# Tobin's $q$ and Intangible Assets

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**Abstract**

In a recent paper Laitner & Stolyarov (2003) assert that measured Tobin's $q$ has usually been well above 1, and use this to back up their conclusion that there are significant quantities of unrecorded intangible assets. This key feature of $q$ turns out however to be entirely due to errors and omissions in the authors' calculations. The corrected $q$ series turns out to be usually well *below* unity.

## 1. Introduction

In a recent paper Laitner & Stolyarov (2003) assert that measured Tobin's $q$ has usually been well above 1, and use this as supporting evidence for their conclusion that there are significant quantities of unrecorded intangible assets. This key feature is referred to repeatedly in the paper to motivate their analysis.[1]

On closer examination this key piece of evidence turns out to be simply incorrect. Laitner and Stolyarov's $q$ estimate is typically above unity solely because of number of clear errors in the way that the authors construct their data. They both overestimate the numerator and underestimate the denominator of $q$. The latter error is most significant: the primary factor being the omission of significant elements of tangible, rather than intangible assets - most notably land. When the calculation is carried out correcting for these errors the resulting $q$ series turns out to be usually well *below* unity.

Even this corrected measure arguably has some major conceptual problems. Laitner & Stolyarov refer to the numerator of $q$ as "stock market value". Yet typically around a half to two thirds of the notional market value series in the numerator is an estimate of the value of unincorporated business for which there is virtually no true market value data. When calculated using market value data for the corporate sector alone, $q$ is typically even further below unity.

In this short note I briefly describe the problems with Laitner & Stolyarov's $q$ data, and illustrate the comparison between the original and corrected series with two charts. I also briefly discuss the puzzle of why $q$ estimates might be so low. An Appendix provides a summary description of data construction and a table with corrected data.[2]

[1] To quote from their abstract; from their opening paragraph; and from Section IV (page 1258, referring to their Figure 1, which shows their $q$ estimate):

*Tobin's average q has usually been well above 1....The stock market value in the numerator of q reflects ownership of physical capital and knowledge, but the denominator measures just physical capital. Therefore q is usually above 1....Figure 1 suggests .. that the stock of applied knowledge (A) is 30-50 percent as large as the physical capital stock (K)...*

[2] A full description of data construction is provided, for the benefit of referees, in Appendix B.

## 2. Comparing $q$ Estimates

### 2.1. Replicating Laitner & Stolyarov

The data used for charts in this paper update Laitner & Stolyarov's original $q$ estimates to include more recent data. In Appendix A, Table A1, I show that the correspondence between the original and recalculated series over their common data sample is extremely close.[3] A key feature of the recalculated $q$ series, as for Laitner & Stolyarov's original series, is that most observations are greater than unity. This feature is unsurprisingly particularly marked in recent years.

### 2.2. Correcting the numerator

The numerator of Laitner & Stolyarov's measure of $q$ is intended to be an estimate of the market value of the entire business sector, made up of both corporate and unincorporated businesses. The authors calculate this value by residual from the net wealth of the personal sector, and the net liabilities of the government, overseas and the monetary authority, taken from the Flow of Funds tables. This apparently simple formula has the following drawbacks:

1. It includes the market value of holdings of equities of overseas corporations by US residents as well as the (empirically trivial) value of gold and SDRs;

2. It includes the value of net overseas direct investment by US corporations, and is therefore not directly comparable with the capital stock data;

3. It imputes all unidentified financial liabilities to the business sector.

The first two of these are clear errors; the third is at best a contentious assumption. An alternative estimate of the numerator that does not have these drawbacks can be constructed relatively straightforwardly (albeit more laboriously) by adding up individual sectoral components using data from the Flow of Funds tables (for details see Appendix).

[3] Although there is an additional caveat that the formula for market value in terms of source data provided by Laitner & Stolyarov (in their Table A1) does not correspond to their own calculation, but itself contains a significant number of errors. See Appendix B for details.

### 2.3. Correcting the denominator

The denominator of $q$ should equal the replacement value of all the physical assets owned by the domestic business sector. The series used by Laitner & Stolyarov is given by the sum of the private nonresidential fixed capital stock and nonfarm inventories. This measure too has very clear-cut drawbacks:

1. It contains some nonresidential fixed assets that should *not* be included, namely those belonging to households and non-profit making institutions.

2. It omits business residential capital (dominated by tenant-occupied housing owned by the noncorporate business sector)

3. It omits the value of land.

The first two defects can be easily rectified using published data.[4] Land presents somewhat more of a problem. The Fed do not publish data on land directly, but instead, for the non-farm, non-financial business sector, replace BEA series for structures with alternative estimates of real estate values: ie, the value of structures plus the land they sit on. Implicit figures for the value of land can be derived as the difference between these two series.[5] For these sectors I use Fed data and thus deal with all three shortcomings simultaneously. For the other business sectors - farms and financial corporations - for which the Fed do not produce real estate data, I have attempted to mimic Fed methodology as closely as possible. However, since the available Fed data cover nearly 90% of total estimated business tangible assets, the results are not sensitive to the inclusion of those missing elements for which they do not provide data.

[4] I again use data from the Federal Reserve Flow of Funds tables where available, but these are derived from, and correspond closely to, the equivalent BEA series.

[5] The Fed used to publish these implied land figures in their balance sheets but not longer do so since they also clearly contain an implicit balancing item between the two different sources.

## 2.4. The impact of the corrections

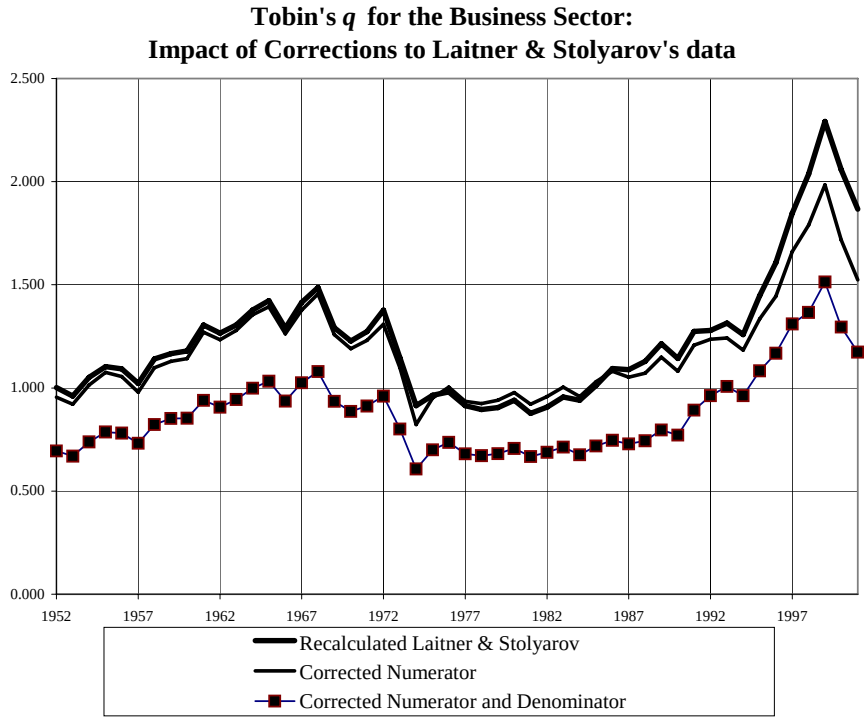


Figure 1

Figure 1 shows the impact of the two sets of corrections alongside the recalculated estimate using Laitner & Stolyarov's approach. The second line shows the impact of amending the formula for the numerator, which has a fairly modest impact except in recent years. The third line shows the much more significant impact of correctly defining the denominator (the inclusion of residential capital and land have a more or less equal impact). The net result of all the corrections is that in 2000, close to the peak of the market, Laitner & Stolyarov's $q$ estimate was around 60% higher than the corrected measure using identified market value.The chart also clearly shows that the corrected $q$ series, far from being predominantly above unity, is predominantly below.[6]

[6]In Appendix B I discuss an alternative treatment of land and residential capital which assumes real estate valuations used by the Fed are directly comparable to stock market valuations. I show that the resulting series is extremely volatile, reflecting the extremely strong assumptions that underlie the calculations. But it too is also typically well below unity.

## 2.5. Sectoral $q$ data

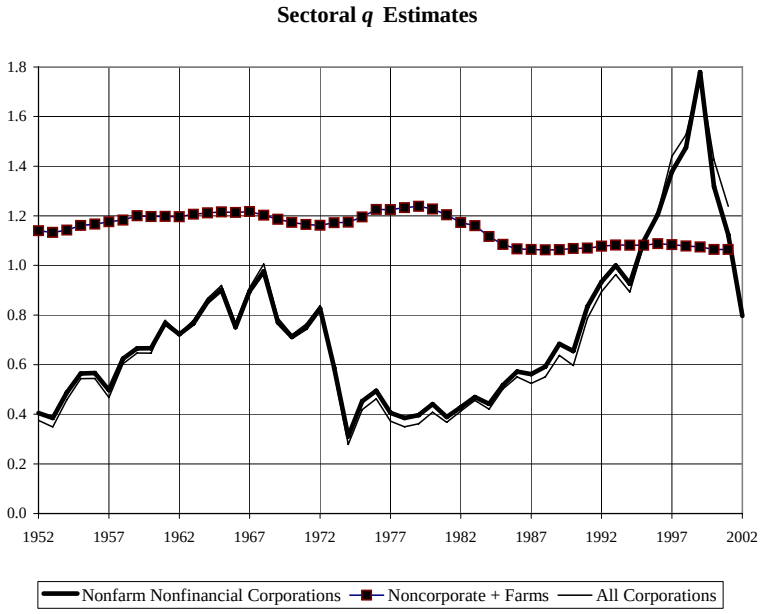


Figure 2

The corrected $q$ series shown in Figure 1 builds up both numerator and denominator from their sectoral components. An advantage of this approach is that it is possible to examine the implied sectoral $q$ estimates that underlie the aggregate figures for the business sector. These, shown in Figure 2, strongly reinforce the picture of $q$ as being predominantly well below unity for those sectors where market valuations are available.

Typically around a half to two thirds of the notional market value of the business sector as a whole is made up of the value of unincorporated business. This series is calculated from net liabilities (of which only a very small fraction are marketed securities) plus the imputed value of equity in unincorporated business. This latter figure (itself the dominant element in total value) is constructed by simply cumulating net investment flows. Unsurprisingly, therefore, the resulting $q$ estimate is very stable (thus dampening fluctuations in the aggregate considerably). It is also systematically somewhat above unity, since the Fed statisticians appear to assume an element of under-recording of net investment in this sector. But this feature tells us precisely nothing about market valuations.

In contrast the market value of the corporate sector is dominated by stock market valuations,[7] and as such corresponds much more closely to the measure of $q$ that Laitner & Stolyarov analyse in their model. Figure 2 shows that this series is distinctly more volatile than the aggregate series in Figure 1.[8] It is also of course distinctly lower: indeed it only rose above unity for the first time at the height of the boom in the 1990s. Thus there is strong evidence that, where market valuations exist, the resulting measure of $q$ is systematically well below, rather than above unity.

## 3. Conclusions

This note has focussed on the empirical basis for the recent claim made by Laitner & Stolyarov (2003) that Tobin's $q$ is typically above unity, which they use as supporting evidence for their conclusion that intangible assets are quantitatively significant.[9] It turns out that, once correctly calculated using available data, Tobin's $q$ has instead typically been well *below* unity. This conclusion is strongly reinforced if $q$ is measured only for sectors where market valuations actually exist.

The fact that, once correctly calculated, measured $q$ is systematically less than unity is a puzzle that deserves attention. It does not, of course, mean that intangible assets do not exist; it just means that their existence or their magnitude cannot be inferred straightforwardly from the properties of $q$. It also means that if they do exist in significant quantities either statisticians or markets must be getting something systematically wrong. Indeed this conclusion is inescapable, whatever the value of intangible assets, since even on the basis of tangible assets alone measured $q$ is systematically below unity.

Two possible problems with the statisticians' approach might be that the

[7] These include Fed estimates of imputed values of unquoted securities.

[8] The chart also shows that movements in $q$ for the corporate sector as a whole are dominated by those of $q$ for the non-farm non-financial corporate sector, which can be derived entirely from Fed data. The implied $q$ estimate for financial corporations, which relies in part on my own estimates (albeit following Fed methodology wherever possible, see Appendix B) is distinctly more volatile but is also typically below unity.

[9] While Laitner and Stolyarov base much of their heuristic arguments on the properties of $q$, their quantitative estimate of the share of intangibles is derived from method of moments estimates of the parameters of a growth model with periodic "revolutions" in technology. These are not directly derived from their $q$ estimates; but in Appendix C (provided for referees) I show that there is a very clear link between the two approaches, both of which are affected by their overstatement of $q$.

BEA's capital stock data may be based on overstated assumed capital lives, or that investment deflators may be incorrectly calculated (as proposed, for example by Gordon, 1990). The alternative explanation, that markets may have systematically under-valued corporate assets over a long period, is probably more worrying for those who believe that market valuations are at least on average rational; but should not be entirely ruled out purely on these grounds. The puzzle of why $q$ is usually less than unity certainly merits further investigation.

A final, and important, if rather uncomfortable conclusion for applied macroeconomists is that Laitner & Stolyarov's errors provide something of a cautionary tale. Data construction, while undeniably tedious, is not something that can be hurried or glossed over. The simple and quick calculations they carried out provided interesting results that appeared to provide significant support for their modelling approach. But these results were incorrect, and, with due care, avoidably so. Since their results were published in a major journal, the key feature they mistakenly ascribed to $q$ is likely to acquire the status of a "stylised fact", if not corrected. That is the purpose of this short note. [10]

[10] My critique of Laitner & Stolaryov's errors and omissions in data construction does not of itself detract from the many other interesting features of their paper, especially those related to the dynamics of Tobin's $q$.

# APPENDIX

## A. Summary of Data Construction

Table A1 provides a summary of the corrections to data in Laitner & Stolyarov (hereafter LS). For full (and very tedious) details the interested reader is referred to Appendix B. A spreadsheet containing all series and their underlying components, the sheets of which correspond to the panels of Appendix B Table A2 is also available on request. All source references below are to Flow of Funds (Z1) tables (Federal Reserve, 2004), except those to TA, which refer to the BEA tangible assets tables, and NIPA, which refer to National Income and Product Accounts tables.

**Column 1, "Business Fixed Capital and Inventories":** This series is taken directly from LS Table A1, and is as defined therein as TA 4.1 R1 + NIPA 5.75 R1. LS refer to this series as "business fixed capital and inventories" but it is more precisely defined as non-residential fixed capital and inventories.

**Column 2, "Market Value of Businesses":** This series is also taken directly from LS Table A1, but the definition given therein contains a significant number of errors and if applied to more recent data results in a series which is radically different. However, the authors have confirmed in email correspondence that the actual data were in fact defined by residual from the net worth of the household sector and non-profit making institutions, less the net liabilities of the government, monetary authorities and overseas. The corrected formula is: L100 R1 -L100 R24 +L105 R1+L105 R17 + L106 R1- L106 R13+ L107 R1 - L107 R23+L108 R1 + L1O8 R15

**Column 3** gives Laitner & Stolyarov's reported $q$ series (ie, Column 1/Column2) from their Table A1, while **Column 4** gives the recalculated series on Laitner & Stolyarov's definition, but using the corrected formula for market value.

**Column 5, Business Tangible Assets:** This series is built up from two elements (for the non-farm, non-financial corporate and non-corporate sectors) that can be taken direct from the flow of funds tables (B102 R2 + B102 R2) plus estimates of equivalent series for financial corporations and farms, the construction of which is summarised in Appendix B Table A2, Panels M and N. The key element in my methodology for these two sectors is to use BEA series to build up reproducible capital, and then to add an estimate of the value of land. I calculate this by assuming that for financial corporations the ratio of real estate to structures is that for nonfarm nonfinancial corporations; for farms I apply the

same ratio as for noncorporate business.[11]

**Column 6, Reproducible Capital:** This series is derived by replacing figures for real estate (ie structures plus land) with those for structures alone. As such it uses only published data. Nonfarm, nonfinancial business structures data are taken from the flow of funds tables but are consistent with BEA data. The precise definition is Column 4 -(B102 R3 + B103 R3 + financial corporations' real estate + farm real estate) +(B102 R 32+ B102 R33+ B103 L32+B103 R33 +TA 4.1 R27 + TA4.1 R6 + TA 5.1 L18). Financial corporations' and farm real estate are defined as in Appendix B Table A2 Panels M and N. In Panel G of the same table I also show that a virtually identical series can be built up directly from BEA tangible assets data.

**Column 7, Market Value of US Business**: In contrast to LS, who construct business market value by residual, I build up this series from its individual components, all of which can be derived from the flow of funds tables. For each sector market value is defined as the sum of net identified liabilities plus the market value of equities. Letting $M$ = market value, Column 6 can be defined as:

$$M_{\text{Domestic Business}} = M_{\text{Nonfinancial}} + M_{\text{Financial}} - \text{Net ODI}$$

where $M_{\text{Nonfinancial}}$ =Nonfinancial net liabilities (L101 R16-L101 R1)+ equities (L5 R23 + L102 R41); $M_{\text{Financial}}$ =Financial corporations' net liabilities[12] (L5 R33-L5 R20+ L100 R1 -L100 R24 +L105 R1+L105 R17 + L106 R1- L106 R13+ L107 R1 - L107 R23+L108 R1 + L1O8 R15-L101 R16+L101 R1) + equities ( L213 R4); and Net ODI = net overseas direct investment (L230 R1-L230 R16). In Appendix B, Table A2 Panel D, I show that Column 2 (corrected definition)= Column 6 plus unidentified net liabilities (L5 R33-L5 R20-L5 R21-L5 R22-L5 R23) plus overseas corporate equities (L213 R3) + all sectors' gold and SDRs (L5 R21), as described in Section 2.2.

**Column 8** gives the corrected series for Tobin's $q$ (as shown in Figure 1), defined by Column 6 / Column 4

[11] I also discuss an alternative method of imputation of real estate values derived from mortgage data, but I show that this has minimal impact on the aggregate figure.

[12] This series is measured, for convenience, by residual from total identified stocks and the equivalent stocks of other sectors. As such it simply inverts the identities used to derive the aggregates, and does not impute any residual items to the financial sector. Note that data for identified financial assets are corrected for an error (confirmed by the Fed) in the published version of Table L5 (Jan 2004 release).

## References


[1] Federal Reserve (2000) *Guide to the Flow of Funds Accounts,* Board of Governors of the Federal Reserve System, Washington DC

[2] Federal Reserve (2004) *Flow of Funds for the United States* January 2004, Board of Governors of the Federal Reserve System, Washington DC

[3] Gordon, R J (1990) "The measurement of durable goods prices" National Bureau of Economic Research Monograph series Chicago and London: University of Chicago Press.

[4] Laitner, J and Stolyarov, D (2003), "Technological Change and the Stock Market", *American Economic Review,* vol. 93, no. 4 1240-67

[5] Wright, S (2004) "Measures of Stock Market Value and Returns for the US Nonfinancial Corporate Sector, 1900-2002" *Review of Income and Wealth*, December 2004

**NOTE TO REFEREES**

**Appendices B and C are provided solely for the benefit of referees; they could also in principle be made generally available in electronic form for any readers interested in such a level of detail, together with the associated dataset.**

## B. Full Details of Data Construction

Table A2 sets out the sources and data construction methods, which allows a direct comparison of the $q$ estimates used in this paper with those in Laitner & Stolyarov (2003) (hereafter LS). The panels of Table A2 correspond to equivalent sections of a spreadsheet that is available on request. All source references therein are to Z1 (flow of funds) tables (Federal Reserve, 2004), except those to TA, which refer to the BEA tangible assets tables, and NIPA, which refer to National Income and Product Accounts tables.

### B.1. Market Value

#### B.1.1. Replicating Laitner & Stolyarov

Panel A of Table A2 reproduces verbatim the formula for market value using the Flow of Funds table references as given in LS (P1261, footnote to Table A2). However, this formula results in a series which is radically different from their own reported series for market value (in Appendix A, col. 2), with the series constructed using their reported formula exceeding their reported series by anything up to 50%.

The authors have however confirmed, in email correspondence, that their reported formula, as published, contains a number of errors, and that their market value series was actually calculated as in Panel B. This uses an alternative, simpler, formula, deriving the value of the business sector by residual from the net worth of the household sector and non-profit making institutions, less the net liabilities of the government, monetary authorities and overseas. Table A1 shows that this results in a series very much closer to that reported in LS in recent years (differing by only fractions of a percent for the past decade).

### B.1.2. Market Value from Identified Components

Panel C provides definitions underlying my alternative market value series that corrects the problems with the LS approach as set out in Section 2.2. It builds up the series by adding up identified components of market value using flow of funds data on equities, assets and liabilities.[13] These sum to a series which measures the identified value of US-owned business. To ensure complete comparability with tangible assets data, the data are then adjusted to provide the market value of domestic US business (ie, including the value of foreign companies' operations in the US, but excluding the value of US companies' operations abroad). Although this last adjustment is required in logic, it turns out to make (surprisingly) little difference to market value estimates.

Panel D shows how this approach can be reconciled with that of Laitner & Stolyarov. Their series does not correct for net ODI, and includes both US holdings of overseas equities and the very small category of other assets (gold and SDRs) not included in liabilities data. Their estimate also includes unidentified liabilities: the gap between total recorded assets and the sum of recorded liabilities and all assets not included in liabilities. This unidentified element in debt may indeed reflect unrecorded liabilities of US business, and thus it is of interest to see its impact; but anyone using this approach should be aware of its inclusion. The table shows that on average inclusion of unidentified debt slightly lowers measured $q$ (since for most of the sample unidentified debt was negative); however, this element switched sign quite dramatically in the 1990s, and by end-2002, unidentified debt was equivalent to 14% of identified business market value.

## B.2. Reproducible Capital at Replacement Cost

Panel F of Table A2 reproduces the formula for business fixed capital plus inventories used in LS: the resulting series is virtually identical.

Panel G provides an alternative formula based on BEA data. There are two key modifications. First, that part of nonresidential fixed capital belonging to the personal sector and non-profit making institutions is excluded, for consistency with the derivation of market value data. Second, a series is constructed for the residential capital stock of the business sector. Following the same methodology

[13] The financial sector's recorded assets and liabilities are measured, for convenience, by residual from total identified stocks and the equivalent stocks of other sectors. As such it simply inverts the identities used to derive the aggregates, and does not impute any residual items to the financial sector.

as outlined by the Federal Reserve in constructing their tangible assets series (Federal Reserve, 2000, p299) for the nonfarm, noncorporate business sector, this subtracts the residential capital stock of non-profit making institutions, and of owner-occupied housing, from the private sector total (unlike the Fed approach tenant-occupied farm housing is retained since the definition here includes all farms). Total private inventories are then added to the figure for fixed capital.

The resulting series for total business reproducible capital is systematically larger than the series used by LS, differing by a factor of around 20% in the early part of the sample, with the difference falling steadily to around 10% by the end of the sample. It is extremely close to the equivalent figure derived on a Z1-equivalent basis, as described in the next section, never differing by more than around 1%.

### B.3. Tangible Assets Including Land

Panel H of Table A2 summarises the construction of a series for business tangible assets including land. The series is built up from two elements (for the non-farm, non-financial corporate and non-corporate sectors) that can be taken direct from the flow of funds tables, plus equivalent series for financial corporations and farms, the construction of which (involving some guesswork) is summarised in Panels M and N.

Figure A1 shows that the implied land series for the two sectors for which the Fed produces tangible assets data both show a distinct discontinuity after 1989, following a change in Fed methodology.[14] The implied share of land for nonfinancial corporates falls virtually to zero in the mid-1990s, but then recovers somewhat. The implied share of land for non-corporates shows a similar fall, but then recovers distinctly more strongly, to a much higher level (reflecting the much more significant proportion of residential capital for the non-corporate sector).

[14]See Wright (2004) for a discussion and a proposed alternative treatment that deals with land more consistently.

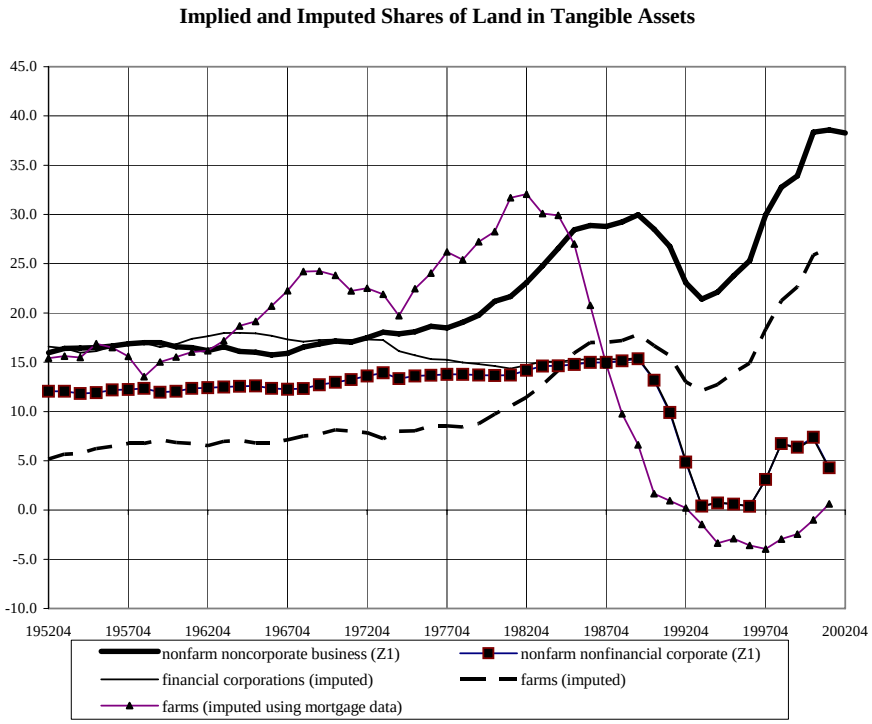


Figure A1

Panel M details the (fairly straightforward) construction of estimates of tangible assets for financial corporations. Data for structures and equipment and software come straight from the BEA nonresidential tangible assets series (Table 4.1). Both inventories and residential capital are set to zero, since these are already fully allocated to nonfinancial corporations in the flow of funds tables. Real estate figures are estimated by scaling BEA structures data using the ratio of real estate to structures for nonfinancial corporations. As Figure **??** shows, the resulting imputed share of land in tangible asesets is very similar to that for non-financial corporations (the somewhat higher figure in earlier years reflecting the lesser importance of equipment and software of financial corporations compared to non-financials).

Panel N details the equivalent calculation for farms (both corporate and non-corporate). BEA tangible assets figures and inventory are available for all categories, except for the very small category of residential plant and equipment (durable goods in rented properties) which is imputed assuming the same ratio as for non-farm non-corporates.. The implied real estate figure is calculated on

the basis of BEA structures figures on the same basis. Figure **??** shows that the resulting figure for land appears distinctly conservative, since it is well below the share of land for non-farm non-corporates, despite the clearly land-intensive nature of farming.

As an alternative method of calculation, to provide at least a basis for comparison, farm real estate values can inferred on the basis of farm and household sector mortgages (lines 5 and 2 of Z1 Table L217, respectively), by working on the assumption that collateral ratios applied by lending institutions are the same, using the ratio of home mortgages to household real estate (Table L100, line 4). Land values are then inferred as the difference between the resulting real estate values and BEA data on farm structures. Figure **??** shows that this results in distinctly higher implied land values in the early part of the sample, but that the implied share of land then falls steadily, such that, by the end of the sample, it is slightly negative - which would imply that the BEA's estimate of the replacement value of farm structures is greater than their market value including the land they sit upon.[15] However, Figure **??** shows that even this very extreme, and distinctly pessimistic implication for farms has very limited implications for the resulting $q$ estimate for the non-corporate plus farm sector in aggregate.

### B.4. An alternative measure of $q$

As noted in Section 2, it is possible to derive an alternative measure of $q$ with a different treatment of land and both residential and nonresidential structures, if real estate valuations used by the Fed can be assumed to be directly comparable to stock market valuations. If the additional assumption is made that $q$ is the same for all elements of capital, this implies the alternative definition:

$$q^* = \frac{\text{total market value - value of real estate}}{\text{total reproducible capital - structures}} \tag{B.1}$$

with an associated estimate of land:

$$\text{land} = \text{value of real estate } - q.\ \text{structures} \tag{B.2}$$

The rationale underlying this alternative estimate is that both elements of the numerator in (B.1) are mutually consistent market valuations. Figure A2 shows the resulting time series for $q^*$.

[15] This somewhat surprising result does not appear to be due to a fall in the importance of farm mortgages as a form of borrowing - on the contrary, the share of mortgages in total farm liabililities was if anything on a somewhat upward trend through the data sample.

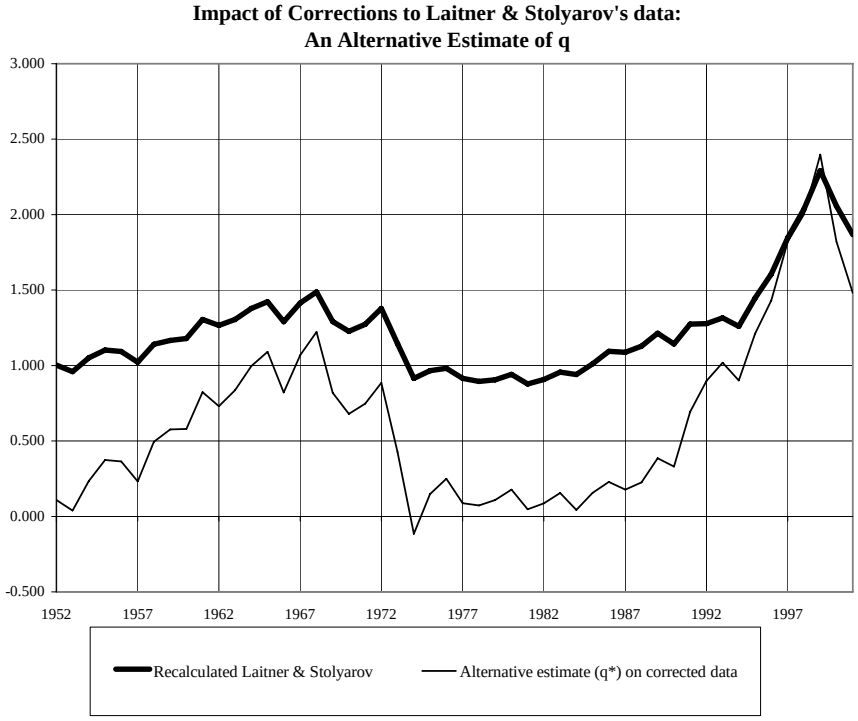


Figure A2

Figure A2 shows that the resulting series is radically more volatile, and includes some extremely low observations (one is negative). The explanation is quite straightforward: the adjusted definition subtracts from both numerator and denominator the largest single component of corporate capital. As the ratio of two generally small numbers, each of which is the difference between two large numbers, the resulting series is extremely volatile.

## C. Method of Moments Estimates of the Share of Intangibles

Laitner & Stolyarov's empirical results in Sections III and IV do not depend directly on recorded capital stock data - a feature of their results they claim is an advantage (see their footnote 22). Their results do however depend on recorded capital consumption data that are calculated using the same methodology as recorded capital stock data. Given this, it is quite easy to show that there is a strong link between their empirical results and the failings of their $q$ data.

While the authors' method of moments estimation procedure estimates six parameters simultaneously, applying cross-equation restrictions across the six equations of their model, there is one key empirical relationship that determines $\theta$, the steady-state share of the intangible capital stock in total capital. In their Cobb-Douglas framework, and working in terms of steady state ratios for simplicity, this is given by

$$\theta = \frac{\alpha}{\alpha + \beta} \equiv \frac{A}{A + K^*} \tag{C.1}$$

where $A$ is intangible capital, $K^*$ is the measured capital stock, and $\alpha$ and $\beta$ are their respective exponents in the production function. Substituting from the authors' equation (19) into (22) implies that in steady state

$$\frac{I^K}{M} = (1 - \theta)\left[g_{\widetilde{M}} + \delta\right] \tag{C.2}$$

where (using their definitions), $I^K$ is gross recorded investment, $M$ is market value, $g_{\widetilde{M}} = \frac{M_t - \widetilde{M}_{t-1}}{\widetilde{M}_{t-1}}$ is a "revolution-adjusted" measure of growth of market value, that strips out the one-off effect of the single technological innovation;[16] and $\delta$ is the true depreciation factor. The authors' estimate of $\theta$ is greater than zero because recorded investment is "too low" in relation to market value to be consistent with growth rates and depreciation.

The authors do not include the measured capital stock $K^*$ in their empirical model directly, but do include it indirectly, since they have an equation for recorded depreciation, given in their model by $D = \overline{\delta} K^*$ where $\overline{\delta} > \delta$ is the average depreciation rate. (In their framework this is higher than true depreciation due to the impact of periodic technological innovations). Using this definition, the evolution of measured tangible capital (excluding land) in steady state implies

$$\frac{I^K}{K^*} = g_{K^*} + \overline{\delta}. \tag{C.3}$$

Combining (C.2) and (C.3) implies

$$1 - \theta = \frac{1}{q^{LS}}\left(\frac{g_{K^*} + \overline{\delta}}{g_{\widetilde{M}} + \delta}\right) \tag{C.4}$$

where $q^{LS} = M/K^*$ is measured Tobin's $q$ on the authors' definition.

[16] $g_{\widetilde{M}} > g_M \equiv \frac{M_t - M_{t-1}}{M_{t-1}}$, actual market value growth.

The second ratio on the right-hand side of (C.4) is very close to unity;[17] indeed it should be precisely unity if $M$ and $K^*$ are not to drift apart indefinitely. Thus, on a single equation basis, the fact that their estimate of $\theta$ greater than zero relates directly to $q^{LS}$ being above unity on average (in practice cross-equation restrictions complicate matters somewhat).

Give the shortcomings of the authors' $q$ data as described in Section 2, one obvious explanation of their estimate of $\theta$ being greater than zero is that it reflects, wholly or in part, omitted elements of tangible, rather than intangible assets, most obviously the exclusion of land. Using Laitner & Stolyarov's own results, it is possible to derive a simple test of the hypothesis that land is the sole explanation.

With three forms of capital, the steady state value of $\theta$ will be given by

$$\theta = \frac{A + L}{A + L + K^*} \tag{C.5}$$

where $A$ is intangible capital and $L$ is land. Deriving implicit land values as described in Section 2.3, we can derive an estimate of $L/\left(L + K^*\right)$, the steady-state share of land in total tangible assets, which has a sample average of 14%. The hypothesis that $A = 0$ thus implies $\theta = .14$, hence, using (C.1) we have $H_0 : \alpha = \left(\frac{0.14}{0.86}\right)\beta \Rightarrow \alpha - .167\beta = 0$, a simple linear restriction. Laitner & Stolyarove have kindly provided an amended set of empirical results (and the *Fortran* program that generates these) in which investment and depreciation figures are adjusted from the original series to include investment in residential capital as described above, but land figures are excluded. Using their estimated parameter covariance matrix, the standard error of the linear combination $\widehat{\alpha} - .167\widehat{\beta}$, results in a t-statistic of just 1.84 for the test of the implicit null that $A = 0$. This seems a very slender statistical thread on which to hang an assertion that there is a significant quantity of unmeasured intangible capital, especially given the other problems with both market value and land data for the business sector discussed above.[18]

[17] My best guess at a point estimate, using the authors' empirical estimates and appropriately adjusted growth rates, is 1.004. In Laitner & Stolyarov's framework $M$ typically grows more rapidly than $K^*$, and depreciates more slowly; but is lowered periodically by structural shocks when "revolutions" happen; the higher depreciation rate for $K^*$ reflects the average effect of these shocks, since the level of $K^*$ is never hit by these negative shocks.

[18] Especially the weaknesses of the land data, and the dominant role of the noncorporate sector as described in Section 2, as well as the impact of cross-equation restrictions.

## Table A2 Detailed Data Definitions for Comparison with Laitner & Stolyarov (2003)

| **A.Market Value as defined in Laitner & Stolyarov** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | less | Less | less | plus | plus | Plus | minus | Minus | plus | minus | Equals |
| Source: | L100, R1 | L100, R25 | L106, R15 | L105, R18 | L105,R7 | L105,R10 | L108, R10 | L106, R14 | L108, R15 | L107, R1 | L107, R23 | |
| | Households and non - profit | Households and non - profit | Fed government | State & local government | State & local government | State & local government | Monetary Authority (excl FRB) | Fed government | Monetary Authority (excl FRB) | Rest of World | Rest of World | Total Business, L&S definition |
| | Financial Assets | total liabilities | Treasury currency (liabilities) | Credit market liabilities | US gov't securities (assets) | municipal securities (assets) | US gov't securities (assets) | SDRs (liabs) | Total liabilities | total financial assets | total liabilities | **Implied market value** |

| **B. Alternative calculation of Market Value as in Laitner & Stolyarov** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Less | plus | less | plus | Less | plus | Minus | Plus | plus | Equals |
| L100, R1 | L100, R25 | L105, R1 | L105, R17 | L106, R1 | L106, R13 | L107, R1 | L107, R23 | L108, R1 | L108, R15 | |
| Households and non - profit | Households and non - profit | State & local government | State & local government | Fed government | Fed government | Rest of World | Rest of World | Monetary Authority (excl FRB) | Monetary Authority (excl FRB) | Total business, L&S definition |
| total financial assets | total liabilities | total financial assets | total liabilities | total financial assets | total liabilities | total financial assets | Total liabilities | Total financial assets | total liabilities | **Implied market value** |

| **C. Market Value from Identified Components** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| minus | plus | plus | plus | equals | Plus | minus | plus | Equals | minus | Plus | equals |
| L101,R1 | L101,R16 | L5,R23 | L102, R41 | | L213,R4 | Panels B, C and D | Panels B, C and D | | L230, R1 | L230, R16 | |
| non-financial business | non-financial business | non-corporate business | non-financial corporations | non-financial business | financial corporations | financial corporations | financial corporations | Total business | total business | total business | total business |
| total financial assets | total liabilities | household equity in noncorp business | value of equities (including farm equities) | market value of US nonfinancial business | value of equities | total financial assets | total liabilities | Identified market value of US business | Stock of US direct investment abroad | Stock of foreign direct investment in US | Identified Market Value of Domestic Business Capital |

| **D: Reconciliation of Identified Market Value with Laitner & Stolyarov Market Value** | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Minus | minus | Equals | plus | plus | Plus | equals |
| L5 row 33 | L5, R20 | L5, R21 to R23 | | Panel C | L213, R3 | L5, R21 | Panel B |
| | all sectors | all sectors | | | rest of world | all sectors | total business |
| Total Identified Assets | Total Liabilities | Assets not included in liabilities (ie equities, gold and sdrs) | Unidentified Liabilities | identified market value of US business | Corporate Equities | gold and SDRs (other assets not included in liabilities) | total market value as in L&S |

| **F. Capital from BEA nonresidential data (as in Laitner & Stolyarov)** | | | |
|---|---|---|---|
| | Plus | equals | Cf |
| TA 4.1, R1 | NIPA 5.75A, R1 | | Laitner & Stolyarov |
| Private | Private | private | |
| non-residential fixed assets | Inventories | **non-residential fixed capital and inventories** | "business fixed capital and inventories" |

| **G. Alternative BEA capital measure data excluding non-business non-residential capital but including residential capital** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Less | less | equals | Plus | Less | Less | | equals | Plus | equals |
| TA 4.1, r1 | TA 4.1, R46 | TA 4.1, R49 | | TA 5.1, R2 | TA 5.1, R6 | TA 5.1, R 14 | | | NIPA 5.75A&B L1 | |
| private | non-profit institutions | persons | business | Private | non-profit institutions | owner-occupied | | business | Private | business |
| non-residential fixed assets | non-residential fixed assets | non-residential fixed assets | non-residential fixed assets | residential fixed assets | residential fixed assets | residential fixed assets | | total fixed assets | Inventories (end-year) | **total reproducible capital** |

| **H. Tangible Assets from flow of funds, BEA data and proxies for land** | | | | | |
|---|---|---|---|---|---|
| | Plus | Equals | Plus | plus | equals |
| b102, r2 | b103, r2 | | Panel M | Panel N | |
| nonfarm non-financial corporate | nonfarm non-corporate | nonfarm non-financial business | financial corporations | farms | business |
| tangible assets (inc land) | tangible assets (inc land) | tangible assets (inc land) | tangible assets (inc land) | tangible assets (inc land) | **Tangible assets (inc land)** |

| **M: Actual and imputed tangible assets of financial corporations** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | equals | Of which: | | | Plus | Plus | Plus | |
| | | | TA 4.1 L 27 | | | TA 4.1 L 26 | | |
| Tangible assets | real estate (using ratio to structures from nonfinancials), o/w | Structures, of which | Non-residential structures, direct from BEA | Residential structures (set to zero, since Z1 imputes all to nonfinancials) | Land (residual) | nonresidential equipment and software, direct from BEA | Inventories (set to zero because Z1 imputes all to nonfinancials) | |

| **N: Actual and imputed tangible assets of farms (corporate and noncorporate) using BEA tangible assets** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Equals | of which: | | | plus | plus | plus | plus |
| | Imputed | | TA 4.1 L6 | TA 5.1, L18 | | TA 4.1 L5 | imputed | NIPA 5.75A L2 |
| Tangible assets | real estate (using ratio to structures from nonfarm noncorporates) | Structures, of which | Non-residential structures, direct from BEA | residential structures (only tenant-occupied) | Land (residual) | nonresidential equipment and software, direct from BEA | residential equipment and software (using ratio for nonfarm noncorporates) | Inventories |