ISSN 1745-8587

School of Economics, Mathematics and Statistics

Birkbeck Working Papers in Economics & Finance

BWPEF 0601

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December 2005

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December 19, 2005

Abstract

This paper analyses the optimal wage contract when firms face demand uncertainty and workers care about employment stability. Workers choose the firm that offers the highest utility taking into account the future lay-off probabilities; firms choose the wage contract that maximises the residual share of the gains from production. For risk-neutral workers this occurs with any efficient wage contract so long as it matches the ex-ante outside option of the workers, i.e. all feasible efficient contracts are optimal. The feasibility is proved for the efficient profit-sharing case. For risk-averse workers with variable effort supply, profit-sharing contracts are further shown to provide effort incentives through both their efficiency wage and performance-related payout effects. The paper thus promotes profit-sharing contracts not only on the grounds of employment stability, but also on the basis of its efficiency and incentive effects.

JEL Classification: J33, J23

*I am grateful to Sir James Mirrlees, Sir Partha Dasgupta, Hamish Low, Alan Manning and Bob Evans for their valuable comments. All errors are the author's. Correspondence to: Kenjiro Hori, School of Economics, Mathematics and Statistics, Birkbeck College, Malet Street, London, WC1E 7HX, UK. e-mail: k.hori@econ.bbk.ac.uk.

Keywords: feasiblity, optimal wage contract, profit-share, efficiency

1 Introduction

This paper investigates the optimal wage contract when firms face product market demand uncertainty and workers care about employment stability. The motivation for the paper comes from my personal experience of working for a large Japanese bank in the 1990s. There our wage contracts were, at least to our understanding, of a profit-sharing form. On the other hand foreign firms competing for the same workers were offering (higher) fixed-wage contracts. These offered no employment guarantee, as opposed to the Japanese firms who, although not explicitly so contracted, were offering a more stable if not a long-term employment environment. The profit-sharing literature spearheaded by Weitzman (1984) explains to great extent this relationship between the form of wage contract and the resulting degree of employment stability. However the question tackled here is whether one form of contract dominates others for profit-maximising firms when they compete for workers, bearing in mind the generally accepted view that at least up until recently, and possibly even now, firms in the two largest economies in the world - the US and Japan - offer broadly speaking very different forms of wage and employment contracts.

The model set-up of this paper is then briefly as follows. Facing different wage contracts offered, the workers, who are initially assumed risk-neutral, choose the firm that offers the highest expected wage income taking into account the probability of being laid-off. There is cost in being laid-off which is the time spent in unemployment, during which time firms compete for workers and the wages adjust to clear the market. In this paper this process is represented by a single-period framework in which all workers are matched to a firm at the beginning of the period ex-ante of product market shocks, but some are laid-off if the realised shock turns out to be negative. A new phase of labour market competition then begins but due to co-ordination problems it takes a whole period for the workers to be fully matched again. Meanwhile production occurs. Firms compete for workers by offering shares of expected surplus gain from production. In equilibrium all workers receive the same pre-shock expected surplus transfer irrespective of the chosen wage contract, and the firms choose the wage contract that provides them with the highest residual share of surplus. This defines the optimal wage contract.

Given a realised market demand then, a firm creates the maximum surplus gain from production when it chooses its output level where the marginal revenue of labour equals its opportunity cost, which in this paper is the unemployment benefit. This defines efficiency. The question is whether a given efficient wage contract is able to provide a high enough expected wage income to match the outside option of the workers in the labour market. This is the question of feasibility. For example if workers have any positive bargaining power (i.e. workers are better off being in work than unemployed), then the efficient fixed wage contract which simply pays the wage equal to the unemployment benefit, is clearly not feasible. Or conversely, a feasible fixed wage contract is not efficient. How about then if we introduce a profit-sharing component and trade-off the fixed-wage component for a higher profit-share? Can we successfully bring the fixed-wage component down to the efficient level without making the workers worse off? The answer to this is not immediately obvious as a decrease in the fixed-wage, which is the marginal cost of labour, has opposing effects on the expected income of a decreased wage level and increased employment stability. The paper however succeeds in showing that the efficient profit-sharing contract is indeed feasible. The proof is an application of the Envelope Theorem. Moreover it is shown more generally that for risk-neutral workers, any feasible efficient contracts are optimal in the set of all linear contracts. As efficiency implies higher employment, this means that facing product market demand uncertainty, firms will optimally choose to offer greater employment stability, without requiring any formal employment commitment.

The analysis is then extended to risk-averse workers. The feasibility of the efficient profit-sharing contract is shown to be preserved. However the optimality is not, as the optimal wage contract is determined by a trade-off between efficiency and risk-sharing. This suggests that a full-insurance wage contract, where the workers receive the same fixed income in or out of work, is the optimal wage contract. However when variable effort supply is introduced, and when the effort exertion is costly, then some level of income variance is required to induce workers to exert higher effort. There are two ways of achieving this: by making unemployment costly, as argued by the efficiency wage literature initiated by Shapiro and Stiglitz (1984), and by making the remuneration performance related, as widely established by the incentives literature such as Salanié (1997). The model here allows me to show that the profit-sharing contracts provide both these incentive effects, and the optimal contract would be the one with the optimal trade-off between efficiency, risk-sharing and incentives. The paper thus promotes the use of profit-sharing contracts not only on the grounds of employment stability, from the workers' point of view as is traditionally argued in the literature (e.g. Weitzman (1984)), but also on the basis of its efficiency and incentive effects, from the view point of the firms.

Empirical studies suggests that there are regional differences for the evidences of the use of profit-share. For example Weitzman (1995) lists Japan, Korea and Taiwan as countries where profit-sharing contracts are offered. He cites studies such as Freeman and Weitzman (1987) who find that in Japan bonuses paid twice a year constitute about 25 percent of an average worker's total pay, with the bonus-to-base-wage ratio statistically significantly correlated with profitability. My personal experience with my bank was closer to a third. Kim (1988), also quoted in Weitzman (1995), finds similar evidence for Korea. In contrast for the West, both Pendleton, Poutsma, van Ommeren and Brewster (2001) and Poutsma (2003) show, based on surveys in the 1990s, that less than 20% of firms in the UK or the EU employ profit-sharing schemes for more than 50% of their workers. This paper limits its analysis to the optimal wage contract under product market demand uncertainty. An extension of the model to further include production uncertainty may explain the observed regional differences. What the paper does is to contribute to the study of optimal wage contracts under uncertainty by developing a simple but comprehensive framework for analysing wage contract determination in a competitive labour market, and in doing so providing a firmer microeconomic analytical basis for the wage theories, especially that of profit-share.

The paper is organised as follows. Section 2 develops a single-period model of labour market competition with demand uncertainty for risk-neutral workers. It establishes the concept of feasibility, and analyses both the first-best and the second-best outcomes. Section 3 extends the analysis to risk-averse workers, and investigates both cases of fixed and variable effort supply. Section 4 then gives concluding remarks.

2 Risk-Neutral Workers and Feasible Wage Contracts

2.1 The Model

An economy consists of homogeneous and infinitely-lived firms and workers. To begin with all agents are assume risk-neutral. As with Weitzman (1985) the firms are thought of as 'competitive monopolists' where a large number of firms operate in individual differentiated product markets. The revenue functions $R = py$ are assumed to be concave, where $p \equiv p(y)$ is the market price, $y \equiv y(N)$ is the firms' production function, and N is the number of employees. To begin with the worker effort supply is assumed fixed. The firms face idiosyncratic uncertainty in their market demand which is represented by a multiplicative parameter ϕ, such that the stochastic market price for the output goods equals ϕp. ϕ is assumed to be independently and identically distributed both intertemporally and between the product markets, with $\phi \in [0, \infty]$ and $E[\phi] = 1$. Its probability density function $f(\phi)$ is known by both the firm and the workers. In this paper I assume no production uncertainty in the worker output.

The firms compete for workers in the labour market. The labour market clears, but because of co-ordination problems due to reasons such as geographical distances or costly search for information, it is not immediate. Therefore once the shocks hit the firms start adjusting their employment levels, but while firing workers is instantaneous, hiring them takes time. Meanwhile production occurs. The workers in the labour market are then matched to the firms at the point of market clearance, after which the whole cycle repeats. Thus unlike in the frictional labour market literature (e.g. Pissarides (2000)), where the labour market does not clear and the wages are

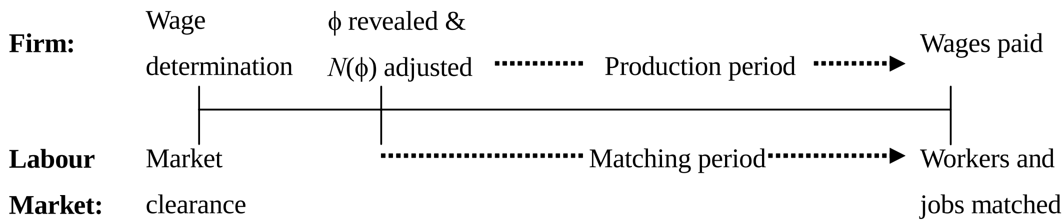

Figure 1: Single-Period Production and Labour Market Process

determined by Nash bargaining of rents from job matches, here wages do clear the job matching market, but only in expectation of future product market demand shocks. Note that a job match does not guarantee a paid wage as if the demand shock turns out to be negative then some of the newly matched workers may be released before production occurs. However job-parting is random in the sense that both existing and new workers have the equal probability of being released. The released workers are assumed to remain unemployed for the whole of the production period, during which time they earn an out-of-work income of w_{out}. This is the workers' ex-post outside option, and can be thought of as the unemployment benefit provided by the state.

The wage contract is then determined in the market-clearing process as firms compete for workers. While post-shock employment is not guaranteed, the firms are committed to the offered wage contracts during the production period. The workers choose the firm that offers the highest expected utility taking into account employment stability. For risk-neutral workers this is the expected income given the probabilities of remaining in-work for each realised shocks. This defines the workers' ex-ante outside option. The pre-shock labour market competition results in a partial transfer of the expected gains of production from the firms to the workers. The ex-ante outside option is therefore higher than the ex-post one. In equilibrium all firms offer the same ex-ante utility, irrespective of the wage contract chosen.

The set-up here is then a simplified single-period representation of a clearing labour market with uncertainty. In reality firms and workers are in a continuous process of job-match, with the wage adjustments leading the labour market in the direction of market clearance, but with the process being affected by random shocks that destroy jobs including those that are in the process of being filled. The simplification allows me to capture the effects of employment stability on the choice of the wage contract in a comprehensible way. Intuitively this framework is comparable to that of Weitzman (1985), whose main analysis is a comparison of the short-run outcomes of two different wage contracts - fixed wage and profit-sharing - when negative shocks hit the long-run full employment equilibria.

I will now formalise this. The cycle of pre-shock job-matching and post-shock production for firm i is modelled as follows,

Stage 2: Post-shock N adjustment Given realised product market demand shock ϕ_i and the wage contract w_i that may or may not depend on ϕ_i and/or N_i, firm i maximises its profit $\pi(w_i, \phi_i)$ with respect to $N_i(\phi_i)$,

$$\max_{N_i} \quad \pi(w_i, \phi_i) = \phi_i R(N_i) - w_i N_i \tag{1}$$

subject to the frictional labour market constraint,

$$N_i(\phi_i) \leq N_i^c \tag{2}$$

where N_i^c is the pre-shock market-clearing employment level. The maximum value function is then given by $\pi^*(w_i, \phi_i)$.

Stage 1: Pre-shock wage determination The firm chooses the wage contract w_i to maximise its expected profit level $E\pi$,

$$\max_{w_i} \quad E\pi(w_i) = \int_0^\infty \pi^*(w_i, \phi) f(\phi) d\phi \tag{3}$$

where E is the mathematical expectation operator over ϕ, subject to a participation constraint,

$$Ew(w_i) = \int_0^\infty E_N w_i \; f(\phi) d\phi \geq \max_{1 \leq j \leq M} \; Ew(w_j) \tag{PC1}$$

where M is the total number of firms, and $E_N w_i$ is the post-shock but pre-employment adjustment expected income for a worker in firm i with a contract w_i when the state of nature turns out to be ϕ_i,

$$E_N w_i = \left(\frac{N_i(\phi)}{N_i^c}\right) w_i + \left(1 - \frac{N_i(\phi)}{N_i^c}\right) w_{out} \tag{4}$$

E_N is the mathematical expectation taken over the Bernoulli distribution with probabilities $\left\{\frac{N_i(\phi)}{N_i^c}, 1 - \frac{N_i(\phi)}{N_i^c}\right\}$. In equilibrium all firms offer the same Ew irrespective of their chosen wage contract w_j. This defines the workers' ex-ante outside option, which is denoted by Ew_{res}. It reflects the supply condition of the labour market, which is taken as exogenous.

Pre-shock Market Clearance Firms compete for workers until all workers are matched to a firm pre-shock,

$$\{w_j\}_{j=1,\ldots,M} \quad \text{such that} \quad \sum_{j=1}^{M} N_j^c(w_j) = N \tag{5}$$

where $N =$ total workforce in the economy.

2.2 First-Best Analysis

I begin by describing the first-best outcome for this problem where there is no uncertainty. The market demands $\{\phi_i\}_{i=1,\ldots,M}$ are therefore known prior to the labour market competition. The optimal outcome is then when the aggregate pay-offs of the firm, workers in-work and the unemployed are maximised,

$$\begin{aligned}\max_{\{N_i\}} \quad & \sum_{i=1}^{M} \{\phi_i R(N_i) - w_i N_i\} + \sum_{i=1}^{M} w_i N_i + w_{out}\left(N - \sum_{i=1}^{M} N_i\right) \\ = \quad & \sum_{i=1}^{M} \{\phi_i R(N_i) - w_{out} N_i\} + w_{out} N\end{aligned}$$

i.e. the workers are allocated amongst the firms in such a way as to maximise the aggregate surplus gain from production. This occurs when the marginal revenue $\phi_i R'$

equals the opportunity cost of labour w_{out} for all i. This then defines the concept of efficiency for a wage contract $w(\mathbf{x})$ with its wage parameters $\mathbf{x}$,

Definition 1 *Given a wage contract $w(\mathbf{x})$, the contract $w(\mathbf{x}^e)$ is **efficient** if it generates the first-best surplus gain from production given the product market demand shock ϕ_i.*

Immediately then for a class of linear wage contracts, an efficient wage contract must have the fixed-wage component equal to the opportunity cost of labour w_{out}. The general form of an efficient linear wage contract is then,

$$w_i = w_{out} + s_{ik} T_i$$

where T_i is a lump-sum transfer of surplus from the firms to the workers in a form that does not affect the firm's employment decision, and s_{ik} is the worker k's share of it in firm i such that $\sum_k s_{ik} = 1$. If the firms have the full bargaining power over the workers then this lump-sum transfer would be zero. Assuming that competition for workers leads to a positive surplus transfer, examples of such transfers are,

1. $T_i = B$, a fixed aggregate-bonus payout irrespective of ϕ_i.
2. $T_i = \lambda\left(\phi_i R - w_{out} N\right)$ where $\lambda \in [0, 1]$, i.e. a profit-sharing scheme.
3. Any combination of the two, such as $T_i = \min\left[\phi_i R - w_{out} N,\ B\right]$.

Now assuming that in equilibrium all firms offer the same wage contract $w_i = w$, for the rest of the paper the subscripts i are suppressed for notational brevity.

2.3 Second-Best Analysis

Let Ω be the set of all linear wage contracts, and $\omega \subset \Omega$ be the set of a particular form of linear contracts, such as fixed wages or profit-sharing contracts. I begin by defining the concepts of optimality and feasibility,

Definition 2 *A wage contract $w(\mathbf{x})$ is **optimal** at $\mathbf{x}^*$ under given constraints if it generates the maximum surplus gain for the firm.*

For example if the constraint is the form of the wage contract $w(\mathbf{x}) \in \omega$, then $w(\mathbf{x}^*)$ is the contract with the parameters that provide the risk-neutral firm with the highest pre-shock expected profit $E\pi$. If on the other hand the referred wage contracts are all contracts $w(\mathbf{x}) \in \Omega$, then the contract $w^*(\mathbf{x}^*)$ that leads to the maximum possible expected profit is the optimal wage contract in the set of all contracts.

Definition 3 *A wage contract $w(\mathbf{x})$ is* ***feasible*** *at $\mathbf{x}$ if the desired level of worker expected utility can be attained at $\mathbf{x}$.*

Take for example the market-clearing expected wage level for risk-neutral workers Ew_{res}. Then immediately one can see that in a labour market where workers have positive bargaining power (i.e. they demand surplus above the ex-post outside option w_{out}), the efficient fixed wage contract $w = w_{out}$ is not feasible. Or putting it conversely, feasible fixed wage contracts $w = F$ are not efficient.

To get a better idea of the feasibility concept, I begin the second-best analysis by considering the following profit-sharing contract as formulated by Weitzman (1985),

$$w(F, \lambda) = F + \lambda \left(\frac{\phi R - FN}{N} \right) \tag{6}$$

where F is a fixed-wage, and λ is as before the workers' share of the firm's aggregate profit $\phi R - FN$ for the particular state of nature ϕ. This is assumed to be shared equally between the N workers. In the extreme case that $\lambda = 0$ this becomes the fixed wage contract. Now it is not immediately obvious that the efficient profit-sharing contract is feasible. To see this start with the feasible fixed wage contract $w(F, 0)$ where $F > w_{out}$, and consider what happens when the profit-share ratio is increased to $\lambda > 0$. For efficiency what we desire is for F to start reducing towards w_{out} as λ increases, while keeping the worker utility level the same. This trade-off would be viable if increases in λ and F were both beneficial to the workers. For λ this is clear as, ceteris paribas, an increase in the profit-share increases a worker's wage level while not affecting his employment, i.e. $\frac{\partial Ew}{\partial \lambda} > 0$. On the other hand when F increases, while it raises the wage level in work, it also affects the firm's post-shock employment decision and decreases the probability of the worker remaining in work. Therefore the sign of the net effect $\frac{dEw}{dF} = \frac{\partial Ew}{\partial F} + E\left[\frac{dN}{dF}\frac{\partial E_N w}{\partial N}\right]$ is ambiguous, and hence so is the sign of $\frac{dF}{d\lambda}$, which for a successful trade-off must be negative. Here however follows an argument that the trade-off is in fact possible unambiguously, and hence a feasible efficient profit-sharing contract exists,

Proposition 1 *The efficient profit-sharing contract is feasible. Moreover for risk-neutral workers, the contract is optimal.*

Proof. To see this, solve the problem outlined in Section 2.1 for the profit-sharing contract (6). First consider the post-shock employment adjustment in Stage 2. By substituting the profit-sharing contract (6) in (1), the post-shock profit is given by,

$$\pi(F, \lambda, \phi) = (1 - \lambda)\{\phi R(N) - FN\} \tag{7}$$

The employment level $N(\phi)$ is then given by the conditions

$$\begin{aligned} R'(N(\phi)) &= \frac{F}{\phi}\ , \quad \phi \leq 1 \\ N(\phi) &= N^c\ , \quad \phi > 1 \end{aligned} \tag{8}$$

Therefore the adjusted employment level $N(\phi)$ depends on λ only insofar as it affects the equilibrium fixed-wage component F through participation constraint (PC1). Next consider Stage 1. The effect of a change in λ on $E\pi$ is given by,

$$\begin{aligned} \frac{dE\pi}{d\lambda} &= \frac{\partial E\pi}{\partial\lambda} + \frac{dF}{d\lambda}\left(\frac{\partial E\pi}{\partial F} + E\left[\frac{dN(\phi)}{dF}\frac{\partial\pi^*}{\partial N}\right]\right) \\ &= -\frac{E\pi}{1-\lambda} - (1-\lambda)\,\overline{N}\frac{dF}{d\lambda} \end{aligned} \tag{9}$$

where I have used the Envelope Theorem to eliminate $\frac{\partial\pi^*}{\partial N} = 0$, and $\overline{N}$ is the average number of employees

$$\overline{N} = \int_0^\infty N(\phi) f(\phi) d\phi \tag{10}$$

Similarly the participation constraint (PC1) implies that,

$$\frac{dEw}{d\lambda} = \frac{\partial Ew}{\partial \lambda} + \frac{dF}{d\lambda}\left(\frac{\partial Ew}{\partial F} + E\left[\frac{dN(\phi)}{dF}\frac{\partial E_N w}{\partial N}\right]\right) \tag{11}$$

$$= \frac{1}{N^c}\frac{E\pi}{1-\lambda} + \frac{1}{N^c}\frac{dF}{d\lambda}\left\{(1-\lambda)\overline{N} + \Delta\overline{N'}\right\} \tag{12}$$

where Δ is the extra income from the fixed-wage component when in work

$$\Delta = F - w_{out} \tag{13}$$

and $\overline{N'}$ is the average change in employment level

$$\overline{N'} = \int_0^\infty \frac{\partial N}{\partial F} f(\phi)d\phi \tag{14}$$

This we know to be negative from the conditions (8) and the concavity of $R(N)$. As for whatever values of λ, F must adjust to provide the outside option Ew_{res}, (12) must equal zero and hence in combining with (9) we have,

$$\frac{dE\pi}{d\lambda} = \Delta\overline{N'}\frac{dF}{d\lambda} \tag{15}$$

Now to prove the feasibility of the efficient profit-sharing contract I require $\frac{dF}{d\lambda}$ to be negative. Using (11) this is,

$$\frac{dF}{d\lambda} = -\left.\frac{\partial Ew}{\partial \lambda}\middle/\frac{dEw}{dF}\right. \tag{16}$$

where from (12) the derivatives are,

$$\frac{\partial Ew}{\partial \lambda} = \frac{1}{N^c}\frac{E\pi}{1-\lambda} \tag{17}$$

$$\frac{dEw}{dF} = \frac{1}{N^c}\left\{(1-\lambda)\,\overline{N} + \Delta\overline{N'}\right\} \tag{18}$$

As expected $\frac{\partial Ew}{\partial \lambda}$ is unambiguously positive while $\frac{dEw}{dF}$ is a sum of positive and negative terms. It is only when the net effect of the latter is positive that the workers

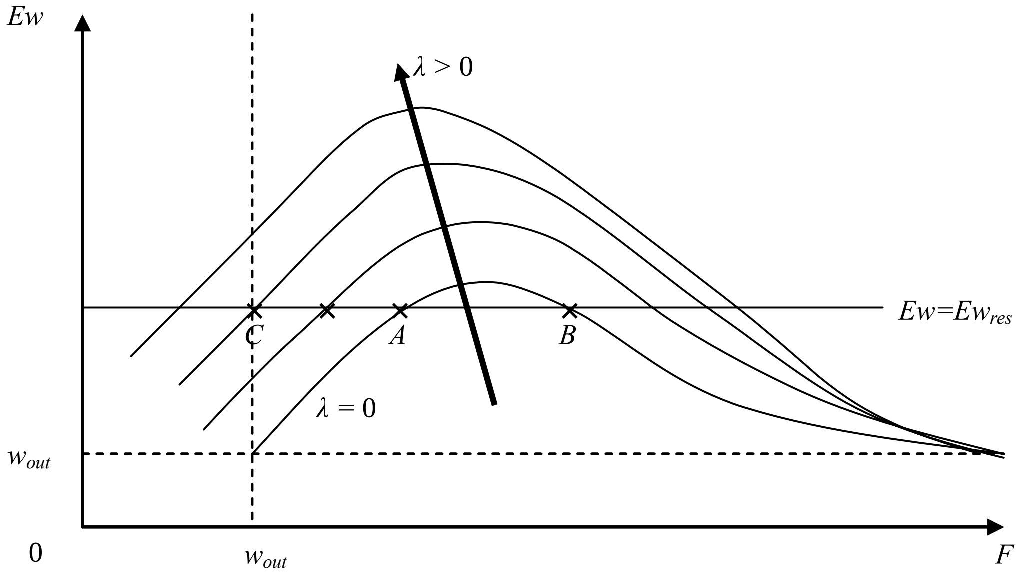

Figure 2: Ew vs. F for different values of λ

will be willing to trade-off the two components F and λ. However we can show that this is indeed the case unambiguously. First rewrite Ew in (PC1) as,

$$Ew(F) = \Delta\frac{\overline{N}}{N^c} + \frac{\lambda}{N^c}\int_0^\infty (\phi R - FN)\, f(\phi)d\phi + w_{out} \tag{19}$$

I denote Ew with the argument F to remind ourselves that we are considering the Ew curve with respect to F, for given values of λ. What we want to investigate is the slope of this, at values of F where Ew equals the given reservation income Ew_{res}, which I call the participation constraint solutions. So first consider the extreme values of $Ew(F)$. For the fixed-wage contract $\lambda = 0$,

$$Ew(F) = \Delta\frac{\overline{N}}{N^c} + w_{out} \begin{cases} = w_{out} & \text{at } F = w_{out} \\ \to w_{out} & \text{as } F \to \infty \end{cases}$$

The latter states that for F very large where N decreases to zero, $Ew(\infty)$ in (19) asymptotically approaches w_{out} from above. This is in fact true for all values of $\lambda < 1$. Next consider the slope of Ew. At $F = w_{out}$, $\frac{dEw}{dF}$ in (18) is unambiguously positive for all values of λ. As F increases and thus N decreases, the first term of (18) decreases while the second term becomes negative, turning $\frac{dEw}{dF}$ negative at some point. Finally $\frac{\partial Ew}{\partial \lambda} > 0$ for any given F in (17) means that the curve shifts upwards as λ increases. Therefore the curves $Ew(F)$ look as depicted in Figure 2. The potential participation constraint solutions are then where the horizontal line Ew_{res} crosses the $Ew(F)$ curves, which for fixed-wage contract $\lambda = 0$ there are two possibilities: A and B in Figure 2. However the firm will always choose to operate at the lower value of F (i.e. at A, the feasible fixed-wage contract) as for a given λ, $\frac{dE\pi}{dF} < 0$ (c.f. (7)). Therefore for $\lambda = 0$ the firm will operate at the upward-sloping part of the $Ew(F)$ curve where $\frac{dEw}{dF} > 0$.

For the efficient profit-sharing contract the curve that we are interested is the one where λ has risen enough such that the participation constraint solution has shifted down from A to C, i.e. where $F = w_{out}$. We know that the slope $\frac{dE\pi}{dF}$ is still positive from (18) when $F = w_{out}$. It thus follows that for the efficient contract, $\frac{dF}{d\lambda}$ in (12) is negative, and the workers are happy to accept lower fixed-wage component for a higher profit-share, i.e. it is feasible. The optimality of this contract then directly follows from (15). ■

The optimality implies that with risk-neutral workers, firms will choose to offer a strictly positive profit-sharing contract as opposed to a fixed-wage contract. This is really an application of the Envelope Theorem. Any changes in F or λ are, in absolute wage level terms, simply surplus transfers between the firm and the workers. However a decrease in F has an additional effect for workers of increased employment stability. For the firm though the marginal effect of this on the ex-post maximised profit level is zero (c.f. (1)). Hence the firm is able to generate higher surplus gain (equalling the employment stability effect) by reducing F and increasing λ, until F is driven down to w_{out}. This is then an example of a wage contract where firms optimally choose to offer greater employment stability, without requiring any formal employment commitment by the firm.

Now it follows from Proposition 1 that, where Weitzman (1985) states that he does not have "a formal theory that would explain ... why a society chooses a particular [profit-sharing] configuration...", I can derive an explicit expression for the optimal profit-share parameter for risk-neutral workers,

Corollary 1 *For risk-neutral workers, the optimal profit-share ratio is given by*

$$\lambda^* = \frac{(Ew_{res} - w_{out})\, N^c}{\int_0^\infty (\phi R - w_{out} N)\, f(\phi) d\phi} \tag{20}$$

Proof. This follows directly from (19) where λ^* is the value of λ at which Ew equals Ew_{res}. ■

Furthermore,

Corollary 2 *Full employment is achieved when $w_{out} = 0$.*

Proof. Proposition 1 implies that with $w_{out} = 0$, λ^* will be at the point where F is driven down to 0 (i.e. a pure profit-share contract). At this point the first-order condition (8) for the profit function implies that for all values of ϕ, the firm will wish to employ the maximum possible employment level, i.e. N^c. Thus full employment is achieved regardless of the fluctuations in the market demand. ■

The full employment profit-share ratio is then (20) when $w_{out} = 0$, i.e.,[1]

$$\lambda^* = \frac{Ew_{res} N^c}{R\,(N^c)} \tag{21}$$

We can now generalise the optimality result to all linear wage contracts,

Proposition 2 *For risk-neutral workers, given a set of wage contracts ω, if an efficient contract $w(\mathbf{x}^e) \in \omega$ is feasible, it is also optimal in ω.*

Proof. Given a wage contract $w(\mathbf{x}) \in \omega$, the optimal wage contract $w(\mathbf{x}^*)$ for risk-neutral workers is one which maximises the following expected profit,

$$E\pi = \int_0^\infty (\phi R - w_{out} N)\, f(\phi) d\phi - (Ew - w_{out})\, N^c \;\text{ such that } Ew = Ew_{res} \tag{22}$$

[1] This in fact follows from the fact that in full employment, $Ew = \lambda \frac{R(N^c)}{N^c}$.

This equation states that the firm's share of surplus gain from production is net of payments $Ew_{res} - w_{out}$ to all N^c matched workers, and an additional w_{out} to those that remain in work post-shock. The parameters $\mathbf{x}$ are chosen so as to maximise this. The feasibility condition asks whether the wage contract permits the market-clearing expected income $Ew = Ew_{res}$ at the efficient parameters $\mathbf{x}^e$. For profit-sharing contracts, the analysis of Proposition 1 established the $F - \lambda$ trade-off that allows this. If this is the case then $\frac{dEw}{d\mathbf{x}} = 0$, and the maximisation solution is given by,

$$\frac{dE\pi}{d\mathbf{x}} = \int_0^\infty \frac{dN}{d\mathbf{x}} \frac{d}{dN}(\phi R - w_{out}N) f(\phi) d\phi = 0 \tag{23}$$

A sufficient condition for this to be true is that $\frac{d}{dN}(\phi R - w_{out}N) = 0$ for all ϕ, i.e. that the wage contract is efficient. Thus given a set of wage contracts ω, the efficient wage contract is also the optimal wage contract in ω. ■

It further follows that,

Proposition 3 *For risk-neutral workers, any feasible efficient wage contract is optimal in the set of all wage contracts Ω.*

Proof. This follows directly from Proposition 2: efficient wage contracts maximise the expected profit level $E\pi$ in (22), but (22) is independent of the choice of the wage contract so long as it is feasible. ■

By definition any efficient wage contract produces the maximum possible surplus gain from production. The labour market competition determines how much of this surplus is transferred to the workers. Then so long as this transfer is possible without affecting the firm's post-shock employment decision (i.e. the wage contract is feasible), the firm is left with the maximum residual surplus gain regardless of its choice of the efficient contract. It follows then that the optimal profit-sharing contract is also optimal in the set of all contracts, but so is, for example, the following efficient fixed aggregate-bonus contract,

$$w(\phi) = w_{out} + \frac{B}{N(\phi)} \tag{24}$$

An analogous argument to Proposition 1 would show that this contract is also feasible.

3 Extension to Risk-Averse Workers

3.1 Fixed Effort Supply

I now extend the analysis to that with risk-averse workers. The worker effort level is still assumed fixed. In contrast to the risk-neutral case where workers' utility was represented by their expected income, risk-averse workers have concave utility functions. For the purpose of analysis then I assume an exponential utility function $U(w) = -e^{-rw}$ for an income w, with the constant absolute risk-aversion parameter r. The expected utility is then given by,

$$EU = -\int_0^\infty \left\{ \left(\frac{N(\phi)}{N^c} \right) e^{-rw} + \left(1 - \frac{N(\phi)}{N^c} \right) e^{-rw_{out}} \right\} f(\phi) d\phi \tag{25}$$

This time then the reservation utility, which is again the maximum utility offered by firms in the labour market competition, is denoted by EU_{res}. The participation constraint for the risk-averse workers is then,

$$EU(w_i) \geq \max_{1 \leq j \leq J} EU(w_j) = EU_{res} \tag{PC2}$$

As with risk-neutral workers, I first investigate the feasibility and the optimality of the efficient profit-sharing contract,

Proposition 4 *If a feasible fixed-wage contract exists, then the efficient profit-sharing contract is feasible. However for risk-averse workers it is not optimal.*

Proof. The proof is analogous to that given for Proposition 1. To prove feasibility, I once again show that the sign of $\frac{dF}{d\lambda}$ is negative. This is given in Appendix A. To prove non-optimality of the efficient profit-sharing contract, I use the certainty equivalent representation of the expected utility which is given by (see for example Milgrom and Roberts (1992) Ch7),

$$CE = Ew - \frac{r}{2} Var[w] \tag{26}$$

where Ew is defined by (PC1) and (4), and the variance term is given by,

$$Var[w] = \int_0^\infty \left\{ \left(\frac{N(\phi)}{N^c} \right) (w - Ew)^2 + \left(1 - \frac{N(\phi)}{N^c} \right) (w_{out} - Ew)^2 \right\} f(\phi) d\phi \quad (27)$$

The binding participation constraint implies then that $\frac{dCE}{d\lambda} = \frac{dEw}{d\lambda} - \frac{r}{2} \frac{dVar[w]}{d\lambda} = 0$, which is using (12),

$$\frac{1}{N^c} \frac{E\pi}{1-\lambda} + \frac{\overline{N}}{N^c} \frac{dF}{d\lambda} \left\{ (1-\lambda)\overline{N} + \Delta \overline{N'} \right\} - \frac{r}{2} \frac{dVar[w]}{d\lambda} = 0 \quad (28)$$

Combining this with $\frac{dE\pi}{d\lambda}$ given in (9) the analogous equation to (15) is then,

$$\frac{dE\pi}{d\lambda} = \Delta \overline{N'} \frac{dF}{d\lambda} - \frac{r}{2} N^c \frac{dVar[w]}{d\lambda} \quad (29)$$

In comparison to (15), for risk-averse workers there is an additional effect of an increase in λ on the wage income variance. The sign of $\frac{dVar[w]}{d\lambda}$ needs a little discussion. A mathematical analysis is outlined in Appendix B. The wage income variance $Var[w]$ comes from two sources as the market demand shock ϕ fluctuates: the variance of the in-work wage w itself, and the dispersion of w from w_{out}, the income when out-of-work. Clearly an increase in λ increases the in-work wage variance as the size of the variable part of the wage increases. However we know that this increase in λ has a secondary effect of a fall in F, which reduces the level of w and hence its dispersion from w_{out}. Therefore this effect alone has a reducing effect on $Var[w]$. There are further tertiary effects of an increase in employment level $N(\phi)$ (as a result of the fall in F), both decreasing the in-work wage variance as it reduces the share of the aggregate profit, and increasing the dispersion from w_{out} from the increased probability of being paid the higher in-work wage. The result of my argument in Appendix B is that the overall sign of $\frac{dVar[w]}{d\lambda}$ is positive at $F = w_{out}$. Then the solution to (29) equals zero occurs at the profit-sharing ratio λ^* where $\Delta > 0$ or $F^* > w_{out}$, and hence the efficient profit-sharing contract is not optimal. ■

The argument here is that for risk-averse workers, the optimal profit-share ratio is derived as a trade-off between efficiency and risk-sharing. To see this in a more general case consider the following analogous equation to (22) for the risk-averse case, which defines the optimal wage contract $w(\mathbf{x})$ as one which maximises with the choice of $\mathbf{x}$,

$$E\pi = \int_0^\infty (\phi R - w_{out}N)\, f(\phi) d\phi - \left\{ CE - \left(w_{out} - \frac{r}{2} Var[w] \right) \right\} N^c$$
$$\text{such that } CE = CE_{res} \tag{30}$$

where CE_{res} corresponds to the certainty equivalent representation of EU_{res}. The CE constraint again requires the wage contract to be feasible. (30) reflects the fact that it is costly for the firm to employ a higher variance wage contract as the workers need to be compensated for the variability with higher expected average income. The optimal wage contract is then one that satisfies,

$$\frac{dE\pi}{d\mathbf{x}} = \int_0^\infty \frac{dN}{d\mathbf{x}} \frac{d}{dN} (\phi R - w_{out}N) f(\phi) d\phi - \frac{r}{2} \frac{dVar[w]}{d\mathbf{x}} = 0 \tag{31}$$

For $\frac{dVar[w]}{d\mathbf{x}} \neq 0$ this implies that $\phi R' \neq w_{out}$, i.e. the optimal wage contracts are in general not efficient. However,

Corollary 3 *For risk-averse workers the efficient full insurance contract is the optimal wage contract in the set of all contracts* Ω*.*

Proof. The full insurance contract is where the firm pays a bonus payment $\frac{B}{N^c}$ which is independent of the market shock ϕ to all workers including those laid-off,

$$w = \begin{cases} F + \frac{B}{N^c} \quad , & \text{if in-work} \\ \frac{B}{N^c} \quad , & \text{if out-of-work} \end{cases} \tag{32}$$

The firm's the marginal cost of an extra worker is then F, and the contract is efficient when $F = w_{out}$. As the bonus payment is independent of ϕ, and the workers receive the same income whether in or out of work, this wage income has zero variance. Therefore the optimal wage contract given by (31) would again be the efficient one, and moreover the firm's expected profit in (30) is the maximum of all contracts for given CE_{res}. Hence the efficient full insurance contract is optimal of all wage contracts. ■

3.2 Variable Effort Supply

We do not, however, observe full-insurance contracts. Apart from the credibility problem where firms do not have an incentive to pay bonuses to laid-off workers, there is a problem of worker incentives. Up until now I have assumed fixed effort supply by the hired workers. However if workers can choose their effort levels, and effort exertion is costly, then they clearly do not have an incentive to provide any effort at all under full insurance. This means that in (30), whilst the full-insurance contract is still "efficient" in terms of allocation of workers, it is no longer optimal.

There are two ways firms can tackle this incentive problem associated with variable effort levels. The first is to make unemployment costly. Then as argued by the efficiency wage literature initiated by Shapiro and Stiglitz (1984), taking into account the probability of being caught workers exert just enough effort so as to be indifferent between shirking and not shirking. A formal treatment of this in my single-period set-up is difficult as by assumption here workers cannot be sacked and replaced instantaneously. However in the understanding that this model is a simplified representation of a multi-period labour market, one can make the following modification for the firms' revenues $R(.)$ and worker effort levels E,

$$R \equiv R(EN) \quad \text{where } E = e(\delta w) \tag{33}$$

where δw is the difference between a worker's incomes in and out of work including the unemployment benefit w_{out}, and $e' > 0$. If by normalisation we assume $e(0) = 0$, then to get any positive revenue we would require the wage variance $Var[w]$ to be positive. This immediately rejects the full-insurance contract as a viable contract. The optimal wage contract would then be one that maximises (30) subject to (33).

The second way to tackle the incentive problem is to use performance-related wage contracts. When the worker effort levels are unobservable and the individual output is subjected to production uncertainty, this is a well established result in the large body of incentives literature including Salanié (1997) and Prendergast (1999). However even in the absence of production uncertainty as assumed in this paper, costly efforts would mean that a remuneration related to performance would enhance productivity, as argued by Weitzman (1995). Whilst recognising that when individual output is unobservable, a wage related to aggregate performance is not the most effective tool for incentivising higher efforts,[2] Weitzman quotes Kruse's (1993) survey of 26 econometric studies which shows that of 265 estimated coefficients measuring the effects of profit-sharing on productivity, 91.7 percent of the estimates are positive, with 51.7 percent having t-statistics greater than +2. Moreover Kruse's own study shows that "profit-sharing adoption is associated with productivity increases of 3.5 to 5 percent". Weitzman concludes then that "there appears to be a very strong statistical association between profit sharing an productivity". Here then the model can be extended to incorporate this by making the effort level exerted by workers remaining at a firm be determined by,

$$E = \arg\max \ w - C(E) \tag{34}$$

where $C(E)$ is the monetary value of effort disutility, with C', $C'' > 0$. The optimal wage contract this time is one that maximises (30) subject to (34). In both this case and the efficiency wage case above then the optimal wage contract is one which provides the optimal trade-off between efficiency, risk-sharing and incentives.

Under variable effort supply then, which wage contract is the most appropriate for this? To explore this further it is useful to decompose the wage income variance

[2] For example Holmström (1982) states, "... moral hazard problems may occur even when there is no uncertainty in output. The reason is that agents who cheat cannot be identified if joint output is the only observable indicator of inputs." Weitzman (1995) however uses the repeated games argument to reason that even in this case, a high productivity Nash equilibrium can be attained.

$Var[w]$ in (27) into two separate terms,

$$\begin{aligned} Var[w] &= \int_0^\infty \left\{ \left(\frac{N(\phi)}{N^c}\right) w^2 + \left(1 - \frac{N(\phi)}{N^c}\right) w_{out}^2 \right\} f(\phi) d\phi - (Ew)^2 \\ &= \int_0^\infty \left[\left\{ \left(\frac{N(\phi)}{N^c}\right) w^2 + \left(1 - \frac{N(\phi)}{N^c}\right) w_{out}^2 \right\} - (E_N w)^2 \right] f(\phi) d\phi \\ &\qquad + \int_0^\infty (E_N w)^2 f(\phi) d\phi - (Ew)^2 \\ &= EVar_N [w] + Var[E_N w] \end{aligned} \tag{35}$$

where $E_N w$ is given by (4), and $Var_N[.]$ is the variance operator for the Bernoulli distribution with probabilities $\left\{\frac{N_i(\phi)}{N_i^c}, 1 - \frac{N_i(\phi)}{N_i^c}\right\}$, i.e.

$$Var_N[w] = \left(\frac{N(\phi)}{N^c}\right)(w - E_N w)^2 + \left(1 - \frac{N(\phi)}{N^c}\right)(w_{out} - E_N w)^2 \tag{36}$$

What this is stating is that the wage income variance is a sum of the average variance of the ex-post wage income, and the variance of the expected ex-post wage income, where ex-post here refers to the post-shock but pre-employment adjustment state. The question is when allowing positive income variance to counter incentives problem, which of the two components is the more effective tool to use. To see this consider first the fixed aggregate-bonus contract suggested in (24),

$$w(\phi) = F + \frac{B}{N(\phi)}$$

The expected ex-post wage income of this contract calculated using (4) is,

$$E_N w(\phi) = \Delta \frac{N(\phi)}{N^c} + \frac{B}{N^c} + w_{out} \tag{37}$$

and therefore its variance is,

$$var[E_N w] = \left(\frac{\Delta}{N^c}\right)^2 var[N(\phi)]$$

which equals zero for the efficient contract $F = w_{out}$. Thus the wage variance for the efficient fixed aggregate-bonus scheme comes solely from the ex-post wage income variance, i.e. employment uncertainty. This suggests that this contract would be most effective in providing incentives under the efficiency wage argument. However due to its non-dependence on the firm's performance, it is still prone to moral hazard issues when effort levels are unobservable.

Consider on the other hand the following contract where the firm pays a profit-sharing bonus payment to both in-work and laid-off workers,

$$w = \begin{cases} F + \lambda \left(\frac{\phi R - FN}{N} \right) & , \quad \text{if in-work} \\ \lambda \left(\frac{\phi R - FN}{N} \right) & , \quad \text{if out-of-work} \end{cases} \tag{38}$$

An example of this kind of wage contract may be one where firms transfer excess workers to their subsidiaries while making up the wage differences. The extreme case is the life-time employment contract where $F \rightarrow 0$. The efficient contract for this unemployment-insurance profit-sharing bonus scheme clearly has no wage variance once the product market demand shock is realised, i.e. $Var_N [w] = 0$. However ex-ante the expected wage income is subject to product market demand uncertainty and hence $Var [E_N w] > 0$. This contract thus provides incentives through performance-dependence of the wage contract, whilst having no efficiency wage effect.

The profit-sharing contract is then the compromise between these two extreme cases. It provides worker incentives through both efficiency wage argument and performance-related remuneration, allowing a more effective trade-off between efficiency, risk-sharing and incentives. This gives a strong case for profit-sharing contracts, more so than the traditional advocacy on the ground of higher employment stability.

4 Concluding Remarks

In concluding this paper I return to my experience with the Japanese bank. My salary there consisted of two components, a fixed salary and bi-annual bonus payments. The bonus payments, at least for us younger cohorts, were independent of individual performance, but varied according to the firm's aggregate performance, albeit with stickiness. This variable component could be as much as a third of my annual income. To our understanding then the wage contract was very much a profit-sharing contract.

Although at the time life-time employment was a predominant practice for large Japanese firms, by mid-1990s many workers began to receive calls from headhunters offering jobs at foreign banks. These invariably offered no employment guarantee but a much higher salary. Although this could partially be because the targeted workers were of higher-ability - Japanese workers in their first 10 to 15 years were paid similar wages depending mostly on their age - the decision these workers had to make was a trade-off between higher wages and employment stability. The result of this paper indicates that if the workers are indifferent between the two offers, the firm offering the profit-sharing contract should expect a higher average profit, by a margin equivalent to the surplus gained from the increased employment stability for the workers. This was shown to be a result of an application of the Envelope Theorem. Indeed for risk-neutral workers the expected profit is maximised with efficient wage contracts, for which feasibility was carefully established.

In practice though the fixed-wage component of our wages in the bank were never as low as the unemployment benefit. However this can be explained by the extension of the model to risk-averse workers and variable effort supply, where the optimal wage contract was shown to be determined as a trade-off between efficiency, risk-sharing and incentives. The profit-sharing scheme was once again argued to be the appropriate tool, both for its feasibility and its ability to provide effort incentives through both the efficiency wage argument and the performance-related payout.

So then if it is beneficial for firms to be offering profit-sharing contracts, as large Japanese firms were, why were foreign firms offering different types of wage contracts? The answer may lie once again with the effort incentive problem. Although as already discussed the incentive effect of a profit-sharing contract is supported by some empirical evidence cited in Weitzman (1995), one can still argue that contracts that pay according to aggregate performance is prone to what is often termed the '$\frac{1}{n}$-problem'. This is where if the workers' individual outputs were affected by a random factor, and their effort levels were unobservable by the firm, paying an equal share of the aggregate profit in a non-cooperative setting would lead to all workers exerting sub-optimal effort levels. This is due to the fact that whilst in shirking a worker gains from saving on his disutility from work, the effect of this on his remuneration is diluted by a factor of n. The problem worsens as n gets larger. The wage contracts offered to us potential recruits head-hunted by foreign firms typically included a possibility of an individual-performance bonus payouts, which can be designed to alleviate this asymmetric information problem (see for example Prendergast (1999)).[3] The analysis of the optimal wage contract under both market demand and production uncertainties will be addressed in future studies.

[3]It can be argued that the Japanese firms tackled this problem in ways other than design of wage contracts, such as small-team based performance evaluation and centrally controlled promotion schemes. The former would strengthen monitoring within the team, while the latter would discourage shirking much in the way analysed in the tournament theory literature initiated by Lazaer and Rosen (1981).

A Feasibility of Efficient Profit-Sharing Contract for Risk-Averse Workers

First the analogous equation to (16) for the binding participation constraint (PC2) is,

$$\frac{dF}{d\lambda} = -\left.\frac{\partial EU}{\partial \lambda}\right/\frac{dEU}{dF} \tag{39}$$

The numerator is unambiguously positive as, using (25),

$$\frac{\partial EU}{\partial \lambda} = \frac{r}{N^c}\int_0^\infty (\phi R - FN)\, e^{-rw} f(\phi)d\phi > 0 \ \ \forall F \tag{40}$$

On the other hand the denominator is again a sum of a positive and a negative term as, in expanding $\frac{dEU}{dF}$,

$$\frac{dEU}{dF} = \frac{\partial EU}{\partial F} + E\left[\frac{dN(\phi)}{dF}\frac{\partial E_N U}{\partial N}\right] \tag{41}$$

where $E_N U$ is the post-shock, pre-employment adjustment expected utility,

$$E_N U = -\left(\frac{N(\phi)}{N^c}\right) e^{-rw} - \left(1 - \frac{N(\phi)}{N^c}\right) e^{-rw_{out}} \tag{42}$$

and the two partial derivatives are given by,

$$\begin{aligned}
\frac{\partial EU}{\partial F} &= r(1-\lambda)\int_0^\infty \frac{N}{N^c} e^{-rw} f(\phi)d\phi > 0 \qquad \forall \lambda < 1 & (43)\\
\frac{\partial E_N U}{\partial N} &= -\frac{1}{N^c}\left[\left\{1 + r\lambda\left(\frac{\phi R - FN}{N}\right)\right\} e^{-rw} - e^{-rw_{out}}\right] & (44)\\
&> -\frac{1}{N^c}\left\{e^{r\lambda\left(\frac{\phi R - FN}{N}\right)} e^{-rw} - e^{-rw_{out}}\right\}\\
&= -\frac{1}{N^c}\left(e^{-rF} - e^{-rw_{out}}\right) \geq 0 \qquad\qquad \forall F \geq w_{out}
\end{aligned}$$

These reflect the fact that increases in the fixed-wage component or the employment probability both increase the workers' expected utility. It is the fact that the increase in F reduces the employment probability that makes it difficult to assess the net effect. To see that this effect is always positive for the contracts we are interested, once again investigate the graphs of $EU(F)$ for different values of λ,

$$EU(F) = -\int_0^\infty \left\{ \left(\frac{N(\phi)}{N^c} \right) \left(e^{-r\left\{F+\lambda\left(\frac{\phi R - FN}{N}\right)\right\}} - e^{-rw_{out}} \right) \right\} f(\phi)d\phi - e^{-rw_{out}} \quad (45)$$

For $\lambda = 0$, this equals $-e^{-rw_{out}}$ when $F = w_{out}$. At the other extreme when $F \to \infty$, $N(\phi) \to 0$ and hence EU asymptotically approaches $-e^{-rw_{out}}$ from above for all values of λ. To see what happens in the middle consider its derivative (41) at $F = w_{out}$. Using (43) and (44) in (41),

$$\left.\frac{dEU}{dF}\right|_{F=w_{out}} = \begin{cases} \left.\frac{\partial EU}{\partial F}\right|_{F=w_{out}} > 0 & \text{at } \lambda = 0 \\ E\left[\frac{dN(\phi)}{dF} \left.\frac{\partial E_N U}{\partial N}\right|_{F=w_{out}} \right] < 0 & \text{as } \lambda \to 1 \end{cases} \quad (46)$$

Therefore the graphs look something like this:

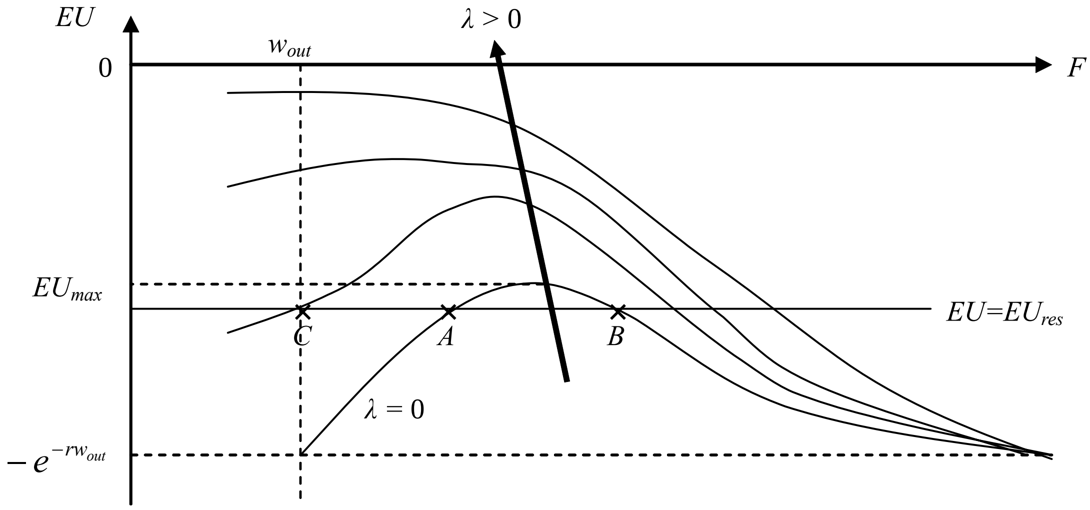

Figure 3: EU vs. F for different values of λ

Once again as $\frac{\partial EU}{\partial \lambda} > 0$ for any given F (c.f. (40)), the curve shifts upwards as λ increases. One major difference between this and the risk-neutral case in Figure 2 is that with risk-neutral workers, the slope of $EU(F)$ at $F = w_{out}$ turns negative as λ approaches 1. This is because compared with (18), risk-averse workers derive negative utility from the increase in the wage income variance that results from an increase in F. This more than offsets the effect of increasing wage-level on the worker utility at higher λ. Now what we require to prove feasibility is that as the firm trades off F with λ from the optimal fixed-wage contract at A (note that once again the firm will choose the lower F solution, and hence not point B), to the efficient profit-sharing contract at C, the slope of the curve remains positive at the participation constraint solutions (i.e. where the horizontal $EU = EU_{res}$ line intersects the $EU(F)$ curves). If this is not the case then $\frac{dF}{d\lambda}$ in (39) would be positive, which contradicts the $F - \lambda$ trade-off process. With risk-neutral workers this was always the case as the slope $\frac{dEU}{dF}$ was positive at $F = w_{out}$ for all values of λ. Here a sufficient condition that the slope is still positive at C is that $EU_{res} \leq EU_{\max}$, where $EU_{\max}$ is the highest achievable expected utility with a fixed-wage contract. Thus if the fixed-wage contract is feasible, then so is the efficient profit-sharing contract. ■

B Discussion on the Derivatives of $Var[w]$

As discussed in the text, as well as its direct effect, a change in λ has secondary and tertiary effects on $Var[w]$ through its effects on F and $N(\phi)$, (with a slight abuse of notation)

$$\frac{dVar[w]}{d\lambda} = \frac{\partial Var[w]}{\partial \lambda} + \frac{dF}{d\lambda}\left(\frac{\partial Var[w]}{\partial F} + \frac{dN}{dF}\frac{\partial Var[w]}{\partial N}\right) \tag{47}$$

Then,

Claim 1 $\frac{dVar[w]}{d\lambda}$ *is positive at* $F = w_{out}$ *for all values of* λ.

Proof. First consider the fixed-wage $\lambda = 0$. Then at $F = w_{out}$, $w = w_{out}$ and hence $Var[w] = 0$. As any increase in λ results in a positive variance, $\left.\frac{dVar[w]}{d\lambda}\right|_{F=w_{out}}$ for $\lambda = 0$ must be greater than zero. Next consider the case $\lambda \to 1$. As discussed in the main text $\frac{\partial Var[w]}{\partial \lambda} > 0$ as an increase in λ increases the size of the variable part of the wage. $\frac{\partial Var[w]}{\partial F}$ is also greater than zero as a rise in F increases the level of w and hence its dispersion from w_{out}.[4] $\frac{\partial Var[w]}{\partial N}$ is however a sum of positive and negative

[4] It can be shown that for a profit-sharing contract (6),

$$\begin{aligned}\frac{\partial Var[w]}{\partial \lambda} &= \frac{2}{N^c(1-\lambda)}\{Cov\left[E_N w, \pi\right] + w\pi\} \\ \frac{\partial Var[w]}{\partial F} &= \frac{2(1-\lambda)}{N^c}\{Cov\left[E_N w, N\right] + wN\}\end{aligned}$$

terms, as higher $N(\phi)$ both increases the dispersion of w from w_{out} from the increased probability of being paid the higher in-work wage, and decreases the in-work wage variance as it reduces the share of the aggregate profit. However at $F = w_{out}$ and large λ the second effect dominates, and hence at this point $\frac{\partial Var[w]}{\partial N} < 0$.[5] As $\frac{dN}{dF} < 0$ then, $\frac{dVar[w]}{dF} = \frac{\partial Var[w]}{\partial F} + \frac{\partial N}{\partial F}\frac{\partial Var[w]}{\partial N}$ is positive. However it was discussed in Appendix A eqn (46) that at $F = w_{out}$ the slope of $EU(F)$ turns negative as λ approaches 1. In other words $\frac{dF}{d\lambda}$ turns positive for large λ. Hence $\frac{dVar[w]}{d\lambda}$ in (47) is again positive at large λ. Thus I claim that $\left.\frac{dVar[w]}{d\lambda}\right|_{F=w_{out}} > 0$ for all values of $\lambda \in [0, 1]$. ■

which are both unambiguously positive.

[5] Again for a profit-sharing contract it can be shown that, with a slight abuse of notation,

$$\frac{\partial Var[w]}{\partial N} = \frac{1}{N^c}\left\{-\lambda^2\left(\frac{\phi R - FN}{N}\right)^2 + \left(F^2 - w_{out}^2\right)\right\}$$

For $F = w_{out}$ then, this is unambiguously negative.

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