ISSN 1745-8587

Birkbeck Centre for Applied Macroeconomics

BCAM 1902

Optimal Monetary and Fiscal Policy Rules, Welfare Gains and Exogenous Shocks in an Economy with Default Risk

OKANO, Eiji
Nagoya City University

Masataka Eguchi
Komazawa University

July 2019

▪ **Birkbeck, University of London ▪ Malet Street ▪ London ▪ WC1E 7HX ▪**

EK-129i

Optimal Monetary and Fiscal Policy Rules, Welfare Gains, and Exogenous Shocks in an Economy with Default Risk *

OKANO, Eiji†
Nagoya City University

Masataka Eguchi
Komazawa University

This Draft: Jul, 2019

Abstract

We develop a class of dynamic stochastic general equilibrium models with nominal rigidities and we introduce default risk in the model. We find that if productivity changes are observed, policy authorities should be aware of default risk, although being aware of such risk is not very important following government expenditure changes. Welfare gains from awareness of default risk are nonnegligible if productivity changes, although welfare gains from awareness of default risk are tiny following government expenditure changes.

Keywords: Sovereign Risk; Optimal Monetary Policy; Fiscal Theory of the Price Level
JEL Classification: E52; E60

†Corresponding Author. Graduate School of Economics, Nagoya City University, Phone and Fax: +81-52-8725721; E-mail: eiji_okano@econ.nagoya-cu.ac.jp

1 Introduction

> The governing Council decided to lower the key ECB interest rates by 25 basis points, following the 25 basis point decrease on 3 November 2011. Inflation is likely to stay above 2% for several months to come, but it will decline to below 2% during 2012. The intensified financial market tensions are continuing to dampen economic activity in the Euro area and the outlook remains subject to high uncertainty and substantial downside risk.

The above is an extract of a speech by Vítor Constâncio, vice-president of the ECB on Dec. 8, 2011.[1] At the time, a sovereign debt crisis hit the eurozone: the Greek 10-year credit default swap premium reached USD 20,404 and HCPI inflation was 2.8% in April 2012. This speech is descriptive of the difficulty of conducting monetary policy amid a sovereign debt crisis. That is, the ECB might give up stabilizing inflation to prevent the spread of the crisis by lowering the policy rates to provide liquidity and promote debt deflation at that time.

In this paper, we try to dissolve difficulties that the ECB faced during the crisis period and we provide important prescriptions for conducting monetary and fiscal policy in an economy with default risk from the viewpoint of minimizing welfare costs as follows: 1) policy authorities should not suppress inflation aggressively if the cause of the default crisis is rooted in a decrease in productivity. At a glance, this is not novel and is consistent with our intuition. However, we have another prescription; 2) when an increase in government expenditure gives rise to default risk, policy authorities should stabilize inflation, similar to a situation where there is no default risk. The latter prescription may be glad tidings for policy authorities, such as central banks and government, and both prescriptions suggest the importance of identification of shocks hitting the economy with default risk, although we do not discuss this topic; 3) welfare gains from conducting optimal monetary and fiscal policy with awareness of default risk when there is an increase in government expenditure are negligible; while 4) welfare gains from conducting optimal monetary and fiscal policy with awareness of default risk when there is a decrease in productivity are nonnegligible.

To derive the above prescriptions, we analyze optimal monetary and fiscal policy in an economy with default risk. First, we develop a class of dynamic stochastic general equilibrium (DSGE) models in which default risk is introduced following Okano and Inagaki[17] who replicate Uribe's[19] fiscal theory of sovereign risk (FTSR) in the DSGE. Calvo pricing is assumed but the steady state is distorted because tax is levied on the output. There are safe assets issued by households and risky assets, namely government debt. The FTSR is applicable, which is derived from the government budget constraint that is iterated forward and an appropriate transversality condition.[2] Let us suppose a decrease in the net present value of the fiscal surplus. Under the fiscal theory of price level (FTPL), this increases the price level while there is the possibility of causing an increase in the default rate, instead of an increase in the price level, under the FTSR. Second, we derive the second-order approximated utility function, which includes not only a quadratic inflation term but also a quadratic premium difference between the (virtual) government debt yield and its coupon rate term, which we call simply the premium difference. This implies that the cost of default risk is summarized as the premium difference.

[1] See ECB[10].

[2] The FTSR is based on the fiscal theory of price level (FTPL), which is advocated by Cochrane[6], Leeper[14] and Woodford[20] and the net present value of the sum of fiscal surplus decides not only price level and inflation but also the default rate.

As well as explaining the reason for a quadratic term for the premium difference in the second-order approximated utility function, we now expand our introduction to our model. In our setting, there are both safe assets and risky assets, namely government debt, as mentioned. If households purchase government debt, households' optimal consumption schedule makes the intertemporal marginal rate of substitution corresponding to the inverse of the gross expected rate of return to holding government debt, which consists of the government debt yield and the expected default rate. Thus, households have to adjust their balance of government debt appropriately. By adjusting the balance, which affects the inverse of the gross expected rate of return on holding government debt through changes in the government debt yield, optimal consumption schedule attains. If the government debt coupon rate precisely corresponds to the government debt yield, such adjustment is not needed. However, that is not necessarily common in an actual economy.[3] Thus, this adjustment of the balance of government debt, namely portfolio rebalancing, is essential. As shown in the text, the premium difference is a function of the expected default rate, and the appearance of the quadratic term on the period welfare cost function implies that the premium difference is the cost of default risk. In other words, default risk generates a cost, forcing households to rebalance their portfolio.

In this paper, we analyze both Exact and False policies. Under the exact policy, the exact welfare cost function is minimized by policy authorities, the central bank, and the government while under the false policy, they minimize a false welfare cost function. The exact welfare cost function is derived exactly, assuming default risk, while the false welfare cost function is derived without assuming default. There are two differences between the exact and the false welfare cost functions, namely the target level of output and the existence of a quadratic term for the premium difference. The difference in the target level of output between the exact and the false welfare cost functions depends on the interest rate spread in the steady state, which decides the steady-state value of the default rate. If the interest rate spread in the steady state is zero (the steady-state value of the default rate is zero simultaneously), the target level of output in both welfare cost functions is the same. Similarly, the existence of the quadratic term of the premium difference in the exact welfare cost function depends on the interest rate spread in the steady state while that term does not appear definitely on the false welfare cost function. If the interest rate spread in the steady state is zero, the premium difference becomes zero and its quadratic term spontaneously disappears from the exact welfare cost function. Those two differences depend on the interest rate spread in the steady state. If the interest rate spread in the steady state is zero, the exact welfare cost function precisely corresponds to the false welfare cost function. Because the interest rate spread in the steady state decides the steady-state value of the default rate, it can be said that default risk affects the period welfare cost function, that is, it affects the policy target. If there is default risk, authorities have to pay attention to the target level of output and consider minimizing the premium difference.

We resort to numerical analysis with plausible parameterization and compare the results under the exact policy with the results under the false policy. The impulse response functions (IRFs) imply that policy authorities should not suppress inflation aggressively if the causation of the default crisis is rooted in a decrease in productivity while policy authorities should stabilize inflation, similar to a situation where there is no default risk if an increase in the government expenditure

[3]The government debt yield is consistent with the coupon rate on benchmark 10-year government bonds in Italy, Spain, Germany, and the US. However, in Portugal, Ireland, and Greece, the yield is not consistent with the coupon rate on the benchmark 10-year government bond. See Okano and InagakiOkanoInagaki17 for details.

gives default risk. Furthermore, we also calculate 'optimal' monetary and fiscal policy rules. When we calculate these, we choose the coefficients on simple rules that are classes of monetary and fiscal policy rules that replicate welfare costs brought about by the optimal monetary and fiscal policy. Interestingly, there are no differences in our fiscal policy rule between exact and false policies to an increase in government expenditure. However, when productivity decreases, the rules are quite different. Finally, we calculate welfare cost from adopting the exact policy rule and find that the welfare gains from conducting the exact policy are 64.4%. The gains are remarkable. Furthermore, concerning changes in productivity, the gains are 49.7%. However, when only government expenditure hits an economy with default risk, the gains are just 1.0% and are negligible.

We now discuss the relationship between our analysis and previous work deriving policy implications in an economy with default risk. Corsetti and Dedola[7] develop a model for a sovereign debt crisis driven by either self-fulfilling expectations or weak fundamentals, and analyze the mechanism through which either conventional or unconventional monetary policy can preclude the former. Their finding that swapping government debt for monetary liabilities can prevent self-fulfilling debt crises is one of several unconventional monetary policies. Elsewhere, and similar to our analysis, Bacchetta, Perazzi, and Wincoop[1] develop a class of DSGE models and analyze both conventional and unconventional monetary policies. They find that the central bank cannot credibly avoid a self-fulfilling debt crisis. Okano and Hamano[16] and Okano and Inagaki[17] analyze stabilizing inflation and suppressing default trade-offs and they find that these do not necessarily exist. Our analysis differs from this earlier body of work in several ways. Although Corsetti and Dedola[7] and Bacchetta, Perazzi, and Wincoop[1] analyze monetary policy, they neither consider fiscal policy nor how to use it as a stabilization or welfare cost-minimization tool. Thus, our purposes are not identical in that we propose monetary and fiscal policies, whereas these related studies propose monetary policy only to suppress default risk.[4] Okano and Hamano[16] and Okano and Inagaki[17] fail to derive implications for welfare costs while in this work we focus on welfare costs.

The remainder of the paper is as follows. Section 2 develops the model, Section 3 derives the welfare cost function and solves the linear quadratic (LQ) problem, Section 4 is devoted to numerical analysis, and Section 5 calculates monetary and fiscal policy rules and analyzes welfare costs. Section 6 concludes the paper. Appendices provide some technical information.

2 The Model

Following Okano and Inagaki[17], we introduce firms into Uribe's[19] FTSR and develop a class of DSGE models with nominal rigidities following Gali and Monacelli[13], although we assume a closed economy.[5] The default mechanism is quite similar to Uribe[19]. We follow Benigno[2] (an earlier working paper version of Benigno[3]) to clarify the households' choice of risky assets. The household i on the interval $i \in [0, 1]$ supplies labor and owns firms. Calvo pricing is adopted and we assume that a tax is levied on output and distorts outcomes. Thus, monopolistic power remains, and the steady state is distorted.

[4] Furthermore, they do not focus on fiscal policy (their models are unsuitable for analyzing fiscal policy regardless), whereas our model can analyze and evaluate the effect of fiscal policy.

[5] Following Ferrero[11], we introduce government into Gali and Monacelli[13]. In other words, the model is a closed economy version of Okano[15].

2.1 Government

We assume that the total government expenditure is given exogenously in each period by $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ where ε denotes the elasticity of substitution among goods. The flow government budget constraint is given by:

$$B_t^n = R_{t-1}^G (1-\delta_t) B_{t-1}^n - \int_0^1 P_t(i) [\tau_t Y_t(i) - G_t(i)] \, di,$$

where $R_t^G \equiv R_t \Gamma(-sp_t)$ denotes the government debt coupon rate, $R_t \equiv 1 + r_t$ denotes the gross (risk-free) nominal interest rate, r_t is the net interest rate, B_t^n is the nominal government debt, δ_t is the default rate, $sp_t \equiv \frac{SP_t}{SP} - 1$ is the percentage deviation of the (real) fiscal surplus from its steady-state value, $SP_t \equiv \tau_t Y_t - G_t$ denotes the (real) fiscal surplus, and τ_t denotes the tax rate. Note that we define $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ where Y_t denotes (aggregated) output. Because government expenditure is given exogenously, fiscal policy consists of choosing the mix between taxes and the one-period nominal debt with default risk to finance the exogenous process of government expenditure.

Here, we discuss the government debt coupon rate $R_t^G \equiv R_t \Gamma(-sp_t)$, where $\Gamma'(-sp_t) > 0$ by assumption. Our assumption implies that the government decides the government debt coupon rate depending on its fiscal situation, such that if the fiscal situation worsens, the government increases the coupon rate. Note that the government debt coupon rate R_t^G is not the government debt yield, which is fully endogenized. In our setting, the government debt yield is decided by households' intertemporal optimal condition, namely the Euler equation. Thus, the government debt yield is decided endogenously, although the government debt coupon rate depends on our assumption.

As mentioned, the function $\Gamma(-sp_t)$ is hinted at by Benigno[2], who develops a two-country model with imperfect financial integration, although the details are somewhat different from Benigno[2]. Benigno[2] assumes that households in the home country face a burden in international financial markets. As borrowers, households in the home country will be charged a premium on the foreign interest rate; as lenders, they will receive remuneration less than the foreign interest rate. Following his setting, Benigno[2] assumes $\Gamma'(\cdot) < 0$, which implies that the higher the foreign country's government debt, the lower the remuneration for holding the foreign country's government debt.[6] However, in contrast, our setting implies that the lower the fiscal surplus, the less the remuneration for holding government debt owing to default, which in turn harms capital and makes households hesitate to hold government debt. The government has to pay additional remuneration for holding government debt, which provides households with a motivation for doing so. Thus, we assume that $\Gamma'(\cdot) > 0$. That is, the lower the fiscal surplus, the higher the interest rate multiplier, and vice versa.

Another assumption that differs from Benigno[2] is that $\Gamma(\cdot)$ is a function of the fiscal surplus, while Benigno[2] assumes that it is a function of current government debt with an interest payment; that is, $R_t B_t$. Our setting for $\Gamma(\cdot)$ follows Corsetti, Kuester, Meier and Mueller[8] indirectly. Corsetti, Kuester, Meier and Mueller[8] assume that the higher the fiscal deficit, the greater the probability of default, and vice versa. If it is given that the higher the probability of default, the

[6] Benigno[2] observes that this function, which depends only on the level of real government bonds in his setting, captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market.

higher the government debt coupon rate, our assumption that $\Gamma(\cdot)$ is a decreasing function of the fiscal surplus is consistent with their analysis because the assumption implies that the higher the fiscal surplus, the higher the government debt coupon rate. That is, if it is given that the higher the probability of default, the higher the government debt coupon rate, it can be said that we indirectly assume that the lower the fiscal surplus, the higher the default rate, which is similar to Corsetti, Kuester, Meier and Mueller[8].[7]

It is worth mentioning here Schabert[18], who argues that the equilibrium allocation cannot be determined if the central bank sets the interest rate in a conventional way while if money supply is controlled, the equilibrium allocation can uniquely be determined under Uribe's[19] FTSR. We adopt Uribe's[19] FTSR while we do not introduce money into our model. However, this does not definitely imply that the equilibrium allocation cannot be determined because we follow Benigno[2], as mentioned. Because we follow Benigno[2], the households' choice of risky assets is determined uniquely, thus the equilibrium allocation can uniquely be determined.

The log-linearized definition of the fiscal surplus is given by:

$$sp_t = \varsigma_\tau \hat{\tau}_t + \varsigma_\tau y_t - \frac{\varsigma_\tau \sigma_G}{\tau} g_t, \tag{1}$$

where $\varsigma_\tau \equiv \frac{\tau}{\frac{SP}{Y}}$ denotes the tax revenue elasticity and $\sigma_G \equiv \frac{G}{Y}$ denotes the steady-state share of the government expenditure to output, and $\hat{\tau}_t \equiv \frac{d\tau_t}{\tau}$ denotes the percentage deviation of the tax rate from its steady-state value. We simply refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_t$ as the tax gap.

By solving cost-minimization problems, the optimal allocation of generic goods is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$ and $G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t$, and the previous flow government budget constraint can be rewritten as:

$$B_t^n = R_{t-1}^G (1-\delta_t) B_{t-1}^n - P_t SP_t,$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}} \tag{2}$$

denotes the price level. Dividing both sides of the equality by P_t yields:

$$B_t = R_{t-1}^G (1-\delta_t) B_{t-1} \Pi_t^{-1} - SP_t, \tag{3}$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ being the gross inflation rate. The first term on the right-hand side (RHS) corresponds to the amount of redemption with the nominal interest payment and shows that the lower the past fiscal surplus, the higher the interest payments, and the higher the default rate, the lower the redemption, and vice versa.

Log-linearizing Eq. (3) yields:

$$b_t = \frac{\tau}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} \hat{r}_{t-1}^G - \frac{\phi\varsigma_\tau\sigma_B}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} \hat{\delta}_t + \frac{\tau}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} b_{t-1} - \frac{\tau}{\beta(\tau + \phi\varsigma_\tau\sigma_B)} \pi_t - \frac{\tau}{\varsigma_\tau\sigma_B} sp_t, \tag{4}$$

with $\hat{r}_t^G \equiv \frac{dR_t^G}{R^G}$ where $\hat{\delta}_t \equiv \frac{d\delta_t}{\delta}$ denotes the default gap.

[7] Our setting on $\Gamma(\cdot)$ follows Okano and Inagaki[17] who analyze whether a fiscal deficit or government debt with interest payment increases the interest rate multiplier $\Gamma(\cdot)$ using Greek data. These data imply that the fiscal deficit but not government debt with interest payment increases $\Gamma(\cdot)$.

Here, we show that the log-linearized definition of the government debt coupon rate is given by:

$$\hat{r}_t^G = \hat{r}_t - \phi sp_t. \tag{5}$$

2.2 Households

2.2.1 The First-Order Necessary Conditions (FONCs) for Households

A representative household's preference is given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t \right), \tag{6}$$

where $U_t \equiv \ln C_t - \frac{1}{1+\psi} N_t^{1+\psi}$ denotes the period utility, E_t is the expectation conditional on the information set at period t, $\beta \in (0,1)$ is the subjective discount factor, C_t is the consumption index, $N_t \equiv \int_0^1 N_t(i)\, dh$ is the hours of labor, and ψ is the inverse of the elasticity of labor supply.

The consumption index of the continuum of differentiated goods is defined as follows:

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods.

The maximization of Eq. (6) is subject to a sequence of intertemporal budget constraints of the form:

$$R_{t-1} D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1 - \delta_t) + W_t N_t + PR_t \geq \int_0^1 P_t(i) C_t(i)\, di + D_t^n + B_t^n, \tag{8}$$

where D_t^n denotes the safe assets issued by households, W_t is the nominal wage, and PR_t denotes profits from the ownership of firms. Furthermore, we define V as the steady-state value of any variables V_t and v_t as the percentage deviation of V_t from its steady-state value.

By solving cost-minimization problems for households, we have the optimal allocation of expenditures as follows:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t. \tag{9}$$

Once we account for Eq. (9), the intertemporal budget constraint can be rewritten as:

$$R_{t-1} D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1 - \delta_t) + W_t N_t + PR_t \geq P_t C_t + D_t^n + B_t^n.$$

The remaining optimality conditions for the household's problem are given by:

$$\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \frac{1}{R_t}, \tag{10}$$

which is the intertemporal optimality condition, namely the Euler equation, and

$$C_t N_t^{\psi} = \frac{W_t}{P_t}, \tag{11}$$

which is the standard intratemporal optimality condition.

There is another intertemporal optimality condition depicting households' motivation to hold government debt with default risk. This is obtained by differentiating the Lagrangian by government nominal debt and is given by:

$$\lambda_t = \beta R_t^H \mathrm{E}_t \left[\lambda_{t+1} \left(1 - \delta_{t+1}\right)\right], \tag{12}$$

with $\lambda_t = \left(P_t C_t\right)^{-1}$ where $R_t^H \equiv R_t \left\{\Gamma\left(-sp_t\right) + B_t \Gamma'\left(-sp_t\right) SP^{-1}\right\}$ is one of the earning rates or marginal revenue of holding government debt that can be interpreted as the (gross) government debt yield (excluding the default risk) requested by households. The definition of government debt yield R_t^H implies that household demand for government debt , which establishes the government debt yield discounting default risk $R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right)$, corresponds to the inverse of the marginal rate of consumption, namely R_t, as long as the additional interest payment for holding government debt is not sufficient to realize the optimal consumption schedule from holding government debt. Hereafter, we dub R_t^H the government debt yield for short.

In fact, log-linearizing Eqs. (10) and (12) and combining them, we have:

$$\hat{r}_t = \hat{r}_t^H - \delta \mathrm{E}_t \left(\hat{\delta}_{t+1}\right), \tag{13}$$

with $\hat{r}_t \equiv \frac{dR_t}{R}$ and $\hat{r}_t^H \equiv \frac{dR_t^H}{R^H}$ being the nominal interest gap and the government debt yield gap, respectively. Eq. (13) shows that the marginal rate of substitution for consumption is the same for households holding either (real) safe assets D_t or (real) government debt B_t because both R_t and $R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right)$ equal the marginal rate of substitution. That is, the consumption schedule is the same whether households hold safe assets D_t or government debt B_t.

Let us define $\hat{r}_t^S \equiv \hat{r}_t^H - \hat{r}_t$, which is the interest rate spread gap for holding government debt, namely risky assets. Then, Eq. (13) can be rewritten as:

$$\hat{r}_t^S = \delta \mathrm{E}_t \left(\hat{\delta}_{t+1}\right), \tag{14}$$

where $\sigma_B \equiv \frac{B}{Y}$ denotes the steady-state share of government debt to output. Eq. (14) shows that the higher the expected default rate, the higher the interest rate spread, and vice versa.

Log-linearizing the definition of government debt yield R_t^H, we have:

$$\hat{r}_t^S = -\frac{\phi\left(\tau + \gamma \varsigma_\tau \sigma_B\right)}{\tau + \phi \varsigma_\tau \sigma_B} sp_t + \frac{\phi \varsigma_\tau \sigma_B}{\tau + \phi \varsigma_\tau \sigma_B} b_t, \tag{15}$$

where $\phi \equiv \Gamma'(0)$ denotes the interest rate spread in the steady state and $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$ denotes the elasticity of the interest rate spread to a one-percent change in the fiscal deficit in the steady state. Following Benigno[2], we define the interest rate spread for government debt ϕ and assume $\Gamma(0) = 1$. The elasticity γ is an unfamiliar parameter, and we assume $\mid \Gamma'(\cdot) \mid < \mid \Gamma''(\cdot) \mid$; thus, $\gamma > 1$. Our assumption implies that a decrease in the fiscal surplus increases the government debt coupon rate via an increase in the interest rate multiplier, and vice versa, and that changes in the government debt coupon rate are larger than the changes in the fiscal surplus in absolute value.[8]

The first term on the RHS of Eq. (15) has a negative sign and implies that an increase in the fiscal surplus decreases the government debt yield, and vice versa. This is intuitively consistent because an increase in fiscal surplus decreases the interest rate multiplier. The second term on the RHS of Eq. (15) has a positive sign. This shows that the government debt yield is an increasing

[8] Our assumption $\gamma > 1$ is supported by the data. See Okano and Inagaki[17] for details.

function of government debt. An increase in government debt coincides with a decrease in the fiscal surplus, and vice versa. Thus, this positive sign in the second term is consistent with the negative sign in the first term.

2.2.2 Fiscal Theory of Sovereign Risk

The appropriate transversality condition for government debt is given by:

$$\lim_{j\to\infty} \beta^{t+j+1}\mathrm{E}_t\left[R^G_{t+j}\left(1-\delta_{t+j+1}\right)\frac{P_{t+j}B_{t+j}}{P_{t+j+1}}\right]=0.$$

By iterating the second equality in Eq. (3) forward, plugging Eq. (10) into this iterated equality, and imposing the appropriate transversality condition for government debt, we have:

$$C_t^{-1}R^G_{t-1}B_{t-1}\Pi_t^{-1}\left(1-\delta_t\right)=C_t^{-1}SP_t+\beta\frac{R^H_t}{R^G_t}\mathrm{E}_t\left(C^{-1}_{t+1}SP_{t+1}\right)+\beta^2\mathrm{E}_t\left(\frac{R^H_t}{R^G_t}\frac{R^H_{t+1}}{R^G_{t+1}}C^{-1}_{t+2}SP_{t+2}\right)+\cdots, \quad (16)$$

which roughly shows that the burden of government debt redemption with interest payment in terms of consumption, or the left-hand side (LHS), corresponds to the expected sum of the discounted value of the fiscal surplus in terms of consumption, or the RHS, because of the transversality condition. Here, $\frac{R^H_t}{R^G_t}$ and so forth appear on the RHS. An increase in the government debt coupon rate R^G_t then worsens the fiscal situation through the increase in the interest payment. Thus, R^G_t is the denominator. An increase in the government debt yield facilitates the purchase of government debt even though consumption decreases. A decrease in consumption then improves the fiscal situation because the decrease in consumption increases the fiscal surplus in terms of consumption. Thus, R^H_t appears as the numerator.

Eq. (16) can be rewritten as:

$$\delta_t=1-\frac{\left(R^R_{t-1}\right)^{-1}\sum_{k=0}^{\infty}\prod_{h=0}^{k}\beta^k\mathrm{E}_t\left(R^R_{t+h-1}C^{-1}_{t+k}SP_{t+k}\right)}{C_t^{-1}R^G_{t-1}B_{t-1}\Pi_t^{-1}}, \quad (17)$$

where $R^R_t \equiv \frac{R^H_t}{R^G_t}$ denotes the (gross) premium difference between the government debt yield and its coupon rate. Eq. (17) is our FTSR and implies that an increase in inflation does not necessarily occur even if the government becomes insolvent, and vice versa, similar to Uribe[19]. Not only inflation but also default can mitigate the burden of government debt redemption with interest payment. Suppose that the price level is constant and there is no inflation. In this situation, if the net present value of the fiscal surplus in terms of consumption (the numerator) is about to fall below the burden of government debt redemption with interest payment in terms of consumption (the denominator), the second term on the RHS is less than unity. Simultaneously, the LHS exceeds zero; that is, default occurs. In other words, if the government becomes insolvent while the price level is strictly stable, default is inevitable. Uribe[19] shows that there is a trade-off between stabilizing inflation and suppressing default (hereafter the SI–SD trade-off) by introducing default, namely default risk, into the central equation of the FTPL. Similar to Uribe[19], at first glance, Eq. (17) also implies that there is an SI–SD trade-off. Furthermore, he calibrates his model and compares this with the monetary policy rule that stabilizes inflation with the interest rate peg. Under the interest rate peg, the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration shows that default ceases just one period after the shock decreasing the fiscal surplus, even though default continues under a monetary policy rule after the shock. This implies that a monetary policy rule to stabilize inflation includes the unwelcome possibility of magnifying default risk, and this calls for an interest rate peg to counter default. Although Uribe[19] ignores the welfare perspective of these actions, his policy implications are persuasive. Paying attention only to Eq. (17), which is similar to that in Uribe's[19] model, we seem to obtain policy implications quite similar to those in Uribe[19].

We now present the relationship between our FTSR, namely Eq. (17), and the FTPL. If there is neither default risk nor an interest rate multiplier in Eq. (17), Eq. (17) reduces to the following because $R_t^G = R_t^H = R_t$:

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1} B_{t-1} \Pi_t^{-1}}, \tag{18}$$

which is our version of the FTPL. On the RHS in this equality, the numerator is the net present value of the sum of the fiscal surplus in terms of consumption, and the denominator is the burden of the government debt redemption with interest payment in terms of consumption divided by inflation. The LHS is unity. If solvency worsens, the price level increases, that is, inflation occurs, such that the burden of government debt redemption is mitigated. For now, we introduce default risk, and this mechanism is no longer fully applicable, as Eq. (17) implies.

2.2.3 Relationship between Default Rate and Fiscal Surplus

By leading Eq. (17) one period and plugging this into Eq. (17) itself, we can rewrite Eq. (17) as a second-order differential equation as follows:

$$\delta_t = 1 - \frac{1}{R_{t-1}^G \Pi_t^{-1} B_{t-1}} \left\{ SP_t + \beta \mathrm{E}_t \left[\left(\frac{C_t}{C_{t+1}} \Pi_{t+1}^{-1} \right) R_t^H \left(1 - \delta_{t+1} \right) B_t \right] \right\}. \tag{19}$$

In Eq. (19), the current government debt B_t appears in the second term on the RHS and the sign is negative. That is, a decrease in current government debt increases the default rate, and vice versa. Why is the sign of government debt B_t in the second term on the RHS negative? This stems from the transversality condition for government debt. Because of the transversality condition, Eq. (17) and its second-order differential version Eq. (19) are strictly applicable. That is, once issued, government debt must be redeemed. Otherwise, the burden of redemption is mitigated by default or inflation. To keep Eq. (17), once government debt is issued, the fiscal surplus must be improved while newly issued government debt is about to reduce the fiscal surplus. Because the fiscal surplus must improve to redeem debt, the default rate declines as a result of an improvement in the fiscal surplus when government debt increases. Thus, the sign is negative.

Log-linearizing Eq. (19) yields:

$$\begin{aligned} c_t \quad = \quad & \mathrm{E}_t \left(c_{t+1} \right) - \hat{r}_t^H + \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\hat{\delta}_{t+1} \right) + \mathrm{E}_t \left(\pi_{t+1} \right) - b_t + \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} \hat{r}_{t-1}^G - \frac{\phi \varsigma_\tau \sigma_B}{\beta \left(\tau + \phi \varsigma_\tau \sigma_B \right)} \hat{\delta}_t \\ & - \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} \pi_t + \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} b_{t-1} - \frac{\tau}{\varsigma_\tau \sigma_B} sp_t, \end{aligned} \tag{20}$$

which is our log-linearized Euler equation.

2.3 Firms

This subsection outlines the production, price setting, marginal cost, and features of the firms, which are quite similar to Gali and Monacelli[13], although here the tax is levied on firm sales and is not constant.[9]

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(i) = A_t N_t(i),$$

where A_t denotes productivity.

By combining the production function and the optimal allocation for goods $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$, we have an aggregate production function relating to aggregate employment as follows:

$$N_t = \frac{Y_t Z_t}{A_t}, \tag{21}$$

where $Z_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ denotes price dispersion.

Log-linearizing Eq. (21) yields:

$$n_t = y_t - a_t. \tag{22}$$

We assume that productivity follows an AR(1) process, namely $\mathrm{E}_t(a_{t+1}) = \rho_A a_t$, similar to government expenditure. Z_t disappears in Eq. (17) because of $o\left(\|\xi\|^2\right)$.

Each firm is a monopolistic producer of one of the differentiated goods. Each firm sets its prices $P_t(i)$ taking as given P_t and C_t. We assume that firms set prices in a staggered fashion, Calvo pricing, according to which each seller has the opportunity to change its price with a given probability $1-\theta$, where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the opportunity to set a new price in period t, it does so to maximize the expected discounted value of its net profits. The FONCs for firms are given by:

$$\tilde{P}_t = \frac{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k}\right)}{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k}\right)}, \tag{23}$$

where $MC_t \equiv \frac{W_t}{(1-\tau_t) P_t A_t}$ denotes the real marginal cost, $\tilde{Y}_{t+k} \equiv \left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$ denotes the demand for goods when firms choose a new price, and $\tilde{P}_t$ denotes the newly set prices. Note that we assume that the government levies a tax on firm sales.

By log-linearizing Eq. (23), we have:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \kappa mc_t, \tag{24}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slope of the New Keynesian Phillips Curve (NKPC). Eq. (24) is the fundamental equality of our NKPC.

Substituting Eq. (11) into the definition of the real marginal cost yields:

$$MC_t = \frac{C_t N_t^{\psi}}{(1-\tau_t) A_t}. \tag{25}$$

Note that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate $1-\tau$ is definitely smaller than one. In such a case, the

[9] Unlike our setting, Gali and Monacelli[13] assume that under constant employment subsidies, monopolistic power completely disappears.

steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it is unable to be completely absorbed through taxation. As we discuss later, we need to derive our welfare criteria following Benigno and Woodford[5] because monopolistic power is no longer removed completely, and the steady state is distorted.

Log-linearizing Eq. (25) yields:

$$mc_t = c_t + \psi n_t + \frac{\tau}{1-\tau}\hat{\tau}_t - a_t. \tag{26}$$

2.4 Equilibrium

The market-clearing condition requires:

$$Y_t(i) = C_t(i) + G_t(i),$$

for all $i \in [0,1]$ and all t. By plugging the optimal allocation for generic goods including Eq. (8) into this market-clearing condition, we have:

$$Y_t = C_t + G_t. \tag{27}$$

By log-linearizing Eq. (27), we obtain:

$$y_t = \sigma_C c_t + \sigma_G g_t, \tag{28}$$

where $\sigma_C \equiv 1 - \sigma_G$ denotes the steady-state ratio of consumption to output.

3 Welfare Costs and the LQ Problem

3.1 Derivation of the Welfare Cost Function

Following Gali[12], the second-order approximated utility function is given by:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{U_t - U}{U_C C}\right) &= \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right) - \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\frac{(1-\Phi)(1+\psi)}{\sigma_C 2}y_t^2 - \frac{(1-\Phi)(1+\psi)}{\sigma_C}y_t a_t\right.\\ &\quad \left.+\frac{\Lambda_\pi}{2}\pi_t^2\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right),\end{aligned} \tag{29}$$

with $\Lambda_\pi \equiv \frac{(1-\Phi)\varepsilon}{\sigma_C \kappa}$ where t.i.p. denotes the terms independent of policy, $o\left(\|\xi\|^3\right)$ are the terms of order three or higher, and $\Phi \equiv 1 - \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. On the RHS, there is a linear term $\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right)$ generating the welfare reversal, which must be eliminated to avoid welfare reversal.

To eliminate this linear term, we follow Benigno and Woodford[4] and Benigno and Woodford[5]. By using the second-order approximated AS equation Eq. (23), the second-order approximated definition of the fiscal surplus SP_t, the second-order approximated definition of the premium difference R_t^R, the second-order approximated market clearing condition Eq. (27), and the second-order approximated government solvency condition Eq. (17), we have:

$$\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right) = -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\tilde{\Omega}_1 y_t^2 - 2y_t\left(\Omega_2 g_t + \tilde{\Omega}_3 a_t\right) + \frac{\Lambda_r}{2}\left(\hat{r}_t^R\right)^2\right] + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

where $\hat{r}_t^R \equiv \frac{dR_t^R}{R^R}$ denotes the premium difference, $\Upsilon_0 \equiv \Theta_1 (1-\beta-\delta)^{-1} \hat{\mathcal{W}}_0 + \Theta_2 \kappa^{-1} \mathcal{V}_0$ denotes the transitory component, $\mathcal{V}_0$ denotes the initial value of the second-order approximated AS equation, $\mathcal{W}_0$ denotes the initial value of the second-order approximated government solvency condition, and $\tilde{\Omega}_1$, Ω_2, $\tilde{\Omega}_3$, Ω_4 are complicated building blocks of parameters with $\hat{\mathcal{W}}_t \equiv \frac{\mathcal{W}_t - \mathcal{W}_{t-1}}{\mathcal{W}_{t-1}}$, $\Lambda_r \equiv \frac{\Theta_1 \beta [1+(\gamma-1)^2]}{(1-\beta-\delta)(\gamma-1)^2}$, $\Theta_1 \equiv \frac{(1-\beta-\delta)\tau\Phi}{\Xi_0}$, $\Theta_2 \equiv -\frac{\Phi(1+2\omega_g)\omega_\phi(1-\tau)}{\Xi_0}$, $\omega_\phi \equiv 1-\beta(1-\phi) - \frac{\phi\varsigma_\tau\sigma_B}{\tau+\phi\varsigma_\tau\sigma_B}$, and Ξ_0 being complicated building blocks of parameters.

Plugging the previous expression into Eq. (29) yields:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(\frac{U_t - U}{U_C C} \right) = \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 (L_t) + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3 \right),$$

which is a second-order approximated utility function and the linear term is appropriately eliminated, where

$$L_t \equiv \frac{\Lambda_y}{2} (y_t - y_t^*)^2 + \frac{\Lambda_\pi}{2} \pi_t^2 + \frac{\Lambda_r}{2} \left(\hat{r}_t^R \right)^2 \tag{30}$$

denotes the period welfare cost function, and

$$y_t^* \equiv \frac{\Omega_2}{\Omega_1} g_t + \frac{\Omega_3}{\Omega_1} a_t \tag{31}$$

denotes the efficient level of output with $\Lambda_y \equiv 2\Omega_1$, $\Omega_1 \equiv \tilde{\Omega}_1 + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$.

The period welfare cost function Eq. (30) has distinctive features. To clarify its distinctive features, we derive the period welfare cost function by using the second-order approximated government solvency condition, which is derived from Eq. (18). Recall that Eq. (18) is our version of the FTPL and can be derived from the government solvency condition Eq. (17) by assuming that there is neither default risk nor an interest rate multiplier. The period welfare cost function derived by using the second-order approximated government solvency condition, which is derived from Eq. (18), is given by:

$$L_t^f = \frac{\Lambda_y^f}{2} \left(y_t - y_t^f \right)^2 + \frac{\Lambda_\pi}{2} \pi_t^2. \tag{32}$$

This is analogous to the period welfare cost function derived by Benigno and Woodford[4] where $y_t^f \equiv \frac{\Omega_2^f}{\Omega_1^f} g_t + \frac{\Omega_3^f}{\Omega_1^f} a_t$ denotes the efficient level of output when there is no default risk with $\Lambda_y^f \equiv 2\Omega_1^f$, $\Omega_1^f \equiv \tilde{\Omega}_1^f + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$, $\Omega_3^f \equiv \tilde{\Omega}_3^f + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$, and Ω_2^f being a complicated block of parameters. There is no quadratic term of $\hat{r}_t^R$. Λf_y replaces Λ_y and the target level of output is not y_t^* but y_t^f in Eq. (30).

3.2 Welfare Costs in an Economy with Sovereign Risk

The most notable feature of welfare costs in an economy with default risk, namely Eq. (30), is the quadratic term of the premium difference between the government debt yield and the government debt coupon rate $\hat{r}_t^R$, which is the third term on the RHS. The appearance of this term suggests that there is an opportunity cost of holding government debt. When households hold government debt, they can obtain interest income that is not necessarily at the government debt coupon rate r_t^G but may be at the government debt yield rate r_t^H. To choose the optimal consumption schedule

Eq. (12), households have to maneuver their government debt position B_t satisfying the definition of the (gross) government debt yield as follows:

$$\begin{aligned} R_t^H &= R_t \left\{ \Gamma(-sp_t) + B_t \Gamma'(-sp_t) SP^{-1} \right\} \\ &= R_t^G + R_t B_t \Gamma'(-sp_t) SP^{-1}. \end{aligned}$$

As shown in the previous expression, the government debt coupon rate does not necessarily correspond to the government debt yield and households have to abandon income at the government debt coupon rate but obtain income at the government debt yield on their own government debt. Thus, there is an opportunity cost to maneuvering the government debt position. If they neglect to maneuver, households can no longer choose an optimal consumption schedule, resulting in welfare costs. This is the reason there is a quadratic of the premium difference between the government debt yield and the government debt coupon rate $\hat{r}_t^R$ in Eq. (30).

The appearance of the quadratic of the premium difference between the government debt yield and the government debt coupon rate in Eq. (30) depends on default risk. The third term on the RHS in Eq. (30) can be rewritten as:

$$\Lambda_r \left(r_t^R \right)^2 = \Lambda_\delta \mathrm{E}_t \left(\hat{\delta}_{t+1} - \hat{\delta}_{t+1}^* \right)^2,$$

because

$$\hat{r}_t^R = \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\hat{\delta}_{t+1} - \hat{\delta}_{t+1}^* \right), \tag{33}$$

which is derived by using Eqs. (5) and (14) with $\Lambda_\delta \equiv \Lambda_r \phi^2 \left(\frac{\varsigma_\tau \sigma_B}{\tau} \right)^2$ where $\hat{\delta}_t^* \equiv -\frac{\tau}{\varsigma_\tau \varsigma_B} sp_{t-1}$ denotes the target level of the default rate. The previous expression shows the welfare costs stemming from the third term of Eq. (30) to be the welfare costs of the deviation of the expected default gap from its target level $\mathrm{E}_t \left(\hat{\delta}_{t+1}^* \right)$ corresponding to the (percentage deviation of) fiscal deficit from its steady-state value $-sp_t$. In addition, the previous expression shows that the higher the steady-state value of the interest spread ϕ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ. Then, we have to pay attention to the steady-state value of interest spread ϕ, which determines the steady-state value of the default rate because

$$\delta = \frac{\phi \varsigma_\tau \sigma_B}{\tau + \phi \varsigma_\tau \sigma_B}.$$

That is, the higher the steady-state value of the interest spread, the higher the steady-state value of the default, and vice versa. Because of that, the higher the steady-state value of the interest spread ϕ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ, the higher the steady-state value of the default rate δ, and the higher the weights on the deviation of the expected default gap from its target level Λ_δ. In addition, when there is no interest spread in the steady state, which means that the steady-state value of the default rate is zero, $\Lambda_\delta = 0$ is applied and the third term in Eq. (30) $\frac{\Lambda_r}{2} \left(\hat{r}_t^R \right)^2$ disappears.

Other distinctive features of Eq. (30) as against Eq. (32) are that the weights on the output deviation from its target level Λ_y replace Λ_f and the target level output y_t^* replaces y_t^f. While Λ_y^f in Eq. (32) does not depend on the steady-state value of the interest spread ϕ, Λ_y depends on ϕ. When $\phi = 0$, $\Lambda_y = \Lambda_y^f$ is applied. That is, the difference of the weights on the output deviation from its target level depends on the steady-state value of the interest spread. In Eq. (30), the target level output y_t^* replaces y_t^f and y_t^* depends on ϕ, although y_t^f does not depend on it. When $\phi = 0$, $y_t^* = y_t^f$. Thus, when $\phi = 0$ in which the steady-state value of the default rate is zero, namely $\delta = 0$, Eq. (32) boils down to Eq. (30), that is,

$$L_t^* = L_t^f$$

is applied. If there is no default risk, the welfare cost function is analogous to one derived by Benigno and Woodford[4] who do not assume the default risk. Thus, it can be said that default risk changes the form of the welfare cost function.

3.3 The LQ Problem

The policy authorities minimize Eq. (30) or Eq. (32) for all t subject to Eqs. (1), (4), (5), (14), (15), (20), (22), (24), (26), and (28) and select the sequence $\left\{y_t, \pi_t, \hat{r}_t^G, \hat{r}_t, \hat{r}_t^H, c_t, b_t, mc_t, n_t, sp_t, \hat{\tau}_t, \hat{\delta}_t\right\}_{t=0}^{\infty}$. We designate the policy minimizing Eq. (30) the 'exact' policy because there is default risk and policy authorities recognize default risk, while we designate the policy minimizing Eq. (32) the 'false' policy because there is default risk but policy authorities do not recognize default risk. Eqs. (30) and (32) are not only distinguished by the quadratic term of the premium difference $\hat{r}_t^R$ but also by the weights on the output deviation from its target level and the target level output. Comparing the outcome of policy minimizing Eq. (30) with minimizing Eq. (30) without the quadratic term of the premium difference, we cannot analyze how default risk affects the outcome of the optimal policy. We have to consider not only the quadratic term of the premium difference but also the weights on the output deviation from its target level and the target level output. Thus, we compare the 'exact' policy, which minimizes Eq. (30) with the 'false' policy, which minimizes Eq. (32).

Under the exact policy, the policy authorities minimize Eq. (30), while they minimize Eq. (32) under the false policy. In the following, we introduce some FONCs that are worth discussing.

The FONCs for the output are given by:

$$\Lambda_y y_t = \varsigma_\tau \rho_{7,t} + \rho_{8,t} - \rho_{10,t} + \Lambda_y y_t^*, \tag{34}$$
$$\Lambda_y^f y_t = \varsigma_\tau \rho_{7,t} + \rho_{8,t} - \rho_{10,t} + \Lambda_y^f y_t^f, \tag{35}$$

where $\rho_{7,t}$, $\rho_{8,t}$, and $\rho_{10,t}$ are Lagrange multipliers on Eqs. (1), (22), and (28), respectively. Eq. (34) is the FONC under the exact policy and shows that the target level output is the efficient level of output. Eq. (35) is the FONC under the false policy and shows that the target level output is the efficient level of output when there is no default risk.[10]

The FONC for inflation is given by:

$$\Lambda_\pi \pi_t = -\frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B}\rho_{1,t} + \frac{1}{\beta}\rho_{1,t-1} - (\rho_{2,t} - \rho_{2,t-1}) - \frac{\tau}{\beta(\tau + \phi\varsigma_\tau\sigma_B)}\rho_{6,t}, \tag{36}$$

where $\rho_{1,t}$ and $\rho_{6,t}$ are Lagrange multipliers on Eqs. (20) and (4), respectively. Because of commitment, lagged Lagrange multipliers appear in Eq. (36). Eq. (36) is common to both the exact and false policies. In Eqs. (34), (35), and (36), $\rho_{2,t}$ appears and those equalities imply that inflation is stabilized by stabilizing output (or stabilizing the welfare-relevant output gap or the difference

[10]To derive Eqs. (34) and (35), we use the FONCs for the marginal cost and the hours of labor and eliminate Lagrange multipliers on Eq. (22) $\rho_{8,t}$.

between output and the target level output). This mechanism is similar to others in the literature on optimal monetary policy and is common to both the exact and false policies.

The FONCs for the government coupon gap are given by:

$$\Lambda_r \hat{r}_t^G = \Lambda_r \hat{r}_t^H - \rho_{5,t} + \frac{\beta(\tau + \varsigma_\tau \sigma_B)}{\varsigma_\tau \sigma_B} \rho_{1,t+1} + \frac{\tau}{\varsigma_\tau \sigma_B} \rho_{6,t}, \tag{37}$$

$$0 = -\rho_{5,t} + \frac{\beta(\tau + \varsigma_\tau \sigma_B)}{\varsigma_\tau \sigma_B} \rho_{1,t+1} + \frac{\tau}{\varsigma_\tau \sigma_B} \rho_{6,t}, \tag{38}$$

where $\rho_{5,t}$ is a Lagrange multiplier on Eq. (3) and Eqs. (37) and (38) are the FONCs under the exact and false policies, respectively. Eq. (37) implies that the policy authorities have to minimize the premium difference $\hat{r}_t^R \equiv \hat{r}_t^H - \hat{r}_t^G$ to minimize the welfare costs, although Eq. (38) implies that there is no explicit incentive to minimize it.

The FONCs for the government debt yield gap are given by:

$$\Lambda_r \hat{r}_t^H = \Lambda_r \hat{r}_t^G - \rho_{1,t} + \rho_{3,t} - \rho_{4,t}, \tag{39}$$

$$0 = -\rho_{1,t} + \rho_{3,t} - \rho_{4,t}, \tag{40}$$

where $\rho_{4,t}$. Eqs. (39) and (42) are the FONCs under the exact and false policies, respectively. Eq. (39) implies that the policy authorities have to minimize the premium difference $\hat{r}_t^R \equiv \hat{r}_t^H - \hat{r}_t^G$ to minimize the welfare costs, although Eq. (42) implies that there is no explicit incentive to minimize it.

4 Numerical Analysis

4.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. The calibrated parameters are shown in Table 1.[11] In addition, we assume that productivity a_t and government expenditure g_t follow AR(1) processes and that persistence is 0.9.

4.2 Impulse Response Functions

We next discuss the IRFs. Figs. 1 and 2 show the IRFs to a unit decrease in productivity and to a unit increase in government expenditure, respectively. First, we discuss IRFs of a one-percent decrease in productivity (Fig. 1). A decrease in the productivity decreases the target level output under both policies as shown in the definition of the target level output. Although target level output depends on the steady-state value of the interest spread under the exact policy, the difference in the target level output can be ignored (Panel 3). Under the false policy, the tax gap falls enough to boost output although under the exact policy it does not (Panel 9). As a result, output and the welfare-relevant output gap under the exact policy fall more under the exact policy, while under the false policy, output corresponds approximately to its target level and the welfare-relevant output gap is close to zero (Panels 1 and 2). Through stabilizing the welfare-relevant output gap, inflation is stabilized under the false policy but not under the exact policy (Panel 4). This is why we have a prescription for conducting monetary and fiscal policy that 1) policy

[11] Creedy and Gremmell[?] report that the tax revenue elasticity ranges from 0.5 to 1 and we choose 1 as the tax revenue elasticity ς_τ.

authorities should not suppress inflation aggressively if the causation of default crisis is rooted by a decrease in productivity (see Section 1).

On the one hand, a drastic decrease in the tax gap worsens the fiscal surplus under the false policy, but on the other, under the exact policy, it does not because the tax gap is not lowered very much. Here, the fiscal surplus under the exact policy has an important role in stabilizing the premium difference, which is:

$$\begin{aligned}
\hat{r}_t^R &\equiv \hat{r}_t^H - \hat{r}_t^G \\
&= \hat{r}_t^H - \hat{r}_t - \left(\hat{r}_t^G - \hat{r}_t\right) \\
&= \hat{r}_t^S + \phi sp_t \\
&= -\frac{\phi \varsigma_\tau \sigma_B \left(\gamma - \phi\right)}{\tau + \phi \varsigma_\tau \sigma_B} sp_t + \frac{\phi \varsigma_\tau \sigma_B}{\tau + \phi \varsigma_\tau \sigma_B} b_t \\
&= \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\hat{\delta}_{t+1} - \delta_{t+1}^*\right).
\end{aligned}$$

Substituting Eq. (15) into the third line, we show that an increase in the fiscal surplus and a decrease in the government debt decreases the premium difference, as long as the steady-state value of the interest rate spread is not high. Because inflation is not so stabilized, government debt decreases more under the exact policy, as shown in Eq. (4) (Panel 10). Because of a small decrease in the tax gap, the effect of a decrease in the fiscal surplus is not severe under the exact policy (Panels 7 and 9). Thus, the premium difference is more aggressively stabilized under the exact policy although it rises considerably under the false policy (Panel 5). As shown in line 5 in the previous expression, the smaller the premium difference, the smaller the deviation of the expected default rate from its target level. Reflecting this fact, the default gap is well stabilized under the exact policy although the default gap does not converge immediately under the false policy (Panel 6).

The default gap under the exact policy, which rises sharply after the shock, is consistent with Uribe[19]'s result. Uribe analyzes the 'interest rate peg' monetary policy that pegs the nominal interest rate for risky assets to that for safe assets under an economy with default risk. His interest peg policy raises the default rate sharply after an exogenous negative fiscal surplus shock although a rise in the default rate is stabilized immediately. His interest peg policy corresponds to the policy minimizing the interest spread for risky assets $\hat{r}_t^S$ in our paper. As Eq. (14) implies, the policy minimizing the interest spread for risky assets is equivalent to the policy minimizing the expected default gap $\mathrm{E}_t\left(\hat{\delta}_{t+1}\right)$. If the policy authorities adopt the policy minimizing the interest spread for risky assets, and they succeed in doing so, the expected default rate becomes zero. Thus, although a rise in the default gap immediately after the shock is inevitable, the default gap then becomes zero. Our exact policy has a feature minimizing the default gap itself, as the previous expression implies, and the default gap rises sharply after the shock and converges immediately, although it takes time to fully converge. This explains why our result is consistent with Uribe[19].

Next, we discuss the IRFs of a one-percent increase in government expenditure (Fig. 2). An increase in government expenditure increases the target level output and applies to inflation pressure (Panel 3). To stabilize inflation, the tax gap is hiked under both exact and false policies although the fiscal surplus worsens. As a result, inflation is well stabilized under both exact and false policies (Panel 4). This is why we have a prescription for conducting monetary and fiscal policy that 2) when an increase in government expenditure gives default risk, policy authorities should stabilize inflation, similar to a situation where there is no default risk (see Section 1). Due to a decrease in the fiscal surplus, the premium difference increases under both policies (Panel 5). The fluctuation of the default gap under both policies is almost the same and not very severe even under the false policy (Panel 6). The reason the fluctuation of the default gap is not very severe is that the interest spread for risky assets r_t^S does not differ much between the exact and false policies. As shown in Eq. (15), the interest spread for risky assets depends on the government debt and the fluctuation of the government debt under the false policy is close to that under the exact policy (Panel 10). Thus, the premium difference $\hat{r}_t^R$ under the false policy is not very severe (recall that the premium difference consists of the coupon premium $\hat{r}_t^G - \hat{r}_t$ and the interest spread for the risky assets r_t^S). Thus, the default gap under the false policy is very close to the exact policy. For reference, we note that the standard deviation of the default gap to a one-percent increase in government expenditure under the exact policy is 1.3839 and under the false policy it is 1.3684 (6th row, Table 2).

4.3 Effects of Differences of Policy on Productivity and Government Expenditure Shocks

Comparing Fig. 1 with Fig. 2, we find that there are nonnegligible differences between policies on productivity and government expenditure shocks. As shown in Panel 9 in Fig. 1, the tax gap is severely reduced under the false policy, whereas it is reduced very little under the exact policy. In addition, while the nominal interest gap is hiked under the false policy, it does not fluctuate under the exact policy, as shown in Panel 8 in Figs. 1 and 2. To cope with the productivity shock, policy instruments are manipulated inversely. However, as shown in Panels 8 and 9 in Fig. 2, both the tax gap and nominal interest rates are similarly manipulated. The nominal interest rate gap decreases and the tax gap increases. Differences are almost zero in response to government expenditure shock between exact and false policies.

The reason that response variation of monetary and fiscal policies depends on the type of shock relates to how each shock shifts the NKPC. Plugging Eqs. (22), (26), and (28) and the definition of the efficient level of output into Eq. (24), we have:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \frac{\kappa(1+\sigma_C\psi)}{\sigma_C}\hat{y}_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_t + \frac{\kappa(1+\sigma_C\psi)\Omega_2}{\sigma_C\Omega_1}g_t - \frac{\kappa[(1+\psi)\sigma_C\Omega_1 - (1+\sigma_C\psi)\Omega_3]}{\sigma_C\Omega_1}a_t, \quad (41)$$

where $\hat{y}_t \equiv y_t - y_t^*$ denotes the welfare-relevant output gap. In the previous equality, the coefficients of the fourth and fifth terms are direct effects that shift the NKPC through changes in government expenditure and through changes in productivity, respectively. Because of the distorted steady state, both government expenditure and productivity appear on the NKPC, unlike in the model assuming that monopolistic competitive power is completely dissolved. Under our benchmark parameterization, the coefficient of government expenditure is 0.0144, although that of productivity is 0.1877. While the direct effects of shifting the NKPC stemming from an increase in government expenditure are negligible, the direct effects stemming from a decrease in productivity are nonnegligible, rather, that decrease (increase) changes inflation strongly.

The reason productivity shifts the NKPC strongly stems from a distorted steady state. In our model, tax is levied on output and monopolistic competitive market power remains as long as the tax rate is set to make the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor Φ. To attain $\Phi = 0$, under our setting that tax is levied on output, the tax rate should be negative as long as the elasticity of substitution among goods is larger than one, namely $\varepsilon > 1$. Under our benchmark parameterization, the steady-state value of the tax rate should be –0.2, which is unrealistic. Because there is a positive tax rate in the steady state, the productivity shock is seen as a cost–push shock. The higher the steady-state value of the tax rate, the lower the contribution of changes in productivity to increase the target level output y_t^*. The complicated parameter Ω_3, which is a function of the steady-state value of τ, appears in the definition of the target level of output Eq. (31) and the NKPC Eq. (43). As mentioned, the higher the steady-state value of the tax rate, the lower Ω_3. A decrease in productivity decreases the target level output. However, if the steady-state value of the tax rate is larger than its optimal value, i.e., –0.2, the target level output does not decrease enough. Then, the output does not decrease enough. In contrast, a decrease in productivity applies pressure to increase marginal cost, as shown in Eq. (26). Now, Ω_3 is small and a decrease in the target level output cannot absorb the decrease in productivity. This can be explained by the coefficient of productivity in Eq. (43) as follows:

$$\kappa\left(1+\psi\right) - \frac{\kappa\left(1+\sigma_C\psi\right)\Omega_3}{\sigma_C\Omega_1}.$$

In this expression, the first term is the direct effect that increases inflation stemming from a decrease in productivity and the second term is the pressure to decrease inflation through a decrease in the target level output. Because the steady-state value of the tax rate exceeds its optimal value, the direct effect, to increase inflation, is larger than the pressure to decrease the target level output. Thus, productivity works as a cost–push shock.

In contrast, as noted, the shift of NKPC through an increase in government expenditure is negligible, due to a 'non-Keynesian effect'. Although government expenditure increases output, consumption decreases simultaneously, as shown in Panel 11 in Fig. 2. The pressure to increase the marginal cost is canceled by the decrease in consumption, and the upward shift of NKPC through an increase in government expenditure is tiny or negligible.

Under the exact and the false policies, policy authorities have quite different period loss functions if the steady-state value of the interest spread is high. While the policy authorities under the exact policy do not necessarily focus only on stabilizing inflation, the policy authorities under the false policy almost solely focus on stabilizing inflation. However, an increase in government expenditure does not shift the NKPC very much, and inflation stabilization policy does not necessarily worsen welfare costs even under the false policy. Thus, as long as government expenditure affects the economy, the policy response does not differ much between the two policies. This implies that even if there is default risk, policy authorities are not necessarily aware of this risk. However, if productivity changes are observed, policy authorities should become aware of the risk. There is strong pressure to increase inflation and to use the false policy to try to stabilize it through a decreasing tax gap, which causes high and long-lasting default because of a worsening fiscal surplus, although the exact policy does not cause this result. Awareness of default risk is important if productivity changes are observed.

5 Optimal Monetary and Fiscal Policy Rules and Welfare Costs

5.1 Optimal Monetary and Fiscal Policy Rules

This section introduces simple policy rules. The monetary policy rule takes a class of Taylor rules:

$$\hat{r}_t = \varphi_\pi \pi_t, \tag{42}$$

while a rule for the government conducting fiscal policy takes the form:

$$\hat{\tau}_t = \varphi_b b_{t-1}. \tag{43}$$

We find both φ_π and φ_b through grid search, which minimizes the consequent difference in welfare costs under optimal monetary and fiscal policy. That is, we find φ_π and φ_b that replicate welfare costs under optimal monetary and fiscal policy. The ranges of grid search are limited to $\varphi_\pi \in [1, 30]$ and $\varphi_b \in [0, 3]$ for the condition of determinacy. The numbers of grid are 25 for both coefficients. Lines 4 and 5 in Table3 show φ_π and φ_b under exact and the false policy rules. As shown in columns 2 and 5, φ_π and φ_b under the false policy rule are larger than those under the exact policy. This implies that inflation is more aggressively stabilized under the false policy, as period loss function Eqs. (30) and (32).

For changes in productivity, although φ_π under the exact policy is higher than it is under the false policy, φ_b under the exact policy is smaller than it is under the false policy and is zero (see columns 3 and 6). This implies that the exact policy is not necessarily the policy that tends to stabilize inflation aggressively following changes in productivity. This is consistent with what Panel 4 in Fig. 1 shows. For changes in government expenditure, φ_π under the exact policy is larger than it is following changes in productivity (see columns 3 and 4). In addition, φ_b under the false policy is the same as it is under the exact policy (see columns 4 and 7). The two facts imply that the exact policy is not necessarily the policy that avoids stabilizing inflation, and awareness of default risk is not so important as long as changes in government expenditure hit an economy with default risk. These implications are consistent with our discussion.

5.2 Welfare Analysis

Now we analyze the welfare properties of both policies. The expected welfare costs are given by:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t \right),$$

which is the first term of a second-order approximated utility function and is the sum of discounted period welfare costs. The last line in Table 3 shows welfare costs under both the exact policy rule and the false policy rule.[12] On average, welfare costs under the exact policy are smaller than they are under the false policy. Welfare gains from conducting the exact policy ((welfare costs under the false policy − welfare costs under the exact policy) / welfare costs under the false policy), namely awareness of default risk, are 64.4% following changes in both productivity and government expenditure and when productivity changes, welfare gains from conducting the exact

[12] Instead of the FONCs for policy authorities, monetary policy rule Eq. (42) and fiscal policy rule Eq. (43) are included in the model to calculate welfare costs.

policy are 49.7%. These results are consistent with our intuition. However, welfare gains from conducting the exact policy are only 1.0% and are thus negligible.

Those results generate our policy prescriptions that 3) welfare gains from conducting optimal monetary and fiscal policy with awareness of default risk when there is an increase in government expenditure are negligible, while 4) welfare gains from conducting optimal monetary and fiscal policy with awareness of default risk when there is a decrease in productivity are nonnegligible (see Section 1).

6 Conclusion

We develop a class of DSGE models with nominal rigidities and we introduce default risk to the model. We find that if productivity changes are observed, policy authorities should be aware of default risk, although being aware of such risk is not very important following government expenditure changes. Welfare gains from awareness of default risk are nonnegligible if productivity changes, although welfare gains from awareness of default risk are tiny if government expenditure changes.

Conclusion

The author acknowledges the financial assistance of a grant from the Ishii Memorial Securities Research Promotion Foundation.

Appendices

A Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_t = 1$ and $\frac{\tilde{P}_t}{P_t} = 1$. Because this steady state is nonstochastic, productivity has unit values; i.e., $A = 1$.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \beta^{-1}.$$

Because $\Gamma(0) = 1$, the definition of the government debt coupon rate boils down to:

$$R^G = R.$$

Notice that $sp_t = 0$ in the steady state.

Eq. (23) can be rewritten as:

$$\frac{\tilde{P}_t}{P_t} = \mathrm{E}_t \left(\frac{K_t}{F_t} \right), \tag{A.1}$$

with

$$K_t \equiv \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \left(P_{t+k} C_{t+k} \right)^{-1} \tilde{Y}_{t+k} MC_{t+k}^n \quad ; \quad F_t \equiv P_t \sum_{k=0}^{\infty} \left(P_{t+k} C_{t+k} \right)^{-1} \tilde{Y}_{t+k},$$

which boils down in the steady state to

$$K = \frac{\frac{\varepsilon}{\varepsilon-1} YMC^n}{(1-\alpha\beta)(PC)} \quad ; \quad F = \frac{PY}{(1-\alpha\beta)(PC)}.$$

Plugging those equalities into the steady-state condition of Eq. (A.1), namely $K = F$, yields:

$$P = \frac{\varepsilon}{\varepsilon - 1} MC^n,$$

which can be rewritten as

$$MC = \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-1}. \tag{A.2}$$

Furthermore, Eqs. (25) and (A.2) imply the following:

$$\frac{U_N}{U_C} = \frac{1-\tau}{\left(\frac{\varepsilon}{\varepsilon-1}\right)\mu^w} = 1 - \Phi,$$

with $U_C = C^{-1}$ and $U_N = N^{\psi}$. Note that because $\tau \in (0,1)$ and $\varepsilon > 1$, this steady state is distorted.

The definition of R_t^H boils down, in the steady state, to

$$R^S = \left[1 + \frac{B}{SP}\Gamma'(0)\right]. \tag{A.3}$$

Eq. (13) boils down in the steady state to

$$R^S = (1-\delta)^{-1}. \tag{A.4}$$

Plugging Eq. (A.4) into Eq. (A.3) and rearranging, we have:

$$\delta = \frac{\phi\varsigma_\tau\sigma_B}{\tau + \phi\varsigma_\tau\sigma_B},$$

where we use $\frac{B}{SP} = \left(\frac{SP}{Y}\right)^{-1}\frac{B}{Y}$.

References

[1] Bacchetta, P., Perazzi, E. and Wincoop, E. (2018), "Self-Fulfilling Debt Crises; What Can Monetary Policy Do?" *Journal of International Economics*, 110, 119–134.

[2] Benigno, P. (2001), "Price Stability with Imperfect Financial Integration," Unpublished Manuscript, New York University.

[3] Benigno, P. (2009), "Price Stability with Imperfect Financial Integration," *Journal of Money, Credit and Banking*, 41, 121–149.

[4] Benigno, P. and Woodford, M. (2003), "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," *NBER Macroeconomics Annual*, 18, 271–333.

[5] Benigno, P. and Woodford, M. (2005), "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economics Association*, 3, 1185–1236.

[6] Cochrane, J.H. (1998), "A Frictionless View of U.S. Inflation," *NBER Macroeconomics Annual*, 13, 323–384.

[7] Corsetti, G. and Dedola, L. (2016), "The Mystery of the Printing Press: Monetary Policy and Self-fulfilling Debt Crises," *Journal of the European Economic Association*, 14, 1329–1371.

[8] Corsetti, G., Kuester, K., Meier, A. and Mueller, G.J. (2013), "Sovereign Risk, Fiscal Policy, and Macroeconomic Stability," *The Economic Journal*, 123, F99–F132.

[9] Creedy, J. and Gremmell, N. (2010), "The Elasticity of Taxable Income and Tax Revenue Elasticity," *Department of Economics*, *The University of Melbourne Working Paper Series*, No. 1110.

[10] European Central Bank (2011),"Challenge to Monetary Policy in 2012," Speech by Vítor Constâncio on Dec. 8, 2011 in Frankfurt. https://www.ecb.europa.eu/press/key/date/2011/html/sp111208.en.html

[11] Ferrero, A. (2009), "Fiscal and Monetary Rules for a Currency Union," *Journal of International Economics*, 77, 1–10.

[12] Gali, J. (2008), *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press, Princeton, NJ.

[13] Gali, J. and Monacelli, T. (2005), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72, 707–734.

[14] Leeper, E.M. (1991), "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 129–147.

[15] Okano, E. (2014), "How Important is Fiscal Policy Cooperation in a Currency Union?" *Journal of Economic Dynamics and Control*, 38, 266–286.

[16] Okano, E. and Hamano, M. (2018), "Inflation Stabilization and Default Risk in a Currency Union," *Macroeconomic Dynamics*, 22, 1790–1807.

[17] Okano, E. and Inagaki, K. (2019), "Revisiting the Fiscal Theory of Sovereign Risk from a DSGE Viewpoint," Revised and Resubmitted to *International Journal of Central Banking*.

[18] Schabert, A. (2010), "Monetary Policy under a Fiscal Theory of Sovereign Default," *Journal of Economic Theory*, 145, 860–868.

[19] Uribe, M. (2006), "A Fiscal Theory of Sovereign Risk," *Journal of Monetary Economics* 53, 1857–1875.

[20] Woodford, M. (1994), "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy," *The Economic Journal*, 4, 345–380.

Table 1: Parameterization

Parameter	Value	Source
β	0.99	Gali and Monacelli[13]
ψ	3	Gali and Monacelli[13]
θ	0.66	Gali and Monacelli[13]
ε	6	Gali and Monacelli[13]
ϕ	0.033	Okano and Inagaki[17]
γ	1.1736	Okano and Inagaki[17]
τ	0.3	Ferrero[11]
σ_G	0.276	Ferrero[11]
σ_B	2.4	Ferrero[11]
ς_τ	1	Creedy and Gremmell[9]

Table 2: Macroeconomic Volatilities

	Exact			False		
Shocks	All	Productivity	Government Expenditure	All	Productivity	Government Expenditure
$y_t - y_t^*$	0.3956	0.3956	0.0046	0.0015	0.0010	0.0011
π_t	0.0483	0.0483	0.0010	0.0000	0.0000	0.0000
$\hat{r}_t^R$	0.6113	0.6113	0.0073	1.9594	1.9593	0.0217
$\hat{\delta}_t$	4.0151	3.7691	1.3839	8.7098	8.6016	1.3684
$\hat{r}_t$	0.2867	0.2691	0.0989	0.9488	0.9444	0.0914
$\hat{\tau}_t$	2.1719	1.4122	1.6501	5.8779	5.6481	1.6274

Table 3: Optimal Monetary and Fiscal Policy Rules and Welfare Costs

	Exact			False		
Shocks	All	Productivity	Government Expenditure	All	Productivity	Government Expenditure
φ_π	14.0556	9.8421	10.1579	23.5556	6.8947	17.7895
φ_b	0.3333	0	2.6842	3	3	2.6842
$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t\right)$	1.3476	1.3270	0.6275	3.7885	2.6545	0.6336

Figure 1: IRFs to Unit Decrease in Productivity

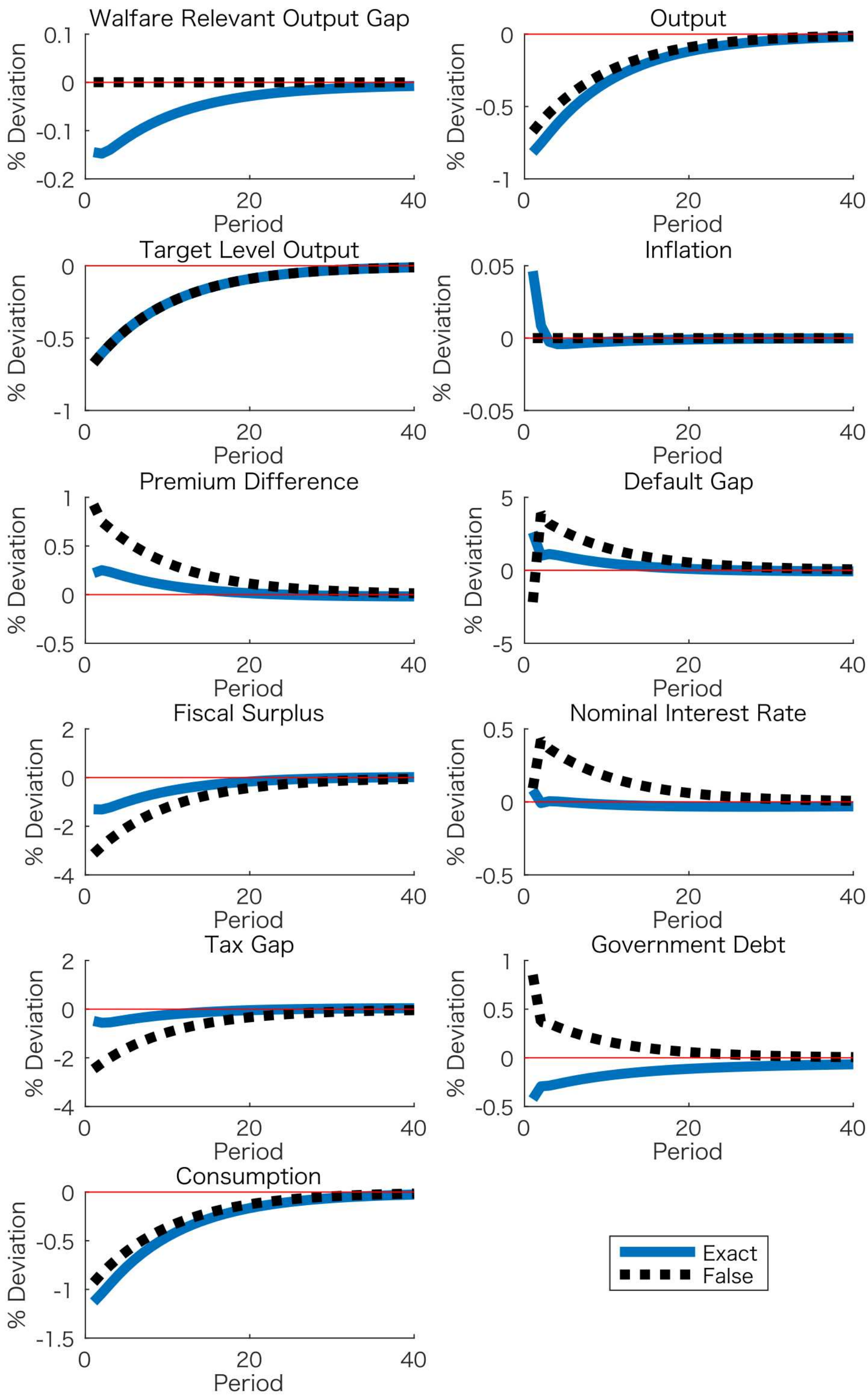

Figure 2: IRFs to Unit Increase in Government Expenditure