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Competitive Externalities in Dynamic Monopolies with Stochastic Demand

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Abstract

This paper analyzes equilibria in sequential take-it-or-leave-it sales when demand is stochastic. It is shown that equilibria in this sales mechanism, unlike in sequential auctions, trade-off allocative efficiency and competing buyers' opportunities to acquire an item to be sold, permitting prices and expected revenue above those of one-shot offers. Hence Coase-type conjectures are invalid in this setting.

KEYWORDS: dynamic monopoly, stochastic demand, Coase conjecture

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1 Introduction

This paper builds on three basic observations: (i) Many economic decisions on markets, such as sellers' pricing and buyers' purchase decisions, are dynamic; (ii) economic agents' preferences and valuations are private information, and hence market demand is stochastic; (iii) in many markets, the number of potential buyers is small. The implications of these conditions are analyzed for a situation in which a monopolistic seller of one item faces a group of n potential buyers with unit demands, whose valuations for the item are private information. There are two periods. If the item does not sell in period 1, it is common knowledge that it may be offered again in the final period 2. The paper considers take-it-or-leave-it sales. Potential buyers compete with each other as it is assumed that the item is randomly assigned among buyers willing to buy at a given price. The paper examines whether the dynamic nature of the transaction game and the intrinsic market uncertainty favor one side of the market, and if so, under what circumstances.

Many markets exhibit the characteristics (i)-(iii), employing take-it-or-leave-it sales mechanisms, and sometimes auctions, with a possibility of re-offering the item, should it not be sold in the first offering. The most prominent online trading platform, eBay, next to its traditional auction sales now offers sellers the option of posted-price sales, under the"Buy It Now" label, and meanwhile earns a quarter of its revenue from it.[1]

The analysis presented in this paper shows that the dynamic nature of potentially sequential offers induces strategic behavior in small markets with stochastic demand which has surprising consequences: It may allow the take-it-or-leave-it seller to benefit from a second period, contrary to Coase-type conjectures; see Coase (1972), as well as formalizations by Bulow (1982), Stokey (1981), Gul, Sonnenschein and Wilson (1986), Ausubel and Deneckere (1989), and Thépot (1998), all under the assumptions of deterministic demand and complete information. Various studies elucidate conditions, which differ from the set-up considered in this paper, under which the conjecture fails: Bagnoli et al. (1989, 1995) show that substituting discrete demands for the assumption of a continuum of buyers reverses Coase's conclusion; Ausubel and Deneckere (1992) demonstrate a no-trade equilibrium if the seller's marginal cost is private information; McAfee and Wiseman (2004) show in a deterministic environment that, if the seller can choose production capacity in every period and buyers get served in order of their valuations, then even small capacity costs

[1]BusinessWeek online, 27 August 2004. Traditional auction houses also offer private sales, such as, for instance, Sotheby's "salon privé" for jewellery.

permit the seller to act as if she could commit to capacity at the outset, inducing non-zero profits. This paper differs from these critiques in three essential ways. First, demand is stochastic, as a consequence of incomplete information about potential buyers' valuations. Second, capacity is understood as the costless and fixed supply of a single item at the outset, which cannot be reproduced in subsequent periods. And, third, there is no assumption that buyers get served in order of their valuations, i.e. that any allocation is necessarily efficient. Instead, it is assumed that the item is assigned randomly among potential those buyers whose equilibrium strategies induce them to accept a given equilibrium price. Under these assumptions, buyers face the risk of not being served if the final allocation is inefficient; with positive probability, the item is not assigned to the highest valuation buyer. In equilibrium under the take-it-or-leave-it mechanism, the increased risk of not obtaining the item in the second period, when the equilibrium price is lower, induces high valuation buyers to accept a higher price in the first period. Competition among buyers with stochastic demand, thus, alters buyers' strategic incentives compared to Coase-type analyses with deterministic demand. This creates a competitive externality in demand which the seller can exploit in take-it-or-leave-it sales. Depending on the degree to which second period pay-offs are discounted and how many potential buyers compete, this competitive externality allows the take-it-or-leave-it seller to achieve higher expected revenue in the dynamic (two-period) than the static (one-period) game.

The paper proceeds as follows. Section two presents the model and the main results. Section 3 briefly compares sequential take-it-or-leave-it mechanisms to sequential auctions, which often serve as alternative mechanisms, as the eBay and Sotheby examples illustrate.[2] Section 4 concludes. All proofs are in an appendix.

2 Sequential Take-it-or-leave-it Sales

The analysis maintains the following assumptions:

Assumption A1: Buyers' valuations X_i are independently and identically distributed with CDF $F(x)$, $i = 1, \cdots, n$.

Assumption A2: $F(x)$ is twice continuously differentiable, with pdf $f(x)$ which is positive on the support $\mathcal{X}$ of F, which is assumed to be a compact set; w.l.o.g., $\mathcal{X} = [0, 1]$.

[2]Bulow and Klemperer (1996) also compare auctions and negotiations, but assume an exogenously, continuously delcining prices and efficient allocations. The broader question of endogenous market regimes is taken up by Haris and Raviv (1981a).

Assumption A3: Marginal revenue $1-F(x)-xf(x)$ is downward sloping.[3]

Assumption A4: The item has zero value for the seller if it is not sold; the seller maximizes expected revenue, and the buyers maximize expected surplus, and buyers and the seller are risk-neutral. If several buyers submit purchase orders at any given price, the item is randomly assigned to any one of them. It is common knowledge that the item will be re-offered in period two if it is not sold in period one. Buyers and the seller have common discount factor $\delta \in [0,1]$.

Since buyers' valuations in the sequential game considered in this paper are private information, the game is a dynamic game with incomplete information. The appropriate equilibrium concept is therefore a Perfect Bayesian Equilibrium (PBE). In the two period case, it is defined as first and second period prices and buyers' strategies to submit purchase orders such that, given prices, buyers' strategies maximize their expected surplus, and given buyers' strategies, the period prices maximize the seller's expected profit.

In the two period game with $\delta > 0$, some buyers will find it optimal to wait until the second period, even though their valuation exceeds the first period price. This is so because in equilibrium the second period price will not be higher than the first period price. The equilibrium valuation of marginal buyers just indifferent between buying at either price, denoted by $y^\star = y(\delta, n)$, can be interpreted as a measure of how strategic buyers behave: If it were the case that $y^\star = 0$, buyers would submit purchase orders according to their true valuations, i.e. there would be no strategic delays. If, on the other hand, it were the case that $y^\star = 1$, all buyers would out-wait the seller; this PBE, if it exists, would be reminiscent of the so-called Coase conjecture. The following results shows that, with stochastic demand, neither one of these two polar cases is a PBE.

Theorem 1: *Consider a two period game with consecutive take-it-or-leave-it sales. Under assumptions A1-A4, there exists a unique PBE with first and second period prices $p_1(y^\star) \geq p_2(y^\star)$, which are monotonic in $y^\star$, and where $y^\star$ is in the interior of $\mathcal{X}$.*

The result is proven in the appendix and has the following corollary.

Corollary 1: *Under assumptions A1-A4, (i) there exists $\check{\delta} \in (0,1)$ such that, for all $\delta \in [\check{\delta}, 1]$, a take-it-or-leave-it seller achieves higher expected revenue in a two period game than in a one period game, and (ii) the equilibrium first period price $p_1(y^\star) > p^\star = \arg\max_p p(1-F(p)^n)$, where $p^\star$ is the optimal*

[3] In auction theory, assumption A3 is sometimes referred to as regularity in the sense of Myerson (1981). See, e.g., Klemperer (2000)

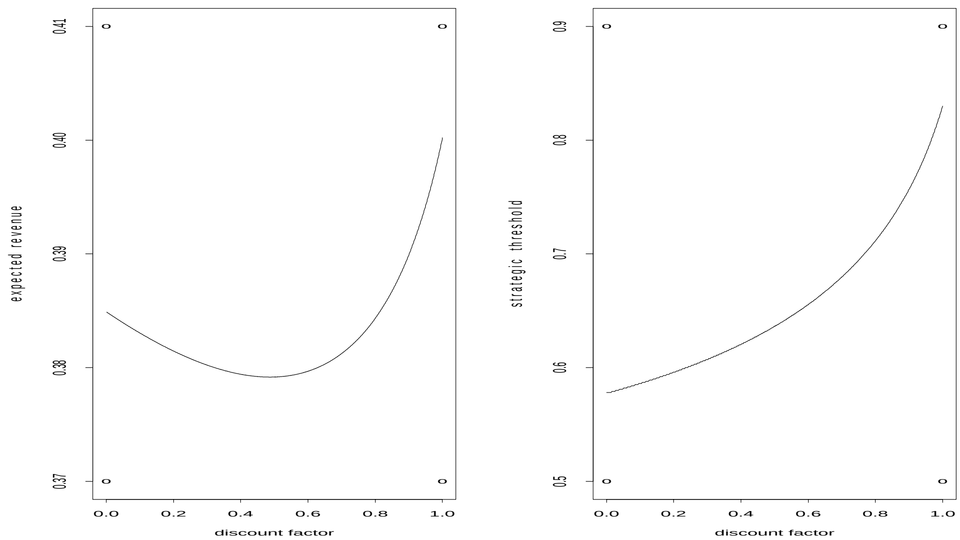

Figure 1: Sequential Sales with Independent Uniform Valuations, $n = 2$.

price in a one-shot offering.

In other words, with stochastic demand a sufficiently patient take-it-or-leave-it seller always benefits from the sales opportunities provided by future periods. Enhanced competition among buyers if the item is not sold in period one induces a competitive externality in demand. This externality allows the seller to charge a first period price that would be higher than optimal in a simple one-shot take-it-or-leave-it offering.

As an example, consider the case of two potential buyers, $n = 2$, whose valuations are $X_i \sim$ i.i.d. $u[0, 1]$, $i = 1, 2$. Figure 1 shows the seller's expected revenue and the buyers' strategic threshold $y^\star$ as a function of the discount factor δ.

3 Comparison with Sequential Auctions

With incomplete information about buyers' valuations, auctions are often another attractive sales mechanism. Compared with take-it-or-leave-it sales, the required coordination to set up an auction often makes them more costly, however. On the other hand, in a setting with symmetric and risk neutral buyers with independent private values and downward sloping marginal revenues, i.e.

under assumptions A1-A4, it is well known that an auction with an optimal reserve price is the optimal mechanism (Vickery (1961), Myerson (1981), Riley and Samuelson (1981), Harris and Raviv (1981)). Hence, there exist sequential auction formats which dominate the take-it-or-leave-it sales mechanism uniformly for any discount factor, but these may be costly to implement.[4]

The analysis considers sequential first-price sealed bid auctions. This facilitates comparison with sequential take-it-or-leave-it sales because equilibrium bidding strategies involve bid shading, which is reminiscent of the strategic delays identified in the preceding section. By the classical revenue equivalence theorem (Vickerey (1961), Myerson (1981), Riley and Samuelson (1981), Milgrom and Weber (1982)) different auction designs induce the same implications regarding expected revenue. In the case of sequential auctions, a PBE involves reserve prices in both periods as well as bidding strategies, such that, given bidders' strategies, the reserve prices maximize the auction seller's expected revenue, and given the reserve prices, the bidding strategies maximize the bidders' expected surpluses. As in the case of sequential take-it-or-leave-it sales, there exists an equilibrium threshold valuation $y^\star$ so that only bidders with valuations above this threshold in the period one auction submit equilibrium bids above the period one equilibrium reserve price. The PBE in such auctions is characterized by

Theorem 2: *Consider a two period game with consecutive first-price sealed bid auctions. Under assumptions A1-A4, there exists a unqiue PBE with first and second period reserve prices $R_1(y^\star)$ and $R_2(y^\star)$ which are monotonic in $y^\star$; furthermore, $y^\star = y(\delta)$ is monotonically increasing in δ, and $y(1) = 1$.*

Note that this implies that, when the future is not discounted, the auction seller effectively collapses the game into a single auction in the second period, by setting a prohibitively high reserve price in the first auction. This result has an important corollary.

Corollary 2: *Under assumptions A1-A4, the expected revenue of a two period game with consecutive first-price sealed bid auctions is no larger than the expected revenue of a single auction in the first period.*

Hence, in the setting considered in this paper, auction sellers, unlike take-it-or-leave-it sellers, never benefit from a second period.

Continuing with the example of the preceding section, with two potential

[4]Optimal sequential auctions are also considered by McAfee and Vincent (1997), for the case of sequential second-price sealed-bid auction designs, and Laffont and Robert (2000), for sequential first-price auctions in the case of affiliated values.

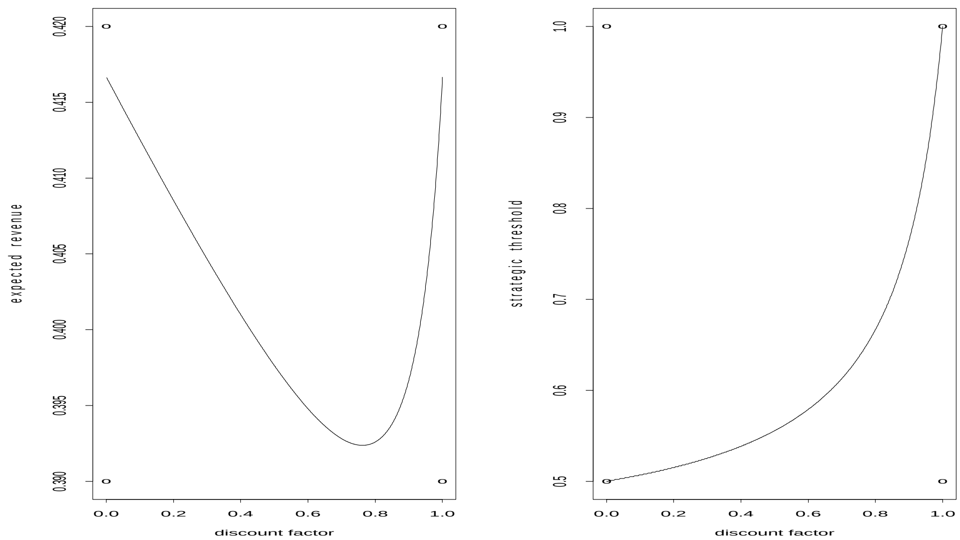

Figure 2: Sequential First-Price Sealed Bid Auctions with Independent Uniform Valuations, $n = 2$.

buyers with independent and uniformly distributed valuations, Figure 2 shows the auction seller's expected revenue and the buyers' strategic threshold $y^\star$ as a function of the discount factor δ.

4 Conclusions

This paper emphasizes the strategic implications of market uncertainty for equilibria in dynamic markets. It highlights the trade-off in take-it-or-leave-it sales between allocative efficiency and opportunities for buyers other than the highest valuation buyer to obtain an item to be sold; this trade-off induces a competitive externality in demand, due to competing buyers who are willing to pay for this opportunity. And it concludes that there exist dynamic situations in which a take-it-or-leave-it seller can inter-temporally exploit this trade-off to raise prices and expected revenue above the one obtained in a one-shot take-it-or-leave-it offer. This cannot happen in sequential auctions in which the allocation is guaranteed to be efficient. The analysis demonstrates that market uncertainty may invalidate Coase-type conjectures.

A Appendix

A.1 Proof of Theorem 1 and Corollary 1

A.1.1 Theorem1

Proceed by backward induction. Given valuation y of the marginal buyer, in period two the seller's problem is

$$\max_{p_2} p_2 \left(1 - \left(\frac{F(p_2)}{F(y)}\right)^n\right),$$

so that $p_2^\star = p_2(y)$ solves

$$F(y)^n - F(p_2^\star)^n - p_2^\star n F(p_2^\star)^{n-1} f(p_2^\star) = 0.$$

Regularity A3 implies that the solution is unique, given y. Also, notice that increasing y shifts up the left-hand side of the last expression, so that A3 implies that $p_2^\star$ must rise to return to equality. Hence, $p_2(y)$ is monotonically increasing in y.

Given y, the first period price $p_1(y)$ is such that the marginal buyer with valuation $y^\star$ is indifferent between buying in either period, hence, her expected surplus $E[s(y^\star, \delta)]$ is

$$\begin{aligned} E[s(y^\star,\delta)] &= (y^\star - p_1(y^\star)) \left[\sum_{i=0}^{n-1} \frac{\binom{n-1}{i}}{i+1} (1 - F(y^\star))^i F(y^\star)^{n-1-i}\right] \\ &= \delta(y^\star - p_2(y^\star)) \left[\sum_{i=0}^{n-1} \frac{\binom{n-1}{i}}{i+1} (F(y^\star) - F(p_2(y^\star)))^i F(p_2(y^\star))^{n-1-i}\right], \end{aligned}$$

where the second term in square brackets results from the product of the probability of the game progressing to the second period, $F(y^\star)^{n-1}$, and the conditional probability of obtaining the item, given that competing buyers' valuations are below $y^\star$, i.e. $\sum_{i=0}^{n-1} \frac{\binom{n-1}{i}}{i+1} \left(1 - \frac{F(p_2(y^\star))}{F(y^\star)}\right)^i \left(\frac{F(p_2(y^\star))}{F(y^\star)}\right)^{n-1-i}$. The seller will choose choose prices that induce $y^\star = y(\delta, n)$ such as to equate expected marginal revenue from selling in periods one and two. Expected revenue in either period is the product of period price and probability of sale. Since an increase in y implies that the probability of selling in period two rises while the probability of selling in period one falls, and since $p_2(y)$ is increasing in y, $p_1(y)$ must be monotonically increasing in y. Moreover, $p_1(y) > p_2(y)$ for all y because otherwise, by continuity, a buyer i with valuation X_i just below y could obtain higher expected surplus by submitting a purchase order in period one: In this case, i's surplus is higher, conditional on being assigned the item, accrues earlier and with higher probability, since in that case $\Pr(y < X_{j\neq i} < p_1(y)) < \Pr(X_{j\neq i} < p_2(y))$.

Finally, suppose $y^\star$ were not unique, i.e. $\exists\, y_1^\star < y_2^\star$, both equating expected marginal revenues and yielding the same expected revenue. Then, for any n and δ,

$$p_1(y_j^\star) \geq p_2(y_j^\star), \;\; j = 1, 2$$

and

$$\begin{aligned} \Pr(\max\{X_i, i=1,\ldots,n\} > y_1^\star) &> \Pr(\max\{X_i, i=1,\ldots,n\} > y_2^\star) \\ \Pr(\max\{X_i, i=1,\ldots,n\} \in S1) &< \Pr(\max\{X_i, i=1,\ldots,n\} \in S_2), \end{aligned}$$

where $S_1 = [p_2(y_1^\star), y_1^\star]$ and $S_2 = [p_2(y_2^\star), y_2^\star]$. Since by hypothesis expected revenue is equal for $y_1^\star$ and $y_2^\star$, it must be that

$$\begin{aligned} p_1(y_2^\star) &> p_1(y_1^\star) \\ p_2(y_2^\star) &< p_2(y_1^\star), \end{aligned}$$

a contradiction to the monotonicity of $p_2(y)$. Hence $y^\star = y(\delta, n)$ is unique.

If $\delta = 0$, then y is irrelevant.[5] Consider $\delta \in (0, 1]$ and suppose $y^\star = 1$. Then, $p_1(1) = 1$ and $p_2(1) < 1$. Therefore, the expected surplus of the marginal buyer, whose valuation is $y^\star = 1$, is zero in period one, and positive in period two. Hence, $y = 1$ cannot be part of a PBE. Alternatively, suppose $y^\star = 0$. Then, $p_2(0) = 0$ and $p_1(0) > 0$. In this case, the expected surplus of the marginal bidder is zero in period two and negative in period one. Hence $y = 0$ cannot be part of a PBE. Therefore, $y^\star$ must be in the interior of $\mathcal{X}$. This completes the proof of the theorem. □

A.1.2 Proof of Corollary 1

The corollary follows immediately. Part (i) is implied by $y^\star$ in the interior of $\mathcal{X}$: In any PBE, since $y^\star < \sup\{x : x \in \mathcal{X}\}$, the seller never finds it optimal to choose a prohibitively high first-period price which would induce all buyers to wait for the second period. Hence collapsing the dynamic game to a one-shot game, which is only possible by annihilating the first period subgame, is not optimal. Since Theorem 1 for $\delta = 1$ then implies that expected revenue is higher in the one-shot game, by continuity of expected revenue in δ, there

[5]In this case, the game collapses to a one-shot game. Haris and Raviv (1981) provide a theory for optimal endogenous pricing regimes in such games.

exists $\check{\delta} \in (0,1)$ such that expected revenues for the sequential game are higher for all $\delta \in [\check{\delta}, 1]$. Part (ii) follows from $y^\star > 0$ in any PBE: In a one shot game, the seller faces all buyers and hence the price $p^\star$ targets all buyers; in the first period of the dynamic game, the seller only faces high valuation buyers with valuations above $y^\star > 0$, and hence the first period price $p_1(y^\star) > p^\star$, targeting only these buyers. □

A.2 Proof of Theorem 2 and Corollary 2

A.2.1 Theorem 2

The game is solved by backward induction. Bidder i's bidding function in the second auction is

$$b(X_i, R_2, n) = X_i - \int_{R_2}^{X_i} F(u)^{n-1} du / F(X_i)^{n-1}.$$

Bidder i's expected surplus in this auction, conditional on it taking place, is $s_2(X_i, R_2, n) = \int_{R_2}^{X_i} F(u)^{n-1} du / F(y)^{n-1}$, while the seller's expected revenue is

$$\pi_2(y, R_2, n) = E\left[X - \int_{R_2}^{X} F(u)^{n-1} du / F(X)^{n-1}; X \in [R_2, y]\right],$$

where the expectation is taken with respect to the distribution of the maximum of the X_i, i.e. $nF(x)^{n-1} f(x)dx$. The seller chooses R_2 so as to maximize this expected revenue, so $R_2^\star = R_2(y, n)$ solves

$$\begin{aligned} n\left[F(R_2^\star)^{n-1}(F(y) - F(R_2^\star)) - R_2^\star f(R_2^\star) F(R_2^\star)^{n-1}\right] &= 0 \\ \Leftrightarrow F(y) - F(R_2^\star) - R_2^\star f(R_2^\star) &= 0. \end{aligned}$$

This implies the well-known result that $R_2^\star \equiv R_2(y)$ does not (directly) depend on n. It does depend on n indirectly, through the dependence of the equilibrium y on n. By A3, the solution exists and is unique. The last equality implies that $y > R_2(y)$, as a consequence of A2. Also, since raising y shifts up the left-hand side, to return to equality $R_2^\star$ has to rise, so that $R_2(y)$ is monotonically increasing in y.

Since the marginal bidder's discounted expected surplus in the second auction is

$$\delta s_2(y, R_2(y), n) F(y)^{n-1} = \delta \int_{R_2(y)}^{y} F(u)^{n-1} du > 0,$$

the bidding function β for the first auction is shaded taking this amount into consideration, beyond conventional shading, so that

$$\beta(X, y, n, \delta) = X - \frac{\int_y^X F(u)^{n-1} du}{F(X)^{n-1}} - \delta \frac{\int_{R_2(y)}^{y} F(u)^{n-1} du}{F(X)^{n-1}}.$$

The reserve price of the first auction equals the bid of the marginal bidder. Hence,

$$R_1(y, n, \delta) = \beta(y, y, n, \delta) = y - \delta \frac{\int_{R_2(y)}^{y} F(u)^{n-1} du}{F(y)^{n-1}}.$$

Also,

$$\frac{d}{dy} R_1(y, n, \delta) = 1 - \delta + \delta \left(\frac{F(R_2(y))}{F(y)} \right)^{n-1} \frac{d}{dy} R_2(y) + (n-1) \frac{\int_{R_2(y)}^{y} F(u)^{n-1} du}{F(y)^{n-2}},$$

i.e. $\frac{d}{dy} R_1(y, n, \delta) > 0$ and $R_1(y) \equiv R_1(y, n, \delta)$ is seen to be monotonically increasing in y, since $R_2(y)$ is. Since $y(\delta) \geq R_1(\delta)$ for any δ and $R_1(\delta)$ is monotonically increasing in δ, it follows that $y^\star = y(\delta)$ is increasing in δ as well.

It remains to be shown that $y^\star = 1$ can be part of a PBE. To demonstrate this, consider $\delta = 1$ and compare the seller's and buyers' strategies for $y = 1$ and $y = 1 - \epsilon$, for some $\epsilon > 0$. In the latter case, it follows from the previous arguments that both reserve prices will be lower than in the former. Because bids are monotonic in valuations, the highest valuation buyer is guaranteed to win the item in either case, but is better off in the latter because his bid can be lower and, therefore, his expected return higher, than in the former and there is no time cost when $\delta = 1$. But this is to the disadvantage of the seller, who, therefore, will choose the higher reserve prices prevailing when $y = 1$. The situation is different when $\delta < 1$ because then buyers with very high valuations will prefer avoiding the cost from waiting instead of a higher expected return accruing in the second period. Hence, when $\delta = 1$, $y^\star = 1$ is part of a PBE. □

A.2.2 Corollary 2

In the auction mechanism, the bidder with the highest valuation is guaranteed to win the item, because bidding functions are strictly increasing in valuations. Hence, the expected revenue of the game equals the discounted expected value of the winning bid. In a single auction, in the case $\delta = 0$, there is no strategic delay on the part of the bidders, and so the expected auction revenue is $\pi = E[\max\{\beta(X_i, R_1(0), n, 0), i = 1, \cdots, n\}] = E[\max\{X_i, i = 1, \cdots, n\}]$. With $\delta > 0$, monotonicity of $y(\delta)$ implies that some bidders strategically delay submitting eligible bids, i.e. bids above the first period reserve price $R_1(\delta)$, so that the bidding function for the first period auction is decreasing in δ, and it exceeds the bidding function for the second period auction. Hence, the expected winning bid cannot exceed π. When $\delta = 1$, then $y^\star = y(1) = 1$, so no bidder submits a bid above $R_1(1)$ in the first period, and the game collapses to a single auction in the second period, which yields expected revenue equal to π.[6] □

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