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Abstract

We derive a golden rule for the level of life-extending health care when the utility of the old depends not only their level of consumption but also on the number of old people alive. While previous work has emphasized the negative pecuniary externality from longevity, we derive the effect of the positive non-pecuniary externality of being able to consume with other members of one's cohort.

1. Introduction

Consumption requires not only time but often good company. The experience of having a meal at a restaurant, going to the theatre, playing golf or taking a vacation depends on the company one has; being alone, with one partner or in a group of people. Old age brings loneliness for many individuals lucky enough to enjoy longevity. The loss of a spouse, relatives and friends brings both grief as well as having fewer people to socialize with. In this paper we incorporate this insight into an overlapping-generations model and show that it can justify spending more resources on extending lives than the individuals themselves would have chosen to do.

A large literature documents the effect of loneliness on the well-being of elderly individuals. Heikkinen et al. (1995) find that loneliness affects health in old age and Green et al. (1992) find that loneliness is correlated with depression. Singh and Misra (2009) used questioners for a sample of 55 elderly people in the age group of 60-80 years and find a significant positive correlation between loneliness and depression for both men and women.[1]

The decision on how to allocate resources between the generally healthy working-age population and expending them on health care for the old is one of the most important taken by society. In a previous paper (Gestsson and Zoega, 2018) we extended the work of Hall and Jones (2007) by deriving the socially optimal level of life-extending health care where the marginal benefit in terms of the utility yielded by each additional year of life was set equal to the marginal cost of such health care that consisted of lower consumption of the working-age population. Our analysis showed that in a private market individuals overinvest in life-extending health care by ignoring the pecuniary negative effect their own longevity has on the income of other retirees by lowering the rate of return on annuities.[2] The question

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[1] The used the UCLA loneliness scale, see Russell et al. (1980) and the Beck depression inventory, see Beck et al. (1961).
[2] See Blanchard (1985) and Heijdra et al. (2011)

concerning the optimal level of health care spending is closely related to the golden rule literature that started with Ramsey (1928) on the optimal level of saving and was extended to the optimal level of research and the optimal level of education (see Phelps, 1966 and 1968, among others). Here, investing in future research ideas, education, or the physical capital stock requires a reduction in current levels of consumption, while more capital, better technology and better education will increase future consumption. When it comes to life extension the question is how much consumption each generation of working-age should sacrifice in order to invest in longevity, which brings more years spent in retirement, hence more retirement consumption.

There is also a literature on the optimal size of the population in philosophy using the utilitarian approach where the optimal size of the population can be determined as the one maximizing total utility – the product of the number of people alive and their personal utility levels. This raises philosophical dilemmas discussed extensively in the literature because an increase in the number of people alive that is accompanied by lower consumption and utility of each of them may increase societal utility by making each person worse off. This gives rise to Parfit's (1984) *Repugnant Conclusion*, according to which a large increase in the number of people alive can increase total utility even though each person's life is barely worth living. Here population size substitutes for average utility and it is the quality-quantity substitution that is repugnant. A related problem is Methuelah's Paradox, see Cowen (1989). Here a longer life welfare dominates a shorter life even though the level of consumption in each year is lower. Thus a very long life during which the individual finds his life almost not worth living is preferred to a shorter live of higher utility per year. We will get around these paradoxes by assuming diminishing marginal utility and diminishing marginal productivity of health care spending.

Andersen and Bhattacharya (2015) show how efficiency considerations can justify public investment in health. First, assuming exogenous mortality risk, they show how publicly-funded health spending on the young can improve welfare if the real rate of interest higher than the rate of population growth. The reason is that the social opportunity costs of public investment in the health care of the young equals the rate of the population growth while the private costs equals the rate of interest.[3] Second, they show that the young may also underinvest in health when mortality risk is endogenous in the presence of mortality-contingent claims. Here public provision of health care lowers the interest rate on life annuities because of reduced mortality risk, which helps the young if they are net-borrowers.

In this paper we add an important element missing from this literature that may offset the negative pecuniary effect of longevity, which is the utility gained in old age from having more old people around to enjoy consumption with. In essence, we make the utility of consumption in old age depend on the company of others. Thus eating in a restaurant is (usually!) more enjoyable in the company of other people and the same applies to many other activities such as playing golf or travelling. The inclusion of this element into our social planner's problem has the added effect of possibly reversing the result that individuals tend to overinvest in health care. We formulate our analysis in a simple model used in our earlier paper as (Gestsson and Zoega, 2018) well as the Hall and Jones (2007) one.

2. A simple model

Imagine a social planner who decides on the optimal steady state in terms of the number of old people alive, consumption per young individual, consumption per old individual and life-extending medical expenditures. He faces a trade-off between having more consumption while young and shorter lives, on the one hand, and having lower consumption and longer

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[3] This result mirrors the standard result that a pay-as-you-go pension system is welfare reducing in a dynamically efficient economy.

lives, on the other hand. We then compare his decision to that of a private individual making a decision on the resources allocated to health care with only his own interests in mind.

Now denote the age of death by A, the age of retirement by R, consumption during working age by c_w, consumption during retirement by c_o and the flow of medial expenditures per old individual by τ. We assume that utility when young is $u(c_w)$, where $u'(c_w) > 0$ and $u''(c_w) < 0$, while utility when old depends on the number of people alive;

$$v(c_o, A - R) \tag{1}$$

where $v_1 > 0$, $v_{11} < 0$, $v_2 > 0$, $v_{22} < 0$ and $v_{12} > 0$. The rationale for the inclusion of A-R in the utility function is that the older generation enjoys both the company of others and also values consumption more when in company with other retirees. Examples include going to restaurants with friends, playing golf and vacation time. Clearly, all of those activities would be less enjoyable when consumed in solitude. The younger generation is less concerned about having company since the number of working-age individuals can be taken for granted because everyone, by assumption, reaches retirement age. The total utility of those currently alive can be written as

$$Ru(c_w) + (A - R)v(c_o, A - R) \tag{2}$$

Utility depends on consumption when young and old and also on the number of old individuals. The social planner can affect longevity, hence also the number of old people alive at each point in time, by spending more on health care τ. The relationship between longevity and health care expenditures are given by

$$A = \gamma B(\tau) \tag{3}$$

where $B'(\tau) > 0 \ and \ B''(\tau) < 0$ and $\gamma > 0$ is a measure of productivity in health care provision. The resource constraint facing the social planner is the following;

$$R(y - c_w) = (A - R)(c_o + \tau) \tag{4}$$

where the left-hand side has the difference between the output of the working-age individuals and their consumption – the surplus output available to the retired older generations – and the right-hand side has the consumption and medical expenditures of the retired population.

The Lagrangian can be written as follows:

$$\Gamma = Ru(c_w) + (\gamma B(\tau) - R)v(c_o, \gamma B(\tau) - R) + \lambda[R(y - c_w) - (\gamma B(\tau) - R)(c_o + \tau)] \quad (5)$$

The first-order conditions with respect to consumption while young, c_w, consumption while old, c_o, and medical expenditures τ follow. We start with the first-order condition with respect to consumption of the working age individuals,

$$\frac{\partial \Gamma}{\partial c_w} = Ru'(c_w) - \lambda R = 0$$

or

$$u'(c_w) = \lambda \quad (6)$$

which sets the marginal utility of consumption equal to the shadow price of wealth. The first-order condition for the consumption of the old is similar except that marginal utility depends on the number of retirees

$$\frac{\partial \Gamma}{\partial c_o} = (\gamma B(\tau) - R)v_1(c_o, \gamma B(\tau) - R) - \lambda(\gamma B(\tau) - R) = 0$$

or

$$v_1(c_o, A - R) = \lambda \quad (7)$$

Combining the (6) and (7) gives,

$$u'(c_w) = v_1(c_o, A - R) \quad (8)$$

It follows that the more people are alive after retirement age, the higher is the level of consumption of each retired individual due to $v_{12} > 0$ and the lower is the level of consumption of each working-age individual.

We now come to the first-order condition with respect to spending on health care τ:

$$\frac{\partial \Gamma}{\partial \tau} = \gamma B'(\tau)[v(c_o, \gamma B(\tau) - R) + (\gamma B(\tau) - R)v_2(c_o, \gamma B(\tau) - R)] - \lambda \gamma B'(\tau)(c_o + \tau)$$

$$- \lambda(\gamma B(\tau) - R) = 0$$

Or using (6):

$$\gamma B'(\tau)[v(c_o, A - R) + (A - R)v_2(c_o, A - R)] = u'(c_w)[\gamma B'(\tau)(c_o + \tau) + (A - R)] \quad (9)$$

The left-hand side has the marginal benefit of spending more on health care. The first term on the left is the utility of those who survive due to the extra expenditures and would have died without them. Secondly, we have the increased utility of those who would have lived in the absence of the increased expenditures but now have more old people to socialize with. The right-hand side then has the marginal cost. The increased costs stem from the utility loss of the working-age population due to the consumption and medical care of the individuals whose lives are extended and also the higher medical costs of the old individuals who would have survived in the absence of increased medical care.

The difference between the social and the private optimum lies in the second term on the left-hand side and the first term on the right-hand side. The first term on the right-hand side is ignored by the private individual who is deciding on his own level of old-age health care because he can rely on the rest of society – annuities in Gestsson and Zoega (2018) – to sacrifice consumption in order to provide him with consumption and health care for the extension of his life. This omission will make him spend more on health care and live longer than the social planner would dictate. But there is also a term on the left-hand side of the equation that he ignores, which is the positive external effect he has on the utility of other old individuals who can now enjoy his company. This will make him underinvest in life-extending health care compared to the social planner. It follows that the private optimum is where

$$\gamma B'(\tau)v(c_o, A - R) = u'(c_w)(A - R) \tag{10}$$

There will only be an overinvestment in health care if

$$u'(c_w)\gamma B'(\tau)(c_o + \tau) > \gamma B'(\tau)(A - R)v_2(c_o, A - R) \tag{11}$$

that is the marginal cost omitted by the private individual exceeds the omitted marginal benefit.

Finally, there is the first-order condition with respect to the Lagrange multiplier:

$$\frac{\partial \Gamma}{\partial \lambda} = R(y - c_w) - (\gamma B(\tau) - R)(c_o + \tau) = 0 \tag{12}$$

For the first-order conditions to represent a maximum the bordered it is sufficient that the Hessian matrix is negative semi-definite. See Appendix I.

3. An overlapping-generations model

We now formulate our insight in a more rigorous model that has overlapping generations in continuous time. We use the model, based on Andersen and Gestsson (2016) and Gestsson and Zoega (2018), to both solve the social planner's problem as well as the private individual's life-time utility maximisation.

Demographics

The population at time t is split into two groups: those working (young), whose ages are between 0 and R ($a \in [0, R]$), and those retired (old), whose ages are between R and A ($a \in (R, A]$), where R is the retirement age and A is the maximum age or longevity. Because the main concern of this paper are the effects of health care expenditure for the elderly, it is assumed that survival probabilities are constant and equal to one when an individual is young, and decreasing and concave in age when he is old. Hence the survival probabilities are:

$$m(a, A) = \begin{cases} 1 \\ f(a, A) \end{cases} if \; \begin{matrix} a \in [0, R] \\ a \in (R, A] \end{matrix} \tag{13}$$

where $\lim_{a \to R^+} f(a, A) = 1$ and $f(A, A) = 0$, which implies that $0 \leq f(a, A) < 1$ must hold

for all $a \in (R, A]$. Further, it is assumed that $f(a, A)$ is strictly decreasing and strictly

concave in a and strictly increasing in A:

$$\frac{\partial f}{\partial a} < 0, \frac{\partial^2 f}{\partial a^2} < 0, \frac{\partial f}{\partial A} > 0$$

The following figure shows the survival function:
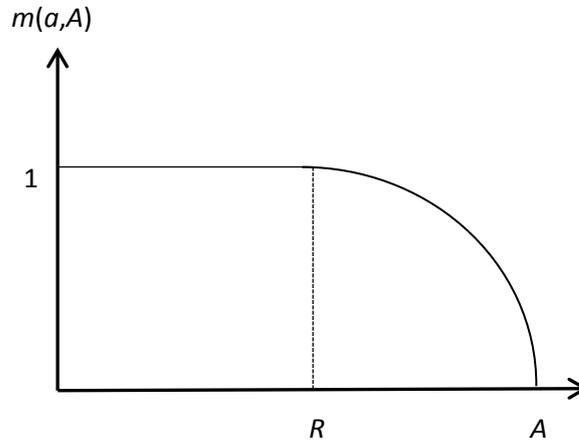
Note that although longevity is uncertain for a given individual the fraction of individuals

reaching a certain age is deterministic for the social planner.

Following Boucekkine *et al.* (2002), the number of individuals born is assumed to grow at

a constant rate n, which affects the age distribution in steady state. The number of individuals

born at time t is φe^{nt}, where $\varphi > 0$. The number of individuals aged a at time t is therefore:

$$l(t, n, a, A) = \varphi e^{n(t-a)} m(a, A) > 0 \tag{14}$$

Note that $l(t, n, 0, A) = \varphi e^{nt} > 0$, $l(t, n, A, A) = 0$ and $\frac{\partial l}{\partial a} < 0$; the number of individuals

born at time t is φe^{nt}, the number of individuals exceeding the maximum age A is zero, and

the number of individuals aged a is strictly decreasing in a for all $a \in (R, A]$, t and A. In

addition, $\frac{\partial l}{\partial A} > 0$ for all $a \in (R, A]$, implying that the number of old individuals at each age

level $a \in (R, A]$ is increasing in longevity. Using (14), the population mass in the economy at

time t can be written as

$$N(t, n, A) = \int_{a=0}^{R} l(t, n, a, A)da + \int_{a=R}^{A} l(t, n, a, A)da \qquad (15)$$

and the number of young and old individuals, respectively:

$$N_w(t, n, R, A) = \int_{a=0}^{R} l(t, n, a, A)da = \frac{\varphi e^{nt}}{n}(1 - e^{-nR}) \qquad (16)$$

where the second uses (13) and (14), and

$$N_o(t, n, R, A) = \int_{a=R}^{A} l(t, n, a, A)da \qquad (17)$$

It follows from (14)-(17) that the population growth rate and the rate of growth of young and

old individuals in the economy follow the growth rate of individuals born: $\dot{N}/N = n$ and

$\dot{N}_w/N_w = \dot{N}_o/N_o = n$ where $\dot{N}$, $\dot{N}_w$ and $\dot{N}_o$ are the time derivatives, while the dependency

ratio, i.e. the ratio between the number of old and young N_o/N_w, is constant over time.

Individual utility

Now denote consumption during working age by c_w and consumption during retirement by

c_o. We assume that utility when young is

$$u(c_w(a)) \; for \; a \in [0, R] \qquad (18)$$

where $u' > 0$ and $u'' < 0$, while utility when old depends on the number of people alive

$$v(c_o(a), N_o) \; for \; a \in (R, A] \qquad (19)$$

where $v_1 > 0$, $v_{11} < 0$, $v_2 > 0$, $v_{22} < 0$ and $v_{12} > 0$.

Health care expenditure and longevity

Health care expenditure per old individual τ is assumed to affect longevity and thus the health of old individuals (since $f(a, A)$ is assumed to be strictly increasing in A) in the following way:

$$A = \gamma B(\tau) \tag{20}$$

where $\gamma > 0$ is a parameter measuring efficiency in health care production, and $B' > 0$ and $B'' < 0$ implying positive but diminishing returns to health care expenditure. Furthermore, it is assumed that $\gamma B(0) > R$ ensuring $A>R$ for all $\tau \geq 0$ implying that all individuals reach retirement age even though there is no spending on old-age health care.

Output

Each young individual produces $y > 0$ at each point in time. National output in the economy at time t can therefore be written as:

$$N_w(t, n, R, A)y \tag{21}$$

It follows that national output can only increase when the number of working-age individuals increases or if output per working individual goes up.

This and constant dependency ratio $(\frac{N_o}{N_w})$ (see above) ensures stationarity in the model and the existence of steady state solutions for the optimal consumption per capita and health expenditure per old individual since it implies that output per capita $\frac{N_w y}{N_w + N_o} = \frac{y}{1 + \frac{N_o}{N_w}}$ is constant over time.

The social planner's problem

We assume a balanced current account growth path in steady state and hence balanced budget constraints for the economy at each point in time. The economy-wide budget constraint is therefore in balance at all points in time:

$$\int_{a=0}^{R} l(t,n,a,A)(y - c_w(a))da = \int_{a=R}^{A} l(t,n,a,A)(c_o(a) + \tau)da$$

which can, after multiplying through by e^{-nt}, be written as:

$$\int_{a=0}^{R} \tilde{l}(t,n,a,A)(y - c_w(a))da = \int_{a=R}^{A} \tilde{l}(t,n,a,A)(c_o(a) + \tau)da \qquad (22)$$

where $\tilde{l}(n,a,A) = e^{-nt}l(t,n,a,A)$.

Because a balanced budget for the economy is assumed, there is no transfer of resources across time in steady state. The social planner's welfare objective can therefore be written as

$$W = \int_{a=0}^{R} l(t,n,a,A)u(c(a))da + \int_{a=R}^{A} l(t,n,a,A)v(c_o(a),N_o)da$$

which can, after multiplying through by e^{-nt}, be written as

$$\widetilde{W} = \int_{a=0}^{R} \tilde{l}(n,a,A)u(c(a))da + \int_{a=R}^{A} \tilde{l}(n,a,A)v(c_o(a),N_o)da \qquad (23)$$

where $\widetilde{W} = e^{-nt}W$.

In essence, the social planner is maximizing the sum of utilities of all living generations at a point in time, and thus gives a golden rule, taking into account that his decision affects both consumption of each working individual through taxes as well as the number of old individuals who get to live due to the provision of health care. He can choose to increase the number of the living by reducing the consumption of each member of the working-age

population and finds the optimal amount of expenditures on health care, hence also the optimal tax rate, by trading off one effect against the other.

The maximization problem solved by the social planner is equivalent to maximizing the expected lifetime utility of a given individual over his life in that by always maximizing the sum of utilities of all living individuals of different ages he manages to maximize the expected lifetime utility of each individual.

The maximization of this objective function subject to the budget constraint in (22) gives the social optimum. Note that the welfare function in (23) is strictly increasing in $c_w(a)$ for all $a \in [0, R]$, $c_o(a)$ for all $a \in (R,A]$ and τ (through A). The budget constraint in (22) ensures that a maximum exists to the constrained maximization problem (increased spending on health care per old individual decreases consumption given output and hence raises the marginal utility of consumption, ensuring that a maximum exists). Also note that time t does not appear in (22) or (23) and, hence, the solutions for consumption per capita and spending on health care per old individual are independent of time.

The Lagrangian for the maximization problem is (after using (13), (14), (16), (17), (20), (22) and (23)):

$$\Gamma = \int_{a=0}^{R} \varphi e^{-na} u(c_w(a)) da + \int_{a=R}^{\gamma B(\tau)} \varphi e^{-na} f(a, \gamma B(\tau)) v\Big(c_o(a), N_o(t, n, R, \gamma B(\tau))\Big) da$$

$$+\lambda \left[\int_{a=0}^{R} \varphi e^{-na} (y - c_w(a)) da - \int_{a=R}^{\gamma B(\tau)} \varphi e^{-na} f(a, \gamma B(\tau))(c_o(a) + \tau) da \right]$$

Assuming an interior solution, the first-order conditions are derived using that $f(A, A) = 0$:

$$u'(c_w(a)) = \lambda \quad for \quad a \in [0, R] \tag{24}$$

$$v_1(c_o(a), N_o) = \lambda \quad for\ a \in (R,A] \tag{25}$$

$$\left(\int_{a=R}^{A} \varphi e^{-na} \frac{\partial f}{\partial A} v(c_o(a), N_o) da \right) \gamma B'(\tau)$$

$$+ \left(\int_{a=R}^{A} \varphi e^{-na} f(a, A) v_2(c_o(a), N_o) \frac{\partial N_o}{\partial A} da \right) \gamma B'(\tau)$$

$$= \lambda \left[\int_{a=R}^{A} \varphi e^{-na} \frac{\partial f}{\partial A} (c_o(a) + \tau) da \right] \gamma B'(\tau) + \lambda \int_{a=R}^{A} \varphi e^{-na} f(a, A) da \quad (25)$$

The last condition is the budget constraint for the economy that sets total output net of consumption when young equal to the sum of consumption of the old and the provision of health care for the old:

$$\int_{a=0}^{R} \varphi e^{-na} (y - c_w(a)) da = \int_{a=R}^{A} \varphi e^{-na} f(a, A) (c_o(a) + \tau) da \quad (26)$$

The conditions in (24) and (25) imply that consumption when young and old are constant, although not necessarily equal:

$$u'(c_w) = \lambda \quad for \quad a \in [0, R] \quad (27)$$

$$v_1(c_o, N_o) = \lambda \quad for \ a \in (R, A] \quad (28)$$

Using (27) and (28) in (25) after multiplying through by e^{nt}, and using (13) – (17), gives our main result:

$$v(c_o, N_o) \frac{\partial N_0}{\partial A} \gamma B'(\tau) + v_2(c_o, N_o) N_o \frac{\partial N_o}{\partial A} \gamma B'(\tau)$$

$$= u'(c_w)(c_o + \tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau) + u'(c_w) N_o \quad (29)$$

The equation gives optimal spending on health care per old individual, which is our golden rule of health care spending. The left-hand side shows increased social welfare in terms of a greater number of old individuals reaching each age level and also more individuals receiving

utility from consumption in each other's companionship. The first term on the right-hand side shows the lost utility for all others, whose consumption is reduced due to the consumption and medical needs of those who now reach higher age levels because of the increased provision of health care. The second term denotes the marginal cost of increased health care for those individuals over retirement age who would have survived in the absence of the increased spending on medical care.

Individual decision making

Due to the uncertain lifetime of an individual, it is assumed that there exists an annuities market (as in Yaari, 1965) providing an individual with an instrument to insure himself against an uncertain lifetime (see, for example, Blanchard (1985) and Sheshinski (2008)). This implies that an individual buys annuities at any age a from an insurance company earning rate of return $r_b(a)$ and the insurance company invests at an exogenous rate of return r for the amount of annuities bought. A zero-profit condition for the insurance company – due to the assumption of free entry into the competitive annuity market – gives the following in equilibrium;

$$r_b(a) = r + \rho(a, A) \tag{30}$$

where $\rho(a, A) = -\dfrac{\frac{\partial m(a,A)}{\partial a}}{m(a,A)}$ is the hazard rate for an individual aged a, i.e. the probability that an individual dies at age a conditional on being alive at age a. It follows that the higher the mortality rate the higher is r_b. Hence, using equation (13) gives

$$\rho(a, A) = \begin{cases} 0 \\ -\dfrac{\frac{\partial f(a,A)}{\partial a}}{f(a,A)} \end{cases} for\ a \in \begin{matrix} [0, R] \\ (R,A] \end{matrix} \tag{31}$$

and, hence, in equilibrium:

$$r_b(a) = \begin{cases} r & \\ r + \rho(a, A) \end{cases} \; for \; a \in \begin{array}{l} [0, R] \\ (R, A] \end{array} \tag{32}$$

The preceding equations show that the higher the mortality rate among retirees, the higher are the earnings on the annuity in equilibrium.

An individual's expected lifetime utility is

$$U = \int\limits_{a=0}^{R} e^{-\delta a} m(a, A) u\big(c_w(a)\big) da + \int\limits_{a=R}^{A} e^{-\delta a} m(a, A) v(c_o(a), N_o) da$$

where $\delta > 0$ is the subjective rate of time preference. Using (13), the equation can be written as

$$U = \int\limits_{a=0}^{R} e^{-\delta a} u\big(c_w(a)\big) da + \int\limits_{a=R}^{A} e^{-\delta a} f(a, A) v(c_o(a), N_o) da \tag{33}$$

An individual consumes when young and old, works and earns labor income when young and spends on old-age health care when old. His budget constraint therefore reads;

$$\int\limits_{a=0}^{R} e^{-\Gamma(a)} \big(y - c_w(a)\big) da = \int\limits_{a=R}^{\cancel{\gamma B(\tau) A}} e^{-\Gamma(a)} \big(c_o(a) + \tau\big) da$$

where $\Gamma(a) = \int_{z=0}^{a} r_b(z) dz$, or (using (32))

$$\int\limits_{a=0}^{R} e^{-ra} \big(y - c_w(a)\big) da = \int\limits_{a=R}^{\gamma B(\tau)} e^{-\Gamma(a)} \big(c_o(a) + \tau\big) da \tag{34}$$

An individual's problem is to choose $\{c_w(a)\}_{a=0}^{R}$, $\{c_o(a)\}_{a=R}^{A}$ and τ such that (33) is maximized subject to (34) and (20) taking the annuity contract r_b and the number of old individuals N_o as given (hence, these are independent of longevity A in an individual's optimization problem). The Lagrangian for the problem is:

$$\Gamma = \int_{a=0}^{R} e^{-\delta a} u\big(c_w(a)\big) da + \int_{a=R}^{\gamma B(\tau)} e^{-\delta a} f\big(a, \gamma B(\tau)\big) v(c_o(a), N_o) da$$

$$+\lambda \left[\int_{a=0}^{R} e^{-ra} \big(y - c_w(a)\big) da - \int_{a=R}^{\gamma B(\tau)} e^{-\Gamma(a)} \big(c_o(a) + \tau\big) da \right]$$

Assuming an interior solution, this gives the following first-order conditions (in addition to the budget constraint in (34)), where it is used that $f(A, A) = 0$:

$$\frac{\partial \Gamma}{\partial c_w(a)} = e^{-\delta a} u'\big(c_w(a)\big) - \lambda e^{-ra} = 0 \quad for \quad a \in [0, R] \tag{35}$$

$$\frac{\partial \Gamma}{\partial c_o(a)} = e^{-\delta a} f(a, A) v_1(c_o(a), N_o) - \lambda e^{-\Gamma(a)} = 0 \quad for\ a \in (R, A] \tag{36}$$

$$\frac{\partial \Gamma}{\partial \tau} = \left(\int_{a=R}^{A} e^{-\delta a} \frac{\partial f}{\partial A} v(c_o(a), N_o) da \right) \gamma B'(\tau)$$

$$-\lambda e^{-\Gamma(A)} (c(A) + \tau) \gamma B'(\tau) - \lambda \int_{a=R}^{A} e^{-\Gamma(a)} da = 0 \tag{37}$$

Using that in equilibrium (using (30) - (32)):

$$e^{-\Gamma(a)} = e^{-\int_{z=0}^{a} [r + \rho(z,A)] dz} = e^{-\int_{z=0}^{a} \left[r + \frac{-\frac{\delta f(z,A)}{\delta z}}{f(z,A)} \right] dz} = e^{-ra + \int_{z=0}^{a} \left[\frac{\frac{\delta f(z,A)}{\delta z}}{f(z,A)} \right] dz}$$

$$= e^{-ra + \int_{z=0}^{a} \left[\frac{d\ln f(a,A)}{dz} \right] dz} = e^{-ra + \int_{z=0}^{a} d\ln f(z,A)}$$

$$= e^{-ra + [\ln f(a,A) - \ln f(0,A)]} = e^{-ra} e^{\ln f(a,A)} = e^{-ra} f(a, A)$$

where it is used that $f(0, A) = 1$, gives the conditions in (35)-(37) as:

$$e^{-\delta a} u'\big(c_w(a)\big) = \lambda e^{-ra} \quad for \quad a \in [0, R]$$

$$e^{-\delta a} v_1(c_o(a), N_o) = \lambda e^{-ra} \quad for\ a \in (R, A]$$

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$$\left(\int_{a=R}^{A} e^{-\delta a} \frac{\partial f}{\partial A} v(c_o(a), N_o) da\right) \gamma B'(\tau) = \lambda \int_{a=R}^{A} e^{-ra} f(a, A) da$$

where it has been used that $f(A, A) = 0$. Since we are only interested in comparing allocation

under individual decision making with the steady state social planner allocation, we ignore

the possibility of a time-varying consumption profile for an individual being optimal and

assume that that the real interest rate r equals the subjective rate of time preference δ and,

hence;

$$u'(c_w(a)) = \lambda \quad for \quad a \in [0, R] \tag{38}$$

$$v_1(c_o(a), N_o) = \lambda \quad for \ a \in (R, A] \tag{39}$$

$$\left(\int_{a=R}^{A} e^{-ra} \frac{\partial f}{\partial A} v(c_o(a), N_o) da\right) \gamma B'(\tau) = \lambda \int_{a=R}^{A} e^{-ra} f(a, A) da \tag{40}$$

The conditions in (38) and (39) imply that consumption when young and old are constant,

although not necessarily equal:

$$u'(c_w) = \lambda \quad for \quad a \in [0, R]$$

$$v_1(c_o, N_o) = \lambda \quad for \ a \in (R, A]$$

Using this in (40) gives:

$$v(c_o, N_o) \left(\int_{a=R}^{A} e^{-ra} \frac{\partial f}{\partial A} da\right) \gamma B'(\tau) = u'(c_w) \int_{a=R}^{A} e^{-ra} f(a, A) da \tag{41}$$

The left-hand side of the equation shows the marginal benefit of increased spending on old-

age health care, which is the increased utility due to longer life expectancy, while the right-

hand side has the marginal cost in the form of lower utility during working years.

Social optimum and individual decision making compared

Equation (41) can be compared to equation (42) below that gives the optimal (that is the social planner) level of spending on the health care. Using equations (13), (14), (15) and (17) in (29), multiplying through with e^{-nt} and dividing by the parameter φ gives that spending on old-age health care in social optimum has to fulfil:

$$
v(c_o, N_o) \left(\int_{a=R}^{A} e^{-na} \frac{\partial f}{\partial A} da \right) \gamma B'(\tau)
$$

$$
+ v_2(c_o, N_o) \left(\int_{a=R}^{A} e^{-na} f(a, A) \frac{\partial N_o}{\partial A} da \right) \gamma B'(\tau)
$$

$$
= u'(c_w)(c_o + \tau) \left[\int_{a=R}^{A} e^{-na} \frac{\partial f}{\partial A} da \right] \gamma B'(\tau) + u'(c_w) \int_{a=R}^{A} e^{-na} f(a, A) da
$$

(42)

Assuming that $r=n$, the two equations in (41) and (42) differ since the second term on the left hand side and the first term on the right hand side of (42) are missing from equation (41). The former captures the external effect of one person's longevity on the utility of other old individuals while the latter captures the cost of consumption and health care of his increased longevity born by other working-age individuals. While the latter would make the individual spend more than the social optimum on life-extending health care, the latter would make him spend less. It is this effect which is novel in our derivation.

4. Conclusions

The literature on the optimal level of life-extending health care compares the marginal cost of increased spending on health care, taking the form of lower consumption of working-age individuals, to the marginal benefit in the form of increased longevity that brings extra

consumption and utility. The main difference between the social and the private optimum is that an individual would ignore the negative pecuniary cost of his increased longevity on others, hence overinvest in life-extending healthcare. We add to this literature the effect of increased longevity of one individual on the utility of others who would have missed her company and hence not enjoyed their consumption to the same extent. The basic idea is that the utility from consumption depends on others sharing the experience.

Appendix I

The matrix of second derivatives of the Lagrangian function is (after inserting the FOC and using that $\gamma B(\tau)=A$);

$$H = \begin{pmatrix} 0 & -R & -(A-R) & \gamma B'(\tau)(c_o + \tau) + (A-R) \\ -R & Ru''(c_w) & 0 & 0 \\ -(A-R) & 0 & (A-R)v_{11} & \gamma B'(\tau)(A-R)v_{12} \\ \gamma B'(\tau)(c_o + \tau) + (A-R) & 0 & \gamma B'(\tau)(A-R)v_{12} & K \end{pmatrix}$$

where

$$K \equiv \gamma B''(\tau)v - \gamma[B''(\tau)(c_o + \tau) + 2B'(\tau)]v_1 + \gamma[2\gamma B'(\tau)^2 + (A-R)B''(\tau)]v_2$$
$$+ (A-R)\gamma^2 B'(\tau)^2 v_{22}$$

Sufficient conditions for a local maximum are then

$$|H_1| = \begin{vmatrix} 0 & -R \\ -R & Ru''(c_w) \end{vmatrix} < 0$$

$$|H_2| = \begin{vmatrix} 0 & -R & -(A-R) \\ -R & Ru''(c_w) & 0 \\ -(A-R) & 0 & (A-R)v_{11} \end{vmatrix} > 0$$

$$|H_3| = \begin{vmatrix} 0 & -R & -(A-R) & \gamma B'(\tau)(c_o + \tau) + (A-R) \\ -R & Ru''(c_w) & 0 & 0 \\ -(A-R) & 0 & (A-R)v_{11} & \gamma B'(\tau)(A-R)v_{12} \\ \gamma B'(\tau)(c_o + \tau) + (A-R) & 0 & \gamma B'(\tau)(A-R)v_{12} & K \end{vmatrix} < 0$$

Here we have

$$|H_1| = -R^2 < 0$$

$$|H_2| = -R(A - R)[Rv_{11} + (A - R)u''(c_w)] > 0$$

and $|H_3| < 0$ is assumed to hold.

The authors declare that they have no conflict of interest.

References

Andersen, Torben M. and Joydeep Bhattacharya (2014), "A dynamic-efficiency rationale for public investment in the health of the young," *Canadian Journal of Economics*, 47 (3), 697-719.

Andersen, Torben M. and Marias H. Gestsson (2016), "Longevity, Growth and Intergeneration Equity – The Deterministic Case," *Macroeconomic Dynamics*, 20 (4), 985-1021.

Beck A. T, Ward C. H, Mendelson M, Mock J, Erbaugh (1961), "An inventory for measuring depression," *Archives of General Psychiatry*, 4), 561–571.

Blanchard, Olivier E. (1985), "Debt, deficits and finite horizons," *Journal of Political Economy*, 93 (2), 223-247.

Cowen, T. (1989), "Normative population theory," *Social Choice and Welfare*, 6, 33-43

Gestsson, M. H. and G. Zoega (2018), "The golden rule of longevity," *Macroeconomic Dynamics*, forthcoming.

Green B. H, Copeland J. R, Dewey M. E, Shamra V, Saunders P. A, Davidson I. A, Sullivan C, McWilliam C. (1992), "Risk factors for depression in elderly people: A prospective study*," Acta Psychiatr Scand*. 86 (3), 213–7.

Hall, Robert E. and Charles I. Jones (2007), "The value of life and the rise in health spending," *The Quarterly Journal of Economics*, 122 (1), 39-72.

Heijdra, Ben J., Jochen O. Mierau and Laurie S.M. Reijnders (2011), "The tragedy of annuitization," HIS Economics Series Working Paper 268.

Heikkinen R, Berg S, Avland K. (1995), "Depressive symptoms in late life," *Journal of Cross Cultural Gerontology*,.10, 315–330.

Parfit, Derek (1984), *Reasons and persons*, Oxford: Oxford University Press.

Phelps, Edmund S. (1966), *Golden Rules of Economic Growth*, New York. W. W. Norton.

Phelps, Edmund S. (1968), "Models of technical progress and the golden rule of research," *Review of Economic Studies*, 33, 133-146.

Ramsey, Frank (1928), "A mathematical theory of saving," *Economic Journal*, 38, 543-559.

Russell D, Peplau L. A, Cutrona C. E. (1980), "The revised UCLA Loneliness Scale: Concurrent & discriminant validity evidence;" *Journal of Personality & Social Psychology*, 39, 472–480.

Singh, Archana and Nishi Misra (2009), "Loneliness, depression and sociability in old age," *Industrial Psychiatry Journal*, 18 (1), 51-55.