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Public employment redux☆

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ABSTRACT

The public sector hires disproportionately more educated workers. To rationalize this finding, we propose a model with a perfectly competitive private sector, and non-Walrasian public sector. Our economy also features heterogeneity across individuals and jobs, and a simple sorting mechanism that generates underemployment - educated workers performing unskilled jobs. We find that the public-sector wage differential and excess underemployment account for 15 percent of the education bias, with the remaining accounted for by technology. In a counterintuitive fashion, we find that more compressed wages in the public sector raise inequality in the private sector.

1. Introduction

The US government spends 60 percent more on compensation of general government employees than on purchases of intermediate goods and services. While purchases of goods and services operate through the output market, employment and wages operate through the market of inputs – the labour market. The large wage bill reflects the fact that the US government hires 16 percent of all employed workers, but also that it hires disproportionately more educated workers. The left panel of Fig. 1 reports the US government employment share for nine educational categories, from few years into primary education until tertiary education. The US government hires fewer than 5 percent of workers without education beyond the 9th grade. At the top, the government hires one third of all employed workers with Masters or Professional degree or who hold a Ph.D.[1]

Our main objective is to understand why the government hires so many educated workers. We consider three explanations. First, the type of services and the technology used to produce them differ from the private sector, and might require more educated workers. Second, a cost-minimizing government constrained to pay a compressed profile of wages (i.e. due to union pressures or political economy reasons), shifts its ideal composition from the (relative more expensive) less qualified workers to the (relative less expensive) more qualified workers. The third explanation is underemployment - educated workers performing unskilled jobs – that we find to be more prevalent in the public sector.[2] Our main contribution is to develop a model that encompasses these mechanisms in a general equilibrium context. By studying what explains the skill mix in the public sector in general equilibrium, we can also analyze the implication of policies for wage inequality in the private sector.

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[1] Although the US data refers to government employment, throughout the paper we prefer to use the term public(-sector) employment that is slightly more encompassing.

[2] In OECD economies a large share of workers are employed in jobs that require qualifications lower than the ones they have, as reported by McGowan and Andrews (2015, 2017). The term underemployment has multiple interpretations in different literatures. It is defined in Wikipedia as "the underuse of a worker because a job does not use the worker's *skills*, is part-time, or leaves the worker idle." Our research focuses on a purely empirical construct reflecting the first dimension. In the mismatch literature, it is sometimes referred as over-qualification, over-education or over-skilling depending on the specific context.

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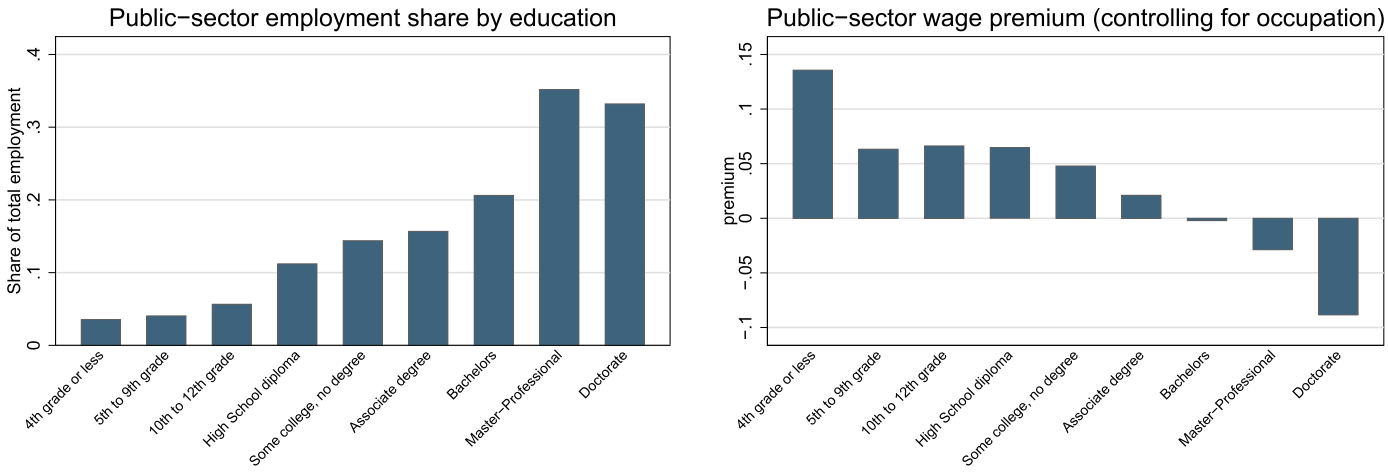


Fig. 1. Public-sector employment share and wage premia by educational levels, US. *Note:* The graph on the left shows the fraction of public-sector employment out of total employment for each educational level. Government workers (Federal, State and Local government), fraction of employment of workers age 16 to 64 with a given level of education. The graph of the right shows the public-sector wage premium, estimated by regressing the log of hourly wage on a public-sector dummy and controls (2-digit occupations, age, gender, region, year and a part-time dummy), separately for workers with different education levels. CPS data, average between 1996 and 2018.

We first provide empirical foundations for our paper. We look at survey data to establish the stylized fact of a public-sector education bias. We summarize education into two categories: college and no-college, and show that the education bias holds across gender, age, states, level of government, as well as over time and for different countries. We then provide empirical evidence underlying the three possible explanations. First, we show that industry and occupation composition of the public sector is important, which supports the idea that public sector needs more educated workers to provide services such as education and health. However, we show that the education bias also holds within industries and in two thirds of 3-digit occupations that are common across the two sectors, so composition alone cannot explain it. Second, we document the wage compression across educational levels in the public sector, with higher (lower) pay for low (high) educated workers vis-à-vis the private sector, as illustrated in the right panel of Fig. 1. Third, we define and report estimates of underemployment across countries and across public and private sectors. A large share of workers are employed in jobs that require qualifications lower than the ones they have, in particular in the public sector. The empirical evidence on wage compression has been documented in previous studies.[3] The originality of the paper lies in providing more detailed evidence on the education bias over different dimensions, evidence on underemployment, and relate them with evidence on the public-sector wage compression.

To understand how the three explanations (technology, wage compression and underemployment) relate to public-sector education bias, we develop a two-sector general equilibrium model with underemployment. The model has two key features. First, alongside a perfectly competitive private sector, our economy features a cost minimizing government facing a wage schedule that does not necessarily equate demand and supply of labour. Given a wage schedule, the government decides how many jobs of different skill requirement it needs to produce a given level of public services. In this sense, our model merges a neoclassical Walrasian private sector with a public sector modeled in the spirit of disequilibrium theories à la Malinvaud (1977) and Barro and Grossman (1971). Second, our economy features heterogeneity across individuals and jobs. Workers can be high- or low-educated while jobs have different skill requirements. Jobs are described through a ladder type mechanism, so that individuals endowed with higher education are able to perform also unskilled jobs, but workers with low education cannot perform skilled jobs.

We assume a variation of the Roy model (Borjas, 1987; Roy, 1951) in which workers attach different "non-pecuniary' value to jobs in different sectors and of different skills. This preference structure generates a non-trivial sorting mechanism that serves two purposes. First, we generate a labor market allocation with endogenous underemployment, that depends on the wage differential between jobs of different skills. Second, it allows for both positive and negative wage premium in the public sector for different workers. On the one hand, when public wages are above the underlying market clearing wages, there would be more workers interested in having a public-sector job than available jobs, with the excess workers driven to the private sector. In this regime, public employment is demand determined and jobs are rationed. On the other hand, if the public-sector wage premium is negative, the government can only fill all of its jobs if there are enough workers with a strong preference for the public sector. Further, if wages for a certain type of workers are below the implicit market clearing wages, the government might be constrained in hiring and forced to substitute to another type of workers to maintain the production of its services. In this regime, public employment is determined by supply. It is also possible that, if both wages decrease below a threshold, the public sector can no longer produce the minimum level of services and breaks down. Our model incorporates the three possible explanations for why public employment is biased towards educated workers: technological, the compression of the wage schedule that shifts the ideal skill-mix in the government, and underemployment which interacts with and amplifies the role of the wage schedule.

We calibrate a variation of the model to match key statistics of the US economy. The model is parsimonious, and seven structural parameters are obtained by matching seven moments, including public employment and the public-private wage differential by education, and a conservative estimate of underemployment. According to the calibration, public jobs are rationed (demand determined). We carry out two quantitative exercises. First, we solve the model under the assumption that wages in the public sector equalize wages in the private sector, which also eliminates excess underemployment in the public sector. We then solve it with the additional assumption that technology is the same in the two sectors. We find that, in the US economy, the excess hiring of educated workers in the public sector is mainly accounted for by technology, with the wage differential and excess underemployment in the public sector accounting for 15 percent of the education bias. In other words, if the

[3] See Katz and Krueger (1991) for the United States, Postel-Vinay and Turon (2007) or Disney and Gosling (1998) for the United Kingdom and Christofides and Michael, 2013 or Castro et al. (2013) for several European countries.

US government aligned its wage schedule with that of the private sector, it would provide the same services, but employing fewer 350 thousand college workers and more 480 thousand workers without college and paying a 4 percent lower wage bill.

In our second exercise, we calculate the elasticities of private wages with respect to public wages. During the Euro Area crisis, many governments reduced public-sector wage dispersion by cutting high wages while protecting low-wage workers. We find that the government wage policy is a crucial driver of private wage inequality, but in an counterintuitive fashion – a more compressed wage schedule in the public sector raises inequality in the private sector. More wage compression alters the skill-mix in the public sector from unskilled to skilled jobs. The skill-mix in the private sector then shifts towards low-educated workers, so their wages fall while wages of high-educated workers go up. A one percent increase in unskilled public wages raises skilled private wages by 0.07 percent and lowers unskilled private wages by 0.06 percent. In European countries the elasticities are up to four times larger. While decreasing wage inequality for workers in the public sector, well-intended policies aiming at reducing wage inequality within the public sector can actually backfire by increasing wage inequality for everyone else in the economy.

Our paper proceeds as follows. Section 2 reviews the recent literature on public employment and how this paper contributes to it. Section 3 documents the main stylized fact that government employment is biased towards educated workers, and evidence supporting each of the three possible explanations. Section 4 constructs the two-sector general equilibrium model with under-employment. Section 5 presents analytical results. Section 6 presents quantitative results. Section 7 concludes.

2. The renewed interest on public employment

Public employment is a particular interesting and complex subject because it lies in the intersection of different literatures: labour economics, macroeconomics of fiscal policy, public economics, political economy and personnel economics. While it attracted large academic interest between the 1970s and the 1990s, well summarized in two Handbook of Labour Economics' chapters by Ehrenberg and Schwarz (1986) and Gregory and Borland (1999), this interest diminished in the following decade.[4] The literature on public employment is now staging a renaissance, as reported in Garibaldi and Gomes (2021).

Sparked by the fiscal policy responses to the Great Recession and the Euro Area crisis, a new wave of theoretical research uses search and matching models to study the effects of public employment and wages on unemployment and other labour market outcomes. Examples include (Afonso and Gomes, 2014; Albrecht et al., 2019; Bradley et al., 2017; Gomes, 2015; 2018; Michaillat, 2014; Postel-Vinay and Turon, 2007; Quadrini and Trigari, 2007), and more recently (Boeing-Reicher and Caponi, 2017; Esteban-Pretel et al., 2021; Geromichalos and Kospentaris, 2020; Navarro and Tejada, 2021). Our approach to model the choice of workers in the public sector - based on a cost minimization - is similar to Gomes (2018). His model has search and matching frictions and is solved quantitatively. Our model has a simple structure summarized by few equations allowing the study of underemployment across sectors and how it interacts and amplifies the role of the government wage schedule.

While search and matching frictions naturally allow the presence of wage differentials and are important to study particular aspects of public employment, such as the role of job security extensively analyzed in Chassamboulli and Gomes (2021); Gomes (2015) or Fontaine et al. (2020), we think that some of its consequences can be more clearly understood with a frictionless labour market. More precisely, the skill mix chosen by the government is bound to affect the skill mix of the private sector, even in a full employment context. The papers taking this approach that are most closely related to ours are (Domeij and Ljungqvist, 2019; Gomes and Kuehn, 2017). Domeij and Ljungqvist (2019) build a neoclassical model, in the spirit of Finn (1998), where the public sector hires an exogenous number of skilled and unskilled workers, to compare the evolution of the skill premium in US and Sweden. They point out that the expansion of the Swedish public sector, that hired more low-skilled workers, can explain the divergence of the skill premium between the two countries. Gomes and Kuehn (2017) study, in a model of occupational choice, the effects of education-biased hiring in the public sector on the occupational choice of entrepreneurs and on firm size. Relative to these two papers, we endogenise the choice of the type of workers the public-sector hires, add underemployment, and allow for different wages across sectors. Our model, without search and matching frictions, can be interpreted as providing a long-run perspective.

Other papers with frictionless labour markets include Cavalcanti and Santos (2020); de Córdoba et al. (2010) or Baerlocher (2020). de Córdoba et al. (2010) and Cavalcanti and Santos (2020) assume that public wages are always higher than the private, so government employment is demand determined and jobs are rationed. In de Córdoba et al. (2010) public jobs are allocated randomly while in Cavalcanti and Santos (2020) they are allocated based on ability. Baerlocher (2020), like Domeij and Ljungqvist (2019) and Gomes and Kuehn (2017), assumes the wages in the two sectors equate. None of these papers allow for the possibility of wages to be lower in the public sector, which is at odds with the fact that some workers do receive lower wages. The way we formalize the acceptance of lower wages by some public-sector workers is novel and originates from the discussion by Musgrave (1982). Musgrave (1982) argued that an argument for public employment arises, when workers themselves prefer to be employed publicly, either because of better working environment or a pure preference. If a sufficient large number of workers have strong preferences for the public sector, hiring might still be satisfied, even with wages below those of the private sector.

The public sector is very different from the private sector, and the usual economic mechanisms that drive the private sector adjustments do not map into the public sector. One of the missing adjustment channels is wages. When governments set their wages (or wage growth), there is a discretionary component that can create the widely documented wage differentials vis-à-vis the private sector. Our assumption that public wages may not necessarily equate demand and supply in the labour market is well grounded in the public economics and political economy literature that has shown that governments use public employment and wage policies for a multitude of objectives, beyond the production of government services. These include: attaining budgetary targets (Gyourko and Tracy, 1989); implementing a macroeconomic stabilization policy (Keynes, 1936); redistributing resources (Alesina et al., 2000); or satisfying interest groups for electoral gains (Gelb et al., 1991). While these papers try to provide a justification for particular policies, the modelling of the labour market and general equilibrium effects tends to be too simplistic or nonexistent. Our approach is complementary. From the perspective of the labour market, the source of wage differentials is not particularly relevant, so we consider it to be determined outside the model and concentrate on studying their general equilibrium implications.

In term of modelling choice, the assumption that public wages do not adjust to equate supply and demand is related to the fixed-price equilibrium literature that followed from Barro and Grossman (1971); Malinvaud (1977). More recent papers in this literature include (Benassy, 1993; Michaillat and Saez, 2015). We think that this is a natural assumption when thinking about the public sector labour

[4] The main themes of the earlier literature were the role of unions, political and budgetary aspects on the wage determination in the public sector, exemplified by Borjas (1980); Courant et al. (1979); Freeman (1986) and Gyourko and Tracy (1989). More related to our theory were papers estimating the length of the queues for Federal jobs, namely Blank (1985) or Krueger (1988), or papers estimating the wage elasticity of demand of government employees, for instance (Ehrenberg, 1973). The 4th Volume of the Handbook of Labour Economics of 2011 does not have a chapter dedicated to public employment.

market, but not for the private sector that we model as Walrasian. One different feature of our framework is that when jobs are rationed in the public sector, workers can always go to the private sector so there is never unemployment in the economy, a dimension that we abstract from entirely.

Finally, a more microeconomic literature on personnel economics of the government analyses the role of financial incentives for recruitment and performance, as well as issues of selection on unobserved characteristics. This literature is well summarized in Finan et al. (2015). While we do not believe we can rationalize a Walrasian public-sector labour market by accounting for selection on unobservable characteristics, we certainly share the view of the literature that the public-sector wage has important implications for which workers with a given education select into the public sector. Our model can be easily extended to include heterogeneous ability in order to illustrate this point. More than the wage compression across education, the issues of selection on unobservables speak more to another well-documented fact that public-sector wages are also compressed within education groups. We show that, if the government does not fully reward their efficiency units as in the private sector, the high-ability educated workers are less likely to go to the public-sector, that has to hire more low-ability workers to compensate, which reinforces the education bias.

3. One stylized fact, three possible explanations

Section 3.1 reports the evidence on the public-sector education bias across various dimensions and countries, which is the main stylized fact we want to explain. Section 3.2 reports empirical evidence on the three possible explanations. First, we show that industry and occupation composition of the public sector is important but alone cannot account for the education bias. Second, we document evidence on the wage compression across educational levels in the public sector. Finally, we define and report estimates of underemployment across countries and across public and private sectors. The originality of the paper lies in documenting the larger underemployment in the public sector and in connecting the stylized fact to the three possible explanations.

The main dataset used is the CPS. This survey provides labor force status, as well as information on demographics, sector, occupation, industry, weeks and hours per week worked. For the calculation of the stocks we use the monthly files from 1996 to 2018. We restrict the sample to individuals aged 16 to 64. When we estimate the public-sector wage premium we use the CPS March Supplement that has information on total income and income components. The distinction between public and private sector jobs is based on a self-reported variable. Each respondent is asked to classify his/her employer. We define public-sector employment as work for the Government (whether Federal, State or Local government). This method is consistent with the statistics published by the BEA.

We also analyse data from the United Kingdom, France and Spain used in Fontaine et al. (2020) and Chassamboulli et al. (2020). We choose these countries because their public sectors ar sizable and encompass different industries and they employ distinct hiring processes. Furthermore, these economies have different labor market institutions and education policies. This guarantees that common findings across these four countries are likely to be intrinsic characteristics of the public sector and are not driven by country specificities. Our analysis is based on microdata and in particular, for each country, we use the representative labor force survey, from which official statistics are drawn: the French *Labour Force Survey* (FLFS), the UK *Labour Force Survey* (UKLFS) and the Spanish *Labour Force Survey* (SLFS). See Fontaine et al. (2020) for details on the definition of the public sector. For the wage regressions, we use microdata from the 2002, 2006, 2010, and 2014 *Structure of Earnings Survey*.

Finally, evidence of underemployment comes from the OECD Survey of Adult Skills, part of the Program for the International Assessment of Adult Competencies (PIAAC). The data were collected between 2011 and 2015. In each country, the survey includes socio-demographic information (gender, education), labor market status and assesses the proficiency of adults aged between 16 and 65 in literacy, numeracy and problem solving.

Table 1
Basic accounting with two sectors and two education categories.

	Public sector	Private sector	Total
College	l_1^g	l_1^p	n
No-college	l_2^g	l_2^p	$1-n$
Total	l^g	l^p	1

Note: Government (g), private (p), college (1), no-college (2). Total employment is normalized to 1. Share of college in total employment (n).

3.1. Public-sector education bias

Fig. 1 reported the public employment share for nine educational categories, illustrating the tendency of the public sector to employ workers with higher degrees of education. For simplicity, throughout the paper, we summarize education into two categories: college and no-college. College includes workers with an Associate degree, Bachelors, Master and Doctorate. We include workers that attended but not completed college in the no-college category. Still, one should keep in mind the further heterogeneity within these groups.

Table 1 reports the accounting definition used in the paper. We normalize the size of the employment pool by 1, and we let n and $1-n$ denote respectively the share of employed workers with and without a college degree. College workers are indicated with subscript 1 while no-college workers with subscript 2. Superscript g refers to the government/public sector while superscript p refers to the private sector. We thus indicate with l_1^g the stock of college workers employed in the public sector (similarly for the other 3 categories).

Fig. 2 shows the bias of the public sector towards workers with higher education in the US, UK, France and Spain. The top-left panel shows the fraction of public employment out of total employment for workers with and without a college degree (l_1^g/n and $l_2^g/(1-n)$). The top-right panel shows the fraction of college graduates out of total public and private employment (l_1^g/l^g and l_1^p/l^p). UK and France have larger public sectors (more than 22 percent of total employment), while Spain has similar levels as the US. In all the four countries the public sector hires significantly more workers with at least a college degree.

Given the two-by-two matrix described in Table 1, we further summarize the education bias in the public sector with one of two indicators. The first indicator is the ratio of public employment shares r^g, simply defined as the ratio of public employment share for college workers over the public employment share for non-college workers. The second statistics is the education intensity ratio ei^g, defined as the ratio of the share of college graduates out of public sector workers over that of the private sector. Formally:

$$r^g = \frac{\frac{l_1^g}{n}}{\frac{l_2^g}{1-n}}, \qquad ei^g = \frac{\frac{l_1^g}{l^g}}{\frac{l_1^p}{l^p}}.$$

These two statistics, shown at the bottom of Fig. 2 are complementary. In the case of perfect symmetry across sectors, both statistics would have a value of 1. The statistics are above 1.4 for the four countries reported. It is lower in France and higher in Spain. The US has a ratio of public employment shares of 2 and an education intensity ratio of 1.5. In the remaining of this section, we focus on the ratio of public employment shares, but we report in Appendix A all the figures with the education intensity ratio (Fig. A.2 and A.3).

In Appendix A, focusing on US data, we show the different statistics across gender, age, US states, and over time (Fig. A.1 and A.2). The ratio of public employment share is constantly around 2 across gender and age. When we disaggregate by US states, the ratio of public employment

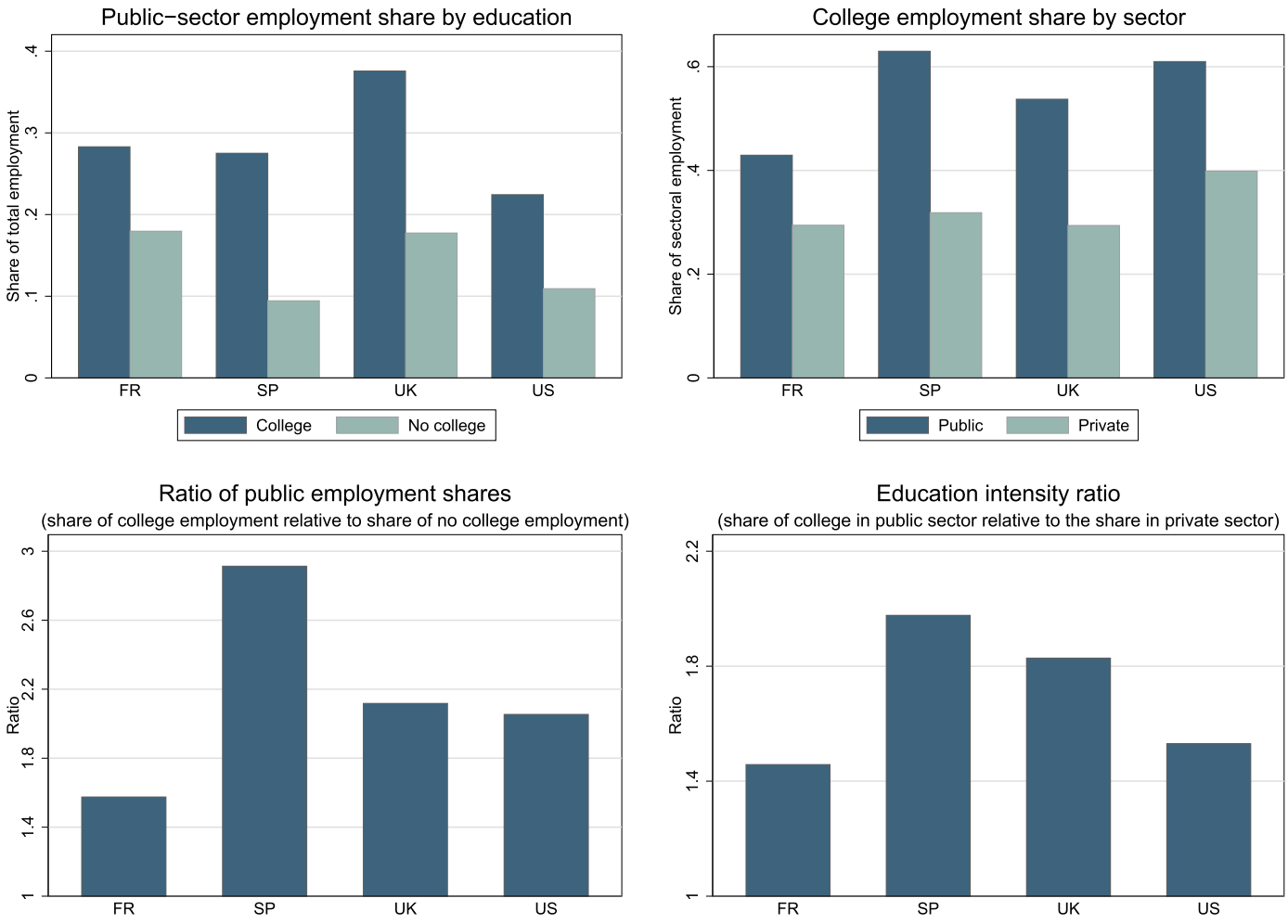


Fig. 2. Public-sector education bias: two simple indicators. *Note:* the top-left graph shows the public employment shares, the fraction of public-sector employment out of total employment for college and not college graduates. The bottom-left graph shows the ratio of public employment shares (r^g). The top-right graph shows the education intensity by sector, the share of public-private workers that have a college degree. The bottom-right graph shows the education intensity ratio (ei^g). For the United States the data is take from CPS (1996–2018), for the United Kingdom from the UK Labour Force Survey (2003–2018), for France for the French Labour Force Survey (2003–2018) and from Spain from the Spanish Labour Force Survey (2005–2018). For details on the methodology for the European economies see Fontaine et al. (2020).

shares varies from 1.4 in Washington DC to 3 in Nevada. The ratio is also persistent over time, even though it fell around the Great Recession, most likely because of large changes in private-sector employment.

3.2. Three possible explanations

3.2.1. Technology: industry and occupation composition

To account for the education bias, a first candidate is to look at the types of services that the government produces. One key empirical finding of this section is that the public-sector education bias holds across industries in the US, France and the UK (Fig. 3). The Spanish LFS does not allow for a disaggregation of public employment by industry. On the one hand, even when excluding the Health and Education industries, industries that naturally employs a large share of graduates, the bias remains, although with lower ratio. The US ratio of public employment shares is 1.8 instead of 2. On the other hand, even within the health and education industries, the public sector hires a larger fraction of graduates than the private sector, leading to a ratio larger than 1.

To dig further into the composition of public-sector jobs, we look at the occupational classification from 3-digit ISCO-08 in the US.[5] We consider only occupations that are common to the two sectors, where the share of public employment in total employment is larger than 5 and below 95 percent.[6] We find that two-thirds of the occupations have ratio of public-employment shares larger than 1. Overall, the distribution across industries and occupations appear important, and indeed will play a key role in the theory that we propose, but it does not explain everything.

3.2.2. Public-sector wage compression

The second possible explanation concerns the wage policy and the tendency to compress wages across educational groups. Specifically, low-educated public-sector workers tends to be paid more than their private-sector counterparts, while the public-sector wage premium of high-educated workers is lower (and sometimes negative). The basic evidence of wage compression comes from a simple Mincer regression on log hourly wages on a variety of controls, including the public-sector dummy.[7]

[5] CPS occupational code is based on 2010 Census 3-digit occupational classification. We use a cross-walk in order to classify occupations based on 3-digit ISCO-08.

[6] In doing so, some top-paid occupations are dropped (such as Manufacturing, mining, construction, and distribution managers; Architects, planners, surveyors and designers) as well as some low-paid jobs (such as Domestic, hotel and office cleaners and helpers, Vehicle, window, laundry and other hand cleaning workers, Waiters and bartenders).

[7] Gregory and Borland (1999) describe different methods to estimate public-private wage differentials of which the dummy variable approach is the most elemental. More

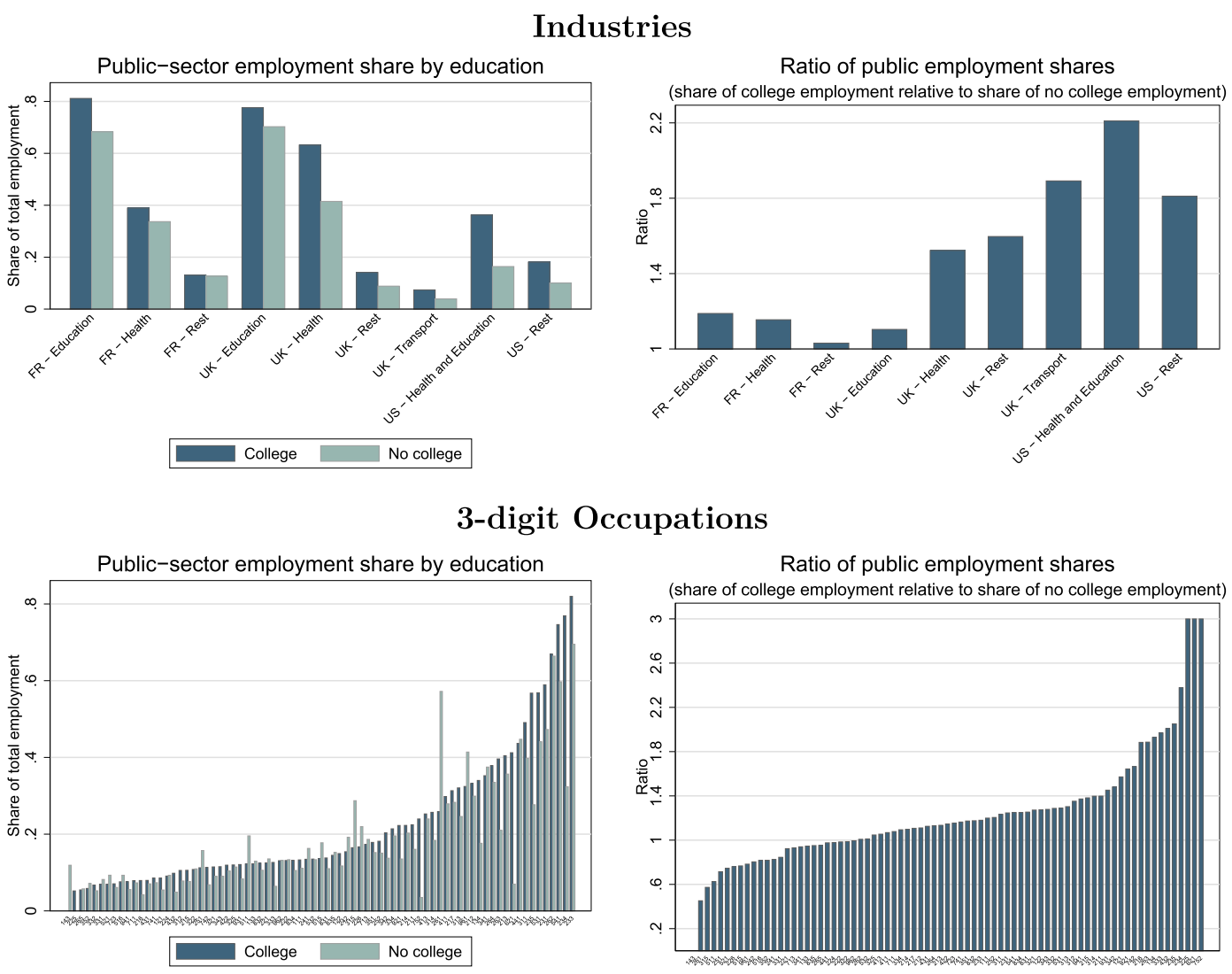


Fig. 3. Public-sector employment share across industries and occupations. *Note:* 1st panel uses French, Spanish, UK Labour Force Surveys and the CPS. 2nd panel: CPS data, average between 1996 and 2018. 3-digit occupations that have an overall share of public-sector employment between 0.05 and 0.95. On the right-hand graph, the ratio was capped at 3 for readability.

Table 2 shows the estimations for our two categories: college and no-college workers. Controlling for 2-digit occupations, the estimate of the public sector wage premium is of 1 percent for college graduate and 7.7 percent for workers with no college. We will use these numbers in the quantitative section. If we do not control for occupation, the public-sector wage premium even becomes negative for college workers. There is substantial variation of pay depending on whether the employer is the Federal, State or Local government, but there is wage compression in all three layers of government. The Federal government pays a premium of 0.10 to college graduates and 0.20 to workers without college. State and Local government offer a negative premium for college workers between −0.06 to −0.03 and a positive premium of 0.008 to 0.018 for workers without college.

To highlight the heterogeneity of the public wage policies, even within a country, we look at regional differences across US states, shown in Fig. 4. The public-sector wage premium for both college and no-college, as well as the difference between the two, varies across states by close to 20 percentage points. While there is large cross sectional variation in policies, the wage compression holds in 50 out of 51 US states. The state with highest compression is Washington DC and the only one with a negative compression is Kentucky.

Finally, we show in Appendix A the evolution of the public-sector wage premia over time, in the US, UK, France and Spain (Fig. A.4). The wage compression across educational group is persistent over time in all countries. The public sector dummy in the Mincer regressions is always larger for workers with low education. Remarkably, the policies on wages can vary substantially in a few years. Most striking is the case of France. Between 2006 and 2010 the estimated premium fell by 15 log points for both workers with and without college. In Spain, we find that the public-sector premium of college graduates fell from 0.10 in 2006 to 0.03 in 2014, while it remained constant for workers without college.

Grounded on the public economics and political economy literature, we interpret the existence of these differentials as having an explanation exogenous to the labour market. Alternatively, there could be two interpretations of the wage compression within a Walrasian framework. The first is that these wage premia correspond to compensating differentials, similar to other intra-industry wage differentials. There are some facts opposing this interpretation. One of the most important non-pecuniary

advanced methods include accounting for selection or decomposing the wage premium into the explained and unexplained components of the gap. These more sophisticated approaches, if anything, reinforce the wage compression fact. See Christofides and Michael, 2013.

Table 2
Regression of the log of hourly wages.

	Controlling for 2-digit occupation		Not controlling for occupation	
	College	No college	College	No college
Public-sector	0.010***	0.077***	−0.0262***	0.095***
	(5.09)	(40.79)	(−14.8)	(51.7)
Controls				
Age and gender	X	X	X	X
Region and year	X	X	X	X
Part-time	X	X	X	X
Occupation	X	X		
Observations	668,287	918,664	668,287	918,664
R-squared	0.294	0.247	0.155	0.167

Note: Estimation by regressing the log of hourly wage on a public-sector dummy and controls (age, gender, region, year and a part-time dummy), separately for workers with and without college graduate. When controlling for occupation we include 2-digit occupation dummies. CPS data between 1996 and 2018.

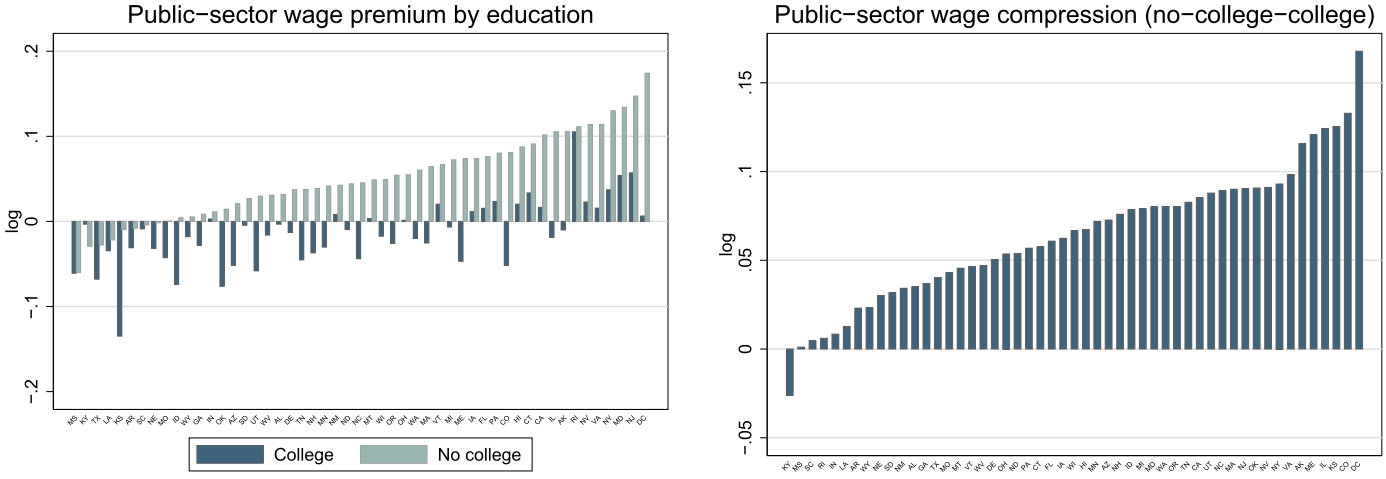


Fig. 4. Public-sector wage compression across US states. *Note:* Estimation by regressing, for each state, the log of hourly wage on a public-sector dummy and controls (age, gender, year and a part-time dummy), separately for workers with and without college graduate. When controlling for occupation we include 2-digit occupation dummies. CPS data between 1996 and 2018.

characteristics of public-sector jobs is its security. In the US, health care insurance is also important. Both these characteristics should be valued more by workers with lower education as they face higher risk of being unemployed or losing health care insurance, which would imply that the public-sector wage premium should be increasing in education. The second interpretation regards unobserved ability. The pattern could be generated if the government *purposefully* wanted to hire the more able workers with primary education and the Ph.D.'s of lower ability, the best janitor and the worst judge. Besides dismissing the existence of other government objectives external to labour market developments that influence public-sector wages, both explanations seem unlikely.

Furthermore, the wage compression suggests public employment is demand determined. If it was supply determined, for lower-educated workers who have a larger premium in the public sector we would observe higher levels of public employment, and for higher-educated workers that have a lower or negative premium, we would observe lower levels of public employment.

3.2.3. Larger underemployment in the public-sector

We refer to *underemployment* u, as to the stock of workers with college employed in jobs typically performed by no-college workers. This is a purely empirical construct.

There is widespread anecdotal evidence of underemployment in the public sector in many countries. In 2017, the Bank of Italy opened 30 jobs whose duties included feeding cash into machines, a rather unskill task. The 85,000 initial applicants were reduced to a "shortlist" of 8000, all of them college graduates that had to face further exams in different subjects to get the post (Geromichalos and Kospentaris, 2020). We look at the prevalence of underemployment in the public and private sectors across countries using survey data.

The CPS data provides some suggestive evidence that underemployment in the public sector contributes to the education bias. The public wage premium for college graduates is lower and negative when we do not control for occupation. This suggests that workers with a college degree in the public sector are more likely to be in lower paid occupations. To corroborate this suggestion, we correlate the ratio of public employment shares in 3-digit occupations (shown in the 2nd panel of Fig. 3) with the gross public-sector premium for no-college in those occupations.[8] Indeed, Fig. 5 indicates a positive and statistically significant relation between the level of public-sector pays for unskilled workers in a given occupation and the education bias within that occupation.

We provide more evidence of underemployment across countries, as well as across public and private sector. We need first some accounting. Similarly as above, n is the stock of employed college workers, and $1-n$ is the stock of non college workers. Let j_1 be the stock of skilled related jobs, only filled by graduates, so that $j_1 = n - u$. Further, j_2 is the stock of unskilled jobs that is filled by workers without college or underem-

[8] We compute hourly wage as the respondent's total pre-tax wage and salary income for the previous calendar year divided by the product of the number of weeks worked last year times the usual hours worked per week last year. We then consider the mean hourly wage in each occupation for the workers with no-college.

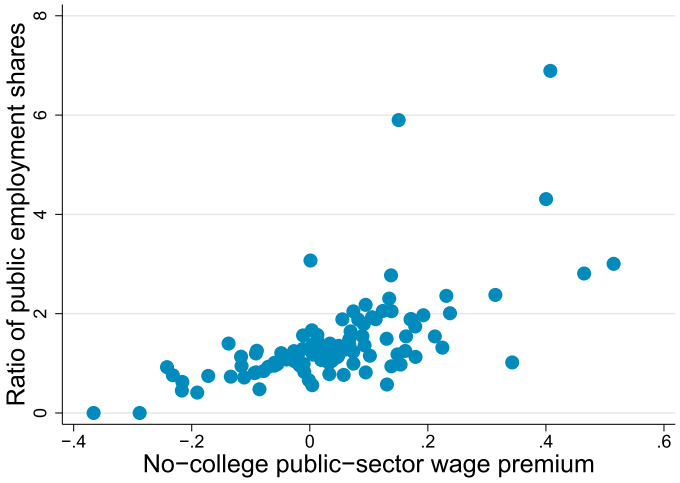


Fig. 5. Ratio of public-sector employment shares and no-college public-sector premia. *Note:* Ratio of public employment shares in 3 digit occupations with no-college public sector wage premium in the occupation. CPS data between 1996 and 2018.

ployed college workers, $j_2 = (1-n)+u$. We define the *underemployment rate*, indicated with $\tilde{u}$, as the fraction of unskilled jobs performed by college graduates. Formally:

$$\tilde{u} = \frac{u}{j_2}.$$

Similarly, we define the underemployment rate in private and public sector as

$$\tilde{u^p} = \frac{u^p}{j_2^p} \quad \tilde{u^g} = \frac{u^g}{j_2^g}.$$

We use PIACC data to calculate the underemployment rates. Supplementary details are provided in Appendix A.1. There is no consensus in the empirical literature on the best way to calculate underemployment. We use different procedures. Our main approach is related to the methodology used by OECD. We identify well-matched individuals as those who neither feel they have the skills to perform a more demanding job nor feel the need for further training to be able to perform their current job satisfactorily. By occupation (isco 1), we calculate the average and standard deviation of the number of years of completed education for (self-reported) well-matched workers. The required educational attainment of a given occupation is calculated as the mean of completed schooling of all well-matched workers (with a symmetric band of 1.96 standard deviation). Workers are defined as underemployed when their years of completed education are 1.96 standard deviation above the mean of well-matched workers in their occupation.

More formally, for individual i in occupation j, with years of schooling e_{ij}, the dummy "underemployed" u_{ij} equals:

$$u_{ij} = \begin{cases} 1 & \text{if} \quad e_{ij} > \bar{e}_j^{wm} + 1.96\sigma_{e_j^{wm}} \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{e}_j^{wm}$ ($\sigma_{e_j^{wm}}$) refers to the mean (standard deviation) years of completed education of well-matched workers in occupation j.

The left graph in Fig. 6 reports underemployment rate across countries. On average, more than 10 percent of unskilled jobs are held by people that have years of education well above those of well-matched people in that occupation. The minimum level is just below 5 percent in countries such as Austria and Ireland. The maximum is above 17 percent in Italy. Our key empirical evidence is in the graph on the right. In 15 out of 21 countries (more than 70% of our sample) the underemployment rate is larger in the public than in the private sector. In the US, the underemployment rate is 10.2 percent in the public sector and 8.7 percent in the private.

Our main measure of underemployment is extremely conservative, and is lower than most estimates from the literature.[9] We show in Appendix A.1 the underemployment rates calculated with three other less conservative methods (Fig. A.5). First, we use the same approach but change the cutoff. We classify as underemployed, workers whose years of completed education are 1 s.d. above the mean years of education of well-matched workers in their occupation (i.e. $u_{ij} = 1$ if $e_{ij} > \bar{e}_j^{wm} + \sigma_{e_j^{wm}}$). The second approach considers as underemployed college graduates that are not well-matched and work in 1-digit occupations that are majority non-college. The third approach is similar to the second but focusses on 2-digit occupations. These approaches give larger underemployment rates: 27 percent of unskilled jobs are held by underemployed workers using the first alternative and around 18 percent for the second and the third. Also, the higher prevalence of underemployment in the public sector is reinformed. In each of the approaches: 95, 90 and 85 percent of the countries display higher underemployment rate in the public sector.

[9] Barnichon and Zylberberg (2019) report that 38% of US college graduates work in lower skill-requirement occupations. Using meta-analysis, Leuven and Oosterbeek (2011) find that a third of American workers are over-schooled which is consistent with estimates by Clark et al., 2017.

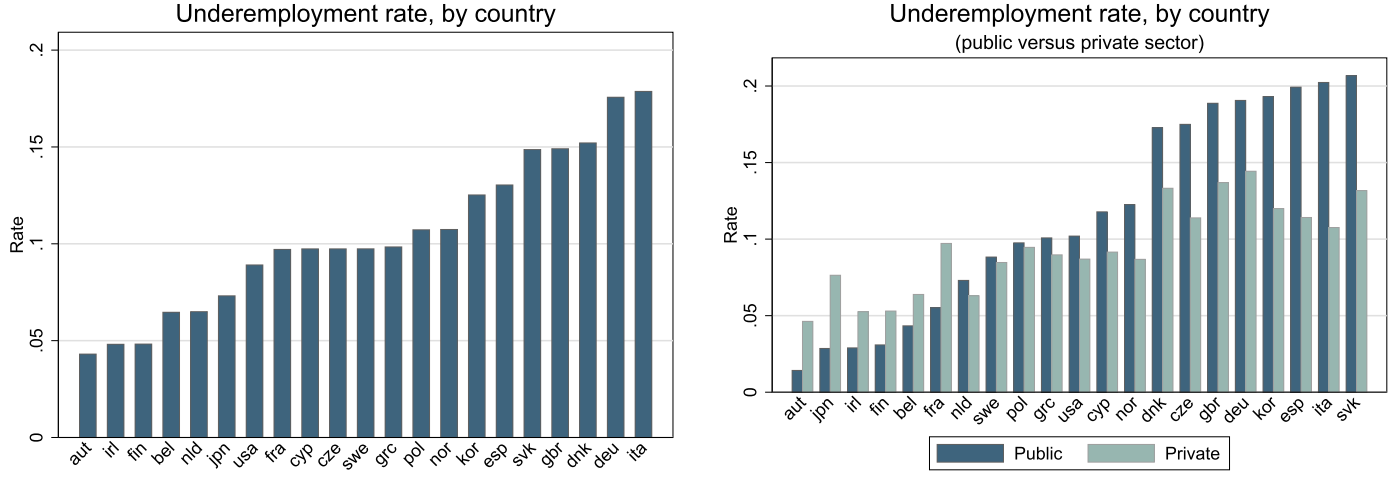


Fig. 6. Underemployment rate across countries. *Note:* PIAAC (Survey of Adult Skills). By occupation, we calculate the mean and standard deviation of the number of years of education of (self-reported) well-matched people. We consider as underemployed workers whose years of education are 1.96 s.d. above the mean years of education of well-matched workers in an occupation.

4. Two-sector model with underemployment

4.1. Technology and preferences

Individuals are endowed with one unit of indivisible labor. There are two types of individuals with high (1) and low (2) education. The supply of educated individuals in the economy is indicated with n, while the supply of the low-educated workers is indicated with $1-n$.

A representative firm and a government offer jobs requiring different skills. The superscript $x = p, g$ refers to the private or public sector and the subscript $e = 1, 2$ refers to both the education of the worker and the skill of the job. The government has j_1^g skilled jobs and j_2^g unskilled jobs, while the private sector has j_1^p and j_2^p. The representative firm produces a private-sector output y - the numeraire of the economy - with a constant return technology. In what follows we use a Cobb Douglas specification,

$$y = (j_1^p)^{\alpha}(j_2^p)^{1-\alpha}, \tag{1}$$

where α is the skill intensity. The government produces government services g – a different good from the private sector for which there is no market (price) – using,

$$g = (j_1^g)^{\beta}(j_2^g)^{1-\beta}. \tag{2}$$

We allow for technology to be different from the private sector $\beta \neq \alpha$ reflecting the fact that these governments services might require more or less skilled jobs.[10]

A key assumption concerns the ability of individuals to perform different jobs. Jobs can be described through a ladder type mechanism, so that individuals with high education are also able to perform unskilled jobs. They can perform at zero effort costs both type of jobs while individuals with low education can only perform at no cost the unskilled job, while we assume that the cost of effort required to perform the skilled job is (infinitely) large.

Individual preferences are linear and the model is static. Each individual worker i has an heterogeneous "non-pecuniary value" over skilled and unskilled jobs in the private and public sector $\epsilon_i^{x,e}$ drawn from a continuous distribution. We assume, for tractability, that they have an Standard Gumbel distribution (extreme type I error distribution).[11]

These "non-pecuniary" value of the job could reflect individual preferences, but also other elements such as location of job, hours, altruism, preference for job stability, or explicit costs of accessing a job. For instance, a worker i of type e, working in sector x, has an utility given by sum of the wage net of taxes and the "non-pecuniary" value, $(1-\tau)w_e^x + \nu\epsilon_i^{x,e}$, where τ is the income tax and ν captures the weight of the "non-pecuniary' value in the individual preferences. Personality traits have been found to be an important determinant in the selection into public employment (Maczulskij and Viinikainen, 2021). The non-pecuniary value, $\epsilon_i^{x,e}$, does not reflect compensating differentials that are equally valued by all workers, but is worker specific. It is a shortcut to capture all possible reasons that pushes people to accept a job with lower wages (labour market conditions, housing market or transport policies, as well as regulation of specific occupations). It is isomorphic to having it as a cost of performing a job i.e. $(1-\tau)w_e^x - \nu\epsilon_i^{x,e}$. As such, the model is not equipped to make any normative statement or to think about optimal policies. Our model accommodates the traditional model in the limit where ν tends to zero and workers always select the highest paying job.

4.2. A malinvaud government...

We assume that the government is required to produce a certain level of government services, $\bar{g}$, taken as exogenous. Given a wage schedule, the government chooses its target (demanded) level and composition of employment ($j_1^{g,d}$ and $j_2^{g,d}$), that minimizes the costs of producing the government services, $\bar{g}$.

For clarity of the model, we assume an exogenous wage schedule for skilled and unskilled jobs (w_1^g and w_2^g). This is not a critical assumption. The critical assumption is that the government wages do not adjust to equate supply and demand. We think this is a realistic assumption, given that the government does not sell its goods and services and finances the wage bill with taxes, so public-sector wages might be influenced by other factors, such as unions, redistribution or elections. Notice that the wages are paid in units of the private-sector good so they are essentially a transfer of resources from private- to public-sector workers.[12] We think this is realistic, given that most hiring decisions are decentralized, with a weak scope to influence the wage schedule.

$$\min_{j_1^g, j_2^g} w_1^g j_1^g + w_2^g j_2^g$$

s.t.

$$\bar{g} = (j_1^g)^{\beta}(j_2^g)^{1-\beta}.$$

Given the level of public wages, the government employs enough workers to maintain an employment level capable of providing its services. Using the production function and the two first-order conditions, we find the optimal ratio of skilled and unskilled target public-sector jobs is:

$$\frac{j_1^{g,d}}{j_2^{g,d}} = \frac{w_2^g}{w_1^g}\frac{\beta}{(1-\beta)}, \tag{3}$$

Plugging in the production function, the target level of jobs of each type is given by:

$$j_1^{g,d} = \bar{g}\left(\frac{w_2^g}{w_1^g}\frac{\beta}{1-\beta}\right)^{1-\beta}, \quad j_2^{g,d} = \bar{g}\left(\frac{w_1^g}{w_2^g}\frac{1-\beta}{\beta}\right)^{\beta}. \tag{4}$$

Lemma 1. *If the government minimizes costs, the target skilled jobs, $j_1^{g,d}$ is increasing in w_2^g and β and decreasing in w_1^g. The target unskilled jobs, $j_2^{g,d}$ is increasing in w_1^g and decreasing in w_2^g and β. They are independent of private sector conditions.*

The first dimension of analysis is the government's preferred choice of which workers to hire. Taking the wage schedule and the production function as given, the government chooses how many workers of the two types to minimize the costs of producing $\bar{g}$. Changes in public wages are going to alter the labour demand choice of the government. Higher unskill wages reduce the demand for unskilled jobs and raise demand for skilled jobs.

In a model without frictions, public-private wage differentials can pose some problems. If it is positive, all workers would prefer the public sector, so one has to assume these jobs are rationed. Perhaps harder to deal is the opposite case, where the differential is negative and no worker would like to work for the government. Our preference structure avoids this problem. It makes the supply of workers to the public sector continuous on the wage, while preserving the different regimes.

[10] With a Cobb-Douglas production function, the elasticity of substitution between skilled and unskilled labor is equal to one. The values provided by Acemoglu and Autor (2011) range between 1.6 and 2.9. However, underemployment disrupts the one-on-one mapping between education and skills used in the production function. The empirical literature does not take into account the existence of underemployment, which biases the empirical estimates. We provide here a benchmark exercise with a Cobb–Douglas function. Using a CES production does not alter the qualitative results of our paper. Results are available upon request.

[11] The probability density distribution is hump-shaped and the cumulative distribution function is an exponential expression. This is useful because the difference of two Gumbel-distributed random variables has a logistic distribution, which allows us to provide analytical results.

[12] For the adamant reader concerned about the assumption of exogenous public wages, we present an extension in Appendix E.1 where the government also chooses wages, but faces an additional union preference constraint. This problem generates an endogenous public-sector premium that depends on an exogenous union power and preference for wage compression, but does not change the labour market analysis. An alternative approach, considered by de Córdoba et al. (2010) is to assume a Leviathan government with preferences to expand both employment and wages with different weights.

If their wages are higher, the government can attain its target level of jobs, that are rationed. If the public-private wage differential is negative, there would still be workers with high enough preference such that their supply is never zero. Still, the supply of workers of a given type might be lower than the target level determined by cost-minimization. In such cases, we assume the government hires more workers of the other type to maintain the production of services.

The final assumption is that an educated worker that applies to an unskilled public job always has priority over low-educated workers. The government is financed through a labour income tax, τ. In the baseline model, we take it as exogenous, but, in Appendix E.3, we discuss one extension in which τ adjusts to satisfy the government budget constraint. In Appendix E.4, we also discuss the differences if we consider the government's dual problem.

4.3. ...And a Walrasian private sector...

The representative firm maximizes profits. The labour market is perfectly competitive such that the wages equate demand and supply and jobs are paid their marginal productivity. The labour demand equations are

$$w_1^p = \alpha\left(\frac{j_2^p}{j_1^p}\right)^{1-\alpha}, \quad w_2^p = (1-\alpha)\left(\frac{j_1^p}{j_2^p}\right)^{\alpha}. \tag{5}$$

These standard inverse labour demand conditions show how the marginal productivity of skilled and unskilled jobs depends only on their relative number. As the market is Walrasian, we do not keep track of demand and supply subscript as they are always equal, regardless of what happens in the public-sector labour market.

4.4. ...With underemployment

The possibility of educated workers to do unskilled jobs creates a dissociation between the number of educated workers and the number of skilled jobs, as well as the number of workers with low education and the number of unskilled jobs. Some of the educated workers might be under-employed in the public or private sector (u^g, u^p) if they choose to. Hence, the market clearing condition in high- and low-educated labour markets are given by

$$n = j_1^g + j_1^p + u^g + u^p, \quad 1-n = j_2^g + j_2^p - u^g - u^p. \tag{6}$$

4.5. Sorting

The total labour supply in the economy is exogenous in the sense that there is no disutility of labour. However the labour supply to the public and private sectors is endogenous and responds to wage differentials across sectors and jobs through two sorting conditions. An educated worker i always has the possibility of going to private or public sector in a skilled job, and might have an opportunity at an unskilled job in either sector. In a general formulation, we consider that a fraction Ξ^g of educated workers has an opportunity at an unskilled job in the public sector. Hence, these workers choose between three options:

$$Max\{(1-\tau)w_1^p + \nu\epsilon_i^{p,1}, (1-\tau)w_1^g + \nu\epsilon_i^{g,1}, (1-\tau)w_2^g + \nu\epsilon_i^{g,2}\}. \tag{7}$$

Likewise, a fraction Ξ^p has one opportunity at an unskilled job in the private sector and chooses between:

$$Max\{(1-\tau)w_1^p + \nu\epsilon_i^{p,1}, (1-\tau)w_2^p + \nu\epsilon_i^{p,2}, (1-\tau)w_1^g + \nu\epsilon_i^{g,1}\}. \tag{8}$$

Finally, a fraction Ξ^b has opportunities at unskilled jobs in both sectors and chooses between four options:

$$Max\{(1-\tau)w_1^p + \nu\epsilon_i^{p,1}, (1-\tau)w_2^p + \nu\epsilon_i^{p,2}, (1-\tau)w_1^g + \nu\epsilon_i^{g,1}, (1-\tau)w_2^g + \nu\epsilon_i^{g,2}\}. \tag{9}$$

For analytical convenience, we now proceed with $\Xi^b = 1$ and $\Xi^g = \Xi^p = 0$, so all educated workers have four available options. We use the more general version for quantitative purposes.

The fact that skilled jobs in the public sector might be rationed is important, given that there might be fewer jobs available than workers wanting to work there at a given public wage. We assume that workers that wished for but could not get a skilled public job, choose the maximum between the three remaining options. Notice that this does not happen for unskilled public jobs because we assume that they have priority over low-educated workers.[13] We include one specific shock for each of the four possible jobs. One alternative would be to consider a preference for public and private sectors and one for complex and simple jobs. Not only would it be less tractable, it would also pose problems on how to identify the ν for the public sector. Notice also that the preference is individual specific and cannot be interpreted as a compensating differential. If there were other non-pecuniary characteristics common to all public- sector workers, these could be added to wages as a separate term.[14]

A worker with low education only has a choice of a private or public unskilled job:

$$Max\{(1-\tau)w_2^p + \nu\epsilon_i^{p,2}, (1-\tau)w_2^g + \nu\epsilon_i^{g,2}\}. \tag{10}$$

If the non-pecuniary value is drawn from a standard Gumbel distribution, the number of high- and low-educated workers whose first choice is a public-sector job with the skill requirement matching their education (denoted by $j_1^{g,s}$ and $j_2^{g,s}$) are given by:

$$j_1^{g,s} = n\left[\frac{e^{\frac{(1-\tau)}{\nu}w_1^g}}{e^{\frac{(1-\tau)}{\nu}w_1^g} + e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{11}$$

$$j_2^{g,s} = (1-n)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{12}$$

These equations follow from the distributional assumption of $\epsilon_i^{x,e}$ and are similar to the well-know econometric multinomial-logit estimation. To better understand the different regimes, for any given level of public-sector employment, we can define two endogenous objects, $\tilde{w}_1^g(j_1^g)$ and $\tilde{w}_2^g(j_2^g)$, the implicit market clearing wages, or threshold wages that allow the government to achieve that level of employment:

$$\tilde{w}_1^g(j_1^g) = \frac{\nu}{(1-\tau)}\left[\log(e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}) - \log(n - j_1^g)\right] \tag{13}$$

$$\tilde{w}_2^g(j_2^g) = \frac{\nu}{(1-\tau)}\left[\log(e^{\frac{(1-\tau)}{\nu}w_2^p}) - \log(1-n+u^g - j_2^g)\right] \tag{14}$$

These two equations are derived using (11) and (12), setting $j_1^{g,s} = j_1^{g,d} = j_1^g$ and $j_2^{g,s} = j_2^{g,d} = j_2^g$ and solving for the wages. If both public wages are above the implicit market clearing wages – regime 1 – the number of interested workers is larger than the number of jobs, so all public jobs are rationed and are determined by demand, $j_1^g = j_1^{g,d} < j_1^{g,s}$ and $j_2^g = j_2^{g,d} < (j_2^{g,s} + u^g)$. If one of the wages is below the threshold, in one market there are fewer interested workers than jobs, so the government is constrained and supply determines either j_1^g or j_2^g and the other adjusts to maintain the production of services (regimes 2 or 3). Finally, if both wages are below the threshold, the government is constrained in both jobs, so it is not able to maintain its government services, $j_1^g = j_1^{g,s} < j_1^{g,d}$ and $j_2^g = (j_2^{g,s} + u^g) < j_2^{g,d}$.

[13] It would technically be possible that the unskilled public wage would be so high that more educated workers would want an unskilled public job than existing jobs; so that these jobs would be rationed too $u^g = j_2^g$. We find that this case is only a theoretical curiosity with little empirical relevance.

[14] If anything, adding a common compensating differentials in the public sector would strengthen the underemployment channel as they would make the unskilled public-sector job even more attractive relative to the private sector. Similarly, if the preferences featured risk aversion, the underemployment channel would also be amplified as small differences in the unskilled wages of public and private sectors would translate into larger differences in utility.

Independently of whether the government jobs are determined by supply or demand, underemployment in the two sectors is pinned down by

$$u^p = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^p}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{15}$$

$$u^g = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{16}$$

Notice that, when $w_2^g = w_2^p$, $u^p = u^g$ independently of the size of the public and private sectors. This means that the underemployment rate in the private sector ($\tilde{u^p} = \frac{u^p}{j_2^p}$) would differ from the one prevailing in the public sector ($\tilde{u^g} = \frac{u^g}{j_2^g}$), unless the two sector were of exactly the same size. In the quantitative exercise, we consider a variation of the sorting problem such that, when $w_2^g = w_2^p$, the model generates $\tilde{u}^p = \tilde{u}^g$.[15]

For the interested reader, we show in Appendix B a version of the two-sector model without underemployment and perfect labour mobility, and in Appendix C a 1-sector model of underemployment where we discuss some comparative statics, namely with respect to the tax rate and the supply of educated workers. These two Appendices develop the intuition and isolate the mechanisms present in the model. Garibaldi et al. (2020), analyse a one-sector model, considering both under and over employment, to measure the output losses of mismatch across OECD economies.

4.6. Equilibrium definition

Definition 1. An equilibrium consists of private-sector wages $\{w_1^p, w_2^p\}$, private-sector jobs $\{j_1^p, j_2^p\}$, public-sector jobs $\{j_1^g, j_2^g\}$, and underemployment in the two sectors $\{u^p, u^g\}$, such that, given some exogenous wage policies, technology and composition of the labour force $\{w_1^g, w_2^g, \tau, \nu, \bar{g}, \alpha, \beta, n\}$, the following apply.

1. Private-sector firms maximizes profits (5).
2. Employment in the government is set either: i) if unconstrained (demand determined), by minimizing the costs of providing government services:

$$\text{Regime 1}\begin{cases} j_1^g = j_1^{g,d} < j_1^{g,s} \\ j_2^g = j_2^{g,d} < (j_2^{g,s} + u^g) \end{cases} \quad \begin{array}{ll} \text{if} & w_1^g \geq \tilde{w}_1^g(j_1^{g,d}) \\ & w_2^g \geq \tilde{w}_2^g(j_2^{g,d}) \end{array} \tag{17}$$

ii) if constrained in one of the markets (partly supply determined), to maintain the production of government services:

$$\text{Regime 2}\begin{cases} j_1^g = j_1^{g,s} < j_1^{g,d} \\ j_2^g = \left[\frac{\bar{g}}{(j_1^g)^\beta}\right]^{\frac{1}{1-\beta}} < (j_2^{g,s} + u^g) \end{cases} \quad \begin{array}{ll} \text{if} & w_1^g < \tilde{w}_1^g(j_1^{g,d}) \\ & w_2^g \geq \tilde{w}_2^g(j_2^g) \end{array} \tag{18}$$

$$\text{Regime 3}\begin{cases} j_1^g = \left[\frac{\bar{g}}{(j_2^g)^{1-\beta}}\right]^{\frac{1}{\beta}} < j_1^{g,s} \\ j_2^g = (j_2^{g,s} + u^g) < j_2^{g,d} \end{cases} \quad \begin{array}{ll} \text{if} & w_1^g \geq \tilde{w}_1^g(j_1^g) \\ & w_2^g < \tilde{w}_2^g(j_2^{g,d}) \end{array} \tag{19}$$

iii) if constrained in both markets (fully supply determined):

$$\text{Regime 4}\begin{cases} j_1^g = j_1^{g,s} < j_1^{g,d} \\ j_2^g = (j_2^{g,s} + u^g) < j_2^{g,d} \end{cases} \quad \begin{array}{ll} \text{if} & w_1^g < \tilde{w}_1^g(j_1^{g,d}) \\ & w_2^g < \tilde{w}_2^g(j_2^{g,d}) \end{array} \tag{20}$$

3. High- and low- educated workers sort across labour markets according to (9) and (10).
4. Markets clear (6).

[15] This requires $\Xi^g = \frac{j_2^g}{j_2^p + j_2^g}$, $\Xi^p = \frac{j_2^p}{j_2^p + j_2^g}$ and $\Xi^b = 0$, but does not allow clear analytical results.

5. Solving the model under different regimes

5.1. Regime 1 - unconstrained government

This equilibrium requires that $w_1^g \geq \tilde{w}_1^g(j_1^{g,d})$ and $w_2^g \geq \tilde{w}_2^g(j_2^{g,d})$. Given that $j_1^g = j_1^{g,d}$ and $j_2^g = j_2^{g,d}$ are only function of the exogenous public-sector wages and technology, the solution of the model under regime 1 can be written in three equations in $u = u^p + u^g$, w_1^p and w_2^p

$$u = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^p} + e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{21}$$

$$w_1^p = \alpha\left(\frac{1 - n - j_2^g + u}{n - j_1^g - u}\right)^{1-\alpha}, \tag{22}$$

$$w_2^p = (1-\alpha)\left(\frac{n - j_1^g - u}{1 - n - j_2^g + u}\right)^{\alpha}, \tag{23}$$

We can further substitute the two wages, and have one equation in one unknown with the left-hand side increasing in u and the right-hand side decreasing in u. We show in Appendix D that the equilibrium exists and is unique, as well as the full system determining the total derivatives of the endogenous variables to the key exogenous variables.

Proposition 1. *Under regime 1, an increase of w_2^g shifts the composition in the public sector towards skilled jobs and in the private sector to unskilled jobs. It raises skilled wages and lowers unskilled wages in the private sector. The effect on underemployment is ambiguous ($\frac{du}{dw_2^g} \lesseqgtr 0$, $\frac{dw_1^p}{dw_2^g} > 0$, $\frac{dw_2^p}{dw_2^g} < 0$, $\frac{dj_1^g}{dw_2^g} > 0$, $\frac{dj_2^g}{dw_2^g} < 0$).*

Proposition 2. *Under regime 1, an increase of w_1^g shifts the composition in the public sector towards unskilled jobs and in the private sector to skilled jobs. It raises unskilled wages and lowers skilled wages in the private sector. It raises underemployment ($\frac{du}{dw_g^1} > 0$, $\frac{dw_1^p}{dw_1^g} < 0$, $\frac{dw_2^p}{dw_1^g} > 0$, $\frac{dj_1^g}{dw_1^g} < 0$, $\frac{dj_2^g}{dw^g 1} > 0$).*

The propositions tell us how public wages affect the private sector. The effect of an increase of w_2^g on underemployment is ambiguous. While there is a direct positive effect on underemployment in the public sector, the higher wage inequality in the private sector, has a negative indirect effect on both private and public underemployment. The effect on underemployment of an increase in w_1^g is unambiguously positive. By reducing private-sector wage inequality it fosters underemployment in both sectors.

We can write expressions for the elasticities of private wages with respect to public wages. For instance, elasticities with respect to unskilled public wages are given by:

$$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p} = (1-\alpha)(1-\beta)\frac{j_1^g}{j_1^p} + (1-\alpha)\beta\frac{j_2^g}{j_2^p} + \frac{du}{dw_2^g}\left[\frac{(1-\alpha)}{j_2^p} + \frac{(1-\alpha)}{j_1^p}\right]w_2^g \tag{24}$$

$$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p} = -\alpha(1-\beta)\frac{j_1^g}{j_1^p} - \alpha\beta\frac{j_2^g}{j_2^p} - \frac{du}{dw_2^g}\left[\frac{\alpha}{j_2^p} + \frac{\alpha}{j_1^p}\right]w_2^g \tag{25}$$

These expressions provide a decomposition of the effects of public wages. Higher unskilled public wages induce the government to open more skilled jobs and fewer unskilled jobs. In turn, this means there is a shortage of educated workers (first term) and an excess of low-educated workers (second term) in the private sector, both pushing skilled wages up and unskilled wages down. Finally, there is an effect on underemployment. If underemployment increases, both the positive effect on skilled wages and the negative effect on unskilled wages are reinforced. If underemployment decreases, they are mitigated.

5.2. Regime 2 - skilled public-sector wages too low

This is a potentially realistic regime. Regime 2 occurs if wages for skilled jobs are too low, $w_1^g < \tilde{w}_1^g(j_1^{g,d})$. The government cannot hire its target level of employment so, to maintain the production of government services it has to open more unskilled jobs (provided it still pays high enough wages, $w_2^g \geq \tilde{w}_2^g(j_2^g)$). The public jobs for the two types are given by

$$j_1^g = j_1^{g,s} \quad , \quad j_2^g = \left[\frac{\bar{g}}{(j_1^g)^\beta}\right]^{\frac{1}{1-\beta}}. \tag{26}$$

The three equations pinning down u, w_1^p and w_2^p are the same as in regime 1, but now they affect both j_1^g and j_2^g that are no longer independent. In Appendix D, we show the full system determining the total derivatives of the endogenous variables to the key exogenous variables.

Proposition 3. *Under regime 2, an increase of w_2^g raises skilled wages and lowers unskilled wages in the private sector. The effect on underemployment and in the skill mix of the public sector is ambiguous ($\frac{du}{dw_g^2} \lesseqgtr 0$, $\frac{dw_1^p}{dw_2^g} > 0$, $\frac{dw_2^p}{dw_2^g} < 0$, $\frac{dj_1^g}{dw_2^g} \lesseqgtr 0$, $\frac{dj_2^g}{dw_2^g} \lesseqgtr 0$).*

Proposition 4. *Under regime 2, an increase of w_1^g shifts the composition in the public sector towards skilled jobs and in the private sector to unskilled jobs. It raises skilled wages and lowers unskilled wages in the private sector. It lowers underemployment ($\frac{du}{dw_g^1} < 0$, $\frac{dw_1^p}{dw_1^g} > 0$, $\frac{dw_2^p}{dw_1^g} < 0$, $\frac{dj_1^g}{dw_1^g} > 0$, $\frac{dj_2^g}{dwg1} < 0$).*

In this case public-sector employment is supply determined so the signs of the effect of public-sector wages on private-sector wages are the opposite of those in regime 1. Increasing wages at the top allows the government to attract more educated workers.

5.3. Regime 3 - unskilled public-sector wages too low

Regime 3 happens if unskilled public wages are too low, $w_2^g < \tilde{w}_2^g(j_2^{g,d})$. The government cannot hire its target level of employment so, to maintain the production of government services, it has to open more skilled jobs (requiring that $w_1^g \geq \tilde{w}_1^g(j_1^g)$). While this case is not realistic, we consider it for completeness. The public employment is given by

$$j_2^g = u^g + j_2^{g,s} \quad , \quad j_1^g = \left[\frac{\bar{g}}{(j_2^g)^{1-\beta}}\right]^{\frac{1}{\beta}}. \tag{27}$$

5.4. Regime 4 - public sector breaks down

Regime 4 occurs if both public wages are too low, $w_1^g < \tilde{w}_1^g(j_1^{g,d})$ and $w_2^g < \tilde{w}_2^g(j_2^{g,d})$. All government jobs are determined by supply. The government cannot hire enough workers to maintain the production of government services, so they have to be scaled down.

$$j_1^g = j_1^{g,s} \quad , \quad j_2^g = u^g + j_2^{g,s}. \tag{28}$$

And the government services that are allowed is given by $g = (j_1^g)^\beta (j_2^g)^{1-\beta}$.

6. Quantitative analysis

6.1. Model with alternative sorting mechanism and exogenous income tax

For quantitative purposes, we consider a more general sorting mechanism. One of the features of the baseline model is that when wages are equal in the two sectors, their level of underemployment is equal. Thus, if the public sector is smaller than the private, their underemployment rate would be larger. As such, even in the case of symmetry between the two sectors in terms of wages and technology, the ratio of public-employment shares is not 1. We thus consider an alternative sorting mechanism whereby the underemployment opportunities are proportional to the dimension of each sector ($\Xi^g = \frac{j_2^g}{j_2^p + j_2^g}$, $\Xi^p = \frac{j_2^p}{j_2^p + j_2^g}$ and $\Xi^b = 0$.). This is sufficient to generate equal underemployment rates and a ratio of public employment shares of 1 when both wages and technology are equal across sectors.

The equations of the model with the alternative sorting mechanism are shown in Appendix D.3. Of all the educated workers, a fraction Ξ^g has an underemployment opportunity only in the public sector. Those workers choose between three options $Max\{w_1^p + \nu\epsilon_i^{p,1}, w_1^g + \nu\epsilon_i^{g,1}, w_2^g + \nu\epsilon_i^{g,2}\}$. The remaining fraction $\Xi^p = 1 - \Xi^g$ has only an underemployment opportunity in the private sector and chooses between $Max\{w_1^p + \nu\epsilon_i^{p,1}, w_1^g + \nu\epsilon_i^{g,1}, w_2^p + \nu\epsilon_i^{p,2}\}$. While Ξ^g and Ξ^p could be in principle any number between 0 and 1, we assume that it is equal to the fraction of unskilled jobs that belong to the government, $\Xi^g = \frac{j_2^g}{j_2^p + j_2^g}$ to generate equal underemployment rates in the two sectors in the symmetric case. The mechanism is similar to the baseline model except that Eq. (21), that determines underemployment, becomes more complex.

Furthermore, we take into account an exogenous income tax τ in the baseline model. The tax rate has the same effect as a change in ν, the weight of the non-pecuniary element of preferences. The income tax rate is taken as a parameter assumed constant even in the quantitative experiments carried out in this section. The justification is that we considered that such policies would be financed with government debt or by adjustments in other spending categories. We take into account the endogenous response of income tax in an extensions shown in section E.3

6.2. Calibration

We calibrate the more general version of model to the United States. The model has seven parameters $\{w_1^g, w_2^g, \frac{\nu}{1-\tau}, \bar{g}, \alpha, \beta, n\}$. As such, we set them to target seven moments of the data, all described in Section 3. Table 3 summarizes the parameter values and target values.

We set n to match 43.2 percent of college graduates. The parameters $\bar{g}$ and β target a public employment of 0.097 and 0.062 of college and non-college, as a proportion of the employed population, taken from the CPS. Notice that the employment of no-college public workers is equal to $j_2^g - u^g$ while the employment of public workers with college is $j_1^g + u^g$. The parameter α targets a college premium of private workers of 58 percent found by regressing the log of hourly wages of private workers on a college dummy, controlling for age, gender, region, year and a part-time dummy, for a sample between 1996 and 2018.

One important point that our model raises is that the observed public wage premium for college workers might be understated if not controlling for occupation, as it includes underemployed workers. We target the coefficient from Table 2, of the regressions in which we control for two digit occupations, meaning a public-private wage rate for both unskilled jobs of $\frac{w_2^g}{w_2^p} = 1.077$ and for skilled jobs of $\frac{w_1^g}{w_1^p} = 1.010$. We cannot dissociate the weight of the preference shock in sorting, ν, from the income tax rate. We set $\frac{\nu}{1-\tau}$, such that the underemployment rate is 0.089, the number found for the US using PIAAC data.

Under the calibration, the US economy is in regime 1, where wages are high enough such that the government hiring is unrestricted. Table 3 also displays additional non-targeted moments : underemployment rates in the private and public sectors. Underemployment rates in both sectors are very close to the data, which suggests that the model provides a good fit along these dimensions.

6.3. What drives the public-sector education bias?

The first exercise shows whether the public-sector education bias is driven by technology or by the combination of the wage policy and ex-

Table 3
Calibration.

Parameter	Value	Variable	Description	Model	Data
		Targeted			
α	0.450	$\frac{w_1^p}{w_2^p}$	College premium (private sector)	1.580	1.580
β	0.657	$j_1^g + u^g$	Public employment of college	0.097	0.097
$\bar{g}$	0.082	$j_2^g - u^g$	Public employment of no-college	0.062	0.062
n	0.432	n	Percentage of college workers	0.432	0.432
w_1^g	0.652	$\frac{w_1^g}{w_1^p}$	Public-sector wage premium (college)	1.010	1.010
w_2^g	0.440	$\frac{w_2^g}{w_2^p}$	Public-sector wage premium (no-college)	1.077	1.077
$\frac{\nu}{1-\tau}$	0.142	$\frac{u}{j_1^p+j_2^p}$	Underemployment rate (economy)	0.089	0.089
		Not Targeted			
		$\frac{u^g}{j_2^g}$	Underemployment rate (public)	0.105	0.102
		$\frac{u^p}{j_2^p}$	Underemployment rate (private)	0.087	0.087

Note: Underemployment rate statistics are calculated from PIACC and are shown in Fig. 6. The remaining data is calculated from the CPS, 1996 to 2018. The public employment of college and no-college is shown in Fig. 2. Public-sector wage premium is shown in the first two columns of Table 2. The college premium in the private sector is estimated by regressing the log of hourly wages of private workers on a college dummy, controlling for age, gender, region, year and a part-time dummy.

Table 4
Decomposition of public-sector employment education bias.

Variable	Data (1)	Baseline (2)	Equating wages (3)	Equating wages and technology (4)
Public employment shares				
Skilled	0.224	0.224	0.218 (10%)	0.166
Unskilled	0.109	0.109	0.115 (11%)	0.166
Ratio	2.054	2.054	1.892 (15%)	1.000
Education intensity				
Public	0.610	0.610	0.590 (11%)	0.432
Private	0.399	0.399	0.402 (10%)	0.432
Ratio	1.530	1.530	1.468 (12%)	1.000
Underemployment rate				
Total	0.089	0.089	0.090	0.116
Public*	0.102	0.105	0.090	0.116
Private*	0.087	0.087	0.090	0.116

Column (2) displays the statistics simulated from the model. Column (3) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs. Column (4) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs and $\beta = \alpha$. In parenthesis we show the percentage of the gap between columns (4) and (2) covered by only equating wages. * statistics not calibrated.

cess underemployment (Table 4). Column (1) shows the values of variables in the data and Column (2) the values under the main calibration. Column (3) shows the counterfactual values when there are no differences across sectors in terms of wages ($w_1^g = w_1^p$ and $w_2^g = w_2^p$). Column (4) equates both wages and technology ($\beta = \alpha$). In that case, the public and private sector have the same skill mix (this would not happen in the baseline model): the government hires 16.6 percent of both types of workers, the underemployment rates in both sectors are equal and the public employment shares ratio and the education intensity ratios are both be equal to 1.

Switching off only the wage differences across sectors, imply cutting public wages by 1.4 percent for skilled and 6.8 percent for unskilled jobs. In this scenario, the underemployment rate is equal in both sectors. This reduces the share of public employment for college graduates by 0.6 percentage point. It would lower the public employment shares ratio from 2.05 to 1.9, roughly 15 percent of the difference to 1. It would lower the education intensity ratio from 1.53 to 1.47, 12 percent of the difference to 1.

In Appendix F we present decomposition for the UK, France and Spain, together with one exercise using the baseline model instead of the model with alternative sorting. In the UK, the wage profile and underemployment account for only 3 percent of the education bias, which might be explained by a larger weight of the health and education industries, that required more qualified workers. In contrast, in France and Spain, the wages schedule and underemployment account for between 13 and 19 percent.

Using the baseline sorting mechanism, the wage schedule and excess underemployment explain more than 80 percent of the difference between the data and the symmetric case. This high number reflects the fact that, with the baseline sorting mechanism, underemployment in the public sector is much more sensitive to unskilled public wages. For instance, for the starting calibration, the underemployment rate in the public sector is larger than 30 percent and that of the private sector lower than 5 percent.

6.4. Elasticities of private-sector wages

We now focus on the price implications of the education bias in the public sector. We calculate the elasticities of private wages, with respect to public wages. To our knowledge, the empirical literature does not provide estimates of these elasticities. Empirical research investigates public-private wage gap or provides evidence on aggregate wage spillovers, rather than focusing on skilled and unskilled workers like in our paper. Our model suggests that the elasticities of private wages, with respect to public wages, depends on the skill-mix in the public sector and the extent of underemployment in each sector, which suggests interesting avenues to explore in future empirical research.

As in Eqs. (24) and (25), we can decompose the elasticities of private wages, with respect to public wages into three components. The first two relate to the adjustment of the skill-mix in the public sector. Higher unskilled public wages alter the government skill-mix towards skilled jobs, hence employing fewer low-educated workers. The first component measures the impact of the shortage of high-educated workers in the private sector. It is positive for private skilled wages and negative for unskilled wages. Similarly, the excess low-educated workers has a positive effect on skilled wages and negative effect on unskilled wages, as measured by the second term. These two effects would exist in a model without underemployment. The contribution of underemployment is measured in the third component, that depends on whether higher unskilled public wages increase or decrease underemployment, which we could not pin down analytically. Hence, we calculate the elasticities and the three components numerically, shown in Table 5.

An increase of one percent in unskilled public wages translates into an increase of 0.07 percent of skilled private wages and a reduction of

Table 5
Elasticities of private-sector wages.

Variable	Elasticity	Decomposition		
		Shortage of skilled	Excess unskilled	Underemployment
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.074	0.059	0.045	−0.029
$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	−0.061	−0.048	−0.037	0.024
Elasticity of private wages w.r.t. skilled public wages				
$\frac{dw_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	−0.046	−0.059	−0.045	0.058
$\frac{dw_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.038	0.048	0.037	−0.047
Elasticity of private wages w.r.t. public wages				
$\frac{dw_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.029	0.000	0.000	0.029
$\frac{dw_2^p}{dw^g}\frac{w^g}{w_2^p}$	−0.023	0.000	0.000	−0.023

Note: the first column is calculated numerically, the decomposition is based on Eqs. (24) and (25).

0.06 percent in unskilled private wages, increasing wage inequality in the private sector. We can see that the presence of underemployment contributes to mitigating the effect. Higher unskilled public wages raise underemployment in the public sector but reduce it in the private sector. The overall effect is negative. Our model suggests that reduced wage inequality in the public sector actually increases wage inequality in the private sector. Borjas (2003) conjectures that public-sector wage compression after the 1970s in the US might have also played a role in the widening of the private-sector wage distribution, in addition to technology and globalization. Our model provides the economic mechanisms to understand Borjas (2003)'s intuition.

An increase of one percent in skilled public wages translates into a reduction of 0.05 percent of skilled private wages and an increase in 0.04 percent of unskilled private wages. Again underemployment mitigates the effect.

The last rows show the elasticity of private wages to an increase of both skilled and unskilled wages. In this case, there is no change in the skill-mix of the government, so all the effects come from underemployment. Still, increasing proportionally wages in the public sector has an asymmetric effect. The increase in underemployment in the public sector is larger than the fall in underemployment in the private sector so overall underemployment increases, which raises skilled wages and lowers unskilled wages in the private sector.

In Appendix F, we present the same exercise for the UK, France and Spain, as well as using the baseline model instead of the model with alternative sorting. Given the higher share of public employment in the UK and France, their elasticities are up to four times larger than in the US. For instance, in UK (France), an increase of one percent in the unskilled public wages raises private skilled wages by 0.21 (0.14) percent and lowers private unskilled wages by 0.06 (0.06).

Using the baseline model, the elasticities are also higher. An increase of one percent in public wages raises private skilled wages by 0.21 and lowers unskilled wages by 0.2. The difference is driven by underemployment. Under the baseline model, higher unskilled public wages raises underemployment, because the direct positive effect on public-sector underemployment largely dominates the negative effect on private underemployment. In the model with alternative sorting, as the set of underemployment opportunities on the public sector is restricted, the positive effect is mitigated and the negative effect is amplified.

6.5. Switching regimes

Although in this quantitative exercise, the US economy is in regime 1, we think the idea of switching regimes is very realistic. Borjas (2003) documents the differential shifts that occurred in the wage structures of the public and private sectors between 1960 and 2000. He concludes that "*as the wage structure in the public sector became relatively more compressed, the public sector found it harder to attract and retain high-skill workers. In short, the substantial widening of wage inequality in the private sector and the relatively more stable wage distribution in the public sector created "magnetic effects" that altered the sorting of workers across sectors, with high-skill workers becoming more likely to end up in the private sector.*" Similar concerns were also raised during the recent experience of European countries subject to austerity packages. Several countries implemented austerity measures that included public sector wage cuts. However, most governments opted for asymmetric cuts, centered on the highest earners, instead of reforms aligning the wage distribution with that of the private sector.[16] As a result of the relative wage compression, the public sector found it increasingly more difficult to attract and retain high-skill workers.[17]

Our definition of equilibrium covered four regimes. Fig. 7 shows which regime is in place depending on the wage policy. The US economy is in the unconstrained regime where both the skilled and unskilled public wages are high enough. Only cuts larger than 25 percent in skilled wages or larger than 50 percent for unskilled wages would push the economy to one of the three other regimes. Still, we perform numerical exercises varying skilled and unskilled public wages across regimes.

Fig. 8 shows the effects of varying skilled public wages. The kink observed for wage cuts above 25 percent is the switching from regime 1 to regime 2. When government employment switches to become supply determined, the sign of the effects on private wages, underemployment, education intensity and public employment shares ratios switches. Particularly interesting is that wage cuts above 25 percent also raise government spending. By lowering the skilled public wages in regime 1, the government reduces spending. But when lowering wages implies that fewer educated workers are attracted to public jobs and the government has to open more unskilled positions (relative more expensive), it generates an inefficient skill mix that is more costly.

Fig. 9 shows the effects of varying unskilled public wages, for the main calibration (regime 1, dark line) and for one where public skilled wages are 35 percent lower (regime 2, light line). In both regimes, higher unskilled public wages raise the education intensity and public employment shares ratios. It also pushes private skilled wages up and unskilled wages down, raising inequality (with a larger slope in regime 1). The one variable that is affected differently by unskilled public wages in the two regimes is underemployment. Higher unskilled wages lower total underemployment in regime 1 because of the large quantitative effects on private-sector inequality, which reduces the incentive of being underemployed. In regime 2, higher unskilled public-sector wages do not reduce directly the number of unskilled jobs of the government (because the government is not able to substitute away from unskilled labour) so they simply foster underemployment in the public-sector.

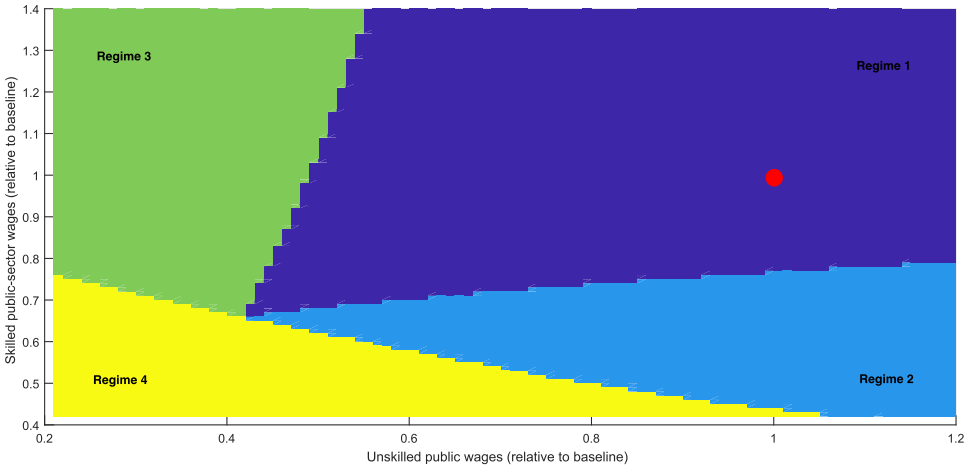


Fig. 7. Regimes as a function of the public-sector wage schedule.

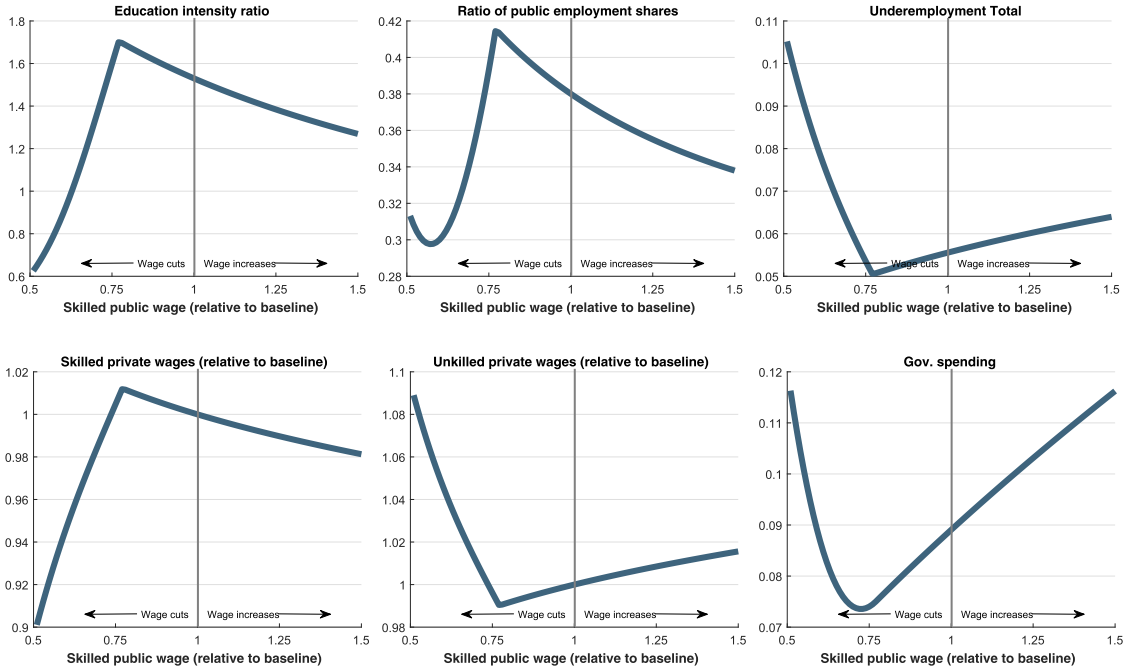


Fig. 8. Effects of public-sector skilled wages. *Note:* The kink observed for wage cuts beyond 25 percent is the switching from regime 1 to regime 2.

6.6. *Extensions*

In Appendix E, we analyse four extensions of the model. We show that these extensions do not modify the key insights from the baseline model, and sometimes reinform them through additional mechanisms. First, we endogeneize public-sector wages, based on the presence of a union constraint. Second, we consider that educated workers have heterogeneous ability. We show that the high-ability educated workers are less likely to be underemployed and, if the government does not fully reward their efficiency units like the private sector, they are less likely to go to the public-sector, which reinforces the education bias. Third, we consider a model with endogenous tax rate, which simply adds another equation reflecting the government budget constraint. Finally, we consider the government's dual problem that gives a slightly different solution.[18]

7. Conclusion

We present a simple two-sector model with underemployment that highlights the main trade-off regarding public wages, without modelling search frictions. The theory highlights three channels to rationalize why public employment is so biased towards educated: technology, the public wage profile and excess underemployment. We find that in the US economy the excess hiring of educated workers in the public sector is mainly accounted for by technology, while the wage policy and excess underemployment account for 15 percent.

We also find that the public wage policy is a crucial driver of private sector inequality: more wage compression in the public sector raises inequality in the private sector. A one percent increase in unskilled public

[16] In Portugal in 2012, the wage cuts were 22 percent on the highest earners and zero percent on the lowest. In Spain in 2010, they were 10 percent on top and zero at the bottom. In Ireland in 2010, the cuts where 15 percent at the top and 5 percent at the bottom.

[17] To illustrate this point, we calibrate the model with a more restricted definition of educated workers, than only includes workers with an M.Cc., Professional or Ph.D. degree. These correspond to 9 percent of the US employed population, of which one third are hired by the government. The public-sector wage premium for these workers is −4 percent. The calibration and the results are shown in Appendix G. We found this economy to be in regime 2, where the government is constrained by the supply of educated workers. This can explain why in Fig. 1 the share of public employment is increasing on education for workers until M.Sc., but falls for workers with Ph.D.

[18] Although we have worked out others, we abstract from discussing the ones that add little to the mechanism (i.e considering a CES production function or introducing capital).

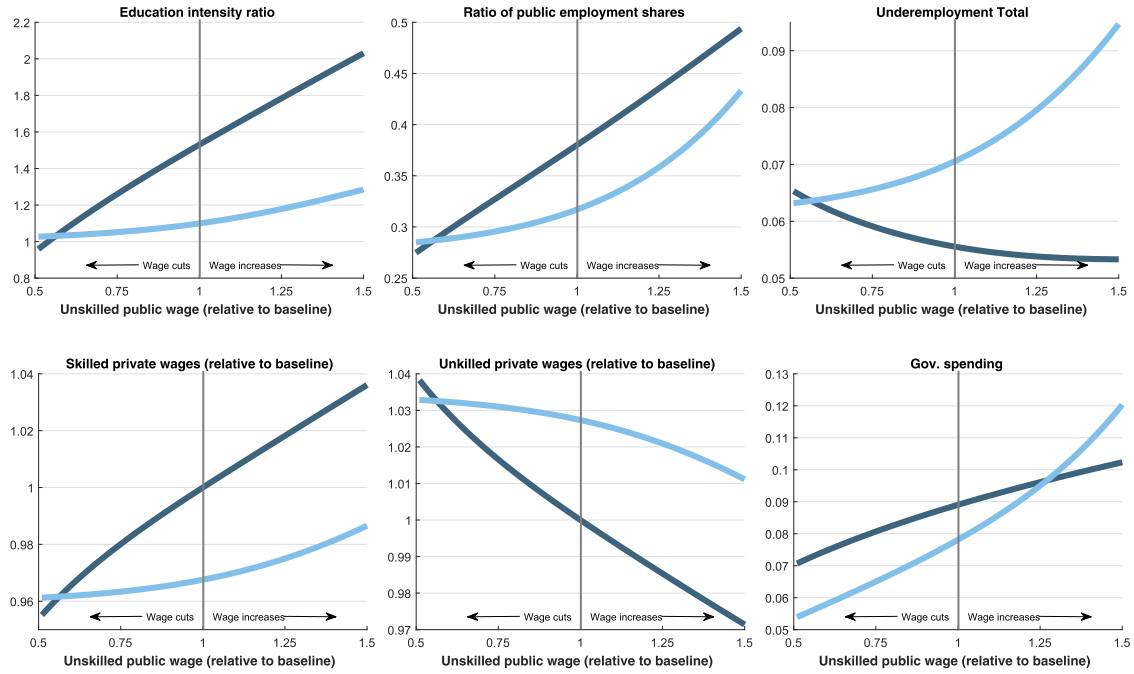


Fig. 9. Effects of public-sector unskilled wages. *Note:* regime 1, dark line. regime 2, light line.

wages raises skilled private wages by 0.07 percent and lowers unskilled private wages by 0.06 percent. Given a variation of the public-sector wage premium of 20 percentage points across US states, the variation of this policy alone can determine a variation of 2.6 percentage points in the college premium. It has been documented that governments are concerned with inequality when setting their wage policies. For instance, during the Euro Area crisis, many governments implemented wage cuts for their highest paid workers, and spared workers with lower wages, on the grounds that further cuts at the bottom would worsen inequalities. We show that this well intended policy can backfire. Higher wage compression shifts demand from workers with low to workers with high education and worsens underemployment in the public sector. As a consequence, the skill-mix in the private sector shifts towards low-educated workers, so their wages fall while skilled private wages go up. While decreasing wage inequality for a sub-set of workers, such policies increase wage inequality for everyone else.

Labour economists were very active, between the 1970s and the 1990s, studying public-sector employment, in particular from an applied angle. We believe that our basic framework can help us think about public employment and revive its study. Our view is that public wages are not set by a Walrasian auctioneer, but are the outcome of various complex decision processes, with consequences in the labour market. Despite its simplicity, the model reveals quite complex mechanisms about the public sector. When public wages do not equate supply and demand of government jobs, different regimes arise. We have shown that the effects of government policies on the private sector are profoundly different, whether we are in a regime where public employment is demand determined or in a regime where public employment is supply determined. While this switching between regimes did not interfere with the quantitative results on the decomposition, we think it is a defining feature of public-sector labour markets. Given the substantial variation of public wage across US states or across countries, we think it could explain variations in labour market and fiscal outcomes.

Declaration of Competing Interest

Authors declare that they have no conflict of interest.

Appendix A. Additional statistics

A1. Details on PIACC data

The OECD Survey of Adult Skills is part of the Program for the International Assessment of Adult Competencies (PIAAC). The data were collected between 2011 and 2015. In each country, the survey includes socio-demographic information (gender, education), labor market status and assesses the proficiency of adults aged between 16 and 65 in literacy, numeracy and problem solving.

The sample includes only respondents who are currently employed (C_D05 Current Employment Status is "Employed"). As in McGowan and Andrews (2015), to identify workers who are neither over-qualified, nor under-qualified, we use 2 questions in the survey asking workers to compare their skill level and that required for their job: "*Do you feel that you have the skills to cope with more demanding duties than those you are required to perform in your current job?*" (F_Q07a) and "*Do you feel that you need further training in order to cope well with your present duties?*" (F_Q07b). Workers who neither feel they have the skills to perform a more demanding job nor feel the need for further training in order to be able to perform their current job satisfactorily are considered as well-matched. These workers provide a reference for the educational attainment that is required to perform the job within each (1-digit) occupation. For a given occupation, in a given country, we compute the mean and standard deviation of years of completed education of well-matched workers. Underemployed workers are those who report years of education that lie 1.96 standard deviation above the average number of years of education of well-matched workers in a given (isco1) occupation. The resulting underemployment rates are reported in Fig. 6.

We repeat the exercise by looking at underemployment in the public and private sectors. Respondents are identified as public-sector workers using the question on "*which sector of the economy do you work?*" (D_Q03). The public sector includes: all parts of the public administration at the national, regional or local levels; public services provided by the state or from state funds (including publicly run schools, hospitals, universities, etc.); and publicly-owned companies.

In the top panel of Fig. A.5, underemployed workers are those who report years of education that lie 1 standard deviation above the average

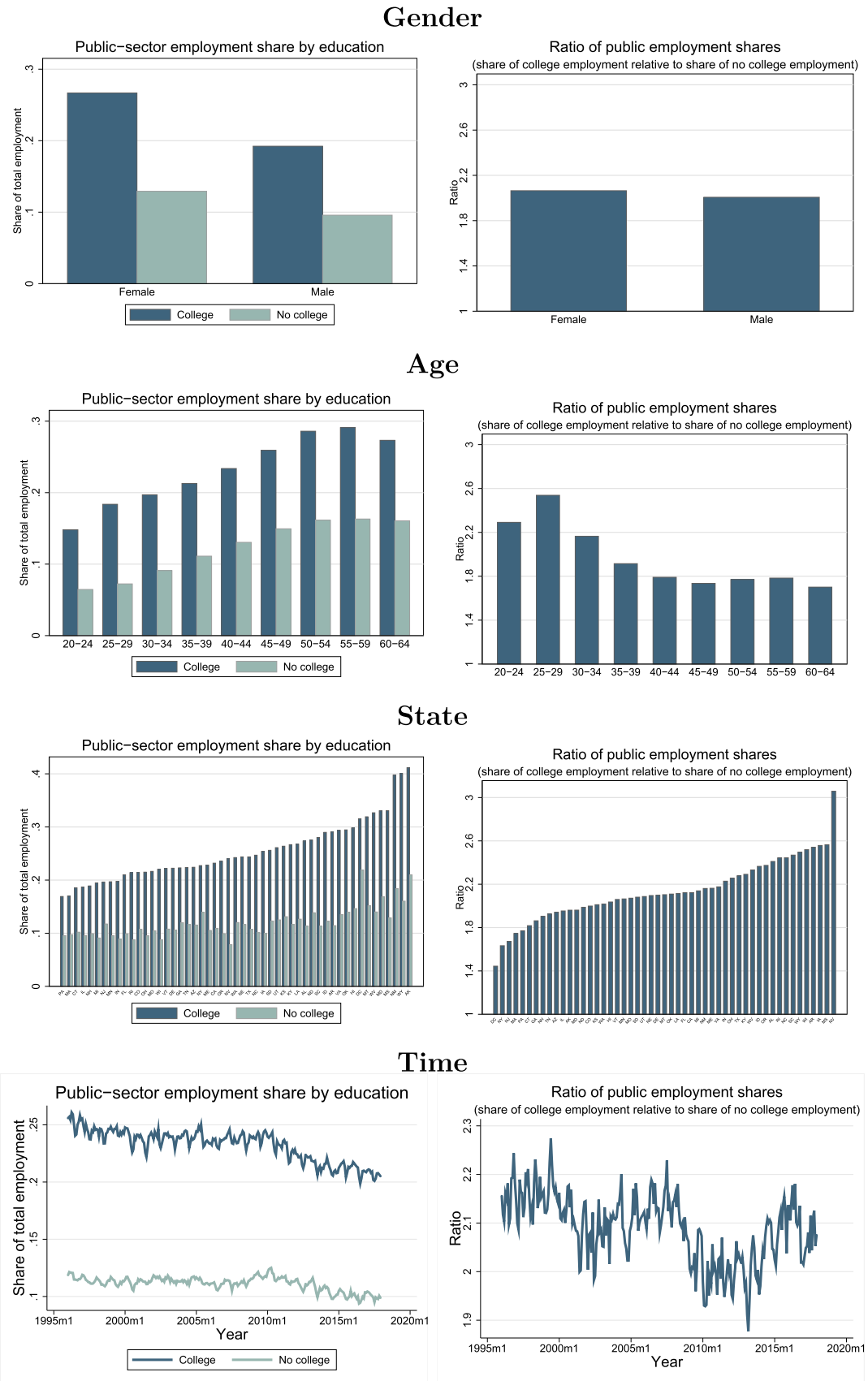


Fig. A.1. Public-sector employment share by education, different dimensions. *Note:* CPS data, average between 1996 and 2018.

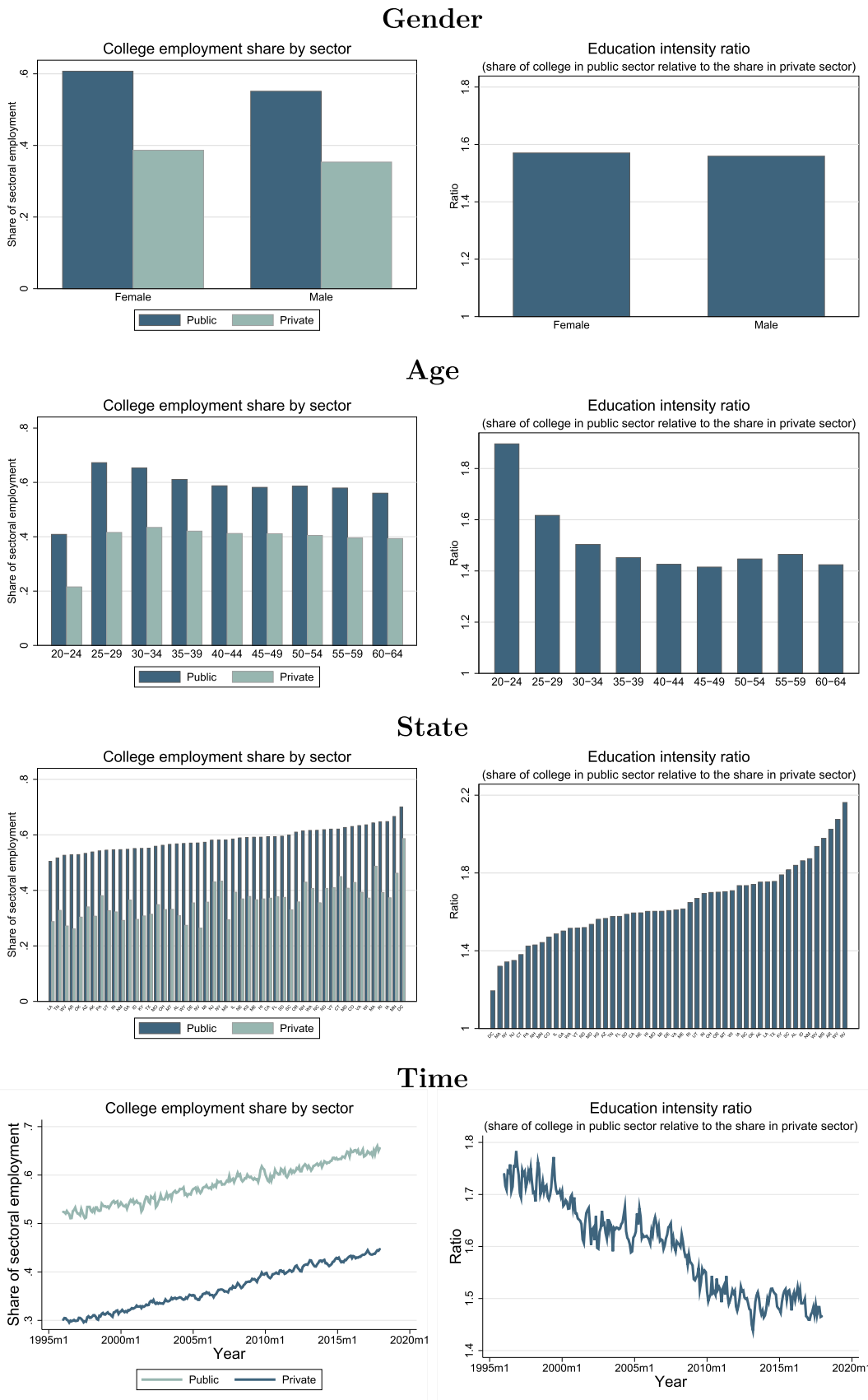


Fig. A.2. College share by sector, different dimensions. *Note:* CPS data, average between 1996 and 2018.

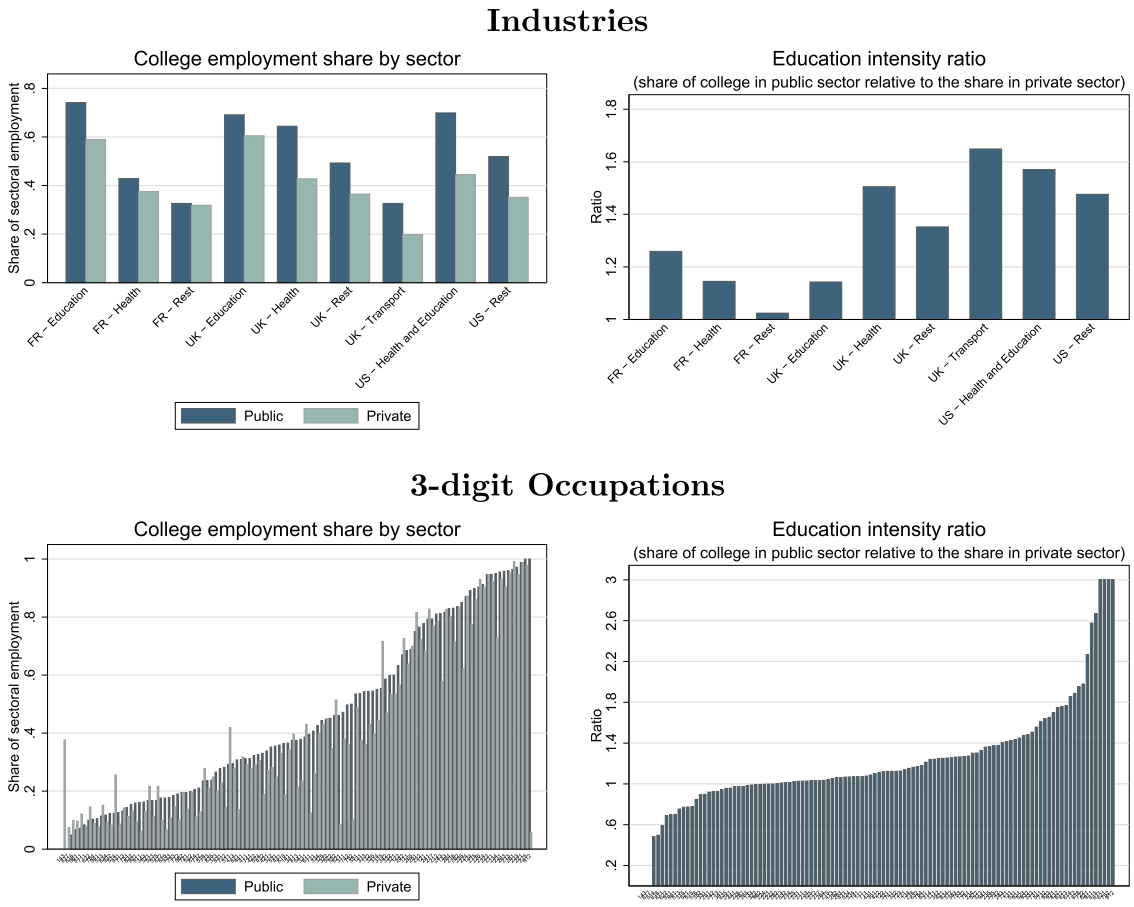


Fig. A.3. College share by sector, across industries and occupations. *Note:* 1st panel uses French, Spanish, UK Labour Force Surveys and the CPS. 2nd panel: CPS data, average between 1996 and 2018. 3-digit occupations that have an overall share of public-sector employment between 0.05 and 0.95.

number of years of education of well-matched workers in a given (isco1) occupation.

In the middle and bottom panel of Fig. A.5, college workers are those who report an educational attainment of ISCED 5B (First stage of tertiary education: typically shorter, more practical, technical specific programmes leading to professional qualifications.) and higher. We then compute the share of non-college workers within each occupation (isco 1 in middle panel of Fig. A.5, isco 2 in bottom panel of Fig. A.5). A college-educated worker is classified as underemployed when 2 conditions are met: (1) she is not well-matched and (2) working in an occupation that is majority non-college.

Appendix B. Two-sector model without underemployment

B1. Technology and preferences

We present a two-sector model that features a labour market with free mobility. There are two types of individuals with high (1) and low (2) education. The supply of educated individuals in the economy is indicated with n, while the supply of the low-educated workers is indicated with $1-n$. The representative firm produces a private sector output y and the government produces services g with constant return technology:

$$y = (j_1^p)^{\alpha}(j_2^p)^{1-\alpha}, \qquad g = (j_1^g)^{\beta}(j_2^g)^{1-\beta}. \tag{B.1}$$

Individuals only value wages, so they chose the highest paying job. If the public sector pays a higher wage than the private sector, these jobs would be preferred and would be rationed. If the public sector pays lower wages, no one would work there. As such, the only equilibrium without rationing, implies that the wages in the two sectors have to equate.

B2. Government

We assume that the government follows the same minimization problem, determining the target (ideal) level and composition of employment ($j_1^{g,d}$ and $j_2^{g,d}$), given by.

$$j_1^{g,d} = \bar{g}\left(\frac{w_2^g}{w_1^g}\frac{\beta}{1-\beta}\right)^{1-\beta}, \qquad j_2^{g,d} = \bar{g}\left(\frac{w_1^g}{w_2^g}\frac{1-\beta}{\beta}\right)^{\beta}. \tag{B.2}$$

B3. Private sector

The representative private sector firm maximizes profits as in the baseline model:

$$w_1^p = \alpha\left(\frac{j_2^p}{j_1^p}\right)^{1-\alpha}, \qquad w_2^p = (1-\alpha)\left(\frac{j_1^p}{j_2^p}\right)^{\alpha}. \tag{B.3}$$

And the market clearing conditions are now

$$n = j_1^g + j_1^p, \qquad 1-n = j_2^g + j_2^p. \tag{B.4}$$

B4. Equilibrium

Definition 2. A steady-state equilibrium consists of private-sector wages $\{w_1^p, w_2^p\}$, private-sector jobs $\{j_1^p, j_2^p\}$, public-sector jobs $\{j_1^g, j_2^g\}$, such that, given an exogenous wage policies, technology and composition of the labour force $\{w_1^g, w_2^g, \bar{g}, \alpha, \beta, n\}$, the following apply.

1. Private-sector firms maximizes profits.
2. Government:
   (a) If unconstrained by supply: minimizes costs of providing government services.
   (b) If constrained by supply: maintains production of government services.

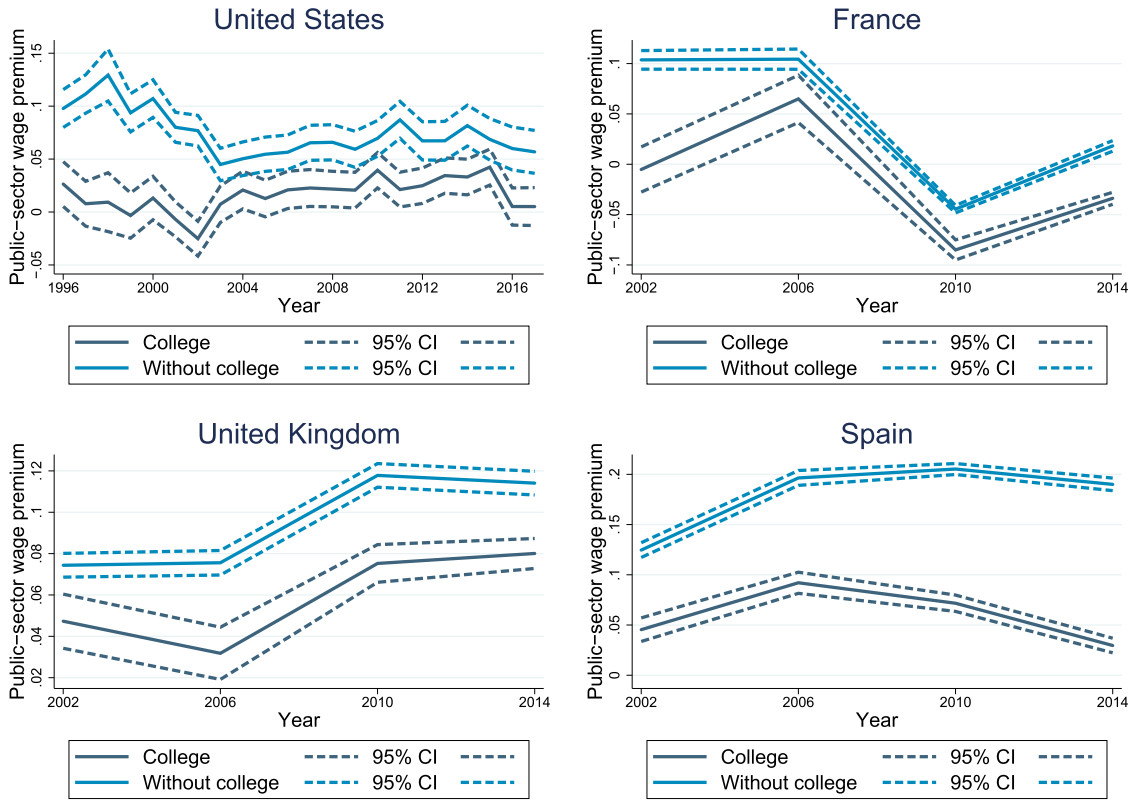


Fig. A.4. Compression, over time, across countries. *Note:* Estimation by, for each year regressing the log of hourly wage on a public-sector dummy and controls (age, gender, region and a part-time dummy), separately for workers with and without college graduate. Structure of Earning Survey (2002, 2006, 2010, 2014) and CPS data between 2006 and 2018.

3. Workers sort across labour markets optimally.
4. Markets clear.

The model can be written in two equations in w_1^p and w_2^p, as a function of public-sector employment j_1^g and j_2^g.

$$w_1^p = \alpha\left(\frac{1-n-j_2^g}{n-j_1^g}\right)^{1-\alpha}, \tag{B.5}$$

$$w_2^p = (1-\alpha)\left(\frac{n-j_1^g}{1-n-j_2^g}\right)^{\alpha}, \tag{B.6}$$

B5. Regime 1: wages are high enough in public sector

This is the case where public employment is demand determined. Jobs are rationed so workers who do not get a job in the public sector work in the private.

$$j_1^g = j_1^{g,d} = \bar{g}\left(\frac{w_2^g}{w_1^g}\frac{\beta}{1-\beta}\right)^{1-\beta}, \tag{B.7}$$

$$j_2^g = j_2^{g,d} = \bar{g}\left(\frac{w_1^g}{w_2^g}\frac{1-\beta}{\beta}\right)^{\beta}. \tag{B.8}$$

For this regime, the wages in the public sector have to be above those in the private.

$$w_1^g > \tilde{w}_1^g = \alpha\left(\frac{1-n-j_2^{g,d}}{n-j_1^{g,d}}\right)^{1-\alpha} \tag{B.9}$$

$$w_2^g > \tilde{w}_2^g = (1-\alpha)\left(\frac{n-j_1^{g,d}}{1-n-j_2^{g,d}}\right)^{\alpha} \tag{B.10}$$

The mechanisms here are the same as in the baseline model, except for the absence of underemployment.

B6. Regime 2: skilled public-sector wages too low

In the case, skilled public wages are below the private wage (when the government hires its target level of workers): $w_1^g < \tilde{w}_1^g$, skilled workers would move away from the public sector. However, not all of them would leave, as doing so would push the private sector wage below the public. Hence, the only equilibrium is that private wages fall until they are equal to public wages ($w_1^p = w_1^g$). This pins down jointly educated private employment, public employment, and unskilled private wages:

$$j_1^p = \left(\frac{\alpha}{w_1^g}\right)^{\frac{1}{1-\alpha}}(1-n-j_2^g), \tag{B.11}$$

$$j_1^g = n - j_1^p, \tag{B.12}$$

$$j_2^g = \left[\frac{\bar{g}}{(j_1^g)^{\beta}}\right]^{\frac{1}{1-\beta}}, \tag{B.13}$$

$$w_2^p = (1-\alpha)\left(\frac{n-j_1^g}{1-n-j_2^g}\right)^{\alpha}, \tag{B.14}$$

provided that $w_2^g \geq \tilde{w}_2^g$. To maintain government services it has to open more low-type jobs. Public employment is supply determined.

B7. Regime 3: unskilled public-sector wages too low

Again, we show this case for completeness. It requires that unskilled public wages are too low and that skilled wages are high enough $w_1^g \geq \tilde{w}_1^g$ and $w_2^g < \tilde{w}_2^g$. Unskilled workers prefer private sector so private wages fall until they are equal to public wages ($w_2^p = w_2^g$). This

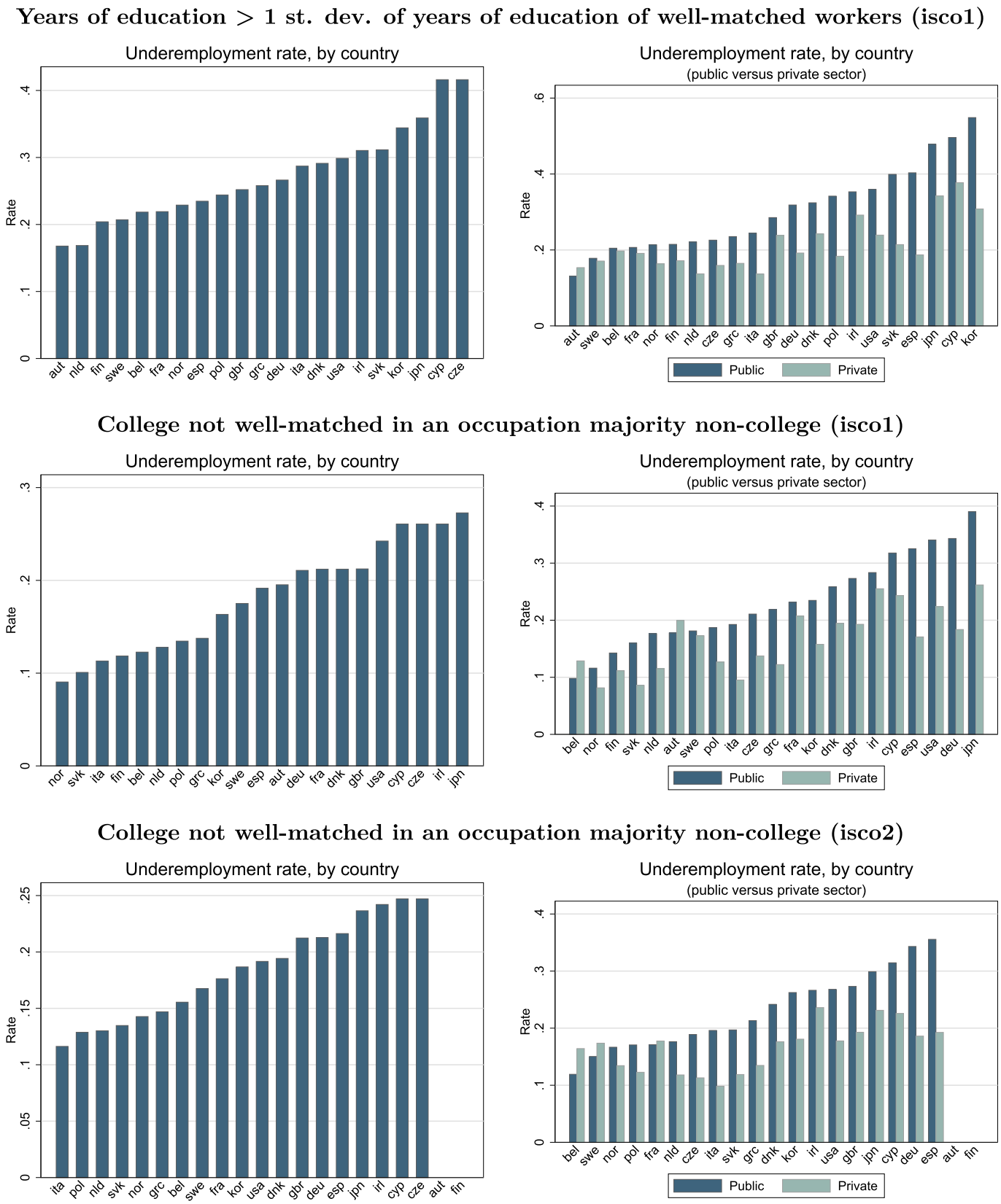


Fig. A.5. Underemployment rates, alternative calculation methods. *Note:* Source: PIAAC. Top panel : underemployed workers are workers whose years of education are 1 s.d. above the mean years of education of well-matched workers in their occupation. Middle panel : underemployed workers are college graduates that are not well-matched and work in 1-digit occupations that are majority non-college. Bottom panel: underemployed workers are college graduates that are not well-matched and work in 2-digit occupations that are majority non-college. 2-digit occupations are not available in Austria and Finland.

pins down jointly educated private employment, public employment, and unskilled private wages

$$j_2^p = \left(\frac{1-\alpha}{w_2^g}\right)^{\frac{1}{\alpha}}(n - j_1^g), \tag{B.15}$$

$$j_2^g = 1 - n - j_2^p \tag{B.16}$$

$$j_1^g = \left[\frac{\bar{g}}{(j_2^g)^{1-\beta}}\right]^{\frac{1}{\beta}}. \tag{B.17}$$

$$w_1^p = \alpha\left(\frac{1-n-j_2^g}{n-j_1^g}\right)^{1-\alpha}, \tag{B.18}$$

To maintain government services it has to open more high-type jobs. Public employment is supply determined.

Appendix C. One-sector model with underemployment

C1. Technology and preferences

Individuals are endowed with 1 unit of indivisible labor and firms have jobs requiring different tasks to produce output. There are two types of individuals with high (1) and low (2) education. The supply of educated individuals in the economy is indicated with n, while the supply of the low educated workers is indicated with $1-n$.

Firms produce with a constant return technology in jobs requiring different skills. There are skilled and unskilled jobs. In what follows we shall use a Cobb Douglas specification.

$$y = (j_1)^{\alpha}(j_2)^{1-\alpha}$$

where $j_1(j_2)$ is the number skilled (unskilled) jobs. Jobs can be described through a ladder type mechanism, so that individuals endowed with higher education are able to perform also unskilled jobs. They can perform at zero effort costs both type of jobs while individuals with low education can only perform the unskilled job.

Individual preferences are linear, and the model is static. The wage paid for the skilled job is indicated with w_1 while the wage paid for unskilled job is indicated with w_2. Each individual worker i has an heterogeneous "non-pecuniary value" over these tasks, ϵ_i^1 and ϵ_i^2, drawn from a continuous distribution with cumulative density Φ and unbounded lower and upper support. For simplicity, we also assume that the expected value of $E[\epsilon^1] = E[\epsilon^2] = 0$. These "non-pecuniary" attributes of the job could reflect preferences, but all other elements such as location of jobs, co-workers, hours, etc. For instance, an educated worker i's utility in the skilled job is given by sum of the wage and "non-pecuniary" shock, $w_1 + \nu\epsilon_i^1$, where ν captures the weight of the "non-pecuniary" shock in the individual preferences. Our model accommodates the traditional model in the limit where ν tends to zero.

C2. Sorting by high-educated workers and underemployment

The key decision rests with the educated workers and concerns the type of sector in which to supply their indivisible unit of labor. An individual i decision is given by

$$U_i^1 = Max\{w_1 + \nu\epsilon_i^1, w_2 + \nu\epsilon_i^2\} \tag{C.1}$$

while type 2 individuals have no choice other than working in the unskilled tasks and their utility is thus $U_i^2 = w_2 + \nu\epsilon_i^2$. Educated individuals join the simple tasks only if $(w_1 + \nu\epsilon_i^1 < w_2 + \nu\epsilon_i^2)+$, or if $\eta_i = \frac{w_2 - w_1}{\nu}$, where $\eta_i = \epsilon_i^1 - \epsilon_i^2$. In what follows, we indicate with Φ_η the probability distribution over the net preference shock $\eta_i = \epsilon_i^1 - \epsilon_i^2$. Educated individuals join the simple job if η is low enough so that $\eta_i < \frac{w_2 - w_1}{\nu}$. This simple sorting condition implies that there is an endogenously determined aggregate number of underemployed defined as

$$u = \Phi_\eta\left(\frac{w_2 - w_1}{\nu}\right) \tag{C.2}$$

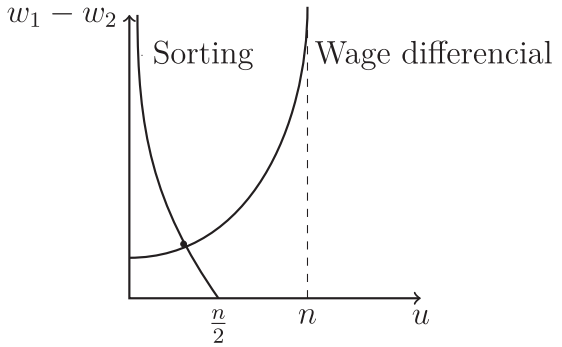


Fig. C.1. Equilibrium underemployment.

C3. Labor demand and market clearing

Firms maximise profits taking as given the wage for both tasks. Labor demand is given by

$$w_2 = (1-\alpha)\left(\frac{j_1}{j_2}\right)^{\alpha}, \quad w_1 = \alpha\left(\frac{j_2}{j_1}\right)^{1-\alpha}. \tag{C.3}$$

Wages adjust until the demand for jobs requiring a particular task is equal to the supply of workers for that task. Market clearing equilibrium imply

$$j_1 = n - u, \quad j_2 = (1-n) + u. \tag{C.4}$$

where labor demand j_1 and j_2 is given by Eq. (C.3) while underemployment u is derived from Eq. (C.2)

C4. Equilibrium

Definition 3. A steady-state equilibrium consists of tasks wages $\{w_1, w_2\}$, jobs in the two tasks $\{j_1, j_2\}$, and underemployment for skilled workers $\{u\}$, such that.

1. Private-sector firms maximizes profits (C.3).
2. Skilled workers sort across labour markets according to (C.2).
3. Markets clear (C.4).

The equilibrium is best summarized in two equations: the sorting condition and a wage differential condition, in u and $w_1 - w_2$:

$$u = n\Phi_\eta\left(\frac{w_2 - w_1}{\nu}\right) \tag{C.5}$$

$$w_1 - w_2 = \alpha\left(\frac{(1-n)+u}{n-u}\right)^{1-\alpha} - (1-\alpha)\left(\frac{n-u}{(1-n)+u}\right)^{\alpha} \tag{C.6}$$

These two conditions are depicted graphically in Fig. C.1. The downward sloping line is the sorting condition (C.2), that crosses the horizontal axis at $\frac{n}{2}$ underemployment. When the wage differential is zero, workers will split equally between the two types of jobs as none offers a wage advantage. As the wage differential increases, there are fewer educated willing to work in unskilled jobs and as this differential increases to infinity underemployment tends to zero. The upward sloping equation is the wage differential condition, obtained from labor demand (C.3), and the market clearing conditions (C.4), is increasing in underemployment. With zero underemployment the intercept represents the wage differential of the typical model where all the educated workers are performing skilled jobs. As underemployment increases, this is reflected on an excess supply of workers to unskilled jobs and a shortage of workers for skilled jobs, thereby increasing the wage differential. As underemployment approaches the total supply of the skilled n, by the Inada conditions, the wage differential tends to infinity. The equilibrium underemployment is the crossing of the two lines, and is given by a single equation in underemployment:

$$u = \Phi_\eta\left(\frac{(1-\alpha)}{\nu}\left(\frac{n-u}{1-n+u}\right)^{1-\alpha} - \frac{\alpha}{\nu}\left(\frac{1-n+u}{n-u}\right)^{\alpha}\right) \tag{C.7}$$

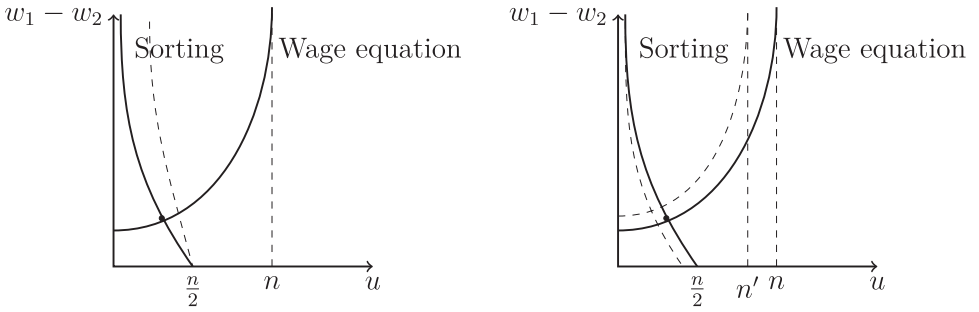


Fig. C.2. Equilibrium underemployment with an income tax and skill shortage.

The equilibrium exists and is unique.

C5. Comparative statics

The simple model can be used to illustrate the effects of two interesting comparative static exercise. Such exercise highlights some features of the public sector that are present in the main model. Suppose first that the government imposes a proportional income tax (Fig. C.2, left panel). Other things equal, the net-wage differential is lower and the sorting condition shifts to the right, and equilibrium underemployment rises. Note that despite the fact that the gross wage differential $(w_2 - w_1)$ rises, the take-home differential actually falls. Next, suppose that the supply of skilled workers available shrinks. As shown in the right panel of Fig. C.2, both curves shift to the left and equilibrium underemployment falls, but the wage gap is now larger.

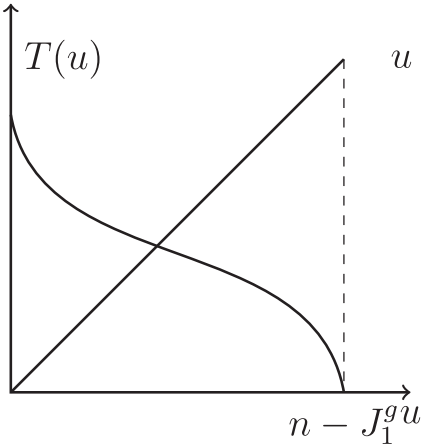
In a companion paper, we generalize this 1-sector model, considering both under and overemployment, and different efficiency units of educated workers in unskilled jobs, to measure the output losses of mismatch (Garibaldi, Gomes, Sopraseuth, 2020).

Appendix D. Baseline model

D1. Regime 1

Substituting the expressions for wages on underemployment, we get one equation that pins down u.

$$u = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}[(1-\alpha)\left(\frac{n-j_1^g-u}{1-n-j_2^g+u}\right)^{\alpha}]} + e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}[\alpha\left(\frac{1-n-j_2^g+u}{n-j_1^g-u}\right)^{1-\alpha}]} + e^{\frac{(1-\tau)}{\nu}[(1-\alpha)\left(\frac{n-j_1^g-u}{1-n-j_2^g+u}\right)^{\alpha}]}}\right] \equiv T(u) \tag{D.1}$$



The LHS is the 45 degree line, from 0 to $n - J_1^g$. The RHS evaluated at zero is positive, evaluated at $n - j_1^g$ is zero and is decreasing in u. We concentrate our analysis on the effects of public-sector wages for both types of workers, the size of the educated population and the level of government services. Under Regime 1, we can write the matrix of marginal effects for the exogenous variables $z \in \{w_g^2, w_g^1, \bar{g}, n\}$ as:

$$\begin{pmatrix} 1 & -\frac{\partial u}{\partial w_p^1} & -\frac{\partial u}{\partial w_p^2} & -\frac{\partial u}{\partial j_g^1} & 0 \\ -\frac{\partial w_p^1}{\partial u} & 1 & 0 & -\frac{\partial w_p^1}{\partial j_1^g} & -\frac{\partial w_p^1}{\partial j_2^g} \\ -\frac{\partial w_p^2}{\partial u} & 0 & 1 & -\frac{\partial w_p^2}{\partial j_1^g} & -\frac{\partial w_p^2}{\partial j_2^g} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \frac{du}{dz} \\ \frac{dw_p^1}{dz} \\ \frac{dw_p^2}{dz} \\ \frac{dj_g^1}{dz} \\ \frac{dj_g^2}{dz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial z} \\ \frac{\partial w_1^p}{\partial z} \\ \frac{\partial w_2^p}{\partial z} \\ \frac{\partial j_1^g}{\partial z} \\ \frac{\partial j_2^g}{\partial z} \end{pmatrix} \tag{D.2}$$

where:

$\frac{\partial u}{\partial w_p^1} = -\frac{1-\tau}{\nu}u(1-\frac{u}{n-j_1^g}) < 0$	$\frac{\partial u}{\partial w_p^2} = \frac{1-\tau}{\nu}u^p(1-\frac{u}{n-j_1^g}) > 0$	$\frac{\partial u}{\partial j_g^1} = -\frac{u}{n-j_1^g} < 0$
$\frac{\partial w_p^1}{\partial u} = (1-\alpha)w_1^p(\frac{1}{j_1^p}+\frac{1}{j_2^p}) > 0$	$\frac{\partial w_p^1}{\partial j_1^g} = \frac{(1-\alpha)w_1^p}{j_1^p} > 0$	$\frac{\partial w_p^1}{\partial j_2^g} = -\frac{(1-\alpha)w_1^p}{j_2^p} < 0$
$\frac{\partial w_p^2}{\partial u} = -\alpha w_2^p(\frac{1}{j_1^p}+\frac{1}{j_2^p}) < 0$	$\frac{\partial w_p^2}{\partial j_1^g} = -\frac{\alpha w_2^p}{j_1^p} < 0$	$\frac{\partial w_p^2}{\partial j_2^g} = \frac{\alpha w_2^p}{j_2^p} > 0$

The right-hand side vector is different depending on which parameter we are doing the comparative statics on

$\frac{\partial u}{\partial w_g^2} = \frac{1-\tau}{\nu}u^g(1-\frac{u}{n-j_1^g}) > 0$	$\frac{\partial u}{\partial w_g^1} = 0$	$\frac{\partial u}{\partial \bar{g}} = 0$	$\frac{\partial u}{\partial n} = \frac{(u)}{n-j_1^g} > 0$
$\frac{\partial w_g^1}{\partial w_g^2} = 0$	$\frac{\partial w_g^1}{\partial w_g^1} = 0$	$\frac{\partial w_1^p}{\partial \bar{g}} = 0$	$\frac{\partial w_1^p}{\partial n} = -(1-\alpha)w_1^p(\frac{1}{j_1^p}+\frac{1}{j_2^p}) < 0$
$\frac{\partial w_2^p}{\partial w_g^2} = 0$	$\frac{\partial w_2^p}{\partial w_g^1} = 0$	$\frac{\partial w_2^p}{\partial \bar{g}} = 0$	$\frac{\partial w_2^p}{\partial n} = \alpha w_2^p(\frac{1}{j_1^p}+\frac{1}{j_2^p}) > 0$
$\frac{\partial j_1^g}{\partial w_g^2} = \frac{(1-\beta)j_1^g}{w_g^2} > 0$	$\frac{\partial j_1^g}{\partial w_g^1} = \frac{-(1-\beta)j_1^g}{w_g^1} < 0$	$\frac{\partial j_1^g}{\partial \bar{g}} = \frac{j_1^g}{\bar{g}} > 0$	$\frac{\partial j_1^g}{\partial n} = 0$
$\frac{\partial j_2^g}{\partial w_g^2} = -\frac{\beta j_2^g}{w_g^2} < 0$	$\frac{\partial j_2^g}{\partial w^g 1} = \frac{\beta j_2^g}{w_1^g} > 0$	$\frac{\partial j_2^g}{\partial \bar{g}} = \frac{j_2^g}{\bar{g}} > 0$	$\frac{\partial j_2^g}{\partial n} = 0$

Solving the matrix system (noticing that $\frac{\partial w_1^p}{\partial j_1^g} \times \frac{\partial w_2^p}{\partial j_2^g} = \frac{\partial w_1^p}{\partial j_2^g} \times \frac{\partial w_2^p}{\partial j_1^g}$, together with $\frac{(u)}{n-j_1^g} < 1$, $-\frac{\partial u}{\partial w_p^1} = \frac{\partial u}{\partial w_g^2} + \frac{\partial u}{\partial w_p^2}$ and that $\frac{\partial j_1^g}{\partial w_g^2} < -\frac{\partial j_2^g}{\partial w_g^2}$ if $w_1^g > w_2^g$. With Matlab Symbolic Toolkit (codes available on request), we show

$\frac{du}{dw_g^2} \lesseqgtr 0$	$\frac{du}{dw^1} > 0$	$\frac{du}{d\bar{g}} \lesseqgtr 0$	$\frac{du}{dn} > 0$
$\frac{dw_1^p}{dw_g^2} > 0$	$\frac{dw_1^p}{dw_g^g} < 0$	$\frac{dw_1^p}{d\bar{g}} \lesseqgtr 0$	$\frac{dw_1^p}{dn} \lesseqgtr 0$
$\frac{dw_2^p}{dw_g^2} < 0$	$\frac{dw_2^p}{dw_g^g} > 0$	$\frac{dw_2^p}{d\bar{g}} \lesseqgtr 0$	$\frac{dw_2^p}{dn} \lesseqgtr 0$
$\frac{dj_1^g}{dw_g^2} > 0$	$\frac{dj_1^g}{dw_g^g} < 0$	$\frac{dj_1^g}{d\bar{g}} > 0$	$\frac{dj_1^g}{dn} = 0$
$\frac{dj_2^g}{dw_g^2} < 0$	$\frac{dj_2^g}{dw^g 1} > 0$	$\frac{dj_2^g}{d\bar{g}} > 0$	$\frac{dj_2^g}{dn} = 0$

D2. Regime 2

Under Regime 2, the last two rows of the matrix of marginal effects for the exogenous variables $z \in \{w_g^2, w_g^1, \bar{g}, n\}$ are different:

$$\begin{pmatrix} 1 & -\frac{\partial u}{\partial w_p^1} & -\frac{\partial u}{\partial w_p^2} & -\frac{\partial u}{\partial j_g^1} & 0 \\ -\frac{\partial w_p^1}{\partial u} & 1 & 0 & -\frac{\partial w_p^1}{\partial j_1^g} & -\frac{\partial w_p^1}{\partial j_2^g} \\ -\frac{\partial w_p^2}{\partial u} & 0 & 1 & -\frac{\partial w_p^2}{\partial j_1^g} & -\frac{\partial w_p^2}{\partial j_2^g} \\ 0 & -\frac{\partial j_1^g}{\partial w_p^1} & -\frac{\partial j_1^g}{\partial w_2^p} & 1 & 0 \\ 0 & 0 & 0 & -\frac{\partial j_2^g}{\partial j_1^g} & 1 \end{pmatrix} \times \begin{pmatrix} \frac{du}{dz} \\ \frac{dw_p^1}{dz} \\ \frac{dw_p^2}{dz} \\ \frac{dj_g^1}{dz} \\ \frac{dj_g^2}{dz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial z} \\ \frac{\partial w_1^p}{\partial z} \\ \frac{\partial w_2^p}{\partial z} \\ \frac{\partial j_1^g}{\partial z} \\ \frac{\partial j_2^g}{\partial z} \end{pmatrix} \tag{D.3}$$

where, in addition

$\frac{\partial j_1^g}{\partial w_1^p} = -\frac{1-\tau}{\nu} j_1^p j_1^g < 0$	$\frac{\partial j_1^g}{\partial w_2^p} = -\frac{1-\tau}{\nu} u^p j_1^g < 0$	$\frac{\partial j_2^g}{\partial j_1^g} = -\frac{\beta}{1-\beta}\frac{j_2^g}{j_1^g} < 0$

The last two rows of the right-hand side vectors are now

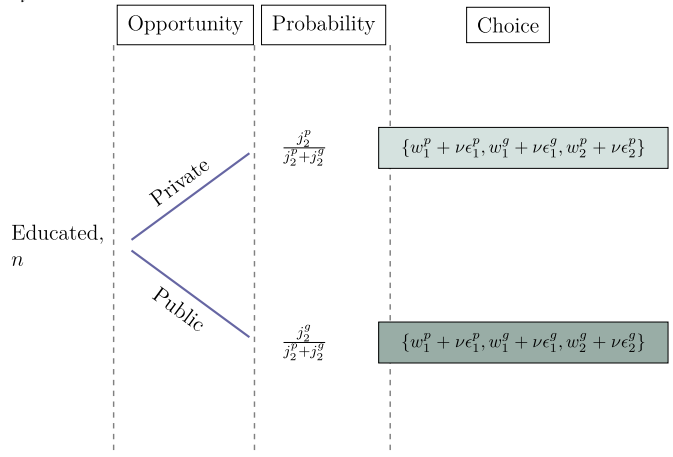
$\frac{\partial j_1^g}{\partial u^g} = -\frac{1-\tau}{n\nu} u^g j_1^g < 0$	$\frac{\partial j_1^g}{\partial u^g} = \frac{1-\tau}{\nu}\frac{j_1^g}{n}(1-\frac{j_1^g}{n}) > 0$	$\frac{\partial j_1^g}{\partial \bar{g}} = 0$	$\frac{\partial j_1^g}{\partial n} = \frac{j_1^g}{n} > 0$
$\frac{\partial j_2^g}{\partial u^g} = 0$	$\frac{\partial j_2^g}{\partial u^g 1} = 0$	$\frac{\partial j_2^g}{\partial \bar{g}} = \frac{1}{\beta}\frac{j_2^g}{\bar{g}}$	$\frac{\partial j_2^g}{\partial n} = 0$

Solving the matrix system (noticing that $\frac{\partial w_1^p}{\partial j_1^g} \times \frac{\partial w_2^p}{\partial j_2^g} = \frac{\partial w_1^p}{\partial j_2^g} \times \frac{\partial w_2^p}{\partial j_1^g}$, together with $\frac{(u)}{n-j_1^g} < 1$, $-\frac{\partial u}{\partial w_p^1} = \frac{\partial u}{\partial w_g^2} + \frac{\partial u}{\partial w_p^2}$ and that $-\frac{\partial j_2^g}{\partial j_1^g} > 1$ if $w_1^g > w_2^g$. With Matlab Symbolic Toolkit (codes available on request), we show

$\frac{du}{dw_g^2} \lessgtr 0$	$\frac{du}{dw_1^g} < 0$	$\frac{du}{d\bar{g}} \lessgtr 0$	$\frac{du}{dn} \lessgtr 0$
$\frac{dw_g^1}{dw_g^2} > 0$	$\frac{dw_1^g}{dw_1^g} > 0$	$\frac{dw_1^p}{d\bar{g}} < 1$	$\frac{dw_1^p}{dn} \lessgtr 0$
$\frac{dw_g^2}{dw_g^2} < 0$	$\frac{dw_2^1}{dw_1^g} < 0$	$\frac{dw_2^p}{d\bar{g}} > 0$	$\frac{dw_2^p}{dn} \lessgtr 0$
$\frac{dj_1^g}{dw_g^2} \lessgtr 0$	$\frac{dj_1^g}{dw_1^g} > 0$	$\frac{dj_1^g}{d\bar{g}} \lessgtr 0$	$\frac{dj_1^g}{dn} \lessgtr 0$
$\frac{dj_2^g}{dw_g^2} \lessgtr 0$	$\frac{dj_2^1}{dw^g 1} < 0$	$\frac{dj_2^g}{d\bar{g}} \lessgtr 0$	$\frac{dj_2^g}{dn} \lessgtr 0$

D3. Baseline model with alternative sorting mechanism

We set up a variation of the model with an alternative sorting mechanism. We consider that the underemployment opportunities are proportional to size of sector. The mechanism is described in the figure below. Of all the educated workers, a fraction $\frac{j_2^g}{j_2^p+j_2^g}$ has an underemployment opportunity only in the public sector. Those workers choose between three options $Max\{w_1^p + \nu\epsilon_i^{p,1}, w_1^g + \nu\epsilon_i^{g,1}, w_2^g + \nu\epsilon_i^{g,2}\}$. The remaining fraction $\frac{j_2^p}{j_2^p+j_2^g}$ has only an underemployment opportunity in the private sector and chooses between $Max\{w_1^p + \nu\epsilon_i^{p,1}, w_1^g + \nu\epsilon_i^{g,1}, w_2^p + \nu\epsilon_i^{p,2}\}$.



The threshold wages $\tilde{w}_1^g$ and $\tilde{w}_2^g$ are defined implicitly by

$$j_1^{g,d} = n\left[\frac{j_2^p}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^p}} + \frac{j_2^g}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^g}}\right] \tag{D.4}$$

$$j_2^{g,d} - u^g = (1-n)\left[\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_2^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_2^g}+e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{D.5}$$

And the different shares in the economy given by:

$$u^g = n\left[\frac{j_2^g}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^g}}\right] \tag{D.6}$$

$$u^p = n\left[\frac{j_2^p}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_2^p}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{D.7}$$

$$j_1^p = n\left[\frac{j_2^p}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_1^p}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^p}} + \frac{j_2^g}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_1^p}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^g}}\right] \tag{D.8}$$

The mechanism is similar to the baseline model but with more complicated solution. The advantage of this extension is that it gives a ratio public employment shares of 1, in the symmetric case, which we think is more realistic. Hence, we use this variation of the model in the quantitative section.

Appendix E. Extensions

E1. Endogenous public-sector wages

Our theory for the endogenous determination of public-sector wage is based on a union constraint. We think the higher unionization rates in the public sector could be one of the causes of significant public-private wage differentials and the compression across education levels. According to the CPS, for our sample period, the unionisation rate is 37 percent in the public sector compared to 8 percent in the private. However, these could be driven by other political economy factors or simply aversion to inequality. It might also be partly the consequence of history dependence and a more sluggish adjustment to technological changes that increased wage inequality in the private sector, as suggested by Borjas (2003). Here we present one possible theory.

We can provide microeconomic foundations for the public employment and wage policies that are taken as exogenous in the baseline model. Consider a government that wants to minimize cost subject to maintaining the production of government services $\bar{g}$. Additionally, it faces a constrain imposed by unions, that arise from political pressure. The preferences of a union represented by $\theta\ln(a_1) + (1-\theta)\ln(a_2)$. Here θ represents the weight of skilled workers in the union's preferences and a_1 and a_2 are the extra payment to public-sector workers on top of the threshold wage for the unconstrained public sector ($w_1^g = \tilde{w}_1^g + a_1$ and $w_2^g = \tilde{w}_2^g + a_2$). The union knows what this minimum required wage is and tries to push the wages above. For convenience, we assume the function expressing the utility of the extra payment to type i workers is $\log(a_i)$. The government's problem can be written as:

$$\min_{j_1^g, j_2^g} w_1^g j_1^g + w_2^g j_1^g$$

s.t.

$$\bar{g} = (j_1^g)^\beta (j_2^g)^{1-\beta}.$$

$$\bar{U} = \theta\ln(a_1) + (1-\theta)\ln(a_2).$$

$$w_1^g = \tilde{w}_1^g + a_1.$$

$$w_2^g = \tilde{w}_2^g + a_2.$$

Where $\bar{U}$ is the required utility of unions. The first order conditions of this problem are:

$$j_1^{g,d} = \bar{g}\left(\frac{w_2^g}{w_1^g}\frac{\beta}{1-\beta}\right)^{1-\beta}, \tag{E.1}$$

$$j_2^{g,d} = \bar{g}\left(\frac{w_1^g}{w_2^g}\frac{1-\beta}{\beta}\right)^{\beta}. \tag{E.2}$$

$$a_1 = \frac{\Omega\theta}{j_1^{g,d}} \tag{E.3}$$

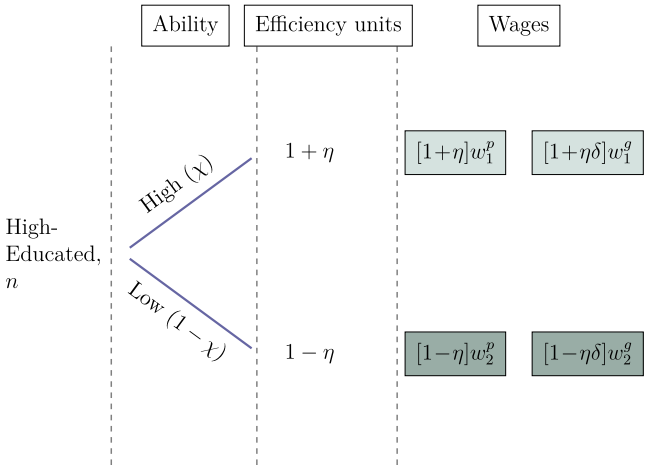
$$a_2 = \frac{\Omega(1-\theta)}{j_2^{g,d}} \tag{E.4}$$

The first two conditions pin down the employment level of the government and are equal to the baseline case. The last two conditions pin down government wages. The additional payment to each type of workers depends on the strength of the union constraint (measured by Ω) and the relative preference of the union over skilled and unskilled workers. Whether it raises more the skilled or unskilled wages, depends on the relative weight on the union preference. This could generate different premia (including negative premia) for different types of workers.

If $\bar{U} = 0$, $a_1 = a_2 = 0$ and the $w_e^g = \tilde{w}_e^g$, the government offers the minimum wage necessary to hire the workers it needs. This would be the outcome of a benevolent government. This is one model of government behaviour, but there could certainly be others. Under this conditions, the economy would be always under Regime 1. To push the government into Regime 2, we would need to add other elements such as budgetary pressures. We think however, when studying the effects of public wages, it is a clearer exercise to take them as exogenous.

E2. Heterogeneous ability of educated workers

We think that heterogeneity of ability of educated workers is an important dimension to understand both underemployment and the selection into the public sector, and how it is affected by the wage compression. Consider a variation of the model where high-educated workers are heterogeneous in their effective units of labour. The setting is described in the figure below. A fraction χ of educated workers have $1+\eta$ efficiency units in skilled jobs, while the remaining only have $1-\eta$. Wages in the private sector reflect perfectly their efficiency units, with the high-ability workers earning $(1+\eta)w_1^p$ and the low-ability workers earning $(1-\eta)w_1^p$. Given that underemployment is a negative function of the wage differential between skilled and unskilled jobs, it is clear that underemployment is concentrated on the low-ability workers.



In the public sector, the payment structure might not reflect entirely the efficiency units of the worker. We assume that the wages of high-ability educated workers is $(1+\eta\delta)w_1^g$ and for low-ability workers $(1-\eta\delta)w_1^g$. The parameter δ reflects the within-group wage compression. If $\delta < 1$, there is lower wage dispersion in the public sector for educated workers. This fact that has been widely documented.[19] At the limit, where $\delta = 0$, the government offers one wage independent of the efficiency units.

This heterogeneity requires that we distinguish the number of workers in terms of headcount and in efficiency units. Furthermore, we assume that the government always prefers the high-ability workers and restrain the analysis to the case χ is small enough so that the government cannot exhaust the high-skilled jobs with high-ability educated workers. We can defined the market clearing in headcount:

$$n\chi = l_{1,h}^g + l_{1,h}^p + u_h \tag{E.5}$$

$$n(1-\chi) = l_{1,\ell}^g + l_{1,\ell}^p + u_\ell \tag{E.6}$$

$$1 - n = j_2^g + j_2^p - u_h - u_\ell. \tag{E.7}$$

where $l_{1,h}^x$ and $l_{1,\ell}^x$ denote the number of high- and low-ability working in sector x. In terms of efficiency units the market clearing is given by

$$j_1^g = (1+\eta)l_{1,h}^g + (1-\eta)l_{1,\ell}^g \tag{E.8}$$

$$j_1^p = (1+\eta)l_{1,h}^p + (1-\eta)l_{1,\ell}^p \tag{E.9}$$

Regarding the sorting, we assume that the government skilled jobs is always high enough such that high-ability workers that want a public-sector job always enter. Hence, for the high ability, the sorting between underemployment, public-sector employment and private-sector employment (remainder) is given by

$$u_h = n\chi\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^p} + e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta\delta)w_1^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta)w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{E.10}$$

$$l_{1,h}^g = n\chi\left[\frac{e^{\frac{(1-\tau)}{\nu}(1+\eta\delta)w_1^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta\delta)w_1^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta)w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{E.11}$$

The low-ability workers take the remaining public-sector jobs. We focus on regime 1 (public-sector wages are high enough) such that for them, jobs are rationed. Hence, the number of low-ability workers in the public sector and underemployed are given by:

$$l_{2,\ell}^g = \frac{j_1^g - (1+\eta)l_{1,h}^g}{(1-\eta)} \tag{E.12}$$

$$u_\ell = \left[n(1-\chi) - l_{2,\ell}^g\right]\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^p} + e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}(1-\eta)w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{E.13}$$

In this version of the model, skilled workers with low efficiency units have lower wages in skilled jobs, and hence are more likely to be underemployed. Our model also helps to understand the implications of the wage compression within education groups. If δ is below 1, the government does not fully reward the efficiency units of high-ability educated workers and rewards too much low-ability workers. As such, fewer high-ability skilled workers work for the government, that is more likely to be constrained by the supply of high-ability workers. Hence, it has to employ more of the low-ability skilled workers whose efficiency units are relatively more expensive, which in turn amplifies the education bias.

E3. Endogenous income tax

One element that we did not develop in the baseline model was the financing side of the government. τ was taken as a parameter in the baseline model. The justification would be that such policies would be financed with government debt or cuts in other spending categories. However, we can easily endogeneize tax rate in the model by introducing an additional budget constraint. τ adjusts in order to balance the

[19] This has been found running quantile regressions and finding that for the bottom of the earnings distribution the public-sector wage premium is large at the bottom very low or negative. See for instance Christofides and Michael, 2013.

government budget. This implies adding a fourth equation to the model and a fourth endogenous variable.

The model can now be written in four equations in u, w_1^p, w_2^p and τ

$$u = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^p} + e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{E.14}$$

$$w_1^p = \alpha\left(\frac{1-n-j_2^g+u}{n-j_1^g-u}\right)^{1-\alpha}, \tag{E.15}$$

$$w_2^p = (1-\alpha)\left(\frac{n-j_1^g-u}{1-n-j_2^g+u}\right)^{\alpha}, \tag{E.16}$$

$$\tau = \frac{w_1^g j_1^g + w_2^g j_2^g}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} \tag{E.17}$$

The solution to the system of total derivatives is:

$$\begin{pmatrix} 1 & -\frac{\partial u}{\partial w_p^1} & -\frac{\partial u}{\partial w_p^2} & -\frac{\partial u}{\partial j_g^1} & 0 & -\frac{\partial u}{\partial \tau} \\ -\frac{\partial w_p^1}{\partial u} & 1 & 0 & -\frac{\partial w_p^1}{\partial j_g^1} & -\frac{\partial w_p^1}{\partial j_g^2} & 0 \\ -\frac{\partial w_p^2}{\partial u} & 0 & 1 & -\frac{\partial w_p^2}{\partial j_g^1} & -\frac{\partial w_p^2}{\partial j_g^2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{\partial \tau}{\partial u} & 0 & 0 & -\frac{\partial \tau}{\partial \partial j_g^1} & -\frac{\partial \tau}{\partial \partial j_g^2} & 1 \end{pmatrix} \times \begin{pmatrix} \frac{du}{dz} \\ \frac{dw_p^1}{dz} \\ \frac{dw_p^2}{dz} \\ \frac{dj_g^1}{dz} \\ \frac{dj_g^2}{dz} \\ \frac{d\tau}{dz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial z} \\ \frac{\partial w_p^1}{\partial z} \\ \frac{\partial w_p^2}{\partial z} \\ \frac{\partial j_g^1}{\partial z} \\ \frac{\partial j_g^2}{\partial z} \\ \frac{\partial \tau}{\partial z} \end{pmatrix} \tag{E.18}$$

$\frac{\partial u}{\partial \tau} = \frac{j_1^g}{\nu}(w_1^p \frac{u}{n-j_1^g} - w_2^p u^p - w_2^g u^g > 0$ $\frac{\partial \tau}{\partial u} = \frac{(w_1^p - w^p 2)\tau}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$ $\frac{\partial \tau}{\partial j_g^1} = \frac{w_1^g(1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$

$\frac{\partial \tau}{\partial j_g^2} = \frac{w_2^g(1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$ $\frac{\partial \tau}{\partial w_2^g} = \frac{j_2^g(1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$ $\frac{\partial \tau}{\partial w_1^g} = \frac{j_1^g(1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$

The tax rate has a same effect as a change in ν, the weight of the non-pecuniary element of preferences. Higher taxes lowers the net income differential between skilled and unskilled jobs, so it raises underemployment. See, for instance, Fig. C.2 in Appendix C for the effects of an increase tax rate in the a one-sector model. An increase of skilled or unskilled wages, by raising government spending, have an additional positive effect on underemployment by raising the income tax.

E4. Dual government problem

In the baseline model, the government minimizes the cost of producing a certain level of government services. There exists a dual government problem, where it maximizes services subject to an exogenous wage bill. Consider a government that, because of budgetary constraints, has a limited amount of spending $\bar{\omega}$, exogenous. Its problem is given by:

$$\max_{j_1^g, j_2^g} (j_1^g)^\beta (j_2^g)^{1-\beta}$$

$$s.t.$$

$$w_1^g j_1^g + w_2^g j_1^g = \bar{\omega}.$$

The first-order conditions pinning employment are given by:

$$j_1^{g,d} = \bar{\omega}\left(\frac{\beta}{w_1^g}\right), \tag{E.19}$$

$$j_2^{g,d} = \bar{\omega}\left(\frac{1-\beta}{w_2^g}\right). \tag{E.20}$$

The two conditions pin down the employment level of the government. Now, the number of workers of a given type only depends on their wage. Given technology and a certain wage, the government spends a constant fraction β of its budget on skilled workers and $1-\beta$ on unskilled workers. Differently from the baseline, j_1^g is increasing in β and decreasing in w_1^g and j_2^g is decreasing in w_2^g and β. The derivatives of employment are given by:

$\frac{\partial j_1^g}{\partial w_2^g} = 0$	$\frac{\partial j_1^g}{\partial w_1^g} = \frac{-j_1^g}{w_1^g} < 0$
$\frac{\partial j_2^g}{\partial w_2^g} = -\frac{j_2^g}{w_2^g} < 0$	$\frac{\partial j_2^g}{\partial w^g 1} = 0$

The solution to this problem is simpler as increases in the unskilled wage lower proportionally the number of unskilled jobs, but do not affect the number of skilled jobs. The expressions for the elasticities of private wages with respect to public wages also simplify, with no cross term. For instance, the elasticities with respect to unskilled public wage are given by:

$$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p} = (1-\alpha)\frac{j_2^g}{j_2^p} + \frac{du}{dw_2^g}\left[\frac{(1-\alpha)}{j_2^p} + \frac{(1-\alpha)}{j_1^p}\right]w_2^g, \tag{E.21}$$

$$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p} = -\alpha\frac{j_2^g}{j_2^p} - \frac{du}{dw_2^g}\left[\frac{\alpha}{j_2^p} + \frac{\alpha}{j_1^p}\right]w_2^g. \tag{E.22}$$

While the decomposition of the elasticity of private wages is different, the intuition is similar.

Appendix F. Additional quantitative results

F1. Calibration for European countries and baseline model

Table F.1 and F.2.

F2. Quantitative results for European countries

Tables F.3 and F.4.

F3. Quantitative results baseline model

Tables F.5 and F.6.

Appendix G. Quantitative results, more restricted definition of educated

In our main quantitative results, the US economy was in Regime 1 and far from Regime 2. However, one should not diminish the importance of modelling the different regimes when studying public employment. To highlight its importance, we do an alternative calibration

Table F.1
Calibration, European countries.

Parameter	UK	France	Spain	Variable	UK	France	Spain
				Targeted			
α	0.224	0.302	0.294	$\frac{w_1^p}{w_2^p}$	1.401	1.474	1.434
β	0.530	0.449	0.624	$j_1^g + u^g$	0.133	0.091	0.101
$\bar{g}$	0.123	0.106	0.082	$j_2^g - u^g$	0.115	0.122	0.060
n	0.354	0.323	0.369	n	0.354	0.323	0.369
w_1^g	0.808	0.700	0.744	$\frac{w_1^g}{w_2^p}$	1.059	0.985	1.060
w_2^g	0.597	0.504	0.580	$\frac{w_2^g}{w_2^p}$	1.096	1.045	1.179
$\frac{\nu}{1-\tau}$	1.645	0.224	0.271	$\frac{u}{j_2^g + j_2^p}$	0.149	0.088	0.124
				Not Targeted			
				$\frac{u^g}{j_2^g}$	0.189	0.055	0.199
				$\frac{u^p}{j_2^p}$	0.137	0.097	0.114

Note: Underemployment rate statistics are calculated from PIACC and are shown in Fig. 6. The public employment of college and no-college is shown in Fig. 2 and is calculated from each countries *Labour Force Survey*, 2003 to 2018. Public-sector wage premia are estimated using the Structure of Earnings Survey (pooled 2002, 2006, 2010 and 2014 data) shown in Fig. A.4. The college premium in the private sector is estimated by regressing the log of hourly wages of private workers on a college dummy, controlling for age, gender, region, year and a part-time dummy.

Table F.2
Calibration, baseline model.

Parameter	Value	Variable	Description	Model	Data
		Targeted			
α	0.483	$\frac{w_1^p}{w_2^p}$	College premium (private sector)	1.580	1.580
β	0.503	$j_1^g + u^g$	Public employment of college	0.097	0.097
$\bar{g}$	0.078	$j_2^g - u^g$	Public employment of no-college	0.062	0.062
n	0.432	n	Percentage of college workers	0.432	0.432
w_1^g	0.641	$\frac{w_1^g}{w_1^p}$	Public-sector wage premium (college)	1.010	1.010
w_2^g	0.431	$\frac{w_2^g}{w_2^p}$	Public-sector wage premium (college)	1.077	1.077
$\frac{\nu}{1-\tau}$	0.089	$\frac{u}{j_2^g + j_2^p}$	Underemployment rate (economy)	0.089	0.089
		Not Targeted			
		$\frac{u^g}{j_2^g}$	Underemployment rate (public)	0.340	0.102
		$\frac{u^p}{j_2^p}$	Underemployment rate (private)	0.043	0.087

Note: Underemployment rate statistics are calculated from PIACC and are shown in Fig. 6. The remaining data is calculated from the CPS, 1996 to 2018. The public employment of college and no-college is shown in Fig. 2. Public-sector wage premium is shown in the first two columns of Table 2. The college premium in the private sector is estimated by regressing the log of hourly wages of private workers on a college dummy, controlling for age, gender, region, year and a part-time dummy.

Table F.3
Decomposition of public-sector education bias, European countries.

Variable	Data (1)	Baseline (2)	Equating wages (3)	Equating wages and technology (4)
Panel A: United Kingdom				
Public employment shares				
Skilled	0.376	0.376	0.374	0.210
Unskilled	0.177	0.177	0.179	0.208
Ratio	2.118	2.118	2.091	1.012
Education intensity				
Public	0.537	0.537	0.534	0.357
Private	0.294	0.294	0.295	0.354
Ratio	1.828	1.828	1.811	1.009
Underemployment rate				
Total	0.149	0.149	0.149	0.190
Public*	0.189	0.152	0.149	0.191
Private*	0.137	0.149	0.149	0.190
Panel B: France				
Public employment shares				
Skilled	0.283	0.283	0.275	0.197
Unskilled	0.180	0.180	0.185	0.197
Ratio	1.575	1.575	1.491	1.000
Education intensity				
Public	0.429	0.429	0.416	0.323
Private	0.295	0.295	0.298	0.323
Ratio	1.458	1.458	1.395	1.000
Underemployment rate				
Total	0.088	0.089	0.090	0.110
Public*	0.055	0.094	0.090	0.110
Private*	0.097	0.087	0.090	0.110
Panel C: Spain				
Public employment shares				
Skilled	0.275	0.274	0.262	0.151
Unskilled	0.094	0.094	0.102	0.151
Ratio	2.913	2.925	2.565	1.001
Education intensity				
Public	0.630	0.631	0.600	0.369
Private	0.319	0.319	0.325	0.369
Ratio	1.977	1.978	1.848	1.001
Underemployment rate				
Total	0.124	0.124	0.125	0.164
Public*	0.199	0.154	0.126	0.164
Private*	0.114	0.121	0.125	0.164

Column (2) displays the statistics simulated from the model. Column (3) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs. Column (4) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs and $\beta = \alpha$. * statistics not calibrated.

Table F.4
Elasticities of private-sector wages, European countries.

Variable	Elasticity	Decomposition		
		Shortage of skilled	Excess unskilled	Underemployment
Panel A: United Kingdom				
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{du_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.206	0.319	0.089	−0.202
$\frac{du_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	−0.059	−0.092	−0.026	0.058
Elasticity of private wages w.r.t. skilled public wages				
$\frac{du_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	−0.182	−0.319	−0.089	0.224
$\frac{du_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.053	0.092	0.026	−0.065
Elasticity of private wages w.r.t. public wages				
$\frac{du_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.023	0.000	0.000	0.022
$\frac{du_2^p}{dw^g}\frac{w^g}{w_2^p}$	−0.007	0.000	0.000	−0.006
Panel B: France				
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{du_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.144	0.169	0.069	−0.094
$\frac{du_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	−0.062	−0.073	−0.030	0.041
Elasticity of private wages w.r.t. skilled public wages				
$\frac{du_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	−0.091	−0.169	−0.069	0.147
$\frac{du_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.039	0.073	0.030	−0.064
Elasticity of private wages w.r.t. public wages				
$\frac{du_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.053	0.000	0.000	0.053
$\frac{du_2^p}{dw^g}\frac{w^g}{w_2^p}$	−0.023	0.000	0.000	−0.023
Panel C: Spain				
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{du_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.094	0.127	0.047	−0.079
$\frac{du_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	−0.039	−0.053	−0.020	0.033
Elasticity of private wages w.r.t. skilled public wages				
$\frac{du_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	−0.059	−0.127	−0.047	0.114
$\frac{du_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.025	0.053	0.020	−0.048
Elasticity of private wages w.r.t. public wages				
$\frac{du_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.036	0.000	0.000	0.035
$\frac{du_2^p}{dw^g}\frac{w^g}{w_2^p}$	−0.015	0.000	0.000	−0.015

Note: the first column is calculated numerically, the decomposition is based on Eqs. (24) and (25).

where the educated workers are defined to have an M.Sc., Professional or Ph.D. degree. These make up close to 10 percent of the employed population. Out of these, more than one third work in the public sector. These workers have a negative public-sector wage premium of about 4 percent. In this particular calibration, we set the same value for $\frac{\nu}{1-\tau}$. Given the education premium for these workers, the model predicts very little underemployment (Figs. G.1-G.3).

This economy is in Regime 2. This means that the government wage policy actually reduces the number of educated workers, so technology explains more than 100 percent of the education bias. The private-sector wage elasticity with respect to public skilled wages, have the opposite sign of the baseline case in Regime 1 (Tables G.1–G.3).

Table F.6
Elasticities of private-sector wages, baseline model.

Variable	Elasticity	Decomposition		
		Shortage of skilled	Excess unskilled	Underemployment
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{du_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.211	0.053	0.047	0.111
$\frac{du_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	−0.196	−0.050	−0.043	−0.104
Elasticity of private wages w.r.t. skilled public wages				
$\frac{du_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	−0.041	−0.053	−0.047	0.058
$\frac{du_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.039	0.050	0.043	−0.054
Elasticity of private wages w.r.t. public wages				
$\frac{du_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.169	0.000	0.000	0.169
$\frac{du_2^p}{dw^g}\frac{w^g}{w_2^p}$	−0.158	0.000	0.000	−0.158

Note: the first column is calculated numerically, the decomposition is based on Eqs. (24) and (25).

Table F.5
Decomposition of public-sector education bias, baseline model.

Variable	Data (1)	Baseline (2)	Equating wages (3)	Equating wages and technology (4)
Public employment shares				
Skilled	0.224	0.224	0.207	0.202
Unskilled	0.109	0.110	0.124	0.127
Ratio	2.054	2.034	1.671	1.593
Education intensity				
Public	0.610	0.607	0.560	0.548
Private	0.399	0.399	0.408	0.410
Ratio	1.530	1.523	1.373	1.336
Underemployment rate				
Total	0.089	0.088	0.085	0.087
Public*	0.102	0.340	0.272	0.273
Private*	0.087	0.043	0.050	0.052

Column (2) displays the statistics simulated from the model. Column (3) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs. Column (4) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs and $\beta = \alpha$. * statistics not calibrated.

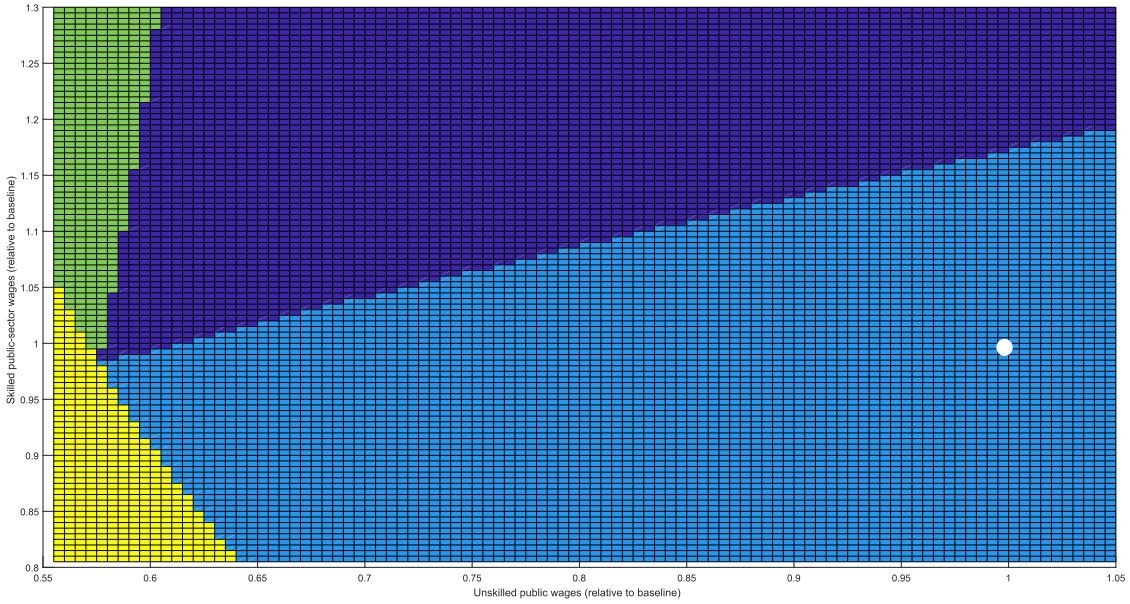


Fig. G.1. Regimes as a function of the public wage schedule, alternative definition of educated.

Table G.1
Calibration, alternative definition of educated.

Parameter	Value	Variable	Description	Model	Data
		Targeted			
α	0.113	$\frac{w_1^p}{w_2^p}$	College premium (private sector)	1.700	1.697
β	0.379	$j_1^g + u^g$	Public employment of Ph.D.-M.Sc.-Professional	0.033	0.032
$\bar{g}$	0.075	$j_2^g - u^g$	Public employment of non Ph.D.-M.Sc.-Professional	0.125	0.125
n	0.091	n	Percentage of Ph.D.-M.Sc.-Professional workers	0.091	0.092
w_1^g	1.049	$\frac{w_1^g}{w_1^p}$	Public-sector wage premium (high-educated)	0.933	0.961
w_2^g	0.700	$\frac{w_2^g}{w_2^p}$	Public-sector wage premium (low-educated)	1.058	1.065
$\frac{\nu}{1-\tau}$	0.142		(kept from main calibration)		
		Not Targeted			
		$\frac{u}{j_2^g + j_2^p}$	Underemployment rate (economy)	0.0023	–
		$\frac{u^g}{j_2^g}$	Underemployment rate (public)	0.0029	–
		$\frac{u^p}{j_2^p}$	Underemployment rate (private)	0.0022	–

Note: $\frac{\nu}{1-\tau}$ is not recalibrated. The remaining parameters are calibrated to match data calculated from the CPS, 1996 to 2018. The high educated are now defined to have an M.Sc.-Ph.D. and low-educated are those with Bachelors or below. The public-sector wage premium and the college premium in the private sector are re-estimated.

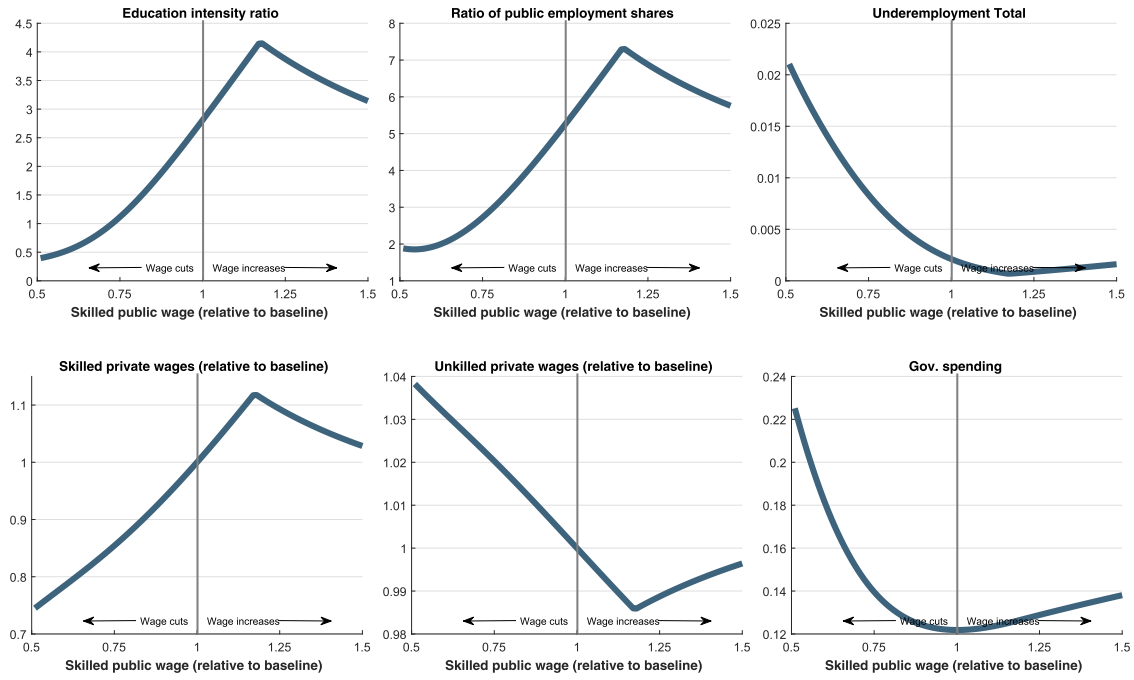


Fig. G.2. Effects of public skilled wages, alternative definition of educated.

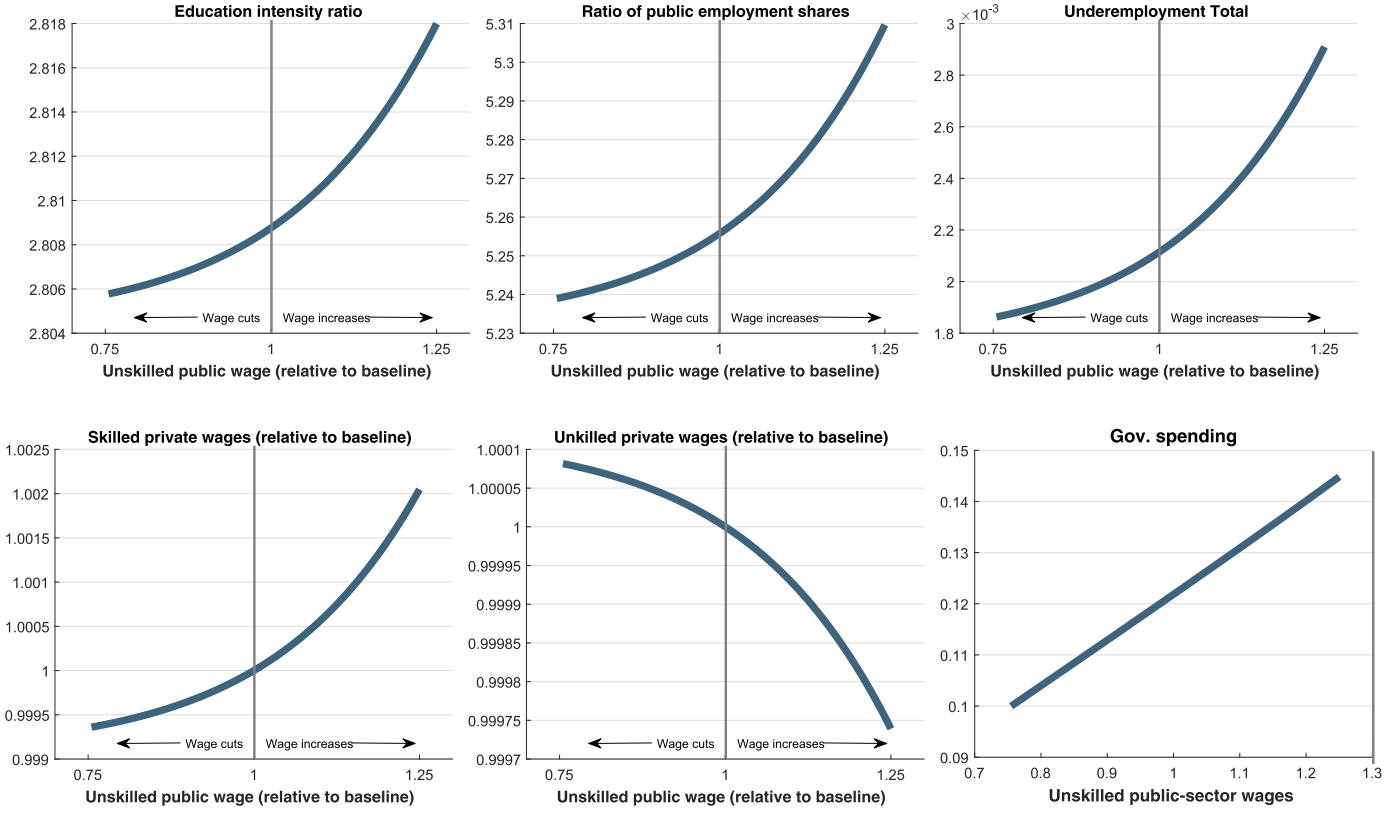


Fig. G.3. Effects of public unskilled wages, alternative definition of educated.

Table G.2
Decomposition of public education bias, alternative definition of educated.

Variable	Data (1)	Baseline (2)	Equating wages (3)	Equating wages and technology (4)
Public employment shares				
Skilled	0.346	0.346	0.360	0.108
Unskilled	0.140	0.140	0.136	0.107
Ratio	2.478	2.478	2.651	1.008
Education intensity				
Public	0.200	0.200	0.211	0.092
Private	0.071	0.071	0.069	0.091
Ratio	2.809	2.809	3.037	1.008
Underemployment rate				
Total		0.0023	0.002	0.010
Public		0.0029	0.002	0.010
Private		0.0022	0.002	0.010

Column (2) displays the statistics simulated from the model. Column (3) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs. Column (4) displays the statistics from a simulation where public-sector wages are equal to private-sector wages for the two types of jobs and $\beta = \alpha$. * statistics not calibrated.

Table G.3
Elasticities of private wages, alternative definition of educated.

Variable	Elasticity	
	Baseline model	Alternative definition of educated
Elasticity of private wages w.r.t. unskilled public wages		
$\frac{du_1^p}{du_2^g}\frac{u_2^g}{u_1^p}$	0.074	0.005
$\frac{du_2^p}{du_2^g}\frac{u_2^g}{u_2^p}$	−0.061	−0.001
Elasticity of private wages w.r.t. skilled public wages		
$\frac{du_1^p}{du_1^g}\frac{u_1^g}{u_1^p}$	−0.046	0.657
$\frac{du_2^p}{du_1^g}\frac{u_1^g}{u_2^p}$	0.038	−0.084
Elasticity of private wages w.r.t. public wages		
$\frac{du_1^p}{du^g}\frac{u^g}{u_1^p}$	0.029	0.662
$\frac{du_2^p}{du^g}\frac{u^g}{u_2^p}$	−0.023	−0.084

Note: calculated numerically, the decomposition is based on Eqs. (24) and (25).

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