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Modelling individual variability in cognitive development

Michael S. C. Thomas[1], Fiona M. Richardson[2], Neil A. Forrester[1], and

Frank D. Baughman[1]

[1] Developmental Neurocognition Laboratory, School of Psychology, Birkbeck, University of London, UK

[2] Functional Imaging Laboratory, Institute of Neurology, University College London, UK

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Address for correspondence:

Dr. M. S. C. Thomas
School of Psychology
Birkbeck College, University of London
Malet Street Bloomsbury
London WC1E 7HX, UK
tel: +44 (0)20 7631 6386
fax: +44 (0)20 7631 6312
email: m.thomas@bbk.ac.uk

Abstract

Investigating variability in reasoning tasks can provide insights into key issues in the study of cognitive development. These include the mechanisms that underlie developmental transitions, and the distinction between individual differences and developmental disorders. We explored the mechanistic basis of variability in two connectionist models of cognitive development, a model of the Piagetian balance scale task (McClelland, 1989) and a model of the Piagetian conservation task (Shultz, 1998). For the balance scale task, we began with a simple feed-forward connectionist model and training patterns based on McClelland (1989). We investigated computational parameters, problem encodings, and training environments that contributed to variability in development, both across groups and within individuals. We report on the parameters that affect the complexity of reasoning and the nature of 'rule' transitions exhibited by networks learning to reason about balance scale problems. For the conservation task, we took the task structure and problem encoding of Shultz (1998) as our base model. We examined the computational parameters, problem encodings, and training environments that contributed to variability in development, in particular examining the parameters that affected the emergence of abstraction. We relate the findings to existing cognitive theories on the causes of individual differences in development.

1. Introduction

The computational modelling of cognitive processes offers several advantages. One of the most notable is theory clarification. Verbally specified theories permit the use of vague, ill-defined terms that may mask errors of logic or consistency, errors that often become apparent when formal implementation forces these terms to be clarified. For example, in the domain of intelligence research, in a verbal theory, one may characterise a more clever cognitive system as being 'faster'; but an implemented model of that system must specify what 'speed' really means and how it relates to the quality of computation. In the domain of developmental research, one may refer to a more developed cognitive system as containing 'more complexity'; but an implemented model must specify what 'complexity' really means in terms of the structure of representations and the processes that act on them. In the domain of atypical development, one may refer to a disordered cognitive system as having 'insufficient processing resources'; but an implemented model must specify what a 'processing resource' really means, and what parametric range constitutes an insufficiency with respect to the range found in the typically developing population.

Connectionist networks have been widely used to model phenomena in cognitive development because they are essentially learning systems (Thomas & Karmiloff-Smith, 2003; Thomas & McClelland, 2008). Connectionist models have been applied to a wide range of developmental phenomena over the last twenty years. These include categorisation and object-directed behaviour in infants, Piagetian reasoning tasks such as the balance scale problem, seriation, and conservation, and other children's reasoning tasks such as learning the relation between time, distance and velocity, and discrimination shift learning. Within the domain of language acquisition, developmental models have been constructed to investigate the categorisation of speech sounds, the segmentation of the speech stream into words, vocabulary development, the acquisition of inflectional morphology, the acquisition of syntax, and learning to read (see Elman et al, 1996; Mareschal & Thomas, 2007, for reviews).

Connectionist models embody a range of constraints or parameters that alter their ability to acquire intelligent behaviours. These include constraints such as the initial architecture of the network (in terms of the number of processing units and the way they are connected), the network dynamics (in terms of how activation flows through the network), the way in which the cognitive domain is encoded within the network (in terms of input and output representations), the learning algorithm used to change the connection weights or architecture of the network, and the regime of training the network will undergo. Only the last of these constraints is derived from the environment; the preceding four are candidates for innate components of the learning system, although in principle these four constraints may themselves be the products of learning or other environmental influences on development.

Decisions about the design of the network directly affect the kinds of cognitive problem it can learn, how quickly and accurately learning will take place, as well as the final level of performance. To the extent that these networks are valid models of cognitive systems, differences in these constraints or parameters provide us with candidate explanations for the variations found both between individuals and within individuals over time. In this paper, we consider the adequacy of the computational constraints within two established connectionist models of cognitive development to provide candidate mechanisms for variability in children's acquisition of reasoning. The models seek to capture variability in two Piagetian reasoning tasks, the *balance scale* task and the *conservation* task.

2. Variability in a developmental model of the balance scale task

In the study of cognitive development, a rich literature has accumulated on the balance scale task. In this task, different numbers of weights are placed at distances either side of a fulcrum and the child is asked whether the scale will balance, tip left, or tip right when released (Inhelder & Piaget, 1958). For both empirical and computational approaches to this domain, the cornerstone is Siegler's initial work (1976, 1981) in which children's decisions at different ages were characterized in terms of four rules of increasing complexity. Rules *I* to *IV* describe the child's performance on each of the six different problem types (see Figure 1), with *Rule IV* representing mastery.

Siegler's rule assessment methodology has provoked much debate, both with regard to whether the rules he postulated are sufficient to capture children's behaviour or are merely descriptive (e.g., Wilkening & Andersen, 1982) and whether rules actually play a causal role in driving behaviour (Hardiman, Pollatsek, & Well, 1986). Rule-based theories of development have traditionally struggled to explain the mechanisms mediating transitions between rule states, leading to theories based on connectionist learning models (e.g., McClelland, 1989). As an example of the debate, it has been argued that children use different rules depending on the torque value (where torque = weight x distance from fulcrum): Ferretti and Butterfield (1986) found that problems with a large difference in the torque acting on each side were likely to draw responses consistent with a more advanced rule; Jansen and van der Maas (1997) later reported that the *torque difference* effect only occurred for problems with extreme torque values.

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Insert Figure 1 about here

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Despite criticism, Siegler's rules have stood the test of time, albeit with proposed additions (and replacements) to the original four core rules. For example, the *smallest distance down* rule (*SDD*, Figure 1) has been proposed as a rule used by children only when in transition between rules *I* and *II* (Jansen & van der Maas, 2002). The majority of new rules have emerged through the scrutiny of behaviour surrounding *Rule III* where, according to Siegler's scheme, children perform well when either weight or distance information unambiguously predicts the side to tip, but then guess when these sources of information conflict. Some of the new rules proposed to account for the variability around *Rule III* include: *Rule IIIa*, the *qualitative proportionality*, *distance dominant*, *addition*, and *buggy* rules (Ferretti & Butterfield, 1986; Jansen & van der Maas, 1997, 2002; Normandeau et al., 1989; van Maanen, Bean & Sijtsma, 1989; Wilkening & Andersen, 1982).

The existence of additional rules has found support from Latent Class Analysis, a statistical technique for categorizing behavioural data into consistent subgroups (e.g., Jansen & van der Maas, 1997, 2002). Though these analyses differ in the number of classes generated (relating to a free parameter in this statistical technique), they converge on the idea that *Rule III* behaviour consists of a variety of strategies that children tend to switch between. Recent work examining reaction times (RT) as well as accuracy has supported the development with age of more complex balance-scale strategies, favouring the *buggy* rule over the *addition* rule as a *Rule III* strategy (van der Maas & Jansen, 2003), although the response patterns for *buggy* and *addition* are co-extensive.

Individual variability in performance on different problem types has been acknowledged in theories of the phases of development. The *staircase model* captures the phases of development by proposing that transitions between rules are quick with relatively little overlap, while transitions in the *overlapping waves model* are more gradual and interleaved, particularly around *Rule III* (Siegler, 2002). A combination of these two models (Jansen & van der Maas, 2002) captures the behavioural data via steep transitions between *Rule I* and *Rule II* but overlap and gradual transitions between subsequent rules (such as *Rule II*, *Rule III*, and the *addition* rule) prior to reaching *Rule IV*.

Computational approaches have sought to specify the mechanisms that generate the behavioural profile of development on the balance-scale task. The models are disparate, ranging from connectionist implementations (Dawson & Zimmerman, 2003; McClelland, 1989; Shultz, Mareschal & Schmidt, 1994) to production systems (van Rijn, Someren & van der Maas, 2003) to decision trees (Schmidt & Ling, 1996). Typically, these models have attempted to capture the sequence of Siegler's four core rules, and have been judged on their ability to capture the complete range of behavioural phenomena (van Rijn et al., 2003). However, Dawson and Zimmerman (2003) have argued that computational modelling has been preoccupied with fitting the data. Since none of the models give a perfect fit and the detailed data are themselves contested, at this stage the contribution of models should be a qualitative understanding of the mechanisms underlying rule transitions (see Quinlan et al., 2007; Shultz & Takane, 2007; Thomas, McClelland et al., 2009, for recent discussions).

Despite the wealth of research on the balance scale task, one area has remained relatively under explored until recently. This is the question of variability. The study of variability in cognitive development is important for three reasons. First, within a single individual, it has been argued that increased variability in performance presages the onset of developmental transitions (Jansen & van der Maas, 2002). Second, variability across individuals of the same age gives a window onto general or specific intelligence. Third, variations in development from the normal pathway are found in disorders, sometimes exhibiting delay, sometimes failure to reach more complex levels of reasoning, and sometimes qualitatively atypical patterns. Implemented models have generally focused on the normative (average) pathway, yet each type of variability must ultimately be explained at a mechanistic level (Thomas & Karmiloff-Smith, 2003).

The following sections report an initial set of simulation results investigating sources of variability in the balance scale task. First we introduce our base or 'normal' model of development. Second, we explore how changes to the model's computational parameters, representations, and training environment alter its behavioural profile. Third, we evaluate variability in a single case study.

2.1 The Normal Model

The normal model was defined as a 3-layer feedforward connectionist network consisting of an input layer of 20 units representing the number of weights placed (up to 5) on each side of the scale (5 distances either side), a hidden layer of 4 units, and an output layer of 2 units (tip left, tip right). The model used McClelland's (1989) input encoding, where weight and distance information were represented on different units. McClelland's original model separated channels for weight and distance processing channels (i.e., a split hidden layer), a design assumption intended to amplify the model's difficulty in integrating these dimensions. In contrast, we used an undifferentiated network because we wished to avoid using a proprietary network architecture for this particular reasoning problem. There are limitations in our simple model but it remains a useful launching pad to begin an exploration of developmental variability (see Quinlan et al., 2007, for criticism of computational models of balance scale; and Schapiro & McClelland, 2009, for a recent extension to the McClelland 1989 model).

The model was trained using back-propagation for 100 epochs, where an epoch is one presentation of the full training set, and the learning rate was set to 0.01. Ten network runs were conducted per manipulation, with initial weights randomized between ±0.5. The standard deviation across runs is depicted in all figures. The training set contained of 621 of the possible 625 balance scale problems for a five-peg scale using up to five weights, and was similar to that of McClelland in that balance and weight problems were duplication / over-sampled in the training set. The training set consisting of 1069 patterns. The 24 problems were held back and used to assess generalization. Performance was measured at 10, 20, 25, 30, 35, 40, 50, 60, 70, 80, 90, and 100 epochs.

The test set consisted of 4 problems from the 6 problem types (see Figure 1). The model's performance on the test set was assessed with 7 test metrics. The metrics captured behavior in line with the following rules: (i) *Rule I*, (ii) *SDD* rule, (iii) *Rule II*, (iv) *QP* rule, (v) *Rule III*, (vi) *addition* rule, and (vii) *Rule IV*. Each metric calculated the percentage of responses consistent with its rule. Note that a given correct response may be consistent with several rules. For example, Figure 2 shows the problem-space and the proportion of patterns consistent with the four core rules.

The normal network learned the training set to an accuracy of 98.0% (SD 0.0%). The mean performance of the normal model on each of the test metrics across training is shown in Figure 3(a), for the one hidden layer (*1HL*) condition. Given that we did not separate distance and weight information in the architecture, the network did not exhibit strong evidence of early *Rule I*, *SDD*, or *Rule II* behavior, confirming that weight-distance integration difficulties require architectural assumptions. However, our focus here is upon the model's balance scale behavior around Rule III, since much of the literature has focused on this phase. The sequence of metrics that best characterized the development of the 'normal' model was: *QP -> Rule III -> addition* rule -> *Rule IV*.

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Insert Figure 3 about here

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2.2 Exploring Variability

Variability was explored by making a series of systematic changes either to the normal model's computational parameters, to the problem encoding, or to its environment.

2.2.1 Variability and Computational Parameters

We varied (i) the number of hidden layers, (ii) the number of hidden units in a single layer, and (iii) the learning rate.

Increasing the number of hidden layers The performance of the model was tested with 2 and 3 hidden layers (*HL*), with 4 units per layer. Additional hidden layers tend to increase the computational complexity of the mappings that can be learned by a network, while slowing down learning since the error signal must filter back through more levels (Beale & Jackson, 1990). So that learning would fall within a 100-epoch window, the learning rate (*lr*) was increased as follows: *1HL*=0.01, *2HL*=0.02, *3HL*=0.2 (these values hold for subsequent use of these architectures unless otherwise stated)[1]. These networks achieved mean accuracy levels on the training set of 98.0, 99.8, and 100.0% respectively. The developmental profiles of the networks are included in Figure 3(a). Increasing the number of hidden layers altered the number

[1] These networks showed qualitatively equivalent results to multi-hidden-layer networks trained on the same learning rate with an extended training time. Learning rate adjustments were utilised for comparison within a common frame of training.

of transitions in behavior made prior to approaching *Rule IV* performance (we define a transition as a shift in the rule that covers the most behavior, as per Figure 2). The standard deviation across runs went up as the number of hidden layers increased but notably, phases of development became less incremental. For example, the sequence of closest fitting metrics for models with *2HL* was: *QP* -> *addition* rule -> *Rule III* -> *Rule IV*, but was just *QP* -> *Rule IV* for networks with *3HL*. (This pattern did not result from *lr* changes, since it did not arise when *1HL* was trained with a learning rate of 0.2). Increasing the power of the network reduced the number of transitional states it went through in reaching mastery.

Increasing the number of hidden units in a single layer Expanding the number of units in a single a layer increases the capacity of the network to learn more patterns of a given complexity (Cybenko, 1989); and allows the network to learn a given problem with smaller weights, thereby requiring less learning. We evaluated networks with 4, 10, and 20 units in the hidden layer for the normal *1HL* network. After training, all the networks had a mean accuracy of 98.0%. Their developmental profiles are shown in Figure 3(b). Increasing the number of hidden units did not change the profiles compared to the normal case. We explored this manipulation in the *2HL* and *3HL* networks and found the same result. If the capacity of the system is measured in parallel processing resources, additional capacity did not alter the transitional stages through which the system passed but altered the rate at which it did so.

Reducing the learning rate Individual differences and developmental disorders are sometimes characterized in terms of *delay* (Thomas, Annaz, et al., 2009). This term is usually descriptive, but one obvious way to implement it is to turn down the learning rate. This would not explain why delay is frequently uneven across problem domains, but we can at least address how learning rate alters the transitions in complexity of reasoning that the system exhibits.

Learning rate was reduced in the normal network in four steps as follows: 0.08, 0.06, 0.04, and 0.02. After 100 epochs, these networks achieved mean accuracies 98.0, 97.1, 94.1, and 56.7% respectively. Slower learning rates caused roughly parallel shifts for all metrics so that characteristic patterns in the developmental profiles appeared after more epochs of training. While development slowed down, the order of the transitions between types of reasoning behaviour remained the same. When the learning rate was insufficient to achieve mastery within the fixed time window of 100 epochs, performance terminated at a less complex level. For example, networks with a learning rate of 0.04 had reached *Rule III* but not *IV*, by 100 epochs, and networks with a learning rate of 0.02 had reached the *addition* rule. However, were training extended, *Rule IV* would be reached in both cases. By contrast, developmental disorders typically exhibit asymptoting performance at less complex levels of reasoning (Thomas, Annaz et al., 2009). For individual differences, it is unclear whether everyone eventually 'catches up'. Reduced learning rate does not, therefore, seem a good (sole) candidate to explain the type of developmental delay found in disorders.

2.2.2 Variability and the Problem Encoding

We explored two variations in the problem encoding. These correspond to alterations in the way in which the problem is presented to the child (perhaps in the salience of different information or options) or to alterations in how the problem is encoded in the part of the cognitive system required to predict outcomes of balance-scale problems. We either: (i) added a further response option so that the scale could either tip left, tip right, or *balance*; or (ii) altered the input coding so that information about the weights was represented with position-specific units.

Changing the response options In the normal model, there were two output units whose activation could vary between 0 and 1. If the left output unit was more active than the right unit by more than 0.33, the response was ‘tip left’, and vice versa for ‘tip right’. If the difference between the units was less than 0.33, the response was taken to be ‘balance’ (McClelland, 1989). However, since balance is a legitimate response for a proportion of the problems, it could reasonably be encoded as a separate output unit. For this condition, a response was considered correct if the activation of the corresponding output unit was >=0.5 and the activation of any other output unit was <0.5. Finally, because encoding of the problem domain could alter its complexity, we contrasted performance on *1HL*, *2HL*, and *3HL* networks with 4 hidden units per layer. After training, these networks achieved mean accuracy levels of 86.3, 87.5, and 99.2%, respectively. The developmental phases are shown in Figure 4(a). Comparison with Figure 3(a) reveals that the additional response option dramatically changed the pattern of transitions. *1HL* and 2HL networks began in *Rule I* and did not exceed *Rule II*. Only the *3HL* network reached *Rule IV*. Changing the response options altered the categorization that the internal representations had to make across problem types. It ramped up the complexity of the task, since balance had to be computed internally rather than left to the competition between left and right output units. More computational power was therefore required for successful acquisition, under this encoding.

<u>Combined Weight-Distance Encoding</u> In McClelland’s (1989) formulation, weight and distance information were encoded separately. However, one could represent the number of weights on each peg locally at each distance. For this manipulation, there were 10 input units, one for each peg on the balance scale. The activation level coded the number of weights placed on a peg. Activation ranged from 0 to 1 and each weight was represented by an increment of 0.2. Thus, three weights on a peg corresponded to an activation of 0.6. The composition of the training set and the output responses remained as in the base condition. Networks with 1, 2, and

3 hidden layers were run to assess the demands of this encoding. The results are in Figure 4(b). The final performance of the models was poorer than with the normal encoding by around 20% (*1HL*=80.2%, *2HL*=65.0%, *3HL*=90.3%). As above, only the *3HL* network achieved *Rule IV* reasoning as the closest fitting metric at the end of training. *1HL* only reached the *QP* rule. Again, a recoding of the problem domain, this time at input, increased the complexity of the task and altered the developmental phases exhibited by the model.

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2.2.3 Variability and the Engaged Environment

Since development in the balance scale task corresponds to the child's active exploration of the domain, we refer to the training set as the *engaged environment*. We created two variations in the environment: (i) an impoverished training set with restricted coverage of the problem space, and (ii) a training set without a bias to increase the salience of the weight dimension. In these cases, the normal architecture and problem encoding was used. *1HL*, *2HL*, and *3HL* networks with 4 units per layer were also contrasted to explore whether additional representational power could overcome limitations in the engaged environment.

An Impoverished Engaged Environment This engaged environment consisted of a subset of 703 training patterns, which excluded any problems where the distances from the fulcrum on either side was >=3. After training, the *1HL*, *2HL*, and *3HL* reached accuracy levels of 97.8, 99.7, and 100.0%, respectively. This environment had an adverse effect on the single hidden layer network, where the closest fitting metric at the end of training was *Rule III* instead of the normal *Rule IV*. The number of closest fitting metrics was also fewer across training, indicating fewer transitions. In contrast, for *2HL* and *3HL* networks, the closest fitting test metric at the end of training was still *Rule IV*, with the *2HL* network making more transitions than the *3HL*. For all models, there was a considerable increase in variability between individual runs compared to the normal environment. This impoverished environment, then, increased developmental variability between individuals. Importantly, with respect to the generalization set at least, the impoverished environment could be compensated for via a more powerful learning system.

An Unbiased Engaged Environment The unbiased engaged environment consisted of 1069 patterns where the original bias for the weight dimension was removed. The duplicated weight problems were replaced with a random selection of patterns already in the training set. All models trained using this environment were able to reach *Rule IV* performance. However, this environment reduced the number of transitions between rules across training.

2.2.4 Individual Variability: A Case Study

Variability also occurs during the development of individual children, including regression to less sophisticated rules. However, averaging across individuals' risks producing variability not found in any one, which may be the case for simulations as well. In this section, we report the rule transitions in the trajectory of a single network (*1HL*, *lr*=0.008, normal encoding and training set). Performance on the training set is shown in Figure 5(a), while 5(b) illustrates performance on the 6 problem types in the generalization set at 25, 40, 60, 70, and 100 epochs. Figure 5(c) depicts the rule transitions shown by this individual network. The model made the following transitions: *QP -> addition -> Rule III -> addition -> Rule IV*. The trajectory confirms variability around *Rule III*, with a jump from *QP* to *addition*, back to the less sophisticated *Rule III*, returning to *addition*, and on to *Rule IV*.

Inspection of Figure 5(b) suggests that this variability was driven by the network's attempts to solve the low-salience distance problems. Performance on balance and weight problems was robust from early on, but the network struggled to accommodate distance and conflict-distance problems, inducing greater variability and more transitions between 60 and 80 epochs. In sum, the variability found in averaged data was not an artifact of averaging but found in individual runs. Apparent rule transitions, including regressions, were a key feature of the network's attempts to integrate weight and distance information in solving balance scale problems.

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2.3 Summary of Balance Scale findings

Simulation of the balance scale task indicated that variations of internal computational parameters, problem encoding, and engaged environment all acted on the complexity of the reasoning exhibited by the network during learning, including the findings that more hidden layers increased complexity but not more units per layer (contrasting reasoning power with plasticity); that a slower learning rate did not reduce complexity per se, and is therefore a poor model of unresolved developmental delay; and that an impoverished environment reduced complexity but could be compensated for (in terms of generalization performance) by a more powerful learning system.

3. The Conservation task

Conservation refers to the understanding or belief in the continued equivalence of two physical sets following a transformation that appears to alter one and not the other. A given transformation may alter a quantity, e.g., by *adding* or *subtracting*, or preserve it, e.g., through *elongation* or *compression.* The acquisition of conservation knowledge involves learning to distinguish between transformations that preserve and those that alter a quantity. For example, in a typical number conservation task, as shown in Figure 6, a child is initially presented with two rows of counters (*pre-transformation*). The child is then asked whether these rows have the same number of counters or whether one has more than the other. A transformation is then applied to one row, and the child is asked again whether the two rows are the same, or whether one now has more counters than the other (*post-transformation*).

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Insert Figure 6 about here

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Piaget (1965) found that young children below 6-7 years are non-conservers, in that when presented with a transformation that preserves number (such as *elongation* or *compression*) they answer that one row has more counters than the other. In contrast children older than 6-7 years are conservers, having learnt that transformations of this type do not alter number. This finding has been corroborated across a range of conservation tasks, such as mass (using modelling clay), liquid quantity (using beakers), and number (using counters) (Brainerd & Brainerd, 1972; Halford & Boyle, 1985; Klah, 1984; Miller & Heldmeyer, 1975; Siegler, 1995; Siegler & Robinson, 1982; Wallach, Wall & Anderson, 1967; Winer, 1974). The shift to conserving may be viewed as the emergence of more abstract knowledge: some properties of the world remain the same, even when their observed perceptual properties have altered. The rich literature on conservation has also established a series of biases that occur as young children learn to conserve, relating to problem size, length, and mode of presentation. These effects are summarized in Figure 6.

While a range of classic Piagetian reasoning tasks such as conservation, seriation and the balance scale, have been subject to computational investigation, Shultz and colleagues have argued that a key implementational feature may be necessary for learning systems to exhibit qualitative changes in the complexity of reasoning (Shultz, 1998; Schultz, Mareschal & Schmidt, 1994). In constructivist networks, networks change their architecture as a function of learning. One example of an algorithm that implements this approach is cascade-correlation (Falham & Lebiere, 1990). During training, network connections are altered but if learning stagnates, the size of the hidden layer is increased. Specifically, connections to existing hidden units are frozen, and a new hidden unit it added. The hidden unit is connected to the input layer and also to all existing hidden units, thereby allowing the additional internal resources to make use of the representations that have already been developed. This allows networks to be constructed which have representational depth (in the sense of multiple hidden layers) rather than just breadth (in the sense of more units in a single layer).

Shultz (1998) used such a constructivist network to model development on the conservation task. He ascribed the ability of his model to capture the abrupt shift from non-conservation (*NC*) to conservation (*C*) to addition of hidden units and an attendant increase in representational power. However, it is possible that other computational parameters have a similar impact upon a model's behavioural profile over the course of development. In the following sections, we employ Shultz's approach to modelling development on the conservation task as our starting point. We once again introduce a normal, base model of development, and then proceed to explore how manipulating the model's computational parameters, input encoding, and training environment alter its developmental behavioural profile.

3.1 The Normal Model

The base model set out to simulate the development of number conservation, as depicted in Figure 6. The normal model was defined as a 3-layer feedforward connectionist network consisting of an input layer of 13 units, a hidden layer of 4 units, and an output layer of 2 units. The problem encoding used by this network was based on Shultz (1998) and is shown in Figure 7. Each row of counters was represented over 2 units, encoding row length and density respectively, as continuously valued activation levels. Both rows are shown represented in their pre- and post-transformation states. The row transformed (either row 1 or row 2) was indicated by the activation (-1 or +1) of a single unit.[2] The transformation type was encoded arbitrarily over 4 units, with the activation of a single unit indicating the type as follows: *addition* (1 -1 -1 -1), *subtraction* (-1 1 -1 -1), *elongation* (-1 -1 1 -1), or *compression* (-1 -1 -1 1). The three possible response options were encoded over 2 binary output units as follows: (i) row 1 longer (1 0), (ii) row 2 longer (0 1), (iii) both rows equal (0 0). The base model differed from Shultz (1998) in that we employed a standard feedforward architecture with a sigmoid rather than hyper-tangent activation function.

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Insert Figure 7 about here

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[2] Input unit activations were bounded between -1 and +1. Hidden and output unit activations were bounded between 0 and 1.

The model was trained using back-propagation for 1500 epochs, with a learning rate of 0.025. Ten network runs were conducted per manipulation, with initial weights randomized between ±0.5. The standard deviation across runs is depicted in all figures. The composition of the training and test sets was drawn from Shultz (1998), with patterns having five levels of row length and five levels of density. A total of 400 training patterns and 100 test patterns were selected from a full set of 600 possible conservation problems (based upon 25 initial rows, 3 possible start states, and 4 possible transformations for each of the 2 rows). Performance was assessed at 5, 25, 50, 100, and 200 epochs, and then at every subsequent 100-epoch interval until the end of training at 1500 epochs.

In order to assess the behavior of the model, the test set was used in conjunction with 4 metrics, each reflecting a target behavioral phenomenon described in Figure 6: (1) the profile of *Acquisition*, (2) the *Problem Size Effect,* (3) the *Length Bias Effect,* and (4) the *Screening Effect.* Metric 1 plotted the development of knowledge of conservation, and calculated the percentage of test patterns correct. Metric 2 calculated the proportion of small vs. large problem types correct. In this case, the test set consisted of 40 patterns, 20 small problem types (<12 items), and 20 large (>24 items). Metric 3 used elongation and compression problems from the test set (a total of 18 patterns, 8 and 10 of each type respectively) to calculate the proportion of patterns where the longer row was selected as having more items than the shorter row. Metric 4 calculated the proportion of unscreened vs. screened problems correct for the complete test set. Test patterns presented to the network were represented as “screened” by replacing post-transformation activation values with zeros.

The base network learned the training set to an accuracy of 99.5%. Training performance exhibited an early shift from NC=>C between 100 and 200 epochs (from 44.6 to 70.4% training patterns correct). The emergence of abstract knowledge was marked by the network's ability to discriminate transformations that altered the (abstract property of) number from those that did not. This shift was preceded by an initial decline in training performance over the first 50 epochs and followed by small incremental improvements in performance as training progressed. The behavioral profile of the model can be seen in Figure 8, where the shift from *NC=>C* (*Acquisition*) on novel patterns occurs between 100-200 epochs and performance leaps from 36.2% to 61.7%. The normal profile represents a non-linear shift to conserving. The model also exhibited a minor performance advantage for small problem sizes (*Problem Size effect*) between 100-700 epochs, the time during which the model was doing the bulk of its learning. Normality is defined as an advantage for small problems (+ve values on the chart) during earlier phases of training. The model's bias for selecting longer rows as having more items (*Length bias effect*) was also found to reduce after this point in learning. Normality is defined as an early positive spike on the length bias chart. Unlike Shultz (1998), our model did not show any preference for "screened" problems early in learning (*Screening effect*), which would appear as an early negative spike on the chart. This shortcoming may relate to our use of sigmoid processing units (see later). Nevertheless, in contrast to Shultz (1998), the model was able to show a shift from non-conserving to conserving within a fixed architecture.

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Insert Figure 8 about here

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3.2 Exploring Variability

With our base model in hand, we then sought to assess the influence of several factors on development. Variability was explored by systematic changes to (1) the base model's computational parameters, (2) its problem encoding, or (3) the training environment.

3.2.1 Variability and Computational Parameters

The computational parameters that were varied included: (i) the number of hidden layers, (ii) the number of hidden units in a single layer, (iii) the learning rate, and (iv) the slope of the sigmoid transfer function for hidden layer units.

Increasing the number of hidden layers The performance of the model was tested over learning with 2 and 3 hidden layers (*HL*), with 4 units per layer. So that learning would fall within a 1500-epoch window, the learning rate (*lr*) was increased as follows: *1HL*=0.025, *2HL*=0.05, *3HL*=0.075 (these values hold for subsequent use of these architectures unless otherwise stated). These networks achieved mean accuracy levels on the training set of 99.5, 99.9, and 99.7%, respectively. The developmental trajectories of the networks are shown in Figure 9. The profiles of networks with *1HL* and *2HL* were very similar. Both *1HL* and *2HL* networks showed a shift from *NC* => *C* between 100-200 epochs. The shift was slightly larger for networks with *1HL* than those with *2HL* (26.5 and 35.5%, respectively). Networks with *3HL* showed a smaller initial shift (19.0%), occurring later between 200-300 epochs, followed by a second successive shift (17.4%) occurring between 300-400 epochs. There was a sustained effect of *Problem size* for networks with *3HL*, as well as an increase in variability. The variability for the *Length bias effect* was very high, particularly for *2HL* and *3HL* networks. As for *Screening*, there was no bias in early learning for screened problems, contrasting with the empirical data. Conversely, there was a developing bias for "unscreened" problems increased over learning.

Increasing the number of hidden units in a single layer We assessed networks with 4, 10, and 20 units in the hidden layer (*HU*) for the normal *1HL* model. At the end of training networks with *4HU* had a mean accuracy of 99.5%; *10HU* and *20HU* networks had reached 100%. *10HU* and *20HU* networks showed earlier *Acquisition* of conservation knowledge (between 50 and 100 epochs). This shift was also larger than networks with *4HU* (30.3% in comparison to 25.5%). The behavioural profile across metrics can be seen in Figure 10. All networks showed a similar profile across testing metrics. Variability was uniformly low across metrics. Interestingly, networks with *4HU* did show a slightly larger *Length Bias effect* of an extended duration, in comparison to *10HU* and *20HU* networks. This suggests that the smaller representational space compromised the network's ability to drive differential output responses when the net input activations were low. In broad terms, however, expanding the capacity of the system in terms of parallel processing resources altered the onset of learning, but not the overall developmental profile, as observed for the balance-scale simulations.

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Insert Figure 9 and 10 about here

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Reducing the learning rate To address the issue of delay, the learning rate was reduced in the normal network in four decrements from 0.025 to 0.02, 0.015, 0.01, and 0.005. After 1500 epochs, these networks achieved mean accuracies 99.8, 98.5, 96.6, and 86.3%, respectively. Lower learning rates slowed down development. As a result, improvements in performance behaviour were more incremental. This was reflected in the gradual decline in the size of the shift from *NC=>C* with decreasing learning rate (from 25.4% for 0.02 to 7.7% for 0.005). Extending this manipulation to networks with *2HL* and *3HL* produced a similar pattern of results.

Though networks with a lower learning rate had a lower level of performance at end of training (at 1500 epochs), performance was steadily increasing, and could have improved further were training time to be extended. There was no indication of a lowering in the maximum performance level simply by virtue of a reduced learning rate. As with the balance-scale simulations, a reduced learning rate does not seem a good (sole) candidate to explain the type of developmental delay found in disorders with learning disability, where reasoning ability frequently asymptotes at a lower level.

Decreasing the sigmoid slope Since we had altered the activation function of the processing units from the original Shultz (1998) of conservation, we investigated the impact of this function on development. The function determines the activation level of hidden and output units, given the net input activation they are receiving. Shultz (1998) used a hyper-tangent function, while we employed a more standard sigmoid function. We explored the impact of the slope of the sigmoid, which changes how much a processing unit can alter its activation level for a given change in the net input it is receiving. This in turn impacts on the nature of the category distinctions a unit can make. For example, a steep sigmoid slope results in sharp category boundaries and is good for tasks where the model is required to make rule-like distinctions, whereas a shallow slope is better suited to fine-grained distinctions and tasks with broad category boundaries. Altering the level of processing unit discriminability has been shown to produce patterns of deficits consistent with those seen in developmental disorders (Thomas, 2005; Thomas & Karmiloff-Smith, 2003). For all processing units, the slope of the sigmoid function was reduced (from a value of 1) in the normal model, by four levels of decreasing discriminability as follows: 0.8, 0.6, and 0.25, to 0.125. Changing the slope of the sigmoid had a negative effect upon the model's ability to learn, even for the 0.8 condition. Networks no longer exhibited a shift from *NC=>C*, resulting in low mean accuracies at the end of training at all levels, as follows: 34.5, 34.0, 38.4, and 28.3%, for the 4 decrements respectively. Performance that was uniformly low for all test metrics. This could not be overcome through the addition of extra hidden layers, suggesting that the model requires processing units with a high level of discriminability in order to achieve task success. This is because conservation requires the network to ignore certain differences in the input. To do so requires a non-linear response. A shallower sigmoid provides the network with a more linear system, more suited to providing proportional responses to proportional changes in the input. However, a *screening effect* (see Figure 6) appeared for the shallowest slope, suggesting that this developmental phenomenon may relate to the nature of the non-linear activation function used in the network's processing units.

3.2.2 Variability and the Problem Encoding

We explored a variation in problem encoding where the salience of transition type was increased. It is this information that drives the emergence of 'abstraction'. The number of units encoding transition information (as shown in Figure 7) was doubled from 4 to 8, thereby increasing the net activation arriving at the hidden layers from this information. The architecture now included an input layer consisting of 17 units, with 8 units encoding pre- and post-transformation information, and 8 units encoding transformation type. This manipulation was carried out for networks with *1HL*, *2HL* and *3HL*. The final performance of the models was found to be similar to that shown for equivalent models trained without increased transition information. The overall profile of development and *Acquisition* of conservation knowledge was also the same as the equivalent models. Therefore, for these simulations, changing the salience of a dimension of information did not have any notable impact upon the developmental trajectory of the model.

3.2.3 Variability and the Engaged Environment

We manipulated the engaged environment, corresponding to the child's active exploration of the domain, by creating a training set with a limited coverage of the problem space of the conservation task. It consisted of 400 problems with a small quantity of items only (<12 items). The base architecture and problem encoding were used. Networks with *1HL*, *2HL* and *3HL* were trained on this environment to explore any interaction between representational power and the engaged environment. Interestingly, this environment did not appear to have notable impact upon the overall performance, irrespective of the number of hidden layers in the model. At the end of training *1HL*, *2HL* and *3HL* networks reached the mean accuracies of 99.8, 99.4, and 97.3%, respectively. The profile of *1HL* and *2HL* networks over metrics was similar to that shown for equivalent models trained on a normal engaged environment. Limiting the engaged environment to problems with a small number of items did not in this case impact upon the developmental trajectory of the model.

3.2.4 Individual Variability: A Case Comparison

Once more, to distinguish individual profiles from group averaged data, we compared two individual cases. These were: (i) a single normal model with *1HL* (henceforth the *Normal* case), and (ii) a *1HL* model with a reduced learning rate (*lr*=0.005, henceforth *Reduced lr* case). Both models were trained using the normal input encoding and engaged environment using the same randomly initialized starting weights. The behavioral profile of each model was assessed using our 4 metrics. In addition, a detailed analysis of the development of conservation according to (i) transformation type, and (ii) problem size was conducted for test items. As expected, the profile of the *Reduced lr* network exhibited a slower developing, more incremental trajectory. The shift from *NC=>C* was markedly later than in the normal case, by approximately 500 epochs. Subsequent improvements in training performance were also smaller.

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Insert Figure 11 about here

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This pattern in training performance is also reflected behavioral profile for the metric *Acquisition* (calculated on novel test items) shown in Figure 11. For the next two metrics, the *Reduced lr* case showed extended *Problem Size* and *Length Bias* effects. These paralleled the protracted learning window of this model. For the *Screening* metric, the trajectory of the *Reduced lr* case deviated from that of the *normal* case, demonstrating a minor preference for "screened" problems at the onset of acquisition of conservation knowledge. An examination of the development of conservation knowledge in the *normal* case across problem types (as shown in Figure 12a) revealed a difference in initial profile for problems that alter number (*addition* and *subtraction*) in comparison to those that preserve number (*elongation* and *compression*). *Addition* and *subtraction* problems showed a static level of performance early in learning, whereas *elongation* and *compression* problems showed an initial dip in performance. As a consequence, performance over learning on transformations that preserved number was poorer than those that altered it. This dip was seen on all problem types in the *lr case*. An initial dip in performance can also be seen for problems of differing sizes (Figure 12b). In the *normal*, case this dip was exaggerated for large problem sizes, resulting in poorer performance on large problems during learning. For the *Reduced lr case*, the converse pattern was observed, where the performance for larger problem types was superior. The atypical case, therefore, exaggerated an early dip in performance and a reliance on large problem sizes in solving conservation problems.

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Insert Figure 12 about here

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3.3 Summary of Conservation findings

A fixed architecture network demonstrated the ability to learn to conserve abstract properties across transformations, within Shultz's (1998) formulation of this Piagetian problem domain. In some cases, changes to the internal computational parameters of the model had a marked impact upon the acquisition of conservation knowledge. Once again, changes in serial resources (hidden layers) altered the profile of development, while changes in parallel resources (within a layer) altered the rate of development. Changes to the internal discriminability of processing units via the slope of the sigmoid activation function resulted in a failure to conserve. Decreasing the learning rate resulted in only a slower rate of acquisition. By contrast, for this model, changes to the problem encoding at input or the engaged environment had little impact on the model's developmental trajectory.

4. Discussion

The advantage of implementation is that it forces theory clarification, while the disadvantage is that models necessarily involve simplification. In this case, we sought to explore the mechanistic basis of individual variability via parametric variations to established models of the development of reasoning in two Piagetian tasks: balance scale and conservation. The former involves learning to integrate two dimensions of information (weight and distance) in determining whether a scale will balance. The latter involves learning that certain transformations do not alter an abstract property (such as number) while others do. In both cases, the base models simulated the development of these abilities via supervised learning and exposure to many examples of these problems (McClelland, 1989; Shultz, 1998). Clearly, these reasoning domains are fairly simple, and far from capturing the full range of skills which developing children exhibit. Nevertheless, both normal models are targeted at tasks of key theoretical interest within a Piagetian framework of the development of abstract reasoning abilities.

Our interest here was to address the mechanisms that explain how children of the same age can differ in their reasoning abilities, in a framework that captures the improvement of these abilities across development. A number of theoretical proposals have been put forward to explain the respective mechanisms of individual differences and cognitive development, either at the cognitive or brain level. For example, with respect to individual differences, several authors have proposed that differences in cognitive abilities between children of the same age can be explained by differences in the speed of processing among basic cognitive components, on the grounds that speed of response in simple cognitive tasks predicts performance on complex reasoning tasks; and at a brain level, that neurophysiological measures such as latency of average evoked potentials and speed of neural conductivity correlate with IQ (Anderson, 1992, 1999; Eysenck, 1986; Jensen, 1985; Nettelbeck, 1987). Sternberg (1983) proposed differences in the ability to control and co-ordinate the basic processing mechanisms, rather than in the functioning of the basic components themselves. Finally, Dempster (1991) proposed differences in the ability to inhibit irrelevant information in lower cognitive processes, since individuals can show large neuroanatomical differences in the frontal lobes, the neural bases of executive function.

With respect to mechanisms that might underlie cognitive development, we once more find speed of processing offered as a factor that may drive improvements in reasoning ability (Case, 1985; Hale, 1990; Kail, 1991). Case (1985) suggested that an increase in speed of processing aids development via an effective increasing in short-term storage space, allowing more complex concepts to be represented. Halford (1999) proposed that the construction of representations of higher dimensionality or greater complexity is driven by an increase in processing capacity where processing capacity is a measure of the 'cognitive resources' allocated to a task. Lastly, Bjorklund and Harnishfeger (1990) proposed improvements in the ability to inhibit irrelevant information, based on evidence from cognitive tasks and changes in the brain that might reduce cross-talk in neural processing, such as the myelination of neural fibres and the decrease with age in neuronal and synaptic density.

Here, we considered three types of manipulation that might influence the development of reasoning ability, within the framework of implemented connectionist models. These were the computational parameters of the learning system, the encoding of the problem domain, and the training set to which the network was exposed (corresponding to the model's engaged environment). Respectively, the first of these manipulations influences the representational states that the learning system is able to acquire, the second influences the complexity of the mapping problem that must be learned, and the third influences the data available to search the representational states of the system. We contrasted results from simulated development in two separate reasoning domains, in order to distinguish patterns emerging from the structure of those domains from those arising from our computational manipulations. In addition, we avoided domain-specific architectural assumptions to focus on the influence of more domain-general processing properties, such as learning rate, number of internal processing units per layer, number of layers, the nature of the activation function, and so forth. Our results revealed the following pattern.

First, increasing the number of hidden unit layers altered the complexity of the reasoning behaviour that a network could acquire, but it did so at the expense of slowing down the rate of acquisition. By contrast, increasing the number of units within a layer did not alter the complexity of the behaviour acquired (in terms of the domain-specific performance metrics) but increased the rate at which they were acquired. Changes in *serial* versus *parallel* representational resources therefore appeared to have contrasting effects.

Changes in the learning rate did not alter the quality of developmental profiles, simply requiring the network to gain greater exposure to the problem domain to reach a given level of performance. Learning rate has been put forward as a neurocomputational parameter (in the guise of 'neuroplasticity') that might explain general intelligence (Garlick, 2002). Our current findings suggest that if this were the case, the less intelligent should catch up with the more intelligent if they are simply given more time to gain relevant experience. There is some evidence that this prediction holds in childhood, through the use of experimental designs where children of disparate chronological age are matched at a given mental age (e.g., a younger high ability group is matched on mental age to an older low ability group). However, this work also suggests subtle differences between performance achieved via greater intelligence versus that achieved via greater chronological age (Baughman, 2009). Moreover, as the comparison is extended into adulthood, the prediction starts to diverge with the prediction. It becomes clearer that differences in intelligence produce asymptotes in development to different levels in functioning. This pattern is particularly apparent in individuals who have developmental disorders involving learning disability, where development is often delayed and asymptotes at a lower level (Thomas, Annaz et al., 2009). Learning rate, therefore, may have limitations as a candidate explanation of individual differences in development. Changes in serial resources appear to offer more promise, as these clearly altered the level at which reasoning performance asymptoted.

The modelling results pointed to other factors that could alter the profile of development in Piagetian reasoning tasks. These included changes to the composition of the training set, changes to the encoding of the problem domain, and changes to the activation function of processing units. In the first of these cases, the impact appeared more domain-specific. For the manipulation we considered (i.e., restricting the training examples to problems specified over a small values), the balance-scale model was more affected than the conservation model. Moreover, as least with respect to extracting the general function of this domain (tested on the novel problem set), some parameters within the balance-scale model demonstrated compensation for restrictions in the nature of the training set. Lastly, since unit activation functions are argued to be modulated by attention which itself varies between children (Cohen, Dunbar, & McClelland, 1990) and since the encoding of problems may differ in more and less intelligent children (Spitz, 1982), these manipulations provide further avenues for explaining individual differences in development.

The current work only points in a direction of advancing our understanding of theories of development and individual differences, via exploring the parameter space of implemented computational models. Paradoxes still remain. For example, as we have seen, cognition in more intelligent individuals is linked to greater processing speed, and at the same time, the utilisation of more complex (perhaps more abstract) representational states. However, as both simulations demonstrated here, systems with greater complexity typically took longer to train; that is, they were slower, not faster to develop. In machine learning terms, such systems possess a richer hypothesis space, and this space takes longer to search given the data in the training set. In order to link greater representational complexity with greater processing speed and faster learning, more detailed models will be required. In particular, it is likely that we will have to utilise attractor networks with activation dynamics that exhibit settling into stable representational states in order to drive responses. This will permit investigations of the relationship between the rate of settling and the parameters that allow more complex representational states to be acquired. Moreover, there are a host of more complex reasoning domains to which models must be applied for a fuller picture of the development of reasoning.

We would argue, in sum, that our understanding of variation in cognition has much to gain from computational implementation, but this work has only just begun.

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Figure captions

Figure 1: The balance scale problem set and developmental 'rules' observed in behavioural data

Figure 2: The problem space for rules *I* to *IV*

Figure 3: (a) Developmental phases of the normal model with one hidden layer (*1HL*) compared with networks with two and three hidden layers (*2HL*, *3HL*); (b) profiles for models with a single hidden layer containing 4 units (normal), 10 units and 20 units

Figure 4: (a) Developmental profiles for networks with 3 response options (left-down, right-down, and balance), and (b) profiles for models with combined distance-and-weight encoding, both shown for *1HL*, *2HL*, and *3HL* conditions

Figure 5: Individual network profile for: (a) training performance; (b) test performance; (c) rule transitions (marked by a dark square)

Figure 6: The number conservation task using counters, and developmental effects observed in behavioural data

Figure 7: The input encoding for the conservation task

Figure 8: Developmental phases of the normal model. The arrow shows shift from not conserving to conserving for the *Acquisition* metric

Figure 9: Profiles for models with one (normal), two and three hidden layers (*HL*). Arrows show shifts from non-conserving to conserving

Figure 10: Profiles for models with 4 (normal), 10 and 20 hidden units (*HU*) in a single layer. Arrows show shifts from non-conserving to conserving

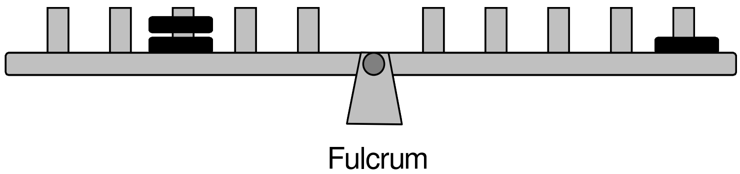
Figure 11: contrasting developmental profiles for two individual networks, for the base model (*Normal*) and a network with a reduced learning rate (*Reduced lr*) models. Arrows show shifts from non-conserving to conserving

Figure 12: (a) Developmental profiles across problem types for the reduced learning rate case study compared to the normal profile; (b) equivalent performance across problems with different numbers of elements

Figures

Figure 1

1. Balance problems	The same weight is positioned at the same distance on both sides
2. Weight problems	Different weights are placed at the same distance on both sides
3. Distance problems	The same weight is placed at different distances either side of the fulcrum
Conflict problems: *weight and distance are placed in conflict on the two sides*	
4. Conflict Weight	The side with the larger weight tips
5. Conflict Distance	The side with the weights the largest distance from the fulcrum tips
6. Conflict Balance	The scale balances



Rule I	Consider weight only; the side with the most weight tips
SDD	Smallest Distance Down: If distance differs, select the side with the weights closest to the fulcrum
Rule II	Consider the distance dimension if weights on either side are equal
QP	Qualitative Proportionality: For conflict problems scale will balance, as larger weight on one side will compensate for greater distance on other
Rule III	Consider information on weight and distance but (as unable to combine them) guess on conflict problems
Rule IIIa	Focus on either the weight or distance and make a perceptual decision on which side will tip
DD	Distant Dominant: side with weights largest distance from the fulcrum tips
Addition	Calculate {*weight + distance*} on each side of the fulcrum; side with highest value tips
Buggy	For side X with more weights but smaller distance, shift weights away from fulcrum until the distance on each side is equal; for each shift, remove one weight from X. Side with greater final weight tips
Rule IV	Mastery: solve problems by calculating the torque {*weight x distance*} on each side of the fulcrum; side with highest torque value tips

Figure 2

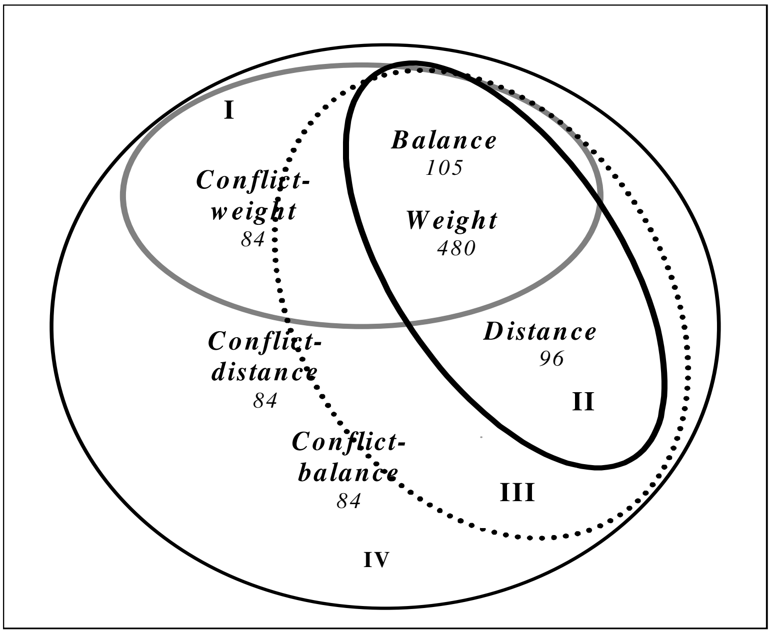
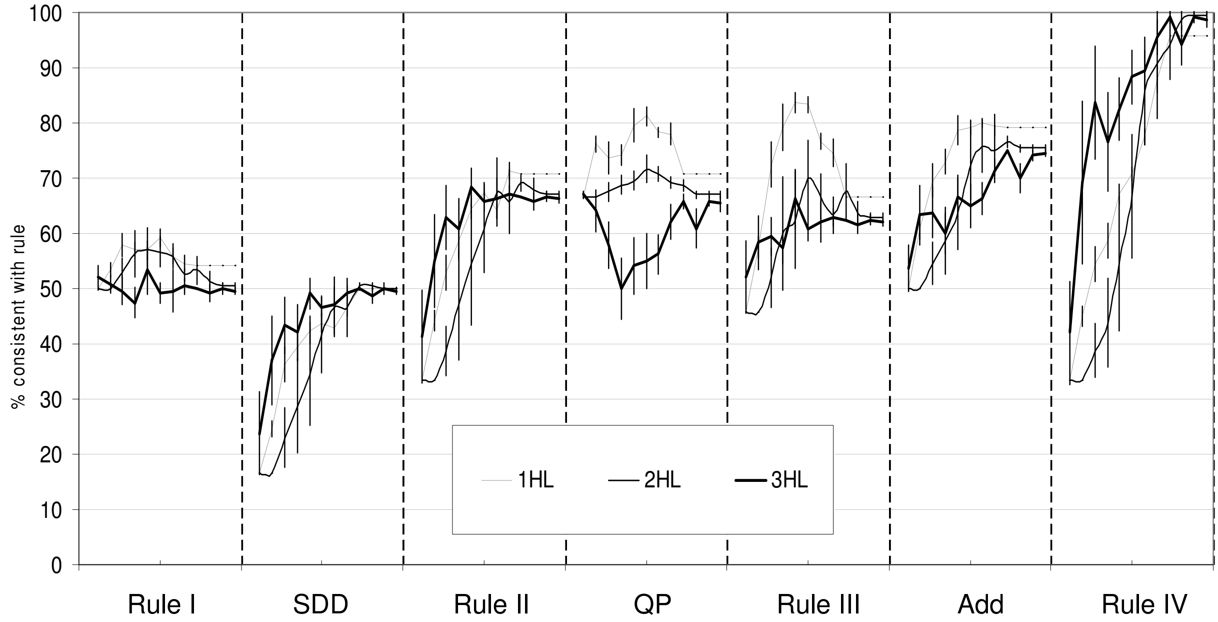

I
Balance
105
Conflict-
weight
84
Weight
480
Conflict-
distance
84
Distance
96
II
Conflict-
balance
84
III
IV

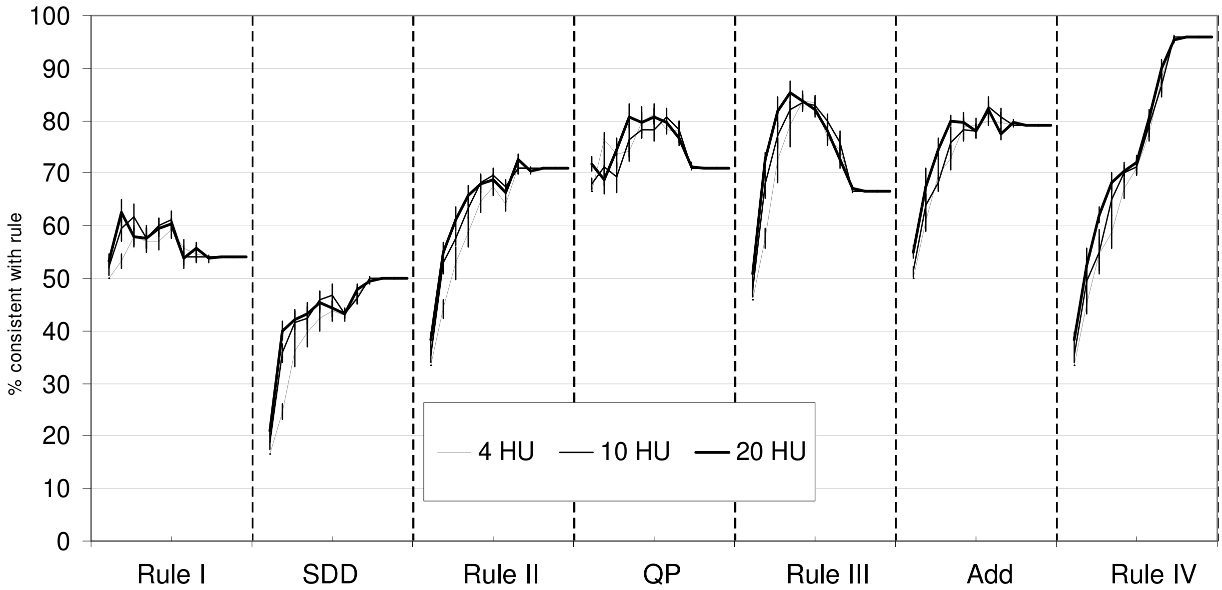
Figure 3

(a)



100
90
80
70
60
50
40
30
20
10
0
% consistent with rule
1HL
2HL
3HL
Rule I
SDD
Rule II
QP
Rule III
Add
Rule IV

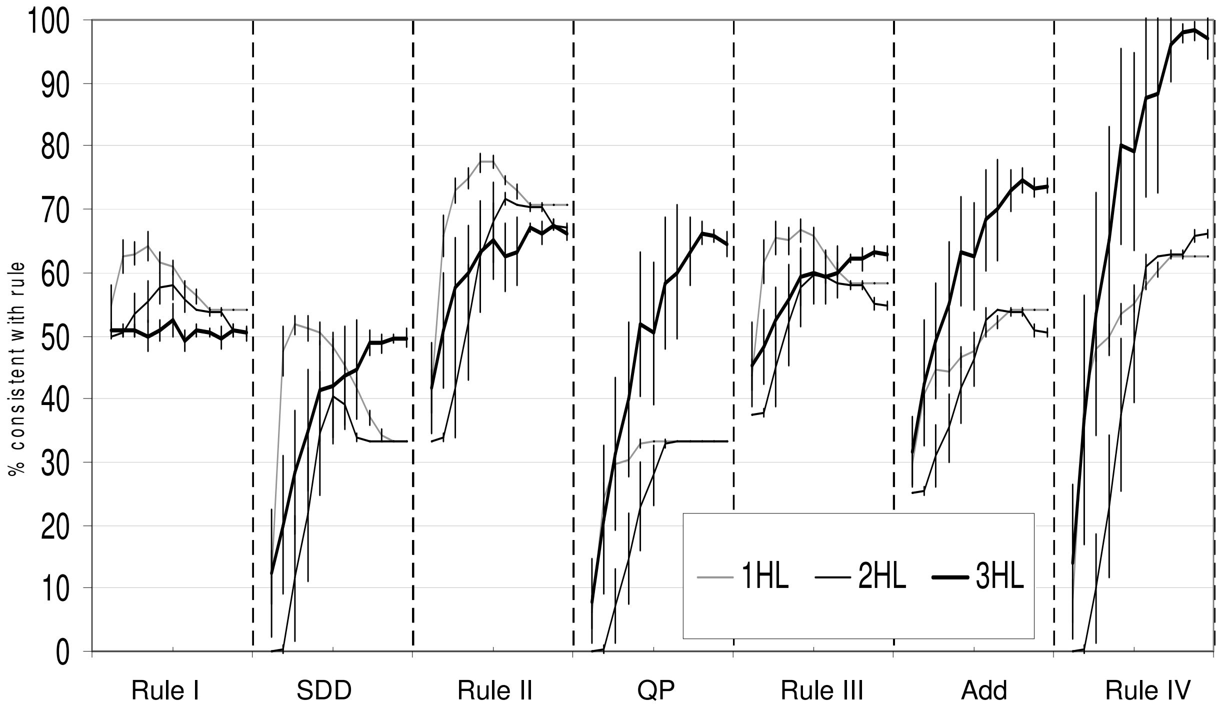
(b)



100
90
80
70
60
50
40
30
20
10
0
% consistent with rule
4 HU
10 HU
20 HU
Rule I
SDD
Rule II
QP
Rule III
Add
Rule IV

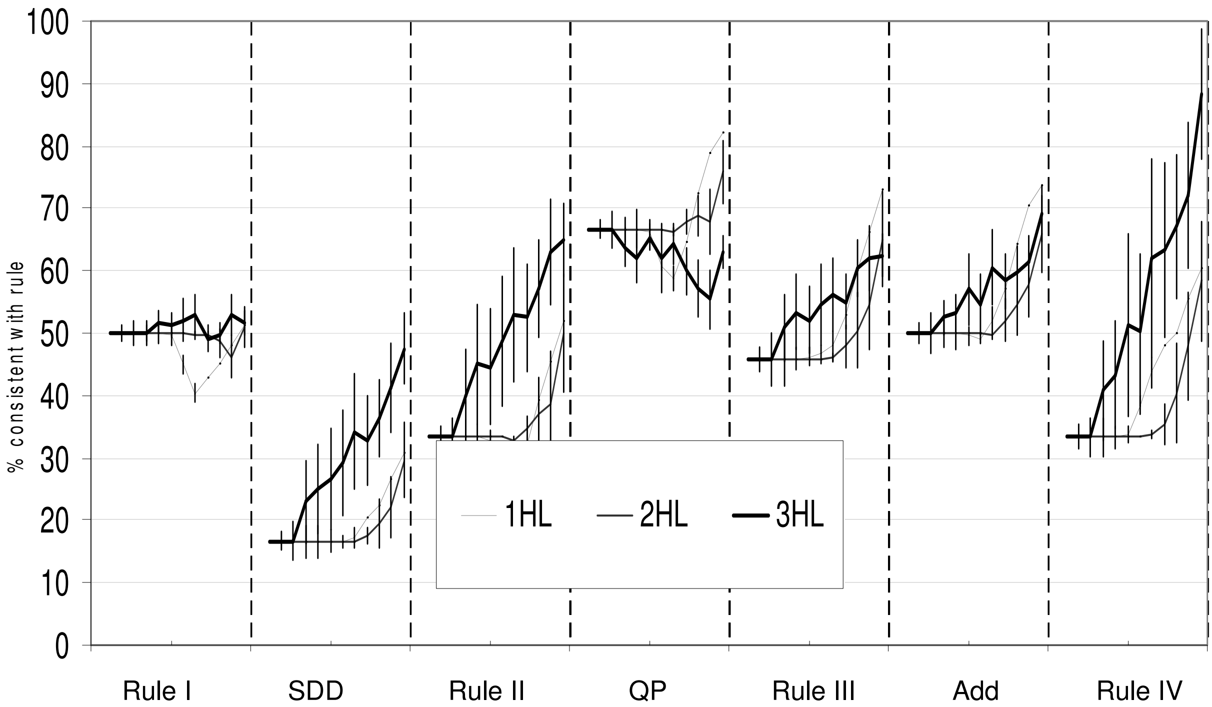
Figure 4

(a)



100
90
80
70
60
50
40
30
20
10
0
% consistent with rule
1HL
2HL
3HL
Rule I
SDD
Rule II
QP
Rule III
Add
Rule IV

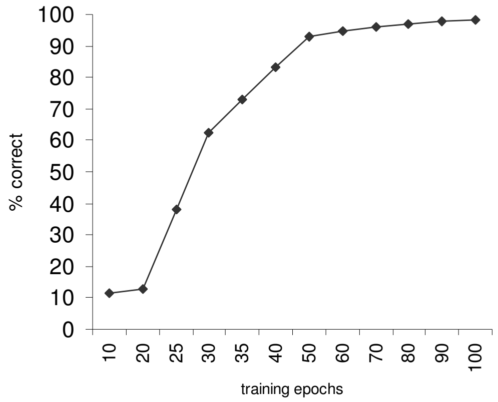
(b)



100
90
80
70
60
50
40
30
20
10
0
% consistent with rule
1HL
2HL
3HL
Rule I
SDD
Rule II
QP
Rule III
Add
Rule IV

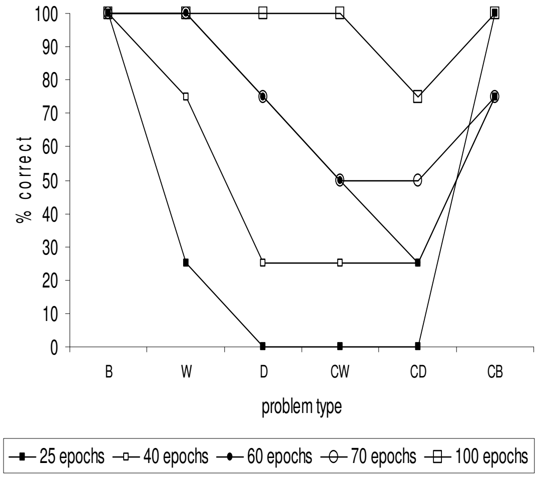
Figure 5

(a)



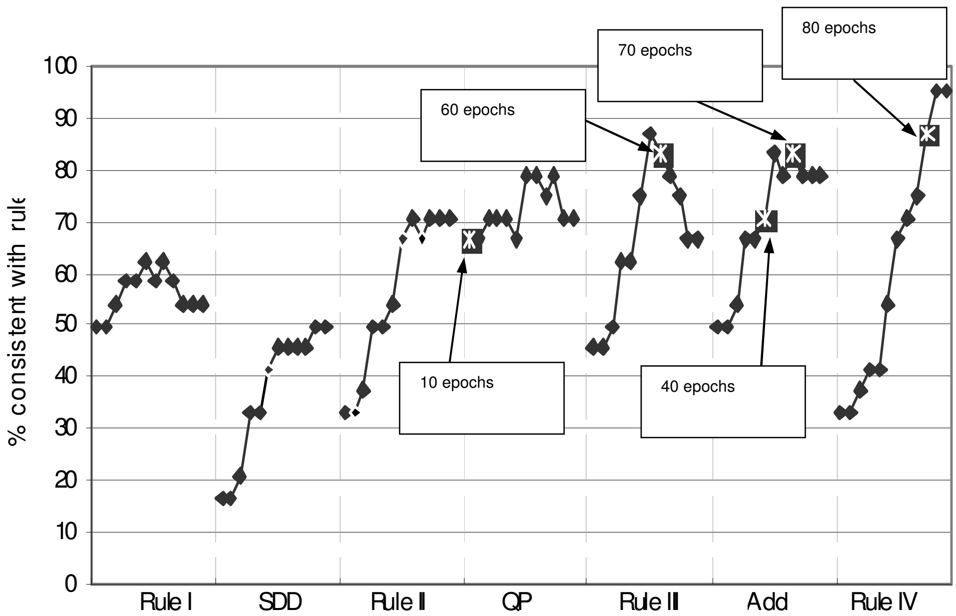
100
90
80
70
60
50
40
30
20
10
0
% correct
10 20 25 30 35 40 50 60 70 80 90 100
training epochs

(b)



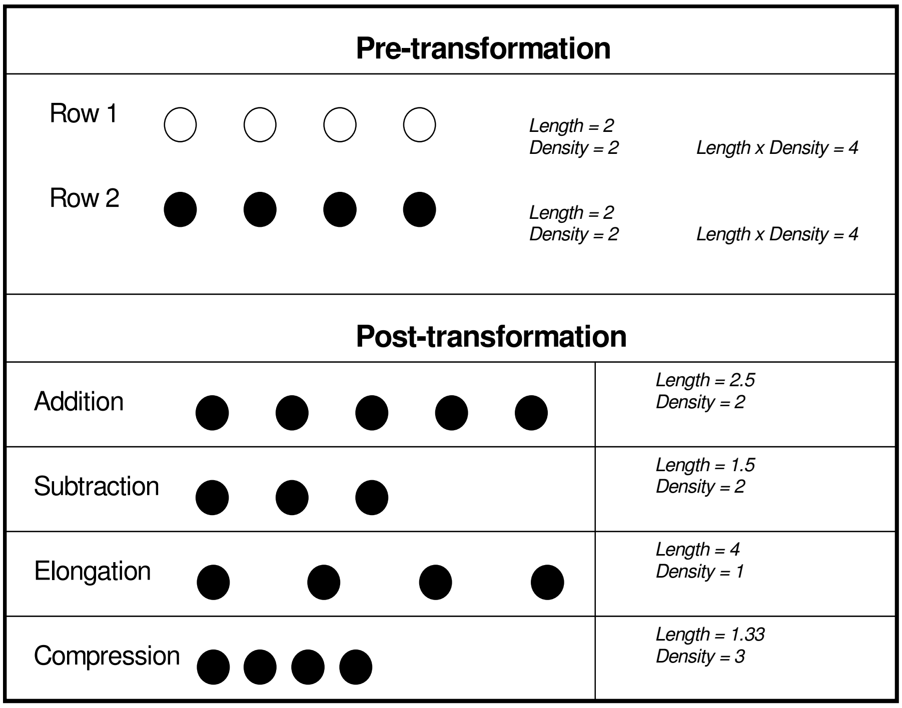
100
90
80
70
60
50
40
30
20
10
0
% correct
B W D CW CD CB
problem type
25 epochs
40 epochs
60 epochs
70 epochs
100 epochs

(c)



80 epochs
70 epochs
60 epochs
10 epochs
40 epochs
100
90
80
70
60
50
40
30
20
10
0
Rule I
SDD
Rule II
QP
Rule III
Add
Rule IV

Figure 6



Acquisition	The shift from non-conservation to conservation (which is typically abrupt).
Problem-size	Knowledge of conservation develops on tasks with smaller numbers of items, prior to those with a larger number of items.
Length-bias	For problems with differing row lengths, non-conservers tend to choose the longer row as having more items.
Screening	Younger children conserve better on problems that are screened, where they cannot see the the result of the transformation.

Figure 7

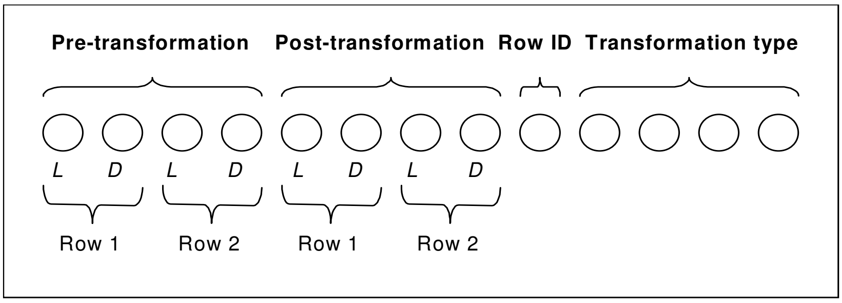
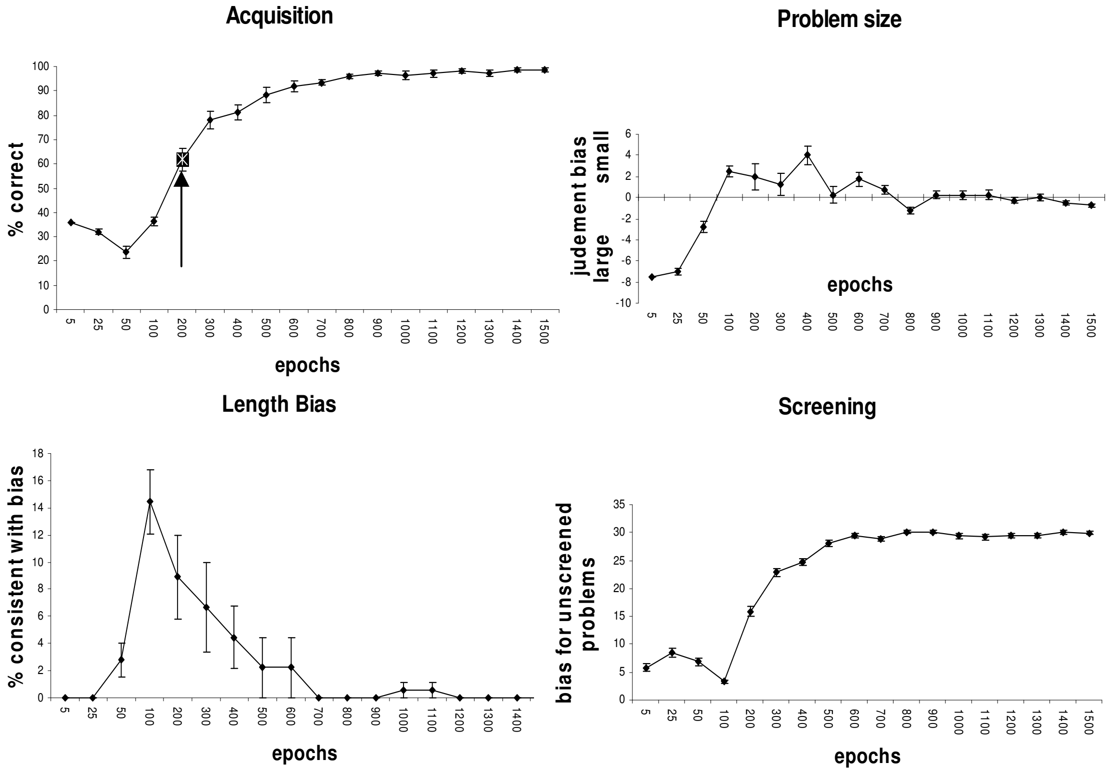

Pre-transformation
Post-transformation
Row ID
Transformation type
L
D
L
D
L
D
L
D
Row 1
Row 2
Row 1
Row 2

Figure 8



Acquisition
% correct
epochs
Problem size
judement bias
large small
epochs
Length Bias
% consistent with bias
epochs
Screening
bias for unscreened problems
epochs

Figure 9

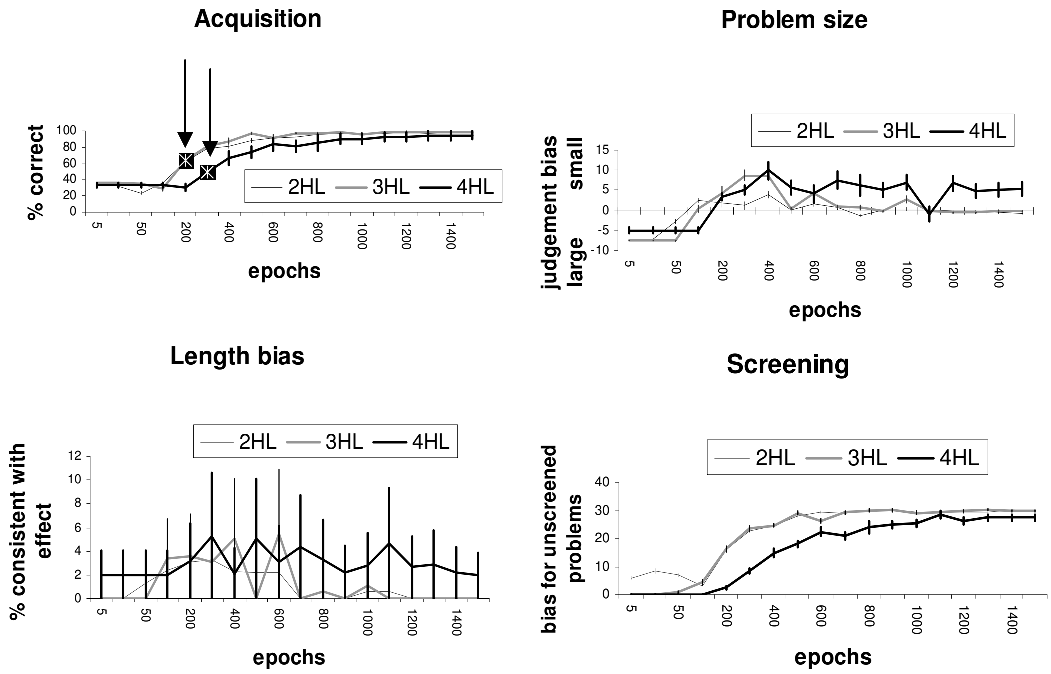

Acquisition
% correct
2HL
3HL
4HL
epochs
Problem size
judgement bias
small
large
epochs
Length bias
% consistent with effect
epochs
Screening
bias for unscreened problems
epochs

Figure 10

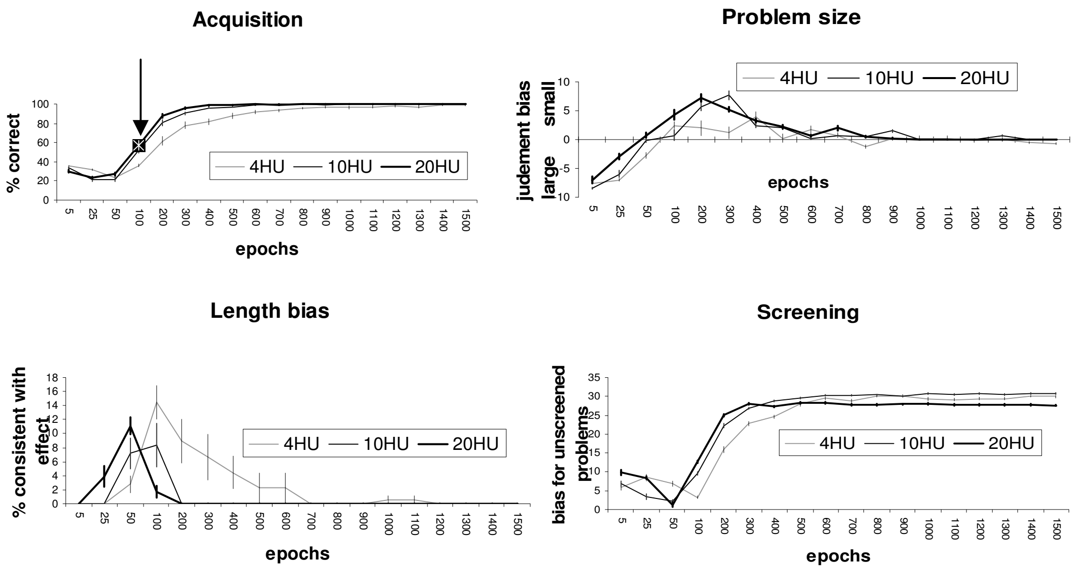
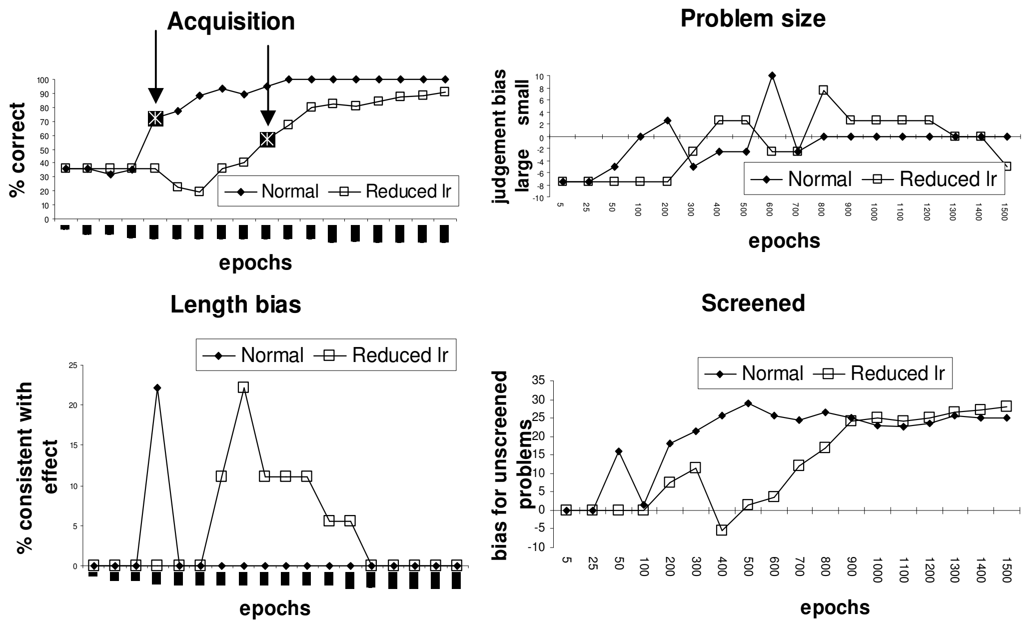

Acquisition
% correct
epochs
4HU 10HU 20HU
Problem size
judement bias
small
large
epochs
4HU 10HU 20HU
Length bias
% consistent with effect
epochs
4HU 10HU 20HU
Screening
bias for unscreened problems
epochs
4HU 10HU 20HU

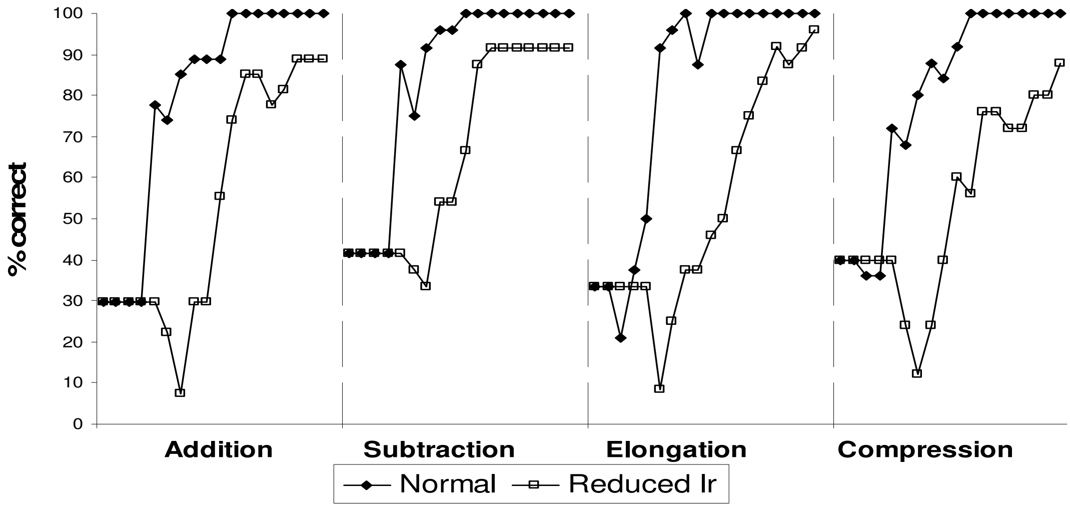
Figure 11



Acquisition
% correct
Normal
Reduced lr
epochs
Problem size
judgement bias
large
small
epochs
Length bias
% consistent with effect
epochs
Screened
bias for unscreened problems
epochs

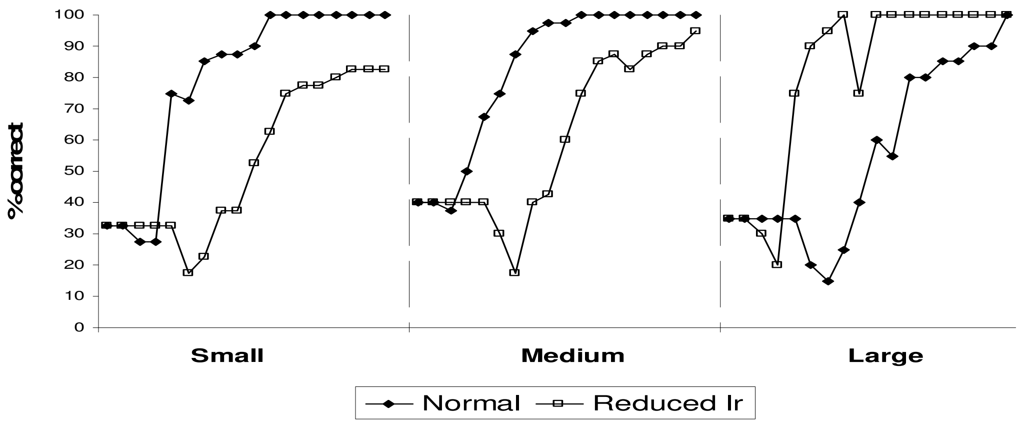
Figure 12

(a)



100
90
80
70
60
50
40
30
20
10
0
% correct
Addition
Subtraction
Elongation
Compression
Normal
Reduced Ir

(b)



100
90
80
70
60
50
40
30
20
10
0
%correct
Small
Medium
Large
Normal
Reduced Ir