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Permanent vs Transitory Components and Economic Fundamentals

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Abstract

Any non-stationary series can be decomposed into permanent (or "trend") and transitory (or "cycle") components. Typically some atheoretic pre-filtering procedure is applied to extract the permanent component. This paper argues that analysis of the fundamental underlying stationary economic processes should instead be central to this process. We present a new derivation of multivariate Beveridge-Nelson permanent and transitory components, whereby the latter can be derived explicitly as a weighting of observable stationary processes. This allows far clearer economic interpretations. Different assumptions on the fundamental stationary processes result in distinctly different results; but this reflects deep economic uncertainty. We illustrate with an example using Garratt *et al's* (2003a) small VECM model of the UK economy.

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An automated routine for *E-Views* that implements the procedure described in this paper can be downloaded from www.econ.bbk.ac.uk/faculty/wright.

1 Introduction

Macroeconomic analysis is largely formulated in terms of stationary processes, yet most economic magnitudes are trending. There is as a result widespread use of a range of de-trending procedures, usually of the "black box" variety, whereby a trend (or permanent component) is extracted by some pre-filtering procedure, usually univariate in nature; with the resulting de-trended series, or transitory component, usually interpreted as a measure of the "cycle".[1] Such methods typically leave unanswered three key questions: How do we know, from the outset, certain key characteristics that we need to feed into the black box?[2] What are the economic mechanisms that pull a given variable towards its trend? And what, if anything, does the extent of the current estimated deviation from trend tell us about the future of that variable?

This paper argues that analysis of economic fundamentals should instead be central to the process of detrending, and that this helps to provide important insights (if not necessarily clear-cut answers) to the questions that black box techniques leave unanswered.

This argument is not, we should stress, new in itself. The alternative approach that we advocate is the multivariate Beveridge-Nelson (henceforth B-N) permanent/transitory decomposition[3] that, in turn, provides the basis for a range of alternative multivariate techniques.[4] We do argue however that B-N trends have been unduly neglected, for which there are probably two explanations. The first arises from the (incorrect) perception that B-N trends are of necessity "too volatile".[5] The second is that the process by which B-N trends are usually derived appears to be even less transparent

[1] Hodrick & Prescott (1997); Baxter & King (1999); Harvey & Trimbur (2003); Ravn & Uhlig (2002); Morley, Nelson & Zivot (2003). We prefer to avoid the use of the term "cycle", which we regard as increasingly a misnomer, since transitory components need not necessarily display periodic movements.

[2] Typical examples of required prior assumptions might be "smoothness" of the trend e.g. Hodrick-Prescott (1997); frequency ranges for the "cycle" e.g. Baxter and King (1999); or orthogonality restrictions on innovations e.g Blanchard and Quah (1989). For a multivariate Hodrick-Prescott example using common trend restrictions see Kozicki (1999).

[3] Beveridge & Nelson (1981); Stock and Watson (1988); Cochrane (1994); Evans & Reichlin (1994); Newbold and Arino (1998).

[4] Since, on reasonable assumptions, the B-N permanent component is a limiting forecast of all possible permanent components derived from a given multivariate representation: for example, the bivariate Blanchard-Quah (1989) decomposition; extended in a larger multivariate framework by King *et al* (1991) Crowder *et al* (1999) and Gonzalez and Granger (1995).

[5] See, for example Massmann and Mitchell (2002); Favero (2001). Such criticisms are often both descriptively incorrect in a multivariate context and, we argue, misplaced, since they prejudge the nature of permanent and transitory components.

than that applied in black box univariate techniques.

Our aim in this paper is to show that, viewed from a new angle, the process that generates B-N permanent and transitory components is *not* a black box. B-N trends are usually derived from the moving average representation. We show that a simple alternative derivation from the vector autoregressive representation has the distinct advantage that transitory components can be related directly to the underlying observable stationary processes that drive the system. The new derivation also helps to illuminate the links with the nature of adjustment processes to disequilibrium.

The advantage of this approach, we argue, is that it should help to focus the mind of applied economists on the key issues involved in the process of detrending. First, we should look for underlying stationary processes, in terms of identifiable economic fundamentals, ideally with a clear basis in theory. Having done this, we need to identify the (necessary) predictive power of such stationary processes for the underlying variables. The transitory components are then simply projections from current values of the underlying stationary processes, and the trends themselves effectively drop out as whatever is left over. The nature of both trends and transitory components must thus depend directly on the nature, and predictive power, of the fundamental stationary processes.

Identifying these processes is of course by no means straightforward. In general, they will reflect deviations - typically due to adjustment costs or other market frictions - from some equilibrium condition. Almost invariably, these equilibrium conditions will link a wider set of variables, hence the process is inherently multivariate in nature. It will usually (but need not always) imply cointegration. There will frequently be competing economic hypotheses about the nature of the equilibrium relationships, both in the long run and (in some cases) in the short run; and very often we cannot avoid a degree of uncertainty about whether the data reject the theory. As a result there may be significant differences in implied permanent and transitory components, that relate directly to the inclusion or exclusion of certain economic relationships from the system.

This may seem like a disadvantage, but we argue that it is not. Indeed it seems to us to make perfect sense. If we assert that a given series is "above trend" by some amount, we must always at least implicitly be positing some underlying disequilibrium, or set of disequilibra that will, in unconditional expectation, be expected to disappear. Crucially, we are also assuming that a fall (or at least below-average growth) in that series will be an important part of that adjustment process, and that this is to some extent predictable. In our framework, we can directly identify the link between deviations from trend and the underlying economic disequlibria. But, since there is almost always doubt about the statistical credentials of any process that is assumed to be stationary (whatever its theoretical credentials), we must inevitably end up with different answers, depending on what we assume are the fundamental stationary processes.

We illustrate our analysis with an empirical example: we derive the permanent and transitory components of real UK GDP from Garratt *et al's* (2003a,b) small VECM model of the UK economy. We show that the nature of the decomposition is very sensitive to assumptions about the underlying stationary processes. But, crucially, the source of these differences can be related directly to economic fundamentals with a clear basis in theory. Thus, we argue, theory helps to illuminate the interior of the black box, since the remaining uncertainty about the nature of permanent and transitory components can be related directly to clear economic hypotheses.

The structure of the paper is as follows. Section 2 sets out the derivation of B-N permanent and transitory components; Section 3 presents our empirical illustration; Section 4 concludes the paper. An appendix provides details of derivations.

2 Beveridge Nelson Trends in the Cointegrating Vector Autoregressive Representation

2.1 A General Definition

The most general definition of Beveridge-Nelson trends is as limiting forecasts, absent deterministic growth, as the forecast horizon goes to infinity.[6] Thus, for a vector process, $\mathbf{x}_t$ define the vector of B-N trends, $\widehat{\mathbf{x}}_t$, by

$$\widehat{\mathbf{x}}_t = \lim_{h\to\infty} E_t \mathbf{x}_{t+h} - \mathbf{g}h \tag{1}$$

where $\mathbf{g}$, the element of deterministic growth, is typically a vector of constants, but may in principle be a deterministic function of h. If $\Delta\mathbf{x}_t$ can be given a stationary moving average representation of the form

$$\boldsymbol{\Delta}\mathbf{x}_t = \mathbf{g} + \mathbf{C}(L)\boldsymbol{\varepsilon}_t \tag{2}$$

then the B-N trends can be expressed as

$$\Delta\widehat{\mathbf{x}}_t = \mathbf{g} + \mathbf{C}(1)\boldsymbol{\varepsilon}_t \tag{3}$$

[6] For multivariate approaches, see, for example, Stock and Watson (1988), Newbold and Arino (1998); Evans & Reichlin (1994).

and are thus by definition correlated random walks with drift.

The random walk feature of B-N trends is sometimes represented as a disadvantage, but is a necessary consequence of their forward-looking nature. Thus, suppose we take any arbitrary partitioning of $\mathbf{x}_t$ into permanent and transitory components, of the form

$$\mathbf{x}_t = \mathbf{x}_t^P + \mathbf{x}_t^T$$

then, since the transitory components must always satisfy

$$\lim_{h\to\infty} E_t \mathbf{x}_{t+h}^T = \mathbf{0}$$

then it must trivially follow that

$$\lim_{h\to\infty} E_t \mathbf{x}_{t+h}^P = \widehat{\mathbf{x}}_t$$

Thus all possible permanent components must converge in expectation on the B-N trends as the forecast horizon increases. Alternative multivariate techniques that introduce additional assumptions (most commonly orthogonality of innovations, as in Blanchard-Quah (1989); King *et al* (1991) Crowder *et al* (1999) and Gonzalez and Granger (1995)) in effect redistribute some additional stationary element between the B-N permanent and transitory components.[7] We focus in this paper solely on B-N trends in the interests of simplicity and clarity.

2.2 Forecasting from a Cointegrating VAR

When a vector of time series can be given a vector autoregressive representation B-N trends can be derived in a form that is readily interpretable in terms of the underlying stationary processes.[8] Assume a cointegrating VAR (VECM) in n variables, of rank r, of the form:

$$\boldsymbol{\Delta}\mathbf{x}_t = \boldsymbol{\Psi} + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_{t-1} + \boldsymbol{\Phi}\boldsymbol{\Delta}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t \tag{4}$$

or equivalently

$$\boldsymbol{\Delta}\mathbf{x}_t = \mathbf{g} + \boldsymbol{\alpha}(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\kappa}) + \boldsymbol{\Phi}(\boldsymbol{\Delta}\mathbf{x}_{\mathbf{t-1}} - \mathbf{g}) + \boldsymbol{\varepsilon}_t \tag{5}$$

[7] See Proietti (1997) for a demonstration of the link between the Gonzalo-Granger and the B-N decompositions.

[8] The approach here can easily be generalised, in principle, to VARMA processes, as in Arino & Newbold (*op cit*); although, as they note, identification problems usually rule out such representations on practical grounds.

where $\mathbf{x}_t$ and $\mathbf{\Psi}$ are $n \times 1$ vectors, $\mathbf{\Phi}$ is an $n \times n$ matrix, $\boldsymbol{\alpha}$ is an $n \times r$ matrix, $\boldsymbol{\beta}'$ is an $r \times n$ matrix and $\boldsymbol{\varepsilon}_t$ is an $n \times 1$ vector of error terms. The $n \times 1$ vector $\mathbf{g}$ and the $r \times 1$ vector $\boldsymbol{\kappa}$ are the trend growth rates in the variables, and the steady state values of the stationary (typically cointegrating) relationships in levels, respectively and represent the deterministic components of the system.[9] Note that these vectors of constants ($\mathbf{g}$ and $\boldsymbol{\kappa}$) can (as shown below) be derived from the intercepts in the estimated VAR as in (4), or can be estimated directly; and hence the data require no pre-filtering (*cf* Newbold and Arino, 1998; Rotemberg and Woodford 1996). Higher order VARs can be dealt with by creating new variables for additional lagged differences (which raise n without raising r).

Two aspects of the representation in (4) and (5) are worth noting. First, $\boldsymbol{\beta}'\mathbf{x}_t$ is typically assumed to pick out cointegrating relationships: stationary combinations of nonstationary series. However $\boldsymbol{\beta}$ may in principle include columns in which there is only a single non-zero element, thus nesting systems in which one or more series is independently stationary (as, for example, in Blanchard and Quah (1989)-type bivariate representations of output growth and unemployment). In this case the relevant elements of $\widehat{\mathbf{x}}_t$ will be time-invariant. In what follows, however, we use the term "cointegrating relations" as a short-hand for any stationary levels relationships. Second, we assume that all the parameters of the model (including r, the number of cointegrating vectors) are known. In our empirical example we provide a discussion of the consequences of uncertainty over r.

The system as specified in (4) or (5) can be reparameterised as the first-order VAR

$$\mathbf{y}_t = \begin{bmatrix} \mathbf{\Psi} \\ \boldsymbol{\beta}'\mathbf{\Psi} \end{bmatrix} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{v}_t \tag{6}$$

where

$$\mathbf{y}_t = \begin{pmatrix} \Delta\mathbf{x}_t \\ \boldsymbol{\beta}'\mathbf{x}_t \end{pmatrix}, \ \mathbf{v}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\beta}'\boldsymbol{\varepsilon}_t \end{pmatrix}; \ \ \mathbf{A} = \begin{bmatrix} \mathbf{\Phi} & \boldsymbol{\alpha} \\ \boldsymbol{\beta}'\mathbf{\Phi} & \mathbf{I}_r + \boldsymbol{\beta}'\boldsymbol{\alpha} \end{bmatrix}$$

where $\mathbf{A}$ is $((n + r) \times (n + r))$ and $\det(\mathbf{I} - \mathbf{A}) \neq 0$. Thus (6) has a unique steady state.

By solving for this steady state, (6) can also be expressed in terms of deviations therefrom as the zero mean system

$$\widetilde{\mathbf{y}}_t = \mathbf{A}\widetilde{\mathbf{y}}_{t-1} + \mathbf{v}_t \tag{7}$$

[9] The system as specified allows for common deterministic trends in all series. If we wish to allow for deterministic trends in the cointegrating space we can augment the vector $\mathbf{x}_t$ with t, with an appropriate set of restrictions. In our empirical example we allow for the presence of deterministic trends in the cointegrating processes.

where

$$\widetilde{\mathbf{y}}_t = \begin{pmatrix} \boldsymbol{\Delta}\mathbf{x}_t - \mathbf{g} \\ \boldsymbol{\beta}'\mathbf{x}_t - \boldsymbol{\kappa} \end{pmatrix}; \quad \begin{pmatrix} \mathbf{g} \\ \boldsymbol{\kappa} \end{pmatrix} = [\mathbf{I} - \mathbf{A}]^{-1} \begin{bmatrix} \boldsymbol{\Psi} \\ \boldsymbol{\beta}'\boldsymbol{\Psi} \end{bmatrix}$$

It is then possible to express h period-ahead expected values of the underlying series as a cumulation of forecasts from the zero mean system:

$$\begin{aligned} E_t\mathbf{x}_{t+h} &= \mathbf{x}_t + \mathbf{g}h + \mathbf{J}\sum_{i=1}^{h}\mathbf{A}^i\widetilde{\mathbf{y}}_t \\ &= \mathbf{x}_t + \mathbf{g}h + \mathbf{B}_h\widetilde{\mathbf{y}}_t \\ &= \mathbf{x}_t + \mathbf{g}h + \boldsymbol{\alpha}_h(\boldsymbol{\beta}'\mathbf{x}_t - \boldsymbol{\kappa}) + \boldsymbol{\Phi}_h(\Delta\mathbf{x}_t - \mathbf{g}) \end{aligned} \tag{8}$$

where $\mathbf{J} = \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \end{bmatrix}$ is a selection matrix that picks out $\boldsymbol{\Delta}\mathbf{x}_t - \mathbf{g}$ from $\widetilde{\mathbf{y}}_t$, and $\mathbf{B}_h = \mathbf{JA}[\mathbf{I}_{n+r} - \mathbf{A}]^{-1}[\mathbf{I}_{n+r} - \mathbf{A}^h]$, can be partitioned into two elements, such that $\mathbf{B}_h = \begin{bmatrix} \boldsymbol{\Phi}_h & \boldsymbol{\alpha}_h \end{bmatrix}$, where $\boldsymbol{\alpha}_h$ and $\boldsymbol{\Phi}_h$ and are of the same dimensions as $\boldsymbol{\alpha}$ and $\boldsymbol{\Phi}$. These capture the expected response over h periods to the current disequilibria in the cointegrating relations and growth rates.[10]

Given deterministic growth, as h goes to infinity, conditional forecasts from period t go to infinity, but in deterministically detrended terms, they will go to limiting values given by the "infinite horizon error correction" representation:

$$\begin{aligned} \lim_{h\to\infty}(E_t\mathbf{x}_{t+h} - \mathbf{g}h) &= \mathbf{x}_t + \mathbf{B}_\infty\widetilde{\mathbf{y}}_t \\ &= \mathbf{x}_t + \boldsymbol{\alpha}_\infty(\boldsymbol{\beta}'\mathbf{x}_t - \boldsymbol{\kappa}) + \boldsymbol{\Phi}_\infty(\Delta\mathbf{x}_t - \mathbf{g}) \end{aligned} \tag{9}$$

where $\mathbf{B}_\infty = \lim_{h\to\infty}\mathbf{B}_h = \mathbf{JA}[\mathbf{I}_{n+r} - \mathbf{A}]^{-1}$

In contrast to the 1- or $h-$period error correction representations in (5) and (8) in which the current disequilibria in terms of cointegrating relations and growth rates will only be partially eliminated, in the infinite horizon representation all disequilibria must in expectation be fully eliminated. We show below that this property is reflected in a set of restrictions satisfied by the two matrices $\boldsymbol{\alpha}_\infty$ and $\boldsymbol{\Phi}_\infty$

[10] If the system is generalised such that $\boldsymbol{\varepsilon}_t$, the underlying error process, contains moving average elements of order p, there will be an additional term that cumulates the impact, up to horizon p, of current and lagged errors. This term will however be constant for $h \geq p$. See Arino and Newbold (1998).

2.3 Beveridge-Nelson Trends as Conditional Cointegrating Equilibrium Values

By inspection the left-hand side of (9) is identical to the right-hand side of (1) and so provides an alternative definition of the multivariate Beveridge-Nelson trends.[11] Hence we can interpret B-N trends as conditional cointegrating equilibrium values: the values to which each series would converge if current disequilibria in the cointegrating relations and in growth rates were eliminated. Thus the trend values always satisfy cointegrating equilibrium:

$$\boldsymbol{\beta}'\widehat{\mathbf{x}}_t = \boldsymbol{\kappa} \tag{10}$$

It also follows immediately that the transitory components of the vector $\mathbf{x}_t$ are simply defined by (9) with signs reversed:

$$\mathbf{x}_t - \widehat{\mathbf{x}}_t = -\boldsymbol{\alpha}_\infty(\boldsymbol{\beta}'\mathbf{x}_t - \boldsymbol{\kappa}) - \boldsymbol{\Phi}_\infty(\Delta\mathbf{x}_t - \mathbf{g}) \tag{11}$$

A forecast of transitional growth towards equilibrium at a rate above (or below) steady-state growth thus implies a low (or high) value of the transitory component for any given variable.

A significant difference between the definitions of the B-N permanent and transitory components given here, and the standard formulation, is that our definitions are entirely in terms of current-dated observable economic magnitudes, in contrast to the standard definition in terms of the moving average representation.[12] An advantage of our approach is that, as we show in the empirical example, it allows a "cointegrating accounting" approach to the analysis of transitory components. The transitory component of any given variable can be decomposed into the contributions of individual cointegrating disequilibria, and disequilibrium growth rates. This is particularly helpful when the cointegrating relations have clear economic interpretations.

Note also that, in the special case that $\boldsymbol{\Phi}_\infty = \mathbf{0}$ there will be an exact static relationship between transitory components and the cointegrating relations. In general this condition will require complicated cross-equation restrictions on the underlying VAR coefficients.[13] Since there are only r cointegrating relations, if the restriction holds the resulting transitory components will in this case be linearly dependent: a special case of the "common

[11] For a formal demonstration of the equivalence of our approach with the standard derivation from the MA representation (as in equation (3)) see the appendix.

[12] Newbold and Arino (*op cit*) and Proietti (*op cit*) also provide equivalent, but rather more opaque, definitions in terms of observables.

[13] It will however hold automatically in the case of a VAR(1) - a special case we examine in the next sub-section.

cycle" approach of Engle & Vahid, 1993; Engle and Kozicki, 1993. Viewed in this light, this version of the common cycle restriction can also be given some theoretical content. Typically there may be little or no theoretical basis for assuming inertia in growth rates.[14] Thus $\mathbf{\Phi}_{\infty}= \mathbf{0}$ can be interpreted as a restriction that deviations from a well-identified set of equilibrium conditions in the model (as captured in cointegrating relations) fully account for observed transitory components of the variables in question.[15] This restriction appears to be close to holding in our empirical example.

2.4 Characteristics of the Infinite Horizon Error Correction Process

While the infinite horizon error correction representation in (9) that determines the nature of the B-N trends and transitory components has the same structure as the underlying one-period-ahead VECM representation in (5), it turns out to contain distinctly fewer parameters, that are themselves combinations of the underlying VAR parameters. This can be shown straightforwardly by considering two special cases in which one or other of the two disequilibrium terms on the right-hand sides of (9) and (11) is zero at time t (and thus where the transitory components are defined by one of the terms alone).

Thus we can consider first the special case where there is an initial disequilibrium in the cointegrating relations ($\boldsymbol{\beta}'\mathbf{x}_t \neq \boldsymbol{\kappa}$) but growth rates are at their steady-state value ($\Delta\mathbf{x}_t = \mathbf{g}$). Pre-multiplying both sides of (11) (with the second term set to zero) by $\boldsymbol{\beta}$, we get

$$\boldsymbol{\beta}'\left(\mathbf{x}_t - \widehat{\mathbf{x}}_t\right) = -\boldsymbol{\beta}'\boldsymbol{\alpha}_{\infty}(\boldsymbol{\beta}'\mathbf{x}_t - \boldsymbol{\kappa})$$

but, since the B-N trends, as noted above, automatically satisfy cointegrating equilibrium, we can substitute from (10), and write

$$\boldsymbol{\beta}'\left(\mathbf{x}_t - \widehat{\mathbf{x}}_t\right) = -\boldsymbol{\beta}'\boldsymbol{\alpha}_{\infty}\boldsymbol{\beta}'\left(\mathbf{x}_t - \widehat{\mathbf{x}}_t\right)$$

implying, by inspection, the adding-up constraint on the elements of $\boldsymbol{\alpha}_{\infty}$:

$$\boldsymbol{\beta}'\boldsymbol{\alpha}_{\infty} = -\mathbf{I}_r \tag{12}$$

[14] An exception to this is sometimes argued to be inflation, although the theoretical basis for inertia in the inflation rate itself is known to be fragile (Kozicki & Tinsley, 2002).

[15] Note also that, since, as noted below, $\mathbf{\Phi}_{\infty}$ only contains $(n-r)\times n$ free parameters, the number of parameter restrictions required to satisfy this condition is not as large as might at first appear (in particular it does not require a VAR(1) representation).

To interpret this condition, recall the contrast drawn in Section 2.2 between the 1- or h-period error correction representations in (5) and (8) and the infinite horizon error correction representation in (9). Over 1 or h periods any current disequilibrium in the cointegrating relations will in expectation be only partially eliminated; but at an infinite horizon it must be expected to be eliminated entirely, since shocks to the cointegrating relations must be transitory. The size of the ultimate adjustment in each variable to a shock to the cointegrating relations is given by the elements of $\boldsymbol{\alpha}_{\infty}$. The restriction in (12) implies that each column of $\boldsymbol{\alpha}_{\infty}$ must add up to -1 because a unit shock to any given cointegrating relation must be expected to disappear entirely at an infinite horizon. Hence, while $\boldsymbol{\alpha}_{\infty}$ contains $n \times r$ elements, the adding up constraint in (12) implies that these are determined by only $(n-r) \times r$ free parameters.

Similarly, consider the alternative special case where there is an initial equilibrium in the cointegrating relations ($\boldsymbol{\beta}'\mathbf{x}_t = \boldsymbol{\kappa}$) but growth rates are *not* at their steady-state values ($\Delta\mathbf{x}_t \neq \mathbf{g}$) . Again, pre-multiply both sides of (11) by $\boldsymbol{\beta}'$ (this time with with the first, rather than second term set to zero). Thus:

$$\boldsymbol{\beta}'\left(\mathbf{x}_t - \widehat{\mathbf{x}}_t\right) = -\boldsymbol{\beta}'\boldsymbol{\Phi}_{\infty}\left(\Delta\mathbf{x}_t - \mathbf{g}\right)$$

which can only be satisfied if

$$\boldsymbol{\beta}'\boldsymbol{\Phi}_{\infty} = \mathbf{0}_{r\times n} \tag{13}$$

hence $\boldsymbol{\Phi}_{\infty}$ is of reduced rank, and only contains $(n-r) \times n$ parameters. The intuition for this second condition is that, even though, in this special case, the system is assumed initially to be in cointegrating equilibrium, with $\boldsymbol{\Phi} \neq 0$, an initial disequlibrium in growth rates will impart some inertial growth effects that will push the system temporarily *away* from cointegrating equilibrium again. But (13) says that to the extent that this causes transitory components to be non-zero, this must still be consistent with cointegrating equilibrium both at period t and at infinity.[16]

2.5 A Simple Example: a Bivariate Cointegrating VAR

Some further insight can be derived by examining the special case of a bivariate cointegrating VAR, with a unit cointegrating vector.

[16]These conditions satisfied by $\boldsymbol{\alpha}_{\infty}$ and $\boldsymbol{\Phi}_{\infty}$ are equivalent to the well-known orthogonality condition (Engle & Granger, 1987; Stock & Watson 1988) on the MA representation in (3), $\boldsymbol{\beta}'\mathbf{C}(1) = \mathbf{0}$ (which is in turn a direct implication of the assumption of cointegration) since, as we show in the appendix $\mathbf{C}(1) = \mathbf{I} + \boldsymbol{\Phi}_{\infty} + \boldsymbol{\alpha}_{\infty}\boldsymbol{\beta}'$.

2.5.1 The VAR(1) Case

We focus initially on the first order case

$$\begin{aligned} \mathbf{\Delta x}_t &= \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_{t-1}+\boldsymbol{\varepsilon}_t \\ &= \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \mathbf{x}_{t-1} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \end{aligned} \tag{14}$$

The implied process for the single cointegrating relation is given by

$$\boldsymbol{\beta}'\mathbf{x}_t = \frac{\boldsymbol{\beta}'\boldsymbol{\varepsilon}_t}{1-\theta L} \tag{15}$$

where $\theta = (1+\alpha_1-\alpha_2) < 1$, by the assumption of cointegration.

The simple structure of the underlying VAR implies equally simple expression for the trends and transitory components. The two underlying matrices in (9) and hence in (11) can be calculated directly as

$$\begin{aligned} \mathbf{\Phi}_\infty &= \mathbf{0} \\ \boldsymbol{\alpha}_\infty &= \begin{bmatrix} \frac{\alpha_1}{\alpha_2-\alpha_1} \\ \frac{\alpha_2}{\alpha_2-\alpha_1} \end{bmatrix} = \begin{bmatrix} -(1-\xi) \\ \xi \end{bmatrix} \end{aligned}$$

The elements of $\boldsymbol{\alpha}_\infty$ (which captures expected infinite horizon error correction) are in this case just scalar multiples of the corresponding elements of $\boldsymbol{\alpha}$ (which captures expected one-period ahead error correction). The single parameter ξ that determines both elements of $\boldsymbol{\alpha}_\infty$ captures the proportion of eventual adjustment towards equilibrium due to changes in x_{2t}. The adding-up constraint that any current disequilibrium must be entirely eliminated, as in (12), ensures that the remainder of the adjustment must occur via changes in x_{1t}.

The implied process for the transitory components is

$$\mathbf{x}_t-\widehat{\mathbf{x}}_t = -\boldsymbol{\alpha}_\infty\boldsymbol{\beta}'\mathbf{x}_t = -\boldsymbol{\alpha}_\infty\frac{\boldsymbol{\beta}'\boldsymbol{\varepsilon}_t}{(1-\theta L)} \tag{16}$$

a restricted VAR(1), with a single error process that is the innovation to the cointegrating relation, a combination of the underlying VAR errors. The transitory components are thus perfectly correlated, since they are hit by the same error process, but for a scaling factor, and both have the same autoregressive coefficient as the cointegrating relation itself: a special case of a "common cycle", also noted by Engle & Vahid (1993).

The process for the trends themselves can be shown[17] to be given by

$$\begin{aligned} \Delta\widehat{\mathbf{x}}_t &= (\mathbf{I} + \boldsymbol{\alpha}_\infty \boldsymbol{\beta}')\,\boldsymbol{\varepsilon}_t \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} (\xi\varepsilon_{1t} + (1-\xi)\varepsilon_{2t}) \end{aligned} \tag{17}$$

Thus the two series have a common B-N trend, the innovation to which (the permanent innovation) is a weighted average of the two VAR innovations.

In this VAR(1) framework there is a simple link between the nature of the permanent and transitory components and the single parameter ξ, which captures Granger Causality relations. Values of $\xi = 1$ or 0 imply one-way causality (from x_{1t} to x_{2t} or vice versa); all other values imply mutual causality.

A value of ξ between zero and unity will imply that the two elements of $\boldsymbol{\alpha}_\infty$ are of opposite sign. In this case the two transitory components will be negatively correlated, and the permanent innovation will be positively related to innovations to both variables, where the weights are inversely proportional to their *relative* speed of error correction. The less a series adjusts to disequilibrium, the more closely it will resemble the common trend. A common example of this form might be the consumption-output relationship. Cochrane (1994) shows that the ratio of US consumption to GDP is a better predictor of output than of consumption so that the common trend much more closely resembles consumption than output.

Values of ξ below zero, or greater than unity are also possible, and will imply that the two elements of $\boldsymbol{\alpha}_\infty$ are of the same sign. Hence the two transitory components will be positively correlated, while the permanent innovation will have a *negative* weight on one of the innovations. An example of a relationship of this form might be a simple real money demand relationship with a unit income elasticity (ignoring interest rate effects for simplicity). Letting x_{1t} be real money balances and x_{2t} be output, we would expect $\xi < 0$, implying that both transitory components will be positively affected by a shock to real money. The weight on ε_{2t} (the innovation to money) in the permanent innovation will be negative; but offset by a greater-than-unit weight on the innovation to output itself.

This simple analytical framework helps to provide two key insights into the nature of B-N trends, and the link with the error correction representation.

First, there is no necessary result that trends are "smooth" in a multivariate context; equally they need not be "noisy" (a common misperception of

[17] By deriving the implied MA process for $\Delta\mathbf{x}_t$, and then either by the conventional B-N approach (see Appendix), or, equivalently, by substitution into (16).

B-N trends that largely arises from univariate estimation that we discuss further below in Section 2.5.2). The variance of the single permanent innovation in this example can be expressed as

$$\sigma_P^2 = \sigma_1^2 \left[\xi^2 + (1-\xi)^2 s^2 + 2\xi(1-\xi)\rho s\right] \tag{18}$$

where ρ is the correlation coefficient between the two residuals, and $s = \sigma_2/\sigma_1$. The smoothness, or otherwise, of the trend will depend both on the relative variances of the two variables, the degree of correlation of individual shocks, and the nature of Granger causality relations (captured by ξ). In the special case of one-way Granger causality (for the sake of argument, from x_{2t} to x_{1t}, implying $\xi = 0$) all but the second term will disappear, implying $\sigma_P^2 = \sigma_1^2 s^2 = \sigma_2^2$ and thus the common trend is simply equal to x_{2t}, and will be relatively "smooth" or "noisy", compared to x_{1t}, depending simply on whether x_{2t} itself is smoother or noisier than x_{1t}.

Second, there is no presumption in general that the permanent innovation will be orthogonal to the innovation to the common transitory component. On the other hand, they are very unlikely to be perfectly correlated (a common misperception that again arises from the nature of univariate B-N trends). The covariance of the permanent and transitory innovations will be given in the above example by

$$\sigma_{PT} = \sigma_1^2 \left[\xi - (1-\xi)s^2 + 2\left(\frac{1}{2} - \xi\right)\rho s\right] \tag{19}$$

which will in general be of indeterminate sign.[18]

2.5.2 The VAR(2) Case, and Univariate B-N Trends

If we supplement the bivariate example above such that is of the same form as (5), and is thus a VAR(2) in levels, we can arrive at some additional insights.

The transitory components can then be expressed, using the reduced rank feature of $\mathbf{\Phi}_\infty$ as:

$$\mathbf{x}_t - \widehat{\mathbf{x}}_t = -\boldsymbol{\alpha}_\infty(\boldsymbol{\beta}'\mathbf{x}_t - \boldsymbol{\kappa}) - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Delta z_t$$

where the scalar process $\Delta z_t = \boldsymbol{\gamma}' (\Delta\mathbf{x}_t - \mathbf{g})$ is a linear combination of the two disequilibrium growth rates. Thus the two transitory components share an identical second element, even without any supplementary restrictions on

[18] It will be equal to zero in the case of symmetric two-way causality, and equal variance of the underlying innovations ($\xi = \frac{1}{2}; s = 1$); but this is a highly special case.

$\mathbf{\Phi}_\infty$. The magnitude of the elements of the 2×1 vector $\boldsymbol{\gamma}$ depend on the nature of Granger causality relations.[19] The trends are given by[20]

$$\Delta\widehat{\mathbf{x}}_t = \mathbf{C}(\mathbf{1})\boldsymbol{\varepsilon}_t = (\mathbf{I} + \boldsymbol{\alpha}_\infty\boldsymbol{\beta}' + \mathbf{\Phi}_\infty)\,\boldsymbol{\varepsilon}_t$$

This decomposition of $\mathbf{C}(1)$ provides a further insight into the common misperception that B-N trends are of necessity noisy, which largely arises from univariate models. If, for example, the two series were *not* cointegrated, but were independent AR(1) processes in differences, then the process for the trends above would reduce to[21]

$$\Delta\widehat{\mathbf{x}}_t = \begin{bmatrix} \frac{1}{1-\phi_{11}} & 0 \\ 0 & \frac{1}{1-\phi_{22}} \end{bmatrix} \varepsilon_t$$

thus the two univariate trends would be more or less noisy than the processes themselves, depending on whether their growth rates were positively or negatively serially correlated. This feature may be present even with cointegration, but, with mutual Granger Causality may be offset by the implicit averaging process described above in relation to the cointegrating VAR(1). Additionally, the long-run impact of persistence in growth rates will be limited by the orthogonality condition on $\mathbf{\Phi}_\infty$ in (13).

3 An Empirical Illustration: Permanent and Transitory Components of GDP in A Small Model of the UK Economy

3.1 The General Framework

We examine the permanent and transitory components of UK GDP, using Garratt *et al's.* (2003a, 2003b) model of the UK economy which considers the following set of quarterly variables over the period 1965q1-1999q4

$$\mathbf{x}_t = (p_t^o, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*, t)'. \quad (20)$$

[19] In the special case of one-way causality from x_{2t} to x_{1t}, for example, the first element of $\boldsymbol{\gamma}$ will be zero, and the second element will be determined solely by the univariate properties of x_{2t}.

[20] Again, exploiting the derivation in the Appendix.

[21] Since in this restricted case $\boldsymbol{\alpha}\boldsymbol{\beta}' = \mathbf{0}$ implying $\mathbf{C}(1) = \mathbf{I} + \mathbf{\Phi}_\infty = \mathbf{I} + \mathbf{\Phi}[\mathbf{I} - \mathbf{\Phi}]^{-1} = [\mathbf{I} - \mathbf{\Phi}]^{-1}$ which has the form given above assuming $\mathbf{\Phi}$ is diagonal.

p_t^o is the oil price, e_t is the nominal exchange rate (the domestic price of a unit of the foreign currency), r_t^* is the foreign short term nominal interest rate, r_t is the domestic short term nominal interest rate, p_t is domestic prices, y_t is real per capita domestic output, p_t^* is foreign prices, y_t^* is real per capita foreign output, h_t is the real per capita money stock and t is a deterministic time trend. All variables are in logarithms.

Garratt *et al.* work on the assumption that all variables are I(1) and estimate a cointegrating VAR(2) model in which they examine the impact of imposing cointegrating relations based on theory. We examine the impact of these restrictions in Section 3.3 below. First, however, for the purposes of illustration, we examine the consequences of simply deriving multivariate B-N trends of the black box variety.

3.2 Pitfalls of Atheoretic Multivariate De-Trending

In a model of this size (eight endogenous variables and one exogenous variable, p_t^o) there is inevitably considerable uncertainty regarding the correct multivariate empirical representation of the data. In particular there are large degrees of uncertainty both about r, the rank of $\boldsymbol{\beta}$, and for any given rank, the nature of the cointegrating relationships. Garratt *et al* (2003) conclude that the weight of the evidence is that $r = 5$; but, since alternative approaches to determining the rank can yield quite different values, there is sufficient uncertainty that it is of interest to see the implications of assuming different values. We briefly illustrate the impact of a range of assumptions regarding the rank of $\boldsymbol{\beta}$ on the properties of the transitory components. The nature of the exercise is deliberately atheoretical in the sense that it does not impose any restrictions on the matrix $\boldsymbol{\beta}$ except those required for exact identification. Thus we focus only on the impact of rank uncertainty.

We analyse a range of rank restrictions, $r = 0$ through to 7, where $\boldsymbol{\beta}$ is exactly identified. Note that the $r = 0$ case where no long run relationship exists provides us with a useful benchmark with which to compare the effect of imposing long run relationships. It also produces results very similar to the univariate Beveridge-Nelson decomposition.

Figure 1 graphs the transitory components in GDP for all eight exactly identified cases (we denote the exactly identified models of ranks 0 through to 7 as $Ex0, Ex1, ...Ex7$). The chart makes clear that an atheoretic approach to the multivariate B-N decomposition provides little or no guidance on the nature of transitory movements in UK GDP. The variance and even signs of the resulting transitory components are not robust to the rank we impose, so that in the absence of any further information, the "black box" approach essentially provides no useful insights at all.

3.3 Cointegrating Relations Based on Theory

Given that the atheoretic approach produces inconclusive results, we now turn to the impact of imposing cointegrating relationships based on theory. Garratt *et al.* (2003a) examine the impact of imposing five long-run relationships which were argued to be important to a small open economy like the UK. In brief, we note here that the five fundamental relationships that are assumed to result in stationary processes (and hence provide the structure for $\boldsymbol{\beta}$) are:

(i) Purchasing Power Parity (PPP), implying $p_t - p_t^* - e_t \sim I(0)$;
(ii) Interest Rate Parity (IRP), implying $r_t - r_t^* - \Delta e_t \sim I(0)$;
(iii) Convergence (CONV), implying $y_t - y_t^* \sim I(0)$;
(iv) Stable Real Money Demand (RMB) implying $h_t - y_t + \zeta_1 r_t + \zeta_2 t \sim I(0)$
(v) Fisher Interest Parity (FIP) implying $r_t - \Delta p_t \sim I(0)$;[22]

In contrast to the atheoretical exercise the attempt to relate the long run to explicit theory implies the presence of over-identifying restrictions. Figure 2 graphs the transitory component in UK GDP derived from the benchmark overidentified model($Ov5$) alongside the atheoretical exactly identified ($Ex5$). From the plot we observe only a limited degree of co-movement between the two series (with a correlation coefficient of only 0.36). The size of the deviations also differs significantly (the standard deviation for $Ov5$ and $Ex5$ are 2% and 3% respectively) and it is clear that imposing the long run restrictions has implications for output deviations over and above just imposing the required rank restriction. [23] Figure 3 graphs the level of UK output alongside its trend, or permanent component, derived using the benchmark model.

3.4 The Transitory Component of UK GDP and Economic Fundamentals

The great advantage of the benchmark restricted model is that it is possible to identify the link between the assumed underlying fundamental processes (which have clear economic interpretations) and the resulting transitory components. Equation (11) showed that the transitory components can be broken

[22] The structure of the model, with both $p_t - p_t^*$ and e_t assumed to be $I(1)$, implies that domestic and overseas inflation rates and interest rates can differ by at most a constant in steady state.

[23] The statistical counterpart to the significant difference between the two series is a rejection of the implied restrictions on a conventional likelihood ratio test (see Garratt *et al,* 2003 for a discussion, and a comparison with bootstrapped test statistics, which do not reject the restrictions).

down into the contribution of the cointegrating relations, and of inertial effects in growth rates. Figure 4 shows that for UK GDP, movements in the transitory component are dominated by movements in cointegrating relations: the role of short-run dynamics is very limited.[24]

Figure 5 provides a more detailed decomposition of the transitory component by breaking it down into the contributions of individual cointegrating relations. Probably the most striking feature of this chart is that there are large, but frequently offsetting contributions from the two cointegrating relations that include output itself, CONV and RMB. To accentuate this feature Figure 6 aggregates the impact of these two relationships: this shows that these two relationships account for most of the variation in the transitory component. The only other relationship that plays a significant role is the PPP relationship.

Figures 7 and 8 show the impact on the transitory and permanent components of excluding from the model any impact of the two most important cointegrating relations, CONV and RMB, in turn.[25] When any impact of the real money demand relationship is removed, both the resulting transitory and (innovations to) the permanent component become distinctly less volatile, while the reverse is the case when there is no role for the convergence relationship. We shall discuss these features in more depth below; but first we focus on the properties of the permanent component.

3.5 The Permanent Component of UK GDP

Figure 3 shows that the permanent component of UK GDP from the Garratt *et al* model is certainly not as "smooth" as trends derived by many black box techniques; indeed its growth rate is as volatile as that of output itself. It is also subject to some fairly significant downward, as well as upward shifts at various points of the sample. This can be explained relatively easily by exploiting our framework. Given the relative unimportance, in quantitative terms, of three out of the five cointegrating relations, and of disequilibrium growth rates shown in Figures 4 and 5, the implied process for trend output can, to a reasonable approximation, be written as

$$\Delta\widehat{y_t} \approx g_y + (1 - \lambda_1 - \lambda_2)\varepsilon_{yt} + \lambda_1\varepsilon_{y^*t} + \lambda_2\left(\varepsilon_{ht} + \zeta_1\varepsilon_{rt}\right) \qquad (21)$$

[24] Suggesting the presence of common cycles. However, it should be borne in mind that we are only examining the transitory component in one of the eight variables in the model.

[25] In each case $\boldsymbol{\alpha}$ and $\boldsymbol{\Phi}$ were reestimated with the relevant column of $\boldsymbol{\beta}$ deleted.

where g_y is the estimated deterministic trend growth rate of output, and ε_{xt} is the innovation to variable x in period t from the underlying VAR.[26] The model implies a value for λ_1 close to one half (0.54). Other things being equal, innovations to trend output would thus be a simple average of innovations to domestic output itself, and to overseas output (as in the case $0 < \zeta < 1$ in our bivariate example). Since overseas output (itself, of course an average) is distinctly smoother than UK output, this would imply that trend output would be distinctly smoother than output itself. However, the impact of the real money demand relationship more than offsets this effect. The implied value of λ_2 is negative (-0.07), capturing the fact that when there is a shock to real money balances this will boost current output (since, as shown in (11), the impact on the transitory component is of opposite sign) but output will be predicted to fall back again at an infinite horizon. This term acts to increase significantly the variability of the growth rate of the permanent component,[27] an issue we discuss further in the next section.

It is important to stress that the relative degree of noisiness of the permanent component of GDP is specific to the Garratt *et al* model, rather than to B-N trends in general. As an example, had Garratt *et al* included consumption as an additional variable in the VAR it is likely (for the reasons discussed above in relation to Cochrane, 1994) that the resulting trend would have been distinctly smoother, both due to the inclusion of an additional variable in the implicit averaging process in (21), and due to the smoothness of consumption itself.[28]

3.6 Interpretation

The differences in permanent and transitory components discussed above can be related to interpretable economic hypotheses. Our results suggest that two relationships: convergence and the real money demand relation, are

---

[26] We exploit the formula in (3) and the relationship (shown in the appendix) that $\mathbf{C}(1) = \mathbf{I} + \mathbf{\Phi}_\infty + \boldsymbol{\alpha}_\infty \boldsymbol{\beta}'$, with the relevant elements of $\mathbf{\Phi}_\infty$ set to zero due to the quantitative unimportance of disequilibrium growth rates. The λ_i are the three elements of $\boldsymbol{\alpha}_\infty$ that capture the long-run response of output to disequilibria in CONV, RMB and PPP, respectively. The adding up constraint on the coefficient on ε_{yt} arises from the restriction on the elements of $\boldsymbol{\alpha}_\infty$ in (12).

[27] In a bivariate case this would correspond to the case $\xi < 0$ in the analytical example of Section 2. Figure 8 also shows that when convergence is excluded from the model, leaving RMB dominant, the trend process becomes distinctly noisier.

[28] In an earlier draft of this paper we also included a second empirical example where we showed that, for certain representations, the permanent component of the US stock market was distinctly smoother than the stock market itself. A similar result is also found in Cochrane (*op cit*).

key; with purchasing power parity being the only other relationship of any significance for output.

The relative lack of importance for output of the other two relationships, international interest parity and Fisher interest parity, is consistent with standard small open economy assumptions. These would suggest, first that both real and nominal interest relations are determined by world markets, and second, as a result, that the role of domestic real interest rate movements in output fluctuations is quite small (as compared to the role of the real exchange rate).[29]

The important role of the convergence relationship is striking, since it suggests a strong link between two strands of macroeconomics - growth and fluctuations. Thus, Figures 5 and 7 show that the convergence relationship made a very large negative contribution to the transitory component in output during the 1970s and early 1980s. The implication was that, after the relatively slow growth of the UK in the 1960s and 1970s, compared to its competitors, the convergence relationship was, by the mid-1970s, expected to make a major contribution to UK growth in the 1980s and 1990s. This catching-up phase did indeed subsequently emerge. While this prediction is consistent with the convergence literature, its impact on shorter-term output fluctuations has been largely ignored. The assumption that relative output levels is a stationary process (which is fundamental to the convergence literature), and, crucially, has such important predictive power for domestic output, puts a very different interpretation on the transitory component to the standard "cycle", since the associated stationary processes are much more long-term in nature than typically assumed.

At the same time the model also suggests a quite significant "monetarist" interpretation. Taken at face value, the decomposition in Figure 5 implies a quite significant role for money (or nominal) shocks in fluctuations in UK output. However, we would argue that this conclusion should be treated with some scepticism.

First, the nature of the real money demand relationship is, in one crucial respect, very distinct, in econometric terms, from that of the other cointegrating relations, since it is the only one that contains any estimated cointegrating parameters. All four of the remaining long-run relationships imply cointegrating vectors of the "(1,-1)" variety, where the coefficients arise straightforwardly from very basic theory.[30] The nature of the associated hypotheses

[29] Note that these relationships are of considerably greater importance to the transitory components of other variables in the system.

[30] Three of the equilibrium relationships: convergence; international interest parity; and purchasing power parity, are effectively no arbitrage conditions. The fourth, Fisher interest parity, requires the absence of nominal illusion, coupled with some stability of intertem-

leaves no scope for the data to provide estimated cointegrating parameters (except for constant terms) under the null. In contrast, the nature of the hypothesis associated with the real money demand relationship is distinctly less constrained. Theory only predicts the sign, but not the magnitude, of the response of money demand to the nominal interest rate; nor does it rule out a role for a deterministic time trend. As a result, in this relationship, two parameters are chosen, not, as in the case of the other relationships, on the basis of theoretical priors, but on the basis of the resulting ability of the model to fit the data. There is therefore a potential data-mining critique of this relationship, that does not apply to the other four.

A second reason for scepticism arises from the characteristics of the implied process for trend output. This can be seen most clearly in Figure 8, which shows that when the convergence relationship is omitted, leaving the real money relationship dominant , this significantly increases the volatility of the trend. Notably, the trend shows some quite sharp falls, particularly in the early 1980s, a feature also evident in the benchmark model. This was a period when, as Figure 5 shows, the transitory component of UK output was apparently being strongly boosted by the strength of real money balances. While this might appear plausible in qualitative terms, Figure 8 makes clear that the quantitative effect is so strong that the strength of the impact on the transitory component in this period more than accounts for the strength of output - thus resulting in a fall in the implied trend.

Given these caveats, we would hesitate before concluding that disequilibrium in money demand has clearly had as important a role in UK output fluctuations as the benchmark model would suggest.[31]

The only other factor that plays a significant role in transitory output fluctuations is the PPP relationship. At certain points, deviations of the real exchange rate from its estimated (constant) equilibrium value imply a contribution to output fluctuations that is non-trivial. Thus in the early 1980s the strength of sterling (which the PPP relationship implies must have been a transitory phenomenon) implied that output was depressed below its long-run trend value; by a maximum amount of around 2% in 1981 when sterling was at its strongest. Perhaps more surprising is the implied role of the real exchange rate in more recent output fluctuations. During the ERM period of the early 1990s, sterling was widely regarded as over-valued; the sharp devaluation at the end of 1992 was at the time regarded as restor-

poral preference parameters.

[31] One area that would repay further investigation is whether the apparent relative importance of this relationship would persist if econometric techniques were employed that were less prone to a data-mining critique: for example, recursive estimation. It would also be interesting to discover if this feature were robust to alternative measures of money.

ing a more sustainable real value of sterling. Figure 6 provides a distinctly different interpretation. The contribution of the PPP relationship to the transitory component of output at the start of the 1990s was essentially zero, for the simple reason that, on the basis of model estimates, sterling was at its equilibrium in terms of PPP. The subsequent devaluation shows in the chart as a move *away* from equilibrium, with the PPP relationship making a distinctly positive contribution to the transitory component of output during the mid-1990s. This contribution must, of its nature, be interpreted as transitory; and indeed, by the end of the 1990s the model suggests that it had evaporated to virtually zero, with sterling, by implication, again at its equilibrium in terms of PPP.

At the very end of the sample period, in 1999, the benchmark model suggests that output was nearly 2% above trend: largely accounted for by the convergence relationship (UK output having by then grown distinctly faster than overseas output for a number of years), with some negative offsetting impact from real money demand. Figure 7 shows that, as on many other occasions, this conclusion is however quite sensitive to the assumptions on the underlying fundamental stationary processes. The chart shows that when the convergence relationship is omitted from the model, output is estimated to have been below, rather than above trend at this time (a feature shared with a number of the atheoretical transitory components plotted in Figure 1). But, as we have argued above, this degree of ambiguity is inevitable. *If* we assume that convergence results in a stationary process for relative output levels, then the nature of output fluctuations must of necessity be different from a world in which we assumed it was not. The lack of robustness regarding the nature of transitory fluctuations cannot be separated from the nature of the underlying fundamental stationary processes that drive the economy.

4 Conclusions

In this paper we present a new derivation of multivariate Beveridge-Nelson trends from the cointegrating vector autoregressive representation, that allows us to relate movements in permanent (or trend) and transitory components directly to the underlying stationary processes. We interpret B-N trends as conditional cointegrating equilibrium values, and show how the nature of the permanent and transitory components can be related to the nature of the error correction process, at both finite and infinite horizons.

We have argued that the role of theory is crucial in suggesting equilibrium relationships; but equally econometric evidence is crucial in revealing the nature of the adjustment towards equilibrium. Neither theory nor econometric evidence can eliminate uncertainty about the nature of permanent and transitory components; but, as our empirical example has demonstrated, both are crucial in clarifying the nature of this uncertainty. As such, we have argued that an approach to the derivation of permanent and transitory components that is based on the analysis of fundamental stationary processes has distinct advantages over the atheoretical (and typically univariate) detrending processes that are still very widely applied. We do not claim that it can provide clear-cut answers to the questions that such atheoretical approaches leave entirely unanswered; but we do claim that it at least provides a coherent framework in which those questions can be investigated.

Appendix

A Equivalence of Trend Definition in (10) to the Standard Multivariate Beveridge-Nelson Definition in (3)

The BN definition of the trend from equation (3) is

$$\mathbf{\Delta}\widehat{\mathbf{x}}_t = \mathbf{g} + \mathbf{C}(1)\boldsymbol{\varepsilon}_t$$

Using the notation of the text we have $\mathbf{\Delta x}_t - \mathbf{g} = \mathbf{J}\widetilde{\mathbf{y}}_t$ and from (7) we have

$$\begin{aligned} \widetilde{\mathbf{y}}_t &= \mathbf{A}\widetilde{y}_{t-1} + \mathbf{K}\boldsymbol{\varepsilon}_t \\ &= (\mathbf{I} - \mathbf{A}L)^{-1}\mathbf{K}\boldsymbol{\varepsilon}_t \end{aligned}$$

where $\mathbf{K} = \begin{bmatrix} \mathbf{I} & \boldsymbol{\beta}' \end{bmatrix}'$. So

$$\mathbf{\Delta x}_t = \mathbf{g} + \mathbf{J}(\mathbf{I} - \mathbf{A}L)^{-1}\mathbf{K}\boldsymbol{\varepsilon}_t$$

hence

$$\mathbf{\Delta}\widehat{\mathbf{x}}_t = \mathbf{g} + \mathbf{J}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{K}\boldsymbol{\varepsilon}_t.$$

From (9) our expression for the trend is

$$\widehat{\mathbf{x}}_t = \mathbf{x}_t + \mathbf{J}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{A}\widetilde{\mathbf{y}}_t$$

(since $\mathbf{A}(\mathbf{I} - \mathbf{A})^{-1} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{A}$) so that

$$\begin{aligned} \mathbf{\Delta}\widehat{\mathbf{x}}_t &= \mathbf{\Delta x}_t + \mathbf{J}(1 - L)(\mathbf{I} - \mathbf{A})^{-1}\mathbf{A}\widetilde{\mathbf{y}}_t \\ &= \mathbf{g} + \mathbf{J}\widetilde{\mathbf{y}}_t + \mathbf{J}(1 - L)(\mathbf{I} - \mathbf{A})^{-1}\mathbf{A}\widetilde{\mathbf{y}}_t \\ &= \mathbf{g} + \mathbf{J}\left(\mathbf{I} + (1 - L)(\mathbf{I} - \mathbf{A})^{-1}\mathbf{A}\right)\widetilde{\mathbf{y}}_t \\ &= \mathbf{g} + \mathbf{J}\left(\mathbf{I} + (1 - L)(\mathbf{I} - \mathbf{A})^{-1}\mathbf{A}\right)(\mathbf{I} - \mathbf{A}L)^{-1}\mathbf{K}\boldsymbol{\varepsilon}_t \\ &= \mathbf{g} + \mathbf{J}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{K}\boldsymbol{\varepsilon}_t \end{aligned}$$

since it is easily shown that $\left(\mathbf{I} + (1 - L)(\mathbf{I} - \mathbf{A})^{-1}\mathbf{A}\right)(\mathbf{I} - \mathbf{A}L)^{-1} = (\mathbf{I} - \mathbf{A})^{-1}$ by premultiplying by $(\mathbf{I} - \mathbf{A})$ and post multiplying by $(\mathbf{I} - \mathbf{A}L)$. Thus the two definitions are equivalent.

Note that this implies

$$\begin{aligned}\mathbf{C}(\mathbf{1}) &= \mathbf{J}(\mathbf{I}_{n+r}-\mathbf{A})^{-1}\mathbf{K} \\ &= \mathbf{I}_n+\mathbf{J}\mathbf{A}(\mathbf{I}_{n+r}-\mathbf{A})^{-1}\mathbf{K} \\ &= \mathbf{I}_n+\left[\begin{array}{cc}\mathbf{\Phi}_\infty & \boldsymbol{\alpha}_\infty\end{array}\right]\mathbf{K} \\ &= \mathbf{I}_n+\mathbf{\Phi}_\infty+\boldsymbol{\alpha}_\infty\boldsymbol{\beta}^{'}\end{aligned}$$

(a feature that, given the equivalence of the two trend definitions, can also be seen directly by deriving the innovations to the right-hand side of (9)) so that, using (12) and (13), it follows that $\boldsymbol{\beta}^{'}\mathbf{C}(1)=\mathbf{0}$

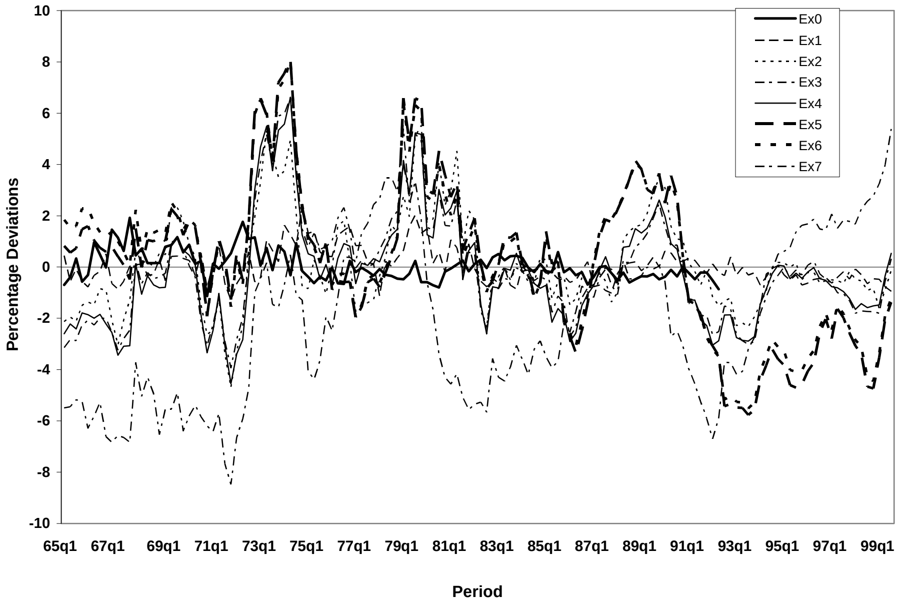

Figure 1: Transitory Component of UK GDP from Exactly Identified Models of Different Rank.

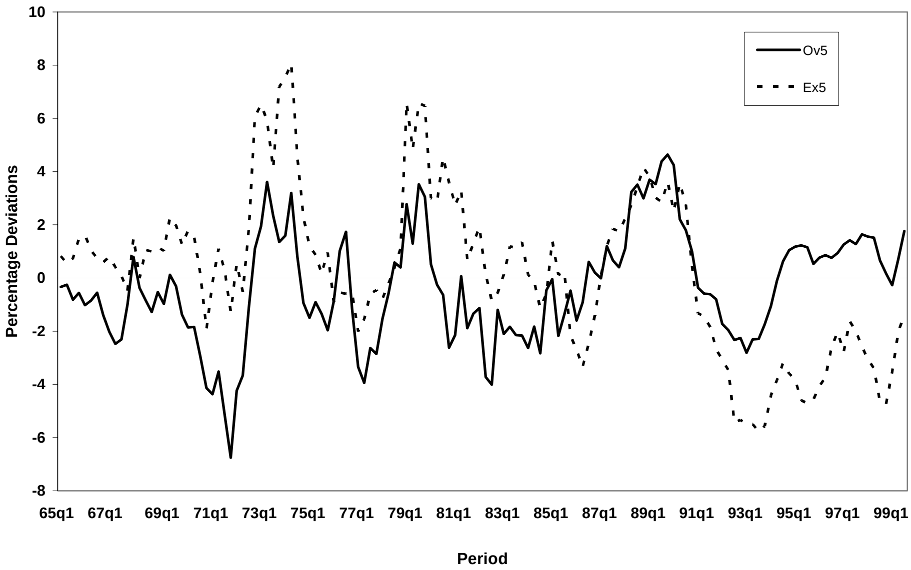

Figure 2: Transitory Component of UK GDP with ($Ov5$) and without ($Ex5$) Long-Run Theory.

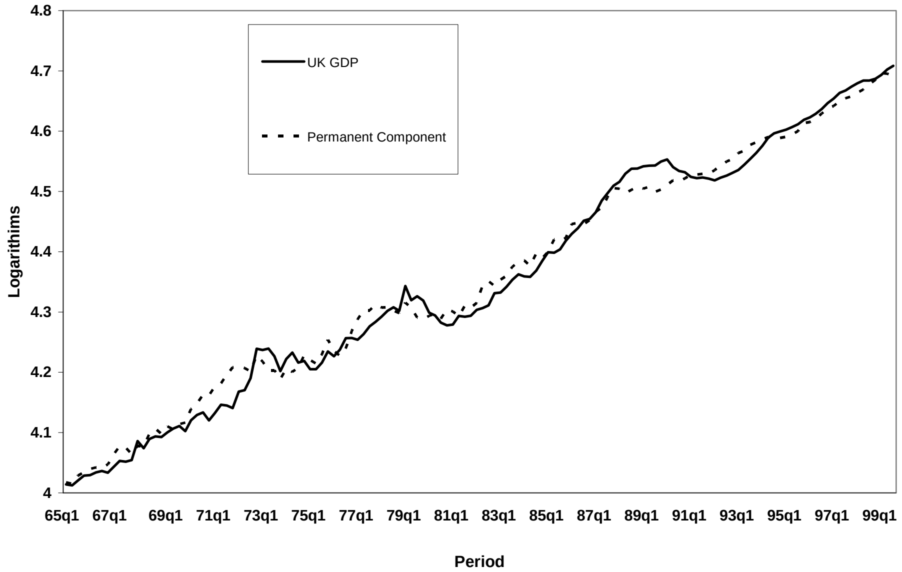

Figure 3: UK GDP and Permanent Component Derived From the Garratt *et al* Model.

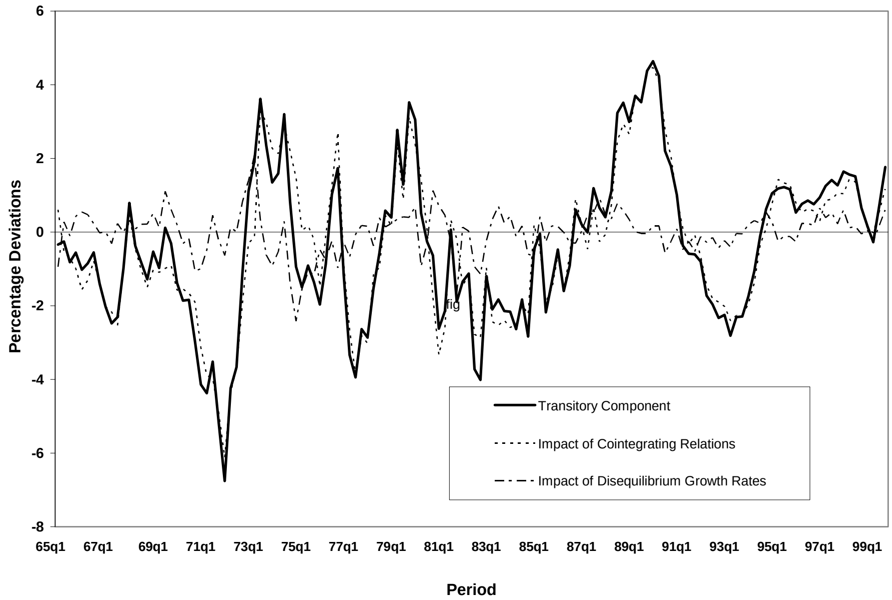

Figure 4: Impact on Transitory Component of UK GDP of Cointegrating Relations & Disequilibrium Growth Rates

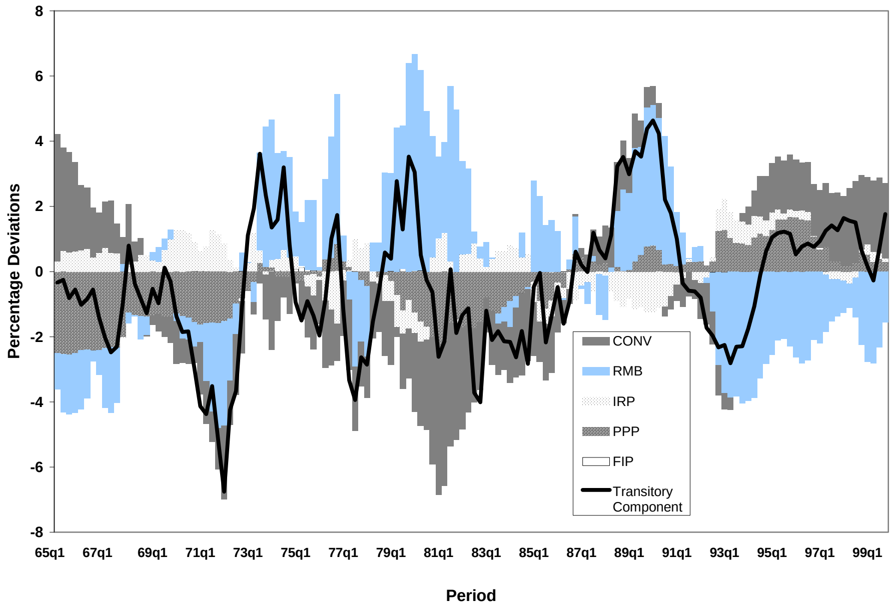

Figure 5: Impact of Cointegrating Relations on the Transitory Component of UK GDP

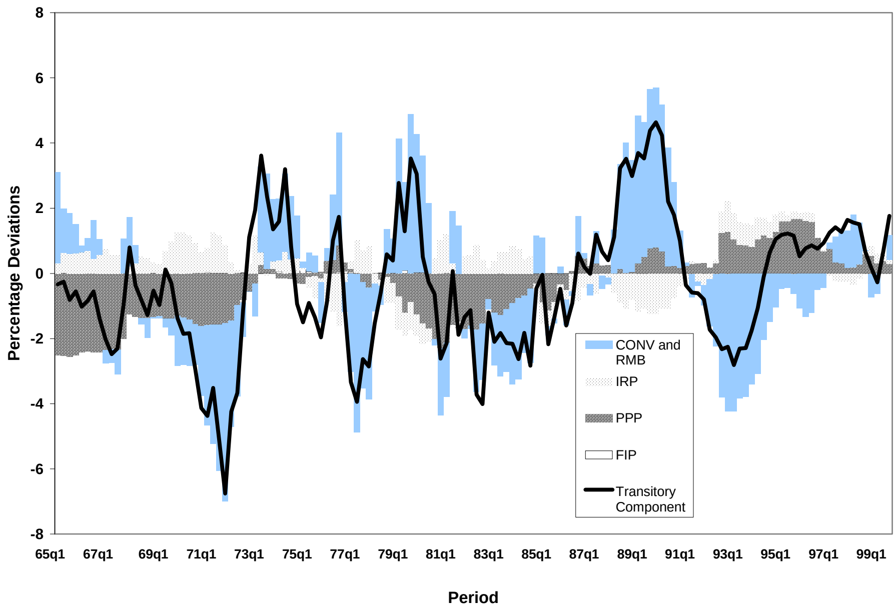

Figure 6: The Combined Impact of Cointegrating Relations Including Output

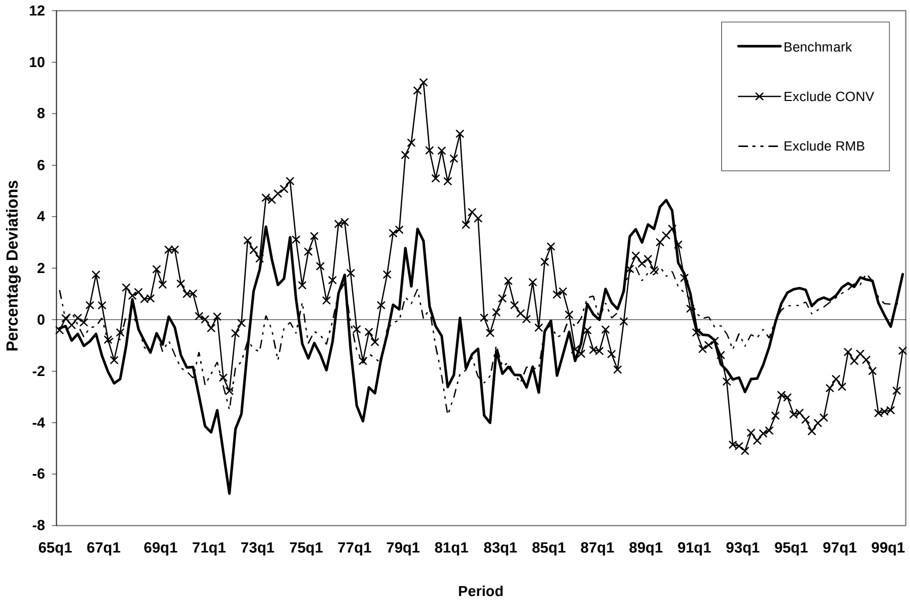

Figure 7: Impact on the Transitory Component of UK GDP of Excluding Cointegrating Relations.

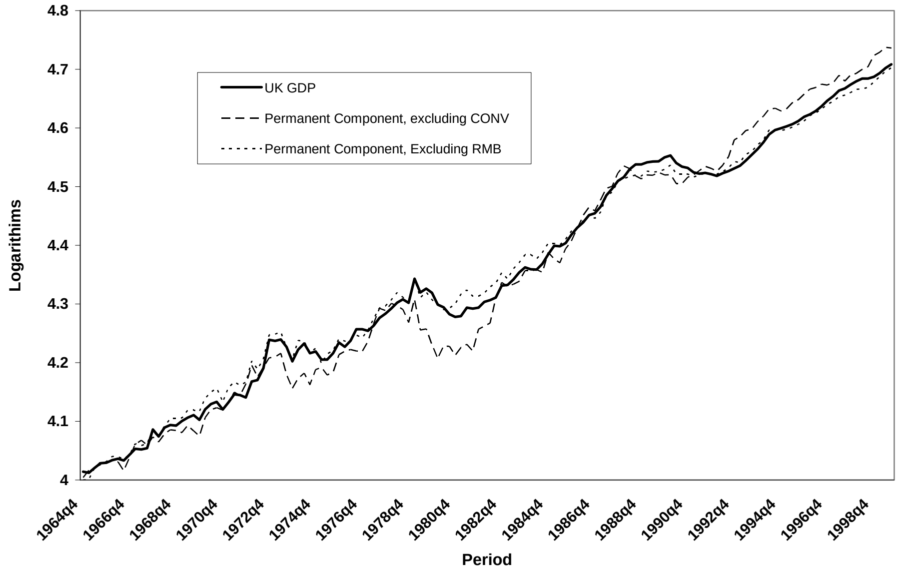

Figure 8: Impact on the Permanent Component of UK GDP of Excluding Cointegrating Relations.

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