the price of the race; instead with respect to point 2) profits go up with y as a second innovation becomes more likely (we remind that the hazard rate of the second innovation is $h_2 = yh_1$). Out of these two divergent effects of y on the follower value, the positive effect dominates, that is the follower value increases with y. In response, as it becomes relatively more profitable to imitate via a second innovation than winning the race (this is particularly true if h_1 is large) the follower delays investment, thus p_{F0} increases with y. It is very interesting that the response of $F(p_t)$ to y affects the entry threshold p_{F0} in a direction which is rather uncommon in real option models. Generally, when an entry threshold is set optimally (as is the case [27] with p_{F0}) an increase in value accelerates entry, while in this case and increase in value delays entry. In our model this should not surprise as the convenience to imitate (or come up with an equally successful variant) makes it profitable to delay.

As to p_{F1}, unlike p_L and p_{F0} an increase in y accelerates entry. This is intuitive as larger spillovers increase the hazard rate of the second innovation, which becomes more likely[28].

To summarise, low appropriability (high y) delays investment in R&D as larger spillovers make it worthwhile for a competitor to wait (p_{F0} increases); in turn this reduces the leader risk of preemption (p_L increases) and delays discovery. In other words when firms cannot easily appropriate the returns from their R&D investment,

[27] The trigger p_{F0} maximises the follower value unlike the trigger p_L which is determined by fear of preemption.

[28] Compared to p_{F0}, the optimal entry trigger p_{F1} behaves in a 'standard' way, that is it declines when the investment value is more profitable.

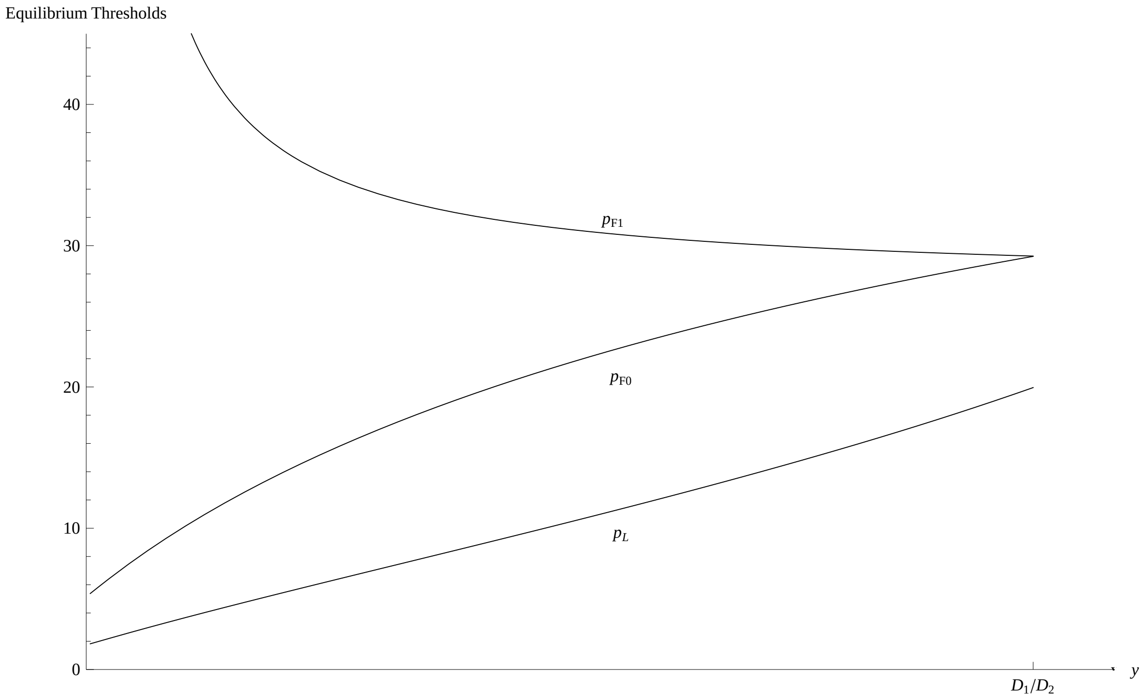

Figure 3: Equilibrium threshold levels, p_L, p_{F0} and p_{F1}, as function of y.

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PM demand should be sufficiently high to make R&D investment worthwhile. On the contrary, with high appropriability (small y) both firms invest in R&D at an earlier stage; this speeds up a first innovation (as both firms start R&D earlier) but leads to a more persistent monopoly -i.e. the social cost for an early innovation. In more general terms, as the independent variable in our setting is the timing of investment, that is, our measure of R&D intensity, the positive relation between the thresholds P_L, P_{F0} and y indicates that there is a positive relation between R&D intensity and PM concentration.

Turning to the effect of h_1 on the equilibrium entry levels, generally all entry triggers decrease when $h1$ increases. However, while p_{F1} continues decreasing for larger h_1, with regards to p_L and p_{F0} there is a range of y values where an increase in h_1 beyond a certain level delays investment. The delaying effect is only marginal for reasonable parameter values and more pronounced on the follower entry threshold p_{F0} than on p_L. Also the delaying effect of $h1$ is stronger for more profitable innovations (i.e. for larger σ, μ, D_1 and smaller r, D_2). We show this result in Figure 4 where the equilibrium thresholds are plotted as functions of h_1 for three levels of y: $y = 0.1$, 1, 1.8. The negative effect of h_1 is due to the fact that beyond a sufficiently high level of h_1 a further increase of h_1 increases the probability of a second discovery more than the probability of a first discovery thus reducing the benefits of innovating first, which in turn delays investment.

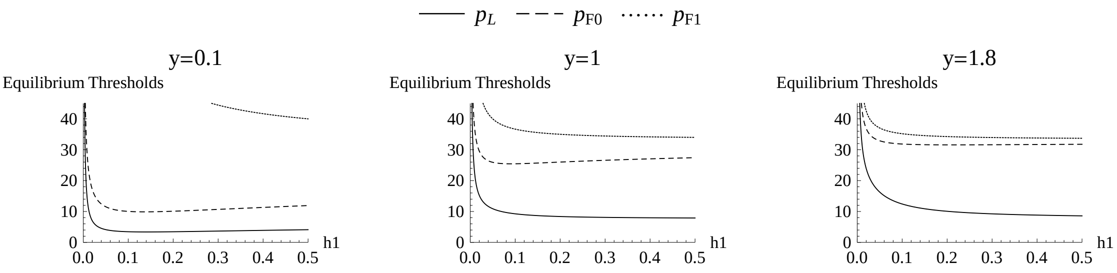

Figure 4: Equilibrium threshold levels, p_L, p_{F0} and p_{F1}, as function of h_1 for different levels of y.

Another interesting implication of the model concerns the time gap between consecutive investments, that is the effect of h_1 on the difference between p_{F0} and p_L. In Figure 5, we plot two sets of equilibrium thresholds for two levels of h_1, $h_1 = 0.004$ and $h_1 = 0.04$ (where the larger h_1 is still sufficiently small so that the increase in h_1 speeds up investment). Interestingly, for very small h_1 the distance between P_L and P_{F0} narrows with the amount of spillovers and instead for larger h_1 it widens with increasing spillovers. In other words, one could say that with regards to industries characterised by high-appropriability, R&D of more challenging innovations would appear more concentrated (as a competitor will invest at or after p_{F0}), while R&D of less challenging innovations would be less concentrated (with smaller time gaps between leader's and competitor's investment). The opposite is true with regards to industries with low appropriability; that is, firms would invest in more difficult R&D shortly after one another, while with respect to less challenging innovations R&D would be more concentrated with a longer time gap between consecutive investments.

In general terms, this result indicates that the relation between competition in R&D and competition in the product market is determined by the joint interaction between h_1 and y. That is, an increase in y can lead to a more or less competitive R&D depending on h_1. This is shown in Figure 6 where the difference $p_{F0} - p_L$ is plotted as function of h_1 and y. While for small levels of h_1 the relation between competition in R&D and PM is positive (as y increases the PM becomes more competitive), the opposite is true for larger h_1.

Therefore the level of appropriability and success rate tell us where a particular industry or a particular product is located on Figure 6 (and thus the speed of innovation and relation between the two stages of competition in R&D and PM). With regards to pharmaceutical, bio-tech and chemical industry, these sectors are characterised by very high appropriability[29] and very low probability of R&D success which, according to our prediction and consistently with empirical finding[30], lead to strong

[29]On the contrary, the computer and software industry are characterised by lower appropriability (see Bessen and Maskin, 2009, also see Fershtman and Markovitch 2010 for a comparison of different R&D technology races and patent regimes).

[30]See next section where we explain in more detail the structure and patenting policy in the

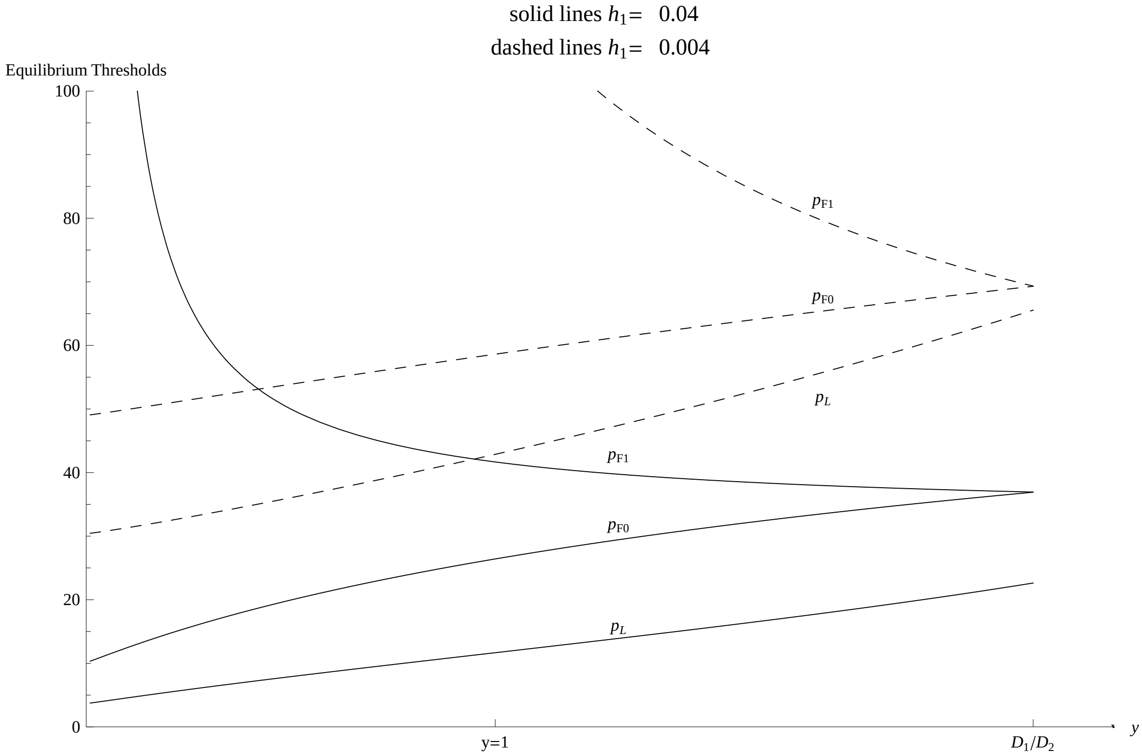

Figure 5: Equilibrium threshold levels, p_L, p_{F0} and p_{F1}, as functions of y for two levels of $h1$: $h_1 = 0.004$ and $h_1 = 0.04$.

market concentration.

Last, we stress that our setting better fits industries, such as the bio-tech, chemical and pharmaceutical, where substitute innovations (i.e. variants) are common and where one innovation becomes one product as opposed to products which embed multiple complementary innovations.[31]

V Pharmaceutical patents

In this section we briefly describe some aspects of pharmaceutical patents which motivate the methodology used in this paper and in particular the relation between the hazard rates of firms competing in R&D.

In the pharmaceutical industry it is common to observe drugs protected by patents competing with other patented drugs in the same therapeutic class. Competition occurs between a very small number of players as, although the industry appears competitive when viewed at aggregate level, concentration is extremely high at ther-

pharmaceutical industry.

[31]These latter types of innovations are common in the computer and software industry where the diffusion of innovation is generally beneficial even when diffusion involves pure imitation (see Bessen and Maskin, 2009).

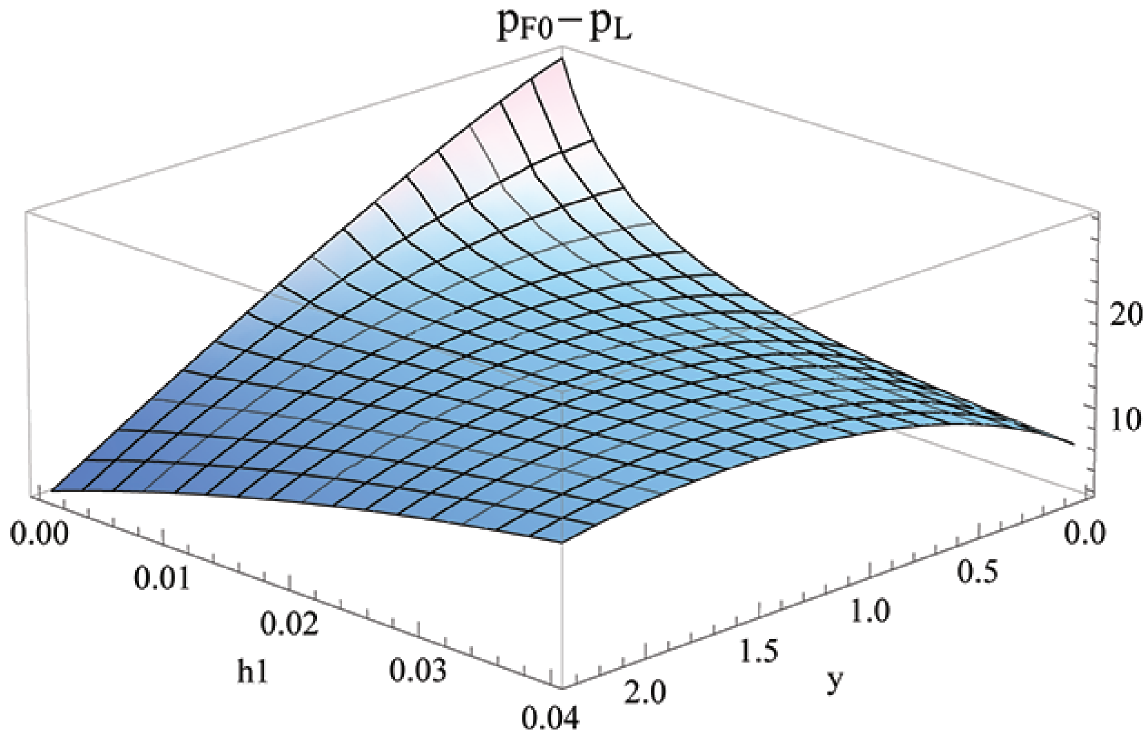

Figure 6: Difference $p_{F0} - p_L$ as function of y and h_1.

apeutic class level, with only a few firms competing within a therapeutic class. For example, according to Schweitzer (2007, pp.26-27) Pfizer and Merck had 8.6% and 4.8% respectively of total prescription sales in 2003, but 50% and 30% of sales in the market of statins in the same year with only four firms accounting for 98% of sales (see also Matraves, 1999 and Schweitzer, 2007).

The bulk of pharmaceutical R&D targeting to creating variants of a drug is so vast that often the industry has been criticized for creating unnecessary variants.[32] The process of creating variants however is far from being a simple one and while variants are broadly perceived as substitute they cannot be thought as pure imitations such as generics which do not involve R&D.[33]

The long and uncertain pharmaceutical R&D heavily relies on patents, generally granted with very ample scope. When a patent for a potential new drug is applied for, the application is written to try and cover as much as possible, far wider than what was actually discovered. So for example, for a new chemical compound with potential drug activity, the applicant will try and cover in a generalized way as many compounds as they think might be active, even if only a few compounds that show activity

[32]Sneader (2005).

[33]As Sneader (2005) points out "It may be easy to discover a substitute drug, but extremely difficult to discover one which is safe".

have been tested. The generalized structure (called Markush formulae) contains a very large number of compounds to allow for variants and the patent agent's job is to try and be as innovative as possible and cover hundreds to billions of new chemical structures. The purpose is to carve out a great area in which the patentee can investigate related structures without a competitor taking opportunities away. Nevertheless, the patentee can miss some combination of compounds which leaves potential gaps for a competitor to investigate.

However, whether a patent is effective ultimately depends on technological possibilities. If a newly patented drug can be altered by changing part of the structure whilst still retaining (or preferably enhancing) the activity, it is possible for competitors to succeed in R&D. If instead it is difficult to alter the drug without losing activity (and if the original patentee has claimed the key basic structure and covered a large number of other substituent patterns), then a competitor can only succeed by finding gaps in the patent thicket. If there are no gaps, the competitor has to look for completely different and novel structures, as many of the compounds in earlier drug cannot be used as a lead compound.

For example, with small molecule drugs (also known as chemical drugs) the patent family will probably cover a vast amount of chemical structures with little space for a second company to develop a similarly chemically structured product, but, as mentioned, there are potential gaps. For example Astra Zeneca developed omeprazole (Losec), a new compound which had a new mechanism of action (proton pump inhibitor), and widely patented related chemical structures. However Takeda found a few gaps in the Markush formula[34] and obtained lansoprazole -a similar structure to omeprazole- and other compounds such as pantoprazole and rabeprazole were separately patented by other companies. Whilst AZ did exceedingly well from omeprazole, competitors did very well with their related drugs, though they came on the market somewhat later.

When instead the focus is on a new biological action rather than chemical structure and the receptor might accommodate different chemical structures, it may be that there are potentially many different compounds that could work. For example cimetidine by SKF, which was regarded as the breakthrough antiulcer drugs H2 antagonist (sold as Tagamet), was a great success, but Glaxo came up with a different chemical compound which had the same effect and marketed ranitidine as Zantac, which was even more successful than Tagamet even though it was second on the market[35]. The statins are another example of patent spillovers, where an initial discovery spawned related chemical structures, so that competitors could substitute part of the structure of the earlier statin to mimic and improve activity. These new compounds

[34]This particular drug worked by in-vivo rearrangement of the compound and competitors had to look for gaps in the Markush formula as they needed the same basic structure for it to work.

[35]With cimetidine, SKF covered a vast amount of structures around cimetidine, but Glaxo discovered that a similar structure but with a different hetero-ring also worked. The Glaxo ring wasnt covered by SKF and hence Zantac (ranitidine) was separately patented.

are new chemicals and are different enough to be separately patentable.

These examples show how the design of a lead patent aims at reducing the probability of rivals' innovations. However, despite the design efforts of a lead innovator, it might occur that the lead patent increases the probability of rivals' innovation. In order to incorporate these features we have i) made the probability of success of a rival's R&D dependent on the occurrence of a lead innovation and ii) allowed the hazard rate of a second innovation to increase or decrease thus incorporating positive or negative spillovers.

VI Conclusion

We have developed a model which builds on Weeds' (2002) study of preemption extending it to model innovations by lead and follower innovators (with dependence between the two) using the Freund extension to the the bivariate exponential distribution. In the past studies, patents are modelled in extreme ways either as providing the holder with IP protection (i.e., blocking competition fully at least for the duration of the patent) or as providing no protection. In our model, we examine intermediate cases by assuming that patents may be more or less effective in protecting IP. By characterising the effect of a lead patent on the probability of a future discovery by a rival, we are able to examine the effect of *R&D* investment on the product market.

We have found that low appropriability (large spillovers) delays R&D investment and innovation, leading to less persistent monopoly, quickly replaced by competition between substitute innovations. Analysing the implication of appropriability on welfare would be a relevant extension of this study.

Furthermore, the interaction between the (ex-ante) probability of success in R&D and the amount and direction of spillovers (positive or negative) has an effect on the gap between consecutive R&D entry thresholds. This can help to better understand certain industries, for instance, why some innovations trigger sudden interest by a number of competitors after a long period of inactivity while other innovations are lead by one innovator for a long period of time before a rival invests in R&D. In this regard, it might be worth extending the model to increase the number of firms.

Also, the inclusion of production costs or a lump sum commercialization cost (which would not add excessive complexity) can be of particular relevance in certain markets, such as biologics, where the production phase is generally very expensive.

Last, the model can be easily extended to account for asymmetries between competitors, regarding investment costs for instance or asymmetries in R&D which can be dealt with by using the Freund extension in its generalised formulation (with a different ex-ante and ex post hazard rates for each innovator).

A Appendix with Proofs

Proof of Proposition 1

Follower' expected profits if $t < T_l$: $F_0(p_t)$ The follower expected profits if the leader has not innovated, that is $F_0(p_t)$ at $t < T_l$, from equation 16 and 5 can be written as the sum of two expectations, that is:

$$\begin{aligned}
F_0(p_t) &= E[R(p_{T_f})e^{-r(T_f-t)} \mid T_l > t] - K \\
&= E[\frac{D_2 p_{T_f}}{r-\mu} e^{-r(T_f-t)} 1_{\{T_f > T_l\}} \mid T_l > t] + \\
&+ E\left[\left(\frac{D_1 p_{T_f}}{r-\mu} - \frac{(D_1 - D_2)p_{T_f}}{r-\mu}\frac{h_2}{r+h_2-\mu}\right) e^{-r(T_f-t)} 1_{\{T_f \le T_l\}} \mid T_l > t\right] - K, \quad (23) \\
&= \frac{p_t}{r-\mu} d_0 - K, \qquad \text{with} \quad d_0 \equiv \frac{h_1}{r+2h_1-\mu}\frac{2D_2 h_2 + D_1(r-\mu)}{r+h_2-\mu} \quad (24)
\end{aligned}$$

If $T_f > T_l$ the joint density of the Freund bivariate exponential is $f(T_l, T_f) = h_1 h_2 e^{-h_2(T_f - T_l) - 2h_1(T_l - t)}$ and the first expectation in equation 23 solves as follows:

$$\begin{aligned}
& E\frac{D_2 p_{T_f}}{r-\mu} e^{-r(T_f-t)} 1_{\{T_f > T_l\}} = \quad (25) \\
&= \int_t^\infty \int_{T_l}^\infty \frac{D_2 p_{T_f}}{r-\mu} e^{-r(T_f-t)} h_1 h_2 e^{-h_2(T_f-T_l)} dT_f e^{-2h_1(T_l-t)} dT_l \quad (26) \\
&= h_1 h_2 \frac{D_2 p_t}{r-\mu} \int_t^\infty \int_{T_l}^\infty e^{(\mu-r-h_2)(T_f-T_l)} dT_f e^{-(2h_1-h_2)(T_l-t)} dT_l \quad (27) \\
&= h_1 h_2 \frac{D_2 p_t}{r-\mu} \int_t^\infty \frac{e^{(\mu-r-2h_1)(T_l-t)}}{r+h_2-\mu} dT_l \quad (28) \\
&= \frac{h_1 h_2 D_2 p_t}{(r-\mu)(r+h_2-\mu)(r+2h_1-\mu)}. \quad (29)
\end{aligned}$$

If $T_f \le T_l$ the joint density of the Freund bivariate exponential is $f(T_l, T_f) = h_1 h_2 e^{-h_2(T_l - T_f) - 2h_1(T_f - t)}$ and the second expectation in equation 23 solves similarly,

$$\begin{aligned}
& E\left(\frac{D_1 p_{T_f}}{r-\mu} - \frac{(D_1 - D_2)p_{T_f}}{r-\mu}\frac{h_2}{r+h_2-\mu}\right) e^{-r(T_f-t)} 1_{\{T_f \le T_l\}} \quad (30) \\
&= \left(D_1 - (D_1 - D_2)\frac{h_2}{r+h_2-\mu}\right) \int_t^\infty \frac{p_{T_f}}{r-\mu} e^{-r(T_f-t)} \int_{T_f}^\infty h_1 h_2 e^{-h_2(T_l-T_f)} dT_l e^{-2h_1(T_f-t)} dT_f,
\end{aligned}$$

where the double integral rearranges as follows,

$$\int_t^\infty \frac{p_{T_f}}{r-\mu} e^{-r(T_f-t)} \int_{T_f}^\infty h_1 h_2 e^{-h_2(T_l-T_f)} dT_l e^{-2h_1(T_f-t)} dT_f \tag{31}$$

$$= \frac{p_t}{r-\mu} h_1 h_2 \int_t^\infty e^{(\mu-r)(T_f-t)} \int_{T_f}^\infty e^{-h_2(T_l-t)} dT_l e^{-(2h_1-h_2)(T_f-t)} dT_f \tag{32}$$

$$= \frac{p_t}{r-\mu} \frac{h_1 h_2}{h_2} \int_t^\infty e^{(\mu-r-2h_1)(T_f-t)} dT_f \tag{33}$$

$$= \frac{p_t h_1}{(r-\mu)(r+2h_1-\mu)}. \tag{34}$$

Taking the sum of revenues in equation 29 and equation 30 (with the integrals given by equation 34) minus cost K yields

$$F_0(p_t) = \frac{D_2 p_t}{r-\mu} \frac{h_1 h_2}{(r+h_2-\mu)(r+h_1+h_2-\mu)} + \frac{p_t}{r-\mu} \frac{h_1}{r+h_1+h_2-\mu} \left(D_1 + \frac{(D_2-D_1)h_2}{r+h_2-\mu} \right) - K \tag{35}$$

which rearranges as

$$F_0(p_t) = \frac{p_t}{r-\mu} \frac{h_1}{r+2h_1-\mu} \frac{2D_2 h_2 + D_1(r-\mu)}{r+h_2-\mu} - K \tag{36}$$

Follower' expected profits if $t > T_l$: $F_1(p_t)$. From 5, the follower expected revenues at time $t > T_l$ when the leader has already innovated, $F_1(p_t)$, is given by:

$$F_1(p_t) = \; = E[R(p_{T_f}) e^{-r(T_f-t)} \mid T_l < t] - K \tag{37}$$

$$= E\left[\frac{D_2 p_{T_f}}{r-\mu} e^{-r(T_f-t)} \mid T_l \leq t \right] - K \tag{38}$$

where the expectation can be rearranged as follows,

$$E\left[D_2 \frac{p_{T_f}}{r-\mu} e^{-r(T_f-t)} \mid t > T_l \right] = \tag{39}$$

$$= D_2 \frac{p_t}{r-\mu} \int_t^\infty e^{-r(T_f-t)} h_2 e^{-h_2(T_f-t)} dT_f \tag{40}$$

$$= D_2 \frac{p_t}{r-\mu} \frac{h_2}{r+h_2-\mu} \tag{41}$$

Proof of equation 14

For $p_t \in (0, p_{F0}]$ the time to a first discovery has a simple exponential density (because the follower has not invested yet). Therefore, the problem is a simple entry decision where the occurrence of a Poisson event with hazard rate h_1 changes the investment payoff; thus, we know that (see Dixit and Pindick (1994, p.305 where a more elaborate version of this problem is explained) F_0 must satisfy:

$$\frac{\sigma^2}{2}F_0''(p_t)p_t^2 + \mu F_0'(p_t)p_t + h_1(F_1(p_t) - F_0(p_t)) = rF_0(p_t). \tag{42}$$

By setting:

$$F_0(p_t) = Ap_t^\alpha + Bp_t^\beta \tag{43}$$

equation 42 becomes:

$$\sigma^2/2[\alpha(\alpha-1)Ap_t^{\alpha-2} + \beta(\beta-1)Bp_t^{\beta-2}]{p_t}^2 + \tag{44}$$
$$+\mu(\alpha A{p_t}^{\alpha-1} + \beta B{p_t}^{\beta-1})p_t + h_1F_1(p_{F1})\left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} = (r+h_1)(Ap_t^\alpha + Bp_t^\beta)$$

which yields the following system of equations:

$$\sigma^2/2\alpha(\alpha-1) + \mu\alpha - (r+h_1) = 0 \tag{45}$$
$$\sigma^2/2\beta(\beta-1)B{p_t}^\beta + \mu\beta B{p_t}^\beta + h_1F_1(p_{F1})\left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} = (r+h_1)Bp^\beta. \tag{46}$$

The above system with usual boundary conditions, $\lim_{p_t\to 0} F_0(p_t) = 0$ and $F_0(p_{F0})$ given by equation 8 gives the following constants:

$$\alpha = \frac{-(\mu-\sigma^2/2) + \sqrt{(\mu-\sigma^2)^2 + 2\sigma^2(r+h_1)}}{\sigma^2} = \lambda_1 \tag{47}$$
$$\beta = \frac{-(\mu-\sigma^2/2) + \sqrt{(\mu-\sigma^2)^2 + 2\sigma^2(r+h_1)}}{\sigma^2} = \lambda_0 \tag{48}$$
$$A = \left(F_0(p_{F0}) - F_1(p_{F1})\left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0}\right)\left(\frac{1}{p_{F0}}\right)^{\lambda_0} \tag{49}$$
$$B = F_1(p_{F1})\left(\frac{1}{p_{F1}}\right)^{\lambda_0} \tag{50}$$

Substituting for the constants into equation 43, one obtains

$$F_0(p_t) = \left(F_0(p_{F0}) - F_1(p_{F1})\left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0}\right)\left(\frac{p_t}{p_{F0}}\right)^{\lambda_0} + F_1(p_{F1})\left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} \tag{51}$$

which after some simple algebra rearranges into equation 14.

Stochastic discount factors

The following expectation rearranges:

$$\begin{aligned}
& Ee^{-r(\tau_0-t)}1_{\{\tau_0<T_l\}} && (52)\\
= \;& E[e^{-r(\tau_0-t)}E(1_{\{\tau_0<T_l\}} \mid \tau_0)] && (53)\\
= \;& E[e^{-r(\tau_0-t)}Pr(\tau_0 < T_l \mid \tau_0)] && (54)\\
= \;& Ee^{-(r+h_1)(\tau_0-t)} && (55)\\
= \;& \left(\frac{p_t}{p_{F0}}\right)^{\lambda_1} && (56)\\
& \text{with} \quad \lambda_1 = \frac{-(\mu-\sigma^2/2)+\sqrt{(\mu-\sigma^2)^2+2\sigma^2(r+h_1)}}{\sigma^2}. && (57)
\end{aligned}$$

Similarly, we can rearrange the expectation below as follows (the algebra below requires using the independence property of non-overlapping stopping times[36] ($\tau_1 - \tau_0$ and $\tau_0 - t$).):

$$\begin{aligned}
& Ee^{-r(\tau_1-t)}1_{\{\tau_0>T_l\}} = && (58)\\
= \;& Ee^{-r(\tau_1-\tau_0)}e^{-r(\tau_0-t)}1_{\{\tau_0>T_l\}} && (59)\\
= \;& Ee^{-r(\tau_1-\tau_0)}Ee^{-r(\tau_0-t)}1_{\{\tau_0>T_l\}} && (60)\\
= \;& \left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0} Ee^{-r(\tau_0-t)}1_{\{\tau_0>T_l\}} && (61)\\
= \;& \left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0} Ee^{-r(\tau_0-t)}(1-e^{h_1(\tau_0-t)}) && (62)\\
= \;& \left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0}\left(\frac{p_t}{p_{F0}}\right)^{\lambda_0} - \left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0} Ee^{-(r+h_1)(\tau_0-t)} && (63)\\
= \;& \left(\frac{p_t}{p_{F1}}\right)^{\lambda_0} - \left(\frac{p_{F0}}{p_{F1}}\right)^{\lambda_0}\left(\frac{p_t}{p_{F0}}\right)^{\lambda_1} && (64)
\end{aligned}$$

[36]the Markov property of a Brownian motion extends to stopping times relative to a Brownian filtration.

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