Longevity Adjustment of Retirement Age and Intra-generational Inequality*

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Abstract: We find that segments of society who have shorter life expectancy can expect a lower income from their pensions and lifetime utility due to the longevity of other groups participating in the same pension scheme. Linking the pension age to average life expectancy magnifies the negative effect on the lifetime utility of those who suffer low longevity. Furthermore, when the income of those with greater longevity increases, those with shorter life expectancy become even worse off. Conversely, when the income of those with shorter life expectancy increases, they end up paying more into the pension scheme, which benefits those who live longer. The relative sizes of the low and high longevity groups in the population determine the magnitude of these effects. We calibrate the model based on data on differences in life expectancy of different socio-economic groups and find that low-income workers suffer from a 10-13 percent drop in pension benefits from being forced to pay into the same scheme as high-income workers.

Keywords: Longevity, pension age, retirement, inequality.
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1. Introduction

Population ageing, driven by increasing life expectancy and decreasing birth rate, is a worldwide phenomenon, which is expected to last for several decades, possibly throughout this century. Indeed, it has become a leading issue not only within social sciences, but also across several scientific disciplines, within policy circles, etc.[1]

From an economic perspective, this prospect raises a number of challenges. In particular, it poses a serious threat to public finances as pension and elderly care expenses increase, all else equal. This is especially the case in countries with a pay-as-you-go (PAYG) pension system, which is the dominant scheme across the OECD area.

A likely response to this development is a policy rule linking the official pension age to average life expectancy. This has already been introduced in Scandinavian countries and the Netherlands (Jensen et al., 2020). Typically, such longevity indexation rules are designed in terms of average measures. In Denmark, for example, the official pension age increases in line with changes in average longevity (Andersen, 2015).

While simple and logical, such rules may well give rise to unintended side effects. For example, due to differences in life expectancy between high-education and low-education workers, or high-income and low-income workers, as well as between men and women, changes in the pension age based on an increase in average life expectancy may affect different socioeconomic groups differently. Such intra-generational differences may have important implications for lifetime utility for different groups in society.

Therefore, if average figures are calculated over a highly heterogeneous population, longevity indexation may widen inequality among the elderly. Such effects may not only seriously jeopardize the egalitarian objectives pursued by, say, Scandinavian welfare states, but they may also undermine the broad political foundation of social policies. Indeed, if longevity indexation would reduce lifetime utility of blue-collar workers with health issues, due to wearing-out following a long working-life with physically demanding routine work, unlike that of white-collar workers with a long education and a shorter working-life, the political and popular backing behind longevity adjustment might well gradually disappear.

[1] Everything points towards an increase in life expectancy for the elderly, i.e., individuals aged 65 and above. A typical estimate today is that the remaining life expectancy for a 65-year old will increase by a year every ten years (OECD, 2017).

In this paper, we study these issues in further detail. Our key theme is the implications of a longevity-indexed pension age in an economy where some segments of the population live longer than others, in order to explore whether a longevity-indexed pension age implicitly exacerbates intra-generational inequality.

Such differences can arise along many dimensions. In our analysis, the classification of the two groups as blue-collar and white-collar is an example based on the observation that those at the lower end of the income distribution have lower life expectancies on average. However, the results hold for any other two groups differing in life expectancy but sharing the same PAYG scheme, such as male versus female, smokers versus non-smokers, indigenous versus immigrant labour and so on.

We believe this fills an important gap in the academic literature. While a few papers have addressed aspects related to intergenerational redistribution following the introduction of longevity adjustment (see, e.g., Andersen, 2014; Jensen and Jørgensen, 2008), the intra-generational dimension has, to our best knowledge, not been studied as extensively so far.

In a recent paper, Laun et al. (2019) study possible retirement reforms in Norway intended to achieve solvency of social security in the face of population ageing taking into account both efficiency and equity across education groups. They find that proportionally lowering old-age retirement benefits as well as disability benefits maximizes average welfare of all education groups although it generates inequality in that recipients of disability benefits are more adversely affected.

Auerbach et al. (2017) study the effects of Social Security and Medicare on intra-generational equity in the U.S. They compare changes in average lifetime benefits received by men in the highest and lowest income quintiles between the 1930 and the 1960 birth cohorts and find that the difference widens considerably over this period.[2] In an earlier paper, Fuster et al. (2003) used an overlapping-generations model to study the heterogeneity in the value households assign to the insurance role of unfunded social security due to differences in mortality risk.[3]

We contribute to this literature by deriving a continuous-time model to show (a) the intra-generational effects of an unfunded pension scheme; (b) how linking the pension age to average

[2] The present value of lifetime benefits at age 50 is equal for those in the highest and lowest quintile of lifetime income for the 1930 cohort while for the 1960 there is a $130,000 gap in benefits.

[3] Households where the children pay high taxes and the father has the shortest lifetime tend to dislike social security. They like it most when the father is transferring income to his children and facing borrowing constraints.

life expectancy magnifies these intra-generational effects by increasing the negative effect on the lifetime utility of those who have short life expectancy; and (c) to show the effect of changes in the income distribution between those with long and short life expectancy on the intra-generational effect of such unfunded pension schemes.

Our findings have potentially important implications for pension policies in countries where PAYG pension schemes prevail. Indeed, the difference in life expectancies could reach a critical level, thus establishing a need for differentiated pension ages. This would involve extending schemes that link the pension age to increases in longevity, which are currently based only on average figures, to incorporate the variability associated with physical and mental disabilities across different socio-economic groups.

From here, the paper proceeds as follows: We start by providing some additional evidence of differences in life expectancy in Denmark of observably different groups of individuals. We then derive an overlapping-generations model for groups that differ in life expectancy but face a common pension age. Next, we use the model to explore the externalities between the groups associated with their differing life expectancies and provide policy recommendations. Thereafter, we calibrate our model based on the different life expectancy of, respectively, blue-collar and white-collar workers in order to get an estimate of the effect of the common pension age on the utility of different socio-economic groups. Finally, we summarize our findings, point out some implications for social and pension policy, and make suggestions for future research.

2. Heterogeneous life expectancy

In order to set the scene for this paper, we provide some additional evidence of differences in life expectancy of observably different groups of individuals who share the same system of a (flow) state pension or belong to the same pension fund. The example that may first come to mind is that of men and women. Differences in the life expectancy of men and women are well known and widely documented. The United Nations published data on developed and developing countries in 2015 where significant differences can be seen, with women having longer life expectancy than men. Thus, the life expectancy for men is 80.9 in Japan, 79.4 in Spain, 81.9 in Sweden and 80.0 in Denmark while the corresponding numbers for women are 86.6 in Japan, 85.1 in Spain, 83.7 in Sweden and 81.9 in Denmark (United Nations, 2015). However, these numbers also reflect infant mortality, which is not a part of the issues addressed in this paper. According to the OECD, the life expectancy at age 65 for men in these countries is 19.6 in Japan, 19.4 in Spain, 19.1 in Sweden and 18.2 in Denmark while the corresponding numbers for women are 24.4 in Japan, 23.6 in Spain, 21.5 in Sweden and 20.8 in Denmark

(OECD, 2019). Thus, a woman at age 65 can expect to live 4.8 more years than a man in Japan, 4.2 in Spain, 2.4 in Sweden and 2.6 in Denmark.

There are also differences in life expectancy across income groups. Table 1 shows life expectancy in Denmark at age 60 by income quantiles. The difference between the life expectancy of men in the top and bottom income group at age 60 was 5.9 years in 1996 and grew to 6.0 years in 2016. Similar numbers for women are 5.2 years in 1996 and 3.8 years in 2016. Thus, the gap between low-income and high-income women was becoming smaller while the gap for men increased slightly.

Table 1. Life expectancy at 60 by income quantiles, Denmark

	Q1 (lowest income)	**Q2**	**Q3**	**Q4 (highest income)**	**Diff. Q4 and Q1**
	Men				
1996	14.9	17.6	18.9	20.8	5.9
2016	18.9	21.3	23.1	24.9	6.0
	Women				
1996	18.8	21.7	22.4	24.0	5.2
2016	23.5	23.9	24.9	27.3	3.8

Source: Danish Ministry of Finance.

In addition, there are differences between skill groups. Table 2 illustrates life expectancy at 60 by skill groups in Denmark.

Table 2. Life expectancy at 60 by education, Denmark

	Unskilled	**Skilled**	**Shorter higher education**	**Longer higher education**	**Diff. long. higher and unskilled**
Men					
2002	18.5	19.1	20.5	21.4	2.9
2016	20.6	22.0	23.3	24.0	3.4
Women					
2002	21.8	22.8	23.6	23.8	2.0
2016	23.8	25.2	26.0	26.3	2.5

Note: Shorter higher education includes educational attainment at Bachelor's level and longer higher education is educational attainment at Master's and PhD level.

Source: Danish Ministry of Finance (2017).

Comparing with income groups, the differences are smaller. For men, the difference in life expectancy between those having longer higher education and those who are unskilled was 3.4 years in 2016 and 2.9 years in 2002. For women, the difference was 2.5 years in 2016 and 2.0 years in 2002. In this case, the gap between the two groups – those unskilled and those with long higher education – is becoming larger. This trend is not necessarily a recent trend, based on Danish registry data. Brønnum-Hansen and Baadsgaard (2012) find that the social life expectancy gap in Denmark has widened since 1987.

Not surprisingly, the differences between the life expectancy of high-income and low-income workers are larger in the US. In an early study, Kitagawa and Hauser (1973) found significant difference in 1969 mortality by education. Waldron (2007) found differences in life expectancy of the rich and the poor in the US and that this is a gradient across the socioeconomic scale. More recently, Chetty et al. (2016) showed that the life expectancy of the richest 1 percent in the U.S. is 14 years longer than that of the poorest 1 percent and the top income quartile can expect to live about a decade longer than the bottom quartile. These are much bigger differences than those found for Denmark (see Table 1).

Furthermore, both studies have found that the spread in life expectancy between income groups is increasing. Case and Deaton (2017) find an increase in mortality and morbidity among white non-Hispanic Americans in midlife (35-59) since the beginning of this century, continuing until at least 2015 due to increases in drug overdoses, suicides and alcohol-related liver mortality. Case and Deaton attribute this development to progressively worsening labour market opportunities of whites with low levels of education at the time of entry into the labour market, which is magnified by the over prescription of opioids and other drugs. Educational differences in mortality among whites are increasing to such an extent that mortality has risen for those without a college degree while decreasing for those with a college degree.

The gap between income groups is evidently not as wide in Europe. The OECD (2017) reported that the average gap in life expectancy in Europe between those with tertiary education and those below upper secondary education is 2.7 years. For example, the gap is only 1.5 years for Denmark. Moreover, Case and Deaton show how mortality rates have continued to drop in Europe, especially for those with lower levels of education. Thus, European countries had an average rate of decline of age-adjusted mortality of 2.0 percent per year between 1990 and 2015, while non-Hispanic whites without a college degree in the US saw that same decline only until the late 1990s, when mortality started to increase for those without a college degree.

Differences in longevity also exist between a wide array of other social groups, such as those defined on the basis of race, country of origin in the case of immigrants, professions, and so forth. However, in this paper we will calibrate our model for socio-economic groups since they are an obvious example of groups that pay the same amount into pension schemes but have different life expectancies.

3. A model with overlapping generations and a heterogeneous population

In this section, we explore the implications of differences in longevity across groups and lifetime utility when there is a common pension age for the whole population. We set up an overlapping generations (OLG) model, stated in continuous time, and with a heterogeneous population based on Andersen and Gestsson (2016) and Gestsson and Zoega (2018). We, as well as these two papers, depart from Blanchard (1985) by assuming that the probability of dying increases with age. Thus, the old differ from the young in facing a higher probability of death and there is a maximum possible age for every cohort.

We depart from the basic model in Andersen and Gestsson (2016) by introducing a more realistic mortality profile and splitting the population into two heterogeneous groups. In that paper, no one dies before reaching pension age and thereafter, the size of a cohort gradually reduces until no one is left. In contrast, we start with data on actual mortality profiles and calibrate our theoretical model to fit this profile. In addition, and this is the key contribution of our paper, one half of the population, denoted by H, enjoys high life expectancy, while the other half, denoted by L, suffers low life expectancy. Different proxies will be used for the two groups. In the previous section, we motivated out analysis by showing differences in life expectancy between groups defined in terms of skills and income and in section 7, we will simulate our model based on differences in life expectancy between income quantiles.

Agents can choose when to retire, making consumption and saving decisions while paying into a pay-as-you-go (PAYG) pension scheme. When they reach the pension age, they become eligible for pension benefits. After retirement, they consume based on their savings and pension transfers. Agents can die at any time, but their instantaneous death probability, or hazard rate, is dependent on their age and societal group. The maximum age possible is A. We can then use the model to explore the effect of increased longevity of one group on the lifetime utility of the other, the effect of changes in the pension effect on the utility of both groups, and, finally, the effect of changes in the income of both groups on their lifetime utility.

3.1. Age-dependent death probability

The cumulative distribution function (CDF) captures the chance of being dead at age a, where $i \in \{H, L\}$. The superscript denotes which group each agent belongs to. We adopt the realistic mortality structure introduced by Boucekkine et al (2002). The CDF of time of death (D) takes the following form:

$$\Phi^i(a) = \Pr(D \leq a) = \int_D^a \varphi^i(D)dD = \frac{e^{\mu_1^i a} - 1}{\mu_0^i - 1}, i = H, L \tag{1}$$

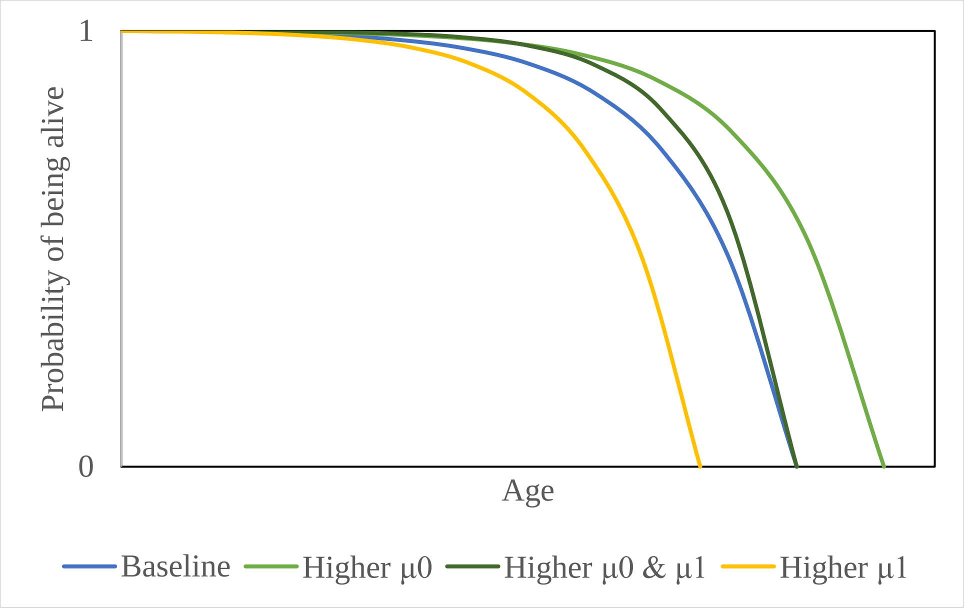
The parameters $\mu_0^i > 1$ and $\mu_1^i > 0$ determine the shape of the CDF. An increase in μ_0^i makes the denominator of the CDF greater, proportionally increasing the probability of being dead at each age. Importantly, any change in μ_1^i has an age-dependent positive effect on the numerator. Therefore, a manipulation of both parameters allows us to change the slope and reach of the CDF, effectively creating a mortality profile that closely resembles reality, see Figure 1. Equation (1) can be used to approximate the empirical survival curves shown in Figure A1 (appendix). In general, we have to assume that the group that suffers from a shorter expected lifespan has either a lower value of μ_0^i or a higher value of μ_1^i (or both) than the high longevity group.

From the CDF we can find that the chance of being alive at a given age, a, denoted by $m^i(a)$, is:

$$m^i(a) = 1 - \Phi^i(a) = \frac{\mu_0^i - e^{\mu_1^i a}}{\mu_0^i - 1} \tag{2}$$

Figure 1 shows the survival function $m^i(a)$.

Figure 1. Effects of changes in μ_0^i and μ_1^i on the shape of the survival function



The maximum age for each group, $A^i = \ln\left(\mu_0^i\right)/\mu_1^i$, can be found through the CDF. We see that the chance of being alive is strictly decreasing and strictly concave in age.

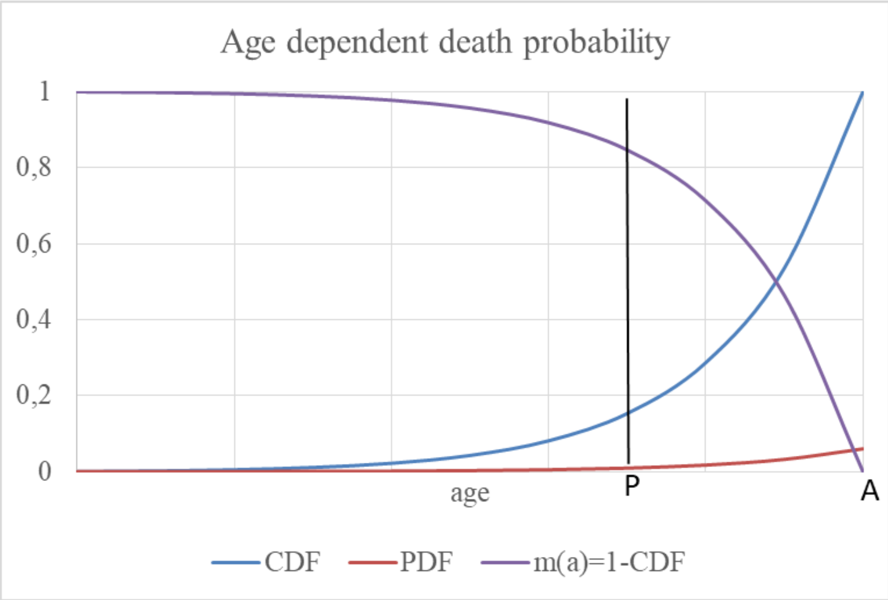
$$\frac{\partial m^i(a)}{\partial a} < 0 \quad , \qquad \frac{\partial^2 m^i(a)}{\partial a^2} < 0 \quad and \quad \frac{\partial m^i(a)}{\partial A^i} > 0$$

Finally, the probability density function (PDF) of death, is found by differentiation:

$$\varphi^i(a) = \frac{\mu_1^i e^{\mu_1^i a}}{\mu_0^i - 1} \tag{3}$$

These functions are depicted in Figure 2 along with a hypothetical pension age, P, and the maximum possible age A.

Figure 2. Mortality over the lifespan



The CDF shows the probability of being dead at each age; the PDF shows the probability of dying at each age; and the m(a) function has probablity of being alive at each age.

3.2. Private insurance/pension system

In line with (Yaari, 1965), we introduce an actuarially fair insurance company that provides agents with actuarial notes. The purchaser of an actuarial note gets a constant stream of payment until his death. The notes are in a sense an annuity, which, at the time of the purchaser's death, leave the insurance company free of any obligations. This mitigates the loss of utility caused by the uncertainty of death, as all of the agent's private assets are held in these notes. These notes pay the rate $r^i(a) = \int_0^a r + \beta^i(z)dz$, where $\beta^i(z)$ is the instantaneous death probability of agent aged a:

$$\beta^i(a) = \frac{\varphi^i(a)}{1 - \Phi^i(a)} = \frac{\mu_1^i e^{\mu_1^i a}}{\mu_0^i - e^{\mu_1^i a}} \tag{4}$$

Here, the small open economy setting, where r is exogenous, has been adopted. Equation (4) implies the insurance company can observe which group each agent belongs to, paying group L a higher rate of return, because their instantaneous death probability is higher than those of group H. Therefore, assets held in these notes can be interpreted as a private pension fund or a life insurance company. From this we get the rate of return:

$$r^i(a) = \int_0^a r + \beta^i(z)dz = ar - \ln\left(\mu_0^i - e^{\mu_1^i a}\right) + \ln\left(\mu_0^i - 1\right) \tag{5}$$

which implies: $e^{-r^i(a)} = e^{-ar}\frac{\mu_0^i - e^{\mu_1^i a}}{\mu_0^i - 1} = e^{-ar}m^i(a).$

It can directly be observed that the mortality profile influences the rate of return and therefore plays a key role in the consumption-saving decisions of agents.

3.3. Beveridgean-type public PAYG pension system

Agents are forced to pay into a government-run PAYG pension scheme. Contrary to the private insurance company, the government, and therefore this pension scheme, cannot "see" which group each agent belongs to.[4] So, all agents pay the same amount into the PAYG system while contributing and receive identical pension benefits after reaching the pension age, providing that the agent is alive. This system represents the social security system of the economy.

The pension system is run on a balanced budget basis:

$$\Pi = \frac{\mathrm{T}N_c}{N_b} \tag{6}$$

Here we operate in the defined contributions (DC) case, where we treat pension benefits, Π, as endogenous, implying a defined contribution scheme, and the pension contributions, T, as exogenous. Conversely, in the defined benefits (DB) case, we treat pension contributions as endogenous and the benefits as exogenous. Here both groups pay the same amount into the pension scheme. In section 5, we will wage-index the pension transfers and allow for an asymmetric income distribution. For simplicity, we start with a uniform income across the groups causing any wage-indexation to yield identical results as in the case where the groups pay the same dollar-amount.

The subscript denotes which group an agent belongs to; N_b and N_c represent the number of pension benefits recipients and contributors, respectively. P represents the pension age. We can find the number of contributing agents and agents receiving benefits from the pension scheme:

$$\begin{aligned} N_c = N_c^L + N_c^H &= \sigma\int_0^P m^L(a)da + (1-\sigma)\int_0^P m^H(a)da \\ &= \sigma\frac{e^{\mu_1^L P} - P\mu_1^L\mu_0^L - 1}{\mu_1^L - \mu_1^L\mu_0^L} + (1-\sigma)\frac{e^{\mu_1^H P} - P\mu_1^H\mu_0^H - 1}{\mu_1^H - \mu_1^H\mu_0^H} \end{aligned} \tag{7}$$

[4] This assumption effectively implies that the government is not allowed to discriminate against agents on the basis of their longevity type, *H* or *L*.

$$N_b = N_b^L + N_b^H = \sigma \int_P^{A^L} m^L(a)da + (1-\sigma) \int_P^{A^H} m^H(a)da$$

$$= \sigma \frac{\mu_0^L - \ln(\mu_0^L)\,\mu_0^L - e^{\mu_1^L P} + P\mu_1^L \mu_0^L}{\mu_1^L - \mu_1^L \mu_0^L} + (1 - \sigma) \frac{\mu_0^H - \ln(\mu_0^H)\,\mu_0^H - e^{\mu_1^H P} + P\mu_1^H \mu_0^H}{\mu_1^H - \mu_1^H \mu_0^H} \tag{8}$$

We impose $0 < \sigma < 1$, where σ and $1 - \sigma$ are the proportional size of the new-born cohorts of group L and H, respectively. Agents in this economy are continuously dying and new agents are continuously being born. The whole population is of size:

$$N = N^L + N^H = \sigma \frac{\mu_0^L - \ln(\mu_0^L)\,\mu_0^L - 1}{\mu_1^L - \mu_1^L \mu_0^L} + (1-\sigma) \frac{\mu_0^H - \ln(\mu_0^H)\,\mu_0^H - 1}{\mu_1^H - \mu_1^H \mu_0^H} \tag{9}$$

Population size is therefore dependent on the life expectancy of each group. Naturally, the population size is not a function of the pension age and is not affected by the pension system. However, the structure of the pension scheme depends on the demographic balance.

3.4. Utility maximization

Agents maximize expected lifetime utility:

$$E(U) = \int_0^{A^i} e^{-\delta a} m^i(a) u\big(c(a)\big)\, da + \int_{R^i}^{A^i} \omega e^{-\delta a} m^i(a) da \tag{10}$$

where $u(c(a))$ is utility derived from consumption at age a, utility derived from leisure during retirement is denoted by ω and δ is the discount rate. We assume that the agent can choose between two alternatives, either he works full time or is retired and doesn't work at all. The agent decides when to retire from the labour force but once retired, the agent cannot transition back into the workforce. This setup of utility during retirement is in the style of Kalemli-Ozcan and Weil (2010). The lifetime budget constraint becomes:

$$\int_0^{R^i} y^i\, e^{-r^i(a)} da + \int_P^{A^i} \Pi e^{-r^i(a)} da = \int_0^{A^i} c(a)\, e^{-r^i(a)} da + \int_0^P \mathrm{T}\, e^{-r^i(a)} da \tag{11}$$

Notice that while the retirement age, R^i, is endogenous the pension age, P, is exogenous. The agent cannot decide the rules of the pension scheme, but he can decide when to retire. Here y^i

is the income of agents. The first order condition of the utility maximization problem w.r.t. $c(a)$ yields:

$$e^{-\delta a}m^i(a)\,u'\big(c(a)\big) = \gamma e^{-ar}m^i(a) \tag{12}$$

where γ is the Lagrange multiplier. By assuming the real interest rate, r, equals the subjective rate of time preference, δ, we get:[5]

$$u'\big(c(a)\big) = \gamma \quad \forall a \in [0, A^i] \tag{13}$$

which implies: $c(a) = c^i \quad \forall a \in [0, A^i]$.

Finally, from this realization, the budget constraint, and applying the identity of the PAYG-pension system ($\Pi = \frac{\mathrm{T}N_w}{N_o}$), we get that each agent consumes according to:

$$c^i \int_0^{A^i} e^{-ar}\, m^i(a)da = y^i \int_0^{R^i} e^{-ar} m^i(a)da - \mathrm{T}\int_0^{P} e^{-ar}\, m^i(a)da + \frac{\mathrm{T}N_w}{N_o}\int_P^{A^i} e^{-ar} m^i(a)da \tag{14}$$

By rearranging we arrive at:

$$c^i = \mathrm{y}^{\mathrm{i}}\,\frac{\int_0^{R^i} e^{-ar} m^i(a)da}{\int_0^{A^i} e^{-ar}\, m^i(a)da} - \mathrm{T}\,\frac{\int_0^{P} e^{-ar}\, m^i(a)da}{\int_0^{A^i} e^{-ar}\, m^i(a)da} + \frac{\mathrm{T}N_w}{N_o}\,\frac{\int_P^{A^i} e^{-ar}\, m^i(a)da}{\int_0^{A^i} e^{-ar}\, m^i(a)da} \tag{15}$$

The consumption of the agent is dependent on the relative portions of his expected lifetime spent working, captured by the first fraction in equation (15), time spent contributing to the pension scheme, captured by the second fraction, and time spent receiving pension benefits, captured by the last fraction. When planning his consumption, the agent accounts for his contributions to the pension scheme and expected pension benefits later in life, both of which are influenced by the demographic structure of society. Therefore, the agent has to account for his own life expectancy and the life expectancy of the whole population when making consumption decisions.

The old-age dependency ratio dictates the contributions/benefits structure of the pension scheme. The demographic structure of society influences the consumption plan of the agent in

[5] We can do this, as we are not interested in the lifetime consumption *path* of each group, but rather the *total* lifetime consumption of each group.

this obvious way. These effects are analysed in sections 4.1 and 4.2. More subtly, as the pension age is uniform across the population, any changes in the life expectancy of one group will raise the average life expectancy of the whole population, which might lead to a rise in the pension age. In sections 4.3 and 4.4 we elaborate further on the implications of pension age increases.

4. Experiments

Having presented our analytical framework, we next study some of the results that can be derived from it. We concentrate on longevity shocks and changes in the pension age.

4.1. Asymmetric longevity shock

We now turn our attention towards the implications of increased longevity by deriving the effect of a widening of the gap between the life expectancies of the two groups. The widening of the gap between the life expectancies of the two groups can either manifest itself in a lowering of μ_1^H or increases in μ_0^H causing the high longevity group (H) to live even longer.

We begin by lowering μ_1^H and keeping μ_0^H, μ_1^L, μ_0^L and P constant. This rise will therefore not affect the mortality profile of the L group but will change the consumption pattern of the L group through the pension system.

$$\frac{\partial N_c}{\partial \mu_1^H} = (1-\sigma)\frac{e^{\mu_1^H P}(1 - P\mu_1^H) - 1}{(\mu_1^H)^2(\mu_0^H - 1)} < 0 \tag{16}$$

$$\frac{\partial N_b}{\partial \mu_1^H} = (1-\sigma)\frac{\mu_0^H - \ln(\mu_0^H)\,\mu_0^H - e^{\mu_1^H P}(1 - P\mu_1^H)}{(\mu_1^H)^2(\mu_0^H - 1)} < 0 \tag{17}$$

We are not only interested in the effect on each population, but also the effect on the ratio of pensioner to contributors. We find that the contributing population grows less than the pensioner population when we raise longevity via μ_1^H. This implies that pension benefits will decrease for both groups. Therefore, group L will receive a lower return for their contributions, because the drop in pension benefits is not caused by an increase in their own lifespan. The expected lifetime consumption of members of the H group will rise, since their expected pension benefits will increase because of their increased life expectancy. Therefore, a positive longevity shock on one group has a negative financial effect on the other.

We can also simulate the asymmetric shock by increasing μ_0^H while keeping all other parameters in the mortality profile, μ_1^H, μ_0^L and μ_1^L, and the pension age, P, constant.

$$\frac{\partial N_c}{\partial \mu_0^H} = (1-\sigma)\frac{e^{\mu_1^H P} - P\mu_1^H - 1}{\mu_1^H(\mu_0^H - 1)^2} > 0 \quad (18)$$

$$\frac{\partial N_b}{\partial \mu_0^H} = (1-\sigma)\frac{\mu_0^H + P\mu_1^H - \ln(\mu_0^H) - e^{\mu_1^H P}}{\mu_1^H(\mu_0^H - 1)^2} > 0 \quad (19)$$

We observe the same effects in this case; the expected lifetime consumption of the H group increases at the cost of the L group.

4.2. Population shock

We have seen that the longevity of one group has an effect on the welfare of the other through the PAYG scheme. This welfare effect depends on the relative sizes of the H and L groups. Let's define the PAYG equality, from equation (6), as:[6]

$$\Pi = \mathrm{T}\frac{N_c}{N_b} = \mathrm{T}\frac{\sigma\tilde{N}_c^L + (1-\sigma)\tilde{N}_c^H}{\sigma\tilde{N}_b^L + (1-\sigma)\tilde{N}_b^H} \quad (20)$$

where: $\tilde{N}_c^L \equiv \int_0^P m^L(a)da$, $\tilde{N}_b^L \equiv \int_P^{A^L} m^L(a)da$, $\tilde{N}_c^H \equiv \int_0^P m^H(a)da$ and $\tilde{N}_b^H \equiv \int_P^{A^H} m^H(a)da$. Now we can see what happens to the PAYG benefits (or contributions in the DB case) when the relative sizes of the group changes:

$$\frac{\partial \Pi}{\partial \sigma} = \mathrm{T}\frac{(\tilde{N}_c^L - \tilde{N}_c^H)\left(\sigma\tilde{N}_b^L + (1-\sigma)\tilde{N}_b^H\right) - (\tilde{N}_b^L - \tilde{N}_b^H)(\sigma\tilde{N}_c^L + (1-\sigma)\tilde{N}_c^H)}{\left(\sigma\tilde{N}_b^L + (1-\sigma)\tilde{N}_b^H\right)^2} > 0 \quad (21)$$

The sign of equation (21) can be determined by solving the numerator:

$$(1-\sigma)\tilde{N}_c^L\tilde{N}_b^H + \sigma\tilde{N}_c^L\tilde{N}_b^H - (1-\sigma)\tilde{N}_c^H\tilde{N}_b^L - \sigma\tilde{N}_c^H\tilde{N}_b^L = \frac{\tilde{N}_b^H}{\tilde{N}_c^H} - \frac{\tilde{N}_b^L}{\tilde{N}_c^L} > 0 \quad (22)$$

This implies that when the size of the L group increases, the pension benefits for both groups increase. This is due to the fact that the L group pays proportionally more into the PAYG scheme than it receives as benefits. So, as the number of L agents increases, there are proportionally fewer long-lived agents who collect benefits in old age. Because the PAYG scheme is balanced at each time, this translates into either a drop in the contributions, T, in the DB case or a rise in the benefits, Π, in the DC case as described in equation (21).

[6] To analyse the effect of the size of one group on the pension transfers we use σ in the definition of the relative size of each population group in contrast to equations (7) and (8) where $N^L = \sigma\tilde{N}^L$.

This implies that when the H group gets smaller the remaining members get even richer through the PAYG scheme. Conversely, as the L group gets smaller its members are even worse off. This finding could have policy relevant implications, as a group in society has fewer members; its members are hurt even more by the existence of the pension scheme. This applies to lifestyle changes, such as the effect of reduced smoking on the relative size of the short-lived smoking population or to the decline of strenuous professions that reduce life expectancy. While this is an interesting avenue to explore, we focus our numerical results on the case where $\sigma = 0.5$ where agents of each type are "born" into the labour market at the same rate.

4.3. Rise in the pension age

Having derived the effects of a longevity shock we now turn to changes in the pension age. At first glance, we see that the probability of surviving until the pension age is always lower for members of the L group than the H group. Furthermore, by differentiation of the survival chance, we get:

$$\frac{\partial m^i(P)}{\partial P} = -\frac{\mu_1^i e^{\mu_1^i P}}{\mu_0^i - 1} < 0 \tag{23}$$

From this expression, we see that any change in the pension age will have a greater impact on the probability of reaching the pension age on members of the L group (since they have a lower value of μ_0^i and a higher value of μ_1^i). The L group already suffers from a lower chance of survival to the pension age, but any raising of the pension age will widen this gap in survival probabilities further.

The relative population size of pensioners and contributors also matters. A rise in the pension age raises the number of people contributing to the pension scheme, N_c, and lowers the number of people receiving benefits from the pension scheme, N_b:

$$\frac{\partial N_c}{\partial P} = \sigma \frac{e^{\mu_1^L P} - \mu_0^L}{1 - \mu_0^L} + (1 - \sigma) \frac{e^{\mu_1^H P} - \mu_0^H}{1 - \mu_0^H} > 0 \tag{24}$$

$$\frac{\partial N_b}{\partial P} = -\frac{\partial N_c}{\partial P} = \sigma \frac{\mu_0^L - e^{\mu_1^L P}}{1 - \mu_0^L} + (1 - \sigma) \frac{\mu_0^H - e^{\mu_1^H P}}{1 - \mu_0^H} < 0 \tag{25}$$

Since we treat the pension contributions, T, as exogenous in the DC case, the pension benefits increase with the pension age since there are now fewer pensioners per contributor. Let's define the dependency ratio, the total number of those receiving pension benefits divided by the total number of those contributing to the pension scheme, as Γ^i for $i \in \{H, L\}$:

$$\Gamma^i \equiv \frac{N_b^i}{N_c^i} = \frac{\mu_0^i - \ln(\mu_0^i)\,\mu_0^i - e^{\mu_1^i P} + P\mu_1^i\mu_0^i}{e^{\mu_1^i R} - P\mu_1^i\mu_0^i - 1} = \frac{\mu_0^i - \ln(\mu_0^i)\,\mu_0^i - 1}{e^{\mu_1^i P} - P\mu_1^i\mu_0^i - 1} - 1 \qquad (26)$$

Next, we find the effect of an increase in the pension age on the dependency ratio:

$$\frac{\partial \Gamma^i}{\partial P} = \frac{\mu_1^i(1 + \ln(\mu_0^i)\,\mu_0^i - \mu_0^i)(e^{\mu_1^i P} - \mu_0^i)}{\left(P\mu_1^i\mu_0^i - e^{\mu_1^i P} + 1\right)^2} < 0 \qquad (27)$$

We conclude that when the pension age, P, increases, the dependency ratio decreases. From the expression above we also see that this impact is greater for the L group, which suffers from high values of μ_1. An increase in the pension age will therefore affect the dependency ratio of both groups, but it will affect the L group disproportionately. Agents in the L group will be forced to contribute longer, and the gap between the H and L group in the probability of reaching the pension age will widen. This leads to the negative financial effect associated with the longevity shock becoming even greater. A rise in the pension age will exacerbate the negative financial effect caused by the disparities in longevity between the groups. The L group will end up contributing more than before to the PAYG scheme while getting lower benefits, since they are less likely to reach the pension age.[7] We arrive at the same results when simulating the asymmetric longevity through μ_0.[8]

4.4. Longevity indexed pension age

In the previous section, we explored the effects of an exogenous rise in the pension age. Now we treat the pension age as endogenous by linking it to the average life expectancy of the whole population. The life expectancy of each agent when entering the labour market, Λ^i, is:

$$\Lambda^{\mathrm{i}} \equiv \frac{\ln(\mu_0^i)\,\mu_0^i + 1 - \mu_0^i}{\mu_1^i(\mu_0^i - 1)} \qquad (28)$$

A longevity indexed pension age follows the rule;

$$P(\Lambda) = \lambda\left(\sigma\frac{\ln(\mu_0^L)\,\mu_0^L + 1 - \mu_0^L}{\mu_1^L(\mu_0^L - 1)} + (1 - \sigma)\frac{\ln(\mu_0^H)\,\mu_0^H + 1 - \mu_0^H}{\mu_1^H(\mu_0^H - 1)}\right) \qquad (29)$$

[7] The members of the L group that reach the pension age also have shorter life expectancies when they reach the pension age.

[8] A numerical exercise was used to explore the effects when the asymmetric longevity structure is driven by μ_0.

where the average life expectancy of the whole population when entering the labour-market is $\Lambda \equiv \sigma\Lambda^{L} + (1-\sigma)\Lambda^{H}$ and λ is the proportional indexation parameter, which determines the average share of adult life contributing to the pension scheme. We generally expect λ to be greater than zero and less than one. For example, a population that has the life expectancy of 80 years, enters the labour market at age 21 and reaches the pension age at 65 would have an indexation parameter of: $\lambda = \frac{65-21}{80-21} = 0.75$. As we treat λ as exogenous, any rise in life expectancy (of either group or both) would affect the pension age through equation (29).

Whether longevity increases of the H group are driven by decreases in μ_1^i or increase in μ_0^i they will lead to higher life expectancy. This will in turn affect the pension age:

$$\frac{\partial P}{\partial \mu_1^H} = \lambda(1-\sigma)\frac{\partial \Lambda^H}{\partial \mu_1^H} < 0 \qquad \text{and} \qquad \frac{\partial P}{\partial \mu_0^H} = \lambda(1-\sigma)\frac{\partial \Lambda^H}{\partial \mu_0^H} > 0$$

An increase in longevity of either group in this setting therefore has the effects on pension income shown in sections 4.1 and 4.3.

We can summarize the findings so far as follows: A positive longevity shock to one group has a negative financial effect on the other. Moreover, any longevity adjustment of the pension age based on the average life expectancy of the whole population will exacerbate the effect on the group that did not enjoy the increased longevity. Both of these effects are then magnified by the relative sizes of the H and L groups. As the L group gets smaller, the members are even worse off. Conversely, as the H group gets smaller, its remaining members benefit.

5. Income inequality and pension transfers

So far, the re-distributional effects of pension schemes that operate based on average life expectancy and old-age dependency ratios of a heterogeneous population have been analysed. For simplicity, we assumed that all agents, regardless of longevity, paid the same contributions and received the same benefits from the pension scheme, provided they were alive. By introducing contribution and benefits proportional to wages, we can further deepen the analysis of intra-generational transfers due to pension systems. This allows us to demonstrate intra-generational transfers imposed by the pension system that are driven by, and exacerbate, income inequality, hence adding to the results of section 4.

5.1. Wage indexed pension contributions

We replace the contribution in the Beveridge case with proportional taxation as introduced by Bismarck in late 19th century Germany. Each agent pays into the PAYG scheme while young, according to τy^i and receives πy^i after reaching the pension age. Here $0 < \pi < 1$ is the replacement rate of the pension benefits and $0 < \tau < 1$ is the proportional wage tax used to finance the benefits to the pensioners. We assume that the L group has the same wage replacement rate as the H group (π and τ are uniform between groups). However, we allow for distinct income levels across groups. The PAYG scheme is balanced each time (aggregate inflows equal aggregate outflows).

$$\pi(y^H N_b^H + y^L N_b^L) = \tau(y^H N_c^H + y^L N_c^L) \tag{30}$$

On the left hand, we have the outflows from the PAYG scheme. This is composed of members of the H group, N_b^H, receiving the pension of πy^H and members of the L group, N_b^L, receiving the pension of πy^L. On the right-hand side, we have flows into the PAYG scheme; both groups pay the same proportion of their wages to the scheme. Total inflows from the H and L groups are $\tau y^H N_c^H$ and $\tau y^L N_c^L$, respectively.

We arrive at a new equation for the relationship between pension benefits and pension contributions:

$$\pi = \tau \frac{y^H N_c^H + y^L N_c^L}{y^H N_b^H + y^L N_b^L} \tag{31}$$

In the case of perfect income equality, $y^H = y^L$, equation (28) reduces to the same PAYG equality as in equation (6). In the case of income inequality, $y^H \neq y^L$, the utility maximization follows near-identical steps as in section 3.4, yielding:

$$c^i = y^i \left(\frac{\int_0^{R^i} e^{-ar}\, m^i(a) da}{\int_0^{A^i} e^{-ar}\, m^i(a) da} - \tau \frac{\int_0^{P} e^{-ar}\, m^i(a) da}{\int_0^{A^i} e^{-ar}\, m^i(a) da} + \tau \frac{y^H N_c^H + y^L N_c^L}{y^H N_b^H + y^L N_b^L} \frac{\int_P^{A^i} e^{-ar}\, m^i(a) da}{\int_0^{A^i} e^{-ar}\, m^i(a) da} \right) \tag{32}$$

In addition to the effects observed in sections 4.1-4.4, we now see from equation (31) and (32) that the income of one group has an effect on the pension contributions paid out to the other, which ultimately affects consumption.

5.2. Income shock to high longevity group

To study this in further detail, let us look first at the PAYG scheme under defined contributions (DC). In this case τ is exogenous and π endogenous. The effect of changes in the demographic or income distribution on the PAYG scheme is therefore captured through changes in π. When the income of the group that enjoys greater longevity increases, we see that:

$$\frac{\partial \pi}{\partial y^H} = \tau \frac{y^L(N_c^H N_b^L - N_c^L N_b^H)}{(y^H N_b^H + y^L N_b^L)^2} < 0 \qquad (33)$$

Notice that if there was no difference in the mortality profiles of the two groups ($N_c^H N_b^L - N_c^L N_b^H = 0$) an increase in the income of one group would not affect the pension benefits/contributions of the other. However, since we impose different mortality profiles, any change in the income of one group imposes externalities on the other through the PAYG scheme.

The old age dependency ratio is higher for the H group because they enjoy higher longevity, implying that the sign of equation (33) is negative. We see that when the income of the H group increases, the pension replacement rate π decreases, causing the pension benefits paid to the L group to decrease – even though they did not enjoy a rise in their income. The H group pays more into the PAYG scheme while young but also receives more benefits during the pension period. This would not have any effect on the replacement rate if the dependency ratio was identical for both groups – the increased benefits would be exactly financed by the increase in contributions. But because the H group enjoys higher longevity than the L group, they collect benefits disproportional to their contributions. When their income goes up they are entitled to more benefits, πy^H, during the pension period than can be financed by their current levels of the contributions, τy^H. In order for the PAYG scheme to remain balanced, the income replacement rate, π, drops for both the H and L group, thereby causing members of the L group to get lower pension benefits than before the H group's income rose.

5.3. Income shock to low longevity group

Now let's look at the effects of an increase in the income of the L group. The derivative is similar to the one we saw above:

$$\frac{\partial \pi}{\partial y^L} = \tau \frac{y^H(N_c^L N_b^H - N_c^H N_b^L)}{(y^H N_b^H + y^L N_b^L)^2} > 0 \qquad (34)$$

Just as above, we see that the sign of the derivative is driven by the relative sizes of the pensioned and contributing cohorts of each group. However, in this case we see that an increase in the wages of the L group actually increases π.

When the L group's income rises, they pay more into the PAYG scheme, just as in the case where the H group's income rose. But the L group pays proportionally more than they receive out of the PAYG scheme. These new funds entering the PAYG scheme allow for an increase in the benefits paid out to the pensioners, causing π to increase. Therefore, the H group pays the same into the PAYG scheme, but receives more as a result of an increase in the income of the L group. The H group is better off when the L group's income rises.

In the defined benefits (DB) case the replacement rate, π, is treated as exogenous and τ as endogenous and is defined as:

$$\tau = \pi \frac{y^H N_b^H + y^L N_b^L}{y^H N_c^H + y^L N_c^L} \tag{35}$$

And the derivatives become:

$$\frac{\partial \tau}{\partial y^H} = \pi \frac{y^L (N_b^H N_c^L - N_b^L N_c^H)}{(y^H N_c^H + y^L N_c^L)^2} > 0 \tag{36}$$

$$\frac{\partial \tau}{\partial y^L} = \pi \frac{y^H (N_b^L N_c^H - N_b^H N_c^L)}{(y^H N_c^H + y^L N_c^L)^2} < 0 \tag{37}$$

We see similar effects here as in the DC case, as the income of the H group (L group) goes up, the L group (H group) is worse off (better off) through negative (positive) externalities of the PAYG system.

To summarize, both in the DC or the DB case, when the income of those that have greater longevity goes up, those that have shorter life expectancy are made worse off. Conversely, when the income of those who have shorter life expectancy goes up they end up paying more into the PAYG scheme, which benefits those that live longer.

6. Public-private pension partnership

We have found that pooling together groups that differ in life expectancy causes a public pension system to redistribute income from the short lived to the long lived. In some cases, such a system is inevitable as policymakers do not have perfect information about the life expectancy of various socio-economic groups. In this section, we propose a policy that would go far in mitigating the distributional effects associated with the PAYG scheme presented in previous sections.

Now let the government collect pension contributions from the working-age population as before but instead of financing the pension benefits with current contributions, the government pays a lump sum to the cohort that reaches retirement at each time while not paying anything to those already retired. This new system is balanced at each time, implying

$$\pi^R\big(y^H m^H(P) + y^L m^L(P)\big) = \tau(y^H N_c^H + y^L N_c^L) \quad (38)$$

where each lump sum recipient receives $y^i\pi^R$ at retirement. As before τ is exogenous and in this case, π^R , which maps the wage into a lump sum, is endogenous in the equation. This sum is transferred to a private pension scheme where the average longevity of members is known.[9] The private pension scheme is balanced at each time, covering benefits by lump-sum transfers of agents of the same type:

$$\pi_{PPP}^i y^i N_b^i = \pi^R y^i m^i(P), \quad i = H, L \quad (39)$$

When maximizing his utility the agent therefore faces the pension rule implied by equations (38) and (39):

$$\pi_{PPP}^i = \tau \frac{y^H N_c^H + y^L N_c^L}{y^H m^H(P) + y^L m^L(P)} \frac{m^i(P)}{N_b^i} \quad (40)$$

Just as before, the agent maximizes his utility taking into account his mortality and the mortality of the other agents paying into the pension scheme.

This public-private partnership eliminates all the intra-generational transfers caused by the different life expectancies after retirement. However, as the members of the L group are also less likely to reach retirement, the policy would not eliminate intra-generational transfers associated with members of the L dying before reaching retirement. The ability of public-private partnership in mitigating the effects of intra-generational transfers is analysed further in the following section.

7. Calibration of the mortality profile

We now turn to comparing the level of pension benefits between two groups that differ in terms of life expectancy and wages and then calculate the effect of increased longevity on their benefits. As demonstrated above, individuals of higher socio-economic statues live on average longer than those of lower and we exploit this difference to estimate the effects of these different groups paying into the same pension scheme.

[9] The scheme is not affected by the so-called „annuitization puzzle“ that makes people prefer a cash treatment to an annuity because no cash transfers to individuals are involved. Instead, the system involves pension funds paying out benefits instead of the government, only difference being that the former can better assess life expectancy. See Benartzi et al. (2011) and Robinson and Comerford (2019), amongst others, on the annuitization puzzle.

7.1. Comparing pension benefits of socio-economic groups

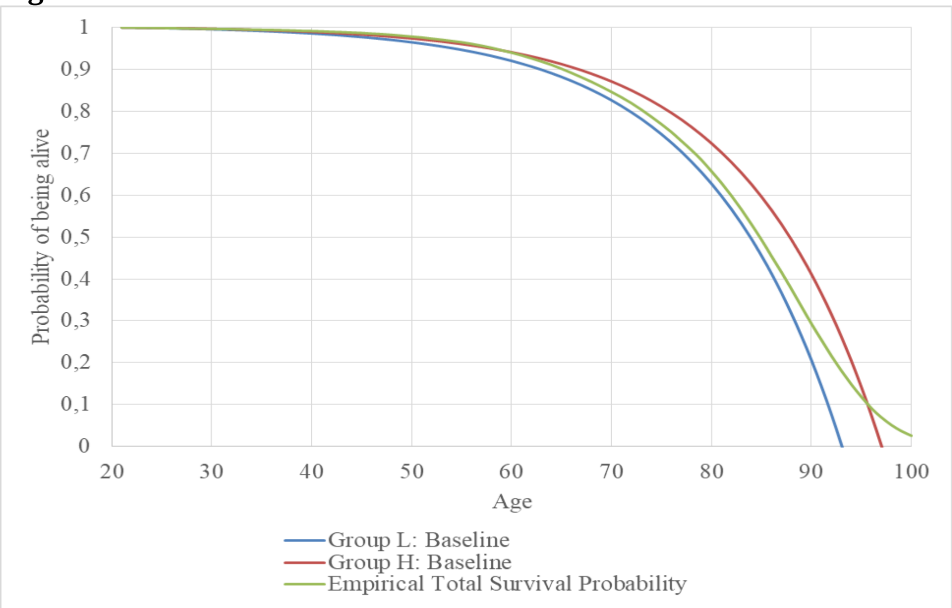
The chief parameters are essentially two, μ_0^i and μ_1^i, which determine the probability of death at each time. We apply parameter values to μ_0^i and μ_1^i, which can be done by matching empirical survival functions. Two calibrations of the mortality profile are needed when comparing the effects on each group since they differ in the value of μ_1^i and μ_0^i. The calibration minimizes the error in total survival probability for all ages across the population while still maintaining each group's observed life expectancy.

We assume agents of both groups enter the labour market at the age of 20 years. The baseline calibration is based on the life expectancy in Denmark by income in 2016 as presented in Table 1 in the introduction. There is approximately a 3.9 year gap in life expectancy between those two groups. The H and L groups are calibrated to match the top and bottom half of the income distribution for men. We refer to them as *white collar* and *blue collar* groups, respectively. Figure 3 depicts the calibrated survival functions of each class and the empirical total survival function.

Table 3. Calibrated parameters

	White collar (H)	Blue collar (L)
μ_1	0.075	0.075
μ_0	299	222
Simulated life expectancy (years)	83.0	79.1
Observed life expectancy (years)	83.0	79.1

Figure 3. Survival functions



Note: Estimated White-/Blue Collar Survival Probability. *Source: Human Mortality Database, 2018*

Now we can calculate to which extent paying into a joint pension scheme will have a negative (positive) effect on the blue collars (white collars) compared to the case where they each pay into their own pension scheme. We assume that the pension age is 66.5 years and both classes earn the same level of wages. Now we can determine the pension benefits received by each group when paying into the different pension schemes, captured in equations (6), (33) and (40). We compare this to the benefits received if the agent paid into a pension scheme where all the agents are of the same type $\left(\pi^i_{type} \equiv \tau N^i_c \backslash N^i_b\right)$. This difference in benefits received when retired is captured by:

PAYG government scheme:
$$\frac{\pi^i}{\pi^i_{type}} = \frac{y^H N^H_c + y^L N^L_c}{y^H N^H_b + y^L N^L_b} \Big/ \frac{N^i_c}{N^i_b} \tag{41}$$

Public-private partnership:
$$\frac{\pi^i_{PPP}}{\pi^i_{type}} = \frac{y^H N^H_c + y^L N^L_c}{y^H m^H(P) + y^L m^L(P)} \frac{m^i(P)}{N^i_b} \Big/ \frac{N^i_c}{N^i_b} \tag{42}$$

We find that blue-collar workers suffer from a 10.4% drop in pension benefits from paying into the same scheme as white-collar workers, while white-collar workers enjoy an increase in the pension benefits by approximately the same amount (equation 41). In line with our model findings, these effects are increasing in the pension age. In the public-private pension scheme, described in section 6, this difference in pension benefits received by the blue collars (equation 42) would drop to 1.6%, implying that even without perfect information governments can substantially mitigate the intra-generational disparities by giving the money to pension funds that have more information on life expectancy and where within-group differences are smaller.

As described in section 5, the level of income has a significant effect on the intra-generational transfers of a PAYG pension scheme. According to Statistics Denmark's Income Statistics, the top half of the Danish population received 69.3% of disposable income in 2016 while the bottom part received 30.7%. By applying this income distribution to the H and L groups in our model we find that the blue-collar group suffers a 13.8% drop in their pension benefits while the white-collar group's benefits rise by 6.1% per month. This is in line with the findings of section 5. When the income of those who live longer increases those who suffer with lower life expectancy are made even worse off. Conversely, when the latter experience rising income, those who enjoy longer life expectancy do gain more from contributing to the same PAYG scheme.

In contrast, the intra-generational effects diminish substantially in the public-private pension scheme described in section 6,. The blue-collar group would now only suffer a 2.2% drop in pension benefits while the white-collar group's benefits would rise by 1.0%. We therefore find that the public-private scheme eliminates around 80-90% of the intra-generational transfers associated with the communal pension scheme. The remaining intra-generational transfers are caused by the fact that members of the L group are more likely to die before reaching retirement.

7.2. Longevity increase with pension age indexed to longevity

Now we increase the longevity of both classes proportionally. This allows us to determine the effects of increased longevity on the benefits received by both classes when the pension age is indexed to longevity. The life expectancy of white- and blue collar workers from equation (30) can be rewritten as:

$$\Lambda^{\mathrm{i}} = \frac{1}{\mu_1^i}\left(\frac{\ln\left(\mu_0^i\right)\mu_0^i + 1 - \mu_0^i}{(\mu_0^i - 1)}\right) \tag{43}$$

In the baseline above, we assigned identical μ_1 values for both groups, making the fraction $1/\mu_1^i$ in equation (43) identical for the groups. Any difference in the lifespan of the groups is then captured by differences in μ_0. We can then simulate a proportional increase in life expectancy through a decrease in μ_1.

We consider two cases. In the first case, the pension age is increased so that individuals spend the same proportion of their lifetime as before, on average, contributing to the pension system. In the second case, which is what is practiced in Denmark, the system is designed to keep the years spent as a pension recipient constant and the pension age therefore increases on a one-to-one basis with life expectancy.

In the proportional indexation setting we arrive at this formula for the pension age;

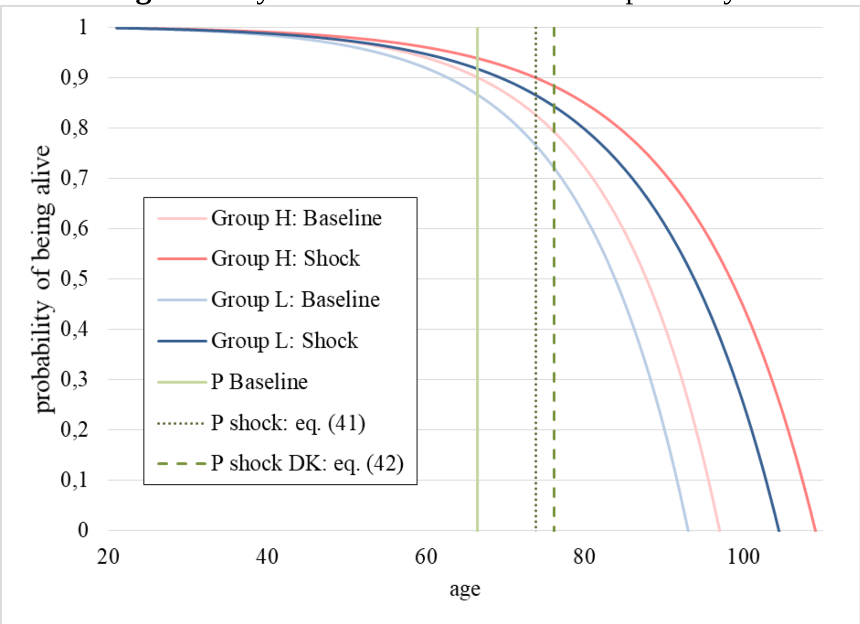
$$P(\Lambda) = \frac{\lambda}{\mu_1}\left(\sigma\frac{\ln(\mu_0^L)\,\mu_0^L + 1 - \mu_0^L}{(\mu_0^L - 1)} + (1-\sigma)\frac{\ln(\mu_0^H)\,\mu_0^H + 1 - \mu_0^H}{(\mu_0^H - 1)}\right) \tag{44}$$

while in the Danish system we find:

$$P^{DK}(\Lambda) = \frac{1}{\mu_1}\left(\sigma\frac{\ln(\mu_0^L)\,\mu_0^L + 1 - \mu_0^L}{(\mu_0^L - 1)} + (1-\sigma)\frac{\ln(\mu_0^H)\,\mu_0^H + 1 - \mu_0^H}{(\mu_0^H - 1)}\right) - 14.5 \tag{45}$$

Now we can impose a symmetric longevity shock by increasing the life expectancy of both groups and the pension age simultaneously. The baseline is the case presented in Table 3 and Figure 3. Assume an increase in life expectancy of ten years for white collar and an increase in the life expectancy of the blue-collar group of 9.4 years.[10] The shift of the mortality curves and the pension age are presented in Figure 4 where the dotted green vertical line corresponds to equation (44) and the dashed green one to the Danish system in equation (45).

Figure 4. Symmetric increases in life expectancy



The results of the calibration exercise are presented in Table 4.

Table 4. Symmetric longevity increase

	White collar (H) (baseline)	Blue collar (L) (baseline)	White collar (H) (shock)	Blue collar (L) (shock)
μ_1	0.075	0.075	0.065	0.065
μ_0	299	222	299	222
Life expectancy (years)	83.0	79.1	93.0	88.4
Pension age ($\lambda = 0{,}76$) (years)	66.5	66.5	73.9	73.9
Pension age (DK style) (years)	66.5	66.5	76.2	76.2

In the case where the retirement age increases proportionally to life expectancy (equation (44)) we see that the loss experienced by blue-collar workers from paying into the same pension fund as white-collar workers has not changed at all. This holds true both if we allow for income disparities between the groups and when we do not.

[10] Notice that even though the longevity increase for the white-collar groups is greater in years compared to the blue-collar group, their life expectancy rose by the same percentage.

The Danish pension age indexation rule does however affect the intra-generational transfers. In the case where both groups pay the same dollar-amount into the pension scheme the loss from paying into the same fund as white collars increases by an additional 1.0%, from 10.4% to 11.4%, compared to paying into their own pension fund. By allowing for income disparities between the groups, the blue collar's loss from the join pension scheme becomes 15.1% (compared to 13.8% in the baseline) and the gain of the white-collar class becomes 6.6% (6.1% before). The Danish style indexation rule therefore exasperates the intra-generational consequences of the pension scheme.

To demonstrate the sensitivity of a longevity-indexed rule based on life expectancy to the shape of the mortality function we perform the same experiment, but with a new baseline in terms of μ_0 and μ_1, for white-collar workers. Now both classes share identical μ_0 values and any differences between the classes are captured by μ_1. We can then simulate longevity increases for both classes by changing the value of μ_0. The results are presented in Table 5.

Table 5. Asymmetric longevity increase

	White collar (H) (baseline)	Blue collar (L) (baseline)	White collar (H) (shock)	Blue collar (L) (shock)
μ_1	0.075	0.080	0.075	0.080
μ_0	299	299	640	640
Life expectancy (years)	83.0	79.1	93.0	88.4
Pension age ($\lambda = 0{,}76$) (years)	66.5	66.5	73.9	73.9
Pension age (DK style) (years)	66.5	66.5	76.2	76.2

In this new baseline, we have the same life expectancies and pension ages as in the previous baseline. As before, the longevity increases are ten years for white-collar workers and 9.4 years for blue-collar workers. The pension age responses to the longevity increase are identical to the experiment presented in Table 4. These two experiments therefore appear identical, but the subtle difference in how the mortality curve shifted have implications for the intra-generational transfers.

Now we see that in the case where the pension age increases proportionally to life expectancy the loss experienced by blue-collar workers from paying the identical dollar-amount into the same pension fund as white-collar workers has increased by an additional 0.4% (compared to no change in the symmetric longevity shock). This demonstrates that the shape of the mortality curve over the lifecycle matters, not just life expectancy. When the pension age follows the

Danish rule the loss from paying into the same fund as white collar workers increases by 2.4% instead of 1% before.

We conclude that even though the life expectancy increases are enjoyed across society, the longevity indexation of the pension age will increase the intra-generational disparities. Furthermore, the type of longevity indexation of the pension age has a substantial effect on the magnitude of the intra-generational transfers due to longevity developments. The Danish system of maintaining a constant length of time receiving benefits penalizes the short-lived group more.

8. Concluding remarks

We have found that groups who have lower life expectancy suffer a drop in lifetime income when forced to share a pension scheme with others who have longer life expectancies. In effect, the lower life-expectancy group pays as much into the PAYG scheme, provided they reach the pension age, but receives less during the pension period due to lower life expectancy.

When the pension age is raised to reflect the increased longevity of the higher life-expectancy group the lower life-expectancy group suffers because even fewer of them will reach the pension age. Also, when contributions are made proportional to wage income, as is typically the case for PAYG schemes, it follows that an increase in the income of the group that enjoys greater longevity will reduce the pension benefits and lifetime utility of the group with less longevity.

While our research has been mainly motivated by observations from the Scandinavian countries, our findings have much more general interest. For example, based on US data, Chetty et al. (2016) observe that the gap in life expectancy between income quartiles has increased dramatically between 2001 and 2014, as the life expectancy of men in the lowest (highest) quartiles grew by 1.1 (2.8) years. If this trend continues, our findings may have important implications for pension policy in economies around the world.

Indeed, the spread between life expectancies between, say, academics and blue-collar workers could reach such high levels that it would hardly be controversial to allow for differentiated pension ages. In practice, schemes designed to link the pension age to changes in longevity and which operate on average figures should be extended to allow for variability in physical and mental disabilities across different groups in society.

A number of models could be used to implement this. One suggestion is to allow people to receive pension benefits before they reach the official pension age, subject to means-testing.

Another might permit people to start receiving pension benefits some years before they reach the official pension age without means-testing. However, such public pensions should be underpinned by an actuarial principle, to ensure that the total pension benefits received by an individual during their life as a retiree is not affected by the time they choose to retire.

Our proposed solution to the problem of intra-generational inequities is to give each retiree a lump sum at a certain age that may coincide with formal retirement from the labour force, which the worker gives to her occupational pension fund. The pension fund then decides on the monthly benefits for its members depending on their average life expectancy. Alternatively, if there is too much heterogeneity in life expectancy within the pension fund or occupation, the retiree could go to an annuity company that would assess his expected longevity.

In effect, the government is giving the money used to fund retirement to an entity that has more information about life expectancy and discontinuing the pooling of a heterogeneous group of workers. In practice, the government would issue a bond and give it to a pension fund, amount depending on the number of members reaching a certain age, providing the pension fund with a fixed income that it can then use to finance the retirement benefit of members.

There is also the possibility that the disutility of work may differ between groups, so that the group with lower life expectancy also has a greater disutility of work, and that this disutility may increase with age.[11] The shorter life expectancy and the rising disutility of work may both stem from the depreciation in health and human capital that workers experience due to more difficult tasks and working conditions. In this case, the short-lived group is adversely affected by a common pension age, due both to the redistribution of pension income shown in this paper to those who, on average, live longer and because their lifetime utility is adversely affected by raising the pension age due to the higher disutility of work. This latter effect remains a topic of future research.

To sum up, introducing a longevity-indexed pension age is critical for keeping fiscal policy on a sustainable track in economies that are subject to population ageing. That is why it is important to maintain a broad support of such schemes, both in the population and across a broad political majority. If the legitimacy and credibility of introducing such key welfare reforms critically depends on easier access to earlier pension age for citizens with lower life expectancy and diminished (physical and mental) ability to work, then such adjustments to the reforms might well be worth advocating.

[11] See Böckerman, Petri and Pekka Ilmakunnas (2019).

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Appendix

Figure A1