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Revisiting the Fiscal Theory of Sovereign Risk from a DSGE Viewpoint *

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Abstract

We revisit Uribe's[32] 'fiscal theory of sovereign risk,' which suggests a trade-off between stabilizing inflation and suppressing default. Unlike Uribe[32], we develop a class of dynamic stochastic general equilibrium models in which the fiscal surplus is endogenous, but where the default mechanism follows Uribe[32] with nominal rigidities. We find that an optimal monetary and fiscal policy, in which both the nominal interest rate and the tax rate are policy instruments, not only stabilizes inflation and the output gap, but also default through stabilizing the fiscal surplus. Thus, there is not necessarily a trade-off between stabilizing inflation and suppressing default.

Keywords: Sovereign Risk; Optimal Monetary Policy; Fiscal Theory of the Price Level
JEL Classification: E52; E60

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1 Introduction

Uribe[32] argues that if a central bank's policy is to peg the price level, government surrenders its ability to inflate away the real value of nominal public liabilities; therefore, public debt default becomes inevitable. Alternatively, if the central bank's policy is to peg the nominal interest rate, the government preserves its ability to suppress public debt default, but is no longer able to stabilize the price level. This argument, known as Uribe's[32] fiscal theory of sovereign risk (FTSR), encompassing a stabilizing of inflation (SI) and a suppressing of default (SD) trade-off (SI–SD trade-off), may be consistent with the intuition of most readers. However, we find that there is not necessarily an SI–SD trade-off, and even if there is, it is not as severe as that suggested by Uribe[32]. Consequently, inflation stabilization is consistent with default suppression, given that default risk could be mitigated through stabilizing inflation, and this result differs markedly from that in Uribe[32]. We can then practically resolve the SI–SD trade-off by adopting an optimal monetary and fiscal (OMF) policy where both nominal interest and tax rates are available as policy instruments to minimize welfare costs, mostly through stabilizing inflation. This is our most important policy contribution. In our model, while we do adopt Uribe's[32] default mechanism, we refocus our attention on the fiscal balance, which is treated as an exogenous shock in Uribe[32]. We then note that it is this exogenous setting that generates Uribe's[32] result that there is an SI–SD trade-off. The most important mechanism in our model is endogenized production, which is a commonplace setting in the literature on optimal monetary policy in the dynamic stochastic general equilibrium (DSGE) established by Woodford[33]. This makes the fiscal balance endogenous and generates a policy implication quite unlike that in Uribe[32]. Thus, the difference in results and/or policy implications between our analysis and that in Uribe[32] depends on the assumption of exogenous or endogenous production.

In the pertinent context of the current European sovereign debt crisis, the conduct of monetary policy appears extremely difficult in such circumstances. For example, even if Greece did not default when it revealed its huge fiscal deficit in October 2009, when its 10-year credit default swap premium began to soar and reached USD 20,404 on April 2012, the European Central Bank (ECB) faced increasing difficulty in conducting monetary policy. Subsequently, the harmonized consumer price index (HCPI) inflation rate started to increase from -0.6% in July 2009, while the ECB's policy interest rate (the short-run buying operation rate) remained at 1% until April 2011, even when HCPI inflation was 2.8%. The ECB thus seemed reluctant to stabilize inflation because of the continuing sovereign debt problem in Greece.[1]

In this paper, we confirm the work in Uribe[32] by developing a class of DSGE models with nominal rigidities. We use this to compare the optimal monetary (OM) and OMF policies with the interest rate spread-minimizing (MIS) policy, a policy that minimizes the interest rate spread; i.e., the difference between the nominal interest rate for safe assets and the government debt yield excluding default risk in an economy with sovereign risk. Note that both the OM and OMF policies correspond to the Taylor rule and the price level targeting in Uribe[32] because they are both de facto inflation stabilization policies, whereas the MIS policy corresponds to the interest rate peg in Uribe[32] because these policies either minimize or set the expected default rate to zero.

[1] In fact, in a speech in December 2011, Vitor Constâncio, vice-president of the ECB, observed that while inflation was likely to remain above 2% for several months, the sovereign debt crisis was ongoing; therefore, the Governing Council of the ECB decided to reduce the monetary policy rate by 25 basis points. It also began to take a series of measures to improve liquidity provision and prevent a possible liquidity crisis. See ECB [17].

We first review Uribe's[32] FTSR. By iterating the government budget constraint forward and imposing an appropriate transversality condition, Uribe[32] demonstrates that the default rate depends on the ratio of the net present value of the real fiscal surplus to real government debt with interest payment. That is, the default rate depends on government solvency. Thus, a decrease in the fiscal surplus, which is exogenous in this setting, decreases government solvency. Facing this case, if the central bank stabilizes inflation, it cannot mitigate the burden of government debt redemption and the default rate increases. Alternatively, if the central bank gives up trying to stabilize inflation, it can mitigate the burden of government debt redemption by inflation. This decreases real government debt, thereby lessening the possibility of default. This is Uribe's[32] FTSR, as hinted at by the 'fiscal theory of price level' in Cochrane[13], Leeper[24], and Woodford[34], and indeed shows that there is an SI–SD trade-off.

How then does endogenized production derive quite different results? First, recall that the fiscal surplus is the difference between tax revenue and government expenditure, and suppose that a tax, which is one of the OMF policy instruments in our analysis, is levied on output and that government expenditure is exogenous. The most important thing here is that the fiscal surplus not only acutely involves the default rate, but also inflation through the output gap. That is, stabilizing the fiscal surplus steadies not only the default rate, but also both inflation and the output gap. Note that the OM and OMF policies are de facto inflation stabilization policies because inflation volatility determines welfare costs stemming from household utility.

Then, suppose that there is an increase in government expenditure, which is exogenous, and the policy authorities, being the government and the central bank, adopt the OMF policy, where the nominal interest and tax rates are policy instruments. Because production is endogenous, the fiscal surplus is now also endogenous. Facing an increase in government expenditure, which applies pressure to increasing inflation because government expenditure increases the GDP gap through an increase in the marginal cost, the government hikes the tax rate to decrease the GDP gap by lowering consumption. As a result, the central bank completely removes the inflation–output gap trade-off because its policy instrument is the nominal interest rate (the basic mechanism for stabilizing inflation in DSGE models with Calvo pricing will be familiar to most readers; thus, we skip to explaining why stabilizing the output gap stabilizes inflation). Although an increase in government expenditure applies pressure to worsening the fiscal deficit, the increased taxation cancels this out, so the fiscal deficit improves. Further, because the fiscal deficit is almost zero as a result, and the fiscal balance stabilizes more than under the OM policy where the tax rate is constant over time, the default rate is roughly zero. In short, the more stabilized is inflation, the more stabilized is the default rate, and vice versa, under the OMF policy. Thus, there is not necessarily an SI–SD trade-off.

We do not necessarily reject Uribe[32] because we can clearly replicate the SI–SD trade-off under the OM policy, which corresponds to the Taylor rule in Uribe[32]. Under the OM policy, facing an increase in government expenditure, inflation is stabilized (it fluctuates more than under the OMF policy because only the nominal interest rate is available to stabilize inflation). However, because the tax rate is constant over time, the tax rate does not increase, the fiscal deficit worsens, and the default rate increases dramatically. Thus, there is an SI–SD trade-off. What about the MIS policy corresponding to the interest rate peg in Uribe[32]? Under the MIS policy, and as in Uribe[32], the interest rate spread is zero. Because the nominal interest rate for safe assets definitely falls in line with the nominal interest rate for risky assets—i.e., government debt yield—the expected default rate is stabilized. In addition, because we assume that the policy authorities commit to their policies, the actual default rate over time is zero. Accordingly, although the default rate is completely stabilized, inflation rises through an increase in the output gap when government expenditure increases. Thus, there is an SI–SD trade-off similar to that in Uribe[32].

However, the SI–SD trade-off that we find is not as severe as that suggested in Uribe[32]. We calculate the volatilities on inflation and the default rate under the OM, OMF, and MIS policies for several plausible levels of price stickiness. First, under the OMF policy, both volatilities are quite low and do not depend on price stickiness (in particular, the volatility on inflation is certainly zero). Second, under the OM policy, the volatility on the default rate is quite high for any plausible price stickiness, even though inflation is well stabilized, unlike the MIS policy. Finally, under the MIS policy, the volatility on the default rate is definitely zero, while the inflation volatility depends on price stickiness, such that the greater the price stickiness, the less the inflation volatility, and vice versa. In addition, if price stickiness is quite high, such as 0.95, which implies that the duration of price revision is 5 years, the volatility on inflation is close to zero. Because the volatility on the default rate is definitely zero, the SI–SD trade-off that we find is not as severe as that suggested in Uribe[32]. Summing up, our results are: i) there is not necessarily an SI–SD trade-off and ii) the trade-off is not as severe as that suggested in Uribe[32]. As policy implications, we argue: i) we can practically solve the SI–SD trade-off by adopting the OMF policy and ii) the MIS policy does not represent an inferior policy from the viewpoint of dissolving the SI–SD trade-off if the price stickiness is sufficiently high.

We now discuss the relationship between our analysis and previous work addressing sovereign risk or crises in the field of macroeconomics. First, Arellano[2] develops a model in which the default probability depends on some stochastic process and shows that default is more likely in recessions. She succeeds in matching her model with Argentinian data and her assumption concerning the default mechanism is subsequently applied by Mendoza and Yue[25] and Corsetti, Kuester, Meier, and Mueller[15]. In their analysis, Mendoza and Yue[25] attempt to explain the negative relationship between output and default observed in the data. That is, they clarify the reason why deep recessions often accompany sovereign default. For their part, Corsetti, Kuester, Meier, and Mueller[15] develop a model including financial intermediaries and demonstrate that sovereign risk may give rise to indeterminacy. They use this to imply that fiscal retrenchment via government spending cuts can help to curtail the risk of macroeconomic instability and, in extreme cases, even stimulate economic activity. Their model stems from Curdia and Woodford[16], and is inclusive of the zero lower bound of nominal interest rates.

Subsequently, Corsetti and Dedola[14] develop a model for a sovereign debt crisis driven by either self-fulfilling expectations or weak fundamentals, and analyze the mechanism through which either conventional or unconventional monetary policy can preclude the former. Their finding that swapping government debt for monetary liabilities can prevent self-fulfilling debt crises is one of several unconventional monetary policies. Elsewhere, and similar to our analysis, Bacchetta, Perazzi, and Wincoop[3] develop a class of DSGE models and analyze both conventional and unconventional monetary policies. They find that the central bank cannot credibly avoid a self-fulfilling debt crisis.

Our analysis differs from this earlier body of work in several ways. Except for Corsetti and Dedola[14] and Bacchetta, Perazzi, and Wincoop[3], the main concern in all these analyses is how sovereign default affects the macroeconomic dynamics, especially those for output, whereas we focus on how the OMF policy affects default risk. In addition, although Corsetti and Dedola[14] and Bacchetta, Perazzi, and Wincoop[3] analyze monetary policy, they do not consider fiscal policy nor how to use it as a stabilization or welfare cost-minimization tool. Thus, our purposes are not identical, and we can say that we propose monetary and fiscal policies to both stabilize inflation and suppress default risk, whereas these related studies propose monetary policy only to suppress default risk.[2]

We also emphasize that while previous work in the area obtains important implications, none examines the SI–SD trade-off in detail. While Uribe[32] certainly discusses the trade-off, we emphasize that there is not necessarily a trade-off. Needless to say, neither Uribe[32] nor Corsetti, Kuester, Meier, and Mueller[15] nor Mendoza and Yue[25] derive this same result. Consequently, examining the trade-off, and deriving useful policy implications from the viewpoint of solving the SI–SD trade-off in this paper, is truly novel.

Finally, we would like to mention that it is not necessarily difficult to talk about default in the closed economy setting we employ. For instance, Burnside, Eichenbaumb, and Rebelo[11] show that a currency crisis stemming from debt deflation, which is a more important source of government income than seigniorage, and introducing indexation does not affect the qualitative results in their calibrations.[3] Elsewhere, Reinhart and Rogoff[29] show that there have been at least 250 sovereign debt defaults worldwide since 1800, with domestic liabilities rapidly increasing five years before default in 89 of these cases. In addition, they find that governments deprived domestic residents more after 1940. Thus, considering default risk in a closed economy setting is not improper.

The remainder of the paper is organized as follows. Section 2 develops the model and Section 3 defines the policy targets under the three policies discussed earlier. Section 4 solves the linear–quadratic (LQ) problem and provides the first-order necessary conditions (FONCs) for the policy authorities. Section 5 calibrates the model under the three policies and Section 6 clarifies the SI–SD trade-off for each. Section 7 concludes the paper. The appendices provide some additional analysis. Appendix A, which provides counterfactual exercises to clarify how the endogenized fiscal surplus yields results different from Uribe[32]. This is because our model differs, not only in this sense, but also in terms of nominal rigidities and elsewhere. Appendix B examines the steady state and Appendices C to F provide some empirical evidence.

2 The Model

We introduce firms into Uribe's[32] model and develop a class of DSGE models with nominal rigidities following Gali and Monacelli[21], although we also assume a closed economy.[4] Thus, the default mechanism is quite similar to Uribe[32]. We follow Benigno[4] (being an earlier working paper version of Benigno[6]) to clarify the households' choice of risky assets. The household i on the

[2]Furthermore, they do not focus on fiscal policy (their models are unsuitable for analyzing fiscal policy regardless), whereas our model can analyze and evaluate the effect of fiscal policy. In terms of other differences, the government in Arellano[2] does not levy taxes on any economic agents, while Mendoza and Yue[25], Corsetti, Kuester, Meier, and Mueller[15] and Bacchetta, Perazzi and Wincoop[3] assume either lump-sum taxes or transfers. Thus, under their settings, it is not possible to analyze fiscal policy. In contrast, in our work, government changes the tax rate to minimize welfare costs, so we can analyze fiscal policy specifically. As a result, we can easily observe the effects of the OMF policy on default. This is the main advantage of our analysis over these existing studies from the viewpoint of model building.

[3]Uribe[32] also cites Burnside, Eichenbaumb, and Rebelo[11] and justifies the closed economy setting when analyzing default.

[4]Following Ferrero[18], we introduce government into Gali and Monacelli[21]. In other words, the model is a closed economy version of Okano[28].

interval $i \in [0,1]$ supplies labor and owns firms that maximize their profit by choosing an optimal price in a monopolistically competitive market. The pricing behavior follows Calvo pricing. We assume that a tax is levied on output and is distorted. Thus, monopolistic power remains, and the steady state is distorted, unlike Gali and Monacelli[21].

2.1 Households

A representative household's preference is given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t \right), \tag{1}$$

where $U_t \equiv \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}$ denotes the period utility, E_t is the expectation conditional on the information set at period t, $\beta \in (0,1)$ is the subjective discount factor, C_t is the consumption index, $N_t \equiv \int_0^1 N_t(i)\, dh$ is the hours of labor, and φ is the inverse of the elasticity of labor supply. The consumption index of the continuum of differentiated goods is as follows:

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods.

The price level is defined as follows:

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}. \tag{3}$$

The maximization of Eq.(1) is subject to a sequence of intertemporal budget constraint of the form:

$$R_{t-1} D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1-\delta_t) + W_t N_t + PR_t \geq \int_0^1 P_t(i) C_t(i)\, di + D_t^n + B_t^n, \tag{4}$$

where $R_t \equiv 1 + r_t$ denotes the gross (risk-free) nominal interest rate, $R_t^G \equiv R_t \Gamma(-sp_t)$ the government debt coupon rate, r_t the net interest rate , $and D_t^n$ is the nominal safety assets issued by households, B_t^n is nominal government debt, W_t is the nominal wage, PR_t denotes profits from the ownership of the firms, δ_t is the default rate, $sp_t \equiv \frac{SP_t}{SP} - 1$ is the percentage deviation of the (real) fiscal surplus from its steady-state value, $SP_t \equiv \tau_t Y_t - G_t$ denotes the (real) fiscal surplus, τ_t denotes the tax rate, $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ denotes (aggregated) output, and $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ denotes (aggregate) government expenditure. Furthermore, we define V as the steady-state value of any variables V_t and v_t as the percentage deviation of V_t from its steady-state value. Thus, SP is the steady-state value of the fiscal surplus. The second term on the left-hand side (LHS) in Eq.(4) implies that the government may default on the share of δ_t and households cannot obtain $R_{t-1}^G B_{t-1}^n \delta_t$ if the government defaults.

Now we discuss the government debt coupon rate $R_t^G \equiv R_t \Gamma(-sp_t)$, where $\Gamma'(-sp_t) > 0$ by assumption. Our assumption implies that government decides the coupon rate for government debt depending on its fiscal situation, such that if this worsens, the government increases the coupon rate. Note that the government debt coupon rate R_t^G is not the government debt yield, which is fully endogenized. In our setting, the government debt yield is decided by households' intertemporal optimal condition; i.e., the Euler equation. Thus, the government debt yield is decided endogenously, although the government debt coupon rate depends on our assumption.

As discussed, the function $\Gamma(-sp_t)$ is hinted at by Benigno[4], although the details somewhat differ. Benigno[4] assumes that households in the home country face a burden in international financial markets in being charged a premium on the foreign interest rate as borrowers and receiving remuneration less than the foreign interest rate as lenders. Following his setting, Benigno[4] assumes $\Gamma'(\cdot) < 0$, which implies that the higher the foreign country's government debt, the lower the remuneration for holding the foreign country's government debt.[5] In contrast, our setting implies that the lower the fiscal surplus, the less the remuneration for holding government debt due to default risk, which in turn harms capital and makes households hesitate to hold government debt. The government then must pay additional remuneration to households to motivate them to hold government debt. Thus, we assume that $\Gamma'(\cdot) > 0$. That is, the lower the fiscal surplus, the higher the interest rate multiplier. Another assumption that differs from Benigno[4] is that $\Gamma(\cdot)$ is a function of the fiscal surplus, whereas Benigno[4] assumes that it is a function of current government debt with an interest payment; i.e., $R_t B_t$. Our setting for $\Gamma(\cdot)$ indirectly follows Corsetti, Kuester, Meier, and Mueller[15]. Corsetti, Kuester, Meier, and Mueller[15] assume that the higher the fiscal deficit, the greater the probability of default, and vice versa. If we are given that the higher the probability of default, the higher the government debt coupon rate, our assumption that $\Gamma(\cdot)$ is a decreasing function of the fiscal surplus is consistent with their assumption in that it implies that the higher the fiscal surplus, the higher the government debt coupon rate. Furthermore, our setting for $\Gamma(\cdot)$ is supported by some empirical evidence. We analyze whether a fiscal deficit or government debt with interest payment increases the interest rate multiplier $\Gamma(\cdot)$ using Greek data. These data imply that the fiscal deficit, but not government debt with interest payment, increases $\Gamma(\cdot)$.[6] Thus, our assumption regarding $\Gamma(\cdot)$ is consistent with both some existing work and the available data.

By solving the cost-minimization problems for households, we obtain the optimal allocation of expenditures as follows:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t. \tag{5}$$

Once we account for Eq.(5), the intertemporal budget constraint can be rewritten as:

$$R_{t-1}D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1-\delta_t) + W_t N_t + PR_t \geq P_t C_t + D_t^n + B_t^n.$$

The households maximize their utility subject to their budget constraint. The optimality conditions for the household's problem are:

$$\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}}\right) = \frac{1}{R_t}, \tag{6}$$

which is the intertemporal optimality condition—i.e., the Euler equation—and:

$$C_t N_t^{\varphi} = \frac{W_t}{P_t}, \tag{7}$$

[5] Benigno[4] observes that this function, which depends only on the level of real government bonds, captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market.

[6] See Appendix D for details.

which is the standard intratemporal optimality condition.

There is another intertemporal optimality condition depicting the households' motivation to hold government debt with default risk. We obtain this by differentiating the Lagrangian by government nominal debt, such that:

$$\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) \quad = \quad \frac{1}{R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right)}. \tag{8}$$

with $R_t^H \equiv R_t \left\{ \Gamma\left(-sp_t\right) + B_t \Gamma'\left(-sp_t\right) \left[B\left(R-1\right)\right]^{-1} \right\}$, and R_t^H can be interpreted as the government debt yield (excluding default risk).

Combining Eqs(6) and (8), we have:

$$R_t = R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right), \tag{9}$$

which shows that the marginal rate of substitution for consumption is the same for households holding either (real) safety assets D_t or (real) government debt B_t because both R_t and $R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right)$ equal the marginal rate of substitution $\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right)$. That is, the consumption schedule is identical irrespective of whether households hold state-contingent claims D_t or government debt B_t. Log-linearizing Eq.(9) yields:

$$\hat{r}_t = \hat{r}_t^H - \mathrm{E}_t \left(\delta_{t+1}\right), \tag{10}$$

with $\hat{r}_t \equiv \frac{dR_t}{R}$ and $\hat{r}_t^H \equiv \frac{dR_t^H}{R^H}$.

Log-linearizing government debt yield R_t^H, we have:

$$\hat{r}_t^H = \frac{\omega_\phi}{1-\beta} \hat{r}_t - \frac{\phi \omega_\gamma}{1-\beta} sp_t + \frac{\phi \beta}{1-\beta} b_t, \tag{11}$$

with $\omega_\phi \equiv 1 - \beta\left(1 - \phi\right)$, and $\omega_\gamma \equiv 1 + \beta\left(\gamma - 1\right)$, where $\phi \equiv \Gamma'\left(0\right)$ denotes the interest rate spread in the steady state and $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$ the elasticity of the interest rate spread to a 1% change in the fiscal deficit in the steady state. Following Benigno[4], we define the interest rate spread for government debtϕ and assume $\Gamma\left(0\right) = 1$. The elasticity γ is an unfamiliar parameter, and we assume $\mid \Gamma'\left(\cdot\right) \mid < \mid \Gamma''\left(\cdot\right) \mid$; thus, $\gamma > 1$. This implies that a decrease in the fiscal surplus increases the government debt coupon rate via an increase in the interest rate multiplier, and vice versa, and that changes in the government debt coupon rate are larger in absolute terms than the changes in the fiscal surplus. Note that our assumption is supported by the data, which we discuss in Appendix B, estimating the elasticity of the interest rate spread given a 1% change in the fiscal deficit γ.

Given our assumption, Eq.(11) implies that an increase in the fiscal surplus decreases the government debt yield, and vice versa. This is intuitively consistent because an increase in fiscal surplus decreases the interest rate multiplier and decreases the government debt yield. In addition, in the third term on the right-hand side (RHS), the sign is positive. This shows that the government debt yield is an increasing function of government debt. An increase in government debt coincides with a decrease in the fiscal surplus, and vice versa. Thus, this positive sign is consistent with the negative sign in the second term. That is, an increase in government debt increases the government debt yield through an increase in the interest rate multiplier $\Gamma\left(\cdot\right)$, which is brought about by a decrease in the fiscal surplus.

We would like to emphasize that a no-arbitrage condition is applied, as shown in Eq.(9). Thus, even if households purchase government debt—i.e., risky assets—households can choose their optimal consumption schedule. Notice that the model includes Eq.(10) which is the log-linearized equality of Eq.(9) and that it will certainly suffice that there is a no-arbitrage condition. To attain the optimal consumption schedule, households need to adjust the balance of the government debt to meet Eq.(9). Because of $R_t^G \equiv R_t \Gamma(-sp_t)$, we can understand that R_t^H consists of the coupon rate and the revenue from holding government debt. Now, suppose that $R_t^H > R_t^G$, which implies that the coupon rate is lower than the government debt yield, and it would seem that there is no incentive to purchase government debt from the government directly. If households purchase government debt, households obtain R_t^G.In addition, households decide their holding of government debt, as shown in the second term of the definition of R_t^H; i.e., $R_t B_t \Gamma'(-sp_t)[B(R-1)]^{-1}$. In this case, R_t^G is low and households purchase much more debt to increase the principal and so obtain more revenue. Purchasing government debt then causes an increase in the interest rate multiplier through a decrease in the fiscal surplus. As a result, household revenue from holding government debt corresponds to R_t^H as households have an incentive to purchase an amount of government debt to meet Eq.(9). Thus, even if $R_t^H > R_t^G$, households purchase government debt because they can choose the amount of government debt outstanding.

2.2 Government

2.2.1 Government Budget Constraint and the Fiscal Theory of the Price Level (FTPL)

Fiscal policy consists of choosing the mix between taxes and the one-period nominal debt with sovereign risk to finance the exogenous process of government expenditure. The flow government budget constraint is given by:

$$B_t^n = R_{t-1}^G (1-\delta_t) B_{t-1}^n - \int_0^1 P_t(i)[\tau_t Y_t(i) - G_t(i)]\, di.$$

Because the optimal allocation of generic goods is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$ and $G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t$, this equality can be rewritten as:

$$B_t^n = R_{t-1}^G (1-\delta_t) B_{t-1}^n - P_t SP_t.$$

Note that government expenditure follows an autoregressive of order one or AR(1) process; i.e., $\mathrm{E}_t(g_{t+1}) = \rho_G g_t$. Dividing both sides of the equality by P_t yields:

$$B_t = R_{t-1}\Gamma\left(-sp_{t-1}\right)(1-\delta_t) B_{t-1}\Pi_t^{-1} - SP_t. \tag{12}$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ being the gross inflation rate. The first term on the RHS corresponds to the amount of redemption with the nominal interest payment and shows that the lower the past fiscal surplus, the higher the interest payments, and the higher the default rate, the lower the redemption, and vice versa.

Log-linearizing Eq.(12) yields:

$$b_t = \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\delta_t - \frac{1}{\beta}\pi_t + \frac{1}{\beta}b_{t-1} - \frac{1-\beta}{\beta}sp_t - \frac{\phi}{\beta}sp_{t-1}, \tag{13}$$

where we use the log-linearized definition of the government debt coupon rate $\hat{r}_t^G = \hat{r}_t - \phi sp_t$ with $\hat{r}_t^G \equiv \frac{dR_t^G}{R^G}$ and $\pi_t \equiv \log\Pi_t$. Eq.(13) implies that not only the higher the current fiscal surplus, but also the higher the past fiscal surplus, the lower the current government debt because an increase in the fiscal surplus decreases the interest payment via a decrease in the interest rate multiplier.

The appropriate transversality condition for government debt is:

$$\lim_{j\to\infty} \beta^{t+j+1} \mathrm{E}_t \left[R_{t+j}^G \left(1-\delta_{t+j+1}\right) \frac{P_{t+j}B_{t+j}}{P_{t+j+1}} \right] = 0.$$

By iterating forward the second equality in Eq.(12), plugging Eq.(6) into this iterated equality, and imposing the appropriate transversality condition for government debt, we have:

$$C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1} \left(1-\delta_t\right) = C_t^{-1} SP_t + \beta \frac{R_t^H}{R_t^G} \mathrm{E}_t \left(C_{t+1}^{-1} SP_{t+1}\right) + \beta^2 \mathrm{E}_t \left(\frac{R_t^H}{R_t^G} \frac{R_{t+1}^H}{R_{t+1}^G} C_{t+1}^{-1} SP_{t+1} \right) + \cdots, \tag{14}$$

which roughly shows that the burden of government debt redemption with interest payment in terms of consumption, or that on the LHS, corresponds to the expected sum of the discounted value of the fiscal surplus in terms of consumption, or the RHS, because of the transversality condition. Here, $\frac{R_t^H}{R_t^G}$ and so forth appear on the RHS. An increase in the government debt coupon rate R_t^G then worsens the fiscal situation through the increase in the interest payment. Thus, R_t^G is the denominator. An increase in the government debt yield facilitates the purchase of government debt, even though consumption decreases. A decrease in the consumption then improves the fiscal situation because it increases the fiscal surplus in terms of consumption. Thus, R_t^H appears as the numerator. If $R_t^G = R_t^H$ is applied to all t, which implies that the government debt coupon rate corresponds to the government debt yield, Eq.(14) reduces to:

$$C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1} \left(1-\delta_t\right) = C_t^{-1} SP_t + \beta \mathrm{E}_t \left(C_{t+1}^{-1} SP_{t+1}\right) + \beta^2 \mathrm{E}_t \left(C_{t+1}^{-1} SP_{t+1}\right) + \cdots.$$

In this case, the burden of government debt redemption with interest payment in terms of consumption simply corresponds to the expected sum of the discounted value of the fiscal surplus in terms of consumption.

Eq.(14) can be rewritten as:

$$\delta_t = 1 - \frac{\frac{R_{t-1}^G}{R_{t-1}^H} \sum_{k=0}^{\infty} \prod_{h=0}^{k} \beta^k \mathrm{E}_t \left(\frac{R_{t+h-1}^H}{R_{t+h-1}^G} C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1}}. \tag{15}$$

Eq.(15) is our FTSR and implies that an increase in inflation does not necessarily occur even if the government's solvency is lost, and vice versa, similar to Uribe[32]. Not only inflation, but also default, can mitigate the burden of government debt redemption with interest payment. Suppose that the price level is constant and there is no inflation. In this situation, if the net present value of the fiscal surplus in terms of consumption (the numerator) is about to fall below the burden of government debt redemption with interest payment in terms of consumption (the denominator), the second term on the RHS is less than unity. Simultaneously, the LHS exceeds zero; i.e., default occurs. In other words, if the government falls insolvent while the price level is strictly stable, default is inevitable. Uribe[32] shows the SI–SD trade-off by introducing default—i.e., sovereign risk—into the central equation of the FTPL. Similar to Uribe[32], at first glance, Eq.(15) also implies that there is an SI–SD trade-off. Furthermore, he calibrates his model and compares the Taylor rule that stabilizes inflation with the interest rate peg. Under the interest rate peg, the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration shows that default ceases just one period after the shock decreasing the fiscal surplus, even though default continues under the Taylor rule after the shock. This implies that a Taylor rule to stabilize inflation includes the unwelcome possibility of magnifying sovereign risk, and this calls for an interest rate peg to counter default. Paying attention to just Eq.(15), which is similar to that in Uribe's[32] model, we seem to obtain policy implications quite similar to those in Uribe[32].

We now present the relationship between our FTSR; i.e., Eq.(15) and the FTPL. If there is neither default risk nor an interest rate multiplier in Eq.(15), Eq.(15) reduces to the following because of $R_t^G = R_t^H = R_t$:

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1} B_{t-1} \Pi_t^{-1}},$$

which is our version of the FTPL. This implies that if solvency worsens, the price level increases; i.e., inflation arises, such that the burden of government debt redemption is mitigated. For now, we introduce sovereign risk, and this mechanism is no longer fully applicable, as Eq.(15) implies.

2.2.2 Default Rule

Because the default rate is decided endogenously, we may not say that the government chooses the default rate following a certain rule. However, although the default rule is endogenous in Uribe[32], Uribe[32] considers the default rule where the government does not default unless the tax-to-debt-ratio falls below a certain threshold.[7] Following this idea, we say that the default rate is decided by the following rule. Let us define $\Psi \equiv \frac{\frac{R_{t-1}^G}{R_{t-1}^H} \sum_{k=0}^{\infty} \prod_{h=0}^{k} \frac{R_{t+h-1}^H}{R_{t+h-1}^G} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1}}$, where Ψ denotes the threshold chosen arbitrarily by the government. Around the steady state, $\Psi = 1$, and we set our threshold to one. The government chooses $\delta_t > 0$ if $\Psi < 1$; i.e., the government defaults if solvency worsens. The government chooses $\delta_t < 0$ if $\Psi > 1$; i.e., the government can afford not to default. The government chooses $\delta_t = 0$ if $\Psi = 1$. [8] Our default rule is different from those proposed by Uribe[32] and Bi, Leeper, and Leith[10]. Unlike Uribe[32], default in our model is consistent with government solvency and is not an ad hoc rule.[9] And unlike Bi, Leeper, and Leith[10], our default rule considers the channel where inflation mitigates default. Their default rule depends on a threshold of the ratio of debt over steady-state GDP, where the threshold is endogenously decided and depends on the government's solvency. However, unlike our default rule, theirs does not consider the situation where inflation mitigates the pressure to increase default. In fact, as pointed out by Reinhart and Rogoff[29], there is a strong observed relationship between default and inflation, with inflation in the year of default usually being quite high. Thus, our default rule is more plausible than those in either Uribe[32] or Bi, Leeper, and Leith[10].

[7] The tax-to-debt ratio in Uribe[32] measures government solvency and corresponds to the second term in Eq.(15).

[8] There is a possibility that the default rate becomes negative; i.e., $\delta < 0$. Uribe[32] interprets a negative default rate as government subsidies for bond holders, and Uribe[31] proposes methods to solve the model with the constraint $\delta \geq 0$. Under this constraint, the government decreases the tax which corresponds to the amount of negative default when the default rate is about to be below zero. We offer another suitable interpretation. The government budget constraint Eq.(12) implies that government debt which will be redeemed increases and the government grants additional government bonds households if the default rate becomes negative. These government bonds are 'subsidy bonds.' In Japan, subsidy bonds are common and often issued to pay for contributions, condolence money, and loss compensation, etc. The total amount of subsidy bonds issued in Japan was 4,250 billion Japanese yen from 1952 to 2017 with the balance of subsidy bonds outstanding being 194 billion Japanese Yen at the end of FY2017. See Ministry of Finance in Japan[26] for details.

[9] Uribe[32] adopts default rules which depend on thresholds, dubbed 'Default Rule 1' and 'Default Rule 2,' the first depending on the ratio of the fiscal surplus over government debt and the second on the ratio of the fiscal surplus over GDP. Under these two rules, default occurs if the ratio exceeds a threshold.

2.2.3 Relationship between Default Rate and Fiscal Surplus

By leading Eq.(15) one period and plugging this back into Eq.(15), we can rewrite Eq.(15) as a second-order differential equation as follows:

$$\delta_t = 1 - \frac{1}{R^G_{t-1}\Pi_t^{-1}B_{t-1}}\left\{SP_t + \beta \mathrm{E}_t\left[\left(\frac{C_t}{C_{t+1}}\Pi_{t+1}^{-1}\right)R_t^H\left(1-\delta_{t+1}\right)B_t\right]\right\}. \tag{16}$$

In Eq.(16), current government debt B_t appears in the second term on the RHS and the sign is negative. That is, a decrease in current government debt increases the default rate, and vice versa. To keep Eq.(15), once government debt is issued, the fiscal surplus must be improved while newly issued government debt is about to reduce the fiscal surplus. Because the fiscal surplus must improve to redeem debt, the default rate declines because of an improvement in the fiscal surplus when government debt increases. Thus, the sign is negative.

In addition, we can easily imagine that the fiscal surplus is a function of the output gap. In fact, the log-linearized fiscal surplus is given by:

$$sp_t = \frac{\beta\tau}{(1-\beta)\varsigma_B}\hat{\tau}_t + \frac{\beta\tau}{(1-\beta)\varsigma_B}y_t - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B}g_t. \tag{17}$$

with $\varsigma_B \equiv \frac{B}{Y}$ and $\varsigma_G \equiv \frac{G}{Y}$ being the steady-state ratio of government debt to output and the steady-state ratio of government expenditure to output, respectively, where $\hat{\tau}_t \equiv \frac{d\tau_t}{\tau}$ denotes the percentage deviation of the tax rate from its steady-state value. We simply refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_t$ as the tax gap. By using Gali and Monacelli's[21] definition of the output gap—i.e., $\tilde{y}_t \equiv y_t - \bar{y}_t$, where $\tilde{y}_t$ and $\bar{y}_t$ denote the output gap and the natural rate of output, respectively—we can recognize that stabilizing the fiscal surplus leads to the stabilization of the output gap.[10]

2.2.4 Log-linearizing the Government Budget Constraint

Log-linearizing Eq.(16) yields:

$$\begin{aligned} c_t \quad = \quad & \mathrm{E}_t\left(c_{t+1}\right) - \beta\hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) - \frac{\omega_\phi}{1-\beta}b_t + \mathrm{E}_t\left(\delta_{t+1}\right) - \frac{\omega_{sp}}{\beta(1-\beta)}sp_t + \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\pi_t \\ & + \frac{1}{\beta}b_{t-1} - \frac{1}{\beta}\delta_t - \frac{\phi}{\beta}sp_{t-1}, \end{aligned} \tag{18}$$

with $\omega_{sp} \equiv (1-\beta)^2 - \phi\omega_\gamma\beta$, where we use the log-linearized definition of the government debt coupon rate. Eq.(18) is our log-linearized Euler equation.

2.3 Firms

This subsection outlines the production, price setting, marginal cost, and other features of the firms, and these are quite similar to Gali and Monacelli[21], although here the tax is levied on firm sales and is not constant.[11]

[10] In our model, the steady state is not efficient because friction stemming from the monopolistically competitive market cannot be dissolved by taxation. Thus, the target level of the output gap (or efficient output gap) is not zero, even though the target level is zero in Gali and Monacelli[21], because the steady state is efficient.

[11] Unlike our setting, Gali and Monacelli[21] assume that under constant employment subsidies, monopolistic power completely disappears.

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(i) = A_t N_t(i),$$

where A_t denotes the productivity.

By combining the production function and the optimal allocation for goods, we have an aggregate production function relating to aggregate employment as follows:

$$N_t = \frac{Y_t Z_t}{A_t}, \tag{19}$$

where $Z_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ denotes the price dispersion.

Log-linearizing Eq.(19) yields:

$$n_t = y_t - a_t. \tag{20}$$

We assume that productivity follows an AR(1) process; i.e., $\mathrm{E}_t(a_{t+1}) = \rho_A a_t$, similar to government expenditure. Z_t disappears in Eq.(15) because of $o\left(\|\xi\|^2\right)$.

Each firm is a monopolistic producer of one of the differentiated goods and sets its prices $P_t(i)$ taking as given P_t and C_t. We assume that firms set prices in a staggered Calvo pricing fashion, according to which each seller has the opportunity to change its price with a given probability $1-\theta$, where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset price. When a firm can set a new price in period t, it does so to maximize the expected discounted value of its net profits. The FONCs for firms are given by:

$$\tilde{P}_t = \frac{\mathrm{E}_t\left(\sum_{k=0}^{\infty}\theta^k\beta^k\tilde{Y}_{t+k}\frac{\varepsilon}{\varepsilon-1}P_{t+k}MC_{t+k}\right)}{\mathrm{E}_t\left(\sum_{k=0}^{\infty}\theta^k\beta^k\tilde{Y}_{t+k}\right)}, \tag{21}$$

where $MC_t \equiv \frac{W_t}{(1-\tau_t)P_t A_t}$ denotes the real marginal cost which is common across firms, $\tilde{Y}_{t+k} \equiv \left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$ denotes the demand for goods when firms choose a new price, and $\tilde{P}_t$ the newly set prices. Note that we assume that government levies a tax on firm sales.

By log-linearizing Eq.(21), we have:

$$\pi_t = \beta\mathrm{E}_t(\pi_{t+1}) + \kappa mc_t, \tag{22}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slope of the New Keynesian Phillips curve (NKPC). Eq.(22) is the fundamental equality of our NKPC.

Substituting Eq.(7) into the definition of the real marginal cost yields:

$$MC_t = \frac{C_t N_t^{\varphi}}{(1-\tau_t)A_t}. \tag{23}$$

Note that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate $1-\tau$ is definitely smaller than one. In such a case, the steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it is unable to be completely absorbed through taxation. As we discuss later, we need to derive our welfare criteria following Benigno and

Woodford[8] because monopolistic power is no longer removed completely, and the steady state is distorted.

Log-linearizing Eq.(23) yields:

$$mc_t = c_t + \varphi y_t + \frac{\tau}{1-\tau}\hat{\tau}_t - (1+\varphi)\, a_t. \tag{24}$$

2.4 Equilibrium

2.4.1 Market-Clearing Condition

The market-clearing condition requires:

$$Y_t(i) = C_t(i) + G_t(i),$$

for all $i \in [0,1]$ and all t. By plugging the optimal allocation for generic goods including Eq.(5) into this market-clearing condition, we have:

$$Y_t = C_t + G_t. \tag{25}$$

By log-linearizing Eq.(25), we obtain:

$$y_t = \varsigma_C c_t + \varsigma_G g_t, \tag{26}$$

where $\varsigma_C \equiv 1 - \varsigma_G$ denotes the steady-state ratio of consumption to output.

2.4.2 Output, Nominal Interest Rate and Inflation Dynamics

Plugging Eq.(26) into Eq.(18) yields:

$$\begin{aligned} y_t \quad = \quad & \mathrm{E}_t(y_{t+1}) - \varsigma_C \hat{r}_t + \varsigma_C \mathrm{E}_t(\pi_{t+1}) - \frac{\varsigma_C \omega_\phi}{1-\beta} b_t + \varsigma_C \mathrm{E}_t(\delta_{t+1}) + \frac{\varsigma_C}{\beta}\hat{r}_{t-1} - \frac{\varsigma_C}{\beta}\pi_t + \frac{\varsigma_C}{\beta} b_{t-1} \\ & + \frac{\varsigma_C}{\beta}\delta_t - \frac{\varsigma_C \omega_{sp}}{\beta(1-\beta)} sp_t - \frac{\phi \varsigma_C}{\beta} sp_{t-1} + \varsigma_G (1-\rho_G)\, g_t, \end{aligned} \tag{27}$$

where we assume that the government expenditure follows an AR(1) process and $\mathrm{E}_t(g_{t+1}) = \rho_G g_t$.

Plugging Eqs(24) and (26) into Eq.(22), we have:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \frac{\kappa[1+\varphi\varsigma_C]}{1-\varsigma_G} y_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_t - \frac{\kappa\varsigma_G}{1-\varsigma_G} g_t - \kappa(1+\varphi)\, a_t. \tag{28}$$

Eq.(28) stemming from the firms' FONC Eq.(16) does not have any notable features.

3 Policy Target

We analyze three policies; i.e., OM, OMF, and MIS. This contrasts with Uribe[32], which analyzes inflation stabilization policy including the Taylor rule and price level targeting, and an interest rate peg that pegs both the nominal interest rate for safe assets and the nominal interest rate for risky assets. Because the OM and OMF policies are both de facto inflation stabilization policies, these clearly correspond to the Taylor rule and the price level targeting in Uribe[32]. At first glance, there is some difference between the interest rate peg in Uribe[32] and the MIS policy, which minimizes the difference between the nominal interest rate $\hat{r}_t$—i.e., the interest rate for safe assets—and the government debt yield $\hat{r}^H_t$. However, both policies are intrinsically the same. The expected default rate converges to zero under the interest rate peg in Uribe[32] and the MIS policy makes the expected default rate zero *ex ante*. In fact, as shown in Eqs(9) and (10), the expected default rate $\mathrm{E}_t(\delta_{t+1})$ should be zero if the nominal interest rate completely corresponds to the government debt yield; i.e., $R_t = R_t^H$. Thus, the MIS policy imitates the interest rate peg in Uribe[32] in this regard. [12]

We now discuss the details of each policy. Under the MIS policy, the policy authorities minimize the interest rate spread between the nominal interest rate and the government debt yield $\hat{r}_t^S \equiv \hat{r}_t^H - \hat{r}_t$ over time. That is, they minimize the following:

$$\mathcal{L}^R \equiv \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t^R\right) \tag{29}$$

with:

$$L_t^R \equiv \frac{1}{2}\left(\hat{r}_t^S\right)^2.$$

Because of Eq.(10), the expected default rate will be zero under the MIS policy. As mentioned, from the viewpoint of minimizing the expected default rate, this policy corresponds to the interest rate peg in Uribe[32]. Note that Uribe[32] shows that the default rate is settled just one period after an exogenous negative fiscal surplus shock under the interest rate peg. Because of the zero expected default rate, default no longer occurs after the second period.

Under the OM and the OMF policies, the policy authorities minimize the welfare cost function over time. We derive the period welfare cost function from the welfare criterion following Gali[19], Benigno and Woodford[8], and Benigno and Woodford[33]. Note that we impose $R_t^G = R_t^H$ when we derive the second-order approximated intertemporal government solvency condition because of the limits of our abilities. However, this restriction has no impact on our analyzing the SI–SD trade-off because our welfare cost function implies that stabilizing inflation is almost the only policy target, and this implies that the OM and OMF policies are de facto inflation stabilization policies.[13] In addition, as shown in Appendix F, our empirical analysis shows that the hypothesis that the government bond yield is consistent with the coupon rate on the benchmark 10-year government bond cannot be rejected for actual data from Italy, Spain, Germany, and the US. This implies that $R_t^G = R_t^H$ cannot be denied, in even countries facing significant sovereign risk, such as Italy and Spain. [14] Thus, we cannot necessarily say that we derive our welfare cost function under a strong assumption.

Following Gali[19], the second-order approximated utility function is given by:

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(\frac{U_t - U}{U_C C}\right) &= \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\frac{\Phi}{1-\varsigma_G} y_t - \frac{(1-\Phi)(1+\varphi)}{\varsigma_C 2} y_t^2 + \frac{(1-\Phi)(1+\varphi)}{1-\varsigma_G} y_t a_t \right. \\ &\quad \left. - \frac{(1-\Phi)\varepsilon}{\varsigma_C 2\kappa} \pi_t^2\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right),\end{aligned} \tag{30}$$

[12] Policy objectives in Uribe[32], such as price level targeting and the interest rate peg, are given exogenously. However, unlike Uribe[32], we do not give policy objective exogenously because this generates indeterminacy. See Gali and Monacelli[21].

[13] In this model, and similar to other DSGE models assuming nominal rigidities, the only practical friction is price stickiness. Thus, our welfare cost function implies that stabilizing inflation is almost the only policy target. In fact, our welfare cost function consists of just the quadratic term for inflation and the output gap from its target level defined later. The value of the coefficient on the quadratic term of inflation is approximately 120.2, although the value of the coefficient on the quadratic term of the output gap from its target level is only approximately 2.4 under our parameterization introduced in Section 5.1.

[14] We find that the results are almost unchanged if we use data on shorter maturity government bonds. A notable exception being Spain because it holds for 10-year maturity bonds, but not those for maturities of 2 and 5 years.

where t.i.p. denotes the terms independent of policy, $o\left(\|\xi\|^3\right)$ are the terms of order three or higher, and $\Phi \equiv 1-\frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. On the RHS, there are linear terms $\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{1-\varsigma_G}y_t\right)$ generating the welfare reversal.[15] To avoid welfare reversal, we need to eliminate the linear terms on the RHS in Eq.(30). Following Benigno and Woodford[8] and Benigno and Woodford[33], the linear terms are rewritten as follows:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{1-\varsigma_G}y_t\right) \quad = \quad & -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\frac{\Phi\left[\left(1-\tau\right)\left(1+\omega_g\right)\omega_{\nu 1}-\tau\omega_{\omega 1}\right]}{2\Gamma\varsigma_C^2}y_t^2\right. \\ & -\frac{\Phi\left[\omega_{\omega 2}\tau-\left(1-\tau\right)\left(1+\omega_g\right)\omega_{\nu 3}\right]}{\Gamma\varsigma_C^2}y_t g_t-\frac{\Phi\left(1-\tau\right)\left(1+\omega_g\right)\omega_{\nu 4}}{\Theta\varsigma_C}y_t a_t \\ & \left.+\frac{\Phi\left(1-\tau\right)\left(1+\omega_g\right)\varepsilon\left(1+\varphi\right)}{2\Theta\kappa}\pi_t^2\right]+\Upsilon_0+o\left(\|\xi\|^3\right),\end{aligned}$$

with $\omega_g \equiv \frac{G}{SP}=\frac{\beta\varsigma_G}{(1-\beta)\varsigma_B}$, $\Theta \equiv \left(1+\omega_g\right)\left(1-\tau\right)\left[1+\varsigma_C\varphi\right]+\tau\left[1-\varsigma_C\left(1+\omega_g\right)\right]$, $\omega_{\nu 1} \equiv \varsigma_C\varphi\left[\varsigma_C\left(1+2\varphi\right)+2\left(2-\varsigma_G\right)\right]$, $\omega_{\omega 1} \equiv \left(1+\varsigma_G\right)\left[1-\varsigma_C\left(1+\omega_g\right)\right]$, $\omega_{\omega 2} \equiv \varsigma_C\left[\varsigma_G\left(1+\omega_g\right)+\omega_g\right]-2\varsigma_G$, $\omega_{\nu 3} \equiv 1-\varsigma_C\left\{\varsigma_G\left(1-2\varsigma_G\right)-\varphi\left[\varsigma_G\left(2-\varsigma_G\right)-2\right]\right\}$, and $\omega_{\nu 4} \equiv \varphi\varsigma_C\left[1+2\left(1+\varphi\right)\right]+\left(1+\varphi\right)\left(2-\varsigma_G\right)$, where $\Upsilon_0 \equiv -\frac{\tau\Phi}{\Gamma(1-\beta)}\omega+\frac{(1-\tau)(1+\omega_g)\Phi}{\Theta\kappa}\nu$ denotes a transitory component and ω and ν are the second-order approximated FONC for firms and the second-order approximated solvency condition for the government. Plugging the previous equality into Eq.(30) yields:

$$\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{U_t-U}{U_C C}\right) \simeq -\mathcal{L}+\Upsilon_0+\text{t.i.p.}+o\left(\|\xi\|^3\right),$$

where:

$$\mathcal{L} \equiv \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(L_t\right) \tag{31}$$

denotes the expected welfare costs:

$$L_t \equiv \frac{\Lambda_x}{2}x_t^2+\frac{\Lambda_\pi}{2}\pi_t^2,$$

with $\Lambda_x \equiv \frac{\omega_{u1}}{\Theta\varsigma_C^2}$ and $\Lambda_\pi \equiv \frac{\varepsilon\left[\Phi(1-\tau)(1+\omega_g)(1+\varphi)\varsigma_C+\Theta(1-\Phi)\right]}{\Theta\kappa\varsigma_C}$, $\omega_{u1} \equiv \Phi\left[\left(1-\tau\right)\left(1+\omega_g\right)\omega_{\nu 1}-\tau\omega_{\omega 1}\right]+\left(1-\Phi\right)\left(1+\varphi\right)\varsigma_C\Theta$, $\omega_{u2} \equiv \varsigma_C\left[\Phi\left(1-\tau\right)\left(1+\omega_g\right)\omega_{\nu 4}+\left(1-\Phi\right)\left(1+\varphi\right)\Theta\right]$, and $\omega_{u3} \equiv \Phi\left[\omega_{\omega 2}\tau-\left(1-\tau\right)\left(1+\omega_g\right)\omega_{\nu 3}\right]$ where L_t denotes the period welfare costs, $x_t \equiv y_t-y_t^*$ denotes the output gap from the target level (OGTL), and $y_t^* \equiv \frac{\omega_{u2}}{\omega_{u1}}a_t+\frac{\omega_{u3}}{\omega_{u1}}g_t$ denotes the target level of output.

Here, we emphasize that analyzing the MIS policy is important. The most important reason we analyze the MIS policy, which corresponds to the interest rate peg in Uribe[32], is to emphasize that one of our results for OMF policy, being that it does not intend to suppress default, can well stabilize the default rate, similarly to the MIS policy. Further, comparing the OM and OMF policies with the MIS policy equates to comparing the inflation stabilization policy with an interest peg in Uribe[32]. That is, we demonstrate that inflation stabilization policy—i.e., the OMF policy—can dissolve the SI–SD trade-off, just as Uribe[32] emphasizes the SI–SD trade-off by comparing the

[15] The presence of linear terms generally leads to the incorrect evaluation of welfare, with a simple example of this result proposed by Kim and Kim[23]. Tesar[30] used the log-linearization method and derived the paradoxical result that an incomplete-markets economy produces a higher level of welfare than the complete-markets economy. Kim and Kim[23] point out that the reversal of welfare ordering implies approximation errors owing to the linearization.

OMF policy with the MIS policy. This method, used to show how the OMF policy dissolves the SI–SD trade-off, makes this easy to understand because we can compare our results with those in Uribe[32].

4 The LQ Problem

4.1 New Keynesian IS (NKIS), NKPC, Government Budget Constraint and the Fiscal Surplus

Plugging the definition of the OGTL into Eq.(27) yields:

$$\begin{aligned} x_t &= \mathrm{E}_t\left(x_{t+1}\right) - \varsigma_C \hat{r}_t + \varsigma_C \mathrm{E}_t\left(\pi_{t+1}\right) - \frac{\varsigma_C\left(1-\beta\right)}{\beta\phi}\mathrm{E}_t\left(\delta_{t+1}\right) + \frac{\varsigma_C}{\beta}\hat{r}_{t-1} - \frac{\varsigma_C}{\beta}\pi_t + \frac{\varsigma_C\omega_o}{\beta^2\phi}\delta_t \\ &\quad - \frac{\varsigma_C\varpi}{\beta}sp_t + \frac{\varsigma_C\left(\omega_\gamma - \phi\beta\right)}{\beta^2}sp_{t-1} + \epsilon_{x,t}, \end{aligned} \tag{32}$$

with $\omega_o \equiv \beta\left(1+\phi\right) - 1$ and $\varpi \equiv 2\left(1-\beta\right) + \beta\gamma$, where $\epsilon_{x,t} \equiv -\frac{\omega_{u2}(1-\rho_A)}{\omega_{u1}}a_t - \frac{\omega_{u3}-\varsigma_G\omega_{u1}(1-\rho_A)}{\omega_{u1}}g_t$ denotes the demand shock. Note that we use Eqs(10) and (11) to derive Eq.(32). Eq.(32) is our version of the NKIS curve. Because of our use of Eqs(10) and (11), the terms for the government debt disappear in Eq.(32).

Plugging the definition of the OGTL into Eq.(28) yields:

$$\pi_t = \beta\mathrm{E}_t\left(\pi_{t+1}\right) + \frac{\kappa\left(1+\varphi\varsigma_C\right)}{\varsigma_C}x_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_t + \epsilon_{\pi,t}, \tag{33}$$

where $\epsilon_{\pi,t} \equiv \frac{\kappa[(1+\varphi\varsigma_C)\omega_{u3}-\varsigma_G\omega_{u1}]}{\varsigma_C\omega_{u1}}g_t - \frac{\kappa[(1+\varphi\varsigma_C)\omega_{u2}-(1+\varphi)\omega_{u1}]}{\varsigma_C\omega_{u1}}a_t$ denotes the cost-push shock. Eq.(33) is our version of the NKPC.

Plugging Eqs(10) and (11) into Eq.(13) yields:

$$sp_t = \frac{1}{\varpi}\hat{r}_{t-1} - \frac{\omega_o}{\phi\beta\varpi}\delta_t - \frac{1}{\varpi}\pi_t + \frac{\omega_\gamma - \phi\beta}{\varpi\beta}sp_{t-1} + \frac{1-\beta}{\phi\varpi}\mathrm{E}_t\left(\delta_{t+1}\right). \tag{34}$$

Plugging the definition of the OGTL into Eq.(17) yields:

$$sp_t = \frac{\beta\tau}{\left(1-\beta\right)\varsigma_B}\hat{\tau}_t + \frac{\beta\tau}{\left(1-\beta\right)\varsigma_B}x_t + \epsilon_{sp,t}, \tag{35}$$

where $\epsilon_{sp,t} \equiv \frac{\beta\tau}{(1-\beta)\varsigma_B}a_t - \frac{\beta(\varsigma_G\omega_{u1}-\tau\omega_{u3})}{(1-\beta)\varsigma_B\omega_{u1}}g_t$ is the fiscal surplus shock.

4.2 FONCs for the Policy Authorities

The policy authorities minimize Eq.(31) under the OM and OMF policies, while they minimize Eq.(29) under the MIS policy, subject to Eqs(32)–(35). Under the OM and MIS policies, the policy instrument is just the nominal interest rate $\hat{r}_t$, and the tax gap $\hat{\tau}_t$ is zero over time; i.e., the tax rate is fixed at its steady-state level. The policy authorities choose the sequence $\{x_t, \pi_t, \hat{r}_t, \delta_t, sp_t\}_{t=0}^{\infty}$.[16] Under the OMF policy, the policy instruments are not only the nominal interest rate $\hat{r}_t$ but also the tax gap $\hat{\tau}_t$. The policy authorities select the sequence $\{x_t, \pi_t, \hat{r}_t, \hat{\tau}_t, \delta_t, sp_t\}_{t=0}^{\infty}$.

[16] Because government debt disappears in our model, at first glance the policy authorities' instrument is merely the nominal interest rate. However, government debt is indirectly chosen by choosing the fiscal surplus and the default rate.

The OM and the OMF policies are then synonyms for an inflation stabilization policy because the weight on the quadratic term of inflation in Eq.(31) is extremely high. However, there is one policy instrument under the OM policy, while there are two policy instruments under the OMF policy. This means that the OM policy regime lacks one of the available policy instruments to conduct policy or to stabilize inflation, while the OMF policy regime is more aggressive in stabilizing inflation than the OM policy regime. Thus, we can find how stabilizing inflation affects the default rate through comparing the dynamics for both inflation and the default rate under both policies.

4.2.1 FONCs under the OM Policy

Now, we show the FONCs under the OM policy. The FONCs for the OGTL and for inflation are given by:

$$\Lambda_x x_t = -\mu_{1,t} + \frac{\kappa\left(1+\varphi\varsigma_C\right)}{\varsigma_C}\mu_{2,t} + \frac{\beta\tau}{\left(1-\beta\right)\varsigma_B}\mu_{4,t} + \frac{1}{\beta}\mu_{1,t-1}, \tag{36}$$

$$\Lambda_\pi \pi_t = -\frac{\varsigma_C}{\beta}\mu_{1,t} - \mu_{2,t} + \frac{1}{\varpi}\mu_{3,t} + \frac{\varsigma_C}{\beta}\mu_{1,t-1} + \mu_{2,t-1}, \tag{37}$$

where $\mu_{1,t}$, $\mu_{2,t}$, $\mu_{3,t}$, and $\mu_{4,t}$ are the Lagrange multipliers on Eqs(32), (33), (34), and (35), respectively. By following Benigno and Benigno[5], we can interpret Eqs(36) and (37) as the targeting rule. Because of default risk, these FONCs are somewhat different from the familiar ones. However, by ignoring the Lagrange multipliers $\mu_{3,t}$ and $\mu_{4,t}$, we can understand that inflation is stabilized via stabilizing the OGTL because the Lagrange multipliers $\mu_{1,t}$ and $\mu_{2,t}$ are multiplied on the NKIS Eq.(32) and NKPC Eq.(33). The mechanism for stabilizing inflation is similar to that in the New Keynesian literature, including Benigno and Benigno[5].

The FONCs for the nominal interest rate and the default rate are given by:

$$\varsigma_C\mu_{1,t} = \varsigma_C \mathrm{E}_t\left(\mu_{1,t+1}\right) + \frac{\beta}{\varpi}\mathrm{E}_t\left(\mu_{3,t+1}\right), \tag{38}$$

$$\frac{\varsigma_C\omega_o}{\phi\beta}\mu_{1,t} = -\frac{\omega_o}{\varpi}\mu_{3,t} - \frac{\varsigma_C\left(1-\beta\right)}{\beta}\mu_{1,t-1} - \frac{1-\beta}{\varpi}\mu_{3,t-1}, \tag{39}$$

where Eqs(38) and (39) are the FONCs for the nominal interest rate and the default rate, respectively. These show that there is a close relationship between the NKIS in Eq.(32) and the government budget constraint in Eq.(34). The FONC for the fiscal surplus is given by:

$$\frac{\varsigma_C\varpi}{\beta}\mu_{1,t} = -\mu_{3,t} - \mu_{4,t} + \frac{\varsigma_C\left(\omega_\gamma - \phi\beta\right)}{\beta}\mathrm{E}_t\left(\mu_{1,t+1}\right) + \frac{\omega_\gamma - \phi\beta}{\varpi}\mathrm{E}_t\left(\mu_{3,t+1}\right), \tag{40}$$

which shows that changes in the fiscal surplus affect the NKIS Eq.(32) and the government budget constraint Eq.(34). In addition, Eqs(39) and (40) imply that changes in the fiscal surplus affect the default rate because of the Lagrange multipliers on the government budget constraint $\mu_{3,t}$ and the definition of the fiscal surplus $\mu_{4,t}$.

4.2.2 FONCs under the MIS Policy

The FONCs for the OGTL and for the inflation are given by:

$$\mu_{1,t} = \frac{\kappa\left(1+\varphi\varsigma_C\right)}{\varsigma_C}\mu_{2,t} + \frac{\beta\tau}{\left(1-\beta\right)\varsigma_B}\mu_{4,t} + \frac{1}{\beta}\mu_{1,t-1}, \tag{41}$$

$$\mu_{1,t} = \mu_{1,t-1} - \frac{\beta}{\varsigma_C}\left(\mu_{2,t} - \mu_{2,t-1}\right) - \frac{\beta}{\varsigma_C\varpi}\mu_{3,t}. \tag{42}$$

Eqs(41) and (42) are equivalent to Eqs(36) and (37) although inflation and the OGTL disappear in Eqs(41) and (42) because the period loss function L_t^R does not include the quadratic terms for inflation and the OGTL. That is, the policy authorities do not intend to stabilize both inflation and the OGTL.

The FONCs for the nominal interest rate and the fiscal surplus are given by Eqs(38) and (40), respectively, even under the MIS policy. The FONC for the default rate has distinctive features and is given by:

$$\delta_t = -\frac{\varsigma_C \omega_o}{\beta \phi}\mu_{1,t} - \frac{\omega_o}{\phi \varpi}\mu_{3,t} - \frac{\varsigma_C (1-\beta)}{\beta \phi}\mu_{1,t-1} - \frac{1-\beta}{\phi \varpi}\mu_{3,t-1}, \tag{43}$$

which can be interpreted as a targeting rule under the MIS policy. As Eq.(10) implies, stabilizing the default rate is essential to minimizing the interest rate spread. Recall that $\mu_{1,t}$ and $\mu_{3,t}$ are Lagrange multipliers on Eqs(32) and (34); i.e., the NKIS and the log-linearized government budget constraint. Thus, to stabilize the default rate, both the NKIS (its LHS is the output gap) and the government budget constraint (its LHS is the fiscal surplus) must shift downward when the default rate is about to increase, and vice versa, because the signs on the first and second terms on the RHS are negative. The negative sign on the first term on the RHS implies that when the default rate is about to increase, the output gap must decrease, and vice versa. If government expenditure does not change, the decrease in the output gap coincides with the decrease in consumption. The decrease in the consumption increases the discounted value of the sum of the fiscal surplus in terms of consumption, or solvency. This improvement in solvency applies pressure to decreasing the default rate and the default rate stabilizes.

The negative sign for the second term on the RHS implies that when the default rate is about to increase, the fiscal surplus decreases. This is consistent with our intuition. If government expenditure is constant, the fiscal surplus decreases when output decreases. A decrease in output coincides with a decrease in consumption. As mentioned, a decrease in consumption increases the fiscal surplus in terms of consumption or solvency and removes the pressure to increase the default rate. Thus, the negative sign for the second term on the RHS is plausible.

4.2.3 FONCs under the OMF Policy

Under the OMF policy, the FONCs are given not only by Eqs(36)–(40), but also by the FONC for the tax gap as follows:

$$\mu_{2,t} = -\frac{(1-\tau)\beta}{(1-\beta)\varsigma_B \kappa}\mu_{4,t}.$$

As mentioned, $\mu_{2,t}$ and $\mu_{4,t}$ are Lagrange multipliers on NKPC Eq.(33) and the definition of the fiscal surplus Eq.(35), respectively. This equality shows that changes in the fiscal surplus affect the NKPC via changes in the tax gap under the OMF policy. In the FONC for inflation Eq.(37), $\mu_{2,t}$ appears with a negative sign. By plugging this FONC into Eq.(37), we can see that the definition of the fiscal surplus Eq.(35) must shift upward to stabilize inflation when inflation is about to increase. Because the LHS of Eq.(35) is the fiscal surplus, this means that the fiscal surplus must increase to stabilize inflation. However, this mechanism to stabilize inflation has another effect. As shown in Eq.(15) (i.e., our FTSR), an increase in the fiscal surplus decreases the default rate and vice versa. Thus, stabilizing the fiscal surplus not only stabilizes inflation, but also suppresses default under the OMF through manipulating the tax gap. Uribe[32] highlights the SI–SD trade-off. However, by endogenizing production, which also endogenizes the fiscal balance, we find that there is not necessarily an SI–SD trade-off. Under the OMF policy, the tax gap works not only to stabilize inflation, but also to suppress default through stabilizing the fiscal balance.

5 Numerical Analysis

5.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. The calibrated parameters mainly follow Ferrero[18], who also analyzes optimal monetary and fiscal policy, except for some unfamiliar parameters, which are estimated. These include the interest rate spread for risky assets ϕ and the elasticity of the interest rate spread to a 1% change in the fiscal deficit γ and an important parameter for analyzing monetary policy, being the price stickiness θ. In addition, we assume that productivity and government expenditure follow AR(1) processes, and we estimate the persistence and standard errors of the innovations from the data.

Following Ferrero[18], the values for the subjective discount factor β, the elasticity of substitution across goods ε, the inverse of the labor supply elasticity φ, the steady-state ratio of government debt to output ς_B, the steady-state ratio of government expenditure to output ς_G, and the steady-state tax rate τ, are set to 0.99, 11, 0.47, 2.4, 0.276, and 0.3, respectively.[17] Using our empirical results for the Greek data reported in Appendices C and E, the spread of the nominal interest rate ϕ, the elasticity of the interest rate spread to the fiscal deficit γ, the price stickiness θ, the persistence of productivity ρ_A, the persistence of government expenditure ρ_G, and the standard errors of the innovations on productivity and government expenditure are set to 0.033, 1.1736, 0.705, 0.976, 0.927, 0.0316, and 0.0728, respectively.

5.2 Macroeconomic Dynamics

5.2.1 Macroeconomic Volatility and Correlation

We first discuss macroeconomic volatility (Tab. 1). The inflation volatility under the MIS policy is 0.8340 and is higher than under the OM policy, where it is 0.0012, even though the default rate volatility under the OM policy is 1.4516 and higher than that under the MIS policy, which is definitely zero. This implies that there is an SI–SD trade-off. If policy authorities choose stabilizing inflation, they must give up suppressing default, and vice versa. This result is consistent with Uribe[32]. However, by comparing the OM policy with the OMF policy, we recognize that there is not necessarily an SI–SD trade-off. Note that both the OM and the OMF policies focus on stabilizing inflation. While the OM policy has just one policy instrument, the OMF policy has two policy instruments. Thus, the volatilities for the OGTL and inflation are definitely zero, which means that the inflation–output gap trade-off is completely dissolved and that these are smaller than the volatilities on the OGTL and inflation under the OM policy (0.0526 and 0.0012, respectively). Notable results are the volatilities on default rate. The volatility under the OMF policy is 0.2372, which is 83.7% smaller than under the OM policy. Because the volatility on inflation under the OMF policy is definitely zero and smaller than under the OM policy, we can say that there is not necessarily an SI–SD trade-off. This result is quite different from Uribe[32].

[17] $\varsigma_B = 2.4$ is consistent with quarterly time periods in the model and implies that the annual steady-state debt–output ratio is 0.6.

We now discuss the correlation between selected variables (Tab. 2). The correlation between inflation and default is -0.9634 under the OM policy. This implies that there is an SI–SD trade-off as long as inflation is stabilized without operating the tax gap. This result is consistent with Uribe[32]. Under the OMF policy, the correlation between the default rate and the fiscal surplus is -0.2306, and the sign is negative. That is, the higher the fiscal surplus, the lower the default rate, and vice versa. The correlation between the fiscal surplus and the tax gap is 0.2582, and the sign is positive. This implies that the tax gap increases facing shocks that increase inflation and that an increase in the tax gap contributes to an increase in the fiscal surplus. As shown in the NKIS Eq.(32) and the NKPC Eq.(33), an increase in the fiscal surplus decreases inflation through a decrease in the OGTL, and vice versa. Thus, inflation is stabilized through an increase in the tax gap. In addition, an increase in the tax gap contributes to decreasing the default rate through an increase in the fiscal surplus, as mentioned in Section 4.2.3. As shown in Eq.(35), an increase in the tax gap increases the fiscal surplus and Eq.(34) shows that the higher the fiscal surplus, the lower the default rate. Thus, the default rate is stabilized through an increase in the tax gap. An increase in the tax gap then stabilizes both inflation and the default rate when facing pressure to inflation. Stabilizing inflation is then consistent with suppressing default, and there is not necessarily the SI–SD trade-off.

5.2.2 Impulse Response Functions

We discuss the impulse response functions (IRFs) and focus on a one-standard-deviation positive change in government expenditure (Fig. 1). An increase in government expenditure applies pressure to decrease the fiscal surplus and to increase the OGTL. Under the MIS policy, the default rate is completely stabilized, while inflation severely fluctuates (Panels 2 and 7). That is, there is clearly an SI–SD trade-off. Similar to the MIS policy, the OM policy generates the SI–SD trade-off. While inflation is more stable than under the MIS policy, the default rate severely rises (Panels 3 and 7). Under the OMF policy, however, while the default rate is not completely stabilized, it is more stable than under the OM policy (Panel 7). In addition, inflation is completely stabilized under the OMF policy (Panel 3). Thus, there is not necessarily an SI–SD trade-off.

How do changes in the steady-state ratio of government debt to output ς_B and the steady-state ratio of government expenditure to output ς_G affects model dynamics? According to OECD[27], the ratio of government debt to output among its member countries varies from 0.127 in Estonia to 2.374 in Japan, while the ratio of government expenditure to output ranges from 0.033 in Columbia to 0.571 in Finland. Thus, we select 0.5 and 9.5 as the low and high cases of the steady-state value of the government debt to output, respectively, and 0.05 and 0.6 as the low and high cases of the steady-state value of government expenditure to output, respectively, and discuss the IRFs under these cases.[18]

We analyze changes in the steady-state ratio of government debt to output ς_B. Generally, the IRFs are not very different between the low and high cases (Fig. 2). In both cases, the tax gap is hiked in a similar way under the OMF policy and both inflation and default are stabilized. Next, we analyze changes in the steady-state ratio of government expenditure to output ς_G. Under the low steady-state ratio of government expenditure to output, an increase in the government expenditure makes the model less volatile due to the low elasticity of government expenditure to output; i.e.,

[18] The timing of the model is quarterly and the ratio of government debt to output 0.127 and 2.374 implies that $\varsigma_B = 0.510$ and $\varsigma_B = 9.496$, respectively.

the low steady-state ratio of government expenditure to output (Fig. 3). Because an increase in government expenditure applies a quite small pressure to increase inflation under the low case of the steady state value of the government expenditure to output, as shown in Eq.(28), the increase in the tax gap needed to cancel this out is also very low, and the fiscal surplus evidently negative, under the OMF policy, unlike the benchmark case (Panels 3 and 5 in Fig. 3). As a result, the default rate severely increases as under the OM policy (Panel 4 in Fig. 3). Under the high steady-state ratio of government expenditure to output, an increase in government expenditure applies quite strong pressure to the increase in inflation, the increase in the tax gap is very high, and the fiscal surplus evidently positive, under the OMF policy, unlike the benchmark case (Panels 8 and 10 in Fig. 3). As a result, the default rate severely decreases (Panel 9 in Fig. 3). In these cases, the fiscal surplus is not stable, unlike the benchmark case, and the default rate not stabilized, even under the OMF policy. Clearly, changes in the steady-state ratio of government debt to output ς_G affect the dynamics under the OMF and generate the SI–SD trade-off.

6 The Trade-off between Stabilizing Inflation and Suppressing the Default Rate

Is the SI–SD trade-off as severe as that highlighted by Uribe[32]? To respond, we calculate the volatilities on both inflation and the default rate under various levels of price stickiness θ ranging from 0.6 to 0.95 in increments of 0.05 (Fig. 4). Note that we just focus on a one-standard-deviation positive change in government expenditure. Under the OM policy, there is clearly an SI–SD trade-off (Panel 1). The higher the price stickiness, the higher the volatility on the default rate and the lower the volatility on inflation, and vice versa. The higher the price stickiness, the higher the weight on inflation in the period welfare costs Λ_π. Thus, the higher the price stickiness, the lower the volatility on inflation. However, as mentioned, aggressively stabilized inflation under the OM policy induces high volatility on the default rate. Thus, there is clearly an SI–SD trade-off. The volatility on inflation depends on the price stickiness under the MIS policy, similar to the OM policy (Panel 2). However, unlike the OM policy, the default volatility does not depend on the price stickiness and is definitely zero. In addition, the standard deviation of inflation is just 0.0070 when the price stickiness is 0.95. When the standard deviation of inflation is nearly zero (3.4×10^{-4}), the standard deviation of the default rate is 1.3649 under the OM policy when the price stickiness is 0.95. Policy authorities may then choose the MIS policy rather than the OM policy because default rate volatility is quite high under the OM policy. Uribe[32] then shows not only the SI–SD trade-off, but also the suggestion of suppressing default by giving up on stabilizing inflation. It seems that Uribe's[32] suggestion is then not totally irrelevant, but may be realistic if the price stickiness is sufficiently high.

What about the SI–SD trade-off under the OMF policy? The inflation volatility is definitely zero, and on the default rate, it is 0.0109, which is constant and does not depend on the price stickiness (Panel 2). Of course, while inflation is completely stabilized, the volatility on the default rate is quite low, but not zero. However, both inflation and default are well and aggressively stabilized, unlike under the OM policy. Thus, we can state that there is not necessarily an SI–SD trade-off. Or if there is an SI–SD trade-off, the SI–SD trade-off is not as severe as that suggested by Uribe[32]. If price stickiness is sufficiently high, and we adopt the MIS policy in place of the OMF policy, both inflation and default rates are well stabilized, although the volatility of the former is not zero.

Next, we discuss the SI–SD trade-off under various steady state ratios of government debt to output ς_B ranging from 0.5 to 9.5 in increments of 0.5 and various steady state ratios of government expenditure to output ς_G ranging from 0.05 to 0.6 also in increments of 0.05 (Figs. 5 and 6). First, we discuss on how changes in the steady state ratio of government expenditure to output ς_B affect the SI–SD trade-off. Similar to the changes in price stickiness, there is an SI–SD trade-off under the OM policy, while under the OMF policy both inflation and default rates are stabilized (Fig. 5).

How do changes in the steady state ratio of government expenditure to output ς_G affect the SI–SD trade-off? Under the OM policy, the higher the ratio, the higher both the standard deviation of the default rate and the standard deviation of inflation and vice versa. The higher the ratio, the greater the pressure to increase inflation. Under a low ratio, inflation is stable because of lesser pressure to increase inflation and the default rate does not rise as much (Panels 1, 2 and 4 in Fig.3). However, if the ratio is high, inflation is not as stabilized as under a low ratio and the default rate rises because of the pressure to increase inflation (Panels 6, 7 and 9 in Fig.3). Thus, the higher the ratio, the higher the standard deviations of the default rate and inflation (Panel 1 in Fig.6). The SI–SD trade-off is even more severe if the ratio is high. Under the MIS policy, the standard deviation of the default rate is always zero, regardless of the ratio. However, the higher the ratio, the greater the pressure to increase inflation and the higher the standard deviation of inflation (Panel 2, Fig.6).

Under the OMF policy, inflation is completely stabilized and the standard deviation of inflation is always zero. If the ratio is sufficiently low, such as 0.05, the increase in the tax gap is quite low and the fiscal surplus becomes obviously negative (Panel 5 in Fig.3). As a result, default is inevitable (Panel 4 in Fig.3). If the ratio is 0.05, the standard deviation of the default rate is 0.1838 (Panel 2 in Fig.6). However, the higher the ratio, the greater the pressure to increase inflation. Thus, an increase in the tax gap is increasing in proportion to an increase in the ratio. An increase in the tax gap improves the fiscal surplus and the fiscal surplus is almost stable if the ratio is 0.276, which is the benchmark value (Panel 9, Fig.1). Because of a stable fiscal surplus, the volatility of the default rate is 0.0817, which is quite low. The SI–SD trade-off then almost dissolves. However, a further increase in the ratio heightens the pressure to increase inflation. To cope with this, the tax gap is aggressively hiked and this increase in the tax gap turns the fiscal surplus positive. As a result, the standard deviation of the default rate increases in proportion to an increase in the ratio. For example, if the ratio is 0.4, the standard deviation of the default rate is 0.8664. Finally, the standard deviation of the default rate is 6.0548 if the ratio is 0.6. The steady-state ratio of government expenditure ς_G affects the SI–SD trade-off and if the ratio is far enough from the benchmark value, the SI–SD trade-off cannot be solved by the OMF policy.

Which policy should we adopt? This cannot be judged unconditionally because the volatility of the default rate is definitely not zero, even under the OMF, and we assume $R_t^H = R_t^G$, which means that the government debt yield equals the government debt coupon rate when we derive the welfare cost function Eq.(29). Thus, we cannot strongly recommend the adoption of the OMF from the viewpoint of minimizing welfare costs. However, if $R_t^H = R_t^G$ is applied, the policy target in our analysis corresponds to the welfare costs. As also discussed, we cannot reject hypothesis $R_t^H = R_t^G$ in both Germany and the US or in Italy and Spain, where the latter face significant sovereign risk. Thus, even in countries such as Italy and Spain facing sovereign risk, we cannot deny that the government debt yield equals the government debt coupon rate. In that case, countries should adopt the OMF policy, but not the MIS policy, from the viewpoint of minimizing welfare cost. As mentioned, the SI–SD trade-off is affected by the steady-state ratio of government expenditure to output and the SI–SD trade-off cannot be solved by the OMF policy if this ratio is sufficiently far from the benchmark value. However, if the ratio ranges from 0.2 to 0.35, the standard deviation of the default rate under the OMF policy is lower than the standard deviation of inflation rate under the MIS policy. Thus, the OMF policy is not an inferior policy if the ratio is around the benchmark value.

7 Conclusion

We develop a class of DSGE models with nominal rigidities and find that: i) there is not necessarily an SI–SD trade-off, and ii) the trade-off is not as severe as what Uribe[32] described. As policy implications, we argue: i) we can practically solve the SI–SD trade-off by adopting the OMF policy, and ii) the MIS policy is not an inferior policy from the viewpoint of dissolving the SI–SD trade-off if the price stickiness is sufficiently high.

While the ECB appears reluctant to stabilize inflation because of smoldering sovereign risk, our results imply that there is another choice for policy authorities without becoming too concerned about the SI–SD trade-off. That is, the OMF policy may be the first option, and the policy authorities should focus on stabilizing inflation through fiscal policy without hesitation even if there is default risk. At the very least, we can surely maintain that the SI–SD trade-off is not as severe as that suggested by Uribe[32]; therefore, we cannot support the assertion that simultaneously stabilizing inflation and suppressing default is impossible.

In terms of future research directions, the welfare criteria and thus the welfare cost function in this paper is not completely consistent with the household utility function. Deriving welfare criteria that is completely consistent with the households' utility function is then a possible avenue of future work.

Appendices

A Counterfactual Exercises to Clarify Endogenized Fiscal Surplus Yields Results Different from Uribe[32]

Unlike Uribe[32], our model includes nominal rigidities and endogenized production. In addition, the tax rate is not constant and debt coupon rate depends on the government's fiscal situation. To clarify how an endogenized fiscal surplus provides different results to Uribe[32], this section provides some counterfactual exercises.

A.1 Price Stickiness

We calculate the standard deviations of the default and inflation rates under price stickiness for values ranging from 0.05 to 0.95 in increments of 0.05. The standard deviations of default rate and inflation are always 0.2372 and 0.0000 under the OMF despite price stickiness, respectively, as shown in Tab.1. The reason is that inflation is completely stabilized under the OMF and the tax gap is available as a policy tool to minimize the welfare costs function. Stabilizing inflation completely implies that the effects stemming from price stickiness are eliminated even if it is very high. Thus, our results are not changed, even if price stickiness changes.

A.2 Constant Tax Rate

The tax rate is constant and the tax gap is not available as a policy tool under the OM, which corresponds to inflation stabilization policies, such as the Taylor rule in Uribe[32], and different to the OMF. The standard deviations of the default rate and inflation under the OMF are always 0.2372 and 0.0000, respectively, and those under the OM are always 1.4516 and 0.0012, respectively. Under the OMF policy, both default and inflation are well suppressed or stabilized. However, under the OM, inflation is not necessarily very well stabilized and default is not suppressed. This means that the role of tax gap is very important.

A.3 Debt Coupon Rate that Depends on the Government's Fiscal Situation

In our model, the government debt coupon rate R_t^G does not necessarily correspond to the government debt yield R_t^H. We show the result when the difference between R_t^G and R_t^H is minimized. Suppose the policy authorities minimizing the period loss function as follows:

$$L_t \equiv \frac{\Lambda_x}{2} x_t^2 + \frac{\Lambda_\pi}{2} \pi_t^2 + \frac{\Lambda_C}{2} \left(\hat{r}_t^H - \hat{r}_t^G \right)^2$$

The policy minimizes this period loss function by controlling both the nominal interest rate and the tax gap, meaning that the policy authorities conducting the OMF policy minimize the difference between the government debt coupon rate and the government debt yield. When we set Λ_C to 78, the mean of $\hat{r}_t^H - \hat{r}_t^G$ is 0.0007 and both the standard error and the variance of $\hat{r}_t^H - \hat{r}_t^G$ are 0.0002 and 0.0000, respectively. This means that the difference between the government debt coupon rate and the government debt yield is negligible, that is $R_t^H = R_t^G$ is applicable. Under this policy, the government issues its debt at the market price and the policy authorities conduct the OMF. We report that the standard deviations of the default and inflation rates under the OMF with a minimized difference between the government debt coupon rate and the government debt yield are 0.0518 and 0.0001, 0.2372 and 0.0000 under the OMF and 1.4516 and 0.0012 under the OM, respectively, as shown in Tab.1. Under the OMF with a minimized difference between the government debt coupon rate and the government debt yield and the OMF, the standard deviations of the default rate and inflation are both smaller than under the OM. Thus, the SI–SD trade-off is dissolved or mitigated, at least under the OMF with a minimized difference between the government debt coupon rate and the government debt yield, similar to the OMF. Further, the standard deviation of the default rate under the OMF with a minimized difference between the government debt coupon rate and the government debt yield is smaller than that under the OMF. This means that our result is robust because the standard deviation of the default rate becomes smaller and the SI–SD trade off more mitigated by assuming a setting closer to Uribe[32].

A.4 Endogenous Production

By assuming that there is no price stickiness and no difference between the coupon rate and the government debt yield, we can obtain the results in a simple endogenous production setting. The results obtained are the same as the results of the OMF with a minimized difference between the government debt yield R_t^H and the government debt coupon rate R_t^G with very low price stickiness. We set the price stickiness θ to 0.001 and Λ_C to 78. The mean of $\hat{r}_t^H - \hat{r}_t^G$ is 0.0007 and the standard error and variance of $\hat{r}_t^H - \hat{r}_t^G$ are 0.0002 and 0.0000, respectively. Note that we cannot solve the model if we set price stickiness θ to zero and θ as 0.001. The standard deviations of the default rate and inflation under the OMF are 0.2372 and 0.0000, respectively, while under the simple endogenous setting they are 0.0518 and 0.0001, which is the same as the results in Section A.3. Our results are thus unchanged even if we assume a simple endogenous production setting.

B Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_t = 1$ and $\frac{\tilde{P}_t}{P_t} = 1$. Because this steady state is nonstochastic, the productivity has unit values; i.e., $A = 1$. We assume that the default rate in the steady state is zero; i.e., $\delta = 0$.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \beta^{-1}.$$

Eq.(21) can be rewritten as:

$$\tilde{P}_t = \mathrm{E}_t \left(\frac{K_t}{P^{-1} F_t} \right) \tag{B.1}$$

with:

$$K_t \equiv \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \left(P_{t+k} C_{t+k} \right)^{-1} \tilde{Y}_{t+k} MC_{t+k}^n \quad ; \quad F_t \equiv P_t \sum_{k=0}^{\infty} \left(P_{t+k} C_{t+k} \right)^{-1} \tilde{Y}_{t+k},$$

which implies that:

$$K = \frac{\frac{\varepsilon}{\varepsilon - 1} Y MC^n}{(1 - \alpha\beta)(PC)} \quad ; \quad F = \frac{PY}{(1 - \alpha\beta)(PC)}.$$

These equalities imply that:

$$P = \frac{\varepsilon}{\varepsilon - 1} MC^n.$$

Thus, we have:

$$MC = \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{-1}. \tag{B.2}$$

Furthermore, Eqs23) and (B.2) imply the following:

$$CN^{\varphi} = \frac{1 - \tau}{\frac{\varepsilon}{\varepsilon - 1}}. \tag{B.3}$$

Eq.(B.3) implies the familiar expression:

$$(1 - \tau) U_C = \frac{\varepsilon}{\varepsilon - 1} U_N.$$

Note that because $\tau \in (0,1)$ and $\varepsilon > 1$, this steady state is distorted.

Eq.(12) yields the following:

$$B\left(\frac{1-\beta}{\beta}\right) = SP, \tag{B.4}$$

with $B \equiv \frac{B^n}{P}$.

Note that $R = R^H$ because of $\delta = 0$ and $R^G = R\Gamma(0)$. Plugging this into Eq.(14) yields:

$$C^{-1}R\Gamma(0)B = C^{-1}SP + \frac{\beta}{\Gamma(0)}C^{-1}SP + \left(\frac{\beta}{\Gamma(0)}\right)^2 C^{-1}SP + \cdots. \quad = \quad \frac{1}{1-\beta[\Gamma(0)]^{-1}}C^{-1}SP,$$

which implies:

$$\Gamma(0)B\beta^{-1} = \frac{1}{1-\beta[\Gamma(0)]^{-1}}SP. \tag{B.5}$$

Plugging Eq.(5) into this equality yields:

$$\Gamma(0) = \frac{1-\beta}{1-\beta[\Gamma(0)]^{-1}},$$

which implies that $\Gamma(0) = 1$. Thus, our assumption that $\delta = 0$ is consistent with $\Gamma(0) = 1$.

Because of $\Gamma(0) = 1$, $R^G = R$. Thus,

$$R^G = R^H. \tag{B.6}$$

In the steady state, Eq.(15) reduces to:

$$1 = \frac{\frac{1}{\left(1-\beta\frac{R^H}{R^G}\right)}\left(C^{-1}SP\right)}{C^{-1}RB}. \tag{B.7}$$

Note that the RHS in Eq.(B.7) corresponds to the steady-state value of Ψ. That is, $\Psi = 1$ is applied in the steady state. This implies that the default rate is zero in the steady state.

C Empirical Evidence for the Calibrated Unfamiliar Parameters and AR(1) Processes

One of our calibrated parameters, the elasticity of the interest rate spread to the fiscal deficit γ, draws on the following regression:

$$\frac{CR^{Risky} - CR_t}{\bar{X}} = \alpha_0 + \alpha_1(1 - DUM_t)df_t + \alpha_2 DUM_t + \alpha_3 DUM_t df_t, \tag{C.1}$$

where CR_t^{risky} corresponding to R_t^G denotes the nominal coupon rate for risky assets, CR_t the nominal coupon rate for safety assets, DUM_t is a Greek crisis dummy variable that takes a value of one for the period from May 2010 to June 2012 and zero otherwise (detailed explanation provided below), and $\bar{X}$ denotes the average of $CR_t^{Risky} - CR_t$ for the period of $DUM_t = 1$. α_1 and α_3 measure how changes in the percentage deviation of the fiscal deficit $df_t \equiv -sp_t$ widen or narrow the interest rate spread (coupon rate based) $CR_t^{risky} - CR_t$. Although these coefficients correspond to γ, we focus on α_3 because it is the elasticity during the severe debt crisis. Specifically, α_3 can be regarded as $\frac{d\left(CR_t^{Risky} - CR_t\right)}{d(df_t)} \frac{1}{CR_t^{Risky} - CR_t}$, which is consistent with our assumption of γ.

Data are monthly and retrieved from Thomson Datastream, and we use the coupon rate spread between the 10-year government bond for Greece and that for Germany and the real government budget balance in Greece.[19] The sample period is from January 2005 to April 2015. Note that the Athens Olympics were in January 2005, at the beginning of the period when the unhealthy fiscal deficit started. The real government budget balance is seasonally adjusted and Hodrick–Prescott (HP) filtered. We assign $DUM_t = 1$ during May 2010 to June 2012, otherwise $DUM_t = 0$. Note that Greece requested fiscal support from both the International Monetary Fund (IMF) and the ECB in April 2010, May 2010 was the following month, and Greece decided to adopt a reduced budget following the results of the poll in June 2012. That is, $DUM_t = 1$ is assigned during the severe debt crisis in Greece.

The estimators on α_0, α_1, α_2, and α_3 are 0.0802, 0.0144, 0.8651, and 1.1736, respectively. The corresponding standard errors are 0.0188, 0.0012, 0.0211, and 0.0955, respectively. All coefficients are significant at the 1% level. The result that α_3 is significant implies that the elasticity of the interest rate spread (coupon rate based) to the fiscal deficit γ is significant during the severe debt crisis when the nominal interest rate rose rapidly, and its elasticity is 1.1736. Thus, we set γ to 1.1736. Because γ is significant during May 2010 to June 2012, we regard the average of the spread $CR_t^{risky} - CR_t$ as the risk premium, and we find that the interest rate spread for risky assets ϕ is 0.033.

AR(1) processes are also estimated from the data for real GDP, the GDP deflator, nominal government expenditure and employment in Greece retrieved from IMF World Economic Outlook, and the sample period is from January 2005 to April 2015. Productivity is GDP divided by employment and real government expenditure is nominal government expenditure divided by the GDP deflator. The generated data are HP filtered. Our results for the persistence of productivity ρ_A and the persistence of government expenditure are 0.976 and 0.927, respectively, and the innovations for productivity and government expenditure are 0.0316 and 0.0728, respectively, as mentioned in Section 5.1.

As we discussed in Section 2.1, our assumption concerning the elasticity of the interest rate spread to the fiscal deficit $\gamma > 1$ is supported by the data. This is because the t-statistic for the null hypothesis $\alpha_3 = 1$ against the alternative hypothesis $\alpha_3 > 1$ is 1.8182, and its corresponding p-value is 0.0359, and thus $\alpha_3 > 1$ is supported statistically. Note that as mentioned, α_3 corresponds to γ.

D Empirical Evidence for Government Debt with Interest Payment as an Argument for $\Gamma(\cdot)$

Similar to Eq.(C.1), we estimate the following:

$$\frac{CR^{Risky} - CR_t}{\bar{X}} = \tilde{\alpha}_0 + \tilde{\alpha}_1 (1 - DUM_t) rb_t + \tilde{\alpha}_2 DUM_t + \tilde{\alpha}_3 DUM_t rb_t,$$

where $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ measure how changes in the percentage deviation of government debt with interest payment from its steady-state value $rb_t \equiv \frac{R_t B_t}{RB} - 1$ widen or narrow the coupon rate spread $CR_t^{risky} - CR_t$. Thus, $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ correspond to γ. Data are quarterly and retrieved from Thomson Datastream, and we use the sum of government debt and the government interest

[19] The original data include the nominal government budget balance, which we deflate using the CPI.

payment divided by the CPI in Greece. The generated data are HP filtered. The sample period runs from Q1, 2005 to Q1, 2015 because data on government debt and interest payment are available in quarterly frequency. We assign $DUM_t = 1$ during Q2, 2010 to Q2, 2012, otherwise $DUM_t = 0$. The estimation procedure is the same as Eq.(C.1).

The estimators on $\tilde{\alpha}_0$, $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, and $\tilde{\alpha}_3$ are 0.0518, -1.5522, 0.9727, and 1.5428, respectively. The corresponding standard errors are 0.0687, 1.5492, 0.0735, and 1.8895, respectively. That $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ are not significant means that γ cannot be estimated if we assume that the argument for $\Gamma(\cdot)$ is government debt with interest payment in Greece. This estimation result and the result on Appendix C imply that the (negative) fiscal surplus as an argument for $\Gamma(\cdot)$ is plausible, although government debt with interest payment as an argument for $\Gamma(\cdot)$ is not plausible.

E Empirical Evidence for Price Stickiness

Following Gali and Gertler[20] and Benigno and Lopez-Salido[7], we estimate an equation as follows:

$$\mathrm{E}_t \left[\theta\pi_t - \theta 0.99\pi_{t+1} - (1-\theta)(1-\theta 0.99)\, mc_t\right] = 0. \tag{E.1}$$

The estimation method is the generalized method of moments developed by Hansen[22]. We use quarterly data for Greece for the GDP deflator and nominal unit labor cost retrieved from Thomson Datastream, both seasonally adjusted. The sample period runs from Q1, 2005 to Q3, 2015. The rate of change in the GDP deflator is regarded as the data series for inflation π_t. We deflate the nominal unit labor cost by the GDP deflator to generate the real unit labor cost. Finally, we calculate the percentage deviation of the marginal cost from its steady-state value following $mc_t = \frac{MC_t - MC_t^{HP}}{MC_t^{HP}}$, where MC_t^{HP} is the HP-filtered real marginal cost.

To estimate, π_{t-1}, π_{t-2}, mc_{t-1}, and mc_{t-2} are designated as instrumental variables. We use heteroscedasticity and autocorrelation-consistent standard errors. The spectral estimation method is the quadratic spectral kernel, and the bandwidth parameter is selected using the Andrews[1] procedure. The J-statistic for the validity of overidentifying restrictions is 2.03, and the associated p-value is 0.56. This suggests that the above equation is successfully estimated.

As estimation results, we obtain the estimator 0.705 and standard error 0.206. Because the p-value is 0.001, our estimator is significant at the 1% level.

F Empirical Evidence for the Relationship between the Redemption Yield and the Coupon Rate

We estimate an equation as follows:

$$r_t^H = \beta_0 + \beta_1 r_t^G,$$

where r_t^H and r_t^G denote the yield and the coupon rate on benchmark 10-year government bonds, respectively. Here, the coupon rate is the monthly average. We use monthly data for the PIIGS—i.e., Portugal, Italy, Ireland, Greece, and Spain—and Germany and the US, and retrieve the data from Thomson Datastream. The sample period runs from January 2005 to September 2015. We verify $\beta_0 = 0$ and $\beta_1 = 1$, which implies that the yield equals the coupon rate on average. Our results for β_0 in Portugal, Italy, Ireland, Greece, Spain, Germany, and the US are 9.501, 0.353, -5.419, 7.939, 0.353, -0.176, and 0.129, respectively, and the corresponding standard errors are

4.349, 0.542, 2.718, 3.898, 0.542, 0.131, and 0.089, respectively. The estimator for β_0 in Portugal, Ireland, and Greece is significant at the 5% level, while the remainder are not significant. Our results for β_1 in Portugal, Italy, Ireland, Greece, Spain, Germany, and the US are -0.919, 0.893, 2.204, 0.350, 0.893, 1.020, and 0.960, respectively, and the standard errors are 0.852, 0.126, 0.659, 1.0418, 0.126, 1.020, and 0.960, respectively. We cannot reject that $\beta_1 = 1$ in Italy, Ireland, Spain, Germany, and the US and the estimators are significant at the 1% level, while the estimator on β_1 in Portugal and Greece is not significant.

We also conduct F-tests for the null hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$, and obtain F-statistics of 2.670, 0.567, 3.036, 5.187, 0.567, 2.584, and 1.082 for Portugal, Italy, Ireland, Greece, Spain, Germany, and the US, respectively. The p-values are 0.073, 0.568, 0.052, 0.007, 0.569, 0.079 and 0.342 for Portugal, Italy, Ireland, Greece, Spain, Germany, and the US, respectively. Because the F-statistics in Greece are significant at the 1% level, we cannot accept our hypothesis $r_t^H = r_t^G$ for Greece.

Summarizing our results, the hypothesis $\beta_0 = 0$ and $\beta_1 = 1$ is supported in Italy, Spain, Germany, and the US. That is, roughly speaking, the yield is consistent with the coupon rate on benchmark 10-year government bonds in these countries. However, in Portugal, Ireland, and Greece, the yield is not consistent with the coupon rate on the benchmark 10-year government bond.

An important issue is that this empirical analysis draws on data for 10-year government bonds whereas our model includes only one-period bonds. To confirm the robustness of the empirical results, we re-estimate the above equation using the data on government bonds with maturities of 2 and 5 years. Unfortunately, coupon rate data on government bonds with maturities shorter than 10 years are not available for Greece. We find that the results remain almost unchanged if we use government bonds with a shorter maturity (an exception is Spain). The results obtained are not provided in this paper, but are available from the authors upon request. For a notable approach to incorporating long-term debt into quantitative analyses of sovereign debt and default, see Chatterjee and Eyingungor[12]. We defer this to future research.

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Table 1: Macroeconomic Volatility

Variable	OM	OMF	MIS
x_t	0.0526	0.0000	0.2126
π_t	0.0012	0.0000	0.8340
$\hat{r}_t$	0.0077	0.0085	0.8124
$\hat{\tau}_t$	NA	0.2336	NA
δ_t	1.4516	0.2372	0.0000
sp_t	2.6390	0.6411	0.7628
$\hat{r}_t^S$	0.1360	0.0085	0.0000

Table 2: Correlation between Selected Variables

Variable	Policy	x_t	π_t	$\hat{r}_t$	$\hat{\tau}_t$	δ_t	sp_t	$\hat{r}_t^S$
x_t	OM	1.0000						
	OMF	1.0000						
	MIS	1.0000						
π_t	OM	-0.1683	1.0000					
	OMF	NA	1.0000					
	MIS	0.4702	1.0000					
$\hat{r}_t$	OM	0.2227	-0.5792	1.0000				
	OMF	NA	NA	1.0000				
	MIS	0.4382	0.9994	1.0000				
$\hat{\tau}_t$	OM	NA	NA	NA	1.0000			
	OMF	NA	NA	0.7961	1.0000			
	MIS	NA	NA	NA	1.0000			
δ_t	OM	0.1571	-0.9634	0.5629	NA	1.0000		
	OMF	NA	NA	-0.2369	-0.3981	1.0000		
	MIS	NA	NA	NA	NA	1.0000		
sp_t	OM	-0.7879	0.2589	-0.7309	NA	-0.2498	1.0000	
	OMF	NA	NA	-0.3790	0.2582	-0.2306	1.0000	
	MIS	0.6670	-0.3432	-0.3767	NA	NA	1.0000	
$\hat{r}_t^S$	OM	-0.6733	0.3531	0.2786	NA	-0.3322	0.1411	1.0000
	OMF	NA	NA	0.9798	0.6592	-0.1630	-0.5562	1.0000
	MIS	NA	NA	NA	NA	NA	NA	1.0000

Figure 1: IRFs to Government Expenditure under the Benchmark

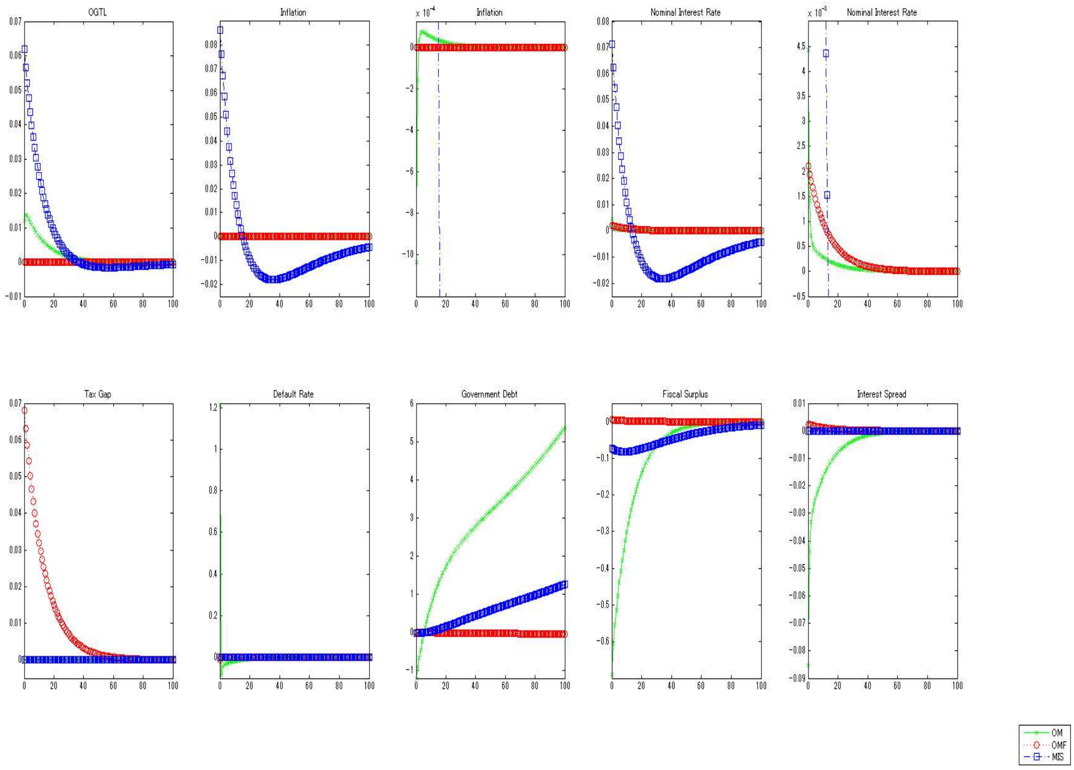

Figure 2: IRFs to Government Expenditure under a Low Ratio of Government Debt to Output ($\varsigma_B = 0.5$, Upper Panels) and a High Ratio of Government Debt to Output ($\varsigma_B = 9.5$, Lower Panels)

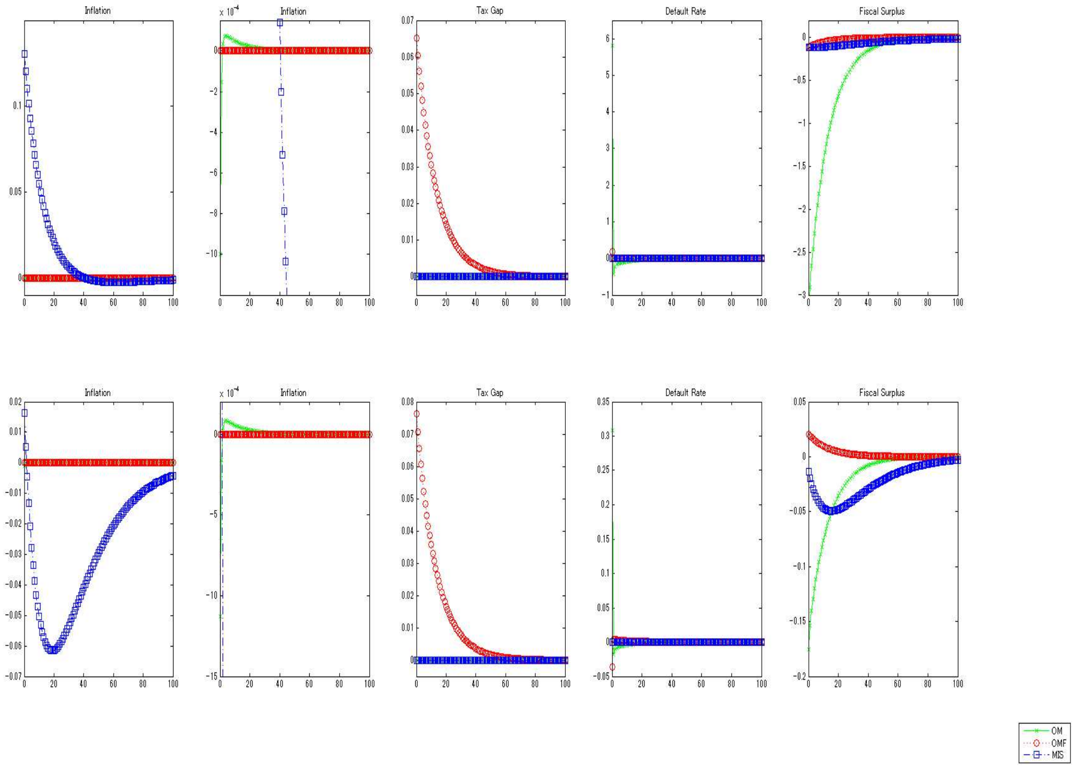

Figure 3: IRFs to Government Expenditure under a Low Ratio of Government Expenditure to Output ($\varsigma_G = 0.05$, Upper Panels) and a High Ratio of Government Expenditure to Output ($\varsigma_G = 0.6$, Lower Panels)

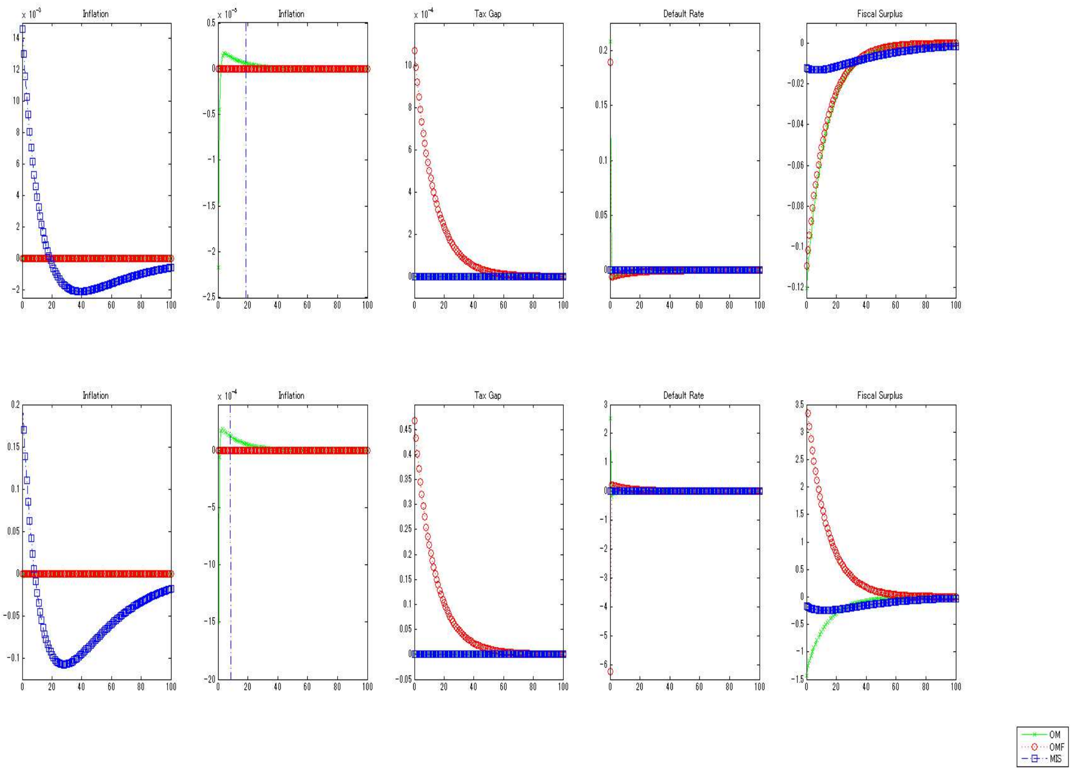

Figure 4: The Trade-off between Stabilizing Inflation and the Default Rate Volatilities under Various Levels of Price Stickiness

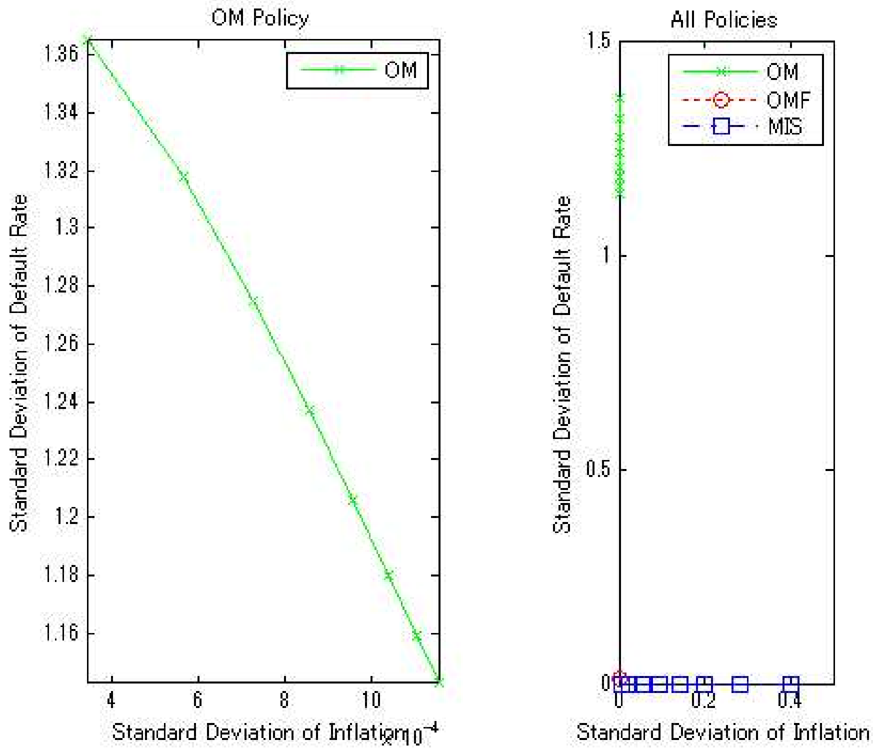

Figure 5: The Trade-off between Stabilizing Inflation and the Default Rate Volatilities under Various Levels of the Steady State Ratio of Government Debt to Output

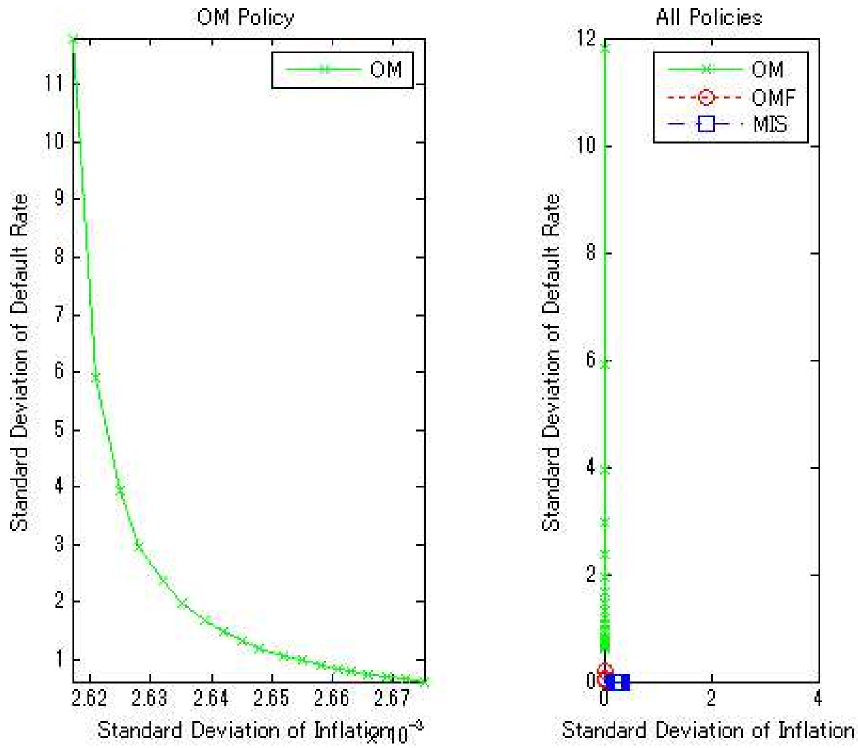

Figure 6: The Trade-off between Stabilizing Inflation and the Default Rate Volatilities under Various Levels of the Steady State Ratio of Government Expenditure to Output