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Buiter, W. and Sibert, Anne (2004) Cross-border tax externalities: are budget deficits too small? Working Paper. Birkbeck, University of London, London, UK.

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Cross-Border Tax Externalities: Are Budget Deficits Too Small?

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March 2004

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30 March 2004

Abstract

In a dynamic optimising model with costly tax collection, a tax cut by one nation creates positive externalities for the rest of the world if initial public debt stocks are positive. By reducing tax collection costs, current tax cuts boost the resources available for current private consumption, lowering the global interest rate. This pecuniary externality benefits other countries because it reduces the tax collection costs for foreign governments of current and future debt service. In the non-cooperative equilibrium, nationalistic governments do not allow for the effect of lower domestic taxes on debt service costs abroad. Taxes are too high and government budget deficits too low compared to the global cooperative equilibrium. Even in the cooperative equilibrium complete tax smoothing is not optimal: current taxes will be lower than future taxes.

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1. Introduction

Does one country's borrowing in an integrated global financial market impose externalities on other countries? If so, are these spillovers welfare-enhancing or welfare-reducing? This issue figures prominently in the debates about the European Union's Stability and Growth Pact and the merits of G3 policy coordination. In this paper we consider cross-border externalities associated with the transmission of national public debt policies through their effect on the global risk-free real interest rate.[1]

We provide a dynamic equilibrium model with optimising households and governments, in which public debt and the government's intertemporal budget constraint provide a link between current and future tax decisions. Such models are analytically difficult, especially if they do not exhibit Ricardian equivalence or debt neutrality. This has led us to focus on the simplest possible supply side for the national economies (a perishable endowment), a representative infinite-lived consumer with log-linear preferences and a simple source of Ricardian nonequivalence, or absence of debt neutrality: *fiscal transfer costs.* We assume that there are increasing and strictly convex real resource costs of administering and collecting taxes.[2,3] We then extend the benchmark model to allow for CES preferences and a production technology.

We focus on real interest rate cross-border spillovers that occur in the absence of

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[1]Another externality is that associated with either the occurrence of sovereign debt default or with actions undertaken by the debtor country or by others to prevent sovereign defaults.

[2]Slemrod and Yitzhaki (2000) report that the administrative cost of the US tax system is 0.6 cents per dollar of revenue raised. Slemrod (1996) estimates compliance costs to be about 10 cents per dollar collected.

[3]Barro (1979) pioneered these strictly convex fiscal transfer costs in a closed-economy setting. He assumed that the government minimises the discounted sum of these costs, rather than maximises the discounted welfare of households.

sovereign default risk and without strategic interactions between a national fiscal authority and a national or supranational monetary authority. We model a non-monetary economy in which every national fiscal authority satisfies its intertemporal budget constraint. Government spending on goods and services is exogenous. We assume that each government can commit to a path of taxes, taking other governments' taxes as given. Thus, there is commitment, but no international cooperation. We show this noncooperative behaviour results in inefficient global equilibria.

We assume that resources are always fully utilised. Financial capital is perfectly mobile across countries. International transmission of national fiscal policy is only through interest rates. Taxes are *lump-sum*; their incidence cannot be altered by changing private behaviour, but because of the strict convexity of the fiscal transfer costs, the timing of taxes matters in this model, just as it would with conventional distortionary taxes on labour income or asset income in models with endogenous labour supply and capital accumulation. With negative taxes, or subsidies, resource costs result from private rent-seeking behaviour. We model the tax collection costs as borne by the public sector. Allowing for compliance costs borne by the private sector would add notational complexity without changing our qualitative conclusions.

Without fiscal transfer costs our model, with its representative private agent, would exhibit Ricardian equivalence: any sequence of lump-sum taxes and debt that satisfies the intertemporal budget constraints would support the same equilibrium for any given sequence of public spending on goods and services. There would be no international spillovers. This is true even if a country is large in the world capital market and exploits its monopoly power.

If we had assumed overlapping generations, instead of a representative agent, then alternative rules for financing a given public spending programme would cause pure *pecuniary* externalities if there were no fiscal transfer costs and taxes were lump sum. Even with symmetric countries, there could be distributional effects between generations, but as long as dynamic

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inefficiency does not occur, any feasible sequence of lump-sum taxes and debt supports a Pareto efficient equilibrium.[4]

In the one-country special case of our model, fiscal transfer costs do not cause inefficiency if one assumes that resource transfers between the private and public sector in the counterfactual command economy are subject to the same fiscal transfer costs as in our market economy. Inefficiency arises when there are multiple countries and each country affects other countries' choice sets in a way that is not adequately reflected in market prices.

With symmetric countries and representative, infinite-lived consumers, symmetric tax policies have no distributional effects. However, they can have welfare consequences if they change the world interest rate. If, for example, countries have outstanding stocks of debt and a change in policy causes the interest rate to rise, then countries must raise taxes, now or in the future, and fiscal transfer costs increase. National governments that maximise their own resident's welfare do not internalise the cost of the higher fiscal transfer costs to other countries. Thus, a national government's financing decision that raises the world interest rate inflicts a negative externality on the rest of the world. This is in line with conventional wisdom.

Where our model departs radically from conventional wisdom is through the mechanism by which financing choices affect interest rates. It is conventional to associate deficit financing of public spending with 'financial crowding out'. That is, for a given public spending programme, larger bond-financed deficits brought about by lower taxes raise interest rates. In our thoroughly neoclassical intertemporal model the opposite is true. Lower taxes and larger deficits early on result in a *lower* global rate of interest.

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[4]See Buiter and Kletzer (1991). If there is dynamic inefficiency, then fiscal policy that causes redistribution from the young to the old can lead to a Pareto improvement. With asymmetric countries, alternative deficit-financing policies would have international, as well as intergenerational, distributional implications.

We show that if a government is too small to affect the global interest rate, it minimises the costs of collecting taxes by smoothing them over time. If it is able to influence the interest rate and has positive initial debt, then it sets a lower tax in the initial period than in future periods. This is because lower fiscal transfer costs in the initial period than in later periods imply higher aggregate consumption in the initial period than in later periods. Thus, the real interest rate in the initial period is lower than with perfectly smooth taxes and this lowers the interest payment on the government's outstanding debt and, hence, lowers future fiscal transfer costs.

Relative to the global (cooperative) optimum, noncooperative countries tax too much and issue too little debt in the initial period. Reducing current taxes has a positive welfare spillover, even though it requires issuing more debt. Lowering the current interest rate by lowering current taxes lowers the cost of servicing all countries' debt and thus reduces all countries' need to collect costly taxes. In a noncooperative equilibrium, countries do not take into account this benefit to other countries and they tax too much in the initial period.

Our conclusion that lack of international cooperation leads to taxes that are initially too high and public deficits that are initially too small seems to contradict the presumption reflected in the debt and deficit ceilings of the Stability and Growth Pact that deficits are apt to be too large. However, we do not want to make too much of the size of the externalities associated with alternative tax and borrowing policies of national governments in EMU; even the larger EMU countries are small fish in the global financial pond. Our analysis is more relevant to interaction between the United States, the European Union as a whole, possibly Japan and soon China.

Our paper analysing the welfare economics of international interest rate spillovers from the tax and borrowing strategies of national governments using a dynamic optimising general equilibrium model is related to the vast literature on other international fiscal policy linkages, the more modest literature on the political economy of the timing of taxes in closed economies

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and the sizable literature on the optimal timing of multiple distortionary taxes in a closed economy.

The literature on the international transmission of fiscal policy has two main strands. First, there is the work on the transmission of government-expenditure shocks with lump-sum taxes and without fiscal transfer costs. Examples are Frenkel and Razin (1985, 1987) and Turnovsky (1988); Turnovsky (1997) provides a survey. The papers in this vein are in sharp contrast to ours. We take government expenditure as exogenous and ask how the financing matters when there are fiscal transfer costs. Second, there are papers on the transmission of tax shocks in models with distortionary taxes in a balanced-budget setting. There is a sizable literature – going back to Hamada (1966) – on the strategic taxation of capital income in a world economy. In this literature, capital-exporting (importing) country can increase its national income by acting as a monopolist (monopsonist) and restricting capital movements. The result is a Nash equilibrium where nations want to tax capital flows. Other papers consider issues of the feasibility of different tax regimes in an integrated world economy, tax harmonisation and tax competition. Examples of such papers are Sinn (1990) and Bovenberg (1994).

In the closed-economy political economy literature, excessive public deficits and debt may result from a political party's desire to tie the hand's of a possible successor (Persson and Svensson (1989) and Alesina and Tabellini (1990)), an incumbent government's incentive to signal its competency prior to an election (Rogoff and Sibert (1988)), or a war-of-attrition game over the allotment of the costs of fiscal adjustment (Alesina and Drazen (1991)).[5] Drazen (2000) provides a discussion of this literature. In this paper, we abstract from political economy concerns; governments are able to commit to policies which maximise national welfare.

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[5]Tabellini (1990) shows that international policy coordination worsens the outcome in Alesina and Tabellini (1990).

Chamley (1981, 1986) pioneered the normative study of dynamic optimal taxation in a closed economy setting, when the government can borrow or lend. He focuses on the choice between distortionary capital and labour income taxation and does not consider fiscal transfer costs of the kind studied here. The state's optimal policy is to impose the maximum possible capital levy on the private sector's initial, predetermined stocks of capital and public debt and then to switch permanently to a zero capital income tax rate (see also Lucas (1988)). Incorporating fiscal transfer costs of the kind considered here would render Chamley's highly uneven time profile of tax receipts suboptimal.

In section 2 we present the model. In section 3 we extend the model to consider small changes in the households' intertemporal elasticity of substitution. We show that as this elasticity falls, the deviation between cooperative and noncooperative taxes rises. In section 4 we consider a production economy and CES preferences. We show that if the world economy is at a steady state with positive debt, a coordinated reduction in the current tax financed by higher future taxes improves welfare. Section 5 concludes.

2. The Model

The model comprises $N \geq 1$ countries, each inhabited by a representative infinite-lived household and a government. Each period, each household receives an endowment of the single private tradeable, non-storable consumption good and each government purchases an exogenous amount of the private good. Governments finance their purchases by issuing debt or by taxing their resident households. We assume that the tax system is costly to administer; governments use up real resources collecting taxes. All savings are in the form of privately or publicly issued real bonds. We assume that households are symmetric and that endowments and government purchases are constant over time. There is perfect international integration of the national financial markets, and hence, a common world interest rate.

2.1 The households

The country-*i* household, $i = 1,...,N$, has preferences over its consumption path given by

$$u^i = \sum_{t=0}^{\infty} \beta^t \ln c_t^i, \tag{1}$$

where c_t^i is its period-*t* consumption and $\beta \in (0,1)$ is its discount factor.

The household's period-*t*, $t = 0,1,...$, budget constraint is

$$c_t^i + a_{t+1}^i = W - \tau_t^i + R_t a_t^i, \tag{2}$$

where a_t^i is the household's stock of assets in the form of real bonds at the start of period *t*, R_t is (one plus) the interest rate between period *t* - 1 and period *t*, *W* is the household's per-period endowment of the good and τ_t^i is its time-*t* tax bill. The household's initial assets, a_0, are given.

In addition to satisfying its within-period budget constraint, the household must satisfy the long-run solvency condition that the present discounted value of its assets be non-negative as time goes to infinity. The transversality condition associated with its optimisation problem ensures that the present discounted value of its assets is not strictly positive. Thus,

$$\lim_{t \to \infty} \left(a_{t+1}^i / \prod_{s=0}^{t} R_s \right) = 0. \tag{3}$$

Equations (2) and (3) imply that the present discounted value of the household's consumption equals the present discounted value of its (after-tax) income plus its initial assets:

$$a_0 + \sum_{t=0}^{\infty} (W - \tau_t^i) / \prod_{s=0}^{t} R_s = \sum_{t=0}^{\infty} c_t^i / \prod_{s=0}^{t} R_s. \tag{4}$$

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The household chooses its consumption path to maximise its utility function (1) subject to its intertemporal budget constraint (4). The solution satisfies the Euler equation

$$c_{t+1}^i = \beta R_{t+1} c_t^i, \quad t = 0,1,\dots \tag{5}$$

Solving the difference equation (5) yields the household's time-t consumption as a function of its initial consumption and the t-period interest factor

$$c_t^i = \beta^t \left(\prod_{s=1}^t R_s \right) c_0^i, \quad t = 1,2,\dots \tag{6}$$

Substituting equation (6) into equation (4) yields the household's initial consumption as a function of its taxes and the interest factors

$$c_0^i = (1 - \beta)\left[R_0 a_0 + W - \tau_0^i + \sum_{t=1}^4 (W - \tau_t^i)/\prod_{s=1}^t R_s \right]. \tag{7}$$

Substituting equation (6) into equation (1) yields the household's indirect utility as a function of initial consumption and the interest factors

$$u^i = \ln c_0^i + (1-\beta)\sum_{t=1}^4 \beta^t \ln\left(\prod_{s=1}^t R_s \right), \tag{8}$$

where constants that do not affect the household's optimisation problem are ignored.

2.2 The government

The country-i, $i = 1,\dots,N$, government's period-t, $t = 0,1,\dots$, budget constraint is

$$\tau_t^i - (\varphi/2)\tau_t^{i^2} + b_{t+1}^i = G + R_t b_t^i, \tag{9}$$

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where b_t^i is the government's outstanding debt at the start of period t and $G > 0$ is its per-period purchase of the good. The fiscal transfer cost associated with a tax τ or a surplus $-\tau$ is $(\varphi/2)\tau^2$, where $\varphi > 0$. The government's initial debt (or credit, if negative), b_0, is given. We restrict the model's parameters so that satisfying equation (9) is feasible; the restrictions are detailed later in this section.

In addition to satisfying its within-period budget constraint, the government also satisfies

$$\lim_{t \to \infty} \left(b_{t+1}^i \Big/ \prod_{s=0}^{t} R_s \right) = 0. \tag{10}$$

As with the household, this is an implication of the long-run solvency constraint and the transversality condition associated with the government's optimisation problem.

Equations (9) and (10) imply that the present discounted value of the government's purchases, plus its initial debt, equals the present discounted value of its tax stream, net of collection costs:

$$\tau_0^i - (\varphi/2)\tau_0^{i2} + g_0 + \sum_{t=0}^{\infty} [\tau_t^i - (\varphi/2)\tau_t^{i2} + g_t] \Big/ \prod_{s=1}^{t} R_s = 0,$$

$$\text{where } g_t := \begin{cases} G + R_0 b_0^i, & t = 0 \\ G, & t > 0. \end{cases} \tag{11}$$

2.3 Market clearing

Market clearing requires that the sum of the N households' asset holdings equals the sum of the N governments' debt. Thus,

$$a_t = b_t, \tag{12}$$

where variables without a superscript denote global averages.

The global resource constraint requires that the sum of total household consumption, total government purchases and total fiscal transfer costs equals total endowments. Thus,

$$c_t = W + G + \frac{\varphi}{2N}\sum_{j=1}^{N}\tau_t^{j2}, \quad t = 0,1,\dots \quad . \tag{13}$$

Equation (13) is, of course, also implied by equations (2), (9) and (12).

Averaging both sides of the Euler equation (6) over the N countries yields

$$\prod_{s=1}^{t}R_s = c_t/(\beta^t c_0), \quad t = 1,2,\dots. \tag{14}$$

Equations (13) and (14) imply that in equilibrium, the time-t interest factor is solely a function of time-0 and time-t taxes.

Equations (13) and (14) imply that lower time-zero taxes financed by higher time-one taxes lower the interest rate between period zero and period one. Does the timing of taxes affect the global risk-free interest rate in the manner that this model predicts? There is a large body of empirical work attempting to quantify the relationship between interest rates and budget deficits.[6] However, it is problematic and the results are hard to interpret for several reasons. First, both taxes and interest rates are endogenous and an apparent relationship between them may be due to the influence of other variables. For example, automatic stabilisers cause tax revenue to be lower and deficits to be higher during recessions. At the same time, an expansionary monetary policy may temporarily lower real interest rates. Thus, the role of monetary policy over the business cycle may cause deficits and real interest rates to be negatively correlated. Second, while lowering taxes may lower the global *risk-free* interest rate, it may also increase sovereign-

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[6]A recent example is Laubach (2003).

default risk premia. If the effect on the risk premium is larger than the effect on the risk-free interest rate, it will cause tax decreases to be associated with higher measured market interest rates. Third, if tax cuts in a country include lower capital taxes, then the country's before-tax interest rate may fall to equate after-tax interest rates. This causes lower taxes to be associated with lower (before-tax) interest rates. Fourth, the scenario here has lower taxes in the current period accompanied by higher future taxes and unchanged government spending. However, households may interpret lower taxes today as a signal of a change in the government's attitude toward fiscal policy and they may expect lower taxes in the medium-run as well. In empirical studies, it is difficult to control for the public's expectation of future tax policy and its beliefs about future spending.

Substituting equation (14) into equation (8) gives the country-*i* household's indirect utility as a function of its initial consumption and the path of aggregate consumption:

$$u^{i} = \ln c_0^{i} + \beta \ln c_0 - (1 + \beta) \sum_{t=1}^{4} \beta^t \ln c_t. \qquad \textbf{(15)}$$

Substituting equations (12) and (14) into equation (7) gives the household's initial consumption as a function of the path of taxes. The predetermined value of initial government debt enters as a parameter.

$$c_0^{i} = (1 + \beta) c_0 \sum_{t=0}^{4} \beta^t (w_t + \tau_t^{i}) / c_t,$$

$$\text{where } w_t := \begin{cases} W - R_0 b_0, & t = 0 \\ W, & t > 0. \end{cases} \qquad \textbf{(16)}$$

Substituting equation (16) into the indirect utility function (equation (15)) yields

$$u^i = \ln\left[\sum_{t=0}^{\infty} \beta^t (w_t - \tau_t^i)/c_t\right] + (1-\beta)\sum_{t=0}^{\infty} \beta^t \ln c_t. \tag{17}$$

Substituting equation (14) into equation (11) yields

$$\sum_{t=0}^{\infty} \beta^t s_t^i = 0, \text{ where } s_t^i = \frac{\tau_t^i - (\varphi/2)\tau_t^{i\,2} - g_t}{c_t}, \ t = 0,1,\ldots. \tag{18}$$

Substituting the global resource constraint (equation (13)) into equations (17) and (18) would allow the household's indirect utility and the government's budget constraint to be expressed solely as functions of the paths of the taxes in the N countries.

2.4 Taxes and revenues

We impose further restrictions on the parameter space to ensure an equilibrium exists:

$$\max\{G,g_0\} \leq 1/(2\varphi), \ w_0 \geq 1/\varphi, \ W - G > 2/\varphi, \ g_0 > 0. \tag{19}$$

The net tax revenue function, $\tau - (\varphi/2)\tau^2$ looks like a Laffer curve, although its shape is the result of tax collection costs and not the distortions associated with non-lump sum taxes. Net revenue has a maximum of $1/(2\varphi)$ at $\tau = 1/\varphi$. Thus, first inequality in assumption (19) ensures balanced budgets are feasible.

The time-0 budget is balanced at $\tau_0 = \tau_0^- := (1 - \sqrt{1 - 2\varphi g_0})/\varphi$ or $\tau_0 = \tau_0^+ := (1 + \sqrt{1 - 2\varphi g_0})/\varphi$. Likewise, the time-$t$, $t>0$, budget is balanced with taxes of $\tau^- := (1 - \sqrt{1 - 2\varphi G})/\varphi$ or $\tau^+ := (1 + \sqrt{1 - 2\varphi G})/\varphi$. The taxes τ_0^- and τ^- are on the "right", or upward-sloping part of the time-0 and time-t, $t > 0$, net tax revenue curves, respectively. The taxes τ_0^+ and τ^+ are on the "wrong" or downward-sloping parts. There is a conventional government budget surplus in country i in period 0 if and only if $\tau_0^i \in [\tau_0^-, \tau_0^+]$ and a primary (that is, net of interest payments)

surplus in period t if and only if $\tau^i \in [\tau^\&, \tau^\%]$.

By the resource constraint, equation (13), the upper bound on feasible taxes in a symmetric equilibrium is $\bar{\tau} := \sqrt{(2/\varphi)(W \& G)}$. Assumption (19) implies that the taxes $\tau_0^\%$ and $\tau^\%$ are strictly less than $\bar{\tau}$, and hence are feasible. By equations (13) and (18), a symmetric equilibrium with constant taxes has taxes of $(1 \pm \sqrt{1 \& 2\varphi[G \% (1 \& \beta)R_0 b_0]})/\varphi$. By assumption (19), this is feasible.

We allow for negative taxes, or subsidies. In this case the collection cost is the cost of administering and disbursing the surplus. We rule out, however, the empirically implausible case of an initial stock of credit that is so large that the government can achieve a balanced budget (including interest payments) in period zero with a subsidy. The necessary condition for this, $g_0 > 0$, is included in (19).

The variable s_t^i in equation (18) is the period-0 value of the government's time-t budget surplus (or deficit, if negative), divided by c_0. For $t = 0$, this surplus is the total surplus and for periods $t > 0$ it is the primary surplus. We will refer to s_t^i as country i's *discounted time-t surplus*.

From the government's budget constraint, equation (18), it appears that a rational government with market power may set taxes on the "wrong" side of the net tax revenue curve, that is, at a tax higher than the net revenue maximising tax $1/\varphi$. To see this, suppose $\tau_t^i \ge 1/\varphi$. With market power, a country can influence the global interest factor. Holding other taxes constant, a marginal rise in τ_t^i causes aggregate period-t consumption to fall. As net revenues are insensitive to taxes at $1/\varphi$, they are unaffected by a marginal increase in the tax at this point. Thus, a marginal increase in τ_t^i above $1/\varphi$ causes the discounted time-t surplus to rise.

We have the following result. All proofs of propositions are in the Appendix.

Proposition 1. *Given τ_t^j, $j \ldots i$, s_t^i has a unique maximum in τ_t^i. The maximising tax is decreasing in the number of countries; as $N \to \infty$ it goes to $1/\varphi$. If $t > 0$, the maximising tax is an element of $[1/\varphi, \tau^\%]$*

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and s_t^i is increasing (decreasing) below (above) the maximising tax on $[\tau^{\&}, \tau^{\%}]$. *If $t = 0$, the maximising tax is an element of* $[1/\varphi, \tau_0^{\%}]$ *and s_0^i is increasing (decreasing) below (above) the maximising tax on* $[\tau_0^{\&}, \tau_0^{\%}]$.

Denote the tax that maximises s_t^i when taxes are symmetric and there are N countries by $\tau_0^{(N}$ if $t = 0$ and by $\tau^{(N}$ if $t > 0$. If $N = 1$ and taxes are symmetric then the maximising tax is $w_0 \& \sqrt{w_0^2 \& \bar{\tau}^2}$ if $t = 0$ and it is $w \& \sqrt{w^2 \& \bar{\tau}^2}$ if $t > 0$. The relationship between important time-t values is shown in Figure 1. The position of the corresponding time-0 values is similar.

We now show that it cannot be part of an equilibrium for any government to ever set a tax above the one that maximises its discounted surplus. The strategy of the proof is to show that if it did so, it could always pick a tax on the other side of the net tax revenue curve that would provide the same discounted surplus and higher utility.

Proposition 2. *It cannot be part of an equilibrium for government i to set its time-t tax higher than the one that maximises s_t^i.*

3. Dynamic Optimal Taxation

We assume that at time zero, the government in country i can commit to a tax plan $\{\tau_t^i\}_{t'0}^4$. It takes the tax plans of the other governments as given and maximises the indirect utility of its household (equation (17)) subject to its budget constraint (equation (18)).

We begin analysing the problem by considering the effects of time-t taxes on welfare and the government's fiscal position. By equations (13) and (17), the marginal change in utility from a marginal increase in the time-t tax is

$$\frac{\mathsf{M} t^i}{\mathsf{M} t_t^i} \cdot \beta^t m_t^i, \text{ where } m_t^i :' \frac{\& 1 \% \dfrac{\varphi \tau_t^i}{N} \dfrac{w_t \& \tau_t^i}{c_t}}{c_t \mathsf{j}_{s'0}^{4} \beta^s \dfrac{w_s \& \tau_t^s}{c_s}} \& \frac{(1 \& \beta) \varphi \tau_t^i}{N c_t}, \quad t ' 0, 1, \dots . \quad \textbf{(20)}$$

If taxes are equal across countries, equation (16) implies $\Sigma_{t'=0}^{4} \beta^{t'}(w_t \& \tau_t)/c_t = 1/(1-\beta)$. Equation (13) and the definition of s_t (in equation (18)) implies $(w_t \& \tau_t)/c_t = 1 \& s_t$. These results and equation (20) imply that with symmetric taxes

$$m_t^i \; ' \; m_t \; ' \; \& \; \frac{1 \& \beta}{c_t}\left(1 \% \frac{\varphi\tau_t s_t}{N} \right), \; t \; ' \; 0,1,... \; . \tag{21}$$

We next show that the marginal (indirect) utility of taxes must be strictly negative.

Proposition 3. *A symmetric equilibrium must have $m_t < 0$, $t = 0,1,...$.*

Holding other taxes constant, raising time-t taxes has the direct effect of lowering disposable income in period t, tending to lower welfare. Suppose $\tau_t > 0$. Then the tax increase raises collection costs, and thus, lowers aggregate consumption and raises the cost of borrowing at time t. This tends to increase (decrease) welfare if consumers lend to (borrow from) the government in period t. Proposition 3 implies that the direct effect dominates the interest rate effect; the tax increase lowers welfare. Suppose $\tau_t < 0$. Then a tax rise increases aggregate consumption and lowers the cost of borrowing in period t because of *lower* collection costs. As consumers must be lending to the government in this case, the interest rate effect tends to lower welfare. The marginal change in utility associated with a tax rise must be negative.

By equations (13) and (18), the marginal increase in the discounted value of the budget surplus (the left-hand side of equation (18)), resulting from a marginal increase in the time-t tax when taxes are identical across countries is

$$\beta^t n_t, \; where \; n_t \; :' \; (1/c_t)(1 \& \varphi\tau_t \% \varphi\tau_t s_t/N), \; t \; ' \; 0,1,... \; . \tag{22}$$

We show that in equilibrium, a marginal increase in the time-t tax must increase the discounted value of the government's stream of budget surpluses.

Proposition 4. *A symmetric equilibrium must have* $n_t > 0$, $t = 0,1 \ldots$.

After the first period, the government smooths taxes.

Proposition 5. *A symmetric Nash equilibrium has constant taxes after period zero.*

Let τ_t^i ' τ^i, c_t^i ' c^i, and s_t^i ' s, $t > 0$. Then equations (17) and (18) imply that the government maximises

$$\ln[(1 \And \beta)(w_0 \And \tau_0^i)/c_0 \% \beta(W \And \tau^i)/c] \% (1 \And \beta)\ln c_0 \% \beta\ln c \tag{23}$$

subject to

$$B^i :' \ (1 \And \beta)s_0^i \% \beta s^i \ ' \ 0. \tag{24}$$

By equations (13) and (23), the marginal utilities associated with τ_0^i and τ^i are

$$m_0^i \ ' \ \And \frac{1 \And \beta}{c_0}\left(\frac{1 \And \dfrac{\varphi\tau_0^i}{N}\dfrac{w_0 \And \tau_0^i}{c_0}}{C} \% \frac{\varphi\tau_0}{N} \right), \ m_1^i \ ' \ \And \frac{\beta}{c}\left(\frac{1 \And \dfrac{\varphi\tau^i}{N}\dfrac{w \And \tau^i}{c}}{C} \% \frac{\varphi\tau}{N} \right), \tag{25}$$

where $C :' \ (1 \And \beta)(w_0 \And \tau_0^i)/c_0 \% \beta(W \And \tau^i)/c,$

respectively.

By equations (13) and (24), the marginal increases in B^i associated with increases in τ_0^i and τ^i are

$$n_0^i \ ' \ \frac{1 \And \beta}{c_0}\left(1 \And \varphi\tau_0^i \% \frac{\varphi\tau_0^i s^i}{N} \right), \ n_1^i \ ' \ \frac{\beta}{c}\left(1 \And \varphi\tau^i \% \frac{\varphi\tau^i s^i}{N} \right), \tag{26}$$

respectively.

If the countries act symmetrically, then equation (16) implies $C = 1$. Equation (13)

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implies $(W - \tau_0)/c_0 = 1$ & s_0 and $(W$ & $\tau)/\tau$ ' 1 & s. These results and equations (25) and (26) imply

$$m_0 \ ' \ \& \ \frac{1 \ \& \ \beta}{c_0}\left(1 \ \% \ \frac{\varphi\tau_0 s_0}{N}\right), \ m_1 \ ' \ \& \ \frac{\beta}{c}\left(1 \ \% \ \frac{\varphi\tau s}{N}\right)$$

$$n_0 \ ' \ \frac{1 \ \& \ \beta}{c_0}\left(1 \ \& \ \varphi\tau_0 \ \% \ \frac{\varphi\tau_0 s_0}{N}\right), \ n_1 \ ' \ \frac{\beta}{c}\left(1 \ \& \ \varphi\tau \ \% \ \frac{\varphi\tau s}{N}\right). \tag{27}$$

The first-order conditions for a maximum imply $m_1/m_0 = n_1/n_0$. This and equations (27) imply[7]

$$\tau_0/(1 \ \% \ \varphi\tau_0 s_0/N) \ ' \ \tau/(1 \ \% \ \varphi\tau s/N). \tag{28}$$

Symmetry and equation (24) imply

$$(1 \ \& \ \beta)s_0 \ \% \ \beta s \ ' \ 0. \tag{29}$$

The second-order condition requires that the bordered Hessian matrix associated with the optimisation problem has a strictly positive determinant. This requires

$$n_0(m_{00}m_1^2 \ \& \ 2m_{01}m_0 m_1 \ \% \ m_{11}m_0^2) \ \& \ m_0(n_{00}m_1^2 \ \% \ n_{11}m_0^2) < 0, \tag{30}$$

where m_{t0} and m_{t1} are the derivatives of m_t^i (as given by equation (25)) with respect to τ_0^i and τ^i, respectively, when taxes are symmetric and where n_{t0} and n_{t1} are the derivatives of n_t^i (as given by equation (26)) with respect to τ_0^i and τ^i, respectively, when taxes are symmetric. It is straightforward, but tedious to demonstrate that symmetric taxes which satisfy equations (28) and

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[7]In deriving equation (28), both sides were divided by φ. If $\varphi = 0$, the timing of taxes is irrelevant as long as the government satisfies its intertemporal budget constraint.

(29) also satisfy equation (30).[8]

Definition 1. *A symmetric* **equilibrium** *is a pair of taxes* $\{\tau_0, \tau\}$ *such that the feasibility condition* (29) *and the optimality condition* (28) *are satisfied.*

We first establish that taxes are always positive.

Proposition 6. *An equilibrium cannot have subsidies.* ($\tau_0 < 0$ *or* $\tau < 0$).

We analyse the equilibrium by graphing equations (29) and (28) in Figure 2. This figure is drawn for strictly positive taxes that are less than the ones that maximise the within-period discounted surpluses as we have shown that no other taxes can be part of an equilibrium.

The feasibility condition (29) is represented by the solid curves F_-, F, and F_+; the different curves representing different initial stocks of debt. By Proposition 4, these curves slope down; an increase in the future tax allows the government to reduce the current tax and still balance its budget.[9] The curve representing the case of zero initial debt, F_0, goes through the point ($\tau^\&, \tau^\&$), labelled A. The curve representing the case of strictly positive initial debt, F_+, lies above F_0 and passes through the point ($\tau_0^\&, \tau^\&$), which is above A. The curve representing the case of strictly negative initial debt, F_-, lies below F_0 and passes through ($\tau_0^\&, \tau^\&$), which is below A.

The curves representing equation (28) in Figure 2 are represented by the dashed lines. The curve O_0 represents the case of no initial debt or an infinite number of countries. The curves labelled $O_\%^{NN}$ and $O_\%^{NO}$ represent the case strictly positive initial debt and $N = N\!N$ and $N = N\!O$, respectively, where $1 \# N\!O < N\!N < 4$; the curves labelled $O_\&^{NN}$ and $O_\&^{NO}$ represent the case strictly negative initial debt when $N = N\!N$ and $N = N\!O$, respectively.

Proposition 7. *The curves representing the optimality conditions in Figure* 2 *have the following*

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[8]Details available on request.

[9]The curves are drawn as convex to the origin. This is true if N is sufficiently large, but need not be true otherwise.

properties:

(*i*) *The curve O_0 is the 45° line.*

(*ii*) *All of the optimality curves are upward sloping and pass through the origin.*

(*iii*) $O_\%^{N\rceil}$ *and* $O_\%^{NO}$ *lie below the 45° line;* $O_\&^{N\rceil}$ *and* $O_\&^{NO}$ *lie above the 45° line.*

(*iv*) $O_\%^{N\rceil}$ *lies above* $O_\%^{NO}$ *when* $\tau > \tau^-$ *and* $\tau_0 < \tau_0^\&$*;* $O_\&^{N\rceil}$ *lies below* $O_\&^{NO}$ *when* $\tau < \tau^-$ *and* $\tau_0 > \tau_0^\&$*.*

The intuition behind the optimality curves in Figure 2 is that the government trades off two objectives. First, it wants to smooth consumption by smoothing fiscal transfer costs over time. If this were its sole objective, optimality would be represented by O_0. Second, it wants to lower the discounted value of the fiscal transfer costs through its influence on the global rate of interest. If it is an initial debtor, it does this by lowering initial taxes and raising future taxes. Through the global resource constraint (equation (13)) this raises initial consumption and lowers future consumption, thus lowering the interest rate between periods zero and one. Thus, its required tax revenue falls. Likewise, if it is an initial creditor it can lower its required discounted tax revenue, and thus its tax collection costs, by raising initial taxes and lowering future taxes, thus raising the interest rate between periods zero and one.

This second objective means that the curve representing the optimality condition in Figure 2 is flatter than O_0 when there is initial debt and it is steeper than O_0 when there is an initial surplus. The more market power a country has (that is, the smaller is N) the greater is its ability to affect the global interest rate and the more important this second motive becomes. Thus, as the number of countries falls, the optimality curve becomes flatter if the country is an initial debtor and steeper if the country is an initial creditor. When $N 6 4$ countries have no market power. Only the first objective matters and the optimality equation is represented by O_0.

Equilibrium occurs at the intersection of the relevant feasibility and optimality curves. We show that a unique intersection must occur.

***Proposition* 8.** *A unique symmetric equilibrium exists.*

Different equilibria are represented by the points *A* - *G* in Figure 2. Point *A* is the equilibrium when there is no initial stock of debt. In this case there is tax smoothing and the budget is balanced each period. Points *B*, *C* and *D* represent equilibria when there is a positive stock of initial debt. If $N = 4$, the equilibrium is represented by point *B* and there is tax smoothing. Points *C* and *D* lie below the 45° line; hence, if $N < 4$ and there is a positive initial stock of debt, $\tau > \tau_0$. As *N* falls, the negative slope of the curve representing equation (29) ensures that the initial tax declines and the future tax rises.

Likewise, points *E*, *F* and *G* represent equilibria when there is an initial negative stock of debt. If $N = 4$ (point *G*), there is complete tax smoothing. Points *E* and *F* lie above the 45° line; hence, if $N < 4$ and there is a negative initial stock of debt, $\tau > \tau_0$. As *N* falls, the initial tax rises and the future tax falls.These results are summarised below.

***Proposition* 9.** *If countries have no market power* ($N = 4$) *or if initial debt is zero, then there is complete tax smoothing. If countries have some market power* ($N < 4$), *then the initial tax is strictly less (greater) than the subsequent tax if there is a strictly positive (negative) initial debt.*

When countries have no market power, we have Barro's (1979) result. Taxes result in resource losses because they are costly to collect. If these costs are convex, then an optimising government smooths them over time. If, however, the government can affect the interest rate and it has a non-zero initial stock of debt, then it lowers the discounted value of its required tax revenue by reducing initial taxes and raising future taxes. If it is an initial creditor it raises its return to its savings by increasing the initial tax and lowering future taxes.[10]

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[10]If the government begins with strictly positive (negative) initial debt, then Proposition 9 says that the initial tax is lower (higher) than subsequent taxes. This implies that the government enters period one with strictly positive (negative) debt. Thus, if the government could re-optimise in period one, Proposition 9 implies that it would set a lower (higher) tax in period one then in later periods. This implies that the equilibrium, which features constant taxes

The case of $N = 1$ corresponds to the social planner's outcome. Hence, we have the following result.

***Proposition* 10.** *Suppose that $N > 1$. If there is a positive (negative) initial stock of debt, then the initial tax is too high (low) relative to the social optimum. The subsequent tax is too low (high) relative to the social optimum.*

If there is a positive stock of initial debt, lowering initial taxes causes a positive externality by decreasing all country's borrowing costs. Countries do not take into account the social benefit and they do not decrease initial costs enough.

The outcome is furthest from the optimal outcome when the number of countries goes to infinity and countries lose their market power. This is in stark contrast to the result in "beggar-thy-neighbour" policy games where nations attempt to exploit their market power to gain at the expense of other countries. In such papers, as the number of countries goes to infinity and nations lose their market power, the noncooperative outcome converges to the cooperative outcome.[11]

The result that the distortion does not vanish, but increases when nations lose their market power, is similar in spirit to that in Kehoe (1987), although the economic mechanism is quite different. In his paper governments balance their budgets and do not fully take into account the negative effect on world capital accumulation of taxing workers to provide current government spending. Here, governments do not take into account the positive effect of current tax reductions and larger budget deficits on the world interest rate. In both papers, as the number of countries increases and the effect of any country on global variables declines, the failure of countries to take into account the effect of their actions on the world economy becomes more severe.

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from period one on, is not time consistent unless $R_0 b_0 = 0$. The time inconsistency arises because the initial debt is taken as exogenous, and hence, unaffected by taxes.

[11]This would occur for, for example, in Hamada (1966).

4. CES Preferences

The log-linear preference specification of the previous section is the special case of CES preferences for an elasticity of intertemporal substitution equal to one. In this section, we look at how small changes in the value of the elasticity of substitution in the neighbourhood of one affect the results of the last section. Let

$$u^i = \frac{1}{1-\theta} \sum_{t=0}^{4} \beta^t ({c_t^i}^{1-\theta} - 1), \quad 0 < \beta < 1, \quad \theta > 0, \tag{31}$$

where θ is the reciprocal of the elasticity of intertemporal substitution. As $\theta \to 1$, the above preferences become the logarithmic specification of the previous sections. We assume that θ is arbitrarily close to one.[12]

In this case, the Euler equation of the consumer's optimisation problem becomes

$$c_{t+1}^i = (\beta R_{t+1})^{1/\theta} c_t^i, \quad t = 0,1,\dots . \tag{32}$$

Solving the difference equation (32) yields the household's time-t consumption as a function of its initial consumption and the interest rate

$$c_t^i = \left(\beta^t \prod_{s=1}^{t} R_s \right)^{1/\theta} c_0^i, \quad t = 1,2,\dots . \tag{33}$$

Averaging both sides of equation (33) across countries yields

$$\prod_{s=1}^{t} R_s = (1/\beta^t)(c_t/c_0)^\theta, \quad t = 1,2,\dots . \tag{34}$$

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[12]It is easy to generalise the results of Section 3 to $\theta < 1$, and by continuity arguments, to θ within a right-hand side neighbourhood of one. It appears analytically intractable to extend them to θ sufficiently greater than one. Here we are concerned with marginal changes at $\theta = 1$.

Substituting equations (33) and (34) into the household's budget constraint yields

$$c_0^i/c_0 = \sum_{t=0}^{\infty} \beta^t \frac{w_t - \tau_t^i}{c_t^\theta} \Big/ \sum_{t=0}^{\infty} \beta^t c_t^{1-\theta}, \quad t > 0. \tag{35}$$

Substituting equations (33) - (35) into equation (31) and ignoring constants that are unimportant to the optimisation problem yields the indirect utility function

$$\frac{1}{1-\theta}\left(\sum_{t=0}^{\infty} \beta^t \frac{w_t - \tau_t^i}{c_t^\theta}\right)^{1-\theta}\left(\sum_{t=0}^{\infty} \beta^t c_t^{1-\theta}\right)^{\theta}. \tag{36}$$

Substituting equation (34) into the government's budget constraint (equation (11)) yields

$$\sum_{s=0}^{\infty} \beta^t \hat{s}_t^i = 0, \text{ where } \hat{s}_t^i = [\tau_t^i - (\varphi/2)\tau_t^{i2} - g_t]/c_t^\theta, \quad t = 1,2,\dots. \tag{37}$$

In the previous section we showed that a symmetric equilibrium has constant taxes after period zero. Substitute $\tau_t = \tau$ into equations (36) and (37). Then the optimisation problem of the government is to choose τ_0 and τ to maximise

$$[(1-\beta)(w_0-\tau_0^i)/c_0^\theta + \beta(W-\tau^i)/c^\theta]^{1-\theta}[(1-\beta)c_0^{1-\theta} + \beta c^{1-\theta}]^\theta/(1-\theta) \tag{38}$$

subject to

$$(1-\beta)\hat{s}_0^i + \beta\hat{s}^i = 0. \tag{39}$$

The first-order conditions evaluated at a symmetric equilibrium imply

$$\tau_0/(1 + \varphi\theta\tau_0 s_0/N) = \tau/(1 + \varphi\theta\tau s/N). \tag{40}$$

By equation (39) and symmetry

$$(1 \& \beta)\hat{s}_0 \% \beta\hat{s} ' 0. \tag{41}$$

The feasibility constraint and the optimality condition are represented graphically in Figure 3. In this figure, F^k represents the feasibility constraint and O^k represents the optimality constraint for the case of $\theta = \theta^k$, $k = 0,1$, where $\theta^0 < \theta^1$. The properties of the curves are summarised in the following proposition.

Proposition 11. *The curves F^0 and F^1 are downward sloping and intersect at $(\tau_0^\&, \tau^\&)$ and on the 45° line. F^0 lies above F^1 to the left of $(\tau_0^\&, \tau^\&)$ and below the 45° line; it lies below F^1 to the right of $(\tau_0^\&, \tau^\&)$ and above the 45° line. The curves O^0 and O^1 are upward sloping and lie below the 45° line. The curve O^1 lies below O^0.*

To see the properties of the feasibility curves, first suppose there is no initial debt. Then increasing θ improves the government's tradeoff over feasible current and future taxes. To see this, suppose that at $t = 0$ the government runs a deficit and borrows. With lower period-zero taxes than future taxes, consumption is higher in period zero than in period one. Consumers smooth their consumption by lending to governments at $t = 0$. The higher is θ, the greater is their desire to smooth their consumption. Thus, the higher is θ, the less costly is it for the government to trade off future tax increases for tax cuts in period zero. The intuition is similar if there is an initial surplus.

When $R_0 b_0 > 0$, the government's tradeoff is more favourable with a higher value of θ than with a lower value of θ if consumption is higher in the period in which the government runs a deficit. With an initial positive stock of debt, however, it is possible for the country to run a deficit in period zero, even though consumption is lower in period zero than in period one. This corresponds to the parts of the curves between the two intersecting points. In this case, reducing

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current taxes requires higher future taxes and the higher is θ the higher are these future taxes. Consumption is made less smooth by the government's borrowing and the higher is θ, the more the government must pay to borrow.

To see the shape of the optimality curves, suppose there is an initial stock of government debt and that taxes are constant across periods. Then the governments run a deficit in the current period and must borrow. If first-period taxes were lowered, this would increase current consumption and lower the interest rate that the government must pay on its debt. If this interest rate effect is taken into account, then taxes will be lower in the first period than if the interest rate effect is not taken into account. This is the argument of the previous section.

The lower is θ, the less consumers want to smooth their consumption and the less is the interest rate effect. Thus, the bigger is θ the greater is the socially optimal reduction in the first-period tax below the feasible constant tax.

Given Proposition 11 we have the following.

Proposition 12. *Suppose that $N > 1$. If there is a positive initial stock of debt, then the initial tax is too high relative to the social optimum and the subsequent taxes are too low relative to the social optimum. An increase in θ causes the socially optimal value of the initial tax to fall.*[13]

As well as considering marginal changes in θ around one, we can consider the polar cases where θ goes to zero and to infinity. In the limit as θ falls to zero, there is no interest rate effect as consumers do not care at all about smoothing their income. Thus, the socially optimal and uncoordinated outcomes coincide and taxes are smoothed over time. In the limit as θ goes to infinity, indifference curves for current and future consumption become right angles and only the minimum consumption matters. If $R_0 b_0 > 0$, cooperating governments should borrow marginally

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[13]As noted, this theorem is for marginal changes at $\theta \simeq 1$. It is easy to generalise it to large changes for $\theta < 1$, but it is not analytically tractable to consider large changes above one.

less than their outstanding debt in period one and set the current tax marginally higher than the one that balances the future primary deficit. Current consumption is then marginally lower than future consumption so the required gross interest rate on the borrowing is zero. The future tax is thus the one that balances the future primary deficit. Minimum consumption over the current and future can be made arbitrarily close to $W \& G \& (\varphi/2)\tau^{\&2}$.

4. Production and capital accumulation

An important simplifying feature of the model is that varying the timing of taxes, and thus fiscal transfer costs, over time is the only way to transfer resources across periods. In equilibrium, net global private and public saving is always zero because the good is perishable. Reducing taxes in any given period increases the resources available that period and increases private consumption. In our benchmark model, the interest rate on current savings falls as a result of the current tax cut. In this section, we allow for production. With capital formation, resources can be transferred across periods not only by changing the path of taxes, but also by capital formation.

Formally, we assume that the household has CES preferences and that the single good in the model is both a capital and a consumption good. The representative households each supply one unit of labour inelastically each period and save both bonds and the output of the current good in the form of capital. The savings of capital are loaned to the firms to be used in the next-period's production process. The firms transform capital and labour into output via a Cobb-Douglas production function where output per unit of labour is $f(k) = Ak^{\alpha}$, where k is the capital-labour ratio, $A > 0$ and $\alpha \in (0,1)$. We suppose that labour is immobile across countries, capital is perfectly mobile and capital depreciates completely. Then perfect mobility of capital and perfect competition imply that capital-labour ratios and wages are equalised across countries and k_t ' $k(R_t)$ ' $[A(1 \& \alpha)/R_t]^{1/\alpha}$.

A symmetric equilibrium is characterised by the Euler equation (32), the government budget constraint (37) and the global resource constraint, which is now

$$f(k(R_t)) \ \& \ G \ \& \ (\varphi/2)\tau_t^2 \ \& \ c_t \ \& \ k(R_{t\%1}) \ ' \ 0, \ t \ ' \ 0,1,... \ . \qquad \textbf{(42)}$$

The model with capital is far more difficult to analyse than the one without. To obtain an analytical result, we restrict ourselves to a simple experiment. Imagine that the world is at a symmetric steady state with constant taxes and a positive initial stock of debt. Can policy makers raise welfare with a coordinated symmetric marginal tax cut?

Proposition 13. *Suppose countries are at a symmetric steady state with constant taxes and strictly positive debt. Then it is possible to increase welfare with a coordinated marginal tax cut in the current period.*

We show in the proof that welfare is improved if the current tax cut is financed with future tax rises that leave consumption constant from period one on. Lowering the current tax and raising future taxes raises current consumption and lowers future consumption, thus lowering the current interest rate as in the previous sections. This lowers the cost of servicing the debt and reduces future tax collection costs. To see that the interest rate must fall, suppose that it did not. Then next period's marginal product of capital will rise so current capital accumulation falls. With lower tax collection costs and fixed current output, this implies current consumption rises. This is inconsistent with the interest rate falling in the current period unless next period's, and hence every future period's, consumption rises by more than current consumption. However, with lower current capital accumulation and higher future tax collection costs this is impossible. Thus, we have a contradiction.

Conclusion.

We have demonstrated that, in our baseline model, optimising governments will perfectly smooth taxes if they have no market power or if they have no initial debt. If countries are large

enough to affect the world interest, then they will set lower taxes in the current period than in the future if they have a positive initial stock of debt. If they have an initial stock of credit they will set higher taxes in the current period than in the future.

We show that, relative to the first-best, cooperative outcome, with positive initial debt, countries set their current taxes too high. Thus, relative to the optimum, initial budget deficits are too low. Similarly, if countries are initial creditors, initial budget deficits are too high.

We extend our baseline model with its log-linear preferences, to the case of CES preferences. We show that a marginal fall in the intertemporal elasticity of substitution increases the deviation between the uncoordinated outcome and the first-best outcome; a marginal rise decreases the deviation. We also consider the case of production and capital accumulation. We show that if there is a steady state with constant taxes and strictly positive debt, then it is possible to increase welfare with a coordinated cut in the current tax.

Appendix

Proof of Proposition 1. We show this for $t > 0$; the proof for $t = 0$ is similar.

A tax that maximises s_t^i must be in $[\tau^-, \tau^+]$ and it must satisfy

$$ds_t^i/d\tau_t^i \equiv (1 \& \varphi\tau_t^i)/c_t \% \varphi\tau_t^i s_t^i/(Nc_t) \equiv 0 \qquad \textbf{(43)}$$

$$d^2 s_t^i/d\tau_t^{i^2} \equiv 2\varphi\tau_t/(Nc_t)\, ds_t^i/d\tau_t^i \& \varphi/c_t \% \varphi s_t^i/(Nc_t) < 0. \qquad \textbf{(44)}$$

When $\tau_t^i \, O\, [\tau^-, \tau^+]$, then $\tau_t s_t^i \$ 0$; hence a solution of (43) must have $\tau_t^i \$ 1/\varphi$. When $\tau_t^i = 1/\varphi$, $ds_t^i/d\tau_t^i > 0$; when $\tau_t^i = \tau^+$, $ds_t^i/d\tau_t^i < 0$; hence (43) has a solution on $[1/\varphi, \tau^+]$. By (43) and (44), $d^2 s_t^i/d\tau_t^{i^2} = 1/(\tau_t^i c_t^i) < 0$ at this solution; hence, the solution is unique and it is a maximum. For $\tau_t^i \, O\, [\tau^-, 1/\varphi]$, $ds_t^i/d\tau_t^i > 0$; hence, s_t^i is increasing below the maximising tax on $[\tau^-, \tau^+]$ and

decreasing above the maximising tax.

By (43) and (44), the maximising tax is decreasing in N and goes to $1/\varphi$ as N goes to ∞.

Proof of Proposition 2. Suppose to the contrary that $\exists$ $t > 0$ such that at least one country sets its tax above $1/\varphi$. Without loss of generality, let country i be the country with the highest tax and let $\tau_t^i = \tau^W$. Let $t > 0$; the proof for $t = 0$ is similar. Let $\hat{\tau}$, the average tax in the other countries in period t, be given. Let s^W be the value of s_t^i at τ^W, τ^* be the tax that maximises s_t^i, s^* be the value of s_t^i at this tax and $\overline{\tau}$ be the positive value of τ_t^i at which $c_t = 0$. Then $\tau^W > \tau^*$ and $s^W \in (-\infty, s^*)$. We have that s_t^i, maps $(\&\tau^*, \tau^*)$ onto $(\&\infty, s^*)$, thus $\exists$ $\tau^R \in (-\overline{\tau}, \tau^*)$ such that $s_t^i = s^W$ at τ^R. A switch from τ^W to τ^R does not affect s_t^i; hence, by (18), it does not require a change in any other tax. Thus, τ^R is preferred to τ^W if indirect utility (given by (17)) is higher at $\tau_t^i \cdot \tau^R$ than at $\tau_t^i \cdot \tau^W$.

We have $\tau^{R^2} < \tau^{W^2}$; hence, $c^R > c^W$, where c^k is c_t when $\tau_t^i \cdot \tau^k$, $k \cdot R, W$. Thus, by (17), indirect utility is higher at $\tau_t^i \cdot \tau^R$ than at $\tau_t^i \cdot \tau^W$ if $(W \& \tau_t^R)/c^R > (W \& \tau^W)/c^W$. By (13),

$$\frac{W \& \tau^k}{c_t^k} \cdot 1 \& s^W \% \frac{N \&1}{N} \frac{\varphi}{2} \frac{\hat{\tau}^2 \& \tau^{k^2}}{c_k}. \qquad \textbf{(45)}$$

Thus, we need to show $(\hat{\tau}^2 \& \tau^{R^2})/c^R > (\hat{\tau}^2 \& \tau^{W^2})/c^W$. By (13), this is true iff $\hat{\tau} < \overline{\tau}$ which must be true as the other countries have lower taxes than country i. As the country with the highest tax cannot set its tax on the wrong side of the net tax revenue curve, neither can any other country.

Proof of Proposition 3. Suppose $t > 0$. By (21), $m_t \#$ unless $\tau_t s_t < 0$. By Proposition 1, $\tau_t \in]-\overline{\tau}, \tau^{(N)}]$; hence, $m_t \#$ unless $\tau_t \in]0, \tau^\&]$. In this case, $m_t < 0$ if $L(\tau_t) :' W \& G \% (\varphi/2)\tau_t^2 - (\varphi^2/2)\tau_t^3 \& \varphi G \tau_t > 0$. The function L has an interior minimum at $\underline{\tau}$ iff $\exists$ $\underline{\tau}$ such that $L\aleph(\underline{\tau}) = \underline{\tau} - (3\varphi/2)\underline{\tau}^2 - G = 0$ and $L\mathbb{Q}(\underline{\tau}) = 1 - 3\varphi\underline{\tau} > 0$. If $1 - 6\varphi G < 0$, then no such $\underline{\tau}$ exits and $L(0) = L(\tau^-) =$

$W - G > 0$ ensures the proposition holds. If $1 - 6\varphi G \geq 0$, then an interior minimum exists. It is sufficient to show that L is strictly positive at this point. Using $(3\varphi/2)\underline{\tau}^2 = \underline{\tau} - G$, we have $L(\underline{\tau}) = W - G + (\varphi/2)\underline{\tau}(\underline{\tau} - \varphi\underline{\tau}^2 - 2G) = (1/3)[3(W - G) + (\varphi/2)\underline{\tau}(\underline{\tau} - 4G)] = (1/9)[9W - 10G + (1 - 6\varphi G)\underline{\tau}]$. Assumption (19) ensures $9W > 10G$; hence this is true. The proof for $t = 0$ is similar.

Proof of Proposition 4. Suppose to the contrary that $\geq u \geq 0$ such that, $n_u < 0$. By Propositions 1 and 2, $\tau_u < \tau^\&$ (or $\tau_0 < \tau_0^\&$ if $u = 0$). Thus, $s_u^i < 0$ and there must be some time v where $s_v^i > 0$. By Propositions 1 and 2, $\tau_v \in (\tau^\&, \tau^{(N)}]$ (or $\tau_0 \in (\tau_0^\&, \tau_0^{(N)}]$ if $v = 0$) and $n_v > 0$. Suppose the government of country i were to lower τ_u^i marginally, changing τ_v^i to satisfy (18). Then $d\tau_v^i = \& \beta^{u\&v}(n_u/n_v)d\tau_u^i < 0$. By marginally lowering taxes in both periods, Proposition 3 ensures utility rises. Thus τ_u cannot be part of an equilibrium. This is a contradiction.

Proof of Proposition 5. Suppose to the contrary that $\tau_u < \tau_v$, $u,v \geq 1$. If the country-i government marginally raises τ_u, changing τ^v to satisfy its budget constraint, then $du^i = \beta^u m_u d\tau_u^i + \beta^v m_v d\tau_v^i$, where $d\tau_v^i = \& \beta^{u\&v}(n_u/n_v)d\tau_u^i$. Then $du^i = \beta^u(m_u - m_v n_u/n_v)d\tau_u^i$. By Proposition 1, $n_u > 0$; hence, country i defects from the equilibrium if $m_u/n_u > m_v/n_v$. This is true if m_t/n_t is strictly decreasing in τ_t. By (21) and (22), this is true if $\tau_t/(1 + \varphi\tau_t s_t/N)$ is rising in τ_t. This is true if $1 > (\varphi\tau_t^2/N)ds_t/d\tau_t$. Proposition 4 and (18) ensure $ds_t/d\tau_t > 0$ in an equilibrium. Thus, if $\tau_t < 0$ the result holds. Suppose $\tau_t > 0$. We show the result holds when $t > 0$; the proof for $t = 0$ is similar.

The result is true if $1 > (\varphi\tau_t^2/N)(W - G - \varphi\tau_t w + \varphi\tau_t^2/2)/c_t^2$ on $[0,\tau^{(1)}] \cup [0,\tau^{(N)}]$. If the left-hand side is negative, this is true. If it is positive, it is true if it is true for $N = 1$. This is true if $G(\tau) := (\overline{\tau}^2 \& \tau^2)^2 \& 2\tau^2(\overline{\tau}^2 \& 2W\tau \% \tau^2) > 0$ on $[0,\tau^{(1)}]$. For G to have an interior minimum on $[0,\tau^{(1)}]$ requires $(3W - \sqrt{9W^2 \& 8\overline{\tau}^2})/2 \# \tau^{(1)}$. By the definition of $\tau^{(1)}$, this is impossible. $G(0) = \overline{\tau}^4 > 0$ and $G(\tau^{(1)}) = (\overline{\tau}^2 \& \tau^{(1)^2})^2 > 0$; hence $G > 0$ on $[0,\tau^{(1)}]$.

Proof of Proposition 6. Assumption (19) rules out negative taxes in both periods. Rearranging (28) yields $N(\tau_0 \& \tau) = \varphi\tau\tau_0(s_0 \& s)$. Substituting in $s_0/s = -\beta/(1 - \beta)$ (from (29)) yields

$$N(1 - \beta)(\tau - \tau_0) = \varphi\tau\tau_0 s. \tag{46}$$

Suppose $\tau > 0 > \tau_0$. Then (19) and (29) imply $s > 0$. The left-hand side of (46) is positive, the right-hand side is negative. This is a contradiction. Suppose $\tau_0 > 0 > \tau$. Then $s < 0$, the left-hand side of (46) is negative and the right-hand side is positive. This is a contradiction.

Proof of Proposition 7. Let the right-hand side of (28) be represented by $h(\tau;N) := \tau(1 + \varphi\tau s/N) > 0$ for $\tau \in [0,\tau^{*1}]$; the left-hand side by $h_0(\tau_0;N) := \tau(1 + \varphi\tau_0 s_0/N) > 0$ for $\tau_0 \in [0,\tau_0^{(1}.]$ These functions have the following properties:

(a) $h(0;N) = h_0(0;N) = 0$

(b) $dh(\tau;N)/d\tau > 0$; $dh_0(\tau_0;N)/d\tau_0) > 0$. (The first inequality follows from an argument in the proof of Proposition 5, the second by a similar argument)

(c) $h(\tau;N') > (= ,<) \ h(\tau;N'')$ when $\tau < (=,>) \tau^-$, $h_0(\tau_0;N') > (= ,<) \ h_0(\tau_0;N'')$ when $\tau_0 < (=,>) \tau_0^-$.

(d) $h_0(\tau;N) > (= ,<) \ h(\tau;N)$ when $R_0 b_0 > (=,<) 0$.

Result (i) follows from (d) and $h_0(\tau) \geq h(\tau)$ when $N \geq 4$. Result (ii) follows from (a) and (b). Result (iii) follows from (b) and (d). Result (iv) follows from (b) and (c).

Proof of Proposition 8. Uniqueness follows from the strictly negative slope of the feasibility curves and the positive slope of the optimality curves.

Suppose $R_0 b_0 \geq 0$. An equilibrium fails to exist iff the F_+ curve lies above $O_\%^N$ at the highest period-1 tax that is consistent with equilibrium, $\tau^{(N}$. By (43), this tax satisfies $1 - \varphi\tau^{(N} + \varphi\tau^{(N}s^{(N}/N = 0$, where $s^{(N}$ denotes s, evaluated at this tax. Thus, the right-hand side of (28) evaluated at $\tau^{(N}$ equals $1/\varphi$. For (28) to hold, (43) implies that $\tau_0 = \tau_0^{(N}$. Assumption (19) implies that surpluses are positive in both periods at the point $(\tau_0^{(N},\tau^{(N})$. Hence, this point must lie above F_+ and an equilibrium exists. The proof for $R_0 b_0 < 0$ is similar.

Proof of Proposition 11. Let $f(\tau) := \tau - (\varphi/2)\tau^2 - g$, $f_0(\tau) := \tau - (\varphi/2)\tau^2 - g_0$, $c(\tau) := W -$

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$G - (\varphi/2)\tau^2$ and $B(\tau_0,\tau;\theta) :' (1 \ \& \ \beta)f_0(\tau_0)/c(\tau_0)^\theta \ \% \ \beta f(\tau)/c(\tau)^\theta$. Then F^* represents the feasibility

constraint when $\theta = \theta^k$, $k = 0,1$. Clearly $B(\tau_0^\&,\tau^\&;\theta^0) ' B(\tau_0^\&,\tau^\&;\theta^1) ' 0$ and $B(\tau,\tau;\theta^0) = B(\tau,\tau,\theta^1)$.

Thus F^0 and F^1 pass through $(\tau_0^\&,\tau^\&)$ and intersect on the 45° line. Let $(\tau_0^),\tau^))$ be such that $B(\tau_0^),\tau^);\theta^0)$

$= 0$. Then $B(\tau_0^),\tau^);\theta^1) ' [\beta f(\tau^))/c(\tau_0^))^{\theta_1}]\{[c(\tau_0^))/c(\tau^))]^{\theta_1} \ \& \ [c(\tau_0^))/c(\tau^))]^{\theta_0}\} > 0$ iff $\tau^) > (<) \tau^-$

and $c(\tau_0^))/c(\tau^)) > (<) 1$. This is true iff $\tau^) > \tau^\&$ and $\tau_0^) < \tau^)$ or $\tau^) < \tau^\&$ and $\tau_0^) > \tau^)$.

Redefine the curves h and h_0 in Proposition 7 as $h(\tau;\theta) := \tau(1 + \theta\varphi\tau s/N)$ and $h_0(\tau_0;\theta) :=$

$\tau(1 + \theta\varphi\tau_0 s_0/N)$. It is easy to establish that property (b) in Proposition continues to hold if $\theta < 1$

and (by a continuity argument) if θ is sufficiently close to one. In addition, property (d) holds and

h is decreasing in θ when $\tau > \tau^\&$ and h_0 is increasing n θ when $\tau_0 < \tau^\&$. These properties ensure

that the O^0 and O^1 have the stated properties.

Proof of Proposition 13. Suppose the initial tax is less than $1/\varphi$. If this were not true, with

constant taxes welfare could be improved by moving to the lower revenue-equivalent tax. Let

the initial period be denoted by $t = 0$. Suppose the coordinated marginal fall in the initial tax

$d\tau_0 < 0$ is financed by a sequence of future tax changes $\{d\tau_t\}_{t'1}^4$ such that $dc_t^i ' dc_t ' dc$, t

$= 1,2,...$. Differentiating (31) and evaluating at the initial steady state yields

$$dU^i ' \sum_{t'0}^4 \frac{\beta^t dc_t^i}{c_t^{i1\&\theta}} ' \frac{dc_0}{c^{1\&\theta}} \ \% \ \frac{\beta}{1\&\beta}\frac{dc}{c^{1\&\theta}} > 0 \ ] \ dc_0 \ \% \ \frac{\beta dc}{1\&\beta} > 0 \tag{47}$$

Differentiating (37) and evaluating at the initial steady state yields

$$\sum_{s'0}^4 \beta^t\frac{(1\&\varphi\tau_t^i)d\tau_t^i}{c_t^\theta} \ \& \ \theta\sum_{s'0}^4 \beta^t\frac{s_t^i dc_t}{c_t^\theta} ' 0 \ ]$$

$$(1\&\varphi\tau)\sum_{t'0}^4 \beta^t d\tau_t \ \& \ \left(s\&\frac{R_0b_0}{c}\right)dc_0 \ \& \ \frac{\beta s dc}{1\&\beta} ' 0, \text{ where } s ' [\tau \ \& \ (\varphi/2)\tau^2 \ \& \ G]/c. \tag{48}$$

At a steady state the gross interest rate must equal $1/\beta$. Thus, evaluating (37) at the steady state yields $s \ ' \ (1 \ \& \ \beta)b_0/(\beta c)$. Substituting this into (48) yields

$$(1 \ \& \ \varphi\tau)\underset{t'\,0}{\overset{4}{\mathrm{j}}} \ \beta^t d\tau_t \ \% \ b_0 dc_0/c \ \& \ b_0 dc/c \ ' \ 0. \tag{49}$$

Differentiating (32) and evaluating at the steady state yields

$$dR_1 \ ' \ \theta(dc \ \& \ dc_0)/(\beta c), \ dR_t \ ' \ 0 \ t \ ' \ 2,3... \ . \tag{50}$$

Differentiating (42), employing $f^{)}(k_t) \ ' \ R_t$ and $dk_t/dR_t \ ' \ \& \ k_t/(\alpha R_t)$, substituting in (42) and (49) and evaluating at a steady state yields

$$\varphi\tau d\tau_0 \ ' \ \theta(dc \ \& \ dc_0)k_1/(\alpha c) \ \& \ dc_0, \ \varphi\tau d\tau_1 \ ' \ \& \ \theta(dc \ \& \ dc_0)k_1/(\alpha\beta c) \ \& \ dc$$
$$\varphi\tau d\tau_t \ ' \ \& \ dc, \ t \ ' \ 2,3... \ . \tag{51}$$

Substituting (51) into (48) yields

$$dc/dc_0 \ ' \ \& \left(\frac{1 \ \& \ \varphi\tau}{\varphi\tau} \ \& \ \frac{b_0}{c} \right) \Big/ \left(\frac{1 \ \& \ \varphi\tau}{\varphi\tau} \frac{\beta}{1 \ \& \ \beta} \ \% \ \frac{b_0}{c} \right). \tag{52}$$

Substituting (52) into (47) yields that utility rises if and only if true iff

$$b_0 > 0, \ \frac{1 \ \& \ \varphi\tau}{\varphi\tau} \frac{\beta}{1 \ \& \ \beta} \ \% \ \frac{b_0}{c} > 0. \tag{53}$$

The second expression is true by $\tau < 1/\varphi$. This gives us our result.

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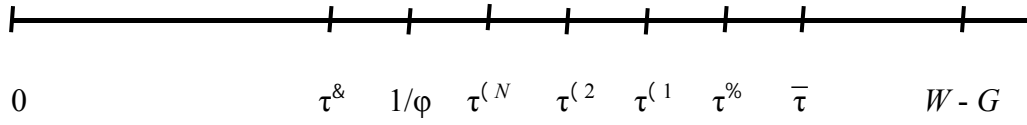
Figure 1. Important Tax Values

$$0 \qquad\qquad \tau^{\&} \quad 1/\varphi \quad \tau^{(N} \quad \tau^{(2} \quad \tau^{(1} \quad \tau^{\%} \quad \bar{\tau} \qquad W\text{-}G$$

Figure 2. Equilibrium Taxes

τ_0

$O_-^{N\prime\prime}$

$O_-^{N\prime}$

O_0

$O_+^{N\prime}$

$O_+^{N\prime\prime}$

B

E
C
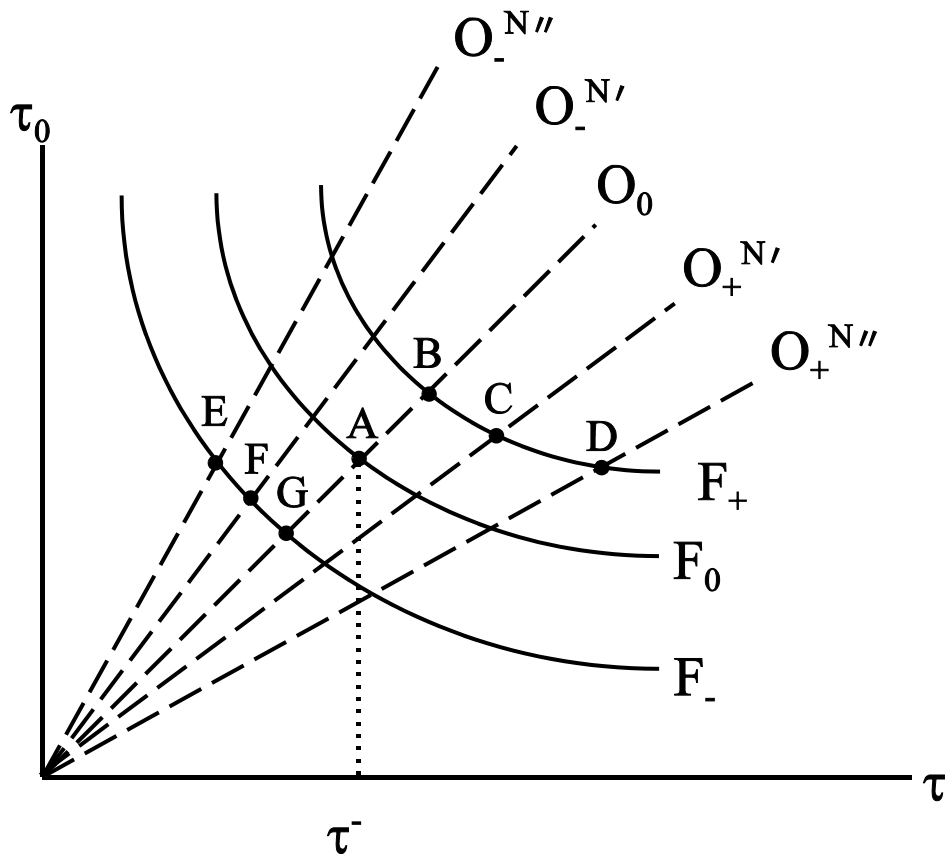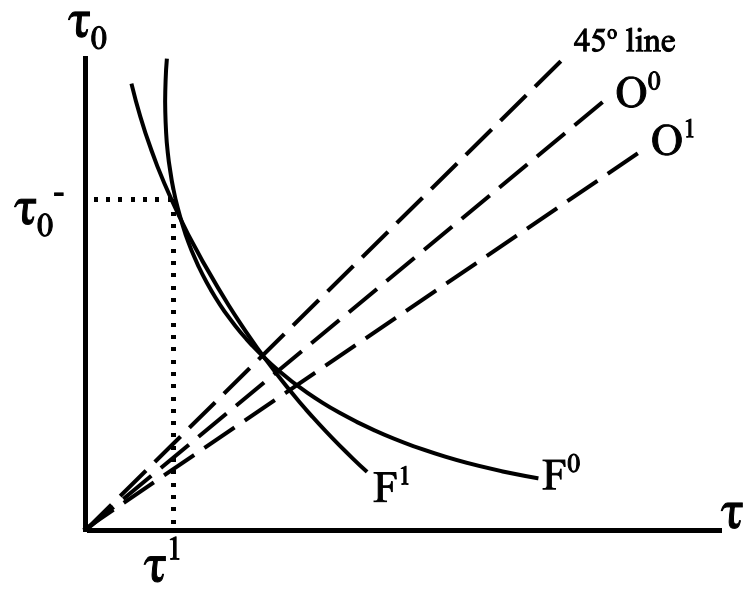A
D

F
F_+
G
F_0

F_-

τ

τ^-

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Figure 3 Equilibrium with CES Preferences and Initial Debt

$\theta^0 < \theta^1$