ISSN 1745-8587

Birkbeck
UNIVERSITY OF LONDON

School of Economics, Mathematics and Statistics

Birkbeck Working Papers in Economics & Finance

BWPEF 0701

Uninformative Equilibrium in Uniform Price Auctions

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January 2007

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Abstract

I analyze the incentive for costly information collection in a multi-unit common-value uniform-price auction in which bidders submit demand functions. I show that so long as there are some bidders who have a very high cost of information collection, even if there are a large number of other bidders who face an arbitrarily small cost of information collection, there are equilibria in which no bidder collects information.

KEYWORDS: *Costly information acquisition, uniform price auction, uninformed bidders, uninformative equilibrium*

JEL CLASSIFICATION: D44

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1 Introduction

I analyze the incentive for costly information collection in a multi-unit common-value uniform-price auction in which bidders submit demand functions. There are some bidders who have the option of receiving a costly signal of the unknown true value. There are also some bidders who face a prohibitively high cost of receiving a signal.

Important work by Pesendorfer and Swinkels (1997) shows that in a common value setting, "large" uniform price auctions aggregate information fully. With n bidders each demanding at most 1 unit, and k units, if both k and $(n-k)$ become large, the auction price converges to the true value. This is true even if only a small proportion of bidders are informed. The equilibrium is also the unique among symmetric equilibria.

The purpose of this note is to show that so long as there are some uninformed bidders (bidders who have a very high cost of information collection), even if there are a large number of bidders who face an arbitrarily small cost of information collection, there are equilibria in which no one collects information so long as the supply is large enough.

I allow demand function bids, but the result would hold with unit demand as well, but with the additional requirement that the number of uninformed bidders rises as supply increases.

The rational expectations (RE) approach provides a systematic way to investigate the informational role of prices. An unappealing feature of these models is that the traders are *naive* price takers. The qualifier *naive* implies that an RE equilibrium is manipulable in the sense that an agent can obtain any allocation in equilibrium by misrepresenting his preferences. Naive price-taking by agents in RE models gives rise to certain well known paradoxes. Grossman and Stiglitz (1980) show that if information is costly, an informationally efficient equilibrium cannot exist. There is a conflict between equilibrium prices fully revealing all relevant information, and the incentive to obtain costly information. Once the price taking assumption is removed and the price formation process is modeled (Milgrom (1981),Dubey, Geanakoplos, and Shubik (1987),Jackson (1991)), there is no necessary conflict between the incentive to acquire costly information and revelation of information through prices.

The examples of uninformative equilibria here point to a different conflict – that between incentive to collect costly information, and the extent to which uninformed bidders attempt make use of the information of the informed by demanding units at high prices, reducing information rent, and therefore the incentive to collect information.

2 The Model

The total supply is normalized to 1. This is supplied in S parts. The normalization implies that each "unit" is a fraction $1/S$ of the total supply, and thus each unit is a smaller fraction of the total supply as supply increases. This serves to capture the idea that for as the auction size gets larger, the value of each unit gets smaller.

The value of the total supply is V. Each unit is valued at V/S. The value V is a random variable with a continuous distribution function $F(\cdot)$ and density function $f(\cdot)$ over a support $[0, \overline{V}]$. The distribution of V is public information.

Some bidders can obtain an informative signal about V at a cost $c > 0$. I assume the standard common values scenario. Let $X_i \in [0,1]$ denote the signal potentially received by informed bidder i, $i \in I$. Let $g(X_i|V)$ denote the conditional density of X_i given the common value V. Conditional on V, the random variables $X_1, \ldots, X_{N_I}$ are independently and identically distributed. For notational convenience I define the "signal" of a bidder who remains uninformed as ϕ. Thus each bidder draws a signal from $[0,1] \cup \{\phi\}$. A bidder with signal ϕ has access to only public information.

There are also some bidders who face a prohibitively high cost of information and therefore never collect information, and only have access to public information.

Let $N_I \geqslant 1$ be the number of potentially informed bidders, and $N_U \geqslant 1$ be the number of bidders who have a prohibitively high cost of information collection and remain uninformed. Let I and U denote the sets of potentially informed and uninformed bidders, respectively. All bidders are risk neutral.

2.1 Bids and Strategies

A bid is any decreasing function $q(p)$ mapping the set of prices $[0, \overline{V}]$ to the set of quantities $\{0, 1, \ldots, S\}$. Since there are S discrete units, a bid function is a step function:

$$q(p) = \begin{cases} 0 & \text{for } p_1 < p \leqslant \overline{V}, \\ 1 & \text{for } p_2 < p \leqslant p_1, \\ \vdots & \vdots \\ S & \text{for } 0 \leqslant p \leqslant p_S, \end{cases}$$

The following representation of a bid function is very useful. Note that the inverse of a bid function can be derived as follows:

$$p(\tilde{q}) = \begin{cases} \max_p\{p|q(p) \geqslant \tilde{q}\} & \text{if this exists,} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting function $p(\cdot)$ is the inverse demand function, and thus a bid can be written as a vector $(p_1, \ldots, p_S)$, such that

$$p(q) = \begin{cases} p_1 & \text{over 1 unit,} \\ p_2 & \text{over 2 units,} \\ \vdots & \vdots \\ p_S & \text{over } S \text{ units,} \end{cases}$$

where

$$\bar{V} \geqslant p_1 \geqslant p_2 \geqslant \ldots \geqslant p_S \geqslant 0. \tag{2.1}$$

Let Ω be the set of vectors $(p_1, \ldots, p_S)$ that satisfy (2.1). Then $\Omega \subset \Re^S_+$ is the set of bid functions. This is compact and convex.

A pure strategy of bidder $i \in I$ has two components. First, he must decide whether to spend c to collect information, and then decide which bid to submit. A bid by $i \in I$ is a mapping from $[0, 1] \cup \{\phi\}$ to Ω. A pure strategy for i is written as $(D, q_i(p)(X_i))$, where $D \in \{Y, N\}$.

A pure strategy for uninformed bidder j, $j \in U$ is simply a bid $q_j(p) \in \Omega$.

I analyze a simultaneous game. Each potentially informed bidder $i \in I$ decides whether to obtain costly information and submits a bid. The decision about information acquisition is private and not observed by others. Simultaneously, other bidders submit bids.

2.2 Market Clearing Price and Allocation Rule

The market clearing price is defined as follows.

Definition 1. *For any $K \leqslant S$, the* market clearing price $m(K)$ *is given by the highest price at which demand exceeds or equals K units. Thus*

$$m(K) = \sup_p \left(p | \sum_{i \in I} q_i(p)(\cdot) + \sum_{j \in U} q_j(p) \geqslant K \right).$$

Finally, the quantity won by a particular bid needs to be specified. Suppose bidder $h \in I \cup U$ submits a demand function $q_h(p)$ specifying positive prices for k units, $k \leqslant S$. Also, let k' be the highest integer below k such that $p_{k'} > p_k$. The winning function $q_h^w(p)$ is specified below.

$$q_h^w(p) = \begin{cases} k & \text{if } p_k > m(S), \\ k' + \alpha_h(k - k') & \text{if } p_k = m(S), \\ 0 & \text{otherwise,} \end{cases}$$

where[1]

$$\alpha_h = \frac{k}{\sum_{i \in I} q_i(m(S))(\cdot) + \sum_{j \in U} q_j(m(S))}.$$

[1]If $\alpha_h(k - k')$ is not an integer the bidder is allocated the greatest positive integer (including 0) less than this. The remainder is allocated randomly according to proportional probabilities.

3 Uninformative Equilibrium

An equilibrium is *uninformative* if no bidder spends c to collect information.

3.1 An equilibrium with zero payoff for all bidders

Consider $K_i \geqslant 0$ for all $i \in I$ and $K_j \geqslant 0$ for all $j \in U$ such that $\sum_{i \in I} K_i = K_I$ and $\sum_{j \in U} K_j = S - K_I - \ell$, where $\ell \geqslant 1$. Now consider the following profile of strategies.

1. For all $i \in I$, bidder i chooses not to collect information and submits the following bid: $p_1 = \ldots = p_{K_i} = \overline{V}$, and $p_{K_i+1} = \ldots = p_S = EV$.
2. Similarly, for all $j \in U$ bidder j submits the following bid: $p_1 = \ldots = p_{K_j} = \overline{V}$, and $p_{K_j+1} = \ldots = p_S = EV$.

The strategy of each bidder is to demand some units at price $\overline{V}$, and demand all available units at the price EV. Given the above strategy profile, the market clears at EV and each bidder earns a zero payoff. I show below that this strategy profile is a Nash equilibrium for large enough S.

Proposition 1. *For any given $\ell \geqslant 1$ and $c > 0$, there exists S_* such that for any $S > S_*$ the strategy profile above is a Nash equilibrium.*

Proof: Let S_* be given by the following:

$$\left(\frac{\max_{i \in I} K_i + \ell}{S_*} \right) E(V|V > EV) = c. \tag{3.1}$$

First, let us see if any $i \in I$ has a profitable deviation to information collection. Note that even if any such bidder reduces demand to zero units, the market clearing price is unaffected and stays at EV. Therefore, the only way information can benefit a bidder is that for high enough signals he could demand an additional ℓ units at a price slightly higher than EV and earn a positive payoff. Therefore an upper bound to the payoff of bidder $i \in I$ from information collection is given by $(\frac{K_i + \ell}{S})E(V|V > EV)$. It is easy to see from equation (3.1) that for $S > S_*$ the payoff from collecting information is less than the cost c. Therefore not collecting information is a best response for each bidder $i \in I$.

From the argument above, we know that given the strategy profile, all bidders must be uninformed. Now let us see whether the bid submitted by any bidder is a best response. Any bidder $h \in I \cup U$, by demanding either fewer units at the price $\overline{V}$ or by reducing some of the prices $(p_1, \ldots, p_{K_h})$ to any other price greater than or equal to EV cannot change the market clearing price. Also by changing some of the prices $(p_1, \ldots, p_{K_h})$ to any price below EV the bidder wins fewer units, but the overall payoff remains zero. Finally, if any such bidder demands more than K_h units at some prices greater than or equal to EV, either this does not affect the market clearing price (if the total demand at prices strictly above EV is still lower than S), in which case the payoff is still zero, or this leads to market clearing at some price strictly above EV in which case payoff is strictly negative. Thus the strategy of each bidder is a best response. This completes the proof.$\|$

3.2 Discrete prices: An equilibrium with positive payoffs for all bidders

In the example above, the market clearing price cannot be any lower than EV. Otherwise, it is always possible for any bidder to benefit by raising the price by a very small amount which increases winning quantity discretely. If price changes in discrete units, another factor starts to have a bite. It is possible to win all marginal units (ℓ units in the example above) by posting a higher price, but this also raises the price paid on all infra-marginal units won by the bidder (K_h for $h \in I \cup U$ in the above example). Using this effect, it is possible to sustain a price lower than EV in equilibrium, earning each bidder a strictly positive payoff.

Suppose price bids must be in units of Δ. Let n be a positive integer such that $n\Delta = \overline{V}$. Therefore any $m\Delta$ with $m \leqslant n$ is a feasible price.

Let $\tilde{p} < EV$ be a feasible price such that

$$EV - \tilde{p} \leqslant 2\Delta. \tag{3.2}$$

I assume that Δ is not too large so that the above condition does hold for some $\tilde{p} < EV$.

Let $S > N_I + N_U$. Consider $K_i > 0$ for all $i \in I$ and $K_j > 0$ for all $j \in U$ be such that $\sum_{i \in I} K_i = K_I \geqslant N_I$ and $\sum_{j \in U} K_j = S - K_I - 1$. The difference with the previous example is that K_i and K_j are now all strictly positive, and ℓ is set to 1.

Now consider the following profile of strategies.

1. For all $i \in I$, bidder i chooses not to collect information and submits the following bid: $p_1 = \ldots = p_{K_i} = \overline{V}$, and $p_{K_i+1} = \ldots = p_S = \tilde{p}$.
2. Similarly, for all $j \in U$ bidder j submits the following bid: $p_1 = \ldots = p_{K_j} = \overline{V}$, and $p_{K_j+1} = \ldots = p_S = \tilde{p}$.

The following result shows that for large enough S, the above strategy profile is a Nash equilibrium.

Proposition 2. *Given any $c > 0$, and any $\tilde{p}$ satisfying* (3.2)*, there exists S_* such that for any $S > S_*$ the strategy profile above is a Nash equilibrium.*

Proof: Let S_* be such that $\left(\frac{\max_{i \in I} K_i + 1}{S_*}\right) E(V|V > \tilde{p}) = c$. The rest of the proof is exactly like the previous proof, the only extra factor is to check whether any bidder has an incentive to raise the price on the marginal unit. The expected payoff of bidder $h \in I \cup U$ under the given strategies is given by $\pi_h = \left(K_h + \frac{1}{N_I + N_U}\right)(EV - \tilde{p})$. The fraction in the first expression arises from the fact that each bidder demands S units at the price $\tilde{p}$ and therefore the marginal unit is allocated to any bidder with probability of $1/(N_I + N_U)$. Next, the payoff from raising the price on the marginal unit is $\widehat{\pi}_h = (K_h + 1)(EV - \tilde{p} - \Delta)$. Equilibrium requires $\pi_h \geqslant \widehat{\pi}_h$, which implies

$$(EV - \tilde{p})\left(\frac{N_I + N_U - 1}{N_I + N_U}\right) \leqslant (K_h + 1)\,\Delta.$$

But (3.2) is sufficient for this to hold.$\|$

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