ISSN 1745-8587

Birkbeck UNIVERSITY OF LONDON

School of Economics, Mathematics and Statistics

Birkbeck Working Papers in Economics & Finance

BWPEF 0411

Formal Bankruptcy: Strategic Debt Service with Senior and Junior Creditors

Adriana Breccia

August 2004

▪ Birkbeck, University of London ▪ Malet Street ▪ London ▪ WC1E 7HX ▪

Formal Bankruptcy: Strategic Debt Service with Senior and Junior Creditors*

Adriana Breccia

Economics Working Papers Series,
Birkbeck August 2004.

Abstract

A crucial aspect of debt restructuring is the redistribution of value among many diverse interests, differing in priority, collateral and bargaining power. Focusing on renegotiable debt contracts in a continuous-time framework, we characterise the U.S corporate bankruptcy renegotiation (Chapter 11) in a game-theoretic framework. In formal bankruptcy, the equity holders can renegotiate with one creditor at a time. Unlike previous studies, this allows us to endogenise not only the bankruptcy threshold but also the impairment strategy. Moreover, our game-theoretic setting allows us to explicitly introduce and accommodate varying bargaining powers of claimants. First, we show that there exists a unique sequence of equilibrium restructuring plans and impairment strategies which allows us to derive simple and intuitive closed-form solutions for pricing different classes of debt. Secondly, the model provides a theoretical explanation for cases of seniority reversal. Thirdly, we derive sufficient conditions for the senior credit spread to be smaller than the junior one at all levels of the state variable.

*Comments are welcome. We are very grateful to Giovanni Barone-Adesi, John Brierley, William Perraudin and Ron Smith for helpful suggestions and discussions. School of Economics, Mathematics and Statistics, Birkbeck College, University of London, Malet Street, London, UK; e-mails: abreccia@econ.bbk.ac.uk; fax: +44 (0)20 7631 6416.

Introduction

This paper examines the role of priority on corporate renegotiable debt under the U.S. bankruptcy code (Chapter 11). Unlike previous literature on renegotiable debt, we endogenise not only the bankruptcy threshold, but also the order in which classes of debt (with different level of priority) are restructured. In fact, according to Chapter 11, some classes of debtholders can be excluded from the renegotiating table as long as they receive the promised contractual payments, that is, such classes are left "unimpaired". Interestingly, we recognise that this aspect of Chapter 11, well known to legal scholars, largely enhances the set of strategic actions available to the equity holders. The equity holders must choose the threshold level to default as well as the class/classes of creditors to impair. Therefore, in this paper, bankruptcy threshold and impairment strategy are jointly and endogenously determined.

In contrast to previous studies on strategic default, introducing impairment strategies allows us to capture absolute priority reversals not just amongst debt and equity, but also among different layers of debt. In fact, we show that what makes the actual debt priority structure is the optimal impairment strategy, which is uniquely defined by the value of collateral, bargaining power and face value of each class of debt. This result is in line with Rajan and Winton (1995), who argue that collateral makes the effective priority of debt[1].

Most important, we show that absolute priority violation amongst senior and junior debtholders can lead to reversal in the credit spreads. Moreover, for spread reversals not to occur we provide a simple and intuitive sufficient condition expressed in terms of face values, collateral and bargaining powers of creditors. Therefore the strategic impairment characterised in our setting is of great relevance when pricing different classes of debt.

Focusing on renegotiable debt contracts, within a continuous-time pricing framework, we formalise the bankruptcy system as a judicially supervised bargaining process between the claimants and the equity holders. Our game-theoretic framework can be compared with Brown (1989), firstly in that our set of bankruptcy rules captures the main features of Chapter 11. Secondly, we both stress the importance of those rules which give the equity holders the option to renegotiate with one creditor at a time [2] in a sort of 'private renegotiation'. Briefly, a Chapter 11 filing triggers two essential rules: i) the equity

[1]In particular, they focus on bank loans, typically highly secured and containing seniority clauses.

[2]Such as the equity holders' exclusivity period and the impairment rules allocating veto power in renegotiation. See below.

holders have the exclusive right to file a first restructuring plan[3] and ii) those claimholders left unimpaired by a plan[4] lose their veto power. Therefore, in formal bankruptcy, the equity holders have the option to renegotiate with one creditor at a time in a sort of 'private renegotiation' while excluding unimpaired creditors from the negotiating table. This possibility is widely exploited in reality, and it largely enhances the equity holders' strategic behaviour in bankruptcy. It is actually rare the case where a debtor files a Chapter 11 plan which impairs all claims at once. Quite commonly, a restructuring plan leaves a certain number of classes unimpaired (most typically, the secured senior classes) and impairs all or some unsecured classes. In Table A, we report the classification of claims, the impairment of classes and the solicitation of votes relative to six companies who proposed Chapter 11 restructuring plans which have all been confirmed by the Bankruptcy Court[5]. In all six cases, there is always one or more classes of claims which are left unimpaired by the plan[6], and therefore, as ruled in the Confirmation Orders, those classes become "non-voting" classes and they are deemed to accept the plan. However, it is feasible to have a plan confirmed even if the impairment structure does not match the priority structure. That is, senior classes are impaired while non-senior classes or junior classes are left unimpaired[7].

We capture the 'impairment strategy' by modelling the first stage of the restructuring as a 'private renegotiation' where the first equity holders plan consists of a take-or-leave-it offer to the impaired class/classes -hence, the 'voting' classes.

Also, according to Chapter 11 rules, on rejection of the first equity holders' plan, any player is allowed to file for competing plans. The bankruptcy code does not set any specific agenda rules concerning subsequent proposals. Many different ways of modelling the second stage of the renegotiation are possible[8]. We use an axiomatic approach and assume that a three-player Nash bargaining outcome is achieved in the case of a disagreement

[3] The plan goes ahead if approved by all claimants.

[4] A creditor is said to be unimpaired if the plan calls for no scaling down of the coupon payment scheduled in the existing contract.

[5] Data in Table A are taken from the 'Plan Disclosure' and 'Confirmation Order' from the U.S. Bankruptcy Courts of: Southern District of New York, District of Delaware and District of Maryland.

[6] Of course, this is without taking into account "non-financial classes", that is classes such as Priority Claims (unpaid wages), Tax-Claims or Administrative Claims.

[7] For instance, in Table A, Vero Electronics plan leaves class 3 -'Other Secured Claims'- unimpaired and impairs class 2 -'Senior Secured Credit Facilities'. Cases like this are not the most common, but not even rare.

[8] For instance, Brown (1989) assumes that the Bankruptcy Court determines the order in which proposals are voted and each proposal might rank first, second or third in the agenda with same probability.

in the 'private renegotiation'. In contrast to Brown (1989), we recognise the limit of a 'refereed' bargaining system. This ideally enables claimants to bring their own rights to a court which cannot be influenced by parties and which automatically enforces rules. Nevertheless, as argued by legal scholars, Chapter 11's rules are idiosyncratic and judicial discretion is granted in many circumstances[9]. Therefore, more realistically, our bargaining setting account for exogenous asymmetries between parties which might reflect the ability of claimholders to influence the court and extract a better package of concession during renegotiation. In line with Welch (1997), such asymmetry can be explained as a reflection of different organisation skills and reputation benefits[10]. Even though we do not attempt to model the source of asymmetry, we introduce an exogenous bias in the Nash game which may encompass a wide range of realistic circumstances whilst keeping the game-theoretic framework simple and flexible. Therefore the importance of claimholders' heterogeneity in terms of bargaining power is stressed through an asymmetric Nash axiomatic bargaining which jointly involves equity holders and creditors (a senior under-secured and a junior creditor).

A number of papers have incorporated strategic interactions between debt and equity holders during renegotiation. In comparison to the literature focusing on strategic debt service, the distinctive feature of our model is the expansion of the set of strategic actions available to the equity holders. As mentioned, the formal reorganisation accomplished under Chapter 11, allows the equity holders to renegotiate vis a vis one creditor at a time. Therefore, the strategic decision concerns not only the timing of bankruptcy but also the creditor type, that is whether defaulting on the junior debt, the senior debt or both. Moreover, the set of strategic actions is additionally expanded by recognising the possibility of filing for formal renegotiation a number of times. Therefore, our setting allows for *sequential restructuring* of different classes of debt.

Our results are as follows.

First, we show that there exists a unique sequence of equilibrium plans which, depending on the parameterization, either impairs the junior creditor first and the senior one later or vive versa. The equity holders' 'impairment strategy' univocally depends on factors such as the face value of claims, the bargaining power of players and the value of collateral. Furthermore, the equilibrium impairment strategy derived is simple and intuitive.

[9]See Kordana and Posner (1999) for a critical and exhaustive analysis of Chapter 11's rules.

[10]As argued by Welch, unlike bond-holders a bank can have a good reputation effect for "tough behaviour" which might prevent other borrowers from opportunistic renegotiation.

Second, our valuation of the senior (under-secured) and junior debt, through simple closed-form solutions, provides some relevant results in terms of credit spreads. Interestingly, the senior credit spread is not necessarily smaller than the junior one at all levels of the state variable. This could not be the case in a pure liquidation scenario *à la* Merton, or under a restructuring system such as the UK Insolvency Law where equity holders have less control over the renegotiation process[11]. The privilege of being senior creditor applies in liquidation where, under absolute priority rule (APR), the line of priority and the collateral are the fundamental and sole factors in determining the claim value. However, in renegotiation players split the surplus generated by escaping inefficient liquidation, therefore the overall package of concession extracted also depends on the bargaining power of creditors and the timing in which claims are restructured, that is the impairment strategy.

Furthermore, we provide a sufficient condition for avoiding spread inversions. Spread inversions does not occur if the junior creditor is restructured/impaired before the senior creditor[12]. In turn, such impairment strategy results to be the equilibrium strategy if the senior claimholder has: (i) sufficiently low unsecured face value relatively to the junior[13] and/or (ii) high liquidation value [14] and/or (iii) sufficiently bargaining power relatively to the junior creditor. Also, it is interesting to note that, in the case where the senior and junior creditors are both unsecured, the priority of claims does not affect the impairment strategy and, hence, the possibility of spread reversals. What matters is only the bargaining power and face values of creditors (that is, point (i) and (iii) above). In particular, the spread is higher for the creditor with higher ratio of face value to bargaining power and this occurs regardless of the priority structure.

[11]The UK Insolvency Law has generally been perceived to allocate substantial privileges to creditors, particularly to senior creditors holding a floating charge – that is, a floating collateral attached to the overall company's assets over which the company retains management autonomy. Yet, recently, new insolvency provisions contained in the 2002 UK Enterprise Act seem to reduce the power of senior creditors in the event of default by circumscribing the administrative receivership. The administrative receiver, appointed by the holder of a floating charge, had the primary duty to realise sufficient assets to pay creditors with higher priority. As argued by Gower (1992), in very few cases the receiver manages so skilfully that the company is restored to solvency. This might be one of the reasons to move the law toward Chapter 11 style – less creditor friendly.

[12]The senior claim will be restructured only if the firm value continues deteriorating and falls below a certain level which triggers a new bankruptcy proceeding.

[13]That is, the unsecured portion of the senior face value is sufficiently small relative to the junior face value.

[14]As in Mella-Barral Perraudin (1997), in our strategic default equilibrium the senior claim value in liquidation is equal to the firm liquidation value.

Third, our in-court restructuring can provide a benchmark for out-of-court restructuring of single claims. We assume that there are no renegotiation costs in the formal bankruptcy proceeding. Therefore, extending this assumption to out-of-court renegotiation implies that our equilibrium in the formal renegotiation also provides the equilibrium in private workouts. In this perspective, our setting justifies the possibility of out-of-court debt forgiveness and strategic default on a single class of claims. This result might be relevant when recognising that, although debt issues often contain cross default provisions, this does not generally prevent private workout and strategic default on a single creditor. Indeed, our equilibrium is consistent with the possibility of out-of-court strategic default on single class of claims accommodated by other classes of claimants.

Fourth, we show that given the overall level of leverage the value of the firm does not depend on the allocation of face value amongst senior and junior creditors because strategic debt service eliminates direct bankruptcy costs. Therefore, our result of irrelevance of debt priority structure is consistent with the Modigliani-Miller theorem. In addition, this result extends and refines Mella-Barral, Perraudin's results (where bankruptcy costs from inefficient liquidation are eliminated through strategic bankruptcy) to a scenario with multiple creditors.

The remainder of the paper is organised as follows. After a brief review of the related literature, in Section 1, we introduce our main assumptions about the value of the firm when it continues to operate and when it is liquidated. In Section 2, the bankruptcy rules defining the game-theoretic structure of the formal renegotiation are set out. In Section 3, we show a simpler version of the model, describing a formal renegotiation between the equity holders and a single creditor. This simplified scenario allows the reader to easily understand the mathematical methodology extensively used in the next sections. This section can also be understood as an application or an extension of MBP technique within our bargaining settings. In Section 4, we present the model with two classes of debt. In Section 5, we summarise and explain our results, in terms of timing of bankruptcy (and impairment strategy), claim values, and restructuring plan. In Section 6, we consider the implications of our previous results on the credit spreads. Section 7 concludes.

Related Literature Overview

Initiated by Merton (1974), the traditional financial option approach to debt valuation does not include the possibility of debt restructuring and has the shortcoming of underestimating corporate spreads. The crucial issue, only recently addressed in the literature, is that financial distress is often accompanied by renegotiations, debt rescheduling, forgiveness, and rarely by liquidations[15].

The new approach to value corporate debt incorporates strategic considerations and, to a more limited extent, bargaining issues. This includes the work of Leland (1994), Leland and Toft (1996), Anderson and Sundaresan (1996) and Mella-Barral and Peraudin (1997). Leland (1994) endogenises the restructuring threshold by allowing the payment of promised coupons through additional equity issues until the equity value is driven to zero. The initial paper by Leland (1994) has been refined and extended in several directions, including finite maturities and cash payout, by Leland and Toft (1996). Strategic bankruptcy has been extensively refined by Mella-Barral and Perraudin (1997) who, in coninuous-time setting, model strategic debt service when bankruptcy is costly[16]. Representing two polar cases, where either the equity holders or the creditor can make take-or-leave-it offer, they conclude that strategic debt service results in deviations from absolute priority intuitively because bankruptcy costs from inefficient liquidation allow equity holders to extract concessions from bond-holders. Moreover, their result significantly increases corporate credit spreads for reasonable parameterization of the model. Exploring two alternative bargaining formulations, such as strategic debt service and debt/equity swap, Fan and Sundaresan (2000) endogenise dividend policy and the optimal value of the firm under the two alternate renegotiations.

Even though to a limited extent, most of the recent works, including papers by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Hege and Mella-Barral (2000), Hege and Mella-Barral (2002) and Fan and Sundaresan (2000), attempt to address the importance of claimholders' bargaining power. This limited literature recognises the possibility that players might be heterogeneous in terms of bargaining power and, in turn, the value of renegotiable claims might reflect such a heterogeneity. Yet, apart from Fan and Sundaresan (2000) who accommodate varying bargaining powers, polar cases are characterised regarding the distribution of bargaining power amongst equity holders and creditors. To a certain extent, our paper can be compared to Fan and Sundaresan (2000) in that, similarly, they characterise a Nash axiomatic solution and explicitly account for varying bargaining power. Nevertheless, in contrast to Fan and Sundaresan (2000), we stress the strategic interaction between three diverse players who can renegotiate the debt service by mean of temporary concessions (strategic debt service, as defined by Anderson

[15]As reported by Franks and Torous (1989), after the introduction of the 1978 U.S. Bankruptcy Act, the number of firms seeking for bankruptcy protection has tremendously increased.

[16]Anderson and Sundaresan (1996) model a similar scenario in a discrete-time framework.

and Sundaresan (1996) and MBP (1997)), while Fan and Sundaresan (2000) focus on two formulations of renegotiation (such as strategic debt service and debt/equity swap) between two players.

Looking at the renegotiation framework, those few papers on strategic bankruptcy accounting for bargaining power considerations, in particular Mella-Barral and Perraudin (1997), Hege and Mella-Barral (2000), Hege and Mella-Barral (2002) and Fan and Sundaresan (2000), incorporate renegotiation by modelling private workouts where generally disagreement triggers liquidation together with the Absolute Priority Rule. Corporate debt valuation models of out-of-court renegotiations provide a useful framework to analyse private workouts. However, we stress that a formal bankruptcy proceeding, such as Chapter 11, differs from private workouts in a number of dimensions which affect the values of corporate securities. Hence the applicability of informal renegotiation models to in-court renegotiation system may be limited.

As argued by Sundaresan (2000), "the existence or absence of a bankruptcy code and its perceived *friendliness* to borrowers or lenders, is a matter of significance in these markets. Yet, we have very few pricing theories that have explicitly addressed this as a structural issue in the determination of spreads". The key role played by the bankruptcy code in the valuation of renegotiable debt is yet to be modelled satisfactorily (Sundaresan (2000)).

Partly, the lack of structural issues can be attributed to the fact that the concern in pricing corporate debt has been more focused on liquidation than renegotiation scenarios, and liquidation rules can be simply accounted for by incorporating the Absolute Priority Rule[17].

Furthermore, it is widely recognised that Chapter 11's rules are quite tedious and modelling the formal renegotiation might add excessive complications without any further insight beyond the trivial fact that 'strong' players may obtain a bigger share of the 'pie' during renegotiation. The recent piece of literature on strategic bankruptcy and strategic debt service seems to share this perspective and, to some extent, we share it as well, at least in the situation where a firm has issued a single class of claims.

[17]See for instance Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995) where the occurrence of default is exogenous, or Leland (1994), Leland and Toft (1996) with endogenous bankruptcy threshold level.

1 Firm Value: Basic Assumptions

The value of the firm is driven by an underlying cash flow process, p_t, which follows a geometric Brownian motion with drift μ and volatility σ. For simplicity, we assume that there are no variable costs and the scrapping value of the firm, γ, is constant. Moreover agents are risk neutral and fully informed, with a risk-free interest rate r.

As the business can be shut down, the value of the firm can be written as

$$V(p_t, \underline{p}) = \frac{p_t}{r-\mu} + \left(\gamma - \frac{\underline{p}}{r-\mu}\right)\left(\frac{p_t}{\underline{p}}\right)^{\lambda}$$

where λ is the negative root of the quadratic equation[18]

$$r - \mu\lambda - \frac{\sigma^2}{2}\lambda(\lambda-1) = 0$$

and is equal to

$$\lambda = \frac{-(\mu-\sigma^2/2) - \sqrt{(\mu-\sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2}.$$

The value of the firm is maximised when the threshold level for shutting down, $\underline{p}$, is such that

$$\underline{p} = \arg\max V(p_t, \underline{p}),$$

which results into

$$\underline{p} = \frac{\lambda}{\lambda-1}\gamma(r-\mu).$$

In order to capture the idea that liquidation is generally inefficient as direct bankruptcy costs arise, we assume that at any time the firm can be liquidated and sold to a potential new owner who is as efficient as the original owner[19]. The liquidation sale of the firm occurs according to a Nash bargaining situation between the initial owner (i.e. the equity holders) and the new owner, where the terms of the agreement define the firm sale price, say $V_L(p_t)$, which is therefore the unknown of the bargaining problem. As well known, the axiomatic solution to the Nash bargaining problem can be found by maximising the "Nash product", say NP, which is the product of the differences between the agreement and disagreement payoff for each player. In our case, the agreements payoffs are: $V_L(p_t)$ to the initial owner (that is, the sale price agreed on and received by initial owner) and

[18] We do not provide here the derivation of the firm value through the stochastic calculus. We refer the reader to Dixit and Pindyck (1994) for a general analysis of entry and exit decision under uncertainty. See also MBP (1997), for the similarity to our analysis.

[19] This means that the new owner can generate a value $V(p_t)$ by running a pure equity firm.

$V(p_t) - V_L(p_t)$ to the new owner[20]. The disagreement payoffs are simply: γ to the initial owner (i.e. if no agreement is reached, the firm is sold piecemeal at its scrapping value) and zero to the new owner. Also, we do not restrict this bargaining situation to be symmetric and therefore one can imagine that the initial owner has a relative bargaining power $\alpha \in [0, 1)$ and, hence, the new owner has bargaining power $1 - \alpha$. Given these specifications, the Nash product can be written as

$$NP = (V_L(p_t) - \gamma)^{\alpha}(V(p_t) - V_L(p_t))^{1-\alpha}$$

and it can be easily found that the agreed sale price which maximises the Nash product is

$$\arg\max NP(V_L(p_t)) = V_L(p_t) = \alpha V(p_t) + (1 - \alpha)\gamma. \tag{1}$$

Therefore from our bargaining formulation, the liquidation value is a weighted average between $V(p_t)$ and γ, and hence it is always greater or equal to the scrapping value. Even though this specification of liquidation value, V_L, might partly resemble the formulation by Leland (1994) and Leland and Toft[21] (1996), our formulation of V_L is more in line with Mella-Barral, Perraudin (1997) in that their liquidation value is always below the maximum firm value $V(p_t)$ but never below the scrapping value of the firm. Moreover their bankruptcy costs become zero as the state variable, p_t, approaches the optimal shutting down trigger, which is also confirmed by our bargaining formulation of[22] V_L.

Furthermore, we assume that the firm has issued a perpetual debt with face value F. The debt is allocated over two classes of claims, a senior and and a junior claim with face values respectively $F_s = b_s/r$ and $F_j = b_j/r$ (and $F_s + F_j = F$) where b_s and b_j are the contractual coupon payments. To make the problem interesting we assume that the senior claim is under-secured, i.e.[23] $F_s > \gamma$.

[20]The new owner receives the firm value (free of debt) which is $V(p_t)$ and pays the agreed price $V_L(p_t)$ to the initial owner.

[21]Their formulation is $V_L = \alpha V$ (in our notation).

[22]Note, the bankruptcy costs correspond to the difference $V(p_t) - V_L(p_t)$ which also rewrites as $(1 - \alpha)(V(p_t) - \gamma)$. Hence the bankruptcy costs converge to zero as p_t tends to the optimal shutting down trigger $\underline{p}$.

[23]If the senior claim is fully secured renegotiation involves only the unsecured or under-secured claim. Even though we do not analyse this case, we show in Section 3 how renegotiation resolves when the firm has issued a single class of under-secured claims. Extending this scenario by adding up a fully secured claim is straightforward.

2 Bankruptcy Rules

This section shortly describes the basic assumptions about the formal bargaining framework. The aim is to provide a set of rules to stylise the renegotiation process in the event of default. Our settings are consistent with the U.S. corporate bankruptcy regulation (Chapter 11).

Timing of Bankruptcy. As in Mella-Barral, Perraudin we assume that the firm can voluntarily go into bankruptcy by ceasing paying the contractual coupon. Strategic bankruptcy implies that the equity holders are free to issue new equity to cover operating losses. As soon as the firm stops meeting its contractual obligation a formal renegotiation procedure is triggered.

First Proposal and Impairment Rule. Following the US bankruptcy code for corporate renegotiation (Chapter 11), we assume that the equity holders have an exclusive right to propose a first reorganisation plan[24], which must be approved by all claimants[25].

Moreover, again as in Chapter 11, we assume that a creditor cannot reject a plan if he/she receives cash equal to the face value of his claim or the plan calls for no scaling down of the coupon payment scheduled in the existing contract. In this case the creditor is said to be unimpaired[26] by the plan and loses his veto power[27]. This 'impairment' rule is useful to equity holders who can thus negotiate with one creditor at a time.

To sum up, at this first stage of the renegotiation, the equity holders have the right to make a take-it or leave-it offer, which impairs one or both creditors. If this offer is accepted by the impaired creditors (as the unimpaired lose veto power) the game ends and restructuring goes ahead. This initial stage is referred to as 'private game'.

Subsequent Proposals. On rejection of the first proposal, without any time delay, the renegotiation moves to a second stage in which any player is allowed to file competing plans.

The bankruptcy code does not set any specific agenda rules concerning subsequent proposals. Therefore, on rejection of the equity holders' proposal, the 'rules' of the game

[24]Bankruptcy Reform Act 1978, Section 1121.

[25]In Chapter 11, more precisely, approval of two-thirds majority within each class is required but to keep the bargaining simple we treat each class of claimants as a single agent.

[26]Bankruptcy Reform Act 1978, Section 1124.

[27]Bankruptcy Reform Act 1978, Section 1126.

become in a way unbiased towards different players. The result of the renegotiation therefore can only depend on the ability of players to propose reorganisation plans. Many different ways of modelling the second stage of the renegotiation are possible[28]. We opt for a simple framework designed to highlight the effect of the different bargaining power of players who might well differ in organisational skills. The formal structure of our framework does not favour any player particularly.

We assume, at this stage, that the outcome of the renegotiation is provided by a Nash axiomatic solution where players split the gains generated from restructuring the business according to their bargaining strength[29]. The generalised Nash axiomatic approach seems to best fit a formal renegotiation which implies the existence of a referee and requires players to commit to their disagreement payoffs. Moreover the Nash axiomatic approach is easy to handle and extends unchanged to multilateral bargaining situations. It also provides a general and flexible result by allowing for asymmetric bargaining power[30].

If no agreement is reached at this stage, without time delay, the firm goes into liquidation, therefore the disagreement payoffs in the Nash bargaining correspond to the liquidation payoff of each claimant.

More formally, let $i = e, s, j$ denote the equity holders, the senior and the junior creditor respectively. As well known[31], in the Nash axiomatic bargaining player 'i' can guarantee his/her disagreement payoff (i.e. liquidation payoff), L_i, plus a share of the surplus, $V(p_t) - V_L(p_t)$, which depends on the relative bargaining power[32]. Therefore, in

[28]For instance, Brown (1989) assumes that the Bankruptcy Court determines the order in which proposals are voted and each proposal might rank first, second or third in the agenda with same probability.

[29]Introducing an exogenous bargaining power simply captures the idea that players might influence the Bankruptcy Court as Chapter 11 is a judicially supervised bargaining process (see A. Schwartz, 2002). Moreover in a mandatory Bankruptcy system where parties must use the state supplied procedure it is more realistic to imagine that courts are not perfectly informed, and therefore the judge's decision is conditional to the information disclosed by the parties. Therefore the bargaining skills of players at influencing the Court's decision are crucial (see Welch, 1997).

[30]Fan and Sundaresan (2000) use the Nash axiomatic approach, with asymmetric bargaining power, to model strategic debt service (as well as debt/equity swap) in a private workout between equity holders and a single class of debt holders. Apart from the similarity in the Nash bargaining, their study crucially differs from the current model. We deal with multiple classes of debt in formal bankruptcy while they compare strategic debt service and debt/equity swap when there is only one class of debt holders.

[31]See Binmore and Dasgupta (1987) for an extensive analysis of the Nash bargaining solution. We also refer to Fan and Sundaresan (2000), for the similarity to the current paper. For a more detailed derivation of the Nash bargaining solution when uncertainty is modelled through a geometric Brownian motion, see also Perraudin and Psillaki (1999).

[32]See Fan and Sundaresan (2000) for a similar characterisation of the Nash bargaining solution.

the Nash bargaining, player 'i''s claim value is equal to

$$\underline{\mathrm{P}}_i = \xi_i(V(p_t) - V_L(p_t)) + L_i, \tag{2}$$

where ξ_i and L_i denote the bargaining power and the liquidation payoff of player 'i' respectively. We will refer to this stage as the 'joint game'.

Liquidation. If the firm is liquidated, the Absolute Priority Rule (APR) applies. Formally, under APR the liquidation payoffs for s, j and e are respectively

$$L_s = \min\{F_s, V_L\} \tag{3}$$

$$L_j = \min\{F_j, V_L - L_s\} \tag{4}$$

$$L_e = \max\{V_L - L_s - L_j, 0\}. \tag{5}$$

Terms of Contracts and Enforceability. In practice, the renegotiation can be implemented by reducing coupon payments while the firm is in a default region, through a variable, state contingent, debt service flow as in MBP (1997). Therefore in what follows we will focus on restructuring plans which consist of piecewise right-continuous service flow function of the state variable p_t, denoted as $b_i(p_t)$ for $i = s, j$ with $b_i(p_t) < b_i$.

As the equity holders cannot commit to remain out of bankruptcy and can trigger renegotiation at any time, creditors will underwrite a state contingent contract only if this is self enforceable on the equity side (in the sense that the equity holders will not have incentive to deviate). It is reasonable then to imagine that players bargain over short term contracts, i.e. instantaneous payoffs, and that they can renegotiate continuously. Though this might be not realistic, a continuous time renegotiation provides the necessary benchmark to derive a self enforceable contract.

We conclude this section by summarising the bankruptcy rules and introducing some notation. When the equity holders trigger bankruptcy the renegotiation is structured as a two-step game without time delay in between the first and the second stage. In the first game, the "private game", having the right to the first proposal, the equity holders make a take-it or leave-it offer which might impair the junior creditor, the senior or both creditors. Let the set of plans proposed by the equity holders be $\mathcal{P} = \{P_e, P_j, P_s : P_e + P_j + P_s = V(p_t)\}$. At the second stage, "joint game", each player can guarantee a payoff, denoted as $\underline{\mathrm{P}}_i$ for $i = e, j, s$ which depends on the bargaining strength of players. Therefore a reorganisation plan will be accepted at the first round by an 'impaired claimant', denoted by 'i', only if $P_i \geq \underline{P}_i$. Panel A shows the two-step game in which, at the first round, the equity holders choose whether to impair: i) the junior creditors by proposing a plan $\mathcal{P}_j \subset \mathcal{P}$ where $\mathcal{P}_j = \{P_e, P_j, P_s : P_e \geq \underline{P}_e, P_j \geq \underline{P}_j, P_s < \underline{P}_s\}$, ii) the senior creditor by proposing a plan $\mathcal{P}_s \subset \mathcal{P}$ where $\mathcal{P}_s = \{P_e, P_j, P_s : P_e \geq \underline{P}_e, P_j < \underline{P}_j, P_s \geq \underline{P}_s\}$ or iii) both the senior and the junior creditors and in this case the only feasible plan, say $\mathcal{P}_{s,j} \subset \mathcal{P}$, must guarantee that each player receives his reservation payoff in the joint game, i.e. $\mathcal{P}_{s,j} = \{P_e, P_j, P_s : P_e = \underline{P}_e, P_j = \underline{P}_j, P_s = \underline{P}_s\}$.

Panel A: Bankruptcy game

Private game		Joint game		Liquidation
'e' offers: $\mathcal{P}_j$ impairing 'j', or $\mathcal{P}_s$ impairing 's', or $\mathcal{P}_{s,j}$ impairing both.	on rejection by: j s at least one player $\longrightarrow$	i=e,j,s, share the 'surplus' according to a payoff $\underline{P}_i$.	on rejection by at least one player $\longrightarrow$	Liquidation payoffs L_i, for $i = e, j, s$ and $\sum L_i = V_L$

3 A Simplified scenario: one creditor only

Before describing the renegotiation with two creditors, we briefly show a simplified scenario in which the firm has issued only one class of under-secured claims, with face value $F_c = b_c/r > \gamma$[33]. This simplified scenario allows the reader to easily understand the mathematical methodology we will extensively use in the next section in order to calculate claims, equity value and restructuring plan (i.e. a debt service flow function $b_c(p_t) < b_c$). The technique shown here is similar to that developed by Mella-Barral, Perraudin[34]. Therefore, this section can also be understood as an application or an extension of Mella-Barral, Perraudin's technique within our bargaining setting.

[33]In this section, we use the subscript 'c' to denote the debt holder.

[34]As the purpose of this paper is quite different from that of Mella-Barral, Perraudin, we limit this section to their mathematical development. Therefore, for an understanding of their economic intuition the reader is referred to their paper.

As there is only one creditor, some of the bankruptcy rules defined in Section 2 are redundant here. More specifically, it is trivial to notice that the equity holders cannot strategically use the impairment rule because there is no other class of claims which can be left unimpaired. Moreover, the private game becomes redundant here, in the sense that the equilibrium shares in the private game guarantee players with their equilibrium shares in the joint game. In fact, given the equilibrium partition in the joint game, i) the claim holder accepts the first equity holders' proposal only if he/she receives at least the equilibrium share of the joint game and ii) the equity holders will offer at most the creditor's equilibrium share of the joint game. Because there is no time delay between the private and the joint game, and hence the value of the 'pie' does not change, it follows that the equilibrium shares in the private and the joint game are the same. This allows us to skip the private game and solve the bankruptcy game by simply solving the two-player Nash axiomatic bargaining.

Apart from these changes, the model develops within the same setting and assumptions introduced in the previous section.

Hypotheses 1A. Assume that there exists a strategy in terms of stopping times, such that i) when the state variable p_t crosses a certain trigger, say p_c, the equity holders renegotiate with the debt holder in a Nash bargaining (through a service flow $b_c(p_t) < b_c$).

Therefore, given that p_c exists, one can formalise the equity holders' decision problem as follows

$$\begin{aligned} &\max_{p_c} E(p_t, p_c) \\ &\text{st. } E(p_t, p_c) = V(p_t) - C(p_t, p_c)) \\ &\qquad\qquad C < F_c, \end{aligned} \tag{6}$$

where $C(p_t, p_c)$ is the debt value for $p_t \geq p_c$ and F_c is the face value of the debt.

Hypotheses 2A. When the state variable is in the range $[\underline{p}, p_c]$ we assume that $V_L < F_c$. Because APR applies, this implies that $L_e = 0$ and $L_c = V_L$.

Renegotiation unfolds through a Nash bargaining where, by 2 and Hypothesis 2A, the shares to the equity holders and the creditor become respectively

$$\begin{aligned}\underline{\text{P}}_e &= \xi_e(V - V_L) & (7)\\ \underline{\text{P}}_c &= \xi_c(V - V_L) + V_L & (8)\\ \text{with} \quad & \begin{cases} \xi_i \in I = [0,1] \\ \sum_i \xi_i = 1 \end{cases} \quad \text{for} \quad i = e, c. \end{aligned}$$

As the renegotiated shares are already defined by 7 and 8, left to determine are the trigger p_c and the service flow $b_c(p_t)$.

In order to derive the bankruptcy trigger p_c, one can solve the equity holders' maximisation problem, or equivalently, the minimisation problem:

$$\begin{aligned} \min_{p_c} \quad & C(p_t, p_c) & (9)\\ \text{st.} \quad & C < F_c \\ \text{with} \quad & \left\{ C(p_t) = \underline{\text{P}}_c \quad \text{for} \quad p_t = p_c \right. \end{aligned}$$

where the debt value, $C(p_t, p_c)$ can be written as

$$C(p_t) = \frac{b_c}{r} + \left(\underline{\text{P}}_c(p_c) - \frac{b_c}{r}\right)\left(\frac{p_t}{p_c}\right)^{\lambda}. \qquad (10)$$

Solving the above problem yields the optimal trigger

$$p_c^* = \frac{\lambda}{\lambda - 1}\frac{b_c/r - \gamma(1 - \alpha_{\xi_c})}{\alpha_{\xi_c}}(r - \mu), \qquad (11)$$

with

$$\alpha_{\xi_c} = \xi_c(1 - \alpha) + \alpha. \qquad (12)$$

Moreover, as shown in Appendix 1, the trigger p_c^* guarantees that Hypotheses 2A holds[35].

What is left to derive is the restructuring plan. Similarly as in MBP (1997), one can derive the service flow function $b_c(p_t)$ as follows. We know that, under risk neutrality, the claim value, $C(p_t)$ is free of arbitrage opportunity if and only if $b_c(p_t)$ solves the differential equation

$$rC(p_t) = s(p_t) + \frac{d}{d\triangle}E_t(C_{t+\triangle}) \mid_{\triangle=0}, \qquad (13)$$

[35] In particular, we prove in Appendix 1 that $F_c > \underline{\text{P}}_c(p_c^*)$, which (by 8) implies $F_c > V_L$, i.e. Hypotheses 2A.

where

$$s_c(p_t) = \begin{cases} b_c & \text{if} \quad p_t \in [p_c^*, \infty) \\ b_c(p_t) & \text{if} \quad p_t \in [\underline{p}, p_c^*) \end{cases}$$

subject to the following conditions. First, $C(p_t)$ must be continuous in level and first derivative, which implies

$$C(p_c^*) = \underline{\mathrm{P}}_c(p_c^*) \tag{14}$$

$$\frac{\partial C(p_t)}{\partial p_t}\,|_{p_c^*} = \frac{\partial \underline{\mathrm{P}}_c(p_t)}{\partial p_t}\,|_{p_c^*}\,. \tag{15}$$

This conditions are also known as smooth-pasting conditions. Secondly, equation 13 must satisfy no arbitrage and no bubbles conditions which correspond respectively to

$$\underline{\mathrm{P}}_c(\underline{p}) = \gamma, \tag{16}$$

$$\lim_{p_t \to \infty} C(p_t) = \frac{b_c}{r}. \tag{17}$$

One can find that the debt value, $C(p_t)$, defined by 10, satisfies all the above conditions[36].

As shown in Appendix 3, there is a unique function, $s(p_t)$, which solves 13, and this is given by

$$s_c(p_t) = \begin{cases} b_c & \text{if} \quad p_t \in [p_c^*, \infty) \\ b_c(p_t) = \alpha_{\xi_c} p_t + (1 - \alpha_{\xi_c}) r\gamma & \text{if} \quad p_t \in [\underline{p}, p_c^*) \end{cases}$$

with α_{ξ_c} defined in 12.

In Figure 1, we show the renegotiated value of the debt under strategic debt service. For $p_t \leq p_c^*$, the debt value is always between $V(p_t)$ and the liquidation value $V_L(p_t)$. Moreover it is equal to either $V(p_t)$ or $V_L(p_t)$ only in the extreme scenarios where either the creditor or the equity holders have full bargaining power. In these extreme cases our result is similar to MBP[37] and, to a certain extent, this section provides a generalisation of their results which accommodate varying bargaining powers.

[36] In fact 10 satisfies conditions 14, 16 and 17 by construction. In Appendix 2, we show that the first order condition to the equity holders' maximisation problem, that is $\partial C(p_t, p_c)/\partial p_c = 0$, is equivalent to the smooth pasting condition 15 which is, therefore, satisfied.

[37] There are marginal differences due to the modelling of direct and indirect bankruptcy costs. In Mella-Barral, Perraudin, when the debt is not renegotiable, on default, the debt holders take over the firm and run the business less efficiently (in terms of net cash flows) than the former management. This assumption allows one to account also for indirect bankruptcy costs, while in our model indirect bankruptcy costs are zero.

Apart from the methodological purpose of this section, we want to stress that the formal setting of the bankruptcy process does not play a crucial role when there are only two players at the negotiating table. In fact, as argued, most of the literature on debt restructuring does not focus on formal bankruptcy but rather on private workouts when there is a single class of claims. In line with this literature, we have pointed out that most of the bankruptcy rules appear to be redundant in this simplified scenario. Moreover, the advantage of formalising the bankruptcy process seems quite limited in terms of economic intuition. Quite different is the purpose of the bankruptcy rules when there are multiple claims. As we show in the next section, all of the formal rules are strategically enforced by the equity holders.

4 The Model

In this section we show how renegotiation is implemented when the firm has issued two classes of claims as defined in Section 1. In this scenario, none of the bankruptcy rules described in Section 2 is redundant. On the contrary, one can notice here how our bankruptcy framework helps to define a unique equilibrium in terms of renegotiating strategy and restructuring plan.

Hypotheses 1 Assume that there exists a strategy in terms of stopping times, such that i) when the state variable p_t crosses a certain trigger, say p_j, the equity holders start renegotiating with the junior creditor while $p_t < p_j$ (by proposing a service flow $b_j(p_t) < b_j$) and ii) when p_t crosses the trigger p_s they start renegotiating with the senior creditor (through a service flow $b_s(p_t) < b_s$).

Now, let $max\{p_j, p_s\} = p^*$ and $min\{p_j, p_s\} = p_*$. Consistently within our bargaining framework the equity holders strategy can be summarised as follows:

$$\text{when} \quad p_* < p_t \leq p^* \text{ propose plan } \mathcal{P}_i,\ b_i(p_t) \text{ with } \begin{cases} i = j \text{ if } p^* = p_j \\ i = s \text{ otherwise} \end{cases} \tag{18}$$

$$\text{when} \quad \underline{p} \leq p_t \leq p_* \text{ propose plan } \mathcal{P}_{s,j}, \begin{cases} b_j(p_t) \\ b_s(p_t) \end{cases} \tag{19}$$

$$\tag{20}$$

Therefore, the problem of the equity holders can be generalised as

$$\max_{p_j, p_s} E(p_t) \tag{21}$$
$$\begin{aligned} \text{st. } E(p_t) &= V(p_t) - S(p_t, p_s, p_j) - J(p_t, p_j, p_s) \\ S &< F_s \\ J &< F_j, \end{aligned}$$

where $E(p_t)$, $S(p_t)$ and $J(p_t)$ denotes the values of the equity, the senior and the junior debt and F_s and F_j are the face values.

Hypotheses 2 We assume that $V_L < F_s$, when the state variable is in the range $[\underline{p}, p_s]$. As absolute priority applies, this implies that $L_e = 0, L_j = 0$ and $L_s = V_L$, therefore the shares in the joint game will be

$$\underline{\mathrm{P}}_e = \xi_e(V - V_L) \tag{22}$$
$$\underline{\mathrm{P}}_j = \xi_j(V - V_L) \tag{23}$$
$$\underline{\mathrm{P}}_s = \xi_s(V - V_L) + V_L \tag{24}$$
$$\text{with} \quad \begin{cases} \xi_i \in I = [0,1] \\ \sum_i \xi_i = 1 \end{cases} \quad \text{for} \quad i = e, j, s.$$

4.1 Equilibrium offers and trigger strategy

The purpose of this section is to determine the equilibrium values, E, J and S and the optimal trigger strategy $\{p_j, p_s\}$. We already know, by 22-24, the values of equity, senior and junior claims when[38] $\underline{p} \leq p_t \leq p_*$, therefore the problem is to determine E, J and S when $p_* < p_t \leq p^*$.

Our argument to solve the renegotiation runs as follows.

We determine E, J, S and the trigger $\{p_j, p_s\}$ under the two possible strategies: CASE A) $p_s < p_j$, i.e. the equity holders impair the junior first, CASE B) $p_s \geq p_j$, the senior creditor is impaired first (if $p_s > p_j$) or jointly with the junior creditor (for $p_s = p_j$).

In order to determine the equilibrium values in both cases, A and B, we use a no arbitrage argument (in the remainder point (i)) and a backward solution when solving for

[38]In fact, if $p_* = min\{p_j, p_s\} = p_s$, then by Hypotheses 2 $V_L(p_s) < F_s$ and therefore, for $p_t < p_s$, players' payoffs are given by 22-24. If $p_* = min\{p_j, p_s\} = p_j$, then it is the case that $p_j < p_s$ and, being $V_L(p_t)$ decreasing in p_t, it holds from Hypotheses 2 that $V_L(p_j) < V_L(p_s) < F_s$, therefore again for $p_t < p_s$, players' payoffs are given by 22-24.

the optimal triggers $\{p_j, p_s\}$ (point (ii)).

Then, we show that depending on the level of the parameters, ξ_i (for $i = e, j, s$), F_j, F_s, α, r and γ, a unique equilibrium strategy exists.

CASE A: $p_s < p_j$**.** The equity holders impair the junior creditor first, when $p_s < p_t \leq p_j$, and then the senior creditor also when $\underline{p} < p_t \leq p_s$.

(i) We define here the junior claim equilibrium value under an accepted offer from the equity holders in the private game.

Proposition 4.1.1 *When $p_s < p_t \leq p_j$, the smallest offer by the equity holders to the junior creditor is accepted if and only if $J(p_t, p_j) = \underline{P}_j + \underline{P}_s - S(p_t, p_s)$.*

Proof Let us define first an arbitrage strategy. In the private game, when $p_s < p_t \leq p_j$, after an equity holders' offer, the junior creditor can: 1) buy the senior claim at $S(p_t, p_s)$, 2) reject the equity holders' plan, therefore losing $J(p_t, p_s)$ and 3) before the start of the joint game, sell the senior claim at $\underline{P}_s(p_t)$. The payoff from such a strategy, say Π, is given by

$$\Pi = \underline{P}_j + \underline{P}_s - J(p_t, p_j) - S(p_t, p_s). \tag{25}$$

We show in Appendix 4 that $\Pi \geq 0$ under an equity holders' proposal belonging to $\mathcal{P}_j$. Therefore an offer from the equity holders, is accepted by the junior creditor, if and only if $\Pi \leq 0$, that is $J(p_t, p_s) \geq \underline{P}_j + \underline{P}_s - S(p_t, p_s)$. Then we conclude that the smallest offer which is accepted by the junior creditor must be such that $\Pi = 0$. ■

Therefore, by Proposition 4.1.1, 'j''s equilibrium value (under an accepted offer) is such that $\Pi = 0$, which, by 25, implies

$$J(p_t, p_j) = \underline{P}_j + \underline{P}_s - S(p_t, p_s). \tag{26}$$

(ii) The triggers p_s and p_j can be found by working backward. When p_t is greater than p_s the senior value is equal to

$$S(p_t, p_s) = \frac{b_s}{r} + \left(\underline{P}_s(p_s) - \frac{b_s}{r}\right)\left(\frac{p_t}{p_s}\right)^{\lambda}, \tag{27}$$

which, minimised with respect to p_s gives[39]

$$p^*_{s2} = \frac{\lambda}{\lambda - 1} \frac{b_s/r - \gamma(1 - \alpha_{\xi_s})}{\alpha_{\xi_s}} (r - \mu), \tag{28}$$

with

$$\alpha_{\xi_s} = \xi_s(1 - \alpha) + \alpha. \tag{29}$$

Then, minimising the junior value with respect to p_j, which, by 26 and 28, is equal to

$$J(p_t, p_j) = \frac{b_j}{r} + \left\{ \underline{\mathrm{P}}_j(p_j) + \underline{\mathrm{P}}_s(p_j) - S(p_j, p^*_{s2}) - \frac{b_s}{r} \right\} \left(\frac{p_t}{p_j} \right)^{\lambda},$$

yields the optimal trigger

$$p^*_{j1} = \frac{\lambda}{\lambda - 1} \frac{(b_s + b_j)/r - \gamma(1 - \alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} (r - \mu), \tag{30}$$

with

$$\alpha_{\xi_{s,j}} = (\xi_s + \xi_j)(1 - \alpha) + \alpha. \tag{31}$$

CASE B: $p_s \geq p_j$**.** The equity holders impair the senior creditor first, when $p_j < p_t \leq p_s$, and then the junior creditor also when $\underline{p} < p_t \leq p_j$.

(i) We derive here the senior claim equilibrium value under an accepted offer in the private game when $p_j < p_t \leq p_s$. One can use a similar argument as the previous case (point (i)). In Proposition 4.1.1, replacing the word 'junior' with 'senior', one can conclude that the senior claim equilibrium value (under an accepted offer) must be such that[40] $\Pi = 0$, which implies

$$S(p_t, p_j) = \underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s - J(p_t, p_j). \tag{32}$$

.

[39]We denote the optimal trigger by using the subscript 'in', with $i = s, j$ and $n = 1, 2$, to stress the order in which claims are impaired. Therefore, for instance, when the senior creditor is the second claimant to be impaired we denote the optimal trigger level as p_{s2} (while when the senior creditor is impaired first, the trigger level is denoted as p_{s1}).

[40]As in the previous case, the gain, Π, is equal to $\underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s - J(p_t, p_j) - S(p_t, p_j)$.

(ii) As before, by working backward one can find p_j first and then p_s. Minimising the junior value, which, at $p_t > p_j$ is equal to

$$J(p_t, p_j) = \frac{b_j}{r} + \left(\underline{\mathrm{P}}_j(p_j) - \frac{b_j}{r}\right)\left(\frac{p_t}{p_j}\right)^{\lambda}, \tag{33}$$

gives

$$p^*_{j2} = \frac{\lambda}{\lambda - 1}\frac{b_j/r + \gamma\xi_j(1-\alpha)}{\xi_j(1-\alpha)}(r - \mu). \tag{34}$$

Then, by minimising the senior value with respect to p_s, which (by 32 and 34), when $p_t > p_s$, is equal to

$$S(p_t, p_s) = \frac{b_s}{r} + \left\{\underline{\mathrm{P}}_j(p_s) + \underline{\mathrm{P}}_s(p_s) - J(p_s, p^*_{j2}) - \frac{b_s}{r}\right\}\left(\frac{p_t}{p_s}\right)^{\lambda},$$

and solving for the optimal trigger yields

$$p^*_{s1} = \frac{\lambda}{\lambda - 1}\frac{(b_s + b_j)/r - \gamma(1 - \alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}}(r - \mu), \tag{35}$$

where $\alpha_{\xi_{s,j}}$ defined in equation 31.

Unique Equilibrium strategy

So far we have identified two optimal trigger strategies, in CASE A) a strategy $\{p^*_{j1}, p^*_{s2}\}$ if the junior is impaired first and, in CASE B) a strategy $\{p^*_{j2}, p^*_{s1}\}$ if the senior creditor is impaired first (that is $p^*_{s1} > p^*_{j2}$) or jointly ($p^*_{s1} = p^*_{j2}$) with the junior creditor.

In the remainder of this section we will show that there is a unique equilibrium strategy. The equilibrium is unique if once the equity holders impair creditor $i = s$ or j, leaving creditor $\underline{i} \neq i$ unimpaired, they will not invert strategy in the future (that is, impairing $\underline{i} \neq i$ and paying the full contractual coupon to creditor i).

First, notice that regardless of the fact that the creditor who is impaired first is the junior (as in CASE A)) or the senior one (CASE B)) the higher bankruptcy trigger does not change. In fact,

$$p^*_{j1} = p^*_{s1}.$$

Therefore the equilibrium strategy will only depend on the triggers p_{s2} and p_{j2}.

Let us denote with $\bar{p}$ the trigger $p^*_{j1} = p^*_{s1}$. One can find, by some simple algebra, that

$$\begin{array}{lll} p^*_{s2} < \bar{p} & \text{if and only if} & \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma \\ p^*_{j2} \leq \bar{p} & \text{if and only if} & \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F - \gamma. \end{array} \tag{36}$$

These two inequalities allow us to state our crucial result in the following Proposition.

Proposition 4.1.2 *The optimal strategy is unique and there is no ambiguity in the path of the renegotiation. The equity holders play the optimal strategy*

$$\begin{array}{lll} \{p^*_{j1}, p^*_{s2}\} & \textit{iff} & \frac{b_j}{r}\frac{\alpha\xi_{s,j}}{\xi_j(1-\alpha)} > F-\gamma \\ \{p^*_{s1}, p^*_{j2}\} & \textit{iff} & \frac{b_j}{r}\frac{\alpha\xi_{s,j}}{\xi_j(1-\alpha)} \leq F-\gamma. \end{array} \tag{37}$$

Proof When $p^*_{s2} < \bar{p}$ (i.e. $\frac{b_j}{r}\frac{\alpha\xi_{s,j}}{\xi_j(1-\alpha)} > F-\gamma$) is also true that $p^*_{j2} > \bar{p}$, but p^*_{j2} is an optimal trigger only if $p^*_{j2} \leq p^*_{s1} = \bar{p}$. Therefore, we can conclude that p^*_{j2} does not exist in this case and hence there is a unique strategy left, that is $\{p^*_{j1}, p^*_{s2}\}$. Under this strategy the equity holders impair the junior creditor first when $p_t = \bar{p}$ and senior later, when $p_t = p^*_{s2}$.

If instead $\frac{b_j}{r}\frac{\alpha\xi_{s,j}}{\xi_j(1-\alpha)} \leq F-\gamma$ then $p^*_{j2} \leq \bar{p}$ and $p^*_{s2} > \bar{p}$. Nevertheless, for p^*_{s2} to be optimal it must be the case that $p^*_{s2} \leq p^*_{j1} = \bar{p}$. Similarly as the previous case, we conclude that p^*_{s2} does not exist and there is only one strategy left corresponding to $\{p^*_{j2}, p^*_{s1}\}$. Therefore the equity holders impair the senior creditor first (i.e. when $p_t = \bar{p}$) and, when $p_t = p^*_{j2}$, the junior creditor also. ■

Last, as shown in Appendix 5, one can notice that under both strategies, $\{p^*_{s2}, p^*_{j1}\}$ and $\{p^*_{s1}, p^*_{j2}\}$, Hypotheses 2 holds, that is $V_L(p_s) < F_s$ (with p_s equal to either p^*_{s2} or p^*_{s1}). This guarantees that the senior claims is never renegotiated when the senior creditor could guarantee the full face value in liquidation.

4.2 Restructuring plan

We have derived in the previous section the equilibrium value of claims and the trigger strategy which defines the optimal timing for restructuring each class of claims. In this section, we will determine the debt service flow function which guarantees each creditor with his/her equilibrium values during renegotiation.

Depending on the set of parameters in our model, we have shown that the equilibrium trigger strategy is either $\{p^*_{s2}, p^*_{j1}\}$ (if $\frac{b_j}{r}\frac{\alpha\xi_{s,j}}{\xi_j(1-\alpha)} \leq F-\gamma$) or $\{p^*_{j2}, p^*_{s1}\}$ (if else) with corresponding claims equilibrium values, $J(p_t)$ and $S(p_t)$, defined in CASE A) and B) (of Section 4.1) respectively. Therefore, according to the two possible sets of parameters there are two corresponding restructuring plans defined in the following propositions. Our derivation of the debt service flow functions follows the same line as the derivation of $b_c(p_t)$ in Section 3. Therefore, we only present here our results and refer the reader to the Appendixes for the technical steps.

Proposition 4.2.1 *If* $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma$ *the equilibrium restructuring plans impair the junior and senior creditor according to service flow functions respectively*

$$b_j(p_t) = \alpha_{\xi_{s,j}} p_t + (1-\alpha_{\xi_{s,j}})\gamma r - s_s(p_t) \quad \textit{for} \quad p_t \leq p^*_{j1} \tag{38}$$

$$s_s(p_t) = \begin{cases} b_s & \textit{for} \quad p^*_{s2} < p_t \\ b_s(p_t) = \alpha_{\xi_s} p_t + (1-\alpha_{\xi_s})\gamma r & \textit{for} \quad p_t \leq p^*_{s2} \end{cases} \tag{39}$$

with $\alpha_{\xi_{s,j}}$ *and* α_{ξ_s} *defined by 31 and 29 respectively.*

Proof See Appendix 6. ■

Proposition 4.2.2 *If* $\frac{b_j}{r}(\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)}) \leq F - \gamma$ *the equilibrium restructuring plans impair the junior and senior creditor according to service flow functions respectively*

$$b_s(p_t) = \alpha_{\xi_{s,j}} p_t + (1-\alpha_{\xi_{s,j}})\gamma r - s_j(p_t) \quad \text{for} \quad p_t \leq p^*_{s1} \tag{40}$$

$$s_j(p_t) = \begin{cases} b_j & \text{for} \quad p^*_{j2} < p_t \\ b_j(p_t) = \xi_j(1-\alpha)(p_t - \gamma r) & \text{for} \quad p_t \leq p^*_{j2} \end{cases} \tag{41}$$

Proof See Appendix 7. ■

5 Comments and Summary of results

In this section we recombine and explicit our results in Section 4. The purpose is to explain our results with particular emphasis on the effect of bargaining powers, face values and allocation of priority. We group our findings into three categories : A) timing of bankruptcy and impairment strategy, B) equity and claims values and C) restructuring plan.

A) Timing of bankruptcy and impairment strategy. The equity holders trigger bankruptcy, and file a plan $\mathcal{P}_i$ or $\mathcal{P}_{s,j}$,[41] according to the following equilibrium strategy:

$$\text{when} \quad p^*_{i2} < p_t \leq \bar{p} \quad \text{file plan } \mathcal{P}_{\neq i}, \text{ with } \begin{cases} i = s & \text{if } \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma \\ i = j & \text{otherwise} \end{cases} \tag{42}$$

$$\text{when} \quad \underline{p} \leq p_t \leq p^*_{i2} \quad \text{file plan } \mathcal{P}_{s,j}, \tag{43}$$

[41] We remind the reader that a plan $\mathcal{P}_i$ impairs only creditor i and $\mathcal{P}_{s,j}$ impairs both creditors.

where

$$p^*_{s2} = \frac{\lambda}{\lambda-1}\frac{b_s/r-\gamma(1-\alpha_{\xi_s})}{\alpha_{\xi_s}}(r-\mu), \tag{44}$$

$$p^*_{j2} = \frac{\lambda}{\lambda-1}\frac{b_j/r+\gamma\xi_j(1-\alpha)}{\xi_j(1-\alpha)}(r-\mu), \tag{45}$$

$$\bar{p} = p^*_{j1}=p^*_{s1}=\frac{\lambda}{\lambda-1}\frac{(b_s+b_j)/r-\gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}}(r-\mu), \tag{46}$$

and

$$\alpha_{\xi_s} = \xi_s(1-\alpha)+\alpha, \tag{47}$$

$$\alpha_{\xi_{s,j}} = (\xi_s+\xi_j)(1-\alpha)+\alpha. \tag{48}$$

First, notice that, given the overall face value of the debt, F, the higher bankruptcy trigger is independent of the priority structure of claims. Bankruptcy is triggered the first time, as soon as the state variable crosses $p^*_{s1}=p^*_{j1}$. In other words, given the overall creditors' bargaining power and the face value F, the default region, $[\underline{p},\bar{p}=p^*_{s1}=p^*_{j1}]$ is independent of the type of creditor impaired first and is not affected by the allocation of debt amongst creditors. In order to highlight the irrelevance of the debt priority structure over the default region, one can compare two firms which differ only for their debt allocation. Imagine, for instance the limiting case of an identical firm (where the equity holders have the same bargaining power, ξ_e) with just one creditor with a claim of face value $F=F_j+F_s$ and bargaining power $\xi_c=1-\xi_e=1-\xi_s-\xi_j$. As shown in Section 3, renegotiation starts when p_t crosses $\frac{\lambda}{\lambda-1}\frac{F/r-\gamma(1-\alpha_{\xi_c})}{\alpha_{\xi_c}}(r-\mu)$ which coincides with the optimal trigger[42] $p^*_{s1}=p^*_{j1}=\bar{p}$.

Therefore, not surprisingly, what defines the default region is the level – not the allocation – of F and the overall bargaining power of creditors vis a vis the equity holders. The higher F and/or $\xi_j+\xi_s$ the earlier bankruptcy occurs[43].

If, on the one hand, the debt priority structure (i.e. the allocation of face value amongst senior and junior creditors) does not alter the default region, on the other hand it crucially determines the timing and the order in which claims will be impaired during bankruptcy. In a way, one could say that, given the overall face value, the priority structure of claims determines the 'default regions of each single claim'. Using this terminology, depending on the claims face values, there are two possible default regions for each debt-holder: i)

[42]Simply replace the notation $b_j/r+b_s/r$ instead of F and notice that $\alpha_{\xi_c}=\alpha_{\xi_{s,j}}$.

[43]In fact, $p^*_{s1}=p^*_{j1}$ increases with F and $\xi_j+\xi_s$.

$[\underline{p}, p^*_{j1} = p^*_{s1}]$ and $[\underline{p}, p^*_{s2}]$, respectively for the junior and the senior, if $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma$ or ii) $[\underline{p}, p^*_{s1} = p^*_{j1}]$ and $[\underline{p}, p^*_{j2}]$, if instead $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F - \gamma$ [44].

The intuition behind this result is immediate. We know that the equity holders always start renegotiating at $p^*_{s1} = p^*_{j1}$ and when the state variable crosses this trigger they would benefit from reducing coupon payments to the creditor whose face value is relatively high with respect to the overall face value F. Therefore, given F, if, for instance, b_j/r is high enough the equity holders would benefit from impairing the junior creditor first.

The former consideration is not the only factor determining the order of the impaired claims. The benefit from impairing the creditor with higher face value must be weighted against the 'strength' of that creditor, that is, the ability to extract a valuable package of concessions in renegotiation. This depends on the liquidation value of the firm, the priority of the claim and the creditor's bargaining power. The higher α and γ, the bigger the liquidation value, which, in turn, strengthens the bargaining position of the senior whilst weakens that of the junior[45]. Therefore, when the liquidation value is sufficiently high, the equity holders impair the junior creditor, who can extract smaller concessions than the senior claimant. The argument runs similarly and the same conclusion holds when the bargaining power of the junior creditor is sufficiently small. If this is the case, again, the equity holders impair first the junior creditor, that is, the 'weak' player. All these factors are captured by the inequality $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \gtreqless F - \gamma$ which determines the order in which claims are impaired. In fact, for instance, it is more likely that $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma$ (and hence the junior is impaired first) for high levels of b_j/r, α and γ and low level of ξ_j, consistently with our intuitive explanation.

Finally, our argument can be clearly summarised by rearranging the above inequality as

$$\frac{\alpha_{\xi_s}}{F_s - \gamma} \gtreqless \frac{\xi_j(1-\alpha)}{F_j}.$$

Here, each side of the inequality measures the 'intensity of the actual bargaining strength'. Precisely, the terms α_{ξ_s} and $\xi_j(1-\alpha)$ can be interpreted as *actual* bargaining strength of creditors in that these terms account for i) the exogenous bargaining power, ξ_i, and ii) α which determines the disagreement/liquidation payoff[46](which is an endogenous compo-

[44]When $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} = F - \gamma$, $p^*_{j2} = p^*_{s1}$, that is claims are jointly impaired and the two default regions are the same.

[45]Because the junior creditor is a residual claimant, he/she purely benefits from receiving a share of the firm continuation surplus, $V - V_L$, while the senior creditor, due to the priority of his/her claim, guarantees also the full liquidation value.

[46]In more details, α represents the percentage liquidation payoff of the unsecured part of the senior

nent of the bargaining strength). Moreover, in the above inequality, the actual bargaining strength is measured in terms of units of unsecured face value (that is, $F_s - \gamma$ and F_j), which explains why one can refer to each side of the inequality as to *intensity* of bargaining strength. By using this terminology, we conclude that the equity holders impair first the creditor with smaller intensity of actual bargaining strength.

B) Equity and claims values. According to the previous trigger strategies, if the junior creditor is a 'weak' player in comparison to the senior one, the equity holders file a plan:

$$\begin{matrix} \mathcal{P}_j, & \forall \quad p_t \in (p^*_{s2}, \bar{p}] \\ \mathcal{P}_{s,j}, & \forall \quad p_t \in [\underline{p}, p^*_{s2}] \end{matrix} \Rightarrow \text{ if } \frac{b_j}{r} \frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma \tag{49}$$

which guarantees creditors with the following claims' values:

$$S(p_t) = \begin{cases} \frac{b_s}{r} + \left(\underline{\mathrm{P}}_s(p^*_{s2}) - \frac{b_s}{r}\right)\left(\frac{p_t}{p^*_{s2}}\right)^{\lambda} & \text{if} \quad p_t > p^*_{s2} \\ \underline{\mathrm{P}}_s(p_t) & \text{if} \quad p_t \leq p^*_{s2} \end{cases} \tag{50}$$

$$J(p_t) = \begin{cases} \frac{b_j}{r} + \left(\underline{\mathrm{P}}_j(\bar{p}) + \underline{\mathrm{P}}_s(\bar{p}) - S(\bar{p}) - \frac{b_j}{r}\right)\left(\frac{p_t}{\bar{p}}\right)^{\lambda} & \text{if} \quad p_t > \bar{p} \\ \underline{\mathrm{P}}_j(p_t) + \underline{\mathrm{P}}_s(p_t) - S(p_t) & \text{if} \quad p^*_{s2} < p_t \leq \bar{p} \\ \underline{\mathrm{P}}_j(p_t) & \text{if} \quad p_t \leq p^*_{s2} \end{cases} \tag{51}$$

If instead the junior creditor is a 'strong' player, the equity holders impair first the senior claimant by filing a plan:

$$\begin{matrix} \mathcal{P}_s, & \forall \quad p_t \in (p^*_{j2}, \bar{p}] \\ \mathcal{P}_{s,j}, & \forall \quad p_t \in [\underline{p}, p^*_{j2}] \end{matrix} \Rightarrow \text{ if } \frac{b_j}{r} \frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F - \gamma \tag{52}$$

which guarantees creditors with the following claims' values:

$$S(p_t) = \begin{cases} \frac{b_s}{r} + \left\{\underline{\mathrm{P}}_j(\bar{p}) + \underline{\mathrm{P}}_s(\bar{p}) - J(\bar{p}) - \frac{b_s}{r}\right\}\left(\frac{p_t}{\bar{p}}\right)^{\lambda} & \text{if} \quad p_t > \bar{p} \\ \underline{\mathrm{P}}_j(p_t) + \underline{\mathrm{P}}_s(p_t) - J(p_t) & \text{if} \quad p^*_{j2} < p_t \leq \bar{p} \\ \underline{\mathrm{P}}_s(p_t) & \text{if} \quad p_t \leq p^*_{j2} \end{cases} \tag{53}$$

$$J(p_t) = \begin{cases} \frac{b_j}{r} + \left(\underline{\mathrm{P}}_j(p^*_{j2}) - \frac{b_j}{r}\right)\left(\frac{p_t}{p^*_{j2}}\right)^{\lambda} & \text{if} \quad p_t > p^*_{j2} \\ \underline{\mathrm{P}}_j(p_t) & \text{if} \quad p_t \leq p^*_{j2} \end{cases} \tag{54}$$

debt. In fact, by definition, α can be written as $(V_L - \gamma)/(V - \gamma)$.

where

$$\underline{\mathrm{P}}_e = \xi_e(V - V_L) \tag{55}$$

$$\underline{\mathrm{P}}_j = \xi_j(V - V_L) \tag{56}$$

$$\underline{\mathrm{P}}_s = \xi_s(V - V_L) + V_L. \tag{57}$$

We show in Figure 2, 3 and 4 the equilibrium claim values resulting from three alternative scenarios where the junior is impaired first (that is, $p^*_{s2} < p^*_{j1} = \bar{p}$, as in Figure 2), together (i.e. $p^*_{s2} = p^*_{j1} = \bar{p}$, Figure 3) or after the senior has been impaired (that is, $p^*_{j2} < p^*_{s1} = \bar{p}$, see Figure 4).

One can notice that, given the creditors' bargaining power $\xi_s + \xi_j$ and the face value F, the priority structure of claims does not affect the equity value. In fact, the debt value, $S(p_t) + J(p_t)$, remains the same regardless of the allocation of face value amongst creditors.

This result is in line with MBP (1997), in that the equity value is affected by the overall face value and the bargaining power of the equity holder. Moreover it extends MBP results to a multiple creditors scenario, in that the allocation of debt amongst classes is irrelevant.

Furthermore, by preventing inefficient liquidation, strategic debt service eliminates direct bankruptcy costs. This result refines the Modigliani-Miller theorem in terms of irrelevance of the debt priority structure.

We conclude that, in a frictionless market, like the one depicted in our model, whether there exists a debt optimal priority structure, this cannot be derived from the path of the renegotiation during bankruptcy.

Further analytical results, directly related to the equilibrium claim values, can be found in terms of credit spreads. We refer the reader to the next section where the implications of the claim values on the spreads are investigated in detail.

C) Restructuring plan. The equity holders file either a restructuring plan $\mathcal{P}_j$ or a plan $\mathcal{P}_j$, depending on the level of our parameters. After impairing either class of claims they will propose an equilibrium plan $\mathcal{P}_{s,j}$.

The equilibrium plan $\mathcal{P}_j$ consists of a promise to pay the following coupon flows:

$$\begin{cases} b_j(p_t) = \alpha_{\xi_{s,j}} p_t + (1 - \alpha_{\xi_{s,j}})\gamma r - b_s \\ b_s \end{cases} \Rightarrow \quad \text{for} \quad p_t \leq \bar{p} \tag{58}$$

which impairs the junior while leaves unchanged the contractual coupon to the senior.

When instead the equity holders file a plan $\mathcal{P}_s$ they promise to pay the pair of coupon flows

$$\begin{cases} b_j \\ b_s(p_t) = \alpha_{\xi_{s,j}} p_t + (1 - \alpha_{\xi_{s,j}})\gamma r - b_j \end{cases} \Rightarrow \quad \text{for} \quad p_t \leq \bar{p} \tag{59}$$

which impairs the senior creditor without rescheduling payments for the junior claimant.

After impairing only one creditor by filing a plan $\mathcal{P}_j$ or $\mathcal{P}_s$, the equity holders will file the equilibrium plan $\mathcal{P}_{s,j}$ which consists of a pair of debt service flow functions:

$$\begin{cases} b_j(p_t) = \xi_j(1-\alpha)(p_t - r\gamma) \\ b_s(p_t) = \alpha_{\xi_s} p_t + (1-\alpha_{\xi_s})\gamma r \end{cases} \Rightarrow \text{ for } p_t \leq \begin{cases} p^*_{s2} & \text{if } \mathcal{P}_j \text{ has been filed} \\ p^*_{j2} & \text{else} \end{cases} \tag{60}$$

This equilibrium plan impairs both creditors at once.

In Figure 5, 6 and 7, we show the three possible pairs of debt service flow functions resulting from three different sequences of bankruptcy plans such as $\{\mathcal{P}_j, \mathcal{P}_{s,j}\}$ (in Figure 5), $\{\mathcal{P}_{s,j}\}$ (in Figure 6, where claims are jointly impaired at the trigger level $p^*_{s2} = p^*_{j2} = \bar{p}$) and $\{\mathcal{P}_s, \mathcal{P}_{s,j}\}$ (in Figure 7).

6 Implications on Credit spreads and risk premia

The purpose of this section is that of showing the effect of our renegotiation framework on the credit spreads of the two classes of debt. It is intuitive that the opportunity of rescheduling debt, by allocating the continuation surplus of an economically viable firm, allows creditors to improve their payoffs as a group. The appropriation of a share of this surplus gives each creditor the incentive to successfully renegotiate his/her claim vis a vis the equity holders. The opportunity to renegotiate should therefore reduce the credit spreads simply because it increases the claim values with respect to a pure liquidation scenario.

In order to isolate the effect of the renegotiation on credit spreads we first define the spread and then we decompose it into two kinds of premia: a default and a renegotiation premium.

Because our coupon bonds are perpetuity the spread, say CS, can be measured as

$$CS_i = \frac{b_i}{B_i} - r \qquad \text{with} \begin{cases} i = s, j \\ B_s = S(p_t) \\ B_j = J(p_t), \end{cases} \tag{61}$$

where the term $\frac{b_i}{B_i}$ represents the yield of the risky bond. By adding and subtracting the liquidation payoff, L_i, the credit spread can be rewritten as

$$r\frac{b_i/r - L_i}{B_i} + r\frac{L_i - B_i}{B_i}. \tag{62}$$

This formulation helps at identifying two kinds of premia. The first term, referred to as LP_i, can be interpreted as a pure liquidation premium in that it measures the loss in liquidation (in terms of difference between the promised contractual coupon payments and the payoff of the claim in liquidation). The second term, shortly denoted as RP_i, can be instead understood as a renegotiation premium, that is, the loss or the gain following from seizing the firm's assets through liquidation instead of continuing and renegotiating debt according to our optimal restructuring plan.

Similarly as in Mella-Barral, Perraudin, renegotiation premia may be measured in terms of percentage contribution to the spread. Denoting the percentage renegotiation premium as rp_i, this can be easily written as[47]

$$rp_i = \frac{RP_i}{CS_i} = \frac{L_i - B_i}{b_i/r - B_i}, \tag{63}$$

which applied to the senior and junior claims yields

$$rp_s = \frac{\min\{b_s/r, V_L(p_t)\} - S(p_t)}{b_s/r - S(p_t)}, \quad rp_j = \frac{\min\{b_j/r, V_L(p_t) - L_s\} - J(p_t)}{b_j/r - J(p_t)}. \tag{64}$$

In order to have a first glance at the shape of the risk premia, in Figure 8, we plot the credit spreads, CS_i, and the associated percentage renegotiation premia, rp_i, for the senior and junior bonds (black and red line respectively). The left and right diagrams in each row of Figure 8 show the spread and the resulting renegotiation premia respectively. Moving from the top row to the bottom one, we show the spreads and the renegotiation premia for different allocations of bargaining power between senior and junior creditor given the bargaining power of the equity holders and all other parameters[48]. Particularly from the top to the bottom of Figure 8, the senior's bargaining power, ξ_s, increases from 0 to 0.6 (and, given the equity bargaining power, $\xi_e = 0.3$, the junior bargaining power diminishes correspondingly).

[47] Once determined the percentage contribution of the renegotiation premium, the effect of the liquidation premium can be written as $LP/CS = 1 - RP/CS$.

[48] The other parameters, which remain unchanged, are: $\xi_e = 0.3$, $\alpha = 0.2$, $\mu = 0.03$, $\sigma = 0.15$, $r = 0.08$, $\gamma = 200$, $F = 600$, $Fs = 400$.

Looking at the spreads in Figure 8, quite interesting is the fact that the senior credit spread is not necessarily smaller than the junior one at all level of the state variable. This could not be the case in a pure liquidation scenario, but it is not surprising within a renegotiation framework. The senior creditor has a privileged position when the firm is liquidated and the priority of his/her claim is the crucial and sole factor at determining the claim value. When instead renegotiation is allowed, holding a senior position plays still a fundamental role[49] but the overall package of concession extracted during renegotiation, and hence the claim value, also depends on the bargaining strength. Therefore, holding a senior but largely under-secured claim[50] does not imply a smaller credit spread compared to a junior claimant. What contributes at reducing the senior credit spread is therefore the combination of a high liquidation payoff[51] and strong bargaining power vis a vis other players. The credit spreads in Figure 8 captures this second factor: the senior spread decreases (and the junior increases) when ξ_s increases.

Looking at the percentage renegotiation premia (second column of Figure 8), these exclusively determine the overall spread for high levels of the state variable. Precisely, by definition of rp_i, when the state variable is high enough (such that $V_L \geq b_s/r$) then $rp_s = 1$ and when p_t is even higher (such that $V_L - b_s/r \geq b_j/r$) then also $rp_j = 1$. Therefore for high levels of the state variable the spreads are purely due to the renegotiation premia.

When p_t decreases, by renegotiating, creditors can extract concession depending on their bargaining power. Therefore the renegotiated claim value becomes greater or at least equal[52] to the liquidation value. This is the positive contribution of a renegotiation scenario in the sense it contributes at reducing the spreads. Notice in fact that both premia, rp_s and rp_j, become negative when p_t and hence the firm's liquidation value decrease.

Moreover, looking at 64 it is clear why the renegotiation premia in Figure 8 converge to zero for small level of the state variable. In fact, when p_t approaches $\underline{p}$, the firm is liquidated and therefore the option to renegotiate disappears. Trivial to say that at $\underline{p}$ the credit spread becomes a pure liquidation premium.

Comparison between senior and junior spread.

[49] In fact, it determines the disagreement payoffs in the Nash bargaining.

[50] This is the case in Figure 8 where the senior face value is 400, the scrapping value of the firm is only 200 and $\alpha = 0.2$.

[51] In fact, the senior spread shift down when α and γ increase.

[52] This is the case when a creditor's bargaining power is equal to zero.

So far, little we have said about the difference between the senior and the junior spread. We have briefly argued that the difference is determined by the bargaining power of players in renegotiation and by the fact that the senior creditor is partially secured in the event of liquidation. The comparison between the two spreads is in a way biased by the fact itself that one creditor is partially secured vis a vis the other creditor.

A more meaningful comparison between the two spreads can be achieved by stripping the senior claim for the firm's scrapping value. By proceeding this way, the junior and the senior claim (stripped for γ) are made more comparable in that they will both receive zero when p_t approaches $\underline{p}$ and the firm goes into liquidation. Stripping the senior claim for γ also helps at isolating the effect of the secured part of the claim on the spread.

Moreover by comparing the junior spread and the spread of the senior 'unsecured' claim[53] we will highlight an interesting property of our model. This provides a sufficient condition for the senior spread to be smaller than the junior for all levels of the state variable.

We start decomposing the senior spread to isolate the effect of the secured part of the claim. Notice first that by 61) the spread of the senior claim can be rewritten as

$$\begin{aligned} CS_s &= r\frac{b_s/r - S}{S} \\ &= r\frac{(b_s/r-\gamma)-(S-\gamma)}{S}\cdot\frac{S-\gamma}{S-\gamma} \\ &= r\frac{\hat{F}_s - \hat{S}}{\hat{S}}\cdot\frac{\hat{S}}{S} \end{aligned} \tag{65}$$

where $\hat{S} = S - \gamma$ and $\hat{F}_s = F_s - \gamma$. The two factors in equation 65) can be interpreted as follows. The first term, denoted as

$$\widehat{CS}_s = r\frac{\hat{F}_s - \hat{S}}{\hat{S}}, \tag{66}$$

represents the credit spread on the senior claim stripped for the scrapping value γ, i.e. the spread of the unsecured part of the claim.

The second term, $\hat{S}/S$, captures the effect of the secured part of the claim on the overall credit spread CS_s. In fact, the bigger is the secured part of the claim the smaller is the ratio $\hat{S}/S$ and the credit spread CS_s.

We can now compare the spreads of the two 'unsecured' claims by calculating the

[53] We will shortly use the term 'unsecured' to refer to a senior claim stripped for the scrapping value γ.

difference $\widehat{CS}_s - CS_j$, which by some simple algebra yields

$$\widehat{CS}_s - CS_j = r\frac{\hat{F}_s J - \hat{S} F_j}{\hat{S} J}. \tag{67}$$

We can now state the main result of this section in the following Proposition.

Proposition 6.0.3 *The difference $\widehat{CS}_s - CS_j$ is positive if and only if the optimal stopping strategy is $\{p^*_{j2}, \bar{p}\}$. While, $\widehat{CS}_s - CS_j$ is negative if and only if the optimal strategy is $\{p^*_{s2}, \bar{p}\}$.*[54].

Proof See Appendix 8. ■

The intuition behind this result is clear. As argued in Section 5), point A, the trigger strategy i) orders creditors on the ground of their actual bargaining strength, and according to this ii) identifies the claims default regions. Therefore, the bigger the actual bargaining strength of a creditor, the smaller the claim default region and the bigger the claim value, which, in turn, reduces the credit spread.

The relevance of this result is immediate. The above proposition provides a sufficient condition for detecting the case where $CS_s < CS_j$ for any level of the state variable. In fact, the inequality $CS_s < CS_j$ rearranges into $\widehat{CS}_s \hat{S}/S < CS_j$, which holds if[55] $\widehat{CS}_s < CS_j$.

These results are shown in Figure 9, where, again, we show the credit spreads of Figure 8 and we add the function $\widehat{CS}_s$ (dashed line) which, depending on the parameterisation and, in turn, the stopping strategy, is above or below the credit spread of the junior creditor (red line). The difference $\widehat{CS}_s \hat{S}/S - CS_j$ appears in the second column of Figure 9. From our analytical result, the difference is positive in the first two plots, where the senior creditor is impaired first, whilst the sign reverts in the last two plots where the junior creditor is the first to be impaired.

[54]The equality holds, and $\widehat{CS}_s - CS_j = 0$ when $p^*_{j2} = p^*_{s2} = \bar{p}$.

[55]As already argued, the term $\hat{S}/S$ is always smaller than one for any under-secured claim.

7 Conclusions

This paper stresses the crucial role of the formal bankruptcy process when there are more than two players at the negotiating table. Obviously, the formal bankruptcy rules -such as the right to file a first proposal and the impairment rule- are redundant in the simple scenario where the equity holders renegotiate vis a vis a single creditor. In contrast, when multiple creditors are involved, the equity holders strategically enforce the set of bankruptcy rules which crucially determines a unique equilibrium restructuring plan. Therefore, to a certain extent, the model discussed above provides a framework for a positive analysis of Chapter 11, which integrates legal and economics features.

The current model provides diverse economic implications. When the equity holders can renegotiate with one creditor at a time, as in Chapter 11, the strategic decision concerns not only the timing of bankruptcy but also the order in which creditors will be impaired. We conclude that the equity holders impair the creditor with a smaller intensity of actual bargaining strength first, that is, the ratio of actual bargaining power to unsecured face value. Therefore, depending on the parameterisation (which determines the above ratio, and, hence, the impairment strategy), in equilibrium the equity holders will strategically default on a single class of claims when the state variable reaches a certain trigger level. If the state variable continues to decrease to a second (lower) trigger level, then the equity holders will default on both claims jointly. This kind of equilibrium, where single claims are restructured individually, is consistent with the fact that private workouts are often targeted to single classes of claims, and that cross default provisions do not prevent a private renegotiation.

Moreover, we show that, given the total debt face value and the overall bargaining power of creditors, the first (higher) bankruptcy trigger and the equity value do not depend on the allocation of face value and bargaining power amongst creditors.

Interestingly, we find that the senior credit spread is not necessarily smaller than the junior one at all levels of the state variable. This could not be the case in a pure liquidation scenario, however in a renegotiation framework it is not surprising. The senior creditor has a privileged position when the firm is liquidated and the priority of his/her claim is the crucial and sole factor at determining the claim value. When renegotiation is allowed, holding a senior position still plays a fundamental role (by affecting the disagreement payoffs), but the overall package of concession extracted during renegotiation also depends on the bargaining strength. Therefore, holding a senior but largely under-secured claim does not guarantee a smaller credit spread with respect to other claimants. What reduces the senior credit spread is therefore the combination of a large liquidation payoff and strong bargaining power vis a vis other players.

Our result on the spreads is crucially related the equilibrium impairment strategy. We have shown that the impairment strategy i) orders creditors on the basis of their actual bargaining strength, and according to this ii) identifies the claims default regions. Therefore, the bigger the intensity of the actual bargaining strength of a creditor, the smaller the claim default region and the bigger the claim value, which in turn, reduces the credit spread. Therefore, depending on the parameterisation, an equilibrium plan which impairs the junior first guarantees that the senior spreads is below the junior one for any level of the state variable.

References

[1] Anderson, Ron W. and Suresh M. Sundaresan. (1996). "The design and valuation of debt contracts." *Review of Financial Studies*, Vol. 9: 37-68.

[2] Binmore, Kennet and P. Dasgupta. (1987). *The Economics of Bargaining.* Blackwell.

[3] Black, F. and J. Cox. (1976). "Valuing Corporate Securities: Liabilities Some Effects of Bond Indenture Provisions." *Journal of Finance*, Vol. 31: 351-367.

[4] Brown, David T. (1989). "Claimholder Incentive Conflicts in Reorganization: The Role of Bankruptcy Law." *Review of Financial Studies*, Vol. 2: 109-123.

[5] Dixit, Avinash K. and Robert S. Pindyck. (1994). *Investment Under Uncertainty.* Princeton University Press.

[6] Fan, Hua, and Suresh M. Sundaresan. (2000). "Debt valuation, renegotiation, and optimal dividend policy." *Review of Financial Studies*, Vol. 13: 1057-1099.

[7] Franks, Julian, and Walter N. Torous. (1989). "An empirical investigation of U.S. firms in reorganization." *Journal of Finance*, Vol. 44: 747-769.

[8] Gower, Laurence C. B. (1992). *Principles of Modern Company Law.* (5th Ed) Sweet Maxwell.

[9] Hege, Ulrich and Pierre Mella-Barral. (2000). "Bargaining power and optimal leverage." *Finance*, Vol. 21: 85-101.

[10] Hege, Ulrich and Pierre Mella-Barral. (2002). "Repeated dilution of diffusely held debt." Mimeo. Forthcoming: 2005, *Journal of Business* Vol. 78, no. 3.

[11] Kordana, K. A. and E. A. Posner. (1999). "A Positive Theory of Chapter 11." *New York University Law Review*, Vol. 74: 161-234.

[12] Leland, Hayne E. (1994). "Corporate debt value, bond covenants and optimal capital structure." *Journal of Finance*, Vol. 49: 1213-1252.

[13] Leland, Hayne E. and K. Toft. (1996). "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads." *Journal of Finance*, Vol. 51: 987-1019.

[14] Longstaff, F., and E. Schwartz. (1995). "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt." *Journal of Finance*, Vol. 50: 789-819.

[15] Mella-Barral, Pierre, and William Perraudin. (1997). "Strategic debt service." *Journal of Finance*, Vol. 52: 531-556.

[16] Merton, R. C. (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance*, Vol. 2: 449-470.

[17] Perraudin, William and Maria Psillaki. (1999). "Corporate Restructuring: The Impact of Loan Sales and Credit Derivatives." *Discussion Papers in Economics*. Birkbeck College.

[18] Rajan, Raghuram and A. Winton. (1995). "Covenants and Collateral as Incentives to Monitor." *Journal of Finance*, Vol. 50: 1113-1146.

[19] Schwartz, Alan. (2002). "The Law and Economics Approach to Corporate Bankruptcy." *Global Corporate Governance Forum Research Network Meeting*, April 5.

[20] Sundaresan, Suresh M. (2000). "Continuous-Time Methods in Finance: A Review and an Assessment." *Journal of Finance*, Vol. 55: 1569-1622.

[21] Welch, Ivo. (1997). "Why is bank debt senior? A theory of asymmetry and claim priority based on influence costs." *Review of Financial Studies*, Vol. 10: 1203-1236.

Appendix

Appendix 1

We prove here that the trigger p_c^* guarantees that Hypotheses 2A in Section 2) holds, that is $F_c > V_L$. In particular, it is convenient to prove that $F_c > \underline{\mathrm{P}}_c(p_c^*)$ which also implies $F_c > \underline{\mathrm{P}}_c(p_c^*) \geq V_L$.

First, one can rearrange $\underline{\mathrm{P}}_c$ as follows

$$\begin{aligned}
\underline{\mathrm{P}}_c &= \xi_c(V - V_L) + V_L \\
&= [\alpha + \xi_c(1-\alpha)]V + (1-\alpha)(1-\xi_c)\gamma \\
&= \alpha_{\xi_c} V + (1-\alpha_{\xi_c})\gamma.
\end{aligned}$$

Then, the inequality $F_c > \underline{\mathrm{P}}_c(p_c^*)$ can be rearranged as

$$\begin{aligned}
F_c &> \alpha_{\xi_c}\left[\frac{p_c^*}{r-\mu} + \left(\gamma - \frac{\underline{\mathrm{p}}}{r-\mu}\right)\left(\frac{p_c^*}{\underline{\mathrm{p}}}\right)^{\lambda}\right] + (1-\alpha_{\xi_c})\gamma \\
F_c &> \alpha_{\xi_c}\left[\frac{\lambda}{\lambda-1}\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\alpha_{\xi_c}} + \left(\gamma - \frac{\underline{\mathrm{p}}}{r-\mu}\right)\left(\frac{p_c^*}{\underline{\mathrm{p}}}\right)^{\lambda}\right] + (1-\alpha_{\xi_c})\gamma \\
\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\alpha_{\xi_c}} &> \frac{\lambda}{\lambda-1}\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\alpha_{\xi_c}} - \frac{\gamma}{\lambda-1}\left(\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\gamma\alpha_{\xi_c}}\right)^{\lambda} \\
\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\gamma\alpha_{\xi_c}} &> \left(\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\gamma\alpha_{\xi_c}}\right)^{\lambda}
\end{aligned}$$

Notice that this last inequality is always satisfied being λ negative and the term

$$\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\gamma\alpha_{\xi_c}} = \frac{p_c^*}{\underline{\mathrm{p}}}$$

is greater than one (that is, $p_c^* > \underline{\mathrm{p}}$) as shown by the following algebra.

$$\begin{aligned}
p_c^* &= \frac{\lambda}{\lambda-1}\frac{F_c - \gamma(1-\alpha_{\xi_c})}{\alpha_{\xi_c}}(r-\mu) > \frac{\lambda}{\lambda-1}\gamma(r-\mu) = \underline{\mathrm{p}} \\
F_c &> \gamma\alpha_{\xi_c} + \gamma(1-\alpha_{\xi_c}) \\
F_c &> \gamma.
\end{aligned}$$

Appendix 2

Here, we show that the first order condition to the equity holders' maximisation problem, that is $\partial C(p_t, p_c)/\partial p_c = 0$, is equivalent to the smooth pasting condition 15), that is $\frac{\partial C(p_t)}{\partial p_t} \mid_{p_c^*} = \frac{\partial \underline{\mathrm{P}}_c(p_t)}{\partial p_t} \mid_{p_c^*}$.

First, develop the first order condition, $\partial C(p_t, p_c)/\partial p_c = 0$, as follows:

$$\begin{aligned} \partial\left[\frac{b_c}{r} + \left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)\left(\frac{p_t}{p_c}\right)^{\lambda}\right]/\partial p_c &= 0 \\ \frac{\partial\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)}{\partial p_c}\left(\frac{p_t}{p_c}\right)^{\lambda} + \frac{\partial\left(\frac{p_t}{p_c}\right)^{\lambda}}{\partial p_c}\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right) &= 0 \\ \frac{\partial\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)}{\partial p_c} &= \lambda {p_c}^{-1}\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right). \end{aligned} \tag{68}$$

Secondly, rearrange the smooth pasting condition, $\frac{\partial C(p_t)}{\partial p_t} \mid_{p_c^*} = \frac{\partial \underline{\mathrm{P}}_c(p_t)}{\partial p_t} \mid_{p_c^*}$, as

$$\begin{aligned} \frac{\partial\left[\frac{b_c}{r} + \left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)\left(\frac{p_t}{p_c}\right)^{\lambda}\right]}{\partial p_t} \mid_{p_c^*} &= \frac{\partial\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)}{\partial p_t} \mid_{p_c^*} \\ \lambda {p_t}^{\lambda-1} {p_c}^{-\lambda}\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right) \mid_{p_c^*} &= \frac{\partial\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)}{\partial p_t} \mid_{p_c^*} \\ \lambda {p_c}^{-1}\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right) &= \frac{\partial\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)}{\partial p_t} \mid_{p_c^*}. \end{aligned} \tag{69}$$

Last notice that equation 68) and 69) are identical because

$$\frac{\partial\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)}{\partial p_c} = \frac{\partial\left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)}{\partial p_t} \mid_{p_c^*}.$$

Appendix 3

We solve here for the unique function, $s(p_t)$, satisfying:

$$rC(p_t) = s(p_t) + \frac{d}{d\triangle} E_t(C_{t+\triangle}) \mid_{\triangle=0},$$

which, by Ito's Lemma, can be rewritten as

$$rC(p_t) = s(p_t) + \mu p_t C'(p_t) + \frac{\sigma^2}{2} {p_t}^2 C''(p_t).$$

i) When $p_t \in [\underline{p}, p_c^*)$, by some easy algebra one can find that C, C' and C'' are equal to

$$\begin{aligned}
C &= \alpha_{\xi_c} V + (1-\alpha_{\xi_c})\gamma \\
C' &= \alpha_{\xi_c} \left[\frac{1}{r-\mu} + \lambda \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \frac{{p_t}^{\lambda-1}}{\underline{p}^{\lambda}} \right] \\
C'' &= \alpha_{\xi_c} \lambda(\lambda-1) \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \frac{{p_t}^{\lambda-2}}{\underline{p}^{\lambda}}.
\end{aligned}$$

By substituting in the last equation for C, C' and C'', one obtains

$$\begin{aligned}
& r\left\{ \alpha_{\xi_c} \left[\frac{p_t}{r-\mu} + \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \left(\frac{p_t}{\underline{p}} \right)^{\lambda} \right] + (1-\alpha_{\xi_c})\gamma \right\} \\
& = s(p_t) + \mu p_t \alpha_{\xi_c} \left[\frac{1}{r-\mu} + \lambda \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \frac{{p_t}^{\lambda-1}}{\underline{p}^{\lambda}} \right] + \\
& + \frac{\sigma^2}{2} {p_t}^2 \alpha_{\xi_c} \lambda(\lambda-1) \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \frac{{p_t}^{\lambda-2}}{\underline{p}^{\lambda}},
\end{aligned}$$

which can be simplified as

$$\begin{aligned}
& r\left\{ \alpha_{\xi_c} \left[\frac{p_t}{r-\mu} + \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \left(\frac{p_t}{\underline{p}} \right)^{\lambda} \right] + (1-\alpha_{\xi_c})\gamma \right\} \\
& = s(p_t) + \mu \alpha_{\xi_c} \left[\frac{p_t}{r-\mu} + \lambda \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \left(\frac{p_t}{\underline{p}} \right)^{\lambda} \right] + \\
& + \frac{\sigma^2}{2} \alpha_{\xi_c} \lambda(\lambda-1) \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \left(\frac{p_t}{\underline{p}} \right)^{\lambda}.
\end{aligned}$$

Using, for brevity, the notation

$$A = \left(\gamma - \frac{\underline{p}}{r-\mu} \right) \left(\frac{p_t}{\underline{p}} \right)^{\lambda}$$

the previous equation can be rearranged as follows

$$\left[r - \mu\lambda - \frac{\sigma^2}{2}\lambda(\lambda-1) \right] \alpha_{\xi_c} A + r\left(\alpha_{\xi_c} \frac{p_t}{r-\mu} + (1-\alpha_{\xi_c})\gamma \right) = s(p_t) + \mu \alpha_{\xi_c} \frac{p_t}{r-\mu},$$

which simplifies into

$$r\left(\alpha_{\xi_c}\frac{p_t}{r-\mu}+(1-\alpha_{\xi_c})\gamma\right)=s(p_t)+\mu\alpha_{\xi_c}\frac{p_t}{r-\mu},$$

because λ is the negative root of the quadratic equation

$$r-\mu\lambda-\frac{\sigma^2}{2}\lambda(\lambda-1)=0.$$

Therefore, solving for $s(p_t)$ yields

$$s_c(p_t)=\alpha_{\xi_c}p_t+(1-\alpha_{\xi_c})r\gamma \quad \text{for} \quad p_t\in[\underline{p},p_c^*).$$

ii) When $p_t\in[p_c^*,\infty)$ one can find that $s(p_t)=b_c$ as shown below. Using the notation:

$$B=\left(\underline{\mathrm{P}}(p_c^*)-\frac{b_c}{r}\right),$$

the term C, C' and C'' can be written as

$$\begin{aligned} C &= \frac{b_c}{r}+B\left(\frac{p_t}{p_c^*}\right)^\lambda \\ C' &= \lambda B\frac{p_t^{\lambda-1}}{p_c^{*\lambda}} \\ C' &= \lambda(\lambda-1)B\frac{p_t^{\lambda-2}}{p_c^{*\lambda}}, \end{aligned}$$

which substituted for into the differential equation $rC=s+\mu pC'+\frac{\sigma^2}{2}p^2C''$, give

$$r\left[\frac{b_c}{r}+B\left(\frac{p_t}{p_c^*}\right)^\lambda\right]=s(p_t)+\mu p_t\lambda B\frac{p_t^{\lambda-1}}{p_c^{*\lambda}}+\frac{\sigma^2}{2}{p_t}^2\lambda(\lambda-1)B\frac{p_t^{\lambda-2}}{p_c^{*\lambda}},$$

which rearranges as

$$\begin{aligned} &\left[r-\mu\lambda-\frac{\sigma^2}{2}\lambda(\lambda-1)\right]B\left(\frac{p_t}{p_c^*}\right)^\lambda+b_c=s(p_t) \\ &b_c=s(p_t). \end{aligned}$$

Appendix 4

The equity holders' plan belongs to the set $\mathcal{P}_j = \{P_e, P_j, P_s : P_e \geq \underline{\mathrm{P}}_e, P_j \geq \underline{\mathrm{P}}_j, P_s < \underline{\mathrm{P}}_s\}$. Therefore

$$P_e \geq \underline{\mathrm{P}}_e,$$

can be rearranged (by adding and subtracting V) into

$$V - P_e - (V - \underline{\mathrm{P}}_e) \geq 0.$$

Replacing $V - P_e$ and $V - \underline{\mathrm{P}}_e$ with $P_j + P_s$ and $\underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s$ gives

$$\underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s - P_j - P_s \geq 0.$$

As $P_j = J$ and $P_s = S$ the above rewrites as

$$\underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s - J - S \geq 0,$$

which is the definition of the arbitrage payoff, Π, therefore one concludes

$$\Pi = \underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s - J - S \geq 0.$$

Appendix 5

Under the two strategies, $\{p^*_{j1}, p^*_{s2}\}$ and $\{p*_{j2}, p*_{s1}\}$, Hypotheses 2) holds.

In fact, if the optimal strategy is $\{p^*_{j1}, p^*_{s2}\}$, one can easily prove that

$$F_s > \underline{\mathrm{P}}_s(p^*_{s2}) \geq V_L(p^*_{s2})$$

by following the same steps as in Appendix 1 (just replace the subscript 'c', in Appendix 1, with the subscript 's').

If instead the optimal strategy is $\{p^*_{j2}, p^*_{s1}\}$, again it is quite straightforward to prove that Hypotheses 2 holds, that is

$$F_s > V_L(p^*_{s1}).$$

By substituting for $V_L(p^*_{s1})$, the above inequality rearranges as

$$\begin{aligned}
F_s &> \alpha\left[\frac{p^*_{s1}}{r-\mu} + \left(\gamma - \frac{\underline{\mathrm{p}}}{r-\mu}\right)\left(\frac{p^*_{s1}}{\underline{\mathrm{p}}}\right)^{\lambda}\right] + (1-\alpha)\gamma \\
F_s &> \alpha\left[\frac{\lambda}{\lambda-1}\frac{F_s+F_j-\gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} + \left(\gamma - \frac{\underline{\mathrm{p}}}{r-\mu}\right)\left(\frac{p^*_{s1}}{\underline{\mathrm{p}}}\right)^{\lambda}\right] + (1-\alpha)\gamma \\
\frac{F_s-\gamma(1-\alpha)}{\alpha} &> \frac{\lambda}{\lambda-1}\frac{F_s+F_j-\gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} - \frac{\gamma}{\lambda-1}\left(\frac{p^*_{s1}}{\underline{\mathrm{p}}}\right)^{\lambda}. \qquad (71)
\end{aligned}$$

One can notice that the left hand side of 71 is such that

$$\frac{F_s-\gamma(1-\alpha)}{\alpha} > \frac{F_s+F_j-\gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}}, \qquad (72)$$

which can be proved as follows. Rearrange 72 as shown below

$$\begin{aligned}
F_s\alpha_{\xi_{s,j}} - \gamma(1-\alpha)\alpha_{\xi_{s,j}} &> (F_s+F_j)\alpha - \gamma(1-\alpha_{\xi_{s,j}})\alpha \\
F_s[(\xi_j+\xi_s)(1-)+\alpha] - \gamma(1-\alpha)[(\xi_j+\xi_s)(1-)+\alpha] &> (F_s+F_j)\alpha - \gamma\alpha \\
\frac{F_s-\gamma}{F_j} &> \frac{\alpha}{(\xi_s+\xi_j)(1-\alpha)}, \qquad (73)
\end{aligned}$$

and notice that 73 holds because under the strategy $\{p^*_{j2}, p^*_{s1}\}$, the second condition in 36 must hold, which, in turn, can be rewritten as

$$\frac{F_s-\gamma}{F_j} > \frac{\xi_s(1-\alpha)+\alpha}{\xi_j(1-\alpha)},$$

where one can easily notice that the right hand side is such that

$$\frac{\xi_s(1-\alpha)+\alpha}{\xi_j(1-\alpha)} > \frac{\alpha}{(\xi_s+\xi_j)(1-\alpha)},$$

which proves therefore 73 and in turn 72.

We can use the result from 72 in 71, which can then be rearranged as follows

$$
\begin{aligned}
\frac{F_s - \gamma(1-\alpha)}{\alpha} &> \frac{F_s + F_j - \gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} \\
&> \frac{\lambda}{\lambda-1}\frac{F_s + F_j - \gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} - \frac{\gamma}{\lambda-1}\left(\frac{p^*_{s1}}{\underline{\mathrm{p}}}\right)^{\lambda}, \\
&= \frac{\lambda}{\lambda-1}\frac{F_s + F_j - \gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} - \frac{\gamma}{\lambda-1}\left(\frac{F_s + F_j - \gamma(1-\alpha_{\xi_{s,j}})}{\gamma\alpha_{\xi_{s,j}}}\right)^{\lambda}.
\end{aligned}
$$

Last it can be proved that the above inequality holds, that is

$$
\frac{F_s + F_j - \gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} > \frac{\lambda}{\lambda-1}\frac{F_s + F_j - \gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}} - \frac{\gamma}{\lambda-1}\left(\frac{F_s + F_j - \gamma(1-\alpha_{\xi_{s,j}})}{\gamma\alpha_{\xi_{s,j}}}\right)^{\lambda},
$$

by following the same calculation as in Appendix 1 (simply replace the term α_{ξ_c} in Appendix 1 with $\alpha_{\xi_{s,j}}$ and the term F_c with $F_s + F_j$).

Appendix 6

We show here that

$$s_j(p_t) = \begin{cases} b_j & \text{for} \quad p^*_{j1} < p_t \\ \alpha_{\xi_{s,j}} p_t + (1-\alpha_{\xi_{s,j}})\gamma r - s_s(p_t) & \text{for} \quad p_t \le p^*_{j1} \end{cases}$$

$$s_s(p_t) = \begin{cases} b_s & \text{for} \quad p^*_{s2} < p_t \\ \alpha_{\xi_s} p_t + (1-\alpha_{\xi_s})\gamma r & \text{for} \quad p_t \le p^*_{s2} \end{cases}$$

As argued in Appendix 3, we know that the claim value, $S(p_t)$ and $J(p_t)$ are free of arbitrage opportunity if the following equation holds:

$$\begin{aligned} rS(p_t) &= s_s(p_t) + \mu p_t S'(p_t) + \frac{\sigma^2}{2} {p_t}^2 S''(p_t), \\ rJ(p_t) &= s_j(p_t) + \mu p_t J'(p_t) + \frac{\sigma^2}{2} {p_t}^2 J''(p_t). \end{aligned}$$

i) For $p_t \le p^*_{s2}$, the function $s_s(p_t)$ can be found by following the same calculation as in Appendix 3, point i), just replace α_{ξ_c} with α_{ξ_s}, which yields

$$s_s(p_t) = \alpha_{\xi_s} p_t + (1-\alpha_{\xi_s}) r\gamma \quad \text{for} \quad p_t \in [\underline{p}, p^*_{s2}).$$

With regard to the function $s_j(p_t)$, using for brevity the notation

$$A = \gamma - \frac{\underline{p}}{r-\mu},$$

one can find that the terms J, J' and J'' are equal to

$$\begin{aligned} J &= \underline{P}_j = \xi_j(1-\alpha)(V-\gamma) \\ J' &= \xi_j(1-\alpha)\left(\frac{1}{r-\mu} + \lambda A \frac{{p_t}^{\lambda-1}}{\underline{p}^\lambda}\right) \\ J'' &= \xi_j(1-\alpha)\lambda(\lambda-1)A\frac{{p_t}^{\lambda-2}}{\underline{p}^\lambda}. \end{aligned}$$

Substituting these terms in the above differential equation yields

$$r\xi_j(1-\alpha)\left[\frac{p_t}{r-\mu} + A\left(\frac{p_t}{\underline{p}}\right)^\lambda - \gamma\right] =$$
$$s_j(p_t) + \mu p_t \xi_j(1-\alpha)\left(\frac{1}{r-\mu} + \lambda A \frac{{p_t}^{\lambda-1}}{\underline{p}^\lambda}\right) + \frac{\sigma^2}{2}{p_t}^2\xi_j(1-\alpha)\lambda(\lambda-1)A\frac{{p_t}^{\lambda-2}}{\underline{p}^\lambda},$$

which rearranges as

$$\begin{aligned}
&r\xi_j(1-\alpha)\left[\frac{p_t}{r-\mu}+A\left(\frac{p_t}{\underline{p}}\right)^\lambda-\gamma\right]= \\
&s_j(p_t)+\mu\xi_j(1-\alpha)\left[\frac{p_t}{r-\mu}+\lambda A\left(\frac{p_t}{\underline{p}}\right)^\lambda\right]+\frac{\sigma^2}{2}\xi_j(1-\alpha)\lambda(\lambda-1)A\left(\frac{p_t}{\underline{p}}\right)^\lambda, \\
&\xi_j(1-\alpha)A\left(\frac{p_t}{\underline{p}}\right)^\lambda\left[r-\mu\lambda-\frac{\sigma^2}{2}\lambda(\lambda-1)\right]+r\xi_j(1-\alpha)\left(\frac{p_t}{r-\mu}-\gamma\right)= \\
&s_j(p_t)+\mu\xi_j(1-\alpha)\frac{p_t}{r-\mu}, \\
&r\xi_j(1-\alpha)\left(\frac{p_t}{r-\mu}-\gamma\right)=s_j(p_t)+\mu\xi_j(1-\alpha)\frac{p_t}{r-\mu}, \\
&s_j(p_t)=\xi_j(1-\alpha)(p_t-r\gamma).
\end{aligned}$$

By simple algebra, $s_j(p_t)$ can also be rewritten as

$$s_j(p_t)=\alpha_{\xi_{s,j}}p_t+(1-\alpha_{\xi_{s,j}})\gamma r-s_s(p_t).$$

ii) For $p_t\in(p^*_{s2},p^*_{j1}]$, the function $s_s(p_t)$ can be simply worked out by following the same calculation in Appendix 3, point ii) (the only change is the trigger p^*_{s2} instead of p^*_c). Therefore the same result holds, that is

$$s_s(p_t)=b_s.$$

In the remainder we will use, for convenience, the following notation:

$$\begin{aligned}
A &= \gamma-\frac{\underline{p}}{r-\mu}, \\
B &= \alpha_{\xi_s}V(p^*_{s2})+(1-\alpha_{\xi_s})\gamma-\frac{b_s}{r}.
\end{aligned}$$

With regard to the function $s_j(p_t)$, first rearrange J as follows

$$\begin{aligned}
J(p_t) &= \underline{P}_j+\underline{P}_s-S(p_t) \\
&= (\xi_s+\xi_j)(V-V_L)+V_L-\frac{b_s}{r}-\left(\xi_s(V(p^*_{s2})-V_L(p^*_{s2}))+V_L(p^*_{s2})-\frac{b_s}{r}\right)\left(\frac{p_t}{p^*_{s2}}\right)^\lambda \\
&= \alpha_{\xi_{s,j}}V+(1-\alpha_{\xi_{s,j}})\gamma-\frac{b_s}{r}-B\left(\frac{p_t}{p^*_{s2}}\right)^\lambda \\
&= \alpha_{\xi_{s,j}}\left[\frac{p_t}{r-\mu}+A\left(\frac{p_t}{\underline{p}}\right)^\lambda\right]+(1-\alpha_{\xi_{s,j}})\gamma-\frac{b_s}{r}-B\left(\frac{p_t}{p^*_{s2}}\right)^\lambda.
\end{aligned}$$

Then, one can work out J' and J'' as follows

$$
\begin{aligned}
J' &= \alpha_{\xi_{s,j}}\left(\frac{1}{r-\mu} + \lambda A \frac{p_t^{\lambda-1}}{\underline{\mathrm{p}}^{\lambda}}\right) - \lambda B \frac{p_t^{\lambda-1}}{{p_{s2}^*}^{\lambda}} \\
J'' &= \alpha_{\xi_{s,j}}\lambda(\lambda-1)A\frac{p_t^{\lambda-2}}{\underline{\mathrm{p}}^{\lambda}} - \lambda(\lambda-1)B\frac{p_t^{\lambda-2}}{{p_{s2}^*}^{\lambda}}.
\end{aligned}
$$

Substituting for J, J' and J'' in the corresponding differential equation yields

$$
\begin{aligned}
& r\left\{\alpha_{\xi_{s,j}}\left[\frac{p_t}{r-\mu} + A\left(\frac{p_t}{\underline{\mathrm{p}}}\right)^{\lambda}\right] + (1-\alpha_{\xi_{s,j}})\gamma - \frac{b_s}{r} - B\left(\frac{p_t}{p_{s2}^*}\right)^{\lambda}\right\} = \\
&= s_j(p_t) + \mu p_t\left[\alpha_{\xi_{s,j}}\left(\frac{1}{r-\mu} + \lambda A\frac{p_t^{\lambda-1}}{\underline{\mathrm{p}}^{\lambda}}\right) - \lambda B\frac{p_t^{\lambda-1}}{{p_{s2}^*}^{\lambda}}\right] + \\
&+ \frac{\sigma^2}{2}{p_t}^2\left[\alpha_{\xi_{s,j}}\lambda(\lambda-1)A\frac{p_t^{\lambda-2}}{\underline{\mathrm{p}}^{\lambda}} - \lambda(\lambda-1)B\frac{p_t^{\lambda-2}}{{p_{s2}^*}^{\lambda}}\right],
\end{aligned}
$$

which simplifies as follows:

$$
\begin{aligned}
& r\left\{\alpha_{\xi_{s,j}}\left[\frac{p_t}{r-\mu} + A\left(\frac{p_t}{\underline{\mathrm{p}}}\right)^{\lambda}\right] + (1-\alpha_{\xi_{s,j}})\gamma - \frac{b_s}{r} - B\left(\frac{p_t}{p_{s2}^*}\right)^{\lambda}\right\} = \\
&= s_j(p_t) + \mu\left[\alpha_{\xi_{s,j}}\left(\frac{p_t}{r-\mu} + \lambda A\left(\frac{p_t}{\underline{\mathrm{p}}}\right)^{\lambda}\right) - \lambda B\left(\frac{p_t}{\underline{\mathrm{p}}}\right)^{\lambda}\right] + \\
&+ \frac{\sigma^2}{2}\left[\alpha_{\xi_{s,j}}\lambda(\lambda-1)A\left(\frac{p_t}{\underline{\mathrm{p}}}\right)^{\lambda} - \lambda(\lambda-1)B\left(\frac{p_t}{\underline{\mathrm{p}}}\right)^{\lambda}\right],
\end{aligned}
$$

$$
\begin{aligned}
\left[\alpha_{\xi_{s,j}}A\left(\frac{p_t}{\underline{\mathrm{p}}}\right)^{\lambda} - B\left(\frac{p_t}{p_{s2}^*}\right)^{\lambda}\right]\left[r - \mu\lambda - \frac{\sigma^2}{2}\lambda(\lambda-1)\right] + & \\
+ r\left[\alpha_{\xi_{s,j}}\frac{p_t}{r-\mu} + (1-\alpha_{\xi_{s,j}})\gamma - \frac{b_s}{r}\right] &= s_j(p_t) + \mu\alpha_{\xi_{s,j}}\frac{p_t}{r-\mu}, \\
r\left[\alpha_{\xi_{s,j}}\frac{p_t}{r-\mu} + (1-\alpha_{\xi_{s,j}})\gamma - \frac{b_s}{r}\right] &= s_j(p_t) + \mu\alpha_{\xi_{s,j}}\frac{p_t}{r-\mu}, \\
\alpha_{\xi_{s,j}}p_t + (1-\alpha_{\xi_{s,j}})\gamma r - b_s &= s_j(p_t).
\end{aligned}
$$

iii) For $p_t \in (p_{j1}^*, \infty)$, the function $s_s(p_t) = b_s$ remains unchanged and by working a similar calculation as in Appendix 3, point ii), it is easy to obtain the result $s_j(p_t) = b_j$. Just replace the term b_c/r with b_j/r and the term $\underline{P}_c(p_c^*)$ with $\underline{P}_j(p_{j1}^*) + \underline{P}_s(p_{j1}^*) - S(p_{j1}^*)$.

Appendix 7

Most of the results of this appendix can be derived and Appendix 6, therefore we will provide only the main guideline of the proof where possible. In this appendix we prove that

$$
\begin{aligned}
s_s(p_t) &= \begin{cases} b_s & \text{for} \quad p^*_{s1} < p_t \\ \alpha_{\xi_{s,j}} p_t + (1-\alpha_{\xi_{s,j}})\gamma r - s_j(p_t) & \text{for} \quad p_t \le p^*_{s1} \end{cases} \\
s_j(p_t) &= \begin{cases} b_j & \text{for} \quad p^*_{j2} < p_t \\ \xi_j(1-\alpha)(p_t - \gamma r) & \text{for} \quad p_t \le p^*_{j2} \end{cases}
\end{aligned}
$$

i) When $p_t \le p^*_{j2}$ the functions $s_s(p_t)$ and $s_j(p_t)$ can be worked out following the same calculation in Appendix 6, point i).

ii) When $p_t \in (p^*_{j2}, p^*_{s2}]$, the function $s_j(p_t)$ can be simply worked out by following the same calculation in Appendix 3, point ii) (the only change is the trigger p^*_{j2} instead of p^*_c). Therefore the same result holds, that is

$$s_j(p_t) = b_j.$$

As already done in Appendix 6, in the remainder we will use, for convenience, the following notation:

$$
\begin{aligned}
A &= \gamma - \frac{\underline{p}}{r-\mu}, \\
B &= \xi_j(1-\alpha)(V(p^*_{j2}) - \gamma) - \frac{b_j}{r}.
\end{aligned}
$$

With regard to the function $s_j(p_t)$, first rearrange S as follows

$$
\begin{aligned}
S(p_t) &= \underline{P}_j + \underline{P}_s - J(p_t), \\
&= (\xi_s + \xi_j)(V - V_L) + V_L - \frac{b_j}{r} - \left(\xi_j(V(p^*_{j2}) - V_L(p^*_{j2})) - \frac{b_j}{r}\right)\left(\frac{p_t}{p^*_{j2}}\right)^\lambda, \\
&= \alpha_{\xi_{s,j}} V + (1-\alpha_{\xi_{s,j}})\gamma - \frac{b_j}{r} - B\left(\frac{p_t}{p^*_{j2}}\right)^\lambda, \\
&= \alpha_{\xi_{s,j}}\left[\frac{p_t}{r-\mu} + A\left(\frac{p_t}{\underline{p}}\right)^\lambda\right] + (1-\alpha_{\xi_{s,j}})\gamma - \frac{b_j}{r} - B\left(\frac{p_t}{p^*_{j2}}\right)^\lambda.
\end{aligned}
$$

Notice that this last equation has been already found in Appendix 6, point ii) when rearranging J, the only difference if that the trigger p^*_{s2} is replaced now by p^*_{j2} and the constant term B is differently defined (but this does not affect the result as the term B, in Appendix 6, point ii) cancels out). We can therefore use the result previously found for the function $s_j(p_t)$ and just invert the subscript j with s. This leads to

$$\alpha_{\xi_{s,j}} p_t + (1 - \alpha_{\xi_{s,j}})\gamma r - b_j = s_s(p_t).$$

iii) For $p_t \in (p^*_{s1}, \infty)$, the function $s_j(p_t) = b_j$ remains unchanged and by working a similar calculation as in Appendix 3, point ii), it is easy to obtain the result $s_s(p_t) = b_s$. Just replace the term b_c/r with b_s/r and the term $\underline{P}_c(p^*_c)$ with $\underline{P}_j(p^*_{s1}) + \underline{P}_s(p^*_{s1}) - S(p^*_{s1})$.

Appendix 8

Notice first that the sign of $\widehat{CS}_s - CS_j$ is positive iff the numerator is positive, that is

$$\frac{(b_s/r - \gamma)}{b_j/r} > \frac{(S-\gamma)}{J}.$$

Notice that regardless of the optimal stopping strategy, for low enough level of the state variable the equity holders impair both creditors jointly. Therefore, when joint renegotiation occurs, i.e. when $p_t \in [\underline{p}, p^*_{i2}]$, we know that the value of the junior and senior debt is given by

$$\begin{aligned} J = \underline{P}_j &= \xi_j(V - V_L) \\ S = \underline{P}_s &= \xi_s(V - V_L) + V_L. \end{aligned}$$

Substituting for the debt values, $\underline{P}_j$ and $\underline{P}_s$, when $p_t \in [\underline{p}, p^*_{i2}]$, the above inequality rearranges as follows

$$\begin{aligned} \underline{P}_j(b_s - \gamma) > (\underline{P}_s - \gamma)b_j/r\xi_j(V - V_L)(b_s - \gamma) &> (\xi_s(V - V_L) + V_L - \gamma)b_j/r \\ \xi_j(1-\alpha)(V-\gamma)(b_s - \gamma) &> [\xi_s(1-\alpha)(V-\gamma) + \alpha(V-\gamma)]b_j/r \\ \xi_j(1-\alpha)(b_s-\gamma) &> [\xi_s(1-\alpha)+\alpha]b_j/r \\ \xi_j(1-\alpha)(b_s-\gamma) &> \alpha_{\xi_s} b_j/r. \end{aligned}$$

Now, by replacing b_s/r with $F - b_j/r$, this last inequality can be easily rearranged as

$$\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} < F - \gamma.$$

We have already found this inequality in comparing p^*_{i2} and $\bar{p}$ and we know that it holds iff the optimal stopping strategy is $\{p^*_{j2}, \bar{p}\}$. The reverse inequality holds instead iff the optimal stopping strategy is $\{p^*_{s2}, \bar{p}\}$. Therefore we can conclude that

$$\text{for } p_t \in [\underline{p}, p^*_{i2}] \begin{cases} \widehat{CS}_s - CS_j > 0 \text{ iff } \frac{(b_s/r-\gamma)}{b_j/r} > \frac{(S-\gamma)}{J} \text{ iff } \{p^*_{j2}, \bar{p}\} \\ \widehat{CS}_s - CS_j < 0 \text{ iff } \frac{(b_s/r-\gamma)}{b_j/r} < \frac{(S-\gamma)}{J} \text{ iff } \{p^*_{s2}, \bar{p}\} \end{cases} \quad (74)$$

So far we have proved that when the stopping strategy is $\{p^*_{j2}, \bar{p}\}$, for $p_t \in [\underline{p}, p^*_{j2}]$, $\widehat{CS}_s - CS_j > 0$ and the reverse inequality holds when the strategy is $\{p^*_{s2}, \bar{p}\}$ and $p_t \in [\underline{p}, p^*_{s2}]$. What is left to prove is that the sign of the spread doesn't revert for the remaining ranges of the state variable, that is for $p_t \in (p^*_{i2}, \bar{p}]$ (we prove this in point i))and $p_t \in (\bar{p}, \infty)$ (see point ii))).

i) We prove here that the sign of $\widehat{CS}_s - CS_j$ does not revert when $p_t \in (p^*_{i2}, \bar{p}]$. In order to do this let us compare our firm with an identical one, say firm A, which only differs for allocation of bargaining power in between creditors, that is:

$$\xi_s + \xi_j = \xi_{As} + \xi_{Aj}, \text{ with } \xi_j \neq \xi_{Aj},$$

where the subscript A denotes the creditors' bargaining powers of firm A.

Particularly, assume that in firm A creditors' bargaining power is allocated such that:

$$\xi_{Aj} : \quad \frac{\alpha_{\xi_{s,j}}}{\xi_{Aj}} = \frac{F - \gamma}{b_j / r},$$

which implies that the optimal stopping strategy is to impair both creditors at $\bar{p}$.

Furthermore, let us assume that the bargaining power of creditors in our firm is allocated such that the optimal stopping strategy is instead $\{p^*_{j2}, \bar{p}\}$ which, therefore, implies

$$\xi_j : \quad \frac{\alpha_{\xi_{s,j}}}{\xi_j} < \frac{F - \gamma}{b_j / r}.$$

First notice that because the sum of creditors' bargaining power in the two firms is the same, the term $\alpha_{\xi_{s,j}}$ is the same for both firms. Therefore, putting together these last two assumptions it follows that

$$\xi_j > \xi_{Aj}.$$

Moreover, from the two above assumptions on $\{\xi_{Aj}, \xi_{As}\}$ and $\{\xi_j, \xi_s\}$ and our previous result in the first part of this Appendix, it follows that

$$\text{for } p_t \in [\underline{\mathrm{p}}, \bar{p}] \rightarrow \frac{b_s/r - \gamma}{b_j} = \frac{\underline{\mathrm{P}}_{As} - \gamma}{\underline{\mathrm{P}}_{Aj}} \tag{75}$$

$$\text{for } p_t \in [\underline{\mathrm{p}}, p^*_{j2}] \rightarrow \frac{b_s/r - \gamma}{b_j} > \frac{\underline{\mathrm{P}}_s - \gamma}{\underline{\mathrm{P}}_j}, \tag{76}$$

where $\underline{\mathrm{P}}_{As}$ and $\underline{\mathrm{P}}_{Aj}$ correspond to the functions $\underline{\mathrm{P}}_s$ and $\underline{\mathrm{P}}_j$ with bargaining powers ξ_{As} and ξ_{Aj} respectively.

Given this scenario it is easy to see that the sign of the spread does not revert. By some straightforward algebra one can prove that

$$\text{for } p_t \in (p^*_{j2}, \bar{p}] \rightarrow \frac{\underline{\mathrm{P}}_{As} - \gamma}{\underline{\mathrm{P}}_{Aj}} = \frac{b_s/r - \gamma}{b_j} > \frac{S - \gamma}{J},$$

where J and S are the debt values in our firm with bargaining power $\{\xi_j, \xi_s\}$. In fact, this last inequality holds if

$$\frac{\underline{P}_{As} - \gamma}{\underline{P}_{Aj}} > \frac{S - \gamma}{J},$$

which can be rearranged as

$$\frac{\underline{P}_{As} - \gamma}{\underline{P}_{Aj}} + 1 > \frac{S - \gamma}{J} + 1,$$

$$\frac{\underline{P}_{As} + \underline{P}_{Aj} - \gamma}{\underline{P}_{Aj}} > \frac{S + J - \gamma}{J}. \tag{77}$$

As we assumed that $\xi_s + \xi_j = \xi_{As} + \xi_{Aj}$, it follows that

$$\underline{P}_{As} + \underline{P}_{Aj} = \underline{P}_s + \underline{P}_j$$

and moreover we know that the equilibrium values J and S are such that

$$\underline{P}_s + \underline{P}_j = S + J.$$

By these two conditions, equation 77) simplifies into

$$J > \underline{P}_{Aj},$$

which holds because the only difference between J and $\underline{P}_{Aj}$ is due to the fact that $\xi_j \neq \xi_{Aj}$ and particularly, as previously shown, $\xi_j > \xi_{Aj}$.

Therefore we conclude that when the stopping strategy is $\{p^*_{j2}, \bar{p}\}$

$$\text{for } p_t \in (p^*_{j2}, \bar{p}] \quad \widehat{CS}_s - CS_j > 0 \;\text{ iff }\; \frac{(b_s/r - \gamma)}{b_j/r} > \frac{(S - \gamma)}{J}.$$

By comparing this result with the previous one, in the first part of the proof, one can conclude that the sign of the spread does not revert over the whole range $[\underline{p}, \bar{p}]$.

When the optimal stopping strategy is $\{p^*_{s2}, \bar{p}\}$, one can repeat symmetrically the above argument. Just compare a firm, which has a stopping strategy $\{p^*_{s2}, \bar{p}\}$, with the same benchmark firm A. The relation between bargaining powers will be reversed now, i.e. $\xi_j < \xi_{Aj}$. The rest of the proof runs symmetrically, and can be easily done by just reverting the sign of the inequalities. We can conclude then that when $\{p^*_{s2}, \bar{p}\}$

$$\text{for } p_t \in (p^*_{s2}, \bar{p}] \quad \widehat{CS}_s - CS_j < 0 \;\text{ iff }\; \frac{(b_s/r - \gamma)}{b_j/r} < \frac{(S - \gamma)}{J}.$$

Again, by comparing this result with the one in the previous part, one can conclude that the sign of the spread does not revert on the range $[\underline{p}, \bar{p}]$.

ii) From some straightforward algebra it is immediate to see that the sign of the spread cannot change when $p_t \in (\bar{p}, \infty)$. We know that the sign of the spread is determined by the following inequality

$$\frac{(b_s/r - \gamma)}{b_j/r} \gtreqqless \frac{(S-\gamma)}{J}.$$

When the stopping strategy is $\{p^*_{j2}, \bar{p}\}$, substituting for the values S and J and rearranging yields

$$\left[\frac{b_j}{r} + \left(\underline{P}_j(p^*_{j2}) - \frac{b_j}{r}\right)\left(\frac{p_t}{p^*_{j2}}\right)^{\lambda}\right]\left(\frac{b_s}{r} - \gamma\right) \gtreqqless$$

$$\left[\frac{b_s}{r} + \left(\underline{P}_s(\bar{p}) + \underline{P}_j(\bar{p}) - J(\bar{p}) - \frac{b_s}{r}\right)\left(\frac{p_t}{\bar{p}}\right)^{\lambda} - \gamma\right]\frac{b_j}{r},$$

$$\left(\underline{P}_j(p^*_{j2}) - \frac{b_j}{r}\right)\left(\frac{p_t}{p^*_{j2}}\right)^{\lambda} \gtreqqless \left(\underline{P}_s(\bar{p}) + \underline{P}_j(\bar{p}) - J(\bar{p}) - \frac{b_s}{r}\right)\left(\frac{p_t}{\bar{p}}\right)^{\lambda},$$

$$\left(\underline{P}_j(p^*_{j2}) - \frac{b_j}{r}\right)\bar{p}^{\lambda} \gtreqqless \left(\underline{P}_s(\bar{p}) + \underline{P}_j(\bar{p}) - J(\bar{p}) - \frac{b_s}{r}\right){p^*_{j2}}^{\lambda},$$

which is independent of p_t. Adding to this result the fact that the difference of the spread is continuous at $p_t = \bar{p}$, we conclude that $\widehat{CS}_s - CS_j$ cannot revert sign in the range $p_t \in (\bar{p}, \infty)$.

We reach the same conclusion when the stopping strategy is $\{p^*_{s2}, \bar{p}\}$, by a similar calculation one can easily prove that the sign of the spread is independent of the state variable.

TABLES AND FIGURES

Table A

Classification of Claims and Voting Solicitations under Chapter 11: Examples of Confirmed Plans.

Company name	Description of Claims	Classes	Impairment	Entitlement to vote	Voting solicitation: acceptance of plan
Petroleum Geo-Services ASA.	Secured Claims	1	Unimpaired	No (deemed to accept)	-
	Priority Non-Tax Claims	2	Unimpaired	No (deemed to accept)	-
	General Unsecured Claims (Class 1 Deficiency Claims .)	3	Unimpaired	No (deemed to accept)	-
	• Bondholder Claims (Senior Notels) • Bank Claims	4	Impaired	Yes	Yes
	Junior Subordinated Debentures Claims	5	Impaired	Yes	Yes
	Securities Law Claims	6	Impaired	No (deemed to reject)	-
	Ordinary Shares	7	Impaired	Yes	Yes
	Other Equity Interests	8	Impaired	No (deemed to reject)	-
360 Networks Ltd.	Administrative Claims	1	Unimpaired	No (deemed to accept)	-
	Priority Tax Claims	2	Unimpaired	No (deemed to accept)	-
	Other priority Tax Claims	3	Unimpaired	No (deemed to accept)	-
	Prepetition Lender Claims	4	Impaired	Yes	Yes
	Nonconsensual Lien Claims	5	Impaired	Yes	Yes
	Secured Claims	6A	Unimpaired	No (deemed to accept)	-
		6B	Impaired	Yes	Yes
	General Unsecured Claims	7	Impaired	Yes	Yes
	Intercompany Claims	8	Impaired	No (deemed to accept)	-
	Interests	9	Impaired	No (deemed to reject)	-
Sun Healthcare Group Inc.	Priority Non-Tax Claims	A	Unimpaired	No (deemed to accept)	-
	Other Secured Claims	B1, B3, B4, B6-B17	Unimpaired	No (deemed to accept)	-
		B2, B5	Impaired	Yes	Yes
	US Health Care Program	C	Impaired	Yes	Yes
	Senior Lender Claims	D	Impaired	Yes	Yes
	General Unsecured	E1	Impaired	Yes	Yes
	Convenience Claims	E2	Impaired	Yes	Yes
	Senior Subordinated Note Claims	F	Impaired	Yes	Yes
	Convertible Subordinated Debenture Claims	G	Impaired	No (deemed to reject)	-

Table A (cont.)

Company name	Description of Claims	Classes	Impairment	Entitlement to vote	Voting solicitation: acceptance of plan
Sun Healthcare Group Inc.	C-TIPS Claims (Convertible Junior Subordinated Debenture)	H	Impaired	No (deemed to reject)	-
	Subsidiary Debtor Intercompany Claims	I	Impaired	No (deemed to reject)	-
	Security Litigation Claims	J	Impaired	No (deemed to reject)	-
	Equity Interests	K	Impaired	No (deemed to reject)	-
National Energy & Gas Transmission, Inc.	Secured Claims	1	Unimpaired	No (deemed to accept)	-
	Priority Claims	2	Unimpaired	No (deemed to accept)	-
	General Unsecured Claims	3	Impaired	Yes	Yes
	Subordinated Claims Interests	4 5	Impaired Impaired	No (deemed to reject) No (deemed to reject)	- -
Williams Telecommunication Group, Inc. and CG Austria, Inc.	Priority Non-Tax Claims	1	Unimpaired	No (deemed to accept)	-
	Prepetition Secured Guarantee Claims	2	Unimpaired	No (deemed to accept)	-
	Other Secured Claims -No claims in this class-	3	(Potentially impaired)	-	-
	TWC Assigned Claims (Senior Notes)	4	Impaired	Yes	Yes
	Senior Redeemable Note Claims	5	Impaired	Yes	Yes
	Other Unsecured Claims	6	Impaired	Yes	Yes
	Subordinated Claims	7	Impaired	No (deemed to reject)	-
	WCG Equity Interests	8	Impaired	No (deemed to reject)	-
	CG Austria Equity Interests	9	Unimpaired	No (deemed to accept)	-
Vero Electronics Inc. and APW Ltd.	Priority Non-Tax Claims	1	Unimpaired	No (deemed to accept)	-
	Senior Secured Credit Facilities	2	Impaired	Yes	Yes
	Other Secured Claims	3	Unimpaired	No (deemed to accept)	-
	Lease Guarantee Claims	4	Unimpaired	No (deemed to accept)	-
	General Unsecured Claims (include Class 2 Deficiency claims)	5A	Impaired	Yes	Yes
		5B	Impaired	Yes	Yes
	Intercompany Claims Securities Litigation Claims	6 7	Unimpaired Impaired	No (deemed to accept) No (deemed to reject)	- -
	Equity Interests	8A	Impaired	No (deemed to reject)	-
		8B	Unimpaired	No (deemed to accept)	-
		8C	Impaired	No (deemed to reject)	-

Data from Chapter 11 Plans and Confirmation Orders provided by the U.S. Bankruptcy Courts of the following districts: Southern District of New York, District of Delaware, District of Maryland.

FIGURE 1: **Restructured debt value with one single creditor**

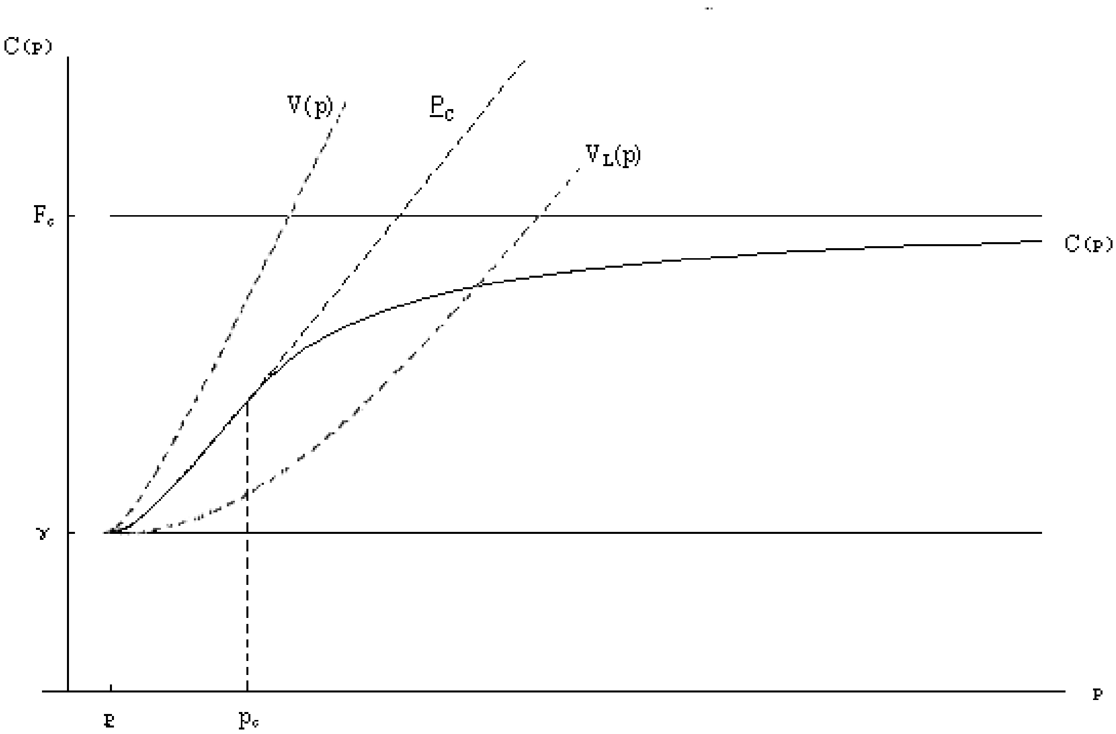


FIGURE 2: **Senior and junior debt values when $p_s<p_j$**

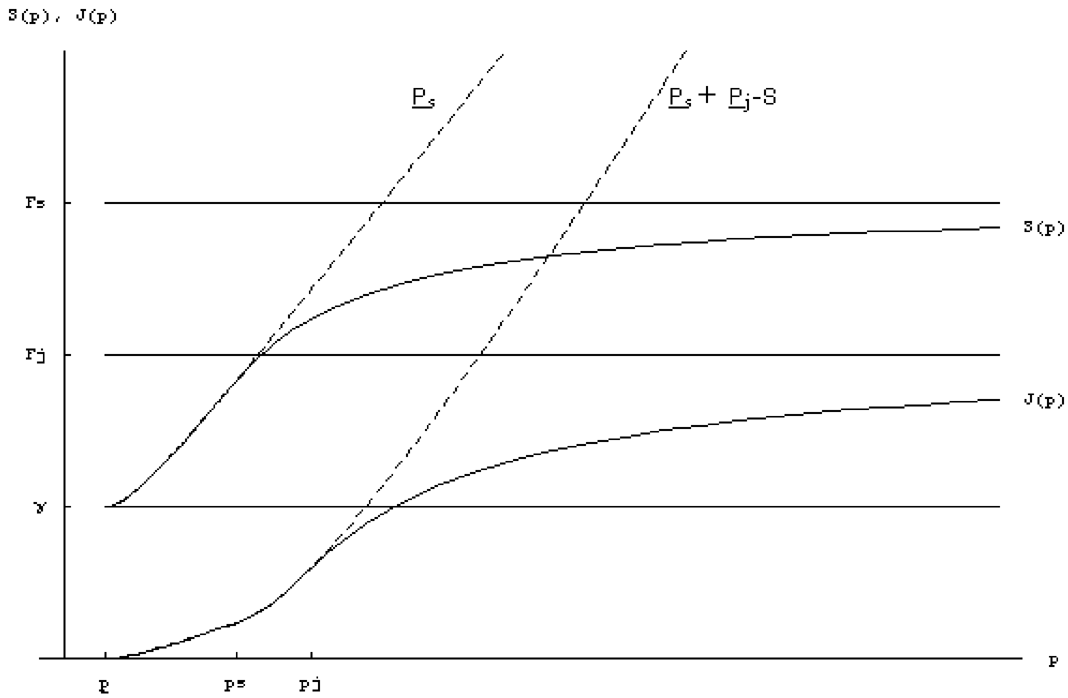


FIGURE 3: **Senior and junior debt values when $p_s = p_j$**

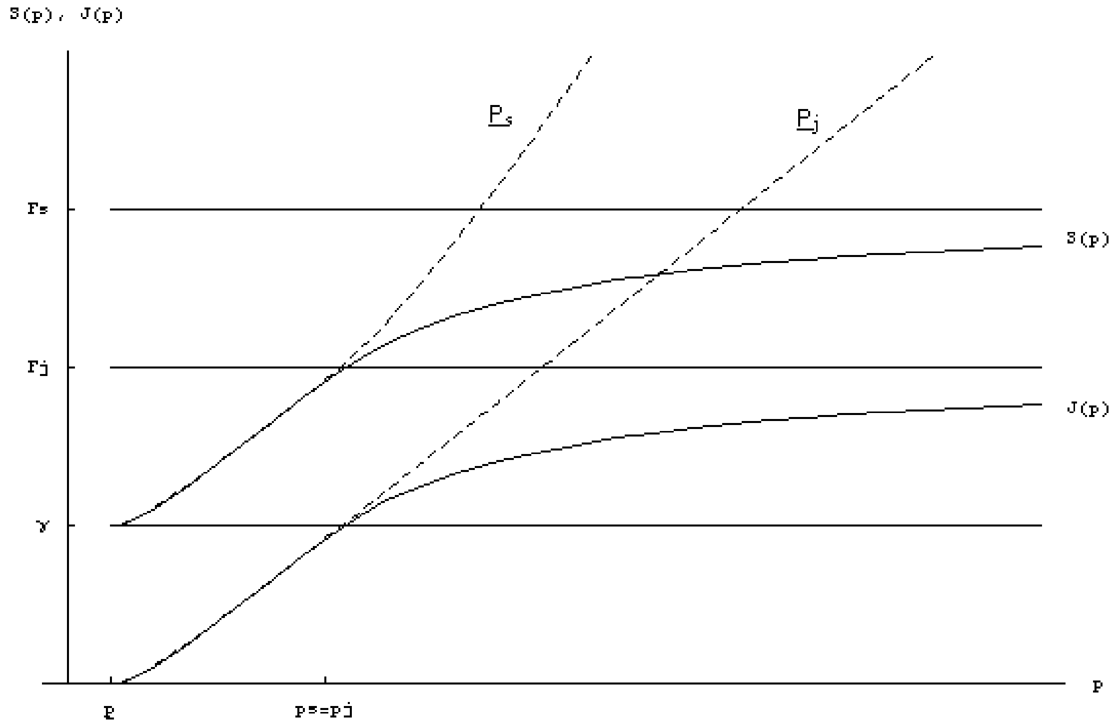


FIGURE 4: **Senior and junior debt values when $p_s > p_j$**

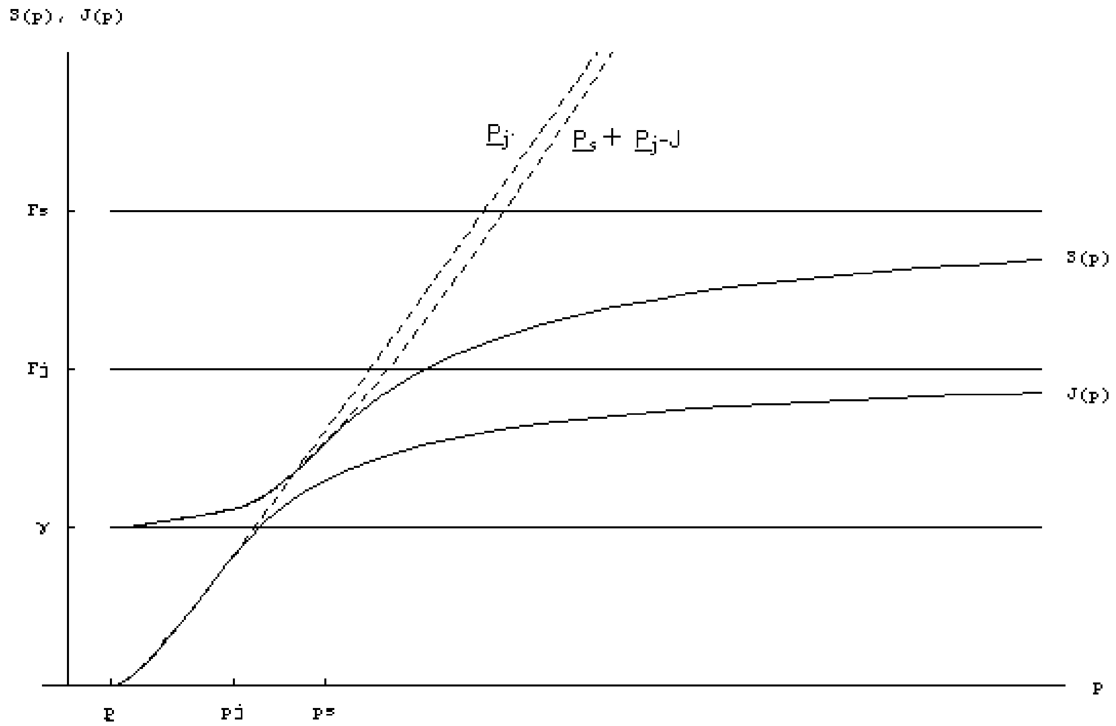


FIGURE 5: Senior and junior debt service when $p_s<p_j$

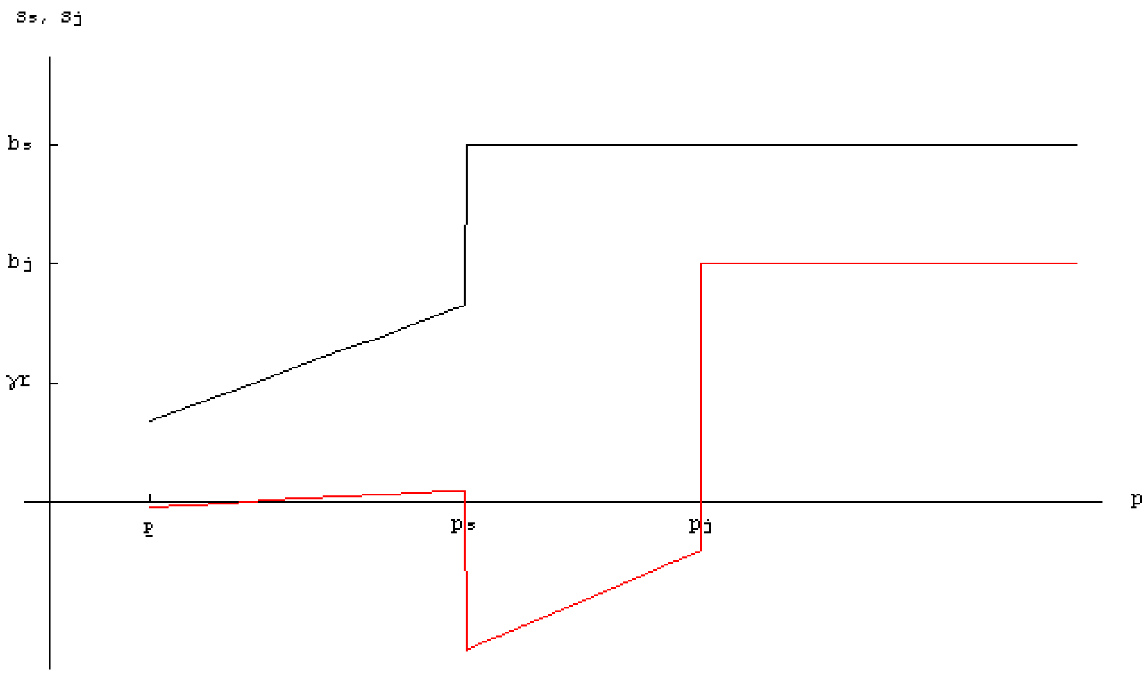


FIGURE 6: Senior and junior debt service when $p_s=p_j$

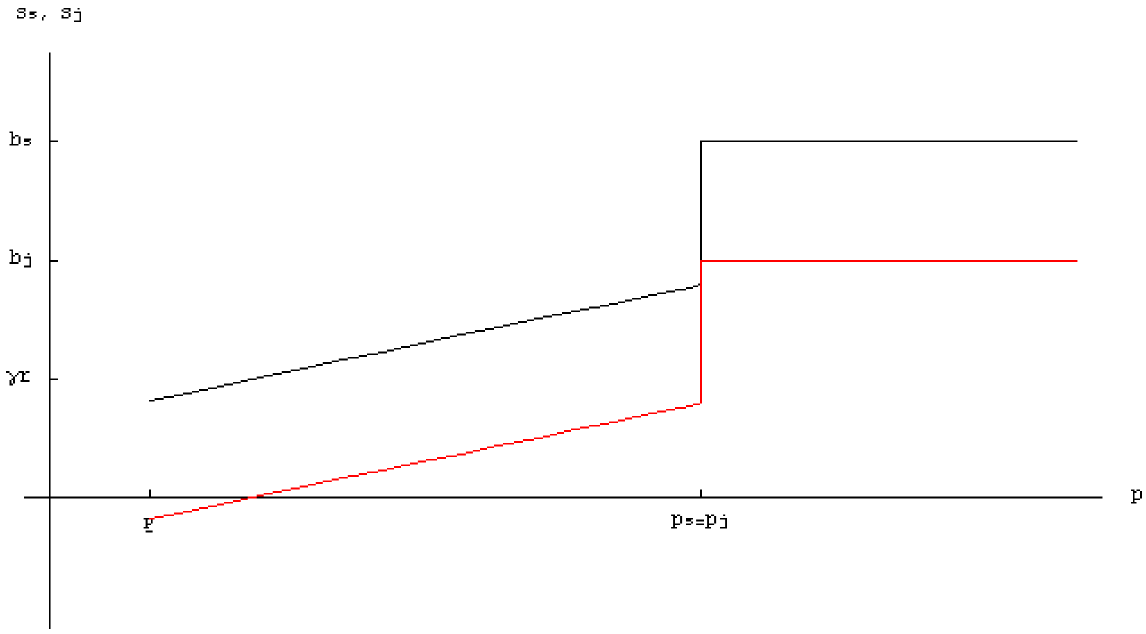


FIGURE 7: Senior and junior debt service when $p_s > p_j$

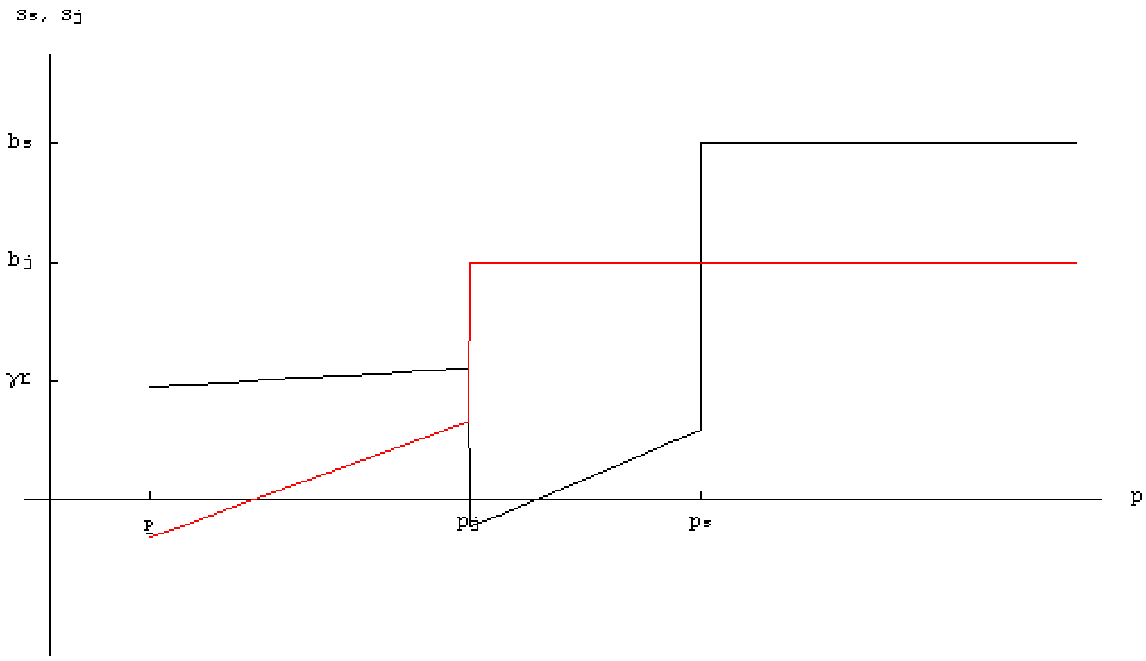
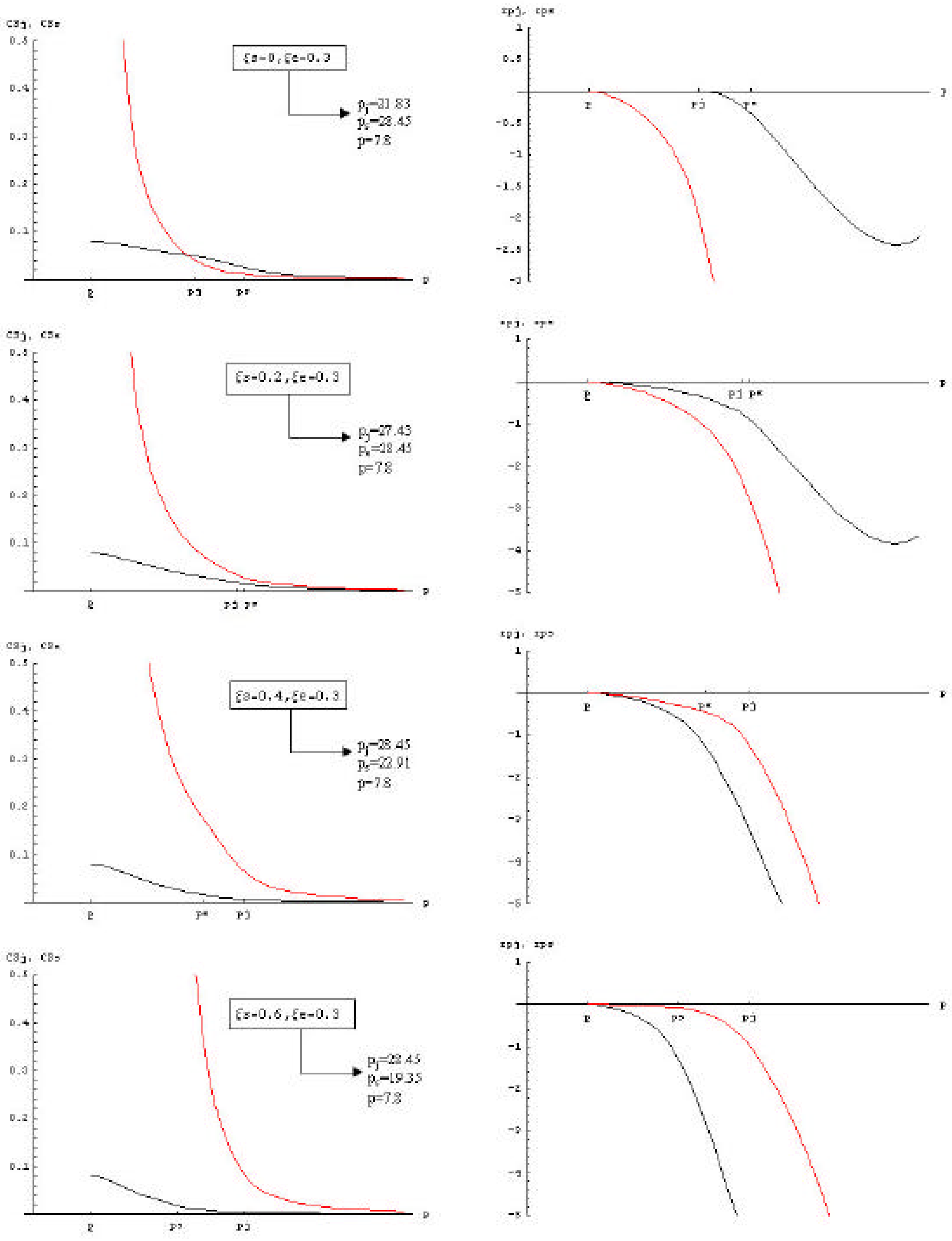


FIGURE 8: Credit Spreads and Renegotiation Premium



Parameter values. The credit spreads and renegotiation premia (right and left column plots respectively) to the senior and the junior creditors (black and red lines respectively) are calculated according to the following parameters: x_e =0.3, a =0.2, m =0.03, s =0.15, r=0.08, g =200, F=600, F_s = 400 and F_j = 200.

FIGURE 9: Senior under-secured/unsecured, junior Credit Spreads and unsecured Spread Differential