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Pricing Multiple Interruptible-Swing Contracts

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Abstract

In this article we price a multiple-interruptible contract for the electricity market in England and Wales under a mean-reverting jump-diffusion model with seasonality. We do so by combining forward contracts with a swing option which can be exercised a pre-specified number of times. We price this swing option by means of an extension of the Least-Squares Monte Carlo methodology for American options. We additionally compute the lower and upper bounds for this contract. For the computation of the lower bound we provide a semi-analytical formula which reduces greatly the required computational time.

Keywords: Energy derivatives, electricity market, Least-Squares Monte Carlo, swing options.

1 Introduction

One of the objectives of NETA (New Electricity Trade Arrangement), introduced on March 27, 2001 was to remove price controls and openly encourage competition by providing greater scope for demand-side participation in the electricity market, thereby ensuring a more efficient means of meeting consumption requirements on the system. By creating a system where the value of short-term portfolio flexibility (to respond to short-term price spikes or to hedge exposure to imbalance charges) was more transparent to market participants, it was anticipated that there would be an increase in the elasticity of price-responsiveness in the market.

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Interruptible contracts, where a supplier has the right to cease supply to a consumer under pre-defined conditions, have been a major tool for introducing demand-side flexibility in other markets, such as the gas market. Until now, such contracts have been utilized by the NGC (National Grid Company) as part of its balancing services obligations, but have not developed significantly as an option in standard commercial contracts. As technology improvements make the use of such arrangements less costly, the key questions are whether such contracts have a real value in electricity markets and how that would translate into contract terms and prices.

In general, an interruptible contract can be considered as a standard supply contract with the following additional conditions on physical delivery: a specified number, or volume, of interruptions that can be called over the life of the contract; a minimum notification period prior to each interruption; and a minimum and maximum period for each interruption called.

The value of the flexibility implied in these conditions would be reflected in the contract price through, for example, discounted unit electricity prices and/or lump-sum payments per interruption. The exact format of the pricing arrangements would depend on both the value to the supplier, the cost to the consumer, and the attitude of each party to the underlying risk in the contract. Moreover, a contract which encompasses all the various flexibilities outlined above has a relatively complex valuation process, but fundamentally stems from valuing a swing option.

Electricity contracts which address interruption of supply have been presented in the past. Gedra [7] introduces the concept of a callable-forward, which is an option that results from taking opposite positions on a forward contract and a call option. A supplier will be able to replicate a simple interruptible contract by shorting a forward with maturity T –which commits the supplier to deliver one unit of energy (denoted by S_t) at time T for the pre-specified price F_t^T, and longing a call option with equal maturity written on the same unit of energy. Hence, the supplier's portfolio at time t is given by $\Pi_t = -F_t^T + C(S_t; K, T)$, where K is the strike price of the option, which is taken as the shortage cost a potential user faces when curtailed.[1]

Since the supplier buys the call, he has the right to exercise the call option at delivery. At expiry, his portfolio is worth $\Pi_T = -S_T + \max(S_T - K, 0)$, since $F_T^T = S_T$. If at expiry the price of electricity is higher than the strike price, the supplier

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[1]In order to avoid confusions at a later stage, it is important at this point to clarify the notation. It is typical in the financial mathematics literature to refer to the forward price of a contract as f_t, whose payoff at delivery time T is $S_T - \chi$, where χ is the price of the forward contract at time T which guarantees that at time t_0, when the forward is contracted, the forward contract f_0 has zero value (since the price of the forward is settled at expiry). The forward F_t^T to which we refer and which is calculated as the expected value of the spot price under an equivalent martingale measure is in this context χ_t and is such that at $t = T$ we recover the spot price.

will exercise the call option; consequently, the unit of energy S_T is canceled and the supplier pays the consumer the strike price K in compensation. On the other hand, if the observed price of electricity is lower than the strike price, the option will not be exercised and the supplier faces the obligation of delivering a unit of energy.

By offering a discount on the forward sold (the value of the call option), the supplier benefits by earning the possibility of calling off supply in case the price of electricity spikes at expiry. On the other hand, the consumer entering this contract benefits by receiving a discount on the value of the forward he is contracting; since his portfolio is worth at any given time t, $\Pi_t = F_t^T - C(S_t; T)$. In addition to this discount, if, at expiry, the supplier decides to exercise the option, he will curtail supply but compensate the consumer with the cost of the shortage, K.

A consumer entering into such a contract must trade off the probability of interruption against the cost of the contract. This means that the consumer must trade off this probability against the shortage cost associated with his particular business. The probability of interruption decreases as the strike price K increases. As a consequence, the call option becomes less valuable, and the discount on the forward price is lower. On the other hand, those consumers with lower shortage costs will be more likely to be interrupted and will receive higher discounts.

Although callable forwards succeed in replicating an interruption strategy, a clear drawback of this type of contract is that there will be consumers whose short-notice interruption costs are too high, and these short-notice interruption costs may not provide a viable strike price for the contracts. Kamat and Oren [13] argue that a possible solution to this would be to introduce an earlier notification date where notice of interruption is given prior to curtailment. This certainly represents an improvement with respect to the simple callable-forward discussed earlier, and principally, under well known frameworks the valuation of such an option is still tractable. Kamat and Oren [13] price this option replicating the payoff with a compound call option on the forward and present results for a geometric Brownian motion GBM, a mean reverting model and an affine-jump-diffusion. However appealing this model is, the assumption of assuming only one early exercise point is still unrealistic and does not seem to capture the real needs and flexibilities that both consumers and generators seek for in these contracts.

We extend these works by substituting the call option on the underlying for an up-swing option (a call-swing option) which allows the holder to exercise N times from a total of M sampling dates. Hence the portfolio the supplier now owns is given by $\Pi_t = -\widetilde{F}_t^T + C^{sw}(S_t; K, T)$, where we can think of the forward $\widetilde{F}_t^T$ in any of two ways. It can either be the market quote for daily delivery across the period $[t, T]$ of one unit of electricity S_u for $t \leq u \leq T$ or a sum of theoretical values of the

forward for which $\widetilde{F}_t^T = \sum_{t_i=t+1}^{t_i=T} F_t^{t_i}$. In any case, when exercising N times the swing option we will be canceling delivery of electricity (and compensating the user with the strike price K) in the corresponding N dates. The user benefits from holding the opposite portfolio, hence the up-swing option represents the discount on the forward contracted for the period.

We also extend this valuation to include putable forwards, which result from a portfolio encompassing a forward and a down-swing option. For instance, let us consider a consumer who owns the following portfolio: $\Pi_t^* = \widetilde{F}_t^T + P^{sw}(S_t; K, T)$, he will be prepared to pay a premium above the forward price in order to have the choice of exercising the down-swing option and therefore selling back to the supplier a unit of electricity at a fixed price K. He will do this only if the price of the spot at the N dates he chooses to exercise is sufficiently low. Since $F_T^T = S_T$ he will then cancel the delivery from the supplier on those dates and buy on the spot market cheap electricity, realizing a profit $K - S_{t_i}$, where t_i is any of the chosen exercise dates. On the other hand, if exercise doesn't take place, he is still guaranteed delivery of a unit of energy.

Regarding the structure of the swing option, we consider an option of the following characteristics: the swing option can be exercised on a single day a number of N-times pre-specified in the contract; in each exercise opportunity the holder will exercise the full volume for a strike price K, which should reflect the customer's cost of interruption.[2]

The pricing of this contract entices two complexities. First, we need to choose a suitable model for the type of market in which we are pricing. As the literature in electricity derivatives now largely suggests, among spot-based models there is strong consensus in that models should include the following characteristics in the spot-price dynamics: mean reversion, seasonality and spikes. These properties have been studied and analyzed by different authors. For instance Escribano et. al. [2] calibrate these models to different electricity markets to find evidence of mean reversion and spikes. Geman and Roncoroni [8] analyze in detail the fine structure of electricity prices and Benth and Koekebakker [3], apart from providing a thorough discussion of how the Nordic power market (Nord Pool) is organized, apply both spot-based models with mean reversion, seasonality and jumps and forward-based models in their discussion. In this application we price under the mean-reverting jump-diffusion (MRJD) model with seasonality presented by Cartea and Figueroa [1], slightly modified from its original version to capture positive and negative exponential jumps, as in Brechner, Cartea and Figueroa [4]. Apart from arguing a case for both positive and negative

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[2]In this case, the interruption is an 'all or nothing' choice -i.e. either all the electricity is delivered on the day or none at all; although this can easily be modified.

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jumps in electricity markets, the inclusion of exponential jumps simplifies the results and enables us to obtain a more tractable option formula.

The second complexity not only involves the American feature of this contract but the additional difficulty introduced by the swing option of allowing for N exercise opportunities within the choice of a set of M different sampling dates. American options, or Bermudan options more precisely since we price in a discrete set of sampling dates, are priced using numerical schemes since analytical solutions are not available. These numerical schemes fall broadly in one of three possibilities: discretizations of the partial differential equation of the problem, which leads to finite-difference schemes; binomial/trinomial-tree-based models; and Monte Carlo techniques. The presence of jumps give rise to non-local operators, or partial-integral differential equations, which make the use of finite difference techniques more complex to apply. Methods based on the tree dynamic programming approach are relatively simpler to implement. For instance, Jaillet, Ron and Tompaidis [12] price swing options based on a multiple-layer tree extension of the classical binomial tree.

Finite-difference schemes and trees are well suited for low dimensional problems and standard dynamics which do not incorporate jumps -i.e. up to three state variables and log-normal or mean-reverting Gaussian processes. In recent years stochastic-mesh methods and regression-based methods based on Monte Carlo techniques (see for example Glasserman [9]) have been developed for the pricing of American options as an alternative. These methods on the other hand, are suitable for problems of higher-dimensions and can handle stochastic parameters as well as jumps. In particular in this paper we will price a swing option by modifying the Least-Squares Monte Carlo (LSM) algorithm proposed by Longstaff and Schwartz [17] to allow for multiple exercise. Finally, we should also mention that within the Monte Carlo methods an alternative to the above LSM methodology has been presented by Ibáñez and Zapatero [11], where they compute directly the optimal exercise frontier rather than estimating by regression the continuation value at each exercise point. Moreover, Ibáñez [10] applies this algorithm to the specific valuation of swing options. In this paper we compare our results obtained through an extended LSM algorithm for swing options to those obtained by Ibáñez [10].

Finally, we present semi-analytical formulæ to price vanilla options under the considered MRJD model which enables us to compute the lower bound for the swing option at least 100 times faster than by pricing the vanilla options with Monte Carlo techniques.

The remaining of this article is organized as follows. In Section 2 we review the model in Cartea and Figueroa [1] and present the semi-analytical pricing formulæ in order to price the lower bound for the price of the swing option more efficiently. In

Section 3 we discuss the valuation of the swing option with an extension of the LSM algorithm. In Section 4 we present the results and finally in Section 5 we conclude.

2 The European Pricing formula

In this section we discuss the pricing of a call option under a MRJD model with seasonality using complex fourier transforms (CFT) in order to set a lower bound for the contract.

Cartea and Figueroa [1] derive a closed-form analytical solution for the forward, however, the pricing of vanilla instruments is not discussed. Here, we extend that work to provide a semi-analytical solution for vanilla options under this model. We provide option formulæ for both an option written on the underlying itself, i.e. the spot price of electricity; and an option written on a forward for a unit of electricity.

Lemma 2.1 *Let Y_t be the underlying state variable representing any class of exponential Lévy process such that $\mathbb{E}_t\left[e^{Y_T}\right] < \infty$, $Y_T(Y_t)$ the underlying at time T conditional on Y_t, $\hat{V}_T(\xi)$ the transformed payoff of the option and $\Psi(\xi)$ the characteristic function, defined as $\Psi(\xi) := \mathbb{E}_t\left[e^{i\xi \ln Y_T}\right]$. Then the price of a T-maturity European option written on the underlying Y_t with strike price K is given by*

$$V(Y_t, t) = \frac{e^{-r(T-t)}}{2\pi} \int_{-\infty+ib}^{\infty+ib} \Psi(-\xi)\hat{V}_T(\xi)d\xi, \tag{1}$$

where $\xi := a + ib$, and $a, b \in \mathbb{R}$. The integration can be performed along any closed-curve within the contour defined by the intersection of the strip of regularities defined by the transformed payoff and the characteristic function.

Lewis [16] shows that this pricing formula can be applied to any class of exponential Lévy processes such that $\mathbb{E}_t\left[e^{Y_T}\right] < \infty$. In particular this encompasses for instance the Black-Scholes model, Merton's jump-diffusion model, the jump-diffusion model of [14] and all other classes of pure jump models.[3]

Cartea and Figueroa [1] show that the stochastic-differential equation (SDE) on the spot price can be integrated to yield

$$S_T = e^{g(T)+(x_t-g_t)e^{-\alpha(T-t)}-\lambda\int_t^T \sigma_s e^{-\alpha(T-s)}ds+\int_t^T \sigma_s e^{-\alpha(T-s)}d\hat{Z}_s+\int_t^T \ln J_s e^{-\alpha(T-s)}dq_s}, \tag{2}$$

where $g(T)$ is the logarithm of the deterministic seasonality function, α is the mean-reversion rate, $\sigma(t)$ the volatility, λ the market price of risk, $d\hat{Z}_t$ the Brownian increment under the risk-neutral measure; J_s the jump at time s which is drawn from a log-normal distribution and dq_t a Poisson process with intensity parameter l.

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[3]Formal proofs can be found in [15].

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They also provide a closed-form solution for the forward at time t maturing at time T', which can be written as

$$F_t^{T'} = e^{g(T')+(x_t-g_t)e^{-\alpha(T'-t)}-\lambda\int_t^{T'}\sigma_s e^{-\alpha(T'-s)}ds+\frac{1}{2}\int_t^{T'}\sigma_s^2 e^{-2\alpha(T'-s)}ds+\int_t^{T'}(\mathbb{E}_t[e^{\alpha_s}]-1)lds}, \quad (3)$$

where $\alpha_s := e^{-\alpha(T'-s)}\ln J_s$.

Cartea and Figueroa [1] assume the logarithm of the jumps are drawn from a normal distribution. Brechner, Cartea and Figueroa [4] have modified this by assuming exponential jumps, as introduced in Kou [14]. This is, it is assumed that $\mathcal{Y} = \ln J$ has an asymmetric double exponential distribution with density

$$f_{\mathcal{Y}}(y) = p\eta_1 e^{-\eta_1 y}\mathbf{1}_{y\geq 0} + q\eta_2 e^{\eta_2 y}\mathbf{1}_{y<0}, \quad (4)$$

where $\eta_1 > 1$, $\eta_2 > 0$, and $p,q \geq 0$ are such that $p + q = 1$. Moreover, p and q represent the probabilities of upward and downward jumps, respectively.

Apart from arguing a case for both positive and negative jumps in electricity markets, the inclusion of exponential jumps simplifies the results and enables us to obtain a more tractable option formula.[4]

2.1 Pricing on a forward

By assuming positive and negative jumps drawn from the double exponential distribution in (4) the forward in (3) becomes

$$F_t^{T'} = G(T)\left(\frac{S(t)}{G(t)}\right)^{h_t}e^{\frac{1}{2}\int_t^{T'}\sigma_s^2 h_s^2 ds-\lambda\int_t^{T'}\sigma_s h_s ds-\frac{kl}{\alpha}(1-h_t)}\left(\frac{\eta_2+h_t}{\eta_2+1}\right)^{\frac{ql}{\alpha}}\left(\frac{h_t-\eta_1}{1-\eta_1}\right)^{\frac{pl}{\alpha}}, \quad (5)$$

where $G(t)$ is the deterministic seasonality function, $\sigma(t)$ is the time-dependant volatility, λ is the market price of risk, $k := \mathbb{E}_t[\ln J_t]$ is the compensator for the Poisson process and $h_t := e^{-\alpha(T'-t)}$ with $\eta_1 > 1$, $\eta_2 > 0$.[5]

In order to apply the CFT technique in this model we need to calculate explicitly $F_T(F_t)$, the forward at time T maturing at time T' conditional on the forward at initial time t maturing at time T'. We then obtain the following pricing formula.

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[4]Even by assuming the logarithm of the jumps belong to a normal distribution we are able to obtain a semi-analytical expression, albeit through some approximations.

[5]Note that in (5) we now include the compensator which was not included in [1] since there it is assumed that $\mathbb{E}_t[J] = 1$.

Proposition 2.1 *Let $F_T^{T'}(F_t^{T'})$ be the forward at time T maturing at time T' conditional on the forward at time t maturing at time T', given by*

$$F_T^{T'} \;\; = \;\; F_t^{T'} e^{-\frac{1}{2}\int_t^T \sigma_s^2 e^{-2\alpha(T'-s)}ds + \int_t^T \sigma_s e^{-\alpha(T'-s)}dZ_s - \int_t^T \Phi(s)lds + \int_t^T e^{-\alpha(T'-s)}\ln J_s dq_s}, \quad \text{(6a)}$$

$$\Phi(s) \;\; := \;\; \frac{q\eta_2}{e^{-\alpha(T'-s)}+\eta_2} - \frac{p\eta_1}{e^{-\alpha(T'-s)}-\eta_1} - 1. \quad \text{(6b)}$$

Then the price of a T-maturity European call option written on the underlying $F_T^{T'}(F_t^{T'})$ with strike price K is given by

$$V(F_t, t) \;\; = \;\; \frac{e^{-r(T-t)}}{2\pi} \int_{-\infty+ib}^{\infty+ib} \Psi(-\xi)\frac{-K^{1+i\xi}}{\xi^2 - i\xi}d\xi, \quad \max(1, \zeta) < b < \theta; \quad \text{(7a)}$$

$$\Psi(-\xi) \;\; := \;\; e^{\phi_t^{T'}} \left(\frac{\eta_2 + h_t}{\eta_2 + H_t}\right)^{\frac{i\xi ql}{\alpha}} \left(\frac{h_t - \eta_1}{H_t - \eta_1}\right)^{\frac{i\xi pl}{\alpha}} \left(\frac{\eta_2 + \hat{h}_t}{\eta_2 + \hat{H}_t}\right)^{\frac{ql}{\alpha}} \left(\frac{\hat{h}_t - \eta_1}{\hat{H}_t - \eta_1}\right)^{\frac{pl}{\alpha}}, \quad \text{(7b)}$$

where $h_t := e^{-\alpha(T'-t)}$, $H_t := e^{-\alpha(T'-T)}$, $\hat{h}_t := -i\xi h_t$, $\hat{H}_t := -i\xi H_t$, $\phi_t^{T'} := -i\xi \ln(F_t^{T'}) + (i\xi - \xi^2)\int_t^{T'} \frac{\sigma_s^2}{2}e^{-2\alpha(T'-s)}ds$ and $\xi := a + ib$, $a, b \in \mathbb{R}$.

A proof of Proposition 2.1 can be found in Appendix A.[6]

2.2 Pricing on the spot-price

We can obtain the pricing formula for the spot price S_t by applying Proposition 2.1 with $Y_T(Y_t) = S_T(S_t)$, as given by (2); which leads to the following pricing formula.

Proposition 2.2 *Let $S_T(S_t)$ be the process given by (2), then the price of a T-maturity European call option written on the underlying S_t with strike price K is given by*

$$V(S_t, t) \;\; = \;\; \frac{e^{-r(T-t)}}{2\pi} \int_{-\infty+ib}^{\infty+ib} \Psi(-\xi)\frac{-K^{1+i\xi}}{\xi^2 - i\xi}d\xi, \quad \max(1, \zeta) < b < \theta; \quad \text{(8a)}$$

$$\Psi(-\xi) \;\; := \;\; e^{\phi_t^T} \left(\frac{\eta_2 + \hat{h}_t}{\eta_2 - i\xi}\right)^{\frac{ql}{\alpha}} \left(\frac{\hat{h}_t - \eta_1}{-\eta_1 - i\xi}\right)^{\frac{pl}{\alpha}}, \quad \text{(8b)}$$

where $h_t := e^{-\alpha(T-t)}$, $\hat{h}_t := -i\xi h_t$, $\phi_t^T := -i\xi g(T) - i\xi(\ln S_t - g_t)h_t + i\xi\lambda\int_t^T \sigma_s h_s ds + i\xi\frac{kl}{\alpha}(1 - h_t) - \frac{\xi^2}{2}\int_t^T \sigma_s^2 h_s^2 ds$ and $\xi := a + ib$, $a, b \in \mathbb{R}$.

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[6]The singularities in $\Psi(-\xi)$ are avoided by the constraints imposed on the amplitudes of the double exponential distribution, $\eta_1 > 1$ and $\eta_2 > 0$; and by further restricting $\eta_2 \neq bH_t$.

The proof can be obtained in a similar manner as to the proof for Proposition 2.1.[7] Alternatively, we can also note that Proposition 2.2 follows from 2.1 by simply specifying $T = T'$; where we recover the known fact that pricing on a forward which matures on the same date as the option is equivalent to pricing on the spot price itself.

2.3 Calibration

In this article we will price subject to the calibration and the same data set used in [1]. However, the jump parameters have been re-calibrated since we are now assuming a different distribution for the jumps in order to account for both positive and negative jumps. Brechner, Cartea and Figueroa [4] have tested this empirically in the Nord-Pool market, by modifying the filter described in [1] in order to discriminate between positive and negative jumps. In this paper, we apply this filter and again find evidence for both type of jumps in the market of England and Wales.

Hence, the mean reversion rate and the average market price of risk parameters are those used in [1], which we reproduce in Table 1. The jumps parameters in (4) have been re-calibrated and are presented in Table 2.

α	$\langle \lambda \rangle (\%)$
0.2853 (0.2431, 0.3274)	-0.2481 (-0.2550, -0.2413)

Table 1: Mean reversion rate α and average (denoted by $\langle \cdot \rangle$) market price of risk; the 95% confidence bounds are presented in parenthesis.

jump	probability	amplitude	jumps p.a.
J^+	$p = 0.6$	$\eta_1 = 1.85$	$l^+ = 5.15$
J^-	$q = 0.4$	$\eta_2 = 2.13$	$l^- = 3.43$

Table 2: $J^\pm$ denotes positive and negative jumps; p and q the probabilities, $\eta_{1,2}$ the amplitudes and $l^\pm$ the annualized frequency of the positive and negative jumps respectively (i.e. $l^+ = pl$, and $l^- = ql$ where l is the total annualized number of jumps per year and $l = l^+ + l^-$).

Another issue which must be reviewed is the impact of the speed of mean reversion in spot-based models. Cartea and Figueroa [1] report for England and Wales values of the mean reverting parameter α such that spot prices mean-revert in approximately

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[7]This time, apart from considering $\eta_1 > 1$ and $\eta_2 > 0$ we must consider additionally $\eta_1 \neq b$ in order to avoid a singularity.

three days. This has marked consequences on the range of applicability of one-factor spot-based MRJD models when pricing on the forward. This can be easily seen in Figures 1 and 2, where in both cases the maturity of the forward T' is fixed and where the surfaces show the variation on the forward as we move in t closer to maturity and for different values of the spot price at time t.
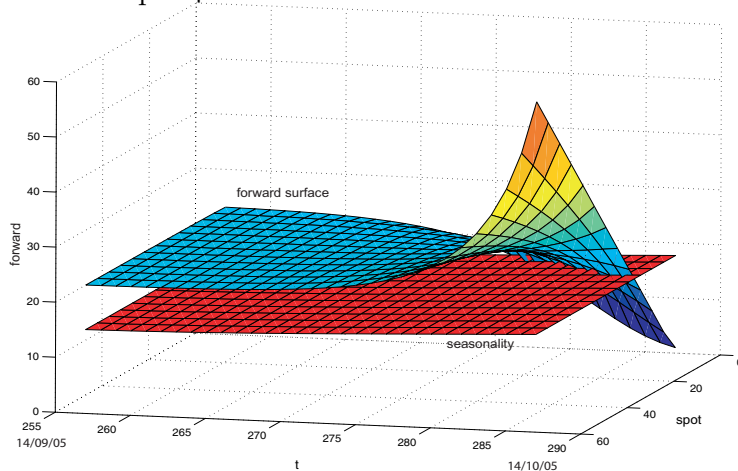

Figure 1: Forward surface with T' fixed and varying t, $\alpha\uparrow$.
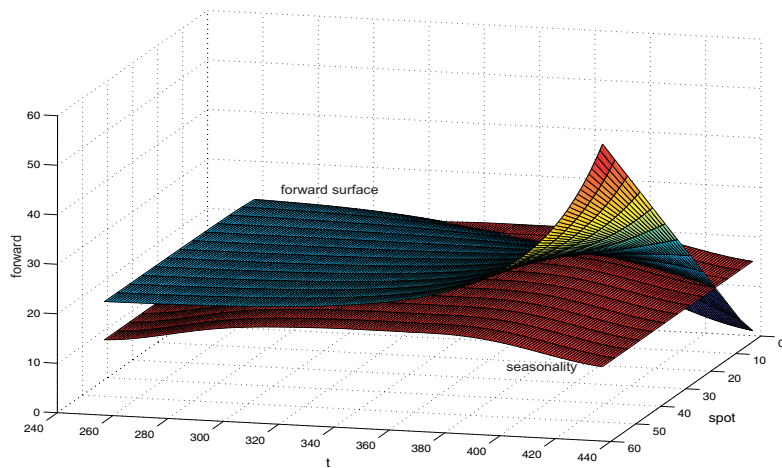

Figure 2: Forward surface with T' fixed and varying t, $\alpha\downarrow$.

In the first figure we observe the calibrated forward surface in (5) for the market of England and Wales.[8] It is easier to interpret this graph from right to left. This is,

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[8]We will refer to the calibrated level of mean reversion as the 'high level', denoted by $\alpha\uparrow$. It implies a daily value of $\alpha = 0.2853$, which we need to multiply times 250 or 365 for an annualized estimate of the mean reversion rate.

at $t = T'$ we observe the initial spot price. As we move towards the left in t we see that the effect of the initial spot price starts to dissipate. After 15 days the forward will not reflect at all the information of the initial spot price. The effect of a high speed of mean reversion in the spot price is to flatten the dynamics of the forward curve towards a level which is a parallel shift of the deterministic seasonality.[9] In the second figure, we observe that for a lower speed of mean reversion the dynamics of the forward curve varies for about 60 days.[10]

The limitations of the short-dynamics of the forward in this model could be improved by considering a second factor. For instance, Schwartz and Smith [19] developed a two-factor model that allows for mean-reversion in short-term prices and uncertainty in the equilibrium level to which prices revert. They obtain good results for oil prices, however this model would not be a realistic choice in electricity where, as has been mentioned already, the presence of large large spikes calls for the incorporation of jump processes.

However, if pricing on forwards on a short-term horizon a one-factor MRJD model might be suitable, since it will describe the underlying dynamics of the forward for short maturities. On the other hand, if we were pricing on the spot-price, as is the case of the swing option considered in this paper, the shortcoming of the fast flattening of the forward curve after a couple of weeks becomes irrelevant, since we price on the spot price and the value of the swing option will be mainly determined by the probability and magnitude of the jumps.

3   Pricing the Swing Option

In Section 2 we presented semi-analytical formulæ for the price of a vanilla option written on a forward and on the spot-price under a MRJD model. These expressions are only applicable for the European option; to price the Bermudan option we need to resort to numerical schemes. As we have discussed in Section 1, there are several approaches and techniques. We follow here the Least-Squares Monte Carlo (LSM) algorithm proposed by Longstaff and Schwartz [17] and price the swing option writing an extension of this algorithm to allow for multiple exercise opportunities.

The key insight of the LSM methodology is to compute the expected payoff from continuation by regressing ex post realized payoffs from continuation on functions

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[9]In this case, this parallel shift is above the seasonality function since the estimated market price of risk in [1] is negative; thus reflecting that in the short term the demand side has greater incentives to hedge their prices and are even willing to pay above the market.

[10]We will refer to this low level of mean reversion by $\alpha\downarrow$; which corresponds to a daily value of $\alpha = 0.027$ (or 10 annualized), which implies it mean reverts in approximately 36 days, which is more typical of gas markets.

of the values of the state variables. The fitted value from this regression provides a direct estimate of the conditional expectation function. By estimating the conditional expectation function for each exercise date, one obtains a complete specification of the optimal exercise strategy along each path.

In order to use the LSM algorithm to price options with multiple-exercise opportunities we need to modify the original algorithm to allow for this extra feature. Similarly to what is done in the binomial/trinomial 'forest' approach, where an extra dimension is added by considering layers of a tree, the extension of the LSM algorithm is based on adding an extra dimension to the LSM matrices.[11]

This is, we work now with cash flow tensors (rather than cash flow matrices) of three dimensions, e.g. $\mathbf{P}_{t_k} \in \mathbb{R}^{N_r \times M \times n}$, where N_r is the total number of replicated paths, M the total number of sampling dates $t_0 < t_1 < t_2 \ldots t_M = T$ and n the number of exercised opportunities (out of a total $N \leq M$). For instance, in the LSM algorithm at each possible exercise date t_k the exercise condition is given by $\mathbf{P}(S_{t_k}, w) > C(w, t_k)$, where $\mathbf{P}(S_{t_k}, w)$ represents the intrinsic value for the w-path at time t_k and where the continuation value $C(w, t_k)$ is given by

$$C(w, t_k) = \mathbb{E}^{\mathcal{Q}} \left[\sum_{j=k+1}^{M} D(t_k, t_j) B(w, t_j, t_k, T) \middle| \mathcal{F}_{t_k} \right], \qquad (9)$$

where $D(t_k, t_j)$ is the discount factor from t_k to t_j and $B(w, t_j, t_k, T)$ represents the cash flow given by path w if the early exercise did not take place.

When extending this algorithm such a condition will now look like: $\mathbf{P}(S_{t_k}, w) + C(w, t_k, n-1) > C(w, t_k, n)$; where $C(w, t_k, n-1)$ is the continuation value obtained by regressing the appropriate future cash flows into their respective basis function on the matrix with one fewer exercise right $(n-1)$.[12]

3.1 Properties of the Swing Option

When valuing an exotic OTC option as the one we are valuing, we often face the problem of knowing if the price we obtain is reasonable or meaningful. One way of testing the performance of the algorithms is to reduce the models to a GBM or any other model where we have a clear benchmark. However, we would often like to have more reassurance about the prices we obtain. It is useful then to review some general

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[11]In the case of the binomial forest approach Jaillet et al. [12] represent this procedure as a multi-layer set of planes, where each plane contains a binomial tree and where moving from the plane i to $i+1$ for instance represents moving from a tree to another tree with one less exercise right.

[12]Dör [6] provides a detailed description of how to extend the LSM for swing options. We refer the reader to this reference for further details.

properties and bounds for a swing option. By explicitly calculating these bounds we can validate our results. In the literature of swing options different approximate bounds have been calculated for different proposed solutions and approximations to the problem. However, we discuss here some general properties and bounds regardless of the model and the numerical scheme. Specifically, we will address the bounds described in Jaillet, Ron and Tompaidis [12].

For a swing option where the holder buys (or sells for a put) the entire volume, i.e. one unit of the underlying commodity at a strike price K with N rights, the following properties and bounds must be satisfied:

1. *Case $N = 1$.* For one exercise right the value of the swing option is that of the Bermudan option.

2. *Case $N = M$.* When the number of rights is equal to the number of exercise dates, the value of the swing option is given by the value of a strip of European options expiring at the exercise dates $t_0, t_1, \ldots t_N$.

3. *Lower bound.* For $N < M$ the lower bound is given by the maximum of the strip of call options among all possible sets of N different exercise dates covering the entire maturity of the contract.

4. *Upper bound.* For $N < M$ the upper bound is given by N identical Bermudan options.

The first two cases are obvious and do not need any further explanation. The lower bound can be interpreted in the following way. When holding a strip of call options, the exercise dates are fixed, whereas with the swing option, we can choose to exercise in those fixed dates of the strip of calls, but we do not need to. This extra flexibility gives a higher value to the swing option. For the upper bound we can say that the owner of N Bermudan options can exercise more than once during the same day. The swing option, although gives the holder the right to exercise N-times in any of those same exercise dates, imposes a restriction by allowing only one exercise per day. This restriction in turn, decreases the value of the swing option with respect to the value of N-Bermudan options.

In Section 4 we show the calculated bounds for the pricing of a swing option with 12 exercise dates and varying number of rights. We will show in effect, that as the number of rights tends to the number of exercise dates, the value of the swing option converges to the value given by the corresponding strip of call options; therefore recovering the case $N = M$.

4 Results

First we price a swing option with 12 exercise points and up to 6 exercise rights. In order to test the reliability of the code we reduce the MRJD model to a GBM and compare the results with those obtained by Ibáñez [10]. We replicate the price of a down-swing, i.e. a put option, for the same set of parameters used in Table 3.1 of [10].

S_0	τ	r	σ	$n_1(\%)$	$n_2(\%)$	$n_3(\%)$	$n_4(\%)$	$n_5(\%)$	$n_6(\%)$
35	0.25	0.0488	0.25	0.1564	0.1766	0.0854	0.0544	0.0952	0.0332
35	0.25	0.0488	0.5	-0.0604	-0.1406	-0.1315	-0.0558	-0.0322	-0.0841
35	0.25	0.0954	0.25	0.0962	0.1419	0.1281	0.1575	0.0940	0.0411
35	0.25	0.0954	0.5	-0.3454	-0.0970	-0.0926	-0.0454	-0.0533	-0.0983
35	0.5	0.0488	0.25	0.0502	0.0093	0.0627	0.0568	0.0420	0.0000
35	0.5	0.0488	0.5	-0.0633	-0.1601	-0.1441	-0.0205	-0.0194	-0.0469
35	0.5	0.0954	0.25	0.2407	0.1973	0.1061	0.0903	0.0891	0.0273
35	0.5	0.0954	0.5	0.0028	-0.0634	-0.0952	-0.0470	-0.0059	-0.0297
40	0.25	0.0488	0.25	-0.4848	-0.1609	-0.0704	-0.1471	-0.1440	-0.1263
40	0.25	0.0488	0.5	-0.3401	-0.1175	-0.0372	-0.1143	-0.0937	-0.1453
40	0.25	0.0954	0.25	-0.2116	-0.2699	-0.2530	-0.1609	-0.1481	-0.1615
40	0.25	0.0954	0.5	-0.2684	-0.2233	-0.1374	-0.1054	-0.0864	-0.1003
40	0.5	0.0488	0.25	-0.2003	-0.1936	-0.1696	-0.1012	-0.1002	-0.1168
40	0.5	0.0488	0.5	-0.1498	-0.1888	-0.1015	-0.0727	-0.0681	-0.0983
40	0.5	0.0954	0.25	-0.1581	-0.4135	-0.2032	-0.1061	-0.1582	-0.1524
40	0.5	0.0954	0.5	-0.1638	-0.1920	-0.1460	-0.1120	-0.0872	-0.0471
45	0.25	0.0488	0.25	0.1460	-0.1554	-0.0092	-0.0295	0.0063	-0.2392
45	0.25	0.0488	0.5	-0.0844	0.0874	-0.0071	-0.1909	-0.1491	-0.1712
45	0.25	0.0954	0.25	0.6052	-0.0461	0.0763	0.1224	0.0600	-0.2571
45	0.25	0.0954	0.5	-0.2612	-0.0717	-0.1797	-0.1644	-0.1962	-0.2218
45	0.5	0.0488	0.25	-0.2899	-0.3615	-0.2484	-0.2925	-0.3314	-0.4300
45	0.5	0.0488	0.5	-0.3147	-0.2227	-0.1367	-0.1540	-0.1549	-0.1578
45	0.5	0.0954	0.25	0.0138	-0.7531	-0.5010	-0.4110	-0.4993	-0.5401
45	0.5	0.0954	0.5	-0.4557	-0.2812	-0.1341	-0.1424	-0.2071	-0.2181

Table 3: Comparison of values reported in Ibáñez [10] (Table 3.1) for a GBM with $M = 12$ exercise points, n exercise rights and strike price $K = 40$. The results in each column labeled $n_i(\%)$, $i = 1, 2, \ldots, 6$ represent the percentage difference between Ibáñez' and our results. S_0 represents the spot price, τ the maturity in years, r the risk free interest rate and σ the volatility.

4.1 Bounds

To calculate the upper bound for the swing option, we need to evaluate the Bermudan option, by employing a LSM algorithm (or by setting $N = 1$ in the algorithm for the swing option). The calculation of the lower bound however, is constrained by the number of exercise rights and sampling dates considered, since these two parameters have a direct impact on the computational time required to compute this bound. For instance, if we are pricing a swing option with 12 exercise dates and 6 exercise rights we have a total of 462 different ways of arranging sets with European options that cover the total life of the contract. In average, in our simulations, as will be detailed below, it takes 17 seconds to price an European call option by Monte Carlo simulations with 100,000 (50,000 plus 50,000 antithetic) paths. This means that to compute the lower bound for one particular value of S_t we would need at least 2 hours. In the present work we dramatically decrease this time by calculating the strip of call options using the pricing formula in Proposition 2.2, where each European call option is obtained in an average of 0.14 seconds, thus requiring around one minute to obtain the same bound.

4.1.1 Pricing with CFT

In order to quantify the accuracy of the CFT results by comparing the semi-analytical solution with an analytical solution we first reduce the proposed MRJD model to a purely mean reverting model with constant level of mean reversion, namely, to the model originally presented by Schwartz [18].

In Table 3 we contrast the results obtained when pricing numerically with the CFT technique and with MC for Schwartz' model.[13] In the first row we show the absolute error, Δ, of the MC simulation and the CFT pricing with respect to the closed-form solution. As we can see from the results, for values of the forward, out-of and in-the-money, the absolute error of the MC simulations after 100,000 (50,000 plus 50,000 antithetic) paths is much larger than that obtained with the CFT technique. Moreover, as seen in the second row of Table 1, the time τ, needed to compute each MC value is one hundred times more than the time needed to compute the corresponding CFT values.

The results for the proposed MRJD model are shown in Figure 3, where we contrast the CFT results obtained with Proposition 2.1 with MC simulations of 100,000 (50,000 plus 50,000 antithetic) averaged paths. We observe that both results match perfectly; however, with the CFT technique we obtain these results at least 100 times faster.

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[13]All the results quoted where computed in a *Pentium* 4, with 3.20 GHz of speed and 2 GB of RAM memory.

	$F = 10$		$F = 25$		$F = 40$	
	CFT	MC	CFT	MC	CFT	MC
Δ	7.66×10^{-12}	5.10×10^{-3}	3.54×10^{-11}	2.86×10^{-2}	7.52×10^{-10}	7.44×10^{-2}
τ	0.13 sec.	17.06 sec.	0.14 sec.	17.00 sec.	0.16 sec.	14.94 sec.

Table 4: Absolute error and computational times when pricing a call option on a forward using CFT and MC methods. Δ is the absolute error of the numerical solution with respect to the analytical price and τ is the time needed to compute each value. The analytical values of the call options for forward values of $F = 10$, $F = 25$ and $F = 40$ are respectively $C(10) = 0.23$; $C(25) = 5.14$ and $C(40) = 14.94$. The parameters used are: strike $K = 25$; initial time $t = 0$; option's maturity $T = 1$ year; forward's maturity $T' = 1.5$ years; mean reversion rate $\alpha = 1.18$; volatility $\sigma = 1.77$; risk-free interest rate $r = 0.15$; number of steps in the MC simulation $n = 100$ and number of paths simulated in the MC approximation $m = 100,000$.
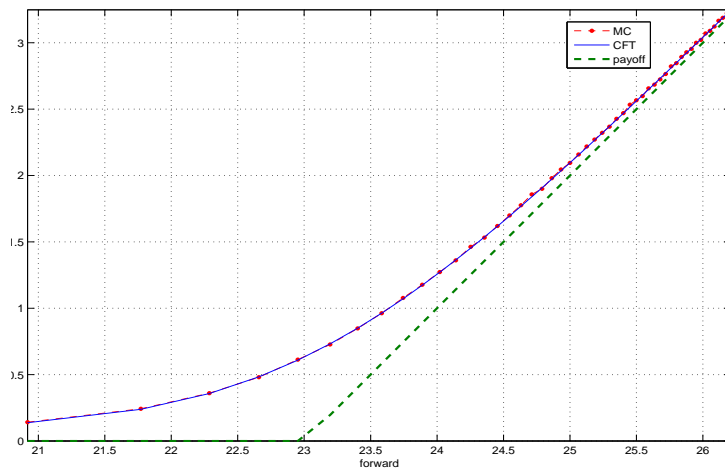

Figure 3: Comparison of CFT pricing and MC simulations for the MRJD model using the calibrated mean reversion rate $\alpha\uparrow$. The dashed line represents the payoff of the option, the solid line the price obtained with the CFT approximation and the solid dots the value obtained with antithetic-MC using 100,000 (50,000 plus 50,000 antithetic) paths.

4.2 The swing option

Figures 4 and 5 show the value of the swing option as a function of the number of exercise rights under a GBM.
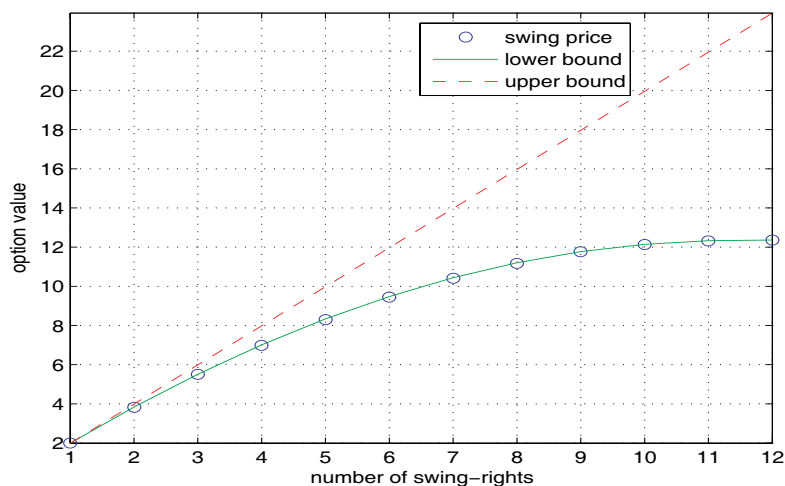

Figure 4: Value of an up-swing under a GBM as a function of the number of exercise rights, N. Parameters: $S_0 = 35$, $K = 40$, $r = 0.0954$, $T = 0.25$, $\sigma = 0.5$, $M = 12$.
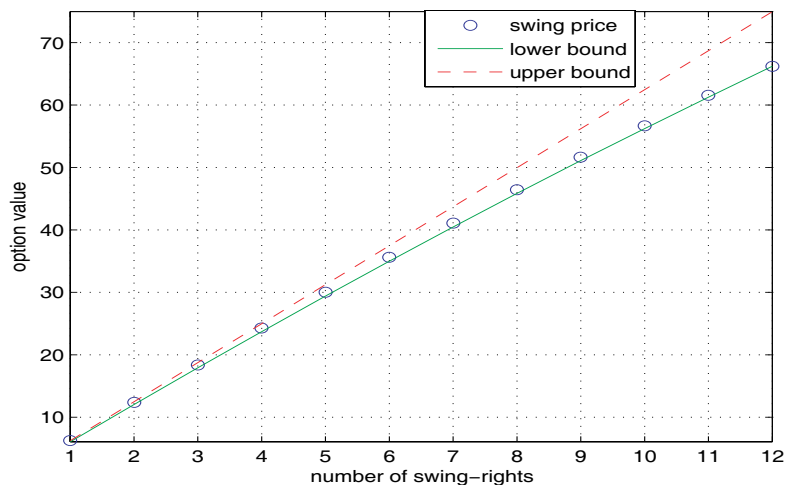

Figure 5: Value of a down-swing under a GBM as a function of the number of exercise rights, N. Parameters: $S_0 = 35$, $K = 40$, $r = 0.0954$, $T = 0.25$, $\sigma = 0.5$, $M = 12$.

We observe that for the call option under a GBM the value of the swing option is given by the lower bound, as expected, since under a GBM a call option which does not pay dividends does not offer the holder any incentive to exercise early. For the put option, we observe that the value of the swing option is above the lower bound,

and this value converges to the lower bound as we increase the number of exercise rights to the limit $N = M$, where as discussed by the second property in Section 3.1, we recover as the price of the swing the lower bound, i.e. the strip of 12 call options.

In Figures 6 and 7 we show the value of the swing options as a function of the number of exercise rights under the MRJD model.
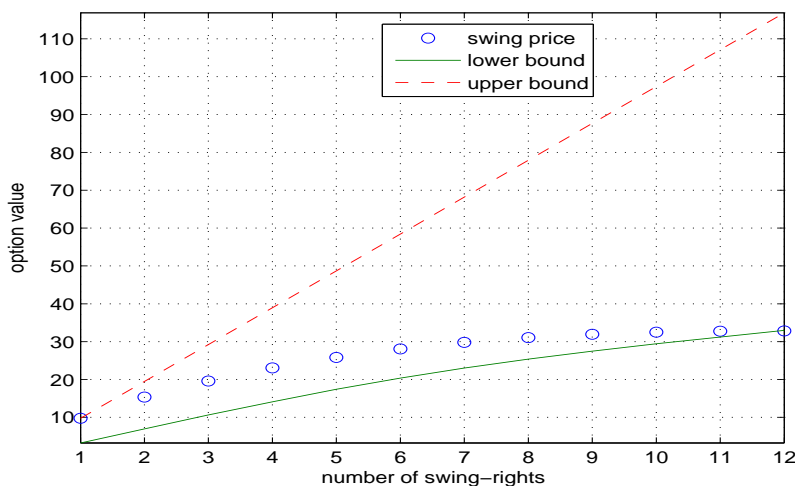

Figure 6: Value of an up-swing option with strike price $K = 18$ under the calibrated MRJD model as a function of the number of exercise rights, N.
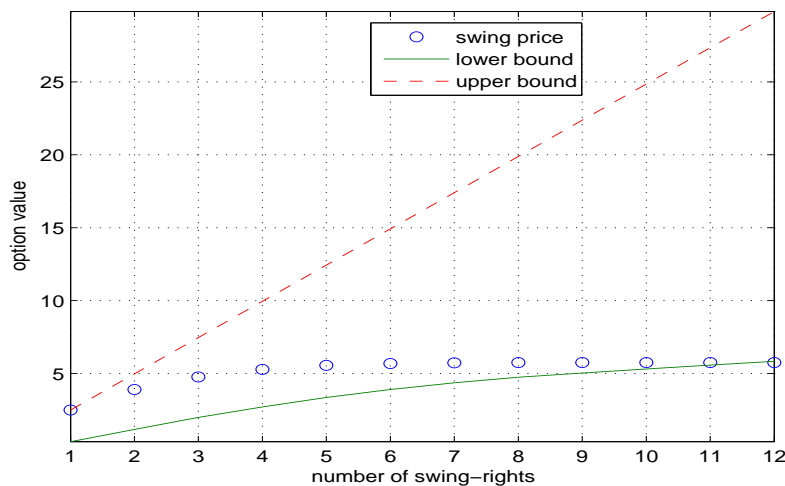

Figure 7: Value of a down-swing option with strike price $K = 18$ under the calibrated MRJD model as a function of the number of exercise rights, N.

For values of $N < M$ we observe now that both the call and the put (the up/down swings) exhibit values which are clearly above the lower bound. The fact that we now

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observe this premium above the lower bound is mainly attributed to the inclusion of jumps in the model. Also as expected, for $N = M$ the price of the swing converges to the lower bound.

In Figure 8 we compare the price of the swing option for different number of exercise rights under the MRJD model, the MRJD model with a lower mean reversion rate ($\alpha = 10$ annualized) and a GBM with $\sigma = 2.5$ (which is the historical volatility for the entire data set we would use if we were to model using Black-Scholes). For the GBM the value of the swing option coincides with its lower bound, as discussed previously. For the MRJD model with both high and low mean reversion rate we observe again that the swing option is clearly above the lower bound. However, for the lower mean reversion rate the value of the swing option is higher, as would be expected since the spot price reverts in a much longer time to its mean. For the case $N = M$, both the GBM and the MRJD models converge to their lower bounds.
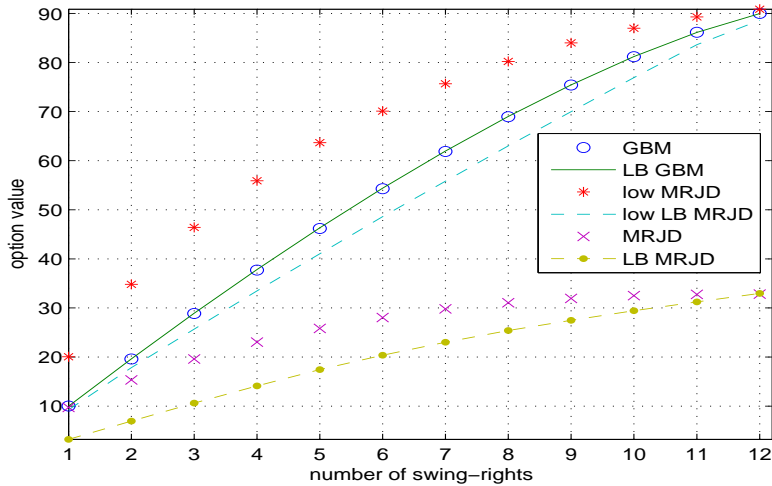

Figure 8: Value of an up-swing and its lower bound as a function of the number of exercise rights n under a GBM with volatility $\sigma = 2.5$, a MRJD model with lower mean reversion rate ($\alpha = 10$ annualized) and the calibrated MRJD; $K = 18$.

Finally, we performed a sensitivity analysis with respect to the strike price K and the number of exercise rights n. On Figure 9 we show a surface plot where the bottom surface is the value of the lower bound and the top surface is the value of the swing option. Examining this plot we can verify three properties. First, the value of the swing option is increasing with the number of exercise rights for the different values of K. Second, the value of the swing option tends to the lower bound on the limit $N \to M$ for every strike considered. Third, the value of the swing option increases inversely with K; thus reflecting that a supplier selling this contract would offer those users who are willing to be curtailed first (and therefore provide a lower strike price)
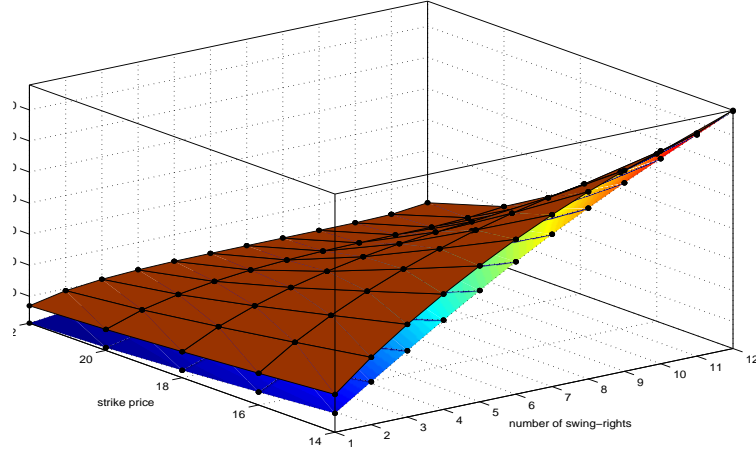
a higher discount on the forward.

Figure 9: Value of an up-swing and its lower bound as a function of the number of exercise rights n and strike price K under the calibrated MRJD model. The bottom surface represents the lower bound and the top surface the price of the swing option.

4.3 The discount on the forward

As we have discussed earlier, a supplier can replicate an interruption strategy by shorting a forward contract (covering a certain period of time) and longing an up-swing option which gives him the right to exercise N times, effectively canceling the obligation of supplying a unit of electricity when the option is exercised and compensating the user with a pre-arranged shortage cost K. The consumer holds the opposite position on this portfolio, namely $\Pi_t = F_t^T - C^{sw}(S_t; K, T)$. The up-swing option in this portfolio clearly represents a net discount on the forward contracted for that period from the consumer's perspective. In Table 5, the second, fourth and sixth columns show respectively the value of the up-swing valued under three different settings: a GBM model, the presented MRJD but with a low level of mean reversion of $\alpha = 10$ (annualized) and the MRJD model under the current calibration.[14]

The discounts shown are computed in each case with respect to a quoted forward for the period, which is $F_{t,T} = 30$MWh per day for daily delivery of one unit of electricity. As expected, the discount increases with the number of exercise rights, hence the customer who is willing to be interrupted more times will receive a higher compensation.

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[14]For the GBM values in Table 5 and 6 we have used $\sigma = 2.5$, which is the resulting historical volatility from the entire data set we would use if we were to model with the Black-Scholes model.

n	GBM	discount	MRJD ($\alpha\downarrow$)	discount	MRJD ($\alpha\uparrow$)	discount
1	10.0120	0.3708%	20.0583	0.7429%	9.7396	0.3607%
2	19.6180	0.7266%	34.8036	1.2890%	15.3450	0.5683%
3	28.8870	1.0699%	46.4124	1.7190%	19.6130	0.7264%
4	37.7090	1.3966%	55.8967	2.0702%	23.0560	0.8539%
5	46.2000	1.7111%	63.667	2.3580%	25.8360	0.9569%
6	54.2950	2.0109%	70.1071	2.5966%	28.0580	1.0392%
7	61.8830	2.2920%	75.657	2.8021%	29.8050	1.1039%
8	68.9630	2.5542%	80.2259	2.9713%	31.0710	1.1508%
9	75.4160	2.7932%	83.9911	3.1108%	31.9560	1.1836%
10	81.1870	3.0069%	86.9844	3.2216%	32.5100	1.2041%
11	86.1450	3.1906%	89.2961	3.3073%	32.7580	1.2133%
12	89.9990	3.3333%	90.8624	3.3653%	32.8290	1.2159%

Table 5: Up-swing option prices and discounts (for n exercise rights) on a contracted forward covering one quarter of a year, where the quoted price is $F_{t,T} = 30$MWh per day for daily delivery of one unit of electricity; the strike price is $K = 18$.

As we can observe from this table, the discounts obtained with a MRJD model with lower mean reversion rate are higher than those obtained with a GBM, since the GBM does not incorporate any jumps. However, for a higher (and more realistic) level of mean reversion the price of the swing option decreases significantly, and the discounts are approximately halved with respect to the other models. This shows that the value of the swing option is sensitive to the estimation of the mean reversion rate.

As we have discussed earlier, we can also replicate an interruption strategy with a down-swing option by considering a consumer who owns a portfolio $\Pi_t^* = \widetilde{F}_t^T + P^{sw}(S_t; K, T)$. In this case the consumer will be prepared to pay a premium above the forward price in order to have the choice of exercising the down-swing option and therefore selling back to the supplier a unit of electricity at a fixed price K, realizing a profit $K - S_{t_i}$, where t_i is any of the chosen exercise dates.

In Table 6, the second, fourth and sixth columns show respectively the value of the down-swing valued under three different settings: a GBM model, the presented MRJD but with a low level of mean reversion of $\alpha = 10$ (annualized) and the MRJD model under the current calibration. The premiums shown are computed in each case with respect to the quoted forward for the period, which is $F_{t,T} = 30$MWh per day for daily delivery of one unit of electricity. As we may observe from this table, the premium increases with the number of exercise rights, as expected.

n	GBM	premium	MRJD ($\alpha\downarrow$)	premium	MRJD ($\alpha\uparrow$)	premium
1	7.7773	0.2880%	4.1265	0.1528%	2.4862	0.0921%
2	15.245	0.5646%	7.7045	0.2854%	3.896	0.1443%
3	22.356	0.8280%	10.7799	0.3993%	4.7622	0.1764%
4	29.1	1.0778%	13.4143	0.4968%	5.2767	0.1954%
5	35.452	1.3130%	15.6334	0.5790%	5.5512	0.2056%
6	41.362	1.5319%	17.4741	0.6472%	5.6765	0.2102%
7	46.811	1.7337%	18.9743	0.7028%	5.7267	0.2121%
8	51.758	1.9170%	20.1636	0.7468%	5.742	0.2127%
9	56.124	2.0787%	21.0579	0.7799%	5.7455	0.2128%
10	59.821	2.2156%	21.7008	0.8037%	5.746	0.2128%
11	62.715	2.3228%	22.1053	0.8187%	5.7461	0.2128%
12	64.546	2.3906%	22.2982	0.8259%	5.7461	0.2128%

Table 6: Down-swing option prices and premiums (for n exercise rights) on a contracted forward covering one quarter of a year, where the quoted price is $F_{t,T} = 30$MWh per day for daily delivery of one unit of electricity; the strike price is $K = 18$.

5 Conclusions

In this paper we have discussed the pricing of multi-interruptible contracts under a MRJD model for the market of England and Wales. The main contribution of this paper is three-fold. First, we have extended the existing literature on interruptible contracts by considering a portfolio composed of a forward and a swing option, which allows for multiple-interruption strategies rather than only a decision of interruption at expiry.

Second, for the valuation of the swing option, we have presented a computationally-efficient way of calculating the lower bound for this option by the use of semi-analytical formulæ which result from inverting the corresponding characteristic function on the complex plane. With this technique we are able to reduce in more than 100 times the computational time required to compute the lower bound if we were using MC techniques. For the valuation of the swing option we have applied an extension of the Longstaff-Schwartz algorithm. In order to test the accuracy of the algorithm we have reduced the MRJD to a GBM and compared our results with those independently obtained by Ibáñez [10], where he uses a Monte Carlo technique which is not based on the LSM algorithm. For every set of parameters used in [10] we were able to replicate those results very accurately, the absolute percentage difference between both results is lower than 0.6% for any of the different set of parameters tested. Moreover, we tested the performance of the algorithm by examining the value of the swing option when the number of swing rights is equal to the total number of exercise opportunities

(the case $N = M$). For the different cases considered it has been shown that the value of the swing option converges convexly as $N \to M$ to its lower bound, as expected.

Third, we have also discussed and priced down-swing options which enable us to replicate a multi-interruption strategy by transferring the decision of interruption to the user which would enable large users or retailers to benefit from sudden falls on the price of electricity.

The results on the discounts (and premiums for the down-swing) reflect that these are increasing with the number of rights, as one would expect, since the owner of the option has greater flexibility. We have also considered the sensitivity of the valuation with respect to the mean reversion rate and the strike price for the contract. Our results confirm that the value of the swing option decreases with increasing mean reversion rate and increases with decreasing strike price, as expected.

Finally, the values of the discounts and premiums obtained with the calibrated MRJD model are relevant, specially considering that the current valuation has been performed for delivery of a single unit of electricity. When taking into account the large volumes typically traded, the discounts may prove a real incentive for market participants to enter into these contracts.

A   Proof of Proposition 2.1

Noting that $-\alpha(T' - t) = \alpha(T - T') - \alpha(T - t)$, we can rewrite (3) as

$$F_t^{T'} = \underbrace{e^{(x_t - g_t)e^{-\alpha(T-t)}}}_{\Omega} e^{e^{\alpha(T-T')}} e^{g(T') - \lambda \int_t^{T'} \sigma_s e^{-\alpha(T'-s)} ds + \frac{1}{2} \int_t^{T'} \sigma_s^2 e^{-2\alpha(T'-s)} ds + \int_t^{T'} \Phi(s) l ds}.$$
(A 1)

Now, solving for Ω from (2) and noting that the integrals in (3) can be written as $\int_t^{T'} f(\cdot) ds = \int_t^T f(\cdot) ds + \int_T^{T'} f(\cdot) ds$ we obtain after rearranging some terms

$$
\begin{aligned}
F_t^{T'} &= e^{g(T') + (x_T - g_T)e^{-\alpha(T'-T)} - \lambda \int_T^{T'} \sigma_s e^{-\alpha(T'-s)} ds + \frac{1}{2} \int_T^{T'} \sigma_s^2 e^{-2\alpha(T'-s)} ds + \int_T^{T'} \Phi(s) l ds} \\
&\times e^{\frac{1}{2} \int_t^T \sigma_s^2 e^{-2\alpha(T'-s)} ds + \int_t^T \Phi(s) l ds - \int_t^T \sigma_s^2 e^{-\alpha(T'-s)} dZ_s - \int_t^T e^{-\alpha(T'-s)} \ln J_s dq_s}.
\end{aligned}
$$
(A 2)

Finally, noting from (3) that the first line of (A 2) is precisely $F_T^{T'}$, we may finally write the forward at time T maturing at time T' conditional on the forward at time t maturing at time T', this is

$$F_T^{T'} = F_t^{T'} e^{-\frac{1}{2} \int_t^T \sigma_s^2 e^{-2\alpha(T'-s)} ds - \int_t^T \Phi(s) l ds + \int_t^T \sigma_s^2 e^{-\alpha(T'-s)} dZ_s + \int_t^T e^{-\alpha(T'-s)} \ln J_s dq_s}.$$
(A 3)

We then calculate the characteristic function as $\Psi(\xi) := \mathbb{E}_t \left[e^{-i\xi \ln F_T^{T'}} \right]$ to arrive after some calculations to (7b).

We now need to transform the payoff of the option and study the region of analyticity of the integrand. These two issues are addressed in the following sub-sections.

A.1   Transforming the payoff of a European Call

The CFT of a function $f(x)$ can be defined as

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} e^{i\xi x} f(x) \, dx = \mathcal{L}\left[f(x) \right],$$
(A 4)

where $\xi = a + ib$ and $a, b \in \mathbb{R}$.

Likewise, the inverse CFT can be defined as

$$f(x) = \frac{1}{2\pi} \int_{-\infty + ib}^{\infty + ib} e^{-i\xi x} \hat{f}(\xi) \, d\xi.$$
(A 5)

The payoff of a European call option written on an underlying F_t is given by

$$C\left(F_T, T \right) = \begin{cases} F_T - K & \text{if } F_T \geq K \\ 0 & \text{if } F_T < 0. \end{cases}$$
(A 6)

By substituting $x_t = \ln F_t$ we may write

$$C\left(x_T, T\right) = \begin{cases} e^{x_T} - K & \text{if } x_T \geq \ln K \\ 0 & \text{if } x_T < \ln K. \end{cases} \tag{A 7}$$

Applying (A 4) and integrating we obtain

$$
\begin{aligned}
\mathcal{L}\left[C\left(x, T\right)\right] &= \int_{-\infty}^{\infty} e^{i\xi x} \max\left(e^x - K, 0\right) dx \\
&= \left. \left(\frac{e^{x(1+i\xi)}}{1+i\xi} - \frac{K e^{i\xi x}}{i\xi} \right) \right|_{\ln(K)}^{\infty} \\
&= \lim_{x \to \infty} \left(\frac{e^{x(1+i\xi)}}{1+i\xi} - \frac{K e^{i\xi x}}{i\xi} \right) - \left(\frac{e^{\ln(K)(1+i\xi)}}{1+i\xi} - \frac{K e^{i\xi \ln(K)}}{i\xi} \right). \tag{A 8}
\end{aligned}
$$

Clearly the first term in the last equation diverges unless we require that $\xi_{Im} > 1$. Hence, since we have defined $\xi = a + ib$, we must restrict $b > 1$, assuring the Fourier transform of the payoff exists.

Hence, we may finally write

$$\mathcal{L}\left[C\left(x_T, T\right)\right] = \frac{-K^{1+i\xi}}{\xi^2 - i\xi}, \qquad \text{for } b > 1. \tag{A 9}$$

A.2 Strip of Regularity

When defining the CFT and its inverse in (A 4) and (A 5) we have not yet said anything about the conditions under which the CFT can be applied.

Following Dettman [5], let us consider any $f(x)$ continuous and with piecewise continuous first derivative in any finite interval.

Let $f(x) = O(e^{\zeta x})$ as $x \to \infty$ and $f(x) = O(e^{\theta x})$ as $x \to -\infty$. Then there exists numbers M, N and R such that the following holds

$$|f(x)| \leq \begin{cases} M e^{\zeta x} & \text{if } x \geq R \\ N e^{\theta x} & \text{if } x \leq -R, \end{cases} \tag{A 10}$$

where M, N and $R > 0$.

We can rewrite (A 4) as

$$\hat{f}(\xi) = \int_{-\infty}^{-R} e^{i\xi x} f(x)\, dx + \int_{-R}^{R} e^{i\xi x} f(x)\, dx + \int_{R}^{\infty} e^{i\xi x} f(x)\, dx \tag{A 11}$$

and using Cauchy's inequality together with (A 10) we have

$$\left| \int_{-\infty}^{-R} e^{i\xi x} f(x) \, dx \right| \leq N \int_{-\infty}^{-R} \left| e^{i\xi x} \right| e^{\theta x} \, dx;$$

$$\leq N \int_{-\infty}^{-R} e^{(\theta - b)x} \, dx$$

$$= \frac{N}{\theta - b} \left(e^{-R(\theta - b)} - \lim_{x \to -\infty} e^{(\theta - b)x} \right). \qquad (A\,12)$$

Clearly (A 12) only exists if $\theta - b > 0 \Leftrightarrow b < \theta$.

Likewise,

$$\left| \int_{R}^{\infty} e^{i\xi x} f(x) \, dx \right| \leq M \int_{R}^{\infty} e^{x(\zeta - b)} \, dx, \qquad (A\,13)$$

which only exists if $\zeta - b < 0 \Leftrightarrow b > \zeta$. Thus, the CFT, as defined in (A 4) is well defined when $\zeta < b < \theta$.

Under rather general conditions for a function $\hat{f}(\xi)$, the inverse transform can be computed by integrating along any line parallel to the real axis lying in the strip of analyticity of $\hat{f}(\xi)$,

$$\frac{1}{2\pi} \int_{-\infty + ib_1}^{\infty + ib_1} e^{-i\xi x} \phi(\xi) \, d\xi = \frac{1}{2\pi} \int_{-\infty + ib_2}^{\infty + ib_2} e^{-i\xi x} \phi(\xi) \, d\xi, \qquad (A\,14)$$

where b_1 and b_2 are any real numbers between ζ and θ.

This last equation can be easily proved by applying Cauchy's theorem in a rectangular contour lying between ζ and θ, of horizontal sides b_1, b_2 and vertical sides $T, -T$, and taking the limit as $T \to \infty$.

Cauchy's theorem states that $\oint_C f(\xi) \, d\xi = 0$, if the integral is taken along a curve which encloses positively an analytical region. The integral cancels on the sides of the rectangle in the limit $T \to \pm\infty$, and since $C = C^1 + C^2 + C^3 + C^4$ we are left with $\int_{C^1} f(\xi) \, d\xi = \int_{C^3} f(\xi) \, d\xi$, which is (A 14).

According to Lewis [15], it can be shown that for most examples involving option pricing, $\zeta < 0$ and $\theta > 1$. Furthermore, as we have seen in (A 9) the transformed payoff is well defined for values of $b > 1$.

Hence, having shown that the CFT and its inverse are well defined in the strip $\mathcal{A} = \{b \colon \zeta < b < \theta\}$, and that $b > 1$ for a European call option, we price in the strip defined by b s.t. $\max(1, \zeta) < b < \theta$.

$\square$

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