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Subsidizing Inventory: A Theory of Trade Credit and Prepayment

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Abstract

We propose a simple theory of trade credit and prepayment. A downstream firm trades off inventory holding costs against lost sales. Lost final sales impose a negative externality on the upstream firm. We show that allowing the downstream firm to pay with a delay, an arrangement known as “trade credit,” is precisely the solution to the problem. Solving a reverse externality accounts for the use of prepayment for inputs, even in the absence of any risk of default by the downstream firm. We clarify previously unexplained facts including the universal presence of a zero-interest component in trade credit terms, and the non-responsiveness of interest charges to fluctuations in the bank rate as well as market demand. We explain why trade credit is short term credit and why the level of provision is negatively related to sales and profit and inventory, but positively related to the profit margin. Finally, we show that under trade credit, inventory investment is invariant to the real interest rate for a wide range of parameters, explaining the puzzle posed by Blinder and Maccini (1991). This implies that standard empirical inventory models would gain explanatory power by including the subsidy effect of accounts payable.

KEYWORDS: *Trade credit, prepayment, externality, subsidy, the Burkart-Ellingsen critique, inventory investment*

JEL CLASSIFICATION: D2, E5

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1 INTRODUCTION

Delivery of inputs to a firm does not always coincide with payment by the firm. A short term delay in payments is a widely observed form of interfirm credit. Delayed payments, usually called trade credit, account for about 15% of the assets of U.S. manufacturing firms. Its importance is further underlined by the role it plays in transmitting monetary policy shocks (e.g. see Brechling and Lipsey (1963), Meltzer (1960) and Nilsen (2002)).

Why does a supplier of inputs provide credit at zero interest¹ to its customers? We propose a simple theory of trade credit as a subsidy mitigating a negative externality. A downstream firm trades off inventory holding costs against lost sales. Lost final sales impose a negative externality on an upstream firm, which supplies inputs. Bank loans cannot solve this problem. However, the upstream firm can induce the downstream firm to internalize the externality by allowing delayed payments, which is the correct instrument to subsidize inventory holding. Our model accounts for the use of delayed payment for inputs (trade credit) as well as prepayment for inputs, which mitigates a reverse externality.

There is a large literature addressing the question under credit market imperfections (arising from asymmetric information problems). To clarify our contribution, let us discuss some aspects of this literature and compare with our approach. A common argument advanced in justifying trade credit is that the input suppliers are better informed/can better monitor their customers compared to banks. An interesting recent paper by Burkart and Ellingsen (2004) reviews this literature concisely and provides a cogent critique. They question the plausibility of such an advantage since typically banks have a greater expertise in evaluating borrowers. The alternative theory they advance argues that it is the nature of the trade credit instrument - illiquid inputs - that is important. The commitment value of illiquid inputs ameliorates the vulnerability of liquid funds to misuse.

In this view trade credit is supplied only when it earns a return as high as real investment at the margin. This ties the trade credit interest rate to be at least as large as the bank

¹In most industries in developed economies, trade credit is provided at zero interest. Trade credit is expensive only when the borrower does not repay early, choosing to forego any discount on the invoice. Further, as noted by Ng et al. (1999), there is widespread use of net terms which are not costly at all when repaid within the specified time period (often 30 days).

rate. However, Ng et al. (1999) report that 88% firms in their survey do not change terms at all when banks change interest rates. Further, to motivate the supply of trade credit, this literature often calculates the interest rate implicit in any missed discount and identifies such credit as a high cost loan. If indeed the objective of trade credit providers were to charge a high interest rate, it would be far simpler to specify a standard loan contract at a high rate. But such terms are not observed anywhere in the developed world. Instead, all trade credit has a zero interest period lasting typically 10-60 days². For net terms, which are widely observed, trade credit has no cost if paid within 30 days. Therefore we believe the observed terms require further explanation.

Next, as Burkart and Ellingsen (2004) point out, the papers invoking a monitoring advantage for the input supplier fail to explain why trade credit is limited only to the value of inputs. Their theory focuses on the advantage derived from the illiquidity of the trade credit instrument, automatically tying the credit to the value of inputs. However, since the illiquidity of inputs is critical, the theory cannot explain the practice of prepayment which is similar to (reverse) trade credit, but is an advance of liquid cash rather than illiquid inputs.

One explanation for prepayment, arising in the context of developing countries, is that it is a response to the presence of default risk and/or the risk of order non-collection. However, in developed western economies, often large, well established firms prepay their suppliers, which cannot be explained by appealing to default risk³.

Therefore it remains a challenge to simultaneously explain (a) both trade credit (input advances by the upstream firm) and prepayment (cash advances by the downstream firm), (b) why all such advances are limited by the value of inputs, and (c) the reason behind the observed terms - in particular why they always have a zero-interest component.

In our model a downstream firm facing stochastic final demand can either produce immedi-

²Brennan et al. (1988) note that while the credit offers by captive finance companies in the auto mobile industry are accompanied by discounts that effectively raise the interest rate above the quoted rate, the “substantial increase in market share gained by the auto finance captives suggests, however, that there remains a net subsidy in their credit terms.”

³As Ferris (1981) notes, prepayment is observed in construction, shipbuilding, aircraft and parts of the defense industries. Typically, in such cases, the probability of default or non-collection is zero. Prepayment is also widely used in the US oil and gas industries.

ately after each sale, or wait for one or more periods before producing. If the downstream firm finds it profitable to follow a waiting strategy, it might lose some sales, generating a negative externality for the upstream firm. We show that delayed payment (trade credit) induces the downstream firm to internalize the negative externality by subsidizing inventory holding.

This subsidy is limited precisely by the value of inputs that is expected to be held. Thus our theory is not subject to the Burkart-Ellingsen critique mentioned above. The theory extends to reverse trade credit - i.e. prepayment. This case arises when the upstream firm wants to wait, generating a negative externality for the downstream firm. This explains why we simultaneously observe trade credit in some cases of upstream-downstream trading, and prepayment in others, and why all such advances are limited by the value of inputs.

Further, the bank rate represents the opportunity cost of inventory holding. Therefore waiting is optimal when the bank rate is relatively high. Since a high bank rate is one of the factors that can *cause* waiting, bank loans cannot solve the problem of negative externality. The solution must be a lower cost subsidy covering at most the value of inputs - and trade credit is precisely that instrument⁴. This clarifies the reason for providing credit at a zero interest rate which is lower than the bank rate, and explains why the zero interest component is present in every trade credit contract. In contrast to the literature, this also explains why the trade credit interest rate does not vary with either the bank rate or market demand fluctuations⁵. A further implication is that trade credit in our model is a complement to bank credit. Existing theories based on imperfect credit markets often imply that such complementarity arises from credit rationing. Our theory, in contrast, predicts complementarity even under perfect capital markets.

While a zero interest component is universal, trade credit terms do vary across sectors. The most common terms are “net 30” (the downstream sector must pay within 30 days to avoid any interest cost). There are also discount terms, typically allowing the downstream

⁴In our simple model, the discounted stream of trade credit and prepayment can be represented as total transfers of a fraction of the input price. Clearly, this can also be achieved by reducing or increasing the price. However, price variation is not an appropriate instrument because it does not target the source of the problem. The issue is discussed in section 9.

⁵See Ng et al. (1999).

firm to take a 2% discount if paid within 10 days. Our model does not directly address this issue. However, our informal explanation for the different net and discount terms observed across sectors is that these serve as repayment incentives⁶, and these incentives should vary across sectors depending on the pattern of final demand and the nature of inputs. Further, Ng et al. (1999) report that 72.4% of firms in their sample at least occasionally allow longstanding customer firms to take unearned discounts. Such weak enforcement is consistent with our view of trade credit. We should expect that as part of ongoing trading relationships, the upstream sector does not always aggressively implement repayment incentives, especially when a delay results from an observable slump in demand or other observable frictions.

Our theory emphasizes the role played by inventories, and therefore our results also relate to the literature on inventories. Blinder and Maccini (1991) point out that contrary to standard theoretical predictions, inventory investment is insensitive to fluctuations in the real interest rate. They describe this as an “open, important and troublesome” issue. Our results offer an explanation for this apparent anomaly. We show that a downstream firm holds a higher level of inventories under trade credit compared to a setting without trade credit, making the level of inventories insensitive to the real interest rate under a wide range of parameters.

A recent paper by Maccini et al. (2004) offers an explanation to the inventory insensitivity puzzle based on the claim that firms care about the long-run level of the interest rate and not about short run rate fluctuations. Our theory complements their work by clarifying why firms might not respond to transitory shocks.

Our results also have implications for the relation between trade credit provision and the supplier’s margin of profit as well as level of profit, the duration of trade credit and business cycle effects. We show that the implications are consistent with evidence.

⁶See also the discussion in section 10.

RELATED LITERATURE

We discussed above the literature exploring trade credit as an antidote to credit market failures arising out of asymmetric information. Other papers assuming perfect financial markets have highlighted sales as motivation for the upstream firm's provision of trade credit. Our theory takes the latter approach, although our analysis is quite different from other papers in this class. An early contribution by Nadiri (1969) simply assumed that total sales is an increasing function of upstream credit provision, and derived optimal trade credit in a simple profit maximization framework. The negative externality we identify provides a reason for such a relation between sales and trade credit to arise. Ferris (1981) suggests that firms utilize trade credit to pool liquidity risk, although, as Burkart and Ellingsen (2004) point out, this does not explain why banks do not perform the pooling function. Our theory, in contrast, rejects bank loans as a solution. Indeed, a high bank rate is one of the factors that induce firms to wait in the first place, generating the negative externality. This provides an explicit reason for delayed payment, which is a loan at a rate lower than the bank rate (and therefore subsidizes inventory holding), to be exactly the right instrument to solve the problem.

A further theory (Schwartz and Whitcomb (1979), Brennan et al. (1988)) explains trade credit as a device for price discrimination. They assume that direct price discrimination (charging different prices to different customers) is not possible. However, offering trade credit at different terms enables the upstream firm to lower the effective price for some customers. If the seller does not have full information about its clients and must set up a menu of contracts, incentive compatibility requires trade credit interest rate to be at least as high as the bank rate (see Brennan et al.). However, as pointed out before, a universal feature of trade credit in developed countries is that it has a zero interest component, which is a rate lower than the bank rate. Further, this theory ties trade credit offers to characteristics of demand facing downstream firms, and cannot account for the observed invariance of the interest charge to market demand fluctuations. Our model, in contrast, accounts for both of these features of trade credit. Finally, price discrimination theory cannot explain the fact that most firms, at least occasionally, allow longstanding customer firms to take unearned discounts. While this is not an issue our model addresses directly, weak enforcement for longstanding customers, as explained earlier, is entirely consistent

with our view of trade credit.

In explaining the role of inventories, Kahn (1992) pointed out that firms must weigh the cost of stockouts (in terms of lost sales) against the cost of carrying inventory in making the optimal production decision. In our model, firms make precisely such a calculation. The extra dimension here arises through the upstream-downstream relationship, and the consequent externalities arising from lost downstream sales. Kahn found evidence of stockouts in U.S. automobile firms, while admitting zero inventories are observed only rarely. Our model suggests this rarity is because trade credit provides the incentive to hold a higher level of inventories under a wide range of parameters. A proper empirical inventory specification should therefore include a trade credit term. This is particularly important in view of the critical role inventories play in business cycles. Blinder and Maccini (1991) note that for the U.S., drop in inventory investment accounts for 87% of the drop in GNP during the average postwar recession.

An earlier paper by Emery (1987) explored the link between inventories and trade credit. In the Emery model, an upstream firm in an industry facing seasonal demand varies credit terms to divide inventory and financing costs between itself and the downstream firm. However, as Ng et al. (1999) document, firms in reality do not vary the credit terms once they have been determined.

The rest of the paper is organized as follows. The next section introduces the model. Section 3 derives the optimal strategies of the firms in the absence of trade credit and clarifies the negative externality generated by the downstream sector. Section 4 then explains the role of trade credit in ameliorating the externality. Next, section 5 extends the model by introducing an upstream cost of production. Section 6 relates our results to evidence. Section 7 analyzes the case of prepayment, and section 8 compares the cases in which trade credit arises to those in which prepayment results. Section 9 discusses why the right instrument is trade credit or prepayment and not price variation. Section 10 discusses extensions, and section 11 concludes. Proofs not in the body of the paper are collected in the appendix.

2 THE MODEL

There is an upstream and a downstream firm. The upstream firm produces an intermediate good which the downstream firm uses as input, and converts into a final consumption good.

Timing is important since - given positive interest rates - a longer inventory holding period implies a higher (opportunity) cost. If a firm purchases inputs early and then experiences low sales, it has to hold inventories over a longer period of time, and therefore incurs a higher cost.

There are an infinite number of periods. Production of 1 unit by either firm takes exactly 1 period. The structure of timing is as follows.

Period -1 is the set-up period for the upstream firm. At the start of period -1, the downstream firm can place an order for a unit of input. The upstream firm has the option of producing 1 unit in period -1⁷. Period 0, which follows, is the set-up period for the downstream firm. It can buy the unit of input from the upstream firm (if the latter has a unit available) and produce a unit of the final good, or choose to wait.

From period 1 onwards, the market is open. At the start of each period, a new customer (who buys a unit of the final good from the downstream firm) arrives with probability p , where $0 < p < 1$.

The arrival of a new customer in any period $t \geq 1$ leads to a successful sale if the downstream firm has a unit available in finished goods inventory. If the firm has no inventory (this can happen, for example, if the downstream firm chooses to wait in period 0 and a customer arrives at the start of period 1), and the customer fails to obtain a unit, he returns with probability q next period (where $0 < q < 1$), and if still not served, does not return.

The return probability q plays an important role. If $q = 0$, firms either produce immediately after each sale, or never produce. If $q > 0$, this gives firms a potential reason for

⁷Note that this period cannot be moved. In other words there is no sense in saying “the upstream firm should start production in a period later than -1.” Whenever the upstream firm produces the first unit, that is period -1.

waiting till a customer arrives to start production. As we clarify later in the paper, if either non-production or waiting to produce is optimal for a firm, a negative externality and consequent scope for trade credit or prepayment arises.

Let P denote the price charged by the downstream firm and C denote the price charged by the upstream firm, which is also the (constant) marginal cost of the downstream firm. Let G denote the (constant) marginal cost of the upstream firm. We assume that $P \geq C \geq G$.

Let r denote the market rate of interest, and let δ denote the discount factor ($\delta = \frac{1}{(1+r)}$), where $0 < \delta \leq 1$.

BENCHMARK EFFICIENCY

We use the term “efficiency” in a limited sense here. We simply require that given the prices, production should not waste any surplus. Since the cost of production of the upstream product is G , and the final price received is P , and since $P \geq G$ (by assumption) efficiency requires that both firms produce immediately after each sale (so that no sale is ever lost). In other words, efficiency requires that all firms pursue the immediate production strategy for all $p > 0$ (i.e. whenever the probability of a customer arriving is non-zero).

STRATEGIES

We consider two types of strategies for each firm. A firm can produce immediately after a sale, or follow a waiting strategy. The details are as follows.

Downstream firm: The “immediate production” strategy is as follows.

- Place an order for a unit of input at the beginning of period -1. Subsequently, produce 1 unit in period 0.
- In any period $t > 0$,
 - if a customer arrives at start of the period, sell and produce again,
 - otherwise carry over inventory to the next period.

The “wait-and-see” strategy is as follows.

- Do not place an order for a unit of input at the beginning of period -1. Do nothing in period 0.
- In any period $t > 0$,
 - produce only if a new customer arrives at the start of period t and this customer is not served (i.e. produce in period $t > 0$ only when there is a strictly positive probability of a returning customer arriving at the start of period $t + 1$).
 - Otherwise restart period 0 strategy.

This strategy says “produce if (1) a customer arrives and (2) there is no unit in the inventory.” If these two conditions are met, the arriving customer is not served and might return next period. Thus the probability of at least one customer arriving next period is higher. Note that waiting beyond this does not make any sense, as any unsatisfied customer today is, by construction, not around two periods from now. More general models could include the possibility of a customer returning many times, and then waiting strategies are more complicated. We have chosen the simplest possible framework in which waiting makes sense and therefore a scope for trade credit arises.

Upstream firm: The immediate production strategy for the upstream firm is as follows: Produce 1 unit in period -1, and then produce 1 unit after every sale of a unit to the downstream firm.

The wait-and-see strategy for the upstream firm is as follows: If the downstream firm places an order to buy a unit in period 0, produce in period -1 and then produce after each sale. If the downstream firm waits (does not place an order to buy in period 0), do not produce in period -1. In any period $t > 0$, produce a unit only if the downstream firm makes a sale at the beginning of period t .

Note that if both firms follow a wait-and-see strategy, production never gets started, and payoffs are zero for all. Thus there are three non-trivial cases: the case in which both firms produce immediately, and two cases in which one of the two firms wait and the other produces immediately. In all these cases, the upstream firm produces in period -1. Thus for purposes of comparison across cases, we can ignore the cost of production in period -1 and simply suppose that the upstream firm starts life in period 0 with an endowment of 1 unit of output.

3 OPTIMAL STRATEGIES AND EXTERNALITY

The upstream cost G does not play a crucial role in making the case for trade credit (except for the uninteresting case in which the cost is so high that the upstream sector does not provide trade credit at all). To clarify the case for trade credit in the simplest possible framework, we assume that the upstream cost

$$G = 0$$

throughout this section. This allows us to ignore the upstream participation constraint and simplifies the exposition. This is then relaxed in section 5, which clarifies the additional upstream participation constraint that arises with a positive cost. This cost is also crucial in the subsequent section which analyzes the case for prepayment.

3.1 DOWNSTREAM OPTIMUM

Let V_0^I be the value at time 0 under immediate production. Clearly,

$$V_0^I = -C + \delta p (P - C) + \delta^2 p (P - C) + \dots = -C + \frac{\delta p (P - C)}{1 - \delta} \quad (3.1)$$

Let V_0^W be the payoff under the wait-and-see strategy. The following result derives this payoff.

Lemma 1 *The payoff of the downstream firm under the wait-and-see strategy is given by*

$$V_0^W = \frac{\delta p [\delta P (p + (1 - \delta)(1 - p)q) - C (1 - \delta(1 - p))]}{(1 - \delta)(1 - \delta + \delta p(2 - q))}. \quad (3.2)$$

Apart from the payoffs from the immediate and wait-and-see strategies, we need to consider the incentive of the downstream firm to participate in production at all. Clearly, if the rate of interest is high relative to the return from investment, the optimal choice is to not invest. The rate of return from investment for the downstream firm is given

by $(P - C)/C$. The following result shows that the rate of interest r must be less than $(P - C)/C$ for the downstream firm to choose to invest at all. Note that $r < (P - C)/C$ implies $1/(1 + r) \equiv \delta > C/P$. The result also shows that this lower bound for δ is tight in the sense that for δ above the lower bound, there does exist some values of p and q such that the downstream firm chooses to invest (using either the wait-and-see or the immediate production strategy).

Proposition 1 *For any $\delta \leq C/P$, no investment takes place for any values of p and/or q . Further, for $\delta > C/P$ there exists $p \in (0, 1)$ and $q \in (0, 1)$ such that $\max\{V_0^I, V_0^W\} > 0$.*

The result shows that $\delta > C/P$ is necessary for investment to take place. We assume this is the case:

Assumption 1 $\delta P > C$.

The following result now derives the optimal strategy, shown in figure 1. Let p_ℓ^D denote the cutoff below which the downstream firm does not produce, and let p_{ic}^D denote the cutoff above which the downstream incentive to follow the immediate production strategy is satisfied.

Proposition 2 *For any given P, C, q and δ satisfying assumption 1, there exist cutoffs p_ℓ^D and p_{ic}^D where $0 \leq p_\ell^D < p_{ic}^D < 1$ such that (a) for any $p \geq p_{ic}^D$, the optimal choice is immediate production, (b) for $p_\ell^D < p < p_{ic}^D$, wait-and-see is optimal, and (c) for $p \leq p_\ell^D$ no investment is optimal. p_ℓ^D and p_{ic}^D are given by:*

$$p_\ell^D = \max \left\{ 0, \frac{(1 - \delta)(C - \delta q P)}{\delta((P - C) - (1 - \delta) q P)} \right\} \quad (3.3)$$

$$p_{ic}^D = \frac{(1 - \delta)C}{\delta((P - C) - q(\delta P - C))} \quad (3.4)$$

3.2 UPSTREAM PAYOFFS AND EXTERNALITY

Note that the upstream firm has a zero cost of production, and therefore does not gain anything by waiting for one or more periods after a sale and then producing a further

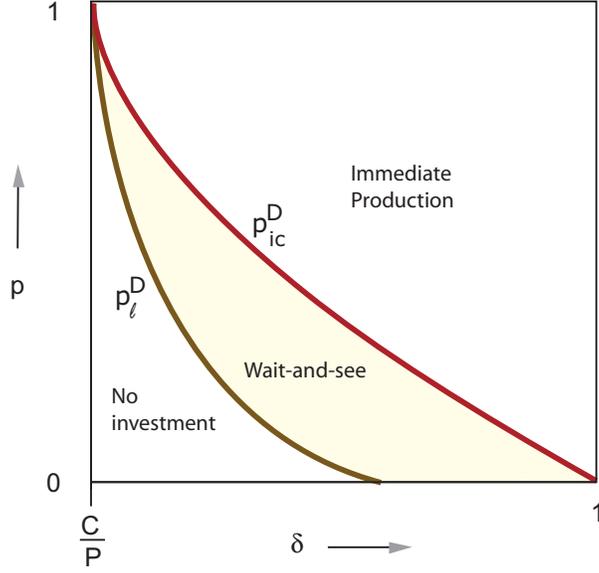


Figure 1: p_{ic}^D is the cutoff above which the downstream firm pursues immediate production. An inefficiency arises for $p < p_{ic}^D$. The downstream firm chooses the wait-and-see strategy for $p \in [p_\ell^D, p_{ic}^D)$, and does not invest at all for $p < p_\ell^D$. p_ℓ^D and p_{ic}^D are drawn here for $C/P = 0.5$ and $q = 0.6$.

unit of input. However, since waiting forces the downstream firm to wait as well, some sales are lost with positive probability, lowering profit in both sectors. Thus the upstream firm always produces immediately after a sale⁸.

Let U_0^I and U_0^W denote the payoffs of the upstream firm when the downstream firm chooses the strategy of immediate production and wait-and-see, respectively.

Lemma 2 *The payoffs of the upstream firm under the downstream strategies of immediate production and wait-and-see are given by:*

$$U_0^I = C + \frac{\delta p C}{1 - \delta} \quad (3.5)$$

$$U_0^W = \frac{\delta p}{(1 - \delta) + \delta p (2 - q)} U_0^I \quad (3.6)$$

⁸Later we introduce a positive upstream cost of production and analyze the upstream incentive to wait.

Since the coefficient of U_0^I on the right hand side of equation (3.6) is strictly lower than 1, any waiting by the downstream firm reduces upstream payoff and generates a negative externality. Thus for $p \in (p_\ell^D, p_{ic}^D)$ the downstream firm generates a negative externality for the upstream firm.

Further, since $U_0^I > 0$ whenever $p > 0$, the upstream firm would want the downstream firm to invest (and follow the immediate production strategy) for all $p > 0$. Since no investment takes place for $p < p_\ell^D$, in this region as well the downstream firm generates a negative externality.

For $p \in [p_\ell^D, p_{ic}^D)$ the extent of externality (here defined as the rate of reduction in upstream payoff) is given by

$$\frac{U_0^I - U_0^W}{U_0^I} = \frac{\delta p (1 - q) + (1 - \delta)}{\delta p (2 - q) + (1 - \delta)} \quad (3.7)$$

For $p \in (0, p_\ell^D)$, no investment takes place (0 upstream payoff), and the externality is given by 1.

4 DELAYED PAYMENT

A trade credit offer is to delay the payment for a fraction τ of the cost inputs till the next order - which occurs when a customer arrives next and a sale takes place. This is therefore an inventory subsidy. We show below that such a subsidy solves the problem of negative externality.

As noted in the introduction, a bank loan can never perform this role. A relatively high bank rate is one of the factors that increases the cost of inventory holding, and causes waiting to be optimal for the downstream firm. Therefore bank loans cannot solve the problem. The solution must be a lower cost subsidy covering at most the value of inputs - and we show below that trade credit is precisely that instrument. This clarifies the reason for a credit at a zero interest rate which is lower than the bank rate, and explains why the zero interest component is present in every trade credit term.

Without any trade credit, the payoff of the downstream firm from immediate production

is given by (from (3.1):

$$V_0^I = -C + \frac{\delta p (P - C)}{1 - \delta}.$$

Suppose a trade credit of τC is offered, and assume that the trade credit offer is such that the downstream firm chooses to produce immediately. Then the downstream firm pays $(1 - \tau)C$ to the upstream firm on delivery of each unit of the input good it purchases. Thus the first period payoff is $-(1 - \tau)C$. In each subsequent period, if there is a sale made by the downstream firm, it receives the price P , repays τC , and incurs a new cost of production of $(1 - \tau)C$. Thus in each period after period 1, the payoff is the same as before, and given by $p(P - C)$. Thus the payoff under trade credit, denoted by V_0^T , is given by

$$V_0^T = -(1 - \tau)C + \frac{\delta p (P - C)}{1 - \delta} = V_0^I + \tau C \quad (4.1)$$

Thus trade credit for a fraction τ of the price can be represented by a total transfer of τC from the upstream firm to the downstream firm over and above the payoff from immediate production, whenever τ is such that under trade credit, the downstream firm chooses to produce immediately.

This proves the following result, which is very useful for later calculations.

Proposition 3 *Suppose trade credit is offered for a fraction τ of the input cost for every sale of input over an infinite horizon, and τ is such that the downstream incentive to produce immediately is restored. Then the trade credit scheme can be represented by a total transfer of τC from the upstream firm to the downstream firm.*

A trade credit offer is feasible if it restores downstream incentive to produce immediately, and satisfies the upstream participation constraint. Using equation (4.1), the downstream incentive constraint is given by

$$V_0^T = V_0^I + \tau C \geq \max\{0, V_0^W\}. \quad (4.2)$$

Next, let U_0^T denote the upstream payoff under trade credit. Using the proposition above, $U_0^T = U_0^I - \tau C$. Therefore the upstream participation constraint is given by $U_0^I - \tau C \geq \max\{0, U_0^W\}$. From (3.6), $U_0^W > 0$. Thus the upstream participation constraint simplifies to

$$U_0^I - \tau C \geq U_0^W \quad (4.3)$$

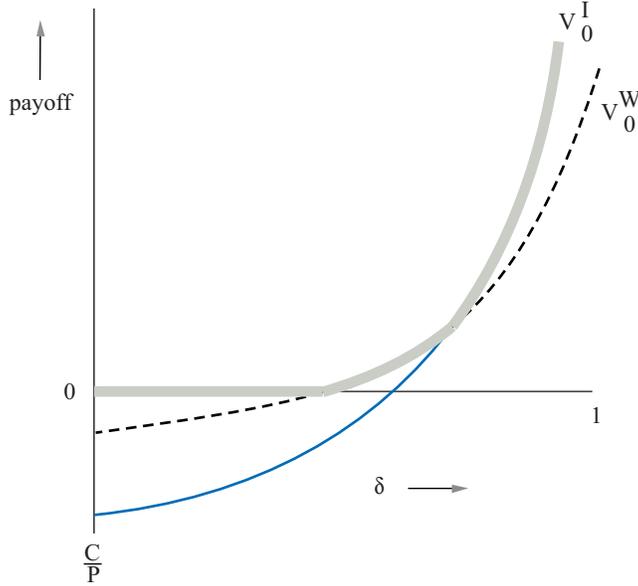


Figure 2: The payoff under trade credit is the upper envelope of the horizontal axis, V_0^W and V_0^I .

Now, the optimal trade credit offer must be such that the downstream firm gets no rent beyond $\max\{0, V_0^W, V_0^I\}$. Let τ^* denote the optimal value of τ and let $V_0^{T^*}$ denote the payoff of the downstream firm under the optimal trade credit offer τ^*C . Then

$$V_0^{T^*} = \max\{0, V_0^W, V_0^I\}. \quad (4.4)$$

This is shown in figure 2. Without trade credit, the downstream firm invests and follows the immediate production strategy for $p \geq p_{ic}^D$, and for $p < p_{ic}^D$ there is a loss of efficiency. The optimal trade credit offer τ^*C is feasible, and raises the payoff of the immediate production strategy to $\max\{0, V_0^W, V_0^I\}$, restoring efficiency for $p < p_{ic}^D$. Thus full efficiency is achieved. The following result confirms this.

Proposition 4 *For any P, C, q and δ satisfying assumption 1, the optimal trade credit fraction $\tau^* < 1$ is given by*

$$\tau^* = \frac{1}{C} (\max\{0, V_0^W, V_0^I\} - V_0^I). \quad (4.5)$$

Optimal trade credit satisfies upstream participation constraint, and attains full efficiency.

5 COSTLY UPSTREAM PRODUCTION

Let us now relax the assumption of zero upstream cost and assume that the upstream firm has a cost of production $G > 0$.

Proposition 1 showed that δ must exceed C/P for downstream investment to take place at all. Now that we have added a cost G for the upstream sector, applying exactly the same reasoning we can show that for upstream investment to be at all viable, we need $\delta > G/C$. To allow for this we replace assumption 1 by the following augmented assumption:

Assumption 2 $\delta > \max(C/P, G/C)$.

The payoff of the upstream firm under the downstream strategy of immediate production is now given by:⁹

$$\widehat{U}_0^I = (C - G) + \frac{\delta p (C - G)}{1 - \delta} \quad (5.1)$$

The payoff if the downstream firm waits is now denoted by \widehat{U}_0^W , and as before given by (3.6), with U_0^I on the right hand side replaced by \widehat{U}_0^I . The extent of the externality is exactly as before.

A trade credit offer is feasible if it satisfies the downstream participation constraint given by (4.2) and the upstream participation constraint, which is now given by

$$\widehat{U}_0^{T*} \equiv \widehat{U}_0^I - \tau^* C \geq \widehat{U}_0^W. \quad (5.2)$$

This now binds at positive levels of p . For any given P, C, q , and δ satisfying assumption 2, suppose the participation-in-trade-credit constraint binds at p_{ptc}^U . Then upstream participation requires $p \geq p_{\text{ptc}}^U$. Figure 3 shows the constraint. We know that trade credit is relevant for $p < p_{\text{ic}}^D$. Therefore the negative externality is now eliminated for $p \in [p_{\text{ptc}}^U, p_{\text{ic}}^D]$. The results are qualitatively the same as in the case of $G = 0$.

⁹As noted at the end of section 2, without loss of generality we can ignore the cost in period -1 (in effect assuming that the upstream firm starts with a unit of endowment in period 0). In period 0, the upstream firm sells this endowment and then produces another unit, making the period payoff $C - G$. Similarly, in any period $t > 0$, a unit is produced if a sale is made to the downstream firm (which happens if the downstream firm sells a unit) so that the expected payoff is $p(C - G)$ in each such period. Thus $\widehat{U}_0^I = (C - G) + \delta p(C - G) + \dots$, which explains the payoff.

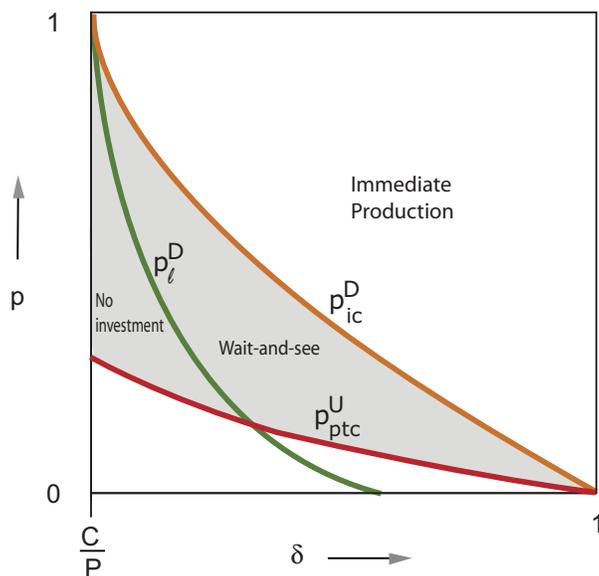


Figure 3: The downstream firm chooses the wait-and-see strategy for $p \in [p_{\ell}^D, p_{ic}^D)$, and does not invest at all for $p < p_{\ell}^D$. The upstream participation-in-trade-credit constraint is satisfied for $p \geq p_{ptc}^U$. Trade credit can restore first best in the shaded region $[p_{ptc}^U, p_{ic}^D)$. p_{ℓ}^D and p_{ic}^D are drawn here for $C/P = 0.5$ and $q = 0.6$. p_{ptc}^U is drawn for $G = 0.3$.

However, for $G > 0$ a new possibility arises. The upstream firm might now have an incentive to delay production. In this case the upstream firm imposes a negative externality on the downstream firm. A natural transfer that can resolve this problem is prepayment by the downstream firm (which is formally similar to negative trade credit, i.e. $\tau < 0$). We explore this after comparing the predictions of our theory with evidence in the next section.

6 RELATING TO EVIDENCE

6.1 TRADE CREDIT AND FIRM MARGIN, PROFIT AND SALES

Petersen and Rajan (1997) find that trade credit provision increases in the supplier's margin. At the same time, firms that experience a decline in sales offer more trade

credit. Further, more profitable firms supply less trade credit. While this might appear anomalous (and the authors do say they find this surprising), so far as higher profits imply greater turnover, this is precisely the prediction of our model.

Corollary 1 *The provision of trade credit is increasing in the supplier's margin ($C - G$) and decreasing in the downstream margin ($P - C$). Further, the supply of trade credit is decreasing in p .*

As $(C - G)$ rises, the upstream firm has more to lose from lost sales (higher externality), and has more of an incentive to give trade credit. As $(P - C)$ rises, the downstream firm has more to lose from lost sales, and has less incentive to wait, reducing the need for trade credit.

For profitable firms turnover is high (i.e. p is high, so that the downstream firm gets many customers resulting in high turnover for the upstream firm). Whenever p is high enough, there is no need for trade credit. Therefore more profitable firms supply less trade credit. Since p parametrizes sales, this also explains why the firms suffering sales declines offer more trade credit.

6.2 INVENTORY INVESTMENT

There is plenty of casual evidence that indicates upstream firms are concerned about the inventories held by downstream firms, which is our central thesis. A liberal returns policy in which the upstream firm allows downstream firms to return unsold goods mitigates downstream risk of holding inventories for too long (see Marvel and Peck (1995)). More closely related to trade credit is the practice of floor planning. This most often occurs when auto manufacturers provide credit to their dealers and is described in Dynan et al. (2002). They find that growth of credit provided by captive finance subsidiaries track growth of inventories of its dealers quite closely in monthly data from 1996 to 2000.

6.2.1 IMPLICATION OF TRADE CREDIT FOR INVENTORY

We define total inventory as final goods inventory plus work-in-progress inventory. The following derives the levels of inventory investment under immediate production as well as under the wait-and-see strategy by the downstream firm.

Proposition 5 *Under immediate production by the downstream firm, the inventory level is equal to 1 per period. Under the wait-and-see strategy by the downstream firm, the average inventory level in each period is given by*

$$\bar{I}^W = \frac{\delta p}{1 - \delta + \delta p(2 - q)} (1 - \delta q(1 - p)) < 1. \quad (6.1)$$

The result shows that the average inventory level under wait-and-see is less than that under immediate production. Further, inventory investment under the latter strategy is insensitive to the interest rate and level of demand, and suffers no loss of sales through stockouts. The incentive for adopting the immediate production strategy comes from two sources. For high demand (high p) and/or low interest rate (high δ), the downstream firm optimally chooses this strategy. If, on the other hand, p is low and/or if the interest rate is high (low δ), the optimal strategy is to wait, allowing lost sales through stockouts. This generates a negative externality for the upstream firm, which now has an incentive to allow the downstream firm to delay payment so that the latter avoids losing sales. Thus for a large range of parameter values, either through own incentives, or through the incentives of the upstream firm transmitted through trade credit, the downstream firm's inventory investment remains insensitive to the rate of interest.

This offers a resolution to the inventory puzzle posed by Blinder and Maccini (1991), who point out that “the question of why inventory investment seems to be insensitive to changes in real interest rates remains open, important, and troublesome.” As they note, firms avoiding stockouts might provide a good explanation. Our model formalizes and clarifies this idea.

A recent paper by Maccini et al. (2004) offers an explanation based on the claim that firms care about the long-run level of the interest rate and not about short run rate fluc-

tuations. Our theory complements their work by clarifying why firms might not respond to transitory shocks.

6.2.2 IMPLICATION OF INVENTORY FOR TRADE CREDIT

Blazenko and Vandezande (2003) show that high margin industries hold higher levels of finished goods inventories, consistent with our argument that high margin firms follow an immediate production strategy. In our theory, if a downstream firm pursues the immediate production strategy, it always has enough finished goods in inventory to serve any arriving customer, and the case for trade credit does not arise. Our theory therefore predicts that high margin firms hold higher levels of finished goods inventories, and should receive less trade credit. This is confirmed by Petersen and Rajan (1997), who find that receipt of trade credit (purchases on account) is negatively related to the ratio of finished goods inventory to total inventory.

6.3 TRADE CREDIT TERMS

6.3.1 INTEREST RATE

In most industries in developed economies, trade credit is provided at zero interest. Trade credit is expensive only when it is not repaid early and the borrower chooses to forego any early repayment discount. Further, as noted by Ng et al. (1999), there is widespread use of net terms which do not have any such incentives and are not costly at all when repaid within the specified time period (often 30 days).

Burkart and Ellingsen (2004) rely on competitive pressure to reduce the trade credit interest rate to the bank rate. Their model cannot explain the a zero trade credit interest rate which is below a positive bank rate, observed even under imperfect competition among suppliers. Also, their model cannot explain why the trade credit interest rate does not respond to changes in the bank rate. Our theory, in contrast, can account for these observations. The model focuses on the negative externality generated by the downstream

firm when it adopts a waiting strategy. Such waiting is optimal if the opportunity cost of inventory holding (part of which is the bank rate) is high. Therefore bank loans cannot help, and the correct instrument is an inventory subsidy. A delayed payment is exactly the right instrument to provide this subsidy. Therefore our theory emphasizes the transfer implicit in trade credit and is consistent with a positive rate of interest (i.e. a discount factor less than 1) along with a zero trade credit interest rate. Since the zero-rate credit is simply a subsidy, it is either offered or not offered, but when offered, the rate does not respond to bank rate changes. This also explains why the trade credit interest rate does not respond to changes in market demand.

6.3.2 DURATION

Trade credit is short term credit. Indeed, it is the most important source of short term funds for US firms. In our theory, trade credit arises as an optimal response to the negative externality generated by the downstream firm when it optimally adopts the wait-and-see strategy. The source of the problem is the inventory holding cost of the downstream firm, and trade credit addresses precisely this problem by subsidizing inventory holding. Further, for any given unit of input, a subsidy is useful only till the final output is sold. Indeed, this is exactly the duration of trade credit in our model. A new unit of input is purchased when a unit of output is sold, and at that time trade credit for the previous unit is settled. The longevity of trade credit is therefore naturally bounded by purchase of input at one end and sale of final good at the other. Thus trade credit is indeed short term credit in our model, consistent with evidence.

6.4 BUSINESS CYCLE EFFECTS

Finally, let us consider the prediction of our theory about the relation between trade credit over the business cycle and size of the firms that receive credit. We take sales (parametrized here by p) and margin as proxy for firm size. Small downstream firms have a greater incentive to wait and see and therefore they are more likely to be already (even before a downturn) receiving trade credit relative to larger firms.

A downturn implies a subset of (or all of) the following: The rate of interest r increases (monetary contraction), so that δ falls. Further, demand falls, i.e. p falls. Finally, P might fall. All these factors make waiting more attractive relative to immediate production for the downstream firm, and expand the scope for improvement through trade credit. For firms already receiving trade credit, the downturn implies that a higher level of provision is needed to restore the incentive to produce immediately, and for large firms who were previously producing immediately, waiting might now become more attractive, generating a scope for trade credit.

While the “demand” for trade credit unambiguously increases after a downturn, to complete the picture we need to consider the effect on the incentive to supply trade credit. A fall in p and/or δ makes it harder to satisfy the upstream participation constraint. A small firm for which p is not very high initially might lose its trade credit if p falls any further. For large downstream firms, p is higher, and therefore the upstream firm has more to lose if such firms choose to wait. Moreover, it takes relatively less to provide such firms with the incentive to produce immediately. Thus for large firms, there is more scope for trade credit expansion after a downturn without hitting the upstream participation constraint. To put it differently, the upstream participation constraint implies that in a downturn, there is less scope of trade credit expansion to existing credit customers relative to new credit customers (typically firms that were cash customers who are now offered credit), and the latter are likely to be larger firms (who previously did not need trade credit in order to choose immediate production). This implies that trade credit should be more countercyclical for large firms. This is consistent with evidence reported by Nilsen (2002), who finds trade credit is more countercyclical for large firms.

7 PREPAYMENT

We now proceed with the assumption that the downstream firm chooses to produce immediately ($P - C$ high) and analyze the upstream firm’s incentive to delay production, making a case for prepayment. In the next section we relax this assumption and look at the different parametric specifications that lead to either trade credit or prepayment.

7.1 OPTIMAL STRATEGY OF THE UPSTREAM FIRM

Let U_0^{UI} and U_0^{UW} denote the payoffs of the upstream firm when the upstream firm chooses the strategy of immediate production and wait-and-see, respectively.

Lemma 3 *The payoffs are given by:*

$$U_0^{UI} = (C - G) + \frac{\delta p (C - G)}{1 - \delta} \quad (7.1)$$

$$U_0^{UW} = \frac{\left(1 - \delta(1 - p)(1 + \delta(1 - \delta)pq)\right) C - \delta p G}{(1 - \delta)(1 + \delta p(1 - \delta q(1 - p)))}. \quad (7.2)$$

We can now derive the optimal strategy of the upstream firm. Let p_{ic}^U denote the cutoff above which the upstream incentive to follow the immediate production strategy is satisfied.

Proposition 6 *For any given C, G and any δ satisfying assumption 2, there exists a cutoff $p_{ic}^U \in (0, 1)$ such that for $p \geq p_{ic}^U$, the optimal choice is immediate production, and for $p < p_{ic}^U$, wait-and-see is optimal.*

7.2 DOWNSTREAM OPTIMUM

Let V_0^{UI} and V_0^{UW} denote the payoffs of the downstream firm when the upstream firm chooses the strategy of immediate production and wait-and-see, respectively. V_0^{UI} is the same as V_0^I given by (3.1). The following result derives V_0^{UW} .

Lemma 4

$$V_0^{UW} = -C + \frac{\delta p (P - \delta C)}{(1 - \delta)(1 + \delta p(1 - \delta q(1 - p)))} \quad (7.3)$$

The following derives the downstream optimum.

Proposition 7 *Whenever the downstream firm prefers immediate production to waiting itself, it also prefers the upstream sector to produce immediately rather than wait and see. Formally, $V_0^{UI} > V_0^{UW}$ for $p \geq p_{ic}^D$, where p_{ic}^D is given by equation (3.4).*

The intuition is simple. Waiting by the upstream firm causes the downstream firm to lose some customers. Since it prefers not to wait itself (i.e. prefers to serve all customers), waiting by the upstream firm generates a lower payoff compared to immediate production by the upstream firm. Thus for $p > p_{ic}^D$, the externality is given by $\frac{V_0^{UI} - V_0^{UW}}{V_0^{UI}}$. (The expanded algebraic form is too complex to be of much help, so we leave it as this.)

7.3 PREPAYMENT

Suppose the downstream firm prepays a fraction $\theta \in [0, 1]$ of the upstream price C in each period in which it places an order for a unit of input from the upstream firm. Since this depends on the arrival of customers, the calculation of payoffs under prepayment is potentially complex. However, the following result shows that any prepayment scheme that restores the upstream incentive to produce immediately at all times over an infinite horizon can be represented very simply as a total transfer. This considerably simplifies the subsequent analysis.

Proposition 8 *Suppose prepayment is offered for a fraction $\theta \in [0, 1]$ of the upstream price at every instance of placing an order for inputs over an infinite horizon, and θ is such that the upstream incentive to produce immediately is restored. Then the prepayment scheme can be represented by a total transfer of θC from the downstream firm to the upstream firm.*

Proof: Without any prepayment, the payoff of the upstream firm from immediate production is given by (from (7.1)):

$$U_0^{UI} = (C - G) + \frac{\delta p (C - G)}{1 - \delta}.$$

If a prepayment of θC is made every period, the first period payoff is $(C - G) + \theta C$. In each subsequent period, if there is a sale made by the downstream firm, the upstream

firm also makes a sale to the downstream firm, and receives the price $(1 - \theta)C$, plus a prepayment of θC and incurs a cost of production of G . Thus in each period after period 1, the payoff is the same as before, and given by $p(C - G)$. Thus the payoff under prepayment, denoted by U_0^{PP} , is given by

$$U_0^{PP} = C - G + \theta C + \delta p(C - G) + \delta^2 p(C - G) + \dots = \theta C + U_0^{UI}$$

Comparing the two payoffs above, it is clear that the payoff under prepayment of θC per period exceeds the no prepayment payoff by exactly θC . Thus the prepayment of a fraction θ of the price can be represented by a total transfer of θC from the downstream firm to the upstream firm.||

For prepayment to be optimal, we need $U_0^{PP} \geq U_0^{UW}$ as well as $V_0^{PP} \geq V_0^{UW}$. U_0^{PP} is given by $U_0^{UI} + \theta C$. Whenever $U_0^{UW} > U_0^{UI}$, let θ^* be such that

$$U_0^{UI} + \theta^* C = U_0^{UW}.$$

From equations (7.1) and (7.2), θ^* is given by

$$\theta^* = \frac{G}{C(1 + \delta p)} - \frac{\delta p(C - G)(1 - \delta(1 - p))}{C(1 - \delta)(1 + \delta p)}. \quad (7.4)$$

Since $C > G$, the first term is less than 1. Since the second term is positive, and subtracted from the first term, clearly $\theta^* < 1$. Therefore the optimal prepayment $\theta^* C$, which restores upstream incentive to produce immediately, is indeed a fraction of the upstream price C .

Finally, the downstream sector participates in prepayment if the following constraint is satisfied:

$$V_0^{UI} - \theta^* C \geq V_0^{UW}. \quad (7.5)$$

The following result characterizes the outcome under prepayment. Let p_{pp}^D denote the cutoff above which the upstream participation-in-prepayment constraint holds.

Proposition 9 *There exists p_{pp}^D , where $p_{ic}^D \leq p_{pp}^D < 1$, such that for $p \geq p_{pp}^D$ the downstream participation-in-prepayment constraint (7.5) holds. For any (P, C, G, q) and any δ satisfying assumption 2, if $p_{pp}^D < p_{ic}^U$, prepayment restores efficiency on $[p_{pp}^D, p_{ic}^U]$.*

8 TRADE CREDIT VERSUS PREPAYMENT

The case for prepayment arises when the upstream markup $C - G$ is low. In this case, the return from production is relatively less attractive compared to the return from waiting (by waiting one period the firm can earn at the bank rate). Therefore if $C - G$ is low, the upstream firm has a greater incentive to wait, which implies that p_{ic}^U is larger. Further, a higher P relaxes the downstream participation-in-prepayment constraint, increasing the scope for prepayment provision. If, on the other hand, $P - C$ is low, the downstream firm has less to lose by waiting, and the case for trade credit arises. Further, a lower G implies a greater willingness of the upstream firm to provide trade credit¹⁰. In all cases, a lower δ (higher interest rate), as well as a higher q make waiting relatively more attractive.

We define the scope of trade credit (prepayment) to be the range of p for which trade credit (prepayment) can improve efficiency.

Proposition 10 (1) *The scope for trade credit decreases with $P - C$, δ and increases with q . Further, for any (P, C, q) , any δ satisfying assumption 2, and any $p < p_{ic}^D$, there exists a cutoff G_c such that for any $G < G_c$, the upstream participation-in-trade-credit constraint is satisfied.*

(2) *The scope for prepayment decreases with $C - G$, δ and increases with q . Further, for any (P, C, q) , any δ satisfying assumption 2, and any $p < p_{ic}^U$, there exists a cutoff P_c such that for any $P > P_c$, the downstream participation-in-prepayment constraint is satisfied.*

The result shows that if the downstream margin is low, and if the upstream cost G is not very high, we are likely to observe trade credit. On the other hand, if the upstream margin is low, and the downstream price P is not very low, we are likely to observe prepayment.

¹⁰Note that while a lower C increases the incentive to make a prepayment, this also reduces the upstream markup $C - G$ and therefore increases the amount of prepayment required to restore upstream incentive to produce immediately. Therefore, a lower C does not necessarily increase the incentive to provide prepayment. Similarly, increasing C need not increase the willingness of the upstream firm to provide trade credit. A higher C increases the incentive to provide trade credit, but also reduces the downstream margin $P - C$ which increases the amount of trade credit required to restore downstream incentive to produce immediately.

9 TRADE CREDIT AND PREPAYMENT VERSUS PRICE VARIATION

In our simple model, we show that the discounted stream of trade credit as well as prepayment can be represented as total transfers of a fraction of the input price. Clearly, this can also be achieved by reducing or increasing the price directly. However, we believe trade credit is the more appropriate instrument for the following reasons.

First, a price change does not target the right source. The source of the negative externality is that inventory holding is costly. So the correct instrument is one that subsidizes this cost, and can change with P , p , q , or r so that the inventory holding incentives remain optimal for each unit of input. A lower input price makes the loss of a downstream firm from a lost sale greater, and hence could provide the incentive to hold inventory. However, if the price is low, this also gives incentive to buy even when there is a unit of final good in the inventory - so as to avoid a future price rise (e.g. when it is expected that r is likely to fall, or p is likely to rise, a lower subsidy - implying a higher price - is expected in future). To forestall this possibility, the upstream firm must monitor the use of any input closely to see which final unit it produces and when this is sold, and only then sell any more input units. This would clearly be unworkable if final sales are not always observable.

In other words, since trade credit is tied to the inputs, its usefulness automatically expires once the final output produced from a unit of input is sold. That is, the trade credit benefit is specific to the unit and cannot be passed on to other units. A price reduction, on the other hand, does not automatically expire once its usefulness in providing incentive to hold inventory ends. This would happen only if the upstream firm has full information about final sales, which is unrealistic.

There are other problems with using price variation as an instrument. In our model there is just one downstream firm. In reality, there could be several downstream firms, each facing a different circumstance (for example, one firm with high final demand (high p), and another with low final demand). If the upstream firm cannot set different prices across customers (as posited by the theories discussed in the introduction that explain trade credit as a device for price discrimination), this would clearly over-subsidize some customers and/or under-subsidize others.

Further, lowering prices could weaken the bargaining power of the upstream firm making it difficult to charge a higher price in future. There could also be menu costs associated with frequent price changes.

In fact, downstream firms might also prefer trade credit over price changes. Frequent price changes, and different prices across customers make for very non transparent pricing, and might increase the market power of upstream firms. Trade credit, on the other hand, is always a subsidy, and cannot be used as a tool to extract a higher price from the downstream firm.

10 EXTENSIONS

Multiple Suppliers Our model has one upstream and one downstream firm. Therefore it cannot say much about the importance of an exclusive relationship. Doing this would require a model with multiple suppliers. Here we discuss informally how the analysis might change under multiple suppliers, and the value of exclusive relationships. Let us suppose there are two input suppliers, A and B.

With multiple suppliers, trade credit or prepayment become more blunt as instruments. The problem is that if supplier A provides trade credit, reducing the cost of holding inventory for inputs supplied by A, the downstream firm might transfer some of the cost saving on A inputs to buy more from supplier B. Thus provision of trade credit by a supplier can generate a positive externality for other suppliers. If this effect is significant, the equilibrium trade credit provision can be low. The same problem applies to the provision of prepayment. An exclusive relationship allows the provider of trade credit or prepayment to enjoy all the benefits of the implied subsidy.

A further issue that arises with multiple suppliers is as follows. Trade credit for a unit of input is useful up to the point a sale is made of the final good produced from that unit of input. If we do not assume full information, then after the sale of a unit by the downstream firm, neither A nor B might know whose input was used to produce this unit. This would make it difficult for the suppliers to determine what the useful period of trade credit is. This would lead the suppliers to use repayment incentives in the form

of discounts for early payments, or penalties for late payment.

In this paper, we have explained discount terms informally as repayment incentives. The discussion above shows that with multiple suppliers, this could be explained formally. Further, we would expect a supplier to be lenient towards long standing customers, and not enforce the penalties very strictly, consistent with evidence gathered by Ng et al. (1999).

Adding a default risk In this paper we set ourselves the challenge of explaining prepayment in developed market economies with little or no chance of default. Our explanation is based on the negative externality arising from upstream optimization, a problem that can be addressed precisely through prepayment, which provides a targeted subsidy. However, if we do include a default risk (perhaps because of poor enforcement of law), the scope for prepayment increases, as this now also serves as insurance against default. This explains the preponderance of prepayment observed in developing countries. In a survey of 115 micro and small firms engaged in metal fabrication and light manufacturing in Nairobi, Vandenberg (2003) finds that firms rely heavily on pre-payment, with 87% of all firms requiring a down payment on large orders. As the author stresses, in such cases prepayment serves the twin purposes of financing and enforcement (the prepaid money is lost if the goods are not collected).

11 CONCLUSION

While theories based on the asymmetric information explain part of the reason for trade credit (input advance by the upstream firm), these leave important aspects uncovered. We take a different route and explain trade credit as inventory subsidy. We believe this motive underlies much of the trade credit exchanged in the economy and has not been explored so far. Our model can also explain the use of prepayment (cash advance by the downstream firm) and clarifies previously unexplained facts including the universal zero interest component in trade credit terms, and the invariance of the interest rate to fluctuations in the bank rate as well as market demand.

Our theory explains trade credit (prepayment) as an inventory subsidy helping to internalize a negative externality that arises whenever the downstream (upstream) firm optimally chooses not to produce or wait before producing.

An inventory subsidy is limited by the value of inputs that is expected to be held. Thus our theory is consistent with the fact that the extent of trade credit is determined by the value of inputs, and not subject to the Burkart-Ellingsen critique. Further, the downstream firm sometimes finds waiting to be optimal because the (opportunity) cost of inventory holding exceeds the expected loss through sales forgone. Given that a relatively high bank rate is one of the factors that generate an incentive to wait, bank loans cannot solve the problem. The solution must be a lower cost subsidy covering at most the value of inputs - and trade credit is precisely that instrument. This clarifies why all trade credit is provided with a zero interest rate component, necessarily lower than the bank rate. This also shows that in our theory trade credit is a complement - not substitute - of bank lending. The literature often implies that such complementarity arises from credit rationing. Our theory, in contrast, predicts complementarity even under perfect capital markets.

Our theory closely links trade credit to the inventory asset. This is particularly important because of the empirical failures of the standard inventory models and the role inventories play in business cycles. Our model predicts that downstream firms hold a higher average level of inventories with trade credit than without. This suggests standard inventory models would perform better if they include trade credit in the specification and take account of the firms' relative supply chain positions. Further, our model predicts that under trade credit, inventory investment is invariant to the real interest rate term for a wide range of parameters. This provides an explanation of the puzzle of insensitivity of inventory investment to the real interest rate pointed out by Blinder and Maccini (1991).

In sum, our model emphasizes the motive for trade credit has no intended "credit" purpose at all - it is a targeted inventory subsidy. The predictions arising from our model are consistent with a broad array of evidence, and throws new light on several aspects of trade credit as well as prepayment. Empirical tests of the success of this approach relative to other theories is on the agenda for future research.

12 APPENDIX: PROOFS

A.1 PROOF OF LEMMA 1

Let $V_1(1)$ be the value starting period 1 if a customer arrives at the start of period 1, and $V_1(0)$ be the value starting period 1 no one arrives at the start of period 1. In the latter case, the situation is exactly as at the start of period 0 and thus $V_1(0) = V_0^W$. Then

$$V_0^W = 0 + \delta \left((1-p) V_0^W + p V_1(1) \right) \quad (\text{A.1})$$

To calculate $V_1(1)$, note that once a customer arrives at the start of period 1, there are 3 possible states at the beginning of period 2:

- State 1: Exactly one customer arrives. Probability of this event is $(p+q-2p q)$. The firm sells and gets P , and then restarts period 0 strategy. Thus payoff is $P + V_0^W$.
- State 2: 2 customers (1 old and 1 new) arrive. Probability of this event is $p q$. The firm sells to the returning customer and gets P , and then onwards gets $V_1(1)$. Thus payoff is $P + V_1(1)$.
- State 3: No one arrives. Probability of this event is $(1-p)(1-q)$. Let the payoff in this state be denoted by $V_2(0)$. This is given by:

$$V_2(0) = 0 + \delta \left(p (P + V_0^W) + (1-p) V_2(0) \right). \quad (\text{A.2})$$

From the above,

$$V_1(1) = -C + \delta \left[(p+q-2p q)(P + V_0^W) + p q(P + V_1(1)) + (1-p)(1-q)V_2(0) \right] \quad (\text{A.3})$$

From equations A.1, A.2 and A.3, we can solve for V_0^W , and obtain the stated value. ||

A.2 PROOF OF PROPOSITION 1

From equation (3.2), we can write V_0^W as $V_0^W = f(\delta)g(\delta)$, where $f(\delta) = \frac{\delta p}{1-\delta}$, and

$$g(\delta) = \frac{P(p + (1-\delta) q (1-p)) - C/\delta + C (1-p)}{D},$$

where $D = (1 - \delta)/\delta + p(2 - q)$.

$g(C/P) = -\frac{C(P-C)(1-p)(1-q)}{(P-C)+p(2-q)C} \leq 0$, with strict inequality for $p < 1$. Thus at $\delta = C/P$, $V_0^W \leq 0$, with strict inequality for $p < 1$. Further, at $\delta = C/P$, $V_0^I = -C(1-p) \leq 0$.

Now, $f(\cdot) > 0$, $f'(\cdot) > 0$. Further,

$$g'(\cdot) = \frac{1}{\delta^2 D^2} (p(1 - \delta^2 q(2 - q)(1 - p))P + (1 - \delta)^2 q(1 - p)P + p(1 - q)C) > 0,$$

where the last step follows from the fact that the maximized value of $q(2 - q)$ is 1, and therefore $1 - \delta^2 q(2 - q)(1 - p) > 0$. Thus $\frac{\partial V_0^W}{\partial \delta} > 0$ whenever $g(\delta) > 0$. But since $f(\delta) > 0$ always, whenever $g(\delta) > 0$ it is also true that $V_0^W > 0$.

Therefore, if $g(\delta_*) > 0$ for any $\delta_* > 0$, then $g(\cdot)$ is positive and increasing for all $\delta > \delta_*$.

Since for $\delta = C/P$, both V_0^I and V_0^W are non-positive, from the above it follows that for all $\delta < C/P$ they are negative. Thus for any investment to take place, a necessary condition is $\delta > C/P$.

Finally, let us show that for $\delta > C/P$ there exists $p \in (0, 1)$ and $q \in (0, 1)$ such that $\max\{V_0^I, V_0^W\} > 0$.

At $p = 1$, V_0^W is given by $\frac{\delta}{(1-\delta)(1+\delta(1-q))}(\delta P - C)$, which is strictly positive for $\delta > C/P$. Thus for any $\delta > C/P$, there exists some $\underline{p} < 1$ such that $V_0^W > 0$ for $p > \underline{p}$. Similarly, for $q = 1$, $V_0^W = \frac{\delta p}{1-\delta}(\delta P - C)$ which is strictly positive for $\delta > C/P$. Thus if $\delta > C/P$, there exists some $\underline{q} < 1$ such that $V_0^W > 0$ for $q > \underline{q}$. This proves for $\delta > C/P$ there exists $p \in (0, 1)$ and $q \in (0, 1)$ such that $\max\{V_0^I, V_0^W\} > 0$.||

A.3 PROOF OF PROPOSITION 2

From equation (3.2), $V_0^W \geq 0$ implies

$$p \geq \frac{(1 - \delta)(C - \delta q P)}{\delta((P - C) - (1 - \delta)qP)}, \quad (\text{A.4})$$

where strict equality implies $V_0^W = 0$. From equation (3.1), $V_0^I \geq 0$ implies

$$p \geq \frac{(1 - \delta)C}{\delta(P - C)}, \quad (\text{A.5})$$

where, similarly, strict equality implies $V_0^I = 0$. It can be easily checked that for any $\delta \in (C/P, 1)$, the expression on the right hand side of (A.5) strictly exceeds that of (A.4).

Thus the participation constraint for investment is given by $p \geq p_\ell^D$ where p_ℓ^D is the maximum of 0 and the expression on the right hand side of (A.4). This gives us the expression for p_ℓ^D given by the proposition.

Next, the optimal choice is immediate production if $V_0^I \geq V_0^W$. Solving from equations (3.1) and (3.2), this is the case if $p \geq p_{ic}^D$ where p_{ic}^D is as in equation (3.4).

Finally, we check that $p_\ell^D < p_{ic}^D < 1$. $p_{ic}^D < 1$ is true if $\frac{(1-\delta)C}{\delta((P-C)-q(\delta P-C))} < 1$, which simplifies to $\delta q < 1$, which is true. Next, since $(P-C)-q(\delta P-C) > (P-C)-(\delta P-C) = (1-\delta)p > 0$, it is clear that $p_{ic}^D > 0$. Thus $p_\ell^D < p_{ic}^D$ is true if

$$\frac{(1-\delta)(C-\delta qP)}{\delta((P-C)-(1-\delta)qP)} < \frac{(1-\delta)C}{\delta((P-C)-q(\delta P-C))},$$

which simplifies to

$$\delta P^2 + (\delta P - C)(C - \delta qP) - CP > 0$$

which implies

$$(\delta P - C)(P(1 - \delta q) + C) > 0$$

which is true, since $\delta > C/P$.

Thus for $p \geq p_{ic}^D$, immediate production is optimal, for $p \in (p_\ell^D, p_{ic}^D)$, the payoff from waiting is higher than immediate production ($V_0^W > V_0^I$), and satisfies the participation constraint ($V_0^W > 0$). Finally, for $p \leq p_\ell^D$, the payoff from investment is negative, and not investing is optimal. ||

A.4 PROOF OF LEMMA 2

If the downstream firm adopts the immediate production strategy, each period a sale occurs with probability p . Thus

$$U_0^I = C + \delta pC + \delta^2 pC + \dots = C + \frac{\delta pC}{1-\delta}$$

If the downstream firm adopts the wait-and-see strategy, the payoff of the upstream firm can be calculated as follows.

In what follows, the term “customer” means a customer for the downstream firm, i.e. a final customer.

Let $U_1(1)$ be the value starting period 1 if a customer arrives at the start of period 1, and $U_1(0)$ be the value starting period 1 no arrives at the start of period 1. In the latter case, the situation is exactly as at the start of period 0 and thus $U_1(0) = U_0^W$. Then

$$U_0^W = 0 + \delta \left((1-p) U_0^W + p U_1(1) \right) \quad (\text{A.6})$$

To calculate $U_1(1)$, note that once a customer arrives at the start of period 1, the upstream firm sells a unit and earns C in period 1. Following this, at the beginning of period 2, there are 3 possible states:

- State 1: Exactly one customer arrives. Probability of this event is $(p + q - 2p q)$. The downstream firm sells and restarts period 0 strategy. Thus upstream payoff is U_0^W .
- State 2: 2 customers (1 old and 1 new) arrive. Probability of this event is $p q$. The downstream firm sells to the returning customer and restarts production. Thus upstream firm is in exactly the same situation as at the start of period 1 if a customer arrives. Thus upstream payoff is $U_1(1)$.
- State 3: No customer arrives. Probability of this event is $(1-p)(1-q)$. Let the payoff in this state be denoted by $U_2(0)$. This is derived as follows. If a customer arrives tomorrow, the downstream firm sells the unit previously produced, and restarts period 0 strategy. The upstream payoff is U_0^W . If no one arrives tomorrow, the upstream firm gets $U_2(0)$. Thus

$$U_2(0) = 0 + \delta \left(p U_0^W + (1-p) U_2(0) \right). \quad (\text{A.7})$$

From the above,

$$U_1(1) = C + \delta \left[(p + q - 2p q) U_0^W + p q U_1(1) + (1-p)(1-q) U_2(0) \right] \quad (\text{A.8})$$

From equations (A.6), (A.7) and (A.8), we can solve for U_0^W , and obtain:

$$\begin{aligned} U_0^W &= \frac{(1 - \delta(1-p)) \delta p C}{(1-\delta)(1-\delta + \delta p(2-q))} \\ &= \frac{\delta p}{(1-\delta) + \delta p(2-q)} U_0^I, \end{aligned}$$

where the second step uses equation (3.5).||

A.5 PROOF OF PROPOSITION 4

From (A.4), $V_0^W \geq 0$ as $p \geq p_\ell^D$, where p_ℓ^D is given by equation (3.3). Thus for $p < p_\ell^D$, τ^* is given by $V_0^I + \tau^*C = 0$, and for $p \in [p_\ell^D, p_{ic}^D]$, τ^* is given by $V_0^I + \tau^*C = V_0^W$, where p_{ic}^D is given by equation (3.4). Finally, from equations (3.1) and (3.2), $V_0^I \geq V_0^W$ if $p \geq p_{ic}^D$. Thus for $p \geq p_{ic}^D$, $\tau^* = 0$. Thus,

$$\tau^* = \frac{1}{C} (\max\{0, V_0^W, V_0^I\} - V_0^I),$$

which is as given.

Let us check that $\tau^* \in (0, 1)$ in all cases. For $p \geq p_{ic}^D$, $\tau^* = 0$. For $p < p_\ell^D$, $V_0^I < V_0^W < 0$. Thus $\tau^* = -V_0^I/C > 0$. Also, from (3.1), $-V_0^I < C$. Thus $\tau^* < 1$.

Next, for $p \in [p_\ell^D, p_{ic}^D]$, $V_0^W > 0$ and $V_0^W > V_0^I$. Thus $\tau^* = (V_0^W - V_0^I)/C > 0$. Also, from equations (3.1) and (3.2), for $p \in [p_\ell^D, p_{ic}^D]$,

$$\frac{V_0^W - V_0^I}{C} = 1 - \frac{\delta p}{1 - \delta} \frac{\delta p(1 - q)(P - C) + (1 - \delta)(1 - \delta q)(1 - p)P}{1 - \delta + \delta p(2 - q)} < 1 \quad (\text{A.9})$$

Thus $\tau^* < 1$.

Next, let us check that τ^* satisfies all the participation and incentive compatibility constraints. From equation (4.4) it is clear that by construction the payoff under τ^* satisfies the downstream incentive constraint (4.2). We finally check the upstream participation constraint (4.3).

The upstream payoff under optimal trade credit, $U_0^{T^*}$, is given by $U_0^I - \tau^*C$.

For $p \in (0, p_\ell^D)$, $\tau^* = -V_0^I/C$. Thus

$$\frac{U_0^W}{U_0^{T^*}} = \frac{U_0^W}{U_0^I - \tau^*C} = \frac{U_0^W}{U_0^I + V_0^I} = \frac{(1 - \delta + \delta p)C}{(1 - \delta + \delta p(2 - q))P} < 1.$$

Next, for $p \in [p_\ell^D, p_{ic}^D]$, $\tau^* = (V_0^W - V_0^I)/C$. Thus

$$\frac{U_0^W}{U_0^{T^*}} = \frac{U_0^W}{U_0^I - (V_0^W - V_0^I)} < \frac{U_0^W}{U_0^I - C} = \frac{(1 - \delta + \delta p)C}{(1 - \delta + \delta p(2 - q))P} < 1,$$

where the second step follows from the fact that from (A.9), $(V_0^W - V_0^I) < C$. The two inequalities above show that the upstream payoff under τ^* satisfies the upstream participation constraint (4.3).

For $p \geq p_{ic}^D$ the immediate production strategy is chosen optimally even without trade credit. The optimal trade credit satisfies feasibility, and ensures that the downstream firm invests and follows the immediate production strategy for $p < p_{ic}^D$. Thus optimal trade credit restores first best. ||

A.6 PROOF OF PROPOSITION 5

If the downstream firm produces immediately after each sale, in each period it either carries 1 unit or produces 1 unit. Thus the average inventory per period is simply 1.

Now let us calculate the average inventory under the wait-and-see strategy. Let M_0 denote the total value of inventories starting period 0 under this strategy. Let $M_1(1)$ denote the value of inventories starting period 1 if a customer arrives at the start of period 1. Note that if no customer arrives at the start of period 1, the situation is exactly like the start of period 0. Thus

$$M_0 = 0 + \delta(p M_1(1) + (1 - p) M_0). \quad (\text{A.10})$$

If a customer arrives at the start of period 1, then a unit is produced in period 1 (inventory value is P in period 1). Then there are three possibilities at the start of period 2. (i) No customer arrives (probability $(1 - p)(1 - q)$), in which case denote the value from period 2 onwards by $M_2(0)$. (ii) Exactly one customer arrives (probability $p + q - 2pq$), in which case the unit is sold and subsequently the situation is exactly like the start of period 0, and the value is given by M_0 . (iii) Two customers arrive (probability pq), in which case the unit is sold to the returning customer, but since there is one new customer who is not served, the subsequent value is given by $M_1(1)$. Therefore,

$$M_1(1) = P + \delta \left((1 - p)(1 - q) M_2(0) + (p + q - 2pq) M_0 + pq M_1(1) \right). \quad (\text{A.11})$$

Finally, $M_2(0)$ can be calculated as follows:

$$M_2(0) = P + \delta(p M_0 + (1 - p) M_2(0)). \quad (\text{A.12})$$

Now, let the average inventory level under the wait-and-see strategy be denoted by \bar{I}^W . Then we can also write M_0 as

$$M_0 = \frac{\bar{I}^W P}{1 - \delta}.$$

Solving for M_0 from equations (A.10)-(A.12), and using the above to solve for \bar{I}^W , we get the required expression for \bar{I}^W .

A.7 PROOF OF LEMMA 3

U_0^{UI} is the same as \hat{U}_0^I given by (5.1) (and explained in footnote 9):

$$U_0^{UI} = (C - G) + \delta p(C - G) + \delta^2 p(C - G) + \dots = (C - G) + \frac{\delta p(C - G)}{1 - \delta} \quad (\text{A.13})$$

Next, let us calculate U_0^{UW} . The upstream firm produces once the downstream firm has sold an unit. This guarantees that as soon as the upstream firm produces a unit of input, it is immediately sold.

In period 0, the downstream firm spends C and starts production. Thus the upstream firm earns C in period 0. In period 1 there are two possible states - either a customer arrives or no customer arrives. Let $U_1(0)$ be the value from period 1 onwards in the second state. If a customer arrives, then the downstream firm makes a sale, and thus the upstream firm produces in period 1. Thus the payoff in period 1 is $-G$, and let the value from period 2 onwards be denoted by $U_2(1)$. Thus:

$$U_0^{UW} = C + \delta(p(-G + \delta U_2(1)) + (1 - p)U_1(0).) \quad (\text{A.14})$$

Now, $U_1(0)$ can be calculated easily as follows:

$$U_1(0) = 0 + \delta p(-G + \delta U_2(1)) + \delta(1 - p)U_1(0). \quad (\text{A.15})$$

Let us now calculate $U_2(1)$. Since the upstream firm produces in period 1, it sells a unit at the start of period 2, and then waits to see if a sale occurs at the start of period 3. If a sale does occur, it is in the same situation as at the start of period 1 if a customer arrives.

If no sale occurs, it is in the same situation as at the start of period 1 if no customer arrives. Thus

$$U_2(1) = C + (\text{prob customer arrives at start of period 3}) \delta (-G + \delta U_2(1)) \\ + (\text{prob no customer arrives at start of period 3}) \delta U_1(0).$$

Now, since the upstream firm produces in period 1, the downstream firm can only produce in period 2. Any customer arriving at the start of period 2 cannot be served.

In period 3, no customer arrives if there is neither any returning customer from period 2, nor any new customer arrives in period 3. The probability of this event is¹¹ $(1-p)(1-pq)$. At least one customer arrives with probability $1 - (1-p)(1-pq)$. Thus

$$U_2(1) = C + \delta (1 - (1-p)(1-pq)) (-G + \delta U_2(1)) + \delta (1-p)(1-pq) U_1(0). \quad (\text{A.16})$$

Solving from equations (A.14)-(A.16), we get the required expression for U_0^{UW} .||

A.8 PROOF OF PROPOSITION 6

Let $\Delta U \equiv U_0^{UW} - U_0^{UI}$. Now, let p_{ic}^U be such that for $p = p_{ic}^U$, $\Delta U = 0$. Using equations (A.13) and (7.2), we have

$$\lim_{p \rightarrow 1} \Delta U = -\frac{\delta C - G}{1 - \delta^2} < 0,$$

where the second step follows from the fact that from assumption 2, $\delta C > G$. Further, $\lim_{p \rightarrow 0} \Delta U = G > 0$. The two limits show that a solution p_{ic}^U exists and that $0 < p_{ic}^U < 1$.||

A.9 PROOF OF LEMMA 4

In period 0, the downstream firm spends C and starts production. In period 1 there are two possible states - either a customer arrives or no customer arrives. Let Y_1 be the value

¹¹The probability that there is a returning customer is the probability that a customer arrives (p) times the probability that he returns (q). Thus the probability of no returning customer is $(1-pq)$. The probability of no new customer arriving in period 3 is $(1-p)$.

in the first state and Y_0 be the value in the second state. Then

$$V_0^{UW} = -C + \delta p Y_1 + \delta (1-p) Y_0. \quad (\text{A.17})$$

Now, Y_0 can be calculated easily as follows: $Y_0 = 0 + \delta (p Y_1 + (1-p) Y_0)$. Thus,

$$Y_0 = \frac{\delta p}{1 - \delta(1-p)} Y_1. \quad (\text{A.18})$$

Let us now calculate Y_1 . In period 1, if a customer arrives, the downstream firm sells the output produced in period 0, and gets P . Following a sale, the downstream firm places an order for an unit of input. Since the upstream firm did not produce in period 0, but delays production for a period to period 1, the downstream firm does not have the required input and cannot produce in period 1. Thus if a customer arrives in period 1, the downstream firm's payoff is simply P today. In period 2, the downstream firm spends C and produces an unit of output. Any customer arriving in this period does not get served, but might return next period. At the start of period 3, the downstream firm has 1 unit of output to sell.

In period 3, no customer arrives if there is neither any returning customer from period 2, nor any new customer arrives in period 3. The probability of this event is¹² $(1-p)(1-pq)$. In this case the situation is exactly like the state in period 1 in which no customer arrives, and the payoff is Y_0 . At least one customer arrives with probability $1 - (1-p)(1-pq)$, and in this case the situation is exactly like the state in period 1 in which a customer arrives. The reason is that even if 2 customers arrive in period 3, since the downstream firm can only sell the next unit of output in period 5 (because of the delay in production by the upstream sector), any customer who does not get served at the start of period 3 is lost completely. Thus there is no difference between 1 customer arriving in period 3 and 2 customers arriving. Thus if at least one customer arrives at the start of period 3, the subsequent payoff is Y_1 .

From the above, we can write $Y_1 = P - \delta C + \delta^2 ((1 - (1-p)(1-pq)) Y_1 + (1-p)(1-pq) Y_0)$.

¹²The probability that there is a returning customer is the probability that a customer arrives (p) times the probability that he returns (q). Thus the probability of no returning customer is $(1-pq)$. The probability of no new customer arriving in period 3 is $(1-p)$.

Using the value of Y_0 from equation (A.18) and solving, we get

$$Y_1 = \frac{(1 - \delta(1 - p))(P - \delta C)}{(1 - \delta)(1 + \delta p(1 - \delta q(1 - p)))} \quad (\text{A.19})$$

Solving from equations (A.17), (A.18), and (A.19), we obtain the stated expression for V_0^{UW} . This completes the proof of the lemma. ||

A.10 PROOF OF PROPOSITION 7

We know that V_0^{UI} is equal to V_0^I given by equation (3.1). Using this and the value of V_0^{UW} from equation (7.3), $V_0^{UI} = V_0^{UW}$ at $p = p_c$ where

$$p_c = \frac{1 - \delta q}{2\delta q} \left(\sqrt{1 + \frac{4q(1 - \delta)C}{(1 - \delta q)^2(P - C)}} - 1 \right).$$

Further, for $p > p_c$, $V_0^{UI} > V_0^{UW}$.

From proposition 2, for $p \geq p_{ic}^D$, the downstream firm prefers immediate production - i.e. $V_0^I > V_0^W$ for $p > p_{ic}^D$, where p_{ic}^D is given by (3.4). Using the value of p_c from above, and the value of p_{ic}^D , it is easy (but laborious) to see that

$$(p_{ic}^D)^2 - (p_c)^2 = \left(\frac{2q(1 - \delta)C}{(1 - \delta q)((P - C) - q(\delta P - C))} \right)^2 \left(\frac{q(\delta P - C)}{(P - C)} \right) > 0,$$

where the last step follows from the fact that $\delta P > C$. Since p_{ic}^D and p_c are both positive, the above shows that $p_{ic}^D > p_c$. It follows that for $p > p_{ic}^D$, $V_0^{UI} > V_0^{UW}$. This completes the proof of proposition 7. ||

A.11 PROOF OF PROPOSITION 9

Since $U_0^{UI} + \theta^*C = U_0^{UW}$, $\theta^* > 0$ whenever $U_0^{UW} > U_0^{UI}$. From proposition 6, this is the case for $p < p_{ic}^U$.

The proof proceeds through the following lemma.

Lemma 5 $V_0^{UI} - V_0^{UW}$ is strictly increasing in p .

Proof: Since $V_0^{UI} = V_0^I$, using the values from equations (3.1) and (7.3), and simplifying, we get

$$V_0^{UI} - V_0^{UW} = \frac{\delta p}{1 - \delta} \left[\frac{(P - C)}{1 + (1/f(p))} - \frac{C(1 - \delta)}{1 + f(p)} \right] \quad (\text{A.20})$$

where $f(p) = p\delta(1 - \delta q) + p^2\delta^2 q$. Note that $f'(p) > 0$. Thus the first term inside the square brackets on the right hand side of the above equation is strictly increasing in p , and the second term is strictly decreasing in p . Thus the term inside the square bracket is strictly increasing in p . The coefficient of this term is also strictly increasing in p . This proves the result. ||

From proposition 7, $V_0^{UI} - V_0^{UW} > 0$ for $p > p_{ic}^D$. The lemma above shows that $V_0^{UI} - V_0^{UW}$ increases in p . Therefore if p_{pp}^D is such that at $p = p_{pp}^D$, $V_0^{UI} - V_0^{UW} = \theta^* C > 0$, then it must be that $p_{pp}^D \geq p_{ic}^D$.

Next, as $p \rightarrow 1$, $V_0^{UI} - \theta^* C - V_0^{UW} \rightarrow \frac{\delta^2 P - G}{1 - \delta^2}$. From assumption 2, $\delta P > C$ and $\delta C > G$. Therefore $\delta^2 P > G$. Thus for p close to 1, the downstream participation constraint, given by equation (7.5) holds with strict inequality. Thus $p_{pp}^D < 1$.

Finally, positive prepayment is needed only for $p < p_{ic}^U$. Thus prepayment restores efficiency on the interval $[p_{pp}^D, p_{ic}^U]$ whenever $p_{pp}^D < p_{ic}^U$. ||

A.12 PROOF OF PROPOSITION 10

1. Trade credit is relevant for $p < p_{ic}^D$. From equation (3.4), it is straightforward to verify that

$$\frac{\partial p_{ic}^D}{\partial P} < 0, \quad \frac{\partial p_{ic}^D}{\partial C} > 0, \quad \text{and} \quad \frac{\partial p_{ic}^D}{\partial q} > 0.$$

Thus p_{ic}^D is decreasing in $(P - C)$ and increasing in q . Finally,

$$\frac{\partial p_{ic}^D}{\partial \delta} = \left(\frac{C}{\delta^2 B^2} \right) (\delta(1 - \delta)qP - B)$$

where $B = (P - C) - q(\delta P - C) > 0$. Now, the first term on the right hand side above is always positive. Let $J(\delta)$ denote the second term. It can be easily checked that $J(C/P) = -(P - C)(1 - qC/P) < 0$ and $J(1) = -(P - C)(1 - q) < 0$. Further, $J'(\cdot) = 2qP(1 - \delta) > 0$. Suppose the second term is positive at some δ . But then $J'(\cdot)$ has

to be negative at some point since the term is negative at $\delta = 1$. This contradicts the fact that $J'(\cdot) > 0$. Therefore the second term is always negative, implying that $\frac{\partial p_{ic}^D}{\partial \delta} < 0$. Further, as δ goes to its lower limit, C/P , $p_{ic}^D \rightarrow 1$, making the scope for trade credit full.

Next, the upstream participation-in-trade-credit constraint is given by 5.2, which can be expanded as

$$(C - G) \frac{(1 - \delta + \delta p)(1 - \delta + \delta p(1 - q))}{(1 - \delta)(1 - \delta + \delta p(2 - q))} \geq \tau^* C$$

where τ^* is given by equation (4.5). Now, the left hand side is decreasing in G and the right hand side does not depend on G . Further, from proposition 4, we know that if $G = 0$, the upstream participation-in-trade-credit constraint does not bind. Therefore, for any P, C, q and any δ satisfying assumption 2, and for any $p < p_{ic}^D$, there exists G_c such that for $G < G_c$ the constraint is satisfied.

2. Next, let us consider the effect on the scope of prepayment of the upstream markup $(C - G)$, q and δ .

Let $\Delta U \equiv U_0^{UW} - U_0^{UI}$. Also let $Z \equiv (1 + \delta p(1 - \delta q(1 - p)))$.

Using equations (A.13) and (7.2),

$$\frac{\partial \Delta U}{\partial C} = - \frac{\delta p}{(1 - \delta)} \frac{1 - \delta(1 - \delta p q) + \delta p(1 - \delta q)}{Z} < 0.$$

$$\frac{\partial \Delta U}{\partial G} = 1 + \frac{\delta^2 p^2 (1 - \delta q(1 - p))}{(1 - \delta) Z} > 0.$$

$$\frac{\partial \Delta U}{\partial q} = \frac{\delta}{(1 - \delta)} \left(\frac{\delta^2 p^2 (1 - p)(\delta C - G)}{Z^2} \right) > 0.$$

The last inequality follows from the fact that from assumption 2, $\delta C > G$. Finally,

$$\frac{\partial \Delta U}{\partial p} = - \frac{\delta}{(1 - \delta)} \left(\frac{(\delta C - G)(1 - \delta^2 p^2 q)}{Z^2} + (C - G) \right) < 0.$$

Now p_{ic}^U is given by $\Delta U = 0$. Thus

$$\frac{\partial p_{ic}^U}{\partial C} = - \frac{\partial \Delta U / \partial C}{\partial \Delta U / \partial p} < 0, \quad \text{and} \quad \frac{\partial p_{ic}^U}{\partial G} = - \frac{\partial \Delta U / \partial G}{\partial \Delta U / \partial p} > 0.$$

From the two inequalities above it follows that p_{ic}^U is decreasing in $(C - G)$. Further,

$$\frac{\partial p_{ic}^U}{\partial q} = - \frac{\partial \Delta U / \partial q}{\partial \Delta U / \partial p} > 0.$$

Finally, as δ falls (i.e. r rises), waiting becomes more attractive relative to immediate production, so that Δ_U rises. Thus $\frac{\partial p_{ic}^U}{\partial \delta} = -\frac{\partial \Delta U / \partial \delta}{\partial \Delta U / \partial p} < 0$. Further, it can be easily checked that as $\delta \rightarrow G/C$, $p_{ic}^U \rightarrow 1$, implying the maximum possible scope for prepayment.

Next, as P increases, the downstream firm has more to lose from lost sales, thus if the upstream firm decides to wait, the downstream firm's incentive to participate in prepayment increases with P . Formally, the downstream sector participates in prepayment if (7.5) holds. From proposition 9, this holds for $p \geq p_{pp}^D$. Thus p_{pp}^D is given by

$$V_0^{UI} - V_0^{UW} = \theta^* C.$$

From equation (A.20) it is straightforward to verify that $V_0^{UI} - V_0^{UW}$ increases in P . On the other hand, it is easy to see from equation (7.4) that $\theta^* C$ is independent of P .

From lemma 5 in section A.11, the left hand side is increasing in p . Also, it can be easily verified from equation (7.4), that $\theta^* C$ is decreasing in p .

Let $X = V_0^{UI} - V_0^{UW} - \theta^* C$. We have just established that

$$\frac{\partial X}{\partial P} > 0 \quad \text{and} \quad \frac{\partial X}{\partial p} > 0.$$

Thus $\frac{\partial p_{pp}^D}{\partial P} = -\frac{\partial X / \partial P}{\partial X / \partial p} < 0$. As P rises, for any given C, G, q and any δ satisfying assumption 2, p_{pp}^D becomes smaller and smaller. Therefore, for any given C, G, q and any δ satisfying assumption 2, and for any $\tilde{p} < p_{ic}^U$, there exists P_c such that for $P > P_c$, p_{pp}^D is lower than \tilde{p} so that the participation-in-prepayment constraint of the downstream firm holds at \tilde{p} . This completes the proof. ||

REFERENCES

- Blazenko, George W. and Kirk Vandezande**, “Corporate Holdings of Finished Goods Inventories,” *Journal of Economics and Business*, 2003, 55, 255–266. 20
- Blinder, Alan S. and Louis J. Maccini**, “Taking Stock: A Critical Assessment of Recent Research on Inventories,” *Journal of Economic Perspectives*, 1991, 5, 73–96. 1, 4, 6, 19, 30
- Brechling, Frank P.R. and Richard G. Lipsey**, “Trade Credit and Monetary Policy,” *Economic Journal*, 1963, 73, 618–641. 1
- Brennan, Michael J., Vojislav Maksimovic, and Josef Zechner**, “Vendor Financing,” *Journal of Finance*, 1988, 43(5), 1127–1141. 2, 5
- Burkart, Mike and Tore Ellingsen**, “In-Kind Finance: A Theory of Trade Credit,” *American Economic Review*, 2004, 94(3), 569–590. 1, 2, 5, 20
- Dynan, Karen E., Kathleen W. Johnson, and Samuel Slowinski**, “Survey of Finance Companies 2000,” *Federal Reserve Bulletin*, 2002. 18
- Emery, Gary W.**, “An Optimal Financial Response to Variable Demand,” *Journal of Financial and Quantitative Analysis*, 1987, 22(2), 209–225. 6
- Ferris, J. Stephen**, “A Transactions Theory of Trade Credit Use,” *Quarterly Journal of Economics*, 1981, 94(2), 243–270. 2, 5
- Kahn, James A.**, “Why is Production More Volatile than Sales? Theory and Evidence on the Stockout Avoidance Motive for Inventory Holding,” *Quarterly Journal of Economics*, 1992. 6
- Maccini, Louis J., Bartholomew Moore, and Huntley Schaller**, “The Interest Rate, Learning, and Inventory Investment,” *American Economic Review*, 2004, forthcoming. 4, 19
- Marvel, Howard P. and James Peck**, “Demand Uncertainty and Returns Policies,” *International Economic Review*, 1995. 18

- Meltzer, Allan H.**, “Mercantile Credit, Monetary Policy, and Size of Firms,” *Review of Economics and Statistics*, 1960, *42(4)*, 429–437. 1
- Nadiri, Ishaq M.**, “The Determinants of Trade Credit in the U.S. Total Manufacturing Sector,” *Econometrica*, 1969, *37(3)*, 408–423. 5
- Ng, Chee K., Janet Kiholm Smith, and Richard L. Smith**, “Evidence on the Determinants of Credit Terms in Interfirm Trade,” *Journal of Finance*, 1999, *54(3)*, 1109–1129. 1, 2, 3, 4, 6, 20, 29
- Nilsen, Jeffrey H.**, “Trade Credit and the Bank Lending Channel,” *Journal of Money, Credit and Banking*, 2002, *34(1)*, 226–253. 1, 22
- Petersen, Mitchell and Raghuram Rajan**, “Trade Credit: Theories and Evidence,” *Review of Financial Studies*, 1997, *10(3)*, 661–691. 17, 20
- Schwartz, Robert A. and David K. Whitcomb**, “The Trade Credit Decision,” in James L. Bicksler, ed., *Handbook of Financial Economics*, North-Holland, 1979, pp. 257–273. 5
- Vandenberg, Paul**, “Adapting to the Financial Landscape: Evidence from Small Firms in Nairobi,” *World Development*, 2003, *31(11)*, 1829–1843. 29