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Abstract

A repo auction is a multi-unit common value auction in which bidders submit demand functions. Such auctions are used by the Bundesbank as well as the European Central Bank as the principal instrument for implementing monetary policy. In this paper, we analyze a repo auction with a uniform pricing rule. We show that under a uniform pricing rule, the usual intuition about the value of exclusive information can be violated, and implies free riding by uninformed bidders on the information of the informed bidders, lowering payoff of the latter. Further, free riding can distort the information content of auction prices, in turn distorting the policy signals, hindering the conduct of monetary policy. The results agree with evidence from repo auctions, and clarifies the reason behind the Bundesbank's decision to switch away from the uniform price format. Our results also shed some light on the rationale behind the contrasting switch to the uniform price format in US Treasury auctions.

KEYWORDS: *Repo auction, Informational Free Riding, Monetary Policy Signals.*

JEL CLASSIFICATION: D44, E50

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1 Introduction

Auctions are now widely recognized as a very important policy tool, and multi-unit auctions of various degrees of complexity are regularly used in a variety of settings including treasury bill sales, allocation of telecom licences, as well as electricity markets[(1)]. Usually, such auctions aim to generate an efficient allocation, and raise as high a revenue as possible. An interesting contrast is presented by auctions for selling money. Known as repo auctions[(2)], these implement monetary policy in several countries, primarily aiming to signal the central bank's monetary policy stance.

Repo auctions constitute the principal instrument of money-market management by the Eurosystem[(3)]. On average, around three-quarters of annual central bank money requirements are met through these auctions. In particular, for the Bundesbank, the importance of repo auctions as a reserve management tool increased dramatically in the mid 80s. Taking after the Bundesbank, The European Central Bank also uses these auctions as the primary instrument for conducting monetary policy. However, very little is known about the theoretical properties of such auctions. Further, the most important aspect of monetary policy as implemented through repo auctions is to signal policy stance to the market. Thus it is very important to know the whether certain auction forms are likely to weaken the signalling ability.

In this paper we explore the structure of bidding in a repo auction (which, as we explain later, is a multi-unit common-value auction) with a uniform pricing rule[(4)], and explore the problems that arise in the presence of heterogeneously informed bidders. Anecdotal evidence from Bundesbank repo auctions using uniform pricing suggests that large banks were unhappy with the pricing rule. Further, an interesting investigation by Nautz (1997) finds evidence of a specific information distortion under this pricing rule: lower repo rates are more informative than higher rates. Our results predict precisely such a distortion,

[(1)]see Klemperer (1999) for a discussion of the recent literature on auctions including multi-unit auctions.

[(2)]We discuss these auctions in detail later in this section.

[(3)]In contrast, the main policy rate in the US is the overnight interbank rate, and the Fed conducts its monetary policy through open market operations by directly dealing with a few primary dealers.

[(4)]Under a uniform pricing rule all winners pay the same price, which is the price at which the market clears. In a repo auction, bidders bid interest rates that they pay to the central bank, rather than prices. However, interest rates map one-to-one into prices - so the use of the term "price" in repo auctions is without loss of generality.

and also show how this pricing rule creates a disadvantage for larger and more informed banks. In clarifying the evidence, the results also explain the reasons for the subsequent change away from a uniform pricing rule. At the same time, our results throw light on the opposite case of the US Treasury, which decided to switch entirely *to* a uniform pricing rule in selling debt.

In terms of the literature on auction theory, our results extend the theory of single unit common value auctions with asymmetrically informed bidders. With multiple units and a uniform pricing rule, the division of profits between the informed and uninformed bidders can be surprising. The uninformed bidders can free ride on the information of the informed reflected in the market clearing price, and thus make a positive profit.

Before discussing the results further, let us clarify the structure of repo auctions. Since the early 1980s many European central banks have used securities repurchase operations (repo) to manage the money market. In a repo, a central bank provides liquidity to banks by buying securities that the banks agree to buy back at a forward date (say two weeks). In other words, a central bank repo provides reserves to banks for a specified period against securities that act as collateral.

The interest rate at which liquidity is provided is decided through an auction. In a fixed-rate repo auction, the interest rate is set by the central bank and banks bid only quantities. A variable rate repo auction, however, is a multi-unit auction in which the bidders submit multiple price-quantity pair bids (demand functions), and the pricing rule is either discriminatory or uniform-price[(5)]. In a discriminatory auction bidders pay their own bids for units they win in an auction. In a uniform price auction, in contrast, all winners pay the market clearing price, which is defined as the highest bid at which aggregate demand equals or exceeds supply (i.e. the lowest price at which some positive quantity is won by some bidder)[(6)].

[(5)] The banking industry often refers to discriminatory auctions as "American" auctions and to uniform-price auctions as "Dutch" auctions. However, the literature on auctions refer to descending auctions (which are very different from uniform-price auctions) as Dutch auctions. To avoid confusion, we simply use the name uniform-price auction.

[(6)] An alternative definition of market clearing price is the highest losing bid. However, the definition used here is the definition used in practice in repo auctions, as well as other multi-unit auctions such as Treasury auctions, index-linked bonds sales, IPO auctions, and corporate bond auctions.

The demand for reserves by banks depends partly on the requirement for settlement balances arising out of the institutional characteristics of payment and settlement arrangements. This part of the demand (which is the private value component) is very insensitive to interest rates[7]. Thus the private value component of the demand is of little importance in the analysis. The other part of the demand for reserves depends entirely on expectations about future interest rates. Since repo rates determine future refinancing conditions for a bank, such expectations matter a great deal[8].

Thus signalling of policy stance, rather than liquidity management, is most important aspect of monetary policy implementation. The importance of expectations also implies that a common value auction model in which the quality of a bidder's guess about the future plays an important role is the natural setting for analyzing repo auctions.

Given the importance of signalling, uniform price repo auctions present a problem. The Bundesbank used uniform price auctions until 1988, but then abandoned the format in favor of discriminatory auctions. In 2001, the Reserve Bank of India introduced uniform price repo auctions for its liquidity adjustment facility. Within a year, however, they were substituted by discriminatory auctions. In both cases, the reason appears to be the fact that some bidders had bid high interest rates[9]. In view of this, an informal argument often put forward goes as follows: in a uniform price auction there is an incentive to bid at very high rates which implies bidders do not bid their true demands, but exaggerate their bids. This in turn implies that repo rates are too high, which therefore give the

[7] See Borio (1997).

[8] See Nautz (1998) for a formal model showing that influence of uncertainty, and the consequent importance of expectations about the future course of monetary policy, is characteristic of a flexible monetary policy design based on repo auctions. Further, As Borio (1997) points out, the remuneration of the reserve requirement of banks is equal to the average main refinancing rate (the repo rate in case of Bundesbank) during the reserve period (1 month for the Bundesbank). This allows a bank to make a profit at the expense of the central bank, as long as it can finance its positions through the central bank auctions. If a bank expects, say, the policy rate to rise during the period, it could borrow from the central bank early in the period and, if the expectation turns out correct, would make an expected profit simply by holding those funds as required reserves. Of course, the profit would be negative if the expectations are incorrect.

[9] In a recent statement on monetary policy, the Governor of Reserve Bank of India said that "an important disadvantage of the uniform price system is that, as bidders are sure to succeed at the most favourable rate, there is a possibility of indiscriminate or irresponsible bidding out of alignment with the market" (see Reserve Bank of India (2002)). See also Ministry of Finance, India (2002).

wrong signals to the money market.

Both strands of this argument are problematic. The idea of a "true" demand makes sense only in a private value setting, and as we showed above, a common value framework is the most appropriate in analyzing repo auctions. In any case, even in a setting with private value elements, uniform price auctions lead to demand reduction, not demand exaggeration. As Ausubel and Cramton (1998) have shown, any equilibrium in a uniform price auction is characterized by demand reduction by bidders (shading of bids to obtain a better price), which results in inefficiency. Further, if indeed repo rates are always too high in equilibrium, bidding at lower rates would be a profitable deviation for any bidder. Thus the informal argument is clearly unsatisfactory, and the nature of distortion arising in a uniform price auction is not well understood. Our results explain the nature of distortion arising in repo auctions through free riding.

Apart from signalling, our analysis applies to other concerns arising in similar auctions. In the longer-term repo auctions, signalling is of little importance, but private information about the future term structure of interest rates is perhaps even more important compared to short-term repos. In March 1999, the ECB switched the auction format for longer-term repos from uniform-price to discriminatory. In a press conference, the governor said[(10)]: "The single rate method was chosen in order to encourage less experienced counterparties to participate in the tender. The Governing Council takes the view that all interested counterparties should by now be sufficiently accustomed to the longer-term refinancing operation also to be in a position to participate in this type of operation under the more market-oriented multiple rate (American) method of allotment."

The concern clearly is that uniform price auctions give an undue advantage to less well informed bidders. Our results show how the choice of auction format can change the division of surplus between informed and uninformed bidders, and explain the policy choice.

Let us now discuss the results. Aside from the bidders, we allow for the presence of outsiders, who would enter if they could earn a strictly positive profit. Thus any equilibrium must satisfy a "no-arbitrage" condition: given the equilibrium strategies of the bidders, an uninformed outsider cannot profitably enter. Such a condition is perhaps a natural

[(10)]Source: European Central Bank (1999) condition in the context of auction markets. Further, we require equilibria to be robust to any cost of entry or bidding. The no-arbitrage condition and robustness to entry cost help us characterize all potential equilibria.

We show that any equilibrium in which uninformed bidders participate is characterized by free riding by the uninformed on the information of the informed. A uniform pricing rule allows uninformed bidders to condition on equilibrium price, and this limits the informed bidders' ability to gain from their information. Thus under a single price rule (which is the natural pricing rule is most markets) the question of information rent is moot, and depends on the extent of free riding by less informed bidders. The result contrasts with the usual intuition about the value of exclusive information.

Further, we show that the presence of large number of uninformed bidders in the repo auctions of the Bundesbank can distort the information content of auction prices so that higher prices are less informative than lower prices. As mentioned above, Nautz (1997) shows that when the Bundesbank used uniform price repo auctions, the money market adjusted almost completely to low repo rates, while very little adjustment occurs when repo rate is high(11). Our results explain this phenomenon.

Our results also apply to other multi-unit common-value auctions such as Treasury auctions. We use our results to shed light on the recent shift to uniform price auctions by the US Treasury, and comment on the potential reasons behind differences in Treasury auction formats across countries.

2 The Model

$S > 1$ units are offered for sale. Each unit has a value V. This is a random variable with a continuous distribution function $F(\cdot)$ and density function $f(\cdot)$ over a support $[0, \overline{V}]$. The distribution of V is public information.

There are N_I informed and N_U uninformed bidders. I and U also denote the sets of informed and uninformed bidders, respectively. Uninformed bidders have access to only

(11)Repo rate is low when the difference between the repo rate and the money market rate before the auction is below median. A high repo rate is defined analogously. Also note that a higher auction "price" corresponds to a higher repo rate. See footnote (4).

public information.

Each informed bidder receives a signal. Let $X = (X_1, \ldots, X_{N_I})$ denote the vector of signals where X_i is the signal received by informed bidder i, $i \in I$. Let $g(X_i|V)$ denote the conditional density of X_i given the common value V. Conditional on V, the random variables $X_1, \ldots, X_{N_I}$ are independently and identically distributed.

2.1 Bids and Strategies

A bid is any decreasing function $q(p)$ mapping the set of prices[12] $[0, \overline{V}]$ to the set of quantities $\{0, 1, \ldots, S\}$. Since there are S discrete units, a bid function is a step function:

$$q(p) = \begin{cases} 0 & \text{for } p_1 < p \leqslant \overline{V}, \\ 1 & \text{for } p_2 < p \leqslant p_1, \\ \vdots & \vdots \\ S & \text{for } 0 \leqslant p \leqslant p_S, \end{cases}$$

The following representation of a bid function is very useful. Note that the inverse of a bid function can be derived as follows:

$$p(\tilde{q}) = \begin{cases} \max_p \{p | q(p) \geq \tilde{q}\} & \text{if this exists,} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting function $p(\cdot)$ is the inverse demand function, and thus a bid can be written as a vector $(p_1, \ldots, p_S)$, such that

$$p(q) = \begin{cases} p_1 & \text{over 1 unit,} \\ p_2 & \text{over 2 units,} \\ \vdots & \vdots \\ p_S & \text{over } S \text{ units,} \end{cases}$$

where

$$\overline{V} \geqslant p_1 \geqslant p_2 \geqslant \ldots \geqslant p_S \geqslant 0. \tag{2.1}$$

[12] As explained in footnote (4), bidders in a repo auction bid interest rates, but the use of the term "price" is without loss of generality. In particular, a higher price corresponds to a higher interest rate in a repo auction. In a Treasury auction, on the other hand, the interest is paid by the Treasury to the bidders - so a higher price corresponds to a lower interest rate.

Let Ω be the set of vectors $(p_1, \ldots, p_S)$ that satisfy (2.1). Then $\Omega \subset \Re^S_+$ is the set of bid functions. Note that this is compact and convex.

Figure 1 shows the set of bids Ω for $S = 2$.

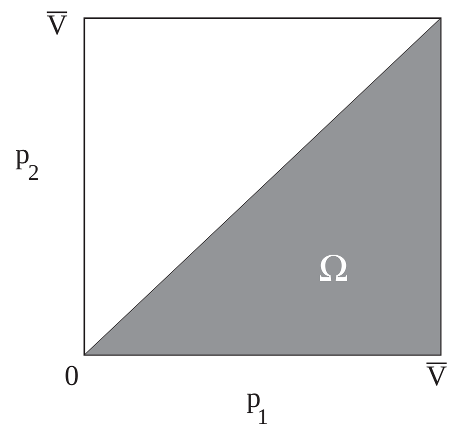

Figure 1: The set of bids for $S = 2$.

Without loss of generality we restrict attention to pure strategies for the informed bidders. A pure strategy for informed bidder i is written as $q_i(p)(X_i)$. A pure strategy for uninformed bidder j, $j \in U$ is simply a bid $q_j(p) \in \Omega$. A mixed strategy for uninformed bidder j is given by a probability distribution μ_j over Ω.

2.2 Market Clearing Price and Allocation Rule

The market clearing price is defined as follows.

Definition 1. For any $K \leqslant S$, the *market clearing price* $m(K)$ is given by the highest price at which demand exceeds or equals K units. Thus

$$m(K) = \sup_p \left(p | \sum_{i \in I} q_i(p)(\cdot) + \sum_{j \in U} q_j(p) \geqslant K \right).$$

Finally, the quantity won by a particular bid needs to be specified. Suppose bidder ℓ submits a demand function $q_\ell(p)$ specifying positive prices for k units, $k \leqslant S$, and $\ell \in I \cup U$. Also, let k' be the highest integer below k such that $p_{k'} > p_k$. Finally, let $\hat{Q}$

be total demand at prices strictly above $m(S)$. The winning function $q_\ell^w(p)$ is specified below.

$$q_\ell^w(p) = \begin{cases} k & \text{if } p_k > m(S), \\ k' + \alpha_\ell(S - \hat{Q}) & \text{if } p_k = m(S), \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\alpha_\ell = \frac{k}{\sum_{i \in I} q_i(m(S))(\cdot) + \sum_{j \in U} q_j(m(S))}$$

Thus if the price bid is above the market clearing price, the bidder wins the associated quantity bid, and if the price bid is equal to the market clearing price, a bidder wins a fraction of the remaining supply of $S - \hat{Q}$ proportional to the ratio of his own demand to total demand at the market clearing price[13].

Each winning bidder pays the market clearing price for each unit won.

2.3 Equilibrium

Definition 2. An *equilibrium* is a strategy profile that is (i) a (Bayesian) Nash equilibrium, (ii) satisfies the following no arbitrage condition: given the equilibrium strategies, no new uninformed bidder can enter and earn a strictly positive payoff, and (iii) is robust to an entry cost - i.e. the strategy profile continues to satisfy the previous two conditions if a small positive entry cost $c > 0$ is introduced.

2.4 Informational Free Riding

We define free riding as follows. Uninformed bidder j is said to free ride on the information of the informed bidders over $K_j > 0$ units if there is some $p_* > 0$ in the support of the market clearing price $m(S)$ such that: **(Condition 1)** uninformed bidder j wins K_j units whenever market clears below p_*, and **(Condition 2)** total uninformed demand is less than total supply, i.e. $\sum_{j \in U} K_j < S$.

[13] If $\alpha_\ell(S - \hat{Q})$ is not an integer, the bidder is allocated the greatest integer less than this. The remaining unit is then allocated randomly according to proportional probabilities.

A special case of free riding occurs if all uninformed bidders demand all their units above even the highest market clearing price. In this case $p_* = \overline{m}(S)$, and the uninformed bidders win $\sum_{j \in U} K_j$ units at market clearing prices dictated only by bids of informed bidders. This case is called **extreme free riding**.

The next section presents an example of equilibrium with free riding. The two following sections characterize equilibria given that some of the participating bidders are uninformed. Proofs not in the body of the paper are collected together in the appendix.

3 Equilibrium with Free Riding: An Example

In this section we analyze a simple common-value multi-unit uniform-price auction and construct an equilibrium with free-riding.

There are 2 informed and 2 uninformed bidders. Let X_i denote the signal of informed bidder i, $i \in \{1, 2\}$. Signals X_1 and X_2 are independently and uniformly distributed on the interval $[0, 1]$. $S > 1$ units are being auctioned. Each unit auctioned has a common value $(X_1 + X_2)/2$.

The following strategies form an equilibrium.

1. Uninformed bidder j submits the following demand function:

$$\text{demand} \begin{cases} K_j \geqslant 1 \text{ units} & \text{at all prices } 0 < p \leqslant 1 \text{, and,} \\ S \text{ units} & \text{at price } 0, \end{cases}$$

   where $K_1 + K_2 = S - 1$.

2. Informed bidder i receiving signal $X_i = x_i$ submits the following demand function:

$$\text{demand } L_i \geqslant 1 \text{ units at price } x_i/2.$$

   where $L_1 + L_2 \geqslant S$.

In this equilibrium the uninformed bidders win all but 1 unit, and informed bidders win only 1 unit. Given the bids of the uninformed bidders, the informed bidders face a single-unit first-price auction. The market clears at the winning bid in this auction. Since the market only clears at prices bid by informed bidders, this is a case of extreme free riding by uninformed bidders over $(S-1)$ units.

We must now show that the strategies above form an equilibrium.

Proof: First, given the strategy profile of uninformed bidders, each informed bidder can win at most 1 unit. The informed bidder with the highest price bid wins 1 unit and his bid is also the market clearing bid. Thus informed bidders face a single-unit first-price auction.

Since only the price bid over the first unit matters for informed bidders, we refer to this as their "bid." Suppose the symmetric equilibrium informed bid is given by $\beta(x_i)$ for signal $X_i = x_i$. Suppose bidder 2 follows the equilibrium strategy and suppose bidder 1 bids a price b. The payoff of bidder 1 is given by

$$Prob(b \geqslant \beta(X_2))E\left(\frac{x_1+X_2}{2} - b \mid b \geqslant \beta(X_2)\right),$$

which simplifies to

$$\left(\beta^{-1}(b)\right)\left(x_1/2 + E(X_2/2|X_2 \leqslant \beta^{-1}(b)) - b\right).$$

Maximizing with respect to b, and substituting the equilibrium condition that $b = \beta(x_1)$ in the first order condition, we have the following equation:

$$\frac{1}{\beta'(x_1)}\left(\frac{3x_1}{4} - \beta(x_1)\right) + x_1\left(\frac{1}{4\beta'(x_1)} - 1\right) = 0.$$

This simplifies to $x_1\beta'(x_1) + \beta(x_1) = x_1$. Solving this first order differential equation, we get $\beta(x_1) = x_1/2$. This proves that the stated strategy for each informed bidder is indeed a best response given the strategies of the uninformed bidders.

Next, we need to show that the strategies of the uninformed bidders are best responses. First, if an uninformed bidder demands a further unit at any positive price p, the market now clears at the maximum of p and the highest price on the first unit submitted by informed bidders, rather than only the latter. Thus there is a loss of payoff over the infra-marginal units. On the marginal unit, the extra payoff is

$$Prob(p \geqslant \max\{X_1/2, X_2/2\})E\left(X_1/2 + X_2/2 - p \mid p \geqslant \max\{X_1/2, X_2/2\}\right) \qquad (3.1)$$

which is clearly non-positive. Second, if an uninformed bidder demands a unit less, that does not lower the market clearing price, and thus lowers the payoff. Thus there is no profitable deviation in terms of quantity demanded. Finally, the market does not clear at uninformed price bids, and by deviating to any price below the market clearing price an uninformed bidder would win no units. These two facts imply that there is no profitable deviation from the price at which uninformed bidders demand their units. These arguments prove that the strategies of the uninformed bidders are indeed best responses.

Finally, we need to show that the candidate equilibrium strategies satisfy no-arbitrage, and are robust to the introduction of an entry cost. First, an entrant, by entering and bidding a price $p > 0$ earns at most the payoff given by (3.1). Since this is non-positive, the no-arbitrage condition is satisfied. Next, the equilibrium is robust to the introduction of an entry cost as every bidder earns a positive payoff. Indeed, the total payoff of the uninformed bidders is simply $(S-1)$ times the payoff of the winning informed bidder. This completes the proof.$\|$

4 Characterizing Equilibria I

The following result derives a condition that must be satisfied by the aggregate demand of informed bidders. This is very useful in proving the theorem that follows.

Let the aggregate demand of informed bidders be denoted by $P_I(Q) = (P_{I1}, \ldots, P_{IS})$.

Lemma 1. *In any equilibrium not characterized by extreme free riding over $(S-1)$ units, the profile of bids by the informed bidders must satisfy*

$$E(V - b | b \geqslant P_{I1}) < 0 \tag{4.1}$$

for any price b in the support of $m(S)$.

The following theorem characterizes equilibria.

Theorem 1. *All equilibria are characterized by free riding by the uninformed bidders.*

While the formal proof is in the appendix, the idea of the proof is as follows. To understand the idea, the following concept is useful: two bidders are said to “compete directly over 1 unit” if the higher of the two prices bid on that unit wins that unit and is also the market clearing bid.

Let us now explain the proof idea. As lemma 1 shows, the no-arbitrage condition imposes a certain lower bound on P_{I1}, the price at which the aggregate informed demand is one unit. We show that given this lower bound, any uninformed bidder competing directly with P_{I1} and winning would earn a negative payoff.

Next, the fact that any equilibrium must be robust to an entry cost implies that each uninformed bidder submits prices above a certain positive lower bound on some units. We show that this implies condition (1) of the definition of free riding (in section 2.4) is satisfied whenever condition (2) holds. Finally, we need to show that condition (2) holds. using the lower bound on uninformed bids, we show that whenever the total demand by uninformed bidders is S units (or more), and they win all S units (which must happen with positive probability), there is some uninformed bidder who wins the S-th unit by competing directly with P_{I1}. But as we said above, this is not profitable. Thus so long as the total demand by uninformed bidders is S units (or more), some uninformed bidder has a profitable deviation to demanding a lower quantity. This implies that the total quantity demanded by uninformed bidders must be below S, which satisfies condition (2) of the definition of free riding, and completes the proof.

5 Characterizing equilibria II

The last section showed that uninformed bidders can free ride and therefore profit at the expense of informed bidders under a uniform price format. In showing this, there is an implicit assumption that the total quantity bid by uninformed bidders at positive prices can be less than total supply - i.e. $\sum_{j \in U} K_j < S$. In this section we consider the other possibility: with a large enough number of uninformed bidders, no single bidder might be able to guarantee that the total uninformed quantity never exceeds total supply.

This case is important to consider because an important feature of the repo auctions conducted by the Bundesbank and the ECB is that there are a large number of bidders.

For the Bundesbank, the number of eligible counterparties (potential participants in repo auction) at the end of 2001 was 1,476. An average of 274 bidders participated in the weekly main repo auctions in 2001, and 166 in the monthly longer-term repo auctions[(14)]. A few of these bidders are large banks who participate regularly[(15)]. These banks are likely to be more experienced in forming expectations about the future course of monetary policy. However, these are not the only participants. For the ECB, for instance, a democratic and open process of liquidity provision, which gives all banks a chance to participate, is a stated goal.

If bidders other than bidder j bid on a total quantity of $S-2$ units or less, bidder j can always ensure that total quantity bid by uninformed bidders is less than supply. In this case, we know from the previous section that an equilibrium exists, and is characterized by free riding. To focus on the other case, we assume the following.

Assumption 1. Any price p at which $\sum_{j\in U} q_j(p) > 0$, $Prob\left(\sum_{j\in U} q_j(p) \geqslant S\right) > 0$.

This condition of course rules out free riding as defined in section 2.4. We modify the definition to **"partial free riding"** which is defined similarly to free riding (in section 2.4), but the second condition now changes to $Prob(\sum_{j\in U} K_j < S) > 0$.

We now characterize equilibria under the above assumption, and show that the twin objectives of signalling monetary policy and open participation can conflict with each other.

Let $m_U(S)$ be the market clearing price from only uninformed bids, and let $\overline{m}_U(S)$ be the highest such price. Second, let $m_I(S)$ be the market clearing price when the probability that some units are being won by informed bidders is positive, and let $\overline{m}_I(S)$ be the highest such price. Thus

$$\overline{m}_U(S) = \sup\left(p \mid Prob\Big(\sum_{j\in U} q_j(p) \geqslant S\Big) > 0\right),$$

[(14)]Source: Deutsche Bundesbank (2001). The numbers for the Eurosystem as a whole at the same time were 2500, 404, 225 respectively.

[(15)]For example, for fine tuning operations, only a small group of institutions are considered on the basis their activity in the money market or the foreign exchange market. At present, the Bank could call, in case of need, on a maximum of 41 counterparties (less than 3% of the number of potential bidders in regular repo auctions) for money-market operations and 15 for foreign exchange market operations. (Source: Deutsche Bundesbank (2001)).

and

$$\overline{m}_I(S) = \sup\Big(p \leqslant \max_i \overline{p}_i(1) \mid Prob\big(\sum_{j \in U} q_j(p) + \sum_{i \in I} q_i(p)(\cdot) \geqslant S\big) > 0\Big).$$

We characterize equilibria with some amount of successful participation by less informed bidders. As the following result clarifies, successful uninformed participation, essential to fulfill the stated goal of open participation, comes at a cost. The fact that uninformed bidders win some units can distort the information content of the auction price, reducing the signalling ability of the central bank.

> **Theorem 2.** *Under assumption 1, any equilibrium with some successful participation by uninformed bidders is characterized by $\overline{m}(S) = \overline{m}_U(S) = \overline{m}_I(S)$, with no atom of uninformed bids at $\overline{m}(S)$.*

Two properties of any such equilibrium are worth noting. First, the equilibrium is characterized by partial free riding (defined above, following assumption 1) by the uninformed bidders. Thus the concerns of the informed bidders with the uniform price format we discussed above apply here as well.

The second, and more important property is that lower prices are more informative. A market clearing price close to $\overline{m}(S)$ cannot signal much - such a price could come about even if informed bidders receive very low signals. A low auction price, on the other hand, definitely implies low signals. Prices in our model directly translate to interest rates - a higher price paid by a bidder corresponds to a higher interest rate paid by a bidder. The result indicates that a low repo rate should have a greater impact on the money market compared to a high repo rate. As Nautz (1997) reports, this is exactly what happened under the Bundesbank uniform price repo auctions. Thus our results clarify the theory behind the Bundesbank experience with uniform price repo auctions and explain the policy choice of central banks to move away from the uniform price format in conducting monetary policy through repo auctions.

6 Discussion

6.1 Treasury Auctions

Treasury auctions are among the most important of all auctions in terms of their impact on the economy. The US Treasury raises billions of dollars every week from sales of Treasury securities.

Like repo auctions, Treasury auctions are multi unit common value auctions. Although, unlike repo auctions, there is no intent of signalling policy stance, free riding has other important implications. In the US, participation in treasury auctions is open, and there are many smaller, relatively uninformed bidders alongside a few primary dealers. In this setting, it is important to provide encouragement to less informed bidders to participate. As the paper shows, uninformed bidders can free ride on the informed bidders in uniform price auctions, and thus this format is appropriate in encouraging wider participation. In discriminatory auctions, on the other hand, bidders pay their own bids for units they win in an auction. This precludes the possibility of free riding and discourages participation by less informed bidders. At the same time, since free-riding tends to shift the distribution of market clearing price upward, it does not harm revenue(16).

As our results show, in repo auctions, the use of a uniform price format to provide an advantage to smaller, less informed bidders comes at a cost - the monetary policy signal gets distorted. In contrast, as the above discussion shows, Treasury auctions do not face this tradeoff. This sheds light on the rationale behind the complete switch to a uniform price format (from a practice of using the discriminatory format for most auctions) by the US Treasury in 1998(17).

While the US now uses only the uniform price format, Treasury auctions in countries such as UK and Germany have always used discriminatory auctions. In these countries, only members of a specific group(18) are allowed to bid in Treasury auctions. To restrict

(16)Whether free riding in uniform price auctions translates into greater revenue compared to a discriminatory auction is an interesting question. However, addressing this requires a detailed characterization of equilibria in discriminatory auctions, which is beyond the scope of this paper.

(17)Ironically, discriminatory auctions are called “American auctions” in the banking industry even though this format is no longer used by the US Treasury.

(18)In the UK, these are the GEMMs (Gilt-Edged Market Makers), and in Germany these are members participation to a group consisting a few larger, informed banks (who are then given incentives to be very active in bidding), it is better to choose a discriminatory framework. This discourages entry in the group by less informed bidders by eliminating the possibility of free riding. This sheds light on the cross-country differences in Treasury auction format.

Finally, it is interesting to note that Treasury auctions have an institutionalized free riding facility to help small, less informed bidders. Treasury auctions as well as other common value multi-unit auctions such as Hambrecht's OpenBook auction[19] allow non-competitive bids which are quantity-only bids to be filled at the auction price (average auction price in case of discriminatory auctions). Such bids are exactly like bids of the associated quantities at the highest possible price, and thus allow free riding. This underlines the importance of free riding in such auctions, and strengthens the explanation we have provided for the switch to uniform price auctions by the US Treasury.

6.2 Auction Theory

Finally, we note our contribution to auction theory. In a uniform price auction, all bidders pay the market clearing price. We define this as the highest price at which the demand equals or exceeds supply[20]. In a single unit auction, the market clearing price coincides with the highest bid. Thus with a single unit, the analysis of a uniform price auction coincides with the analysis of a single-unit first-price common-value auction. Engelbrecht-Wiggans, Milgrom and Weber (1983) (EMW) analyze a single-unit common-value first-price auction with a single informed bidder and several uniformed bidders and derive the unique equilibrium. In this equilibrium uninformed bidders earn a zero profit.

The results here show that going from one unit to even two changes the flavor of the results substantially when a uniform-price rule is used. With more than one unit, free riding becomes possible, and thus uninformed bidders can earn a positive profit. Further, the equilibrium derived in EMW is not robust to any cost of entry or bidding. Any

of the "Bund Issues Auction Group."

(19) This is an online auction set up by the investment banker W.R. Hambrecht for selling corporate bonds. See Hall (2001) for details.

(20) As noted before in footnote (6), Some authors define the uniform price as the highest losing bid. However, this is never used in policy applications. Our definition is the one used in all practical applications.

such positive cost implies that deviation to non-participation is better for an uninformed bidder. Here, on the other hand, equilibria characterized by free riding are robust to entry/bidding costs. Moreover, under any such cost, if we also exclude the possibility of an outsider entering a earning a strictly positive profit, all equilibria are necessarily characterized by free riding.

Finally, in a multi-unit uniform-price auction, the auction price depends on the entire profile of demand functions (i.e. vectors of prices and associated quantities submitted). This makes the possible set of equilibria large and not very tractable. Here, the no-arbitrage condition and robustness to entry cost help us characterize all potential equilibria.

7 Conclusion

We characterize equilibria in a repo auction - which is a multi-unit common value auction - under a uniform price rule. We show that all equilibria in which some uninformed bidders participate are characterized by the latter free riding on the information of the informed bidders. The result shows that the usual intuition about the value of exclusive information in strategic settings may not hold in a uniform price auction, and explains the concerns of large banks participating in repo auctions conducted by the Bundesbank. Further, we show that the presence of a large number of uninformed bidders can distort the signalling ability of auction prices so that higher auction prices are less informative than lower prices. This impairs the ability of the central bank to signal monetary policy stance. The results explain the problem central banks face in using the uniform price format to implement monetary policy through repo auctions. The Bundesbank in the 80s as well as the Reserve bank of India in 2001 abandoned uniform price auctions in favor discriminatory auctions. The US Treasury, on the other hand, abandoned the discriminatory framework and switched to using only the uniform price format in selling government bonds. In a Treasury auction, there is no signalling intent, and the possibility of free riding creates an advantage for less informed bidders, encouraging wider participation. This sheds light on the rationale behind US Treasury policy.

8 Appendix

A.1 Proof of Lemma 1

Note that the market clearing price $m(K)$, $K \leqslant S$, is a random variable with a distribution induced by the mixed strategies of the uninformed bidders and the distribution of signals received by informed bidders[21].

Suppose an outsider enters and bids a price-quantity pair $(b, 1)$ (which is a demand function of the simplest form) where b is in the support of the market clearing price $m(S)$.

For any given strategy profile of the bidders, the entrant's profit from the price-quantity pair $(b, 1)$ is given by:

$$\begin{aligned} \pi_{\mathrm{E}}(\{b,1\}) &= Prob(m(S-1) \geqslant b \geqslant m(S))E(V-b|m(S-1) \geqslant b \geqslant m(S)) \\ &\qquad + Prob(b > m(S-1))E(V-m(S-1)|b > m(S-1)) \\ &\geqslant Prob(b \geqslant m(S))E(V-b|b \geqslant m(S)) \end{aligned} \tag{A.1}$$

Recall that the aggregate demand of informed bidders is denoted by $P_I(Q) = (P_{I1}, \ldots, P_{IS})$. Let the aggregate demand of the uninformed bidders be denoted by $P_U(Q) = (P_{U1}, \ldots, P_{US})$.

Now, suppose there is extreme free riding in equilibrium over $(S-1)$ units. Then aggregate demand is $(S-1)$ at prices above the highest market clearing price $\overline{m}(S)$. Thus $m(S-1) \nless \overline{m}(S)$ - i.e. any price b in the support of $m(S)$ never exceeds $m(S-1)$. Thus

$$Prob(b \geqslant m(S)) = Prob\left(b \geqslant P_{US}\right) Prob\left(b \geqslant P_{I1}\right).$$

Therefore the payoff of the outsider from the bid $(b, 1)$ is given by

$$\begin{aligned} \pi_{\mathrm{E}}(\{b,1\}) &= Prob(b \geqslant m(S))E(V-b|b \geqslant m(S)) \\ &= Prob\left(b \geqslant P_{US}\right) Prob\left(b \geqslant P_{I1}\right) E(V-b|b \geqslant P_{I1}) \end{aligned}$$

Thus to ensure absence of arbitrage, $E(V-b|b \geqslant P_{I1}) \leqslant 0$.

In the absence of extreme free riding over $(S-1)$ units, the supports of $m(S-1)$ and $m(S)$ overlap, and for any b in the support of $m(S)$ we have $Prob(b > m(S-1)) > 0$.

[21] Let Γ denote the distribution of aggregate demand. The market clearing price $m(K)$ is a random variable with a distribution given by the marginal distribution of Γ over the K-th unit.

Then (A.1) holds with strict inequality, and therefore, satisfying no-arbitrage requires $E(V - b|b \geqslant P_{I1}) < 0.\|$

A.2 Proof of theorem 1

Suppose an uniformed bidder $j \in U$ bids strictly positive prices on k units with strictly positive probability. Since a demand function of bidder j can be written as a vector of prices $(p_1^{(j)}, p_2^{(j)}, \ldots, p_S^{(j)})$ where $p_1^{(j)} \geqslant p_2^{(j)} \geqslant \ldots \geqslant p_S^{(j)} \geqslant 0$, another way of saying the above is that $p_k^{(j)} > 0$ with strictly positive probability.

Then let $\underline{p}_j(k)$ denote the infimum of the set of strictly positive prices $p_K^{(j)}$ that are submitted by bidder j with strictly positive probability. Formally, $\underline{p}_j(k) = \inf(p_k^{(j)}|p_k^{(j)} > 0)$.

Let us proceed to prove the theorem. In equilibrium, for any uninformed bidder j, and for any k, either $\underline{p}_j(k) \geqslant \overline{m}(S)$, or $\underline{p}_j(k) < \overline{m}(S)$.

If the former condition holds for all $j \in U$, equilibria are characterized by extreme free riding, and the proof is done.

Next, suppose for some j, $\underline{p}_j(k) < \overline{m}(S)$. We claim that in any such equilibrium, for each uninformed bidder j, $\underline{p}_j(k) > 0$ for some $k \geqslant 1$. To see this, suppose not. Then for any j, $\underline{p}_j(k) = 0$ for all $k \leqslant S$. But a bid at 0 must earn zero payoff in equilibrium (otherwise outsiders can enter and bid 0 and share the positive payoff to be had at a bid of 0) - and since uninformed bidders are playing a mixed strategy equilibrium, all their bids must earn a zero payoff. But then the strategies would not survive introduction of any positive entry cost. Thus any equilibrium strategy profile that is robust to entry cost must have the property that for each uninformed bidder j, $\underline{p}_j(k) > 0$ for some $k \geqslant 1$.

Let $K_j = \max(k|\underline{p}_j(k) > 0)$. Thus bidder j free rides on K_j units. For all participating uninformed bidders to successfully free ride, market must clear with positive probability at bids by informed bidders even below the lowest $\underline{p}_j(K_j)$. This is ensured if $\sum_j K_j < S$, which we now proceed to show.

Consider the aggregate demand of uninformed bidders other than uninformed bidder j. Let this be denoted by $p_{-j}(q) = (p_{(-j),1}, p_{(-j),2}, \ldots)$.

Let $\underline{p}_{(-j)}(k)$ be the lowest price at which the aggregate demand of uninformed bidders other than 1 equal or exceed k, and let K_{-j} denote the total amount on which uninformed bidders other than j free ride (thus $K_{-j} = \sum_{\ell \in U, \ell \neq j} K_\ell$).

For the result to hold, we must have $K_j + K_{-j} < S$ for all $j \in U$. This requires $\underline{p}_{(-j)}(S - K_j) = 0$. Suppose not. Then suppose $\underline{p}_{(-j)}(S - K_j) > 0$.

Now, there is always some j such that $\underline{p}_j(K_j) \leqslant \underline{p}_{(-j)}(S - K_j)$. Without loss of generality, let this be true for $j = 1$. Thus the rest of the proof refers to uninformed bidder 1.

Further, to reduce notation, let the quantity demanded by uninformed bidder 1 be $K_1 = K$.

Case 1: There is no atom at $\underline{p}_{(-1)}(S - K)$.

Note that we have started with the supposition that $\underline{p}_1(K) \leqslant \underline{p}_{(-1)}(S - K)$.

Then any $p_{1,K} \in [\underline{p}_1(K), \underline{p}_{(-1)}(S - K)]$ wins only if $p_{(-1),(S-K+1)} \leqslant p_{1,K}$ and $P_{I1} \leqslant p_{1,K}$[22]. Compare two demand functions of bidder 1: $p^{(1)}(q) = \left(p_1^{(1)}, \ldots, p_S^{(1)}\right)$, and $p^{(2)}(q) = \left(p_1^{(2)}, \ldots, p_S^{(2)}\right)$ such that the demand functions are identical up to $K - 1$ units, but differ on the K-th unit (and do not demand any further units at any positive price):

$$\begin{array}{rcccl} p_\ell^{(1)} & & = & & p_\ell^{(2)} \quad \text{for} \quad \ell \in \{1, \ldots, K-1\}, \\ p_K^{(1)} & = b & > & 0 = & p_{1K}^{(2)} \\ p_m^{(1)} & = 0 & = & & p_m^{(2)} \quad \text{for} \quad m \in \{K+1, \ldots, S\}, \end{array}$$

where $b \leqslant \underline{p}_{(-1)}(S - K)$.

Let D denote the amount by which the payoff of bidder 1 from $p^{(1)}(\cdot)$ exceeds that from $p^{(2)}(\cdot)$:

$$\begin{aligned} D &= Prob(P_{I1} \leqslant b) Prob(p_{(-1),(S-K+1)} \leqslant b) \Big[K E(V - b | P_{I1} \leqslant b) \\ &\qquad - (K-1) E\left(V - \max\left(P_{I1}, p_{(-1),(S-K+1)}\right) | P_{I1} \leqslant b\right) \Big] \\ &< Prob(P_{I1} \leqslant b) Prob(p_{(-1),(S-K+1)} \leqslant b) E(V - b | P_{I1} \leqslant b) \\ &< 0, \end{aligned}$$

[22] Note that if there is an atom at $\underline{p}_{(-1)}(S - K)$, $p_{1,K} = \underline{p}_{(-1)}(S - K)$ could win even if these conditions are not satisfied.

where the last step follows as before from (4.1). Thus $p^{(2)}(\cdot)$ yields a strictly greater payoff. Since this is true for any $0 < b \leqslant \underline{p}_{(-1)}(S-K)$, and thus $p_{1K} = \underline{p}_{(-1)}(S-K)$ is strictly worse than $p_{1K} = 0$, there exists $\xi > 0$ such that in equilibrium

$$p_{1,K} \notin (0, \underline{p}_{(-1)}(S-K) + \xi].$$

This implies that $\underline{p}_1(K) > \underline{p}_{(-1)}(S-K)$, which contradicts the original supposition.

Thus there is no equilibrium with $\underline{p}_{(-j)}(S-K_j) > 0$ for any $j \in U$.

Case 2: There is an atom at $\underline{p}_{(-1)}(S-K)$.

The same argument as in case 1 above now applies to $p_{1,K} \in (0, \underline{p}_{(-1)}(S-K))$. Thus, by the argument for case 1, $p_{1K} \notin (0, \underline{p}_{(-1)}(S-K))$. The only remaining possibility is that $\underline{p}_1(K) = \underline{p}_{(-1)}(S-K)$.

Suppose $\underline{p}_1(K) = \underline{p}_{(-1)}(S-K)$ is preferred to $p_{1K} = 0$ (otherwise the proof is already done). But then by raising p_{1K} by a very small amount 1 can improve his payoff as price paid increases continuously while expected quantity won changes discontinuously. Thus there is no atom at $\underline{p}_1(K)$. But then, by an argument analogous to case 1, bids at $\underline{p}_{(-1)}(S-K)$ (by some bidder j) earn a negative payoff. Thus if in equilibrium $\underline{p}_1(K) = \underline{p}_{(-1)}(S-K)$, there cannot be an atom at $\underline{p}_{(-1)}(S-K)$. Therefore if there is an atom at $\underline{p}_{(-1)}(S-K)$, the only possibility is $\underline{p}_1(K) > \underline{p}_{(-1)}(S-K)$ which contradicts the original supposition.

Thus again there is no equilibrium with $\underline{p}_{(-j)}(S-K_j) > 0$ for any $j \in U$. This proves that $\sum_{j \in U} K_j < S$. Therefore any equilibrium must be characterized by free riding.$\|$

A.3 Proof of theorem 2

The proof proceeds through the following lemma.

Lemma 2. *Under assumption 1,* $\overline{m}_U(S) \ngtr \overline{m}_I(S)$.

Proof: Suppose not. Then it is possible to have $\overline{m}_U(S) > \overline{m}_I(S)$ in equilibrium. In this case, $\overline{m}(S) = \overline{m}_U(S) > \overline{m}_I(S)$.

Case 1: $\overline{m}_U(S) > EV$ In this case, whenever market clears at prices in the interval $(\max(\overline{m}_I(S), EV), \overline{m}_U(S)]$ the uninformed bidders earn a strictly negative payoff. In this case, an uninformed bidder has a profitable deviation as follows.

There is some bidder j whose demand function involves prices above $\overline{m}_I(S)$. Suppose j demands a total K_j units. Consider a deviation by j to a flat demand of K_j at price $\overline{m}_I(S)$. Then j wins nothing whenever market clears above $\overline{m}_I(S)$ (thus reducing negative payoff) - and still wins all K_j units when market clears at bids made by informed bidders below $\overline{m}_I(S)$. Thus the deviation is profitable.

Case 2: $\overline{m}_U(S) \leqslant EV$ Consider a price-quantity pair $(EV, 1)$ bid by an outsider. If uninformed bidders bid so that market clears below EV with positive probability, then this bid makes a strictly positive profit, violating the no-arbitrage condition. Suppose uninformed bidders bid at EV so that the market never clears below EV. Then if $\overline{m}_U(S) > \overline{m}_I(S)$, informed bidders make a zero payoff. But then for a sufficiently high signal, an informed bidder can bid just above EV, win with probability 1, and make a positive profit. Thus it is not possible to have $\overline{m}_U(S) > \overline{m}_I(S)$.‖

Next, the fact that uninformed bidders win a positive quantity implies that $\overline{m}_U(K) \nless \overline{m}_I(K)$. From this, and from lemma 2, in any equilibrium with some successful participation by uninformed bidders, we must have $\overline{m}(S) = \overline{m}_U(S) = \overline{m}_I(S)$.

Finally, we need to show that there is no atom of uninformed bids at $\overline{m}(S)$.

Suppose, on the contrary, that the distribution of market clearing bid has an atom of uninformed demand at $\overline{m}_U(S)$.

Now, if $\overline{m}(S) < EV$, an outsider could bid a price-quantity pair $(EV, 1)$ and make a strictly positive profit. Thus we must have $\overline{m}(S) \geqslant EV$, which implies $\overline{m}_U(S) \geqslant EV$. Thus whenever market clears at $\overline{m}_U(S)$ the uninformed bidders earn a non-positive gross payoff (and a strictly negative payoff net of entry cost). Consider a deviation by uninformed bidder j to a price just below $\overline{m}_U(S)$. This suffers a second order loss compared to the original bid whenever market clears below $\overline{m}_U(S)$, but enjoys a first order gain whenever market clears exactly at $\overline{m}_U(S)$. Thus the deviation is profitable. Contradiction.‖

References

Ausubel, Lawrence and Peter Cramton, "Demand Reduction and Inefficiency in Multi-Unit Auctions," 1998. working paper, University of Maryland.

Borio, Claudio E.V., "The Implementation Of Monetary Policy In Industrial Countries: A Survey," 1997. BIS Economic Papers, No. 47.

Deutsche Bundesbank, "Annual Report," 2001. www.bundesbank.de/vo/vo_gb.en.php.

Engelbrecht-Wiggans, Richard, Paul Milgrom, and Robert Weber, "Competitive Bidding and Proprietary Information," *Journal of Mathematical Economics*, 1983, *11:2*, 161–169.

European Central Bank, "The President's Introductory Statement, Press Conference, March 4," 1999. www.ecb.int/key/st990304.htm.

Hall, Robert E., *Digital Dealing*, New York: Texere, 2001.

Klemperer, Paul, "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys*, 1999, *13*, 227–286.

Milgrom, Paul and Robert Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica*, 1982, *50:5*, 1089–1122.

Ministry of Finance, India, "Economic Survey 2001-02," 2002. http://indiabudget.nic.in/es2001-02, especially chapter 48 in the section on capital and money markets.

Nautz, Dieter, "How Auctions Reveal Information: A Case Study on German REPO Rates," *Journal of Money, Credit, and Banking*, 1997, *29*, 17–25.

——, "Banks Demand for Reserves When Future Monetary Policy is Uncertain," *Journal of Monetary Economics*, 1998, *42*, 161–183.

Reserve Bank of India, "Statement by Dr. Bimal Jalan, Governor, Reserve Bank of India on Monetary and Credit Policy for the year 2001-2002," 2002. www.rbi.org.in.