Public-sector employment, wages and education decisions*

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Abstract

We set up a search and matching model with a private and a public sector to understand the effects of employment and wage policies in the public sector on unemployment and education decisions. The effects on the educational composition of the labor force depend crucially on the structure of the labor market. An increase of skilled public-sector wages has a small positive impact on educational composition and larger negative impact on the private employment of skilled workers, if the two sectors are segmented. If there are movements across the two sectors, it has large positive impacts on education and on skilled private employment. We highlight the usefulness of the model for policymakers by calculating the value of public-sector job security for skilled and unskilled workers.

JEL Classification: E24; J31; J45; J64.

Keywords: Public-sector employment; public-sector wages; unemployment; skilled workers; education decision; public-sector job security premium.

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1 Introduction

The government is a major employer of qualified workers, hiring more than 25 per cent of all college graduates in most OECD economies. However, in the public sector, the returns of education are lower. On average, the public sector pays higher wages than the private sector but the premium is higher for workers without a college degree.[1] Our objective is to study how public-sector employment and wage policies, heterogeneous across education groups, influence incentives to invest in formal education and employment of workers with and without college.

We set up a search model with a private and a public sector. Employment and wages in the private sector are determined through the usual channels of free entry and Nash bargaining. In the public sector, by contrast, employment and wages are policy variables and, hence, taken as exogenous. We incorporate an education decision. Prior to entering the labor market, individuals decide whether to invest in education. Doing so is costly but yields returns, as highly-educated workers benefit from higher job-finding rates and wages in both sectors. Workers are heterogeneous with respect to their education costs. Thus, only a fraction of individuals – those whose benefits exceed the costs – invest in education.

We find that wage and employment policies affect the educational composition and unemployment rate of skilled and unskilled workers. The mechanisms and the quantitative effects depend on the labour market structure - segmented markets or random search. Under segmented markets, workers choose a career in a particular sector and might face an entry cost for public-sector jobs. Under random search, unemployed workers apply randomly to jobs in both sectors, and do not face an entry cost in the public sector. We argue that the two sectors are segmented when barriers to entering the public sector exist. These barriers may represent national entry exams or jobs that are restricted to workers with political capital. We think this assumption portrays a realistic mechanism of selection into the public sector in several countries, documented empirically by Nickell and Quintini (2002) or experimentally by Bó et al. (2013), lying at the heart of current policy discussions. High wages attract many unemployed to queue for public-sector jobs. Conversely, if wages are too low, few unemployed search in the public sector, which then faces recruitment problems. In several countries, like France or Spain, the general rule is that civil servants are recruited through competitive exams early in their careers which clearly suggests that markets are segmented. However, we also examine the scenario where workers search randomly for jobs in both sectors. This could be a realistic assumption for some types of public-sector jobs or for some

[1]See Katz and Krueger (1991) for the United States; Postel-Vinay and Turon (2007) or Disney and Gosling (1998) for the United Kingdom; and Christofides and Michael (2013), Castro et al. (2013) and Giordano et al. (2011) for several European countries.

countries, where there are no barriers to entering the public sector. In the US, for instance, the majority of federal government jobs are filled through an examination of the applicant's background, work experience, and education, not through a written civil service test. While this in itself does not rule out that markets are segmented, it is consistent with unemployed workers not specifically searching for a public-sector job, but getting one by chance.

If the two sectors are segmented, an increase of skilled public-sector wages has a small positive impact on the proportion of highly educated and a larger negative impact on skilled private employment. If search across the two sectors is random, it has a large positive impact on both education and skilled private employment. In segmented markets, when skilled public-sector wages increase, more people queue for these jobs. The consequent decrease in the job-finding rate partially offsets the extra gains of education. However, if workers search randomly for jobs in both sectors such offsetting decrease in job-finding rate is not possible. The value of education goes up by more, leading to larger increases in the proportion of high-educated in the labor force. As unemployed search randomly for jobs, a higher pool of educated workers leads to a higher level of skilled private employment, but a lower level of unskilled private employment. Quantitatively, in the model calibrated to four economies - United States, United Kingdom, France and Spain - a 10 per cent increase in skilled wages raises the number of educated workers by 0 to 0.18 per cent under segmented markets, and by 2.5 to 6.9 per cent under random search. Our results are consistent with a recent empirical paper by Somani (2021) that exploits a public-university expansion in Ethiopia together with a shift of demand of public organizations from workers without college to workers with a college degree. Education attainment increased after the expansion, but only in districts where the public-wage premium is large.We also consider a third labour market structure with segmented markets, but in which a worker can choose the sector to apply to whenever in unemployment. This structure allows movements of workers between sectors and shares features of both the segmented markets and random search models.

In policy discussions over public-sector pay, it is frequently argued that the jobs offer compensating benefits, like job-security. We highlight the usefulness of our model by quantifying the value of job security in the public-sector, for workers with and without college. We calculate what fraction of their wage would private-sector workers sacrifice for the job-separation rate of the public sector, or conversely, how much would public-sector workers have to be compensated in order to have the job-security of the private sector. As our model is populated by risk-neutral workers, this is a lower bound for the value of job security. We find that these premia vary across countries and across workers. It is larger in countries where unemployment is more persistent and the unemployment benefits are low, and it is larger for unskilled workers. The job-security premium in the US varies between 0.5 to 1.1

per cent for skilled workers and between 1.5 to 5.9 per cent for unskilled workers. In Spain, the values range from 2.3 to 7.8 per cent.

We contribute to the recent literature that analyzes the role and effects of public employment and wages. Some papers assume that the two sectors' labor markets are segmented, and that the unemployed choose which of the sectors to search in, depending on the government's hiring, separation and wage policies. Hörner et al. (2007) study the effect of turbulence on unemployment when wages in the public sector are insulated from this volatility. Quadrini and Trigari (2007) show how different exogenous business cycle rules affect unemployment volatility. Gomes (2015) emphasizes the role of public-sector wages in achieving the efficient allocation, while Afonso and Gomes (2014) highlight the interactions between private and public wages. Chassamboulli and Gomes (2021) study the role of nepotism in public sector hiring. In a model with workers heterogeneous in ability, Geromichalos and Kospentaris (2022) study how workers' selection into the public sector affects macroeconomic aggregates.

Other papers assume that the unemployed search randomly across sectors, and, hence, policies affect the equilibrium through the outside option of the unemployed and their reservation wage. Burdett (2012) includes the public sector in a job-ladder framework where firms post wages. Bradley et al. (2017) further introduce job-to-job transitions between the two sectors to study the effects of public-sector policies on the distribution of private-sector wages. Albrecht et al. (2017) consider heterogeneous human capital and match specific productivity in a Diamond-Mortensen-Pissarides model. Michaillat (2014) shows that the crowding-out effect of public employment is lower during recessions, giving rise to higher government spending multipliers. These papers' objective is to determine how public employment and wage policies affect private employment, the unemployment rate and private wages. Feng and Guo (2021) focus on employment of state-owned enterprizes in China. Because there is no clear reason why one assumption is more relevant and their relevance might vary across countries, we compare the results under the two structures.

With the exception of Albrecht et al. (2017), none of these papers consider heterogeneity in education, an oversight, given that the government predominantly hires workers with college degree and that the premia it pays vary with education. Two papers consider this dimension of heterogeneity in a search and matching model. Gomes (2018) examines the effects of a public-sector reform that eliminates the wage premium for all workers. Navarro and Tejada (2022) study the interaction between public employment and the minimum wage. Other papers model this heterogeneity in a frictionless labour market. Domeij and Ljungqvist (2019) study how the public-sector hiring of skilled and unskilled workers in Sweden and the US can explain the diverging evolutions of the skill premium in the two countries. Garibaldi et al. (2021) document that underemployment is more salient in the public sector in OECD countries and disentangle whether the higher education intensity in the public sector is due to technology, wage compression or a higher level of underemployment. In a model of occupational choice, Gomes and Kuehn (2017) analyze the effects of skill-biased hiring in the public sector on the occupational choice of entrepreneurs and on firm size. Wilson (1982) studies public-sector employment within a neoclassical optimal taxation model. With the exception of Wilson (1982) and Navarro and Tejada (2022) who formalize the education decision but do not analyse it in detail, all other papers take the education endowment as exogenous. By endogenizing the choice of education and showing under which conditions and how government employment and wage policies affect it, we contribute to the literature on the determinants of education that started with the seminal contributions of Mincer (1958), Ben-Porath (1967), Weisbrod (1962) and Becker (1975). More closely related contributions using search models include Charlot and Decreuse (2005), Charlot et al. (2005) and Charlot and Decreuse (2010).

2 Evidence from microdata

2.1 Worker stocks and flows, by sector and education

Fontaine et al. (2020) establish key facts about the US, UK, French and Spanish public and private labour market flows, using data from the US *Current Population Survey* (CPS) and the UK, French and Spanish *Labour Force Surveys* (LFS) over the past 15 years. We use the same data to analyse the stocks and flows by education in more detail. These four countries have sizable levels of public employment, variable industry composition of the public sector, different hiring methods and labour market institutions. All the common patterns that we find can then be considered general characteristics of the public sector. By focussing on a representative survey, used to calculate official labour market statistics, we can pin down accurately the stocks of public and private employment, but more importantly the worker flows, particularly job-finding and job-separation rates. We use these statistics in the calibration of the model in Section 7.

We first extract the stocks and transition probabilities between private and public employment and unemployment, for workers with at least a college degree and for workers without a college degree. We follow the procedures described in Fontaine et al. (2020), with a few exceptions. First, instead of dividing the population into three groups, we focus on two education levels: college and no college.[2] Second, we restrict the sample to workers aged 20

[2]For the United States, college graduates includes individuals with a Bachelors, Masters, Professional or Doctorate degree, but exclude individuals with Associate degrees or with some college but no degree.

Table 1: Public-sector employment stock and transition rates, by education

Variable	United States		United Kingdom		France		Spain	
	College	No college	College	No college	College	No college	College	No college
Public-sector employment (share of total employment)	0.254	0.116	0.358	0.192	0.281	0.181	0.275	0.095
Unemployment rate	0.032	0.073	0.033	0.057	0.057	0.103	0.110	0.208
Job-finding rate								
Private sector	0.213	0.328	0.325	0.268	0.285	0.217	0.239	0.214
Public sector	0.052	0.008	0.085	0.029	0.041	0.024	0.047	0.018
Job-separation rate								
Private sector	0.007	0.028	0.012	0.014	0.016	0.023	0.031	0.051
Public sector	0.005	0.016	0.005	0.005	0.005	0.010	0.014	0.036
Share in the labour force	0.27	0.73	0.46	0.54	0.32	0.68	0.34	0.66
Unemp. duration of new hires (private over public)	1.248	1.009	0.744	0.735	0.948	0.767	0.988	0.794

Note: Data extracted from the French, UK and Spanish LFS and the US CPS. Sample: 2003:1 to 2015:4 for France and UK, 2005:1 to 2015:4 for Spain, and 2003 Jan to 2017 Dec. Population aged 20 to 64. The job-finding and -separation rates are monthly for the US and quarterly for the UK, France and Spain.

to 64, whereas they focussed on workers aged 16 to 64. As our model does not incorporate a labour market participation decision, we abstract from the flows in and out of inactivity. Also, the direct flows between public and private sector are small, so we also abstract from on-the-job search and job-to-job transitions, as shown by Chassamboulli et al. (2020).[3]

Table 1 shows the average stocks and transition rates. The unconditional job-finding rate is the probability that an unemployed is employed in a sector in the following quarter. The job-separation rate is the probability that a worker employed in a given sector is unemployed three months after. The public sector is a sizable employer, especially of educated workers. In the US, it employs 25 per cent of the employed population with a college degree and only 12 per cent of people without one. In the UK, the difference is even larger with 36 and 19 per cent. In France, these are 29 and 18 per cent. In Spain, these are 28 and 10 per cent. Labour turnover is lower in the public sector. On the one hand, jobs are safer: the job-separation rate is roughly two to three times higher if working in the private sector. On the other hand, there are fewer hires in the public sector. The probability of finding a job in the private sector for college graduates is four to seven times the probability of finding a job in the public sector, while it is 8 to 12 times higher for workers without college. Job-finding rates are increasing and job-separation rates are decreasing in education in both sectors.

How does the recruitment takes place in these countries? In France, civil servants are

[3] This is particularly true in France and Spain where workers employed in the private sector in the previous quarter represent only 10 to 15 per cent of inflows into public employment. In the US and UK these numbers are slightly higher: around 30 per cent of inflows into public employment come from the private sector. Similar magnitudes hold for outflows. While not negligible, these shares still represent the minority of all inflows into and outflows from public employment. See Appendix E.

recruited through competitive exams, either external (reserved to competitors fulfilling certain conditions of diplomas or professional experience and age) or internal (reserved to civil servants in certain positions). See Meurs and Puhani (2018) for a detailed description of the process. The Spanish civil service has a similar recruitment method. Currently in Spain, the rule of thumb is that one candidate for a position requiring high-school needs 9 to 12 months to prepare for the exam, while for a position targeting a college graduate requires 18 to 24 months of preparation. In some specific occupations, such as notaries, the average time to prepare for the exam is more than four years. But the public sector is wider than the civil service, and also includes workers in local government or in industries of education and health, each with specific channels of entry. In the particular case of France and Spain, like their private-sector counterparts, many public-sector workers are hired with temporary contracts. In the US, civil service exams are required for certain groups including foreign service officers, customs, some secretarial and clerical, air traffic control, law enforcement, postal service, and for some entry level government jobs, but these are a minority. More than 80 per cent of federal government jobs are filled through an examination of the applicant's background, work experience, and education, not through a written civil service test. It is important to read the job announcement thoroughly to determine if a civil service exam, self certification, or a skill is necessary. In the UK, there are also no civil service exams, but standard recruitment methods alongside specific entry channels such as apprenticeships, graduate or internship programmes.

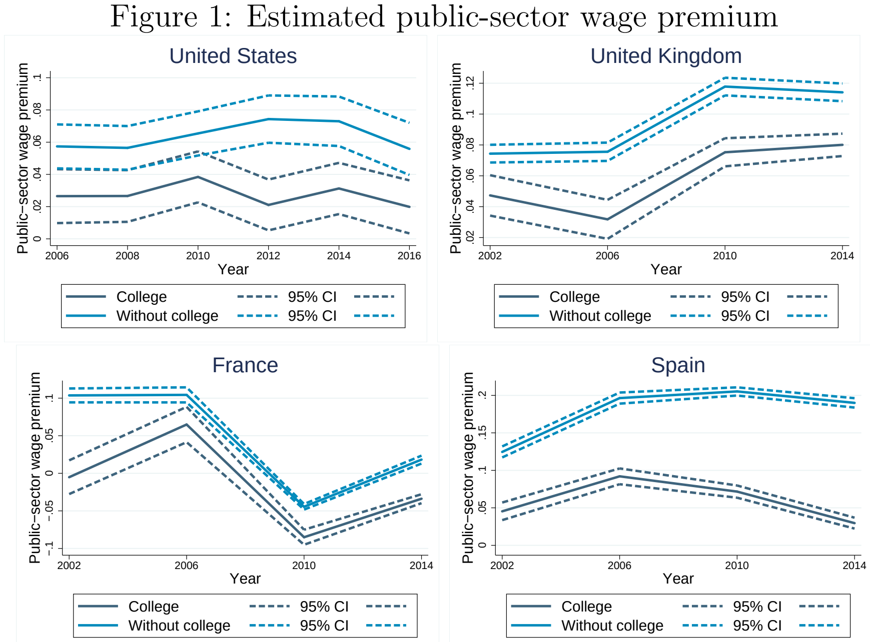
One of the difficulties with distinguishing whether search across sectors is random or directed towards one sector or the other is that the behaviour of the unemployed is unobservable. Hence, we cannot calculate the job-finding rates conditional on searching in a particular sector. As an alternative, in the last row of Table 1, we calculate the ratio of unemployment durations of new hires in the private over that of the new hires in the public sector. For UK, France and Spain the number is lower than one, meaning that the unemployment duration is lower in the private sector, or in other words, queues are longer in the public sector. In the US the number is close to one. Notice that under random search, the unemployment duration should be equal for workers who just found a job in either the public or the private sector, so the ratio should be close to one. However, it could also be close to one under segmented markets. Consequently, differences in unemployment durations between new hires in the private and public sectors, as well as the existence of public-sector entry restrictions can give an indication as to whether the two sectors are segmented or not, but cannot give a definite answer. Without a better way to distinguish between the two labour market structures in the data, we work out a model under both assumptions in order to better understand the mechanisms and their quantitative implications.

2.2 Public-sector wage premium by education

We use microdata from the CPS and the *Structure of Earnings Survey* (SES) for the waves of 2002, 2006, 2010 and 2014, to reproduce some findings of the empirical literature estimating public-private wage differentials. We calculate the public-sector wage premium by education splitting the sample for college graduates and workers without college. Along the lines of Castro et al. (2013) and Giordano et al. (2011), we run regressions of the log gross hourly earnings on a dummy for the public sector, controlling for region, gender, age, occupation, finer education categories and a part-time dummy for each the two groups and for each year of the survey. The estimated premia alongside with a 95 per cent confidence intervals are shown in Figure 1.

The premium for workers with lower qualifications is higher than for college graduates in all countries. This reflects the wage compression across education groups found in the literature. Still, the average premium and the compression vary substantially. For instance, the estimated premia for the UK are consistently 3 to 4 percentage points higher than for the US. The premia also vary across time, reflecting either a different evolution in the private sector of skilled and unskilled wages, that was not incorporated in the public-sector pay scale, or a deliberate policy. In the beginning of the Euro-Area crisis in Spain, the highest

Figure 1: Estimated public-sector wage premium



Source: SES and CPS. We run regressions of the log gross hourly earnings on a dummy for the public sector, controlling for region, gender, age, occupation, education and a part-time dummy for workers with and without a college degree. For the European countries it is ran for 2002, 2006, 2010, 2014. For the US, each point is the estimation results for a 2-year sample.

public-sector wages of top earners were cut 10 per cent while the lower earners did not face direct cuts. One can see the effects of this policy in the graph. The estimated premium for college graduates in Spain fell from about 10 per cent in 2006 to 3 per cent in 2014, while the estimated premium for workers without college stayed roughly constant. In France, both premia fell by 15 percentage points between 2006 and 2010.

The fact that the premia can change rather quickly is relevant for our interpretation of the estimates of the public-private wage differentials, and brings a note of caution in the quantitative section and the drawing of policy conclusions. One should be aware that they refer to an average of the policy between 2002 and 2014 and do not reflect the current policy.

3 Model with segmented markets

We first describe the benchmark model in which, entry of job searchers into the public sector is costly, the two sectors are segmented and workers can choose either a career in the public or the private sector. We begin by discussing some characteristics of the public sector, which help explain the main features of our model. We emphasize that the public sector has a number of specificities, which explains why our model must depart from standard models with a binary choice between labor-market sectors/careers. In subsequent subsections we lay out the model assumptions and define the model equilibrium. Section 4 describes the main results of the paper. Section 5 analyzes the setting in which search between the private and public sectors is random. Section 6 presents a third model where the choice of sector in not an absorbing state. In Section 7, we perform quantitative exercises.

3.1 Preliminary considerations

The defining characteristic of the public sector is that it does not sell its goods or services - it supplies them directly to the population. There is no market price. Governments finance employment, not by selling goods, but by using the power of taxation. As such, the public sector does not maximize profits and the decisions regarding employment can reflect different government objectives. Even in determining wages (or wage growth) there is a discretionary component that can create wage differentials vis-à-vis the private sector, documented in the previous section. The usual mechanisms that drive the private sector adjustments studied by economists do not map into the public sector.

Our modeling choices reflect this view of the public sector. As in other papers in the literature on public employment, i.e. Bradley et al. (2017) or Albrecht et al. (2017), we assume that the government wage schedule is exogenous: they are not an equilibrium outcome (i.e. private wages) but a policy variable (i.e. unemployment benefits or government spending). Public-sector wages is a payment in units of private-sector goods (financed with taxation), not in units of public-sector goods, hence they are not necessarily dependent of the (marginal) productivity. As such, they might be influenced by several factors, such as unions, redistribution or elections. We do not take a stance as to why public-sector wages of skilled or unskilled workers are high or low. We take them as exogenous to match the data. Also, we assume that for the public sector to function, it requires an exogenous number of skilled and unskilled workers. Given a wage for each of these two skill groups, the government must maintain its employment level constant by hiring enough new workers to replace those that separate into unemployment or retire.[4]

We think it is important to model frictional unemployment in addition to the choice of sector. First, in a model without frictions identical workers must receive the same wage due to arbitrage. With frictions, the labour market tolerates a wage differential. Second, job security is a key feature of the public sector, and its value varies with education, so to understand its role we must consider unemployment risk.

3.2 General setup

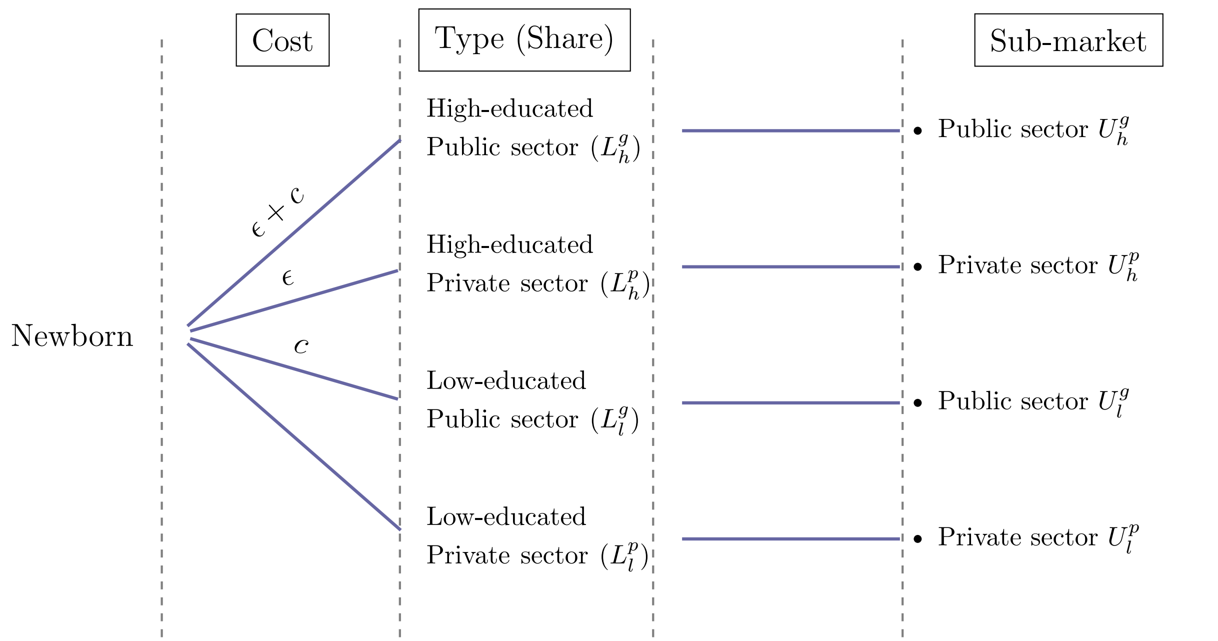
We consider a search and matching model with private-sector firms and a public sector. Workers can be either employed and producing or unemployed and searching for a job. Each firm can post a single vacancy that can be vacant or filled (job). At each instant, τ individuals are born (enter the labor market) and die (retire) so the working population is constant and normalized to unity. All agents are risk-neutral and discount the future at a common rate $r > 0$. Time is continuous.

An agent is either low- or high-educated. All individuals are born low-educated, but prior to entering the labor market, they can become high-educated by paying a schooling cost ϵ. The schooling cost is distributed across individuals according to the cumulative function $\Xi^{\epsilon}(\cdot)$ on $[0, \bar{\epsilon}]$. Heterogeneity with respect to schooling cost reflects either different learning abilities or the existence of financial constraints. High-educated workers are more productive. Education is observable and there are no other dimensions of human capital, such as unobserved ability, match quality or on-the-job learning.

In parallel, prior to entering the labor force individual decide whether to pursue a career

[4]We show in Appendix D one simple model where government maximizes an objective function that incorporates government services and union preferences, subject to a budget constraint. The model endogeneizes both public-sector employment and wages as a function of the technology of production of government services, the budget, the power of unions and the relative union preference for unskilled public-sector workers. The exogenous changes in public-sector wages and employment of low- and high-educated workers that we consider, can be interpreted as driven by changes in these factors.

Figure 2: Decision of newborn



in the public sector or not. To access jobs in the public sectorthey must pay cost c. The cost is distributed according to the cumulative function $\Xi^c(\cdot)$ on $[0, \bar{c}]$.[5] This cost may represent the cost of passing a civil service exams, networking or investing in political connections. Taking exams may be more costly to some individuals than to others, while having access to jobs in the public sector may be easier for workers whose family members work in the public sector. If such barriers to entry exist, the two sectors must be segmented, because for the workers who choose to pay the entry cost, pursuing a career in the public sector strictly dominates all other options. Our choice of segmented markets is akin to a Roy model with frictions in the labour market.[6] In Section 5, we also explore the case where no barriers to entry into the public sector exist and workers search randomly for both public- and private-sector jobs, and one where the decision of sector is done after the education decision and every time the worker is unemployed. We abstract from on-the-job search following the evidence by Chassamboulli et al. (2020) that 70 to 85 per cent of new hires in the public-sector come from non-employment.

An endogenous proportion of the population (those whose schooling cost is low) become

[5]We assume that the distributions of education and public-sector costs are independent. As we will show, the model endogenously generates complementarity or substitutability between education and search in the public sector. Assuming exogenously a relation between the two distributions would simply tilt the equilibrium towards one or the other.

[6]A traditional Roy model ignores that it takes time to find a job, potentially differently across sectors. Modeling labour market frictions explicitly also allows us to study the role (and value) of job security which is a key feature of the public sector and varies over education.

high educated; another fraction (those whose entry cost is low) attaches to the public sector. If both costs are low, workers become high-educated in the public sector, while if both costs are high, workers remain low-educated in the private sector. The rest may remain low educated and search in the public sector or become high educated and search in the private sector. The shares of each type in the population are L_l^p, L_l^g,L_h^p and L_h^g, and add up to 1. Variables are indexed by the superscript $x = [g, p]$, where g refers to the public (government) sector and p to the private sector, and the subscript $i = [l, h]$, where h to high- and l to low-educated. Figure 2 depicts these four choices. In each sector there are two labor markets segmented by education. In the "high-education" market, both firms and government open vacancies suited for high-educated workers, whereas in the "low-education" market, vacancies are suited for low-educated workers; high-educated individuals in either sector (private or public) direct their search towards type-h jobs, whereas low-educated workers direct their search towards type-l jobs. The output y_i of any match depends only on the worker's education: high-educated individuals are more productive than low-educated individuals. That is, ($y_h > y_l$). A searching (unemployed) worker of type-i receives a flow of income b_i, the opportunity cost of employment.

We model the choice of sector to be an absorbing state, preventing ex-post mobility between sectors. This seems a reasonable assumption for a model focussing on the education decision, which is a major determinant of one's career. In many occupations, such as teachers or judges, the public sector is close to a monopsonist, while other occupations have few employment opportunities in the public sector. The models in sections 4 and 6 allow for more mobility between sectors.

3.3 The Private sector

Firms in the two submarkets open vacancies and search for workers until all rents are exhausted. The rate at which type-i workers find private-sector jobs depends positively on the tightness, $\theta_i = \frac{v_i^p}{u_i^p}$, where v_i^p is the measure of vacancies, and u_i^p is the number of unemployed searching there. Workers of type i are hired into jobs (of type i) at Poisson rate $m(\theta_i)$, and firms fill vacancies at rate $q(\theta_i) = \frac{m(\theta_i)}{\theta_i}$.

Wages, denoted as w_i^p, depend on match surplus, so they differ by education. They are determined by Nash bargaining, such that the worker gets a share β of the match surplus. With higher surplus, firms expect to generate larger profits from creating jobs; firm entry is higher; and workers can more easily find jobs and earn higher wages.

A vacant firm bears a recruitment cost κ_i, related to the expenses of keeping a vacancy open. When a vacancy and a worker are matched, they bargain over the surplus. Matches in the private sector of type i dissolve at the rate s_i^p. Following a job destruction, the worker searches for a new match in the same submarket.

3.4 Government

To produce its services, the government employs a certain number of high and low educated workers (e_h^g, e_l^g), which together with their wages, (w_h^g,w_l^g), are the exogenous policy variables. In each instant, the government must hire enough workers to replace the workers that exogenously separate or retire. That means hiring $(s_h^g + \tau)e_h^g$ skilled and $(s_l^g + \tau)e_l^g$ unskilled workers, where s_i^g is the separation rate. The matching function in the public sector is $M_i^g = \min\{v_i^g, u_i^g\}$. To maintain its employment level, the government must attract a number of searchers in each segment, u_i^g, at least equal to the number of job openings, v_i^g, meaning that $M_i^g = v_i^g$. Otherwise, public-sector services break down. As we show in Lemma 2, this imposes a condition on wages to be high enough to attract at least the same number of searchers as of vacancies. We assume that the recruitment is part of the role of the government and is done by its workforce. Since the government's objective is to maintain employment levels (e_h^g, e_l^g) by hiring enough workers to replace those that separate or retire, it follows that $v_i^g = (s_i^g + \tau)e_i^g$. Workers of type-$i$ find public-sector jobs at rate $m_i^g = \frac{(s_i^g+\tau)e_i^g}{u_i^g}$.

We choose this matching function for simplicity and clarity. First, it makes the concept of queues in the public sector clearer. When unemployed surpass vacancies, the vacancy filling rate for the government is 1, and all the unemployed in excess are queuing. This makes the minimum wage required for the existence of the public sector an intuitive object, easy to calculate. Second, this assumption has been used in other papers, i.e. Quadrini and Trigari (2007) and there is evidence that the elasticity of matches with respect to unemployed is lower in the public sector than in the private (Gomes, 2015). This does not mean that there are no matching frictions, only that they are one-sided.[7]

We assume that a worker of type i produces y_i in both sectors, but because public employment is exogenous, their productivity does not matter for the results that remain unchanged even if productivity was different across the two sectors. In this setting, where the government has a fixed employment level, the separation rates s_i^g play a double role: they reflect the expected duration of the match but also determine the number of new hires. Higher separations reduce the value of employment in the public sector but, at the same time,

[7]Nothing substantial would change in the model if the matching function in the public sector was Cobb Douglas: $M_i^g = (v_i^g)^\eta (u_i^g)^{1-\eta}$. In this case, the vacancy filling probability of the government would no longer be 1, and it would need to set endogenously the number of vacancies such that the total number of matches would equate exactly the number of workers that retire or separate – that is, $M_i^g = e_i^g(s_i^g + \tau)$. Solving for v_i^g we would obtain $v_i^g = (e_i^g(s_i^g + \tau))^{\frac{1}{\eta}} / (u_i^g)^{\frac{1-\eta}{\eta}}$. Still, the job-finding rate of the unemployed would be exactly the same, as well as the size of the queue, measured by $u_i^g - M_i^g$.

increase the vacancies and make an unemployed more likely to find a job there. We assume that the separation rates, as well as other labor market friction parameters, are exogenous. We ignore the issue of how the government finances its wage bill and assume that it can tax its citizens in a non-distortionary lump-sum tax.

3.5 Value functions, Free entry, Wages

Let U_i^p and E_i^p be the (discounted lifetime) values associated with unemployment (searching for a job) and employment in the private sector of a worker with education $i = [h, l]$, defined by:

$$(r+\tau)U_i^p = b_i + m(\theta_i)\left[E_i^p - U_i^p\right], \quad (1)$$
$$(r+\tau)E_i^p = w_i^p - s_i^p\left[E_i^p - U_i^p\right]. \quad (2)$$

The values associated with unemployment and employment in the public sector are:

$$(r+\tau)U_i^g = b_i + m_i^g\left[E_i^g - U_i^g\right], \quad (3)$$
$$(r+\tau)E_i^g = w_i^g - s_i^g\left[E_i^g - U_i^g\right], \quad (4)$$

For a firm, let J_i^p be the value of a job and V_i^p be the value associated with posting a vacancy and searching for a type i worker to fill it, given by

$$rJ_i^p = y_i - w_i^p - (s_i^p + \tau)\left[J_i^p - V_i^p\right], \quad (5)$$
$$rV_i^p = -\kappa_i + q(\theta_i)\left[J_i^p - V_i^p\right]. \quad (6)$$

In equilibrium, free entry drives the value of a vacancy to zero:

$$V_i^p = 0, \quad i = [h, l]. \quad (7)$$

Wages are determined by Nash bargaining. The outside options of the firm and the worker are the value of a vacancy and the value of unemployment, respectively. Let $S_i^p \equiv J_i^p - V_i^p + E_i^p - U_i^p$ denote the surplus of a match of type i. The wage w_i^p is such that the worker gets a share β of the surplus, and the share $(1-\beta)$ goes to the firm. This implies two equilibrium conditions:

$$\beta S_i^p = E_i^p - U_i^p \qquad (1-\beta)S_i^p = J_i^p - V_i^p. \quad (8)$$

Setting $V_i^p = 0$ in (6) and imposing the Nash bargaining condition in (8) gives:

$$\frac{\kappa_i}{q(\theta_i)} = (1-\beta)S_i^p. \tag{9}$$

Using (1)-(5) together with (8) and the free-entry condition $V_i^p = 0$, we can write:

$$S_i^p = \frac{y_i - b_i}{r + \tau + s_i^p + \beta m(\theta_i)}, \tag{10}$$

and the free-entry condition as

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(y_i - b_i)(1-\beta)}{r + \tau + s_i^p + \beta m(\theta_i)}. \tag{11}$$

The equation in (11) gives the two job-creation conditions that set the expected costs of having a vacancy (left-hand-side) equal to the expected gains from a job (right-hand-side). It determines the equilibrium market tightness θ_i, the matching rates, $m(\theta_i)$ and, in turn, the size of private employment,

$$e_i^p = \frac{m(\theta_i)L_i^p}{s_i^p + \tau + m(\theta_i).} \tag{12}$$

Imposing the free-entry condition (9), the Nash bargaining solution implies that

$$w_i^p = b_i + \beta(y_i - b_i + \kappa_i\theta_i), \quad i = [h, l]. \tag{13}$$

Lemma 1 *Tightness and wages in the private sector, in both the low- and the high-education submarkets, are independent of the government employment and wage policies (e_i^g and w_i^g).*

This lemma is a useful intermediate result and follows from equations (11) and (13). It implies that government employment and wage policies affect the equilibrium only through the education decisions of the newborns or through the scale of the private sector (L_i^p). Given a constant tightness, policies that make the public sector more attractive drain the unemployed from the private sector and reduce, one-to-one, the number of vacancies, leaving private wages unchanged.

3.6 Newborn's Decisions

We can summarize the four options of the newborn, depicted in Figure 2, as

$$(r+\tau)U_i^p = b_i + \frac{m(\theta_i)}{r+\tau+s_i^p+m(\theta_i)}[w_i^p - b_i], \quad i=[h,l], \tag{14}$$

$$(r+\tau)U_i^g = b_i + \frac{m_i^g}{r+\tau+s_i^g+m_i^g}[w_i^g - b_i], \quad i=[h,l]. \tag{15}$$

The newborn chooses the option that, given her ϵ and c, has the highest value:

$$Max\{U_l^p, U_h^p - \epsilon, U_l^g - c, U_h^g - c - \epsilon\}. \tag{16}$$

A worker of type i and entry cost c searches in the public sector only if the benefit exceeds the cost, $U_i^g - U_i^p \geq c$. The threshold level of c at which the worker is indifferent between searching for a job in the public or in the private sector is, therefore, given by

$$\tilde{c}_i = U_i^g - U_i^p. \tag{17}$$

A worker chooses the public sector if $c \leq \tilde{c}_i$. Otherwise, he searches in the private sector.

Lemma 2 *There exist a public sector with two markets with employment levels e_h^g and e_l^g, provided that it pays sufficiently high wages $w_h^g \geq \underline{w}_h^g$ and $w_l^g \geq \underline{w}_l^g$.*

The expressions for the minimum level of wages in the public sector, $\underline{w}_h^g$ and $\underline{w}_l^g$, are in Appendix A. This lemma states that the public sector needs to pay a sufficiently high wages to attract enough job seekers to fill its vacancies and maintain constant employment levels e_h^g and e_l^g. These thresholds, $\underline{w}_h^g$ and $\underline{w}_l^g$, depend positively on private-sector wages, w_h^p and w_l^p, and unemployment benefits, b_h and b_l and is high enough to compensate workers for paying the entry cost c.

Lemma 3 *If $w_i^g \geq \underline{w}_i^g$ then $U_i^g > U_i^p$ and the two sectors of type i, the public and the private, are segmented.*

The proof is in Appendix A. This lemma implies that if a public sector with costly entry exists, which means that its wage is high enough to attract enough job searchers ($w_i^g \geq \underline{w}_i^g$), then it must be that the value of searching for a job in that public sector is higher than the value of searching in the private sector. Hence, the option of searching for a job in the public sector strictly dominates all others. The workers who pay the cost c, will therefore seek a career in the public sector; the rest will search for jobs in the private sector.[8]

[8]Because the value of searching in the public sector is higher, for workers who chose to pay the cost c,

3.7 Equilibrium Allocations

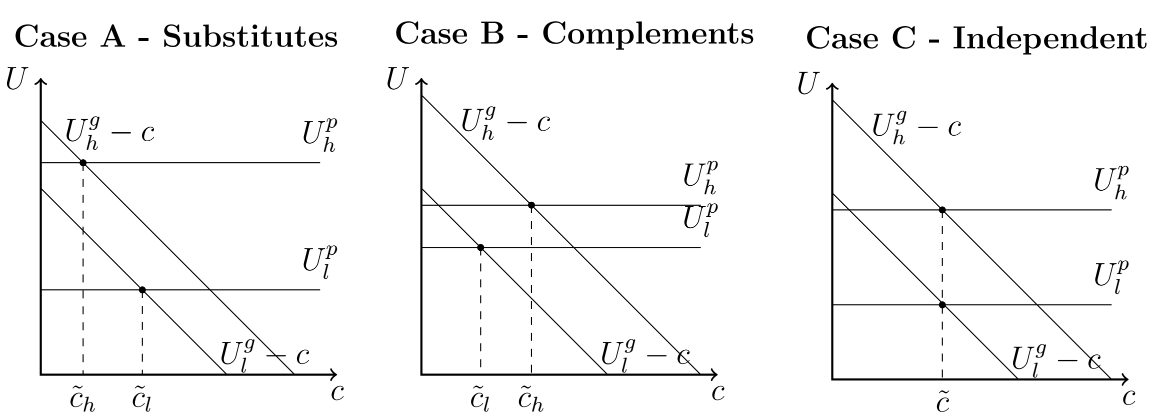
The model endogenously generates complementarity or substitutability between education and search in the public sector. As shown in Figure 3, we can have three cases, each with different implications for how the existence of a public sector alters workers' incentives to invest in education.

Case A in Figure 3 describes a scenario in which a career in the public sector substitutes investing in education. The benefit from investing in education is smaller if the worker enters the public sector than if not. That is, $U_h^p - U_l^p > U_h^g - U_l^g$, and low-educated workers have more incentive than high-educated workers to join the public sector labour market ($\tilde{c}_h < \tilde{c}_l$). Case A could occur when public-sector wages are flat across worker qualifications (wage compression) relative to the private sector, or when the value of public-sector job security is higher for low-educated workers. In such cases, those seeking jobs in the public sector have less incentive to invest in education, while those whose entry cost is high, have more incentive to opt for education.

The two thresholds $\tilde{c}_h$ and $\tilde{c}_l$ can be used to divide workers into three groups that differ in their incentives to obtain higher education. In the first group are workers whose entry cost c is low: $c < \tilde{c}_h (< \tilde{c}_l)$. For these workers, targeting jobs in the public sector always dominates, regardless of their education, and the net benefit from investing in education is given by $\tilde{\epsilon}_g = U_h^g - U_l^g$. Next is the group of workers whose entry cost c lies between $\tilde{c}_h$ and $\tilde{c}_l$. For these workers, a job in the public sector is worthwhile only if they remain low-educated. If

randomizing or splitting their search effort between the two sectors yields lower returns than directing their search towards the public sector only. However, eliminating the entry cost does not automatically imply that workers will be searching randomly across the two sectors. On the contrary, if public-sector entry is free and public jobs are better, then more (newborn) workers will choose to direct their search towards these jobs. In subsection 3.8 we present one such special case.

Figure 3: Decision Thresholds



they invest in eduction they are better off searching in the private sector. Thus, their benefit from education is $\tilde{\epsilon}_m(c) \equiv U_h^p - (U_l^g - c)$, which is increasing in c. In the last group are the workers with $c > \tilde{c}_l (> \tilde{c}_h)$, who never opt for jobs in the public sector because the cost is too high. If they choose to become high-educated, they obtain a payoff of $\tilde{\epsilon}_p \equiv U_h^p - U_l^p$.

In the opposite case – case B in Figure 3 – education "complements" search in the public sector. Workers seeking a career in the public sector have more incentive to become high-educated ($U_h^g - U_l^g \geq U_h^p - U_l^p$), which also implies that high-educated workers have more incentive to join the public sector ($\tilde{c}_h > \tilde{c}_l$). Case B could arise when the public sector has many skilled jobs. As above, the low-entry-cost workers ($c < \tilde{c}_l (< \tilde{c}_h)$) always choose public-sector jobs. Conversely, workers with $c > \tilde{c}_h (> \tilde{c}_l)$, never opt for government jobs. In between are the workers with $\tilde{c}_l < c < \tilde{c}_h$. They enter the public sector if they become high-educated but will not if they remain low-educated. Investing in education brings them a benefit of $\tilde{\epsilon}_m(c) \equiv U_h^g - c - U_l^p$, which is decreasing in c.

Finally, case C is the knife-edge case in which the payoff from being high-educated is the same in both sectors. Targeting jobs in the public sector does not alter a worker's payoff from investing in education ($U_h^g - U_l^g = U_h^p - U_l^p$), and high- and low-educated workers both have equal incentives to search in the public sector ($\tilde{c}_h = \tilde{c}_l$). In this case, all workers obtain a payoff of $\tilde{\epsilon} = U_h^g - U_l^g = U_h^p - U_l^p$ from investing in education.

A worker invests in education only if the benefit exceeds the cost (ϵ). The education benefit can be either $\tilde{\epsilon}_p, \tilde{\epsilon}_m$ or $\tilde{\epsilon}_g$, depending on the entry cost (c). These thresholds are given by:

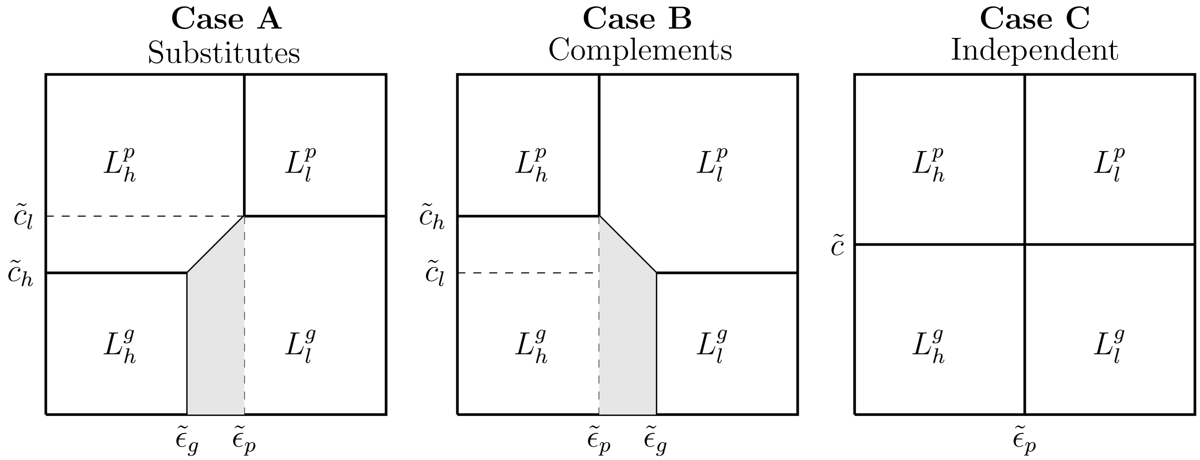
$$\tilde{\epsilon}_g = \tilde{\epsilon}_p + \tilde{c}_h - \tilde{c}_l, \tag{18}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_p + c - \tilde{c}_l \quad c \in [\tilde{c}_h, \tilde{c}_l], \text{ if } \tilde{c}_h < \tilde{c}_l \text{ (case A)}, \tag{19}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_p + \tilde{c}_h - c \quad c \in [\tilde{c}_l, \tilde{c}_h], \text{ if } \tilde{c}_h > \tilde{c}_l \text{ (case B)}. \tag{20}$$

Figure 4 illustrates how the education and public-sector entry cutoffs relate under the three cases. In case A, education substitutes for search in the public sector, and those most likely to invest in education have a high public-sector entry cost: $\tilde{\epsilon}_g \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_p$ and $\tilde{c}_h < \tilde{c}_l$. In this case, $\tilde{\epsilon}_m$ is increasing one-to-one with c. In case B, education complements search in the public sector, and the benefit from education is higher for those whose public-sector entry cost is low: $\tilde{\epsilon}_g \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_p$ and $\tilde{c}_h > \tilde{c}_l$. In this case, $\tilde{\epsilon}_m$ is decreasing one-to-one with c. In case C, incentives to invest in education are independent of workers' public-sector entry cost, and equal fractions of workers searching in both sectors invest in education: $\tilde{\epsilon}_g = \tilde{\epsilon}_m = \tilde{\epsilon}_p$ and $\tilde{c}_h = \tilde{c}_l = \tilde{c}$. Workers' cutoffs determine their selection into four groups: the high- and low-educated who target public-sector jobs (L_h^g and L_l^g), and the high- and low-educated

Figure 4: Cutoffs and allocations



who search in the private sector (L_h^p and L_l^p), as depicted in Figure 4. For each of the cases A, B and C, these four groups' share are:

$$\text{Case A, } \tilde{c}_h < \tilde{c}_l, \quad \begin{cases} L_h^g = \Xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_h) \\ L_l^g = (1-\Xi^\epsilon(\tilde{\epsilon}_g))\Xi^c(\tilde{c}_h) + \int_{\tilde{c}_h}^{\tilde{c}_l}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c) \\ L_h^p = \int_{\tilde{c}_h}^{\tilde{c}_l}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_p) \\ L_l^p = (1-\Xi^\epsilon(\tilde{\epsilon}_p))(1-\Xi^c(\tilde{c}_l)) \end{cases} \tag{21}$$

$$\text{Case B, } \tilde{c}_h > \tilde{c}_l \quad \begin{cases} L_h^g = \Xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_l) + \int_{\tilde{c}_l}^{\tilde{c}_h}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) \\ L_l^g = (1-\Xi^\epsilon(\tilde{\epsilon}_g))\Xi^c(\tilde{c}_l) \\ L_h^p = (1-\Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_p) \\ L_l^p = (1-\Xi^\epsilon(\tilde{\epsilon}_p))(1-\Xi^c(\tilde{c}_h)) + \int_{\tilde{c}_l}^{\tilde{c}_h}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c) \end{cases} \tag{22}$$

$$\text{Case C, } \tilde{c}_h = \tilde{c}_l = \tilde{c} \quad \begin{cases} L_h^g = \Xi^\epsilon(\tilde{\epsilon}_p)\Xi^c(\tilde{c}) \\ L_l^g = (1-\Xi^\epsilon(\tilde{\epsilon}_p))\Xi^c(\tilde{c}) \\ L_h^p = (1-\Xi^c(\tilde{c}))\,\Xi^\epsilon(\tilde{\epsilon}_p) \\ L_l^p = (1-\Xi^c(\tilde{c}))(1-\Xi^\epsilon(\tilde{\epsilon}_p)) \end{cases} \tag{23}$$

$L_h = L_h^g + L_h^p$ gives the share of high-educated in the labor force and $L_l = 1 - L_h = L_l^g + L_l^p$ the share of low-educated. $L^g = L_h^g + L_l^g$ is the share attached to the public sector and $L^p = L_h^p + L_l^p = 1 - L^g$ attached to the private sector.

Using (8)-(11) and (14)-(15), we can write the cutoffs as:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[\frac{\frac{(s_i^g+\tau)e_i^g}{u_i^g}}{r+\tau+s_i^g+\frac{\mu(s_i^g+\tau)e_i^g}{u_i^g}}[w_i^g - b_i] - \frac{\beta\kappa_i\theta_i}{(1-\beta)}\right], \quad i=[h,l], \tag{24}$$

$$\tilde{\epsilon}_p = \frac{1}{r+\tau}\left[b_h - b_l + \frac{\beta\kappa_h\theta_h}{(1-\beta)} - \frac{\beta\kappa_l\theta_l}{(1-\beta)}\right]. \tag{25}$$

Definition 1 *A steady-state equilibrium consists of a set of cut-off costs* $\{\tilde{c}_h, \tilde{c}_l, \tilde{\epsilon}_p, \tilde{\epsilon}_g\}$, *private sector tightness* $\{\theta_h, \theta_l\}$, *and unemployed searching in each submarket* $\{u_h^p, u_l^p, u_h^g, u_l^g\}$, *such that, given the exogenous government policies* $\{w_h^g, w_l^g, e_h^g, e_l^g\}$, *the following apply.*

1. *Private-sector firms satisfy the free-entry condition (11)* $i=[h,l]$.

2. *Private-sector wages are the outcome of Nash Bargaining (13)* $i=[h,l]$.

3. *Newborns decide optimally their investments in education and choice of sector (equation 16), and the population shares are determined by the equations (21), (22) or (23), depending on the case.*

4. *Flows between private employment and unemployment are constant:*

$$(s_h^p+\tau)e_h^p = m(\theta_h)u_h^p, \tag{26}$$

$$(s_l^p+\tau)e_l^p = m(\theta_l)u_l^p. \tag{27}$$

5. *Population add-up constraints are satisfied:*

$$L_i^p = e_i^p + u_i^p, \tag{28}$$

$$L_i^g = e_i^g + u_i^g, \tag{29}$$

$$L_h^p + L_l^p + L_h^g + L_l^g = 1. \tag{30}$$

3.8 A special case: no entry costs and segmented markets

We first consider the case where the two sectors are segmented, but there are no costs to entering the public sector. In this special case, the presence of a public sector or its policies do not distort newborns' decision to invest in education and generate only employment effects. We examine this case for two reasons. First, it demonstrates how mobility between the two sectors neutralizes the effects of policies, when the two sectors are segmented. Second, it allows us to isolate the pure employment effects of the public sector on the low- and high-educated workers, from the changes in employment that arise due to changes in the educational composition of the labor force.

Free entry of searchers into the public sector ensures

$$U_i^g = U_i^p, \quad i = [h, l]. \tag{31}$$

This condition determines the number of searchers in the public sector, u_i^g, which is the variable that compensates any asymmetry in the value of the job in the two sectors. An increase of the value of a public-sector job, E_i^g, (driven by higher wages or lower separations) raises the unemployed searching for these jobs and lowers their matching rate (m_i^g), such that its effect on U_i^g is neutralized. This condition also ensures that $U_h^g - U_l^g = U_h^p - U_l^p$ meaning that $\tilde{\epsilon}_g = \tilde{\epsilon}_p$ and the payoff from being high-educated is the same in both sectors. The presence of the public sector does not distort newborns' decision to invest in education and the population shares are determined by equations:

$$L_h = \Xi^{\epsilon}(\tilde{\epsilon}_p), \tag{32}$$

$$L_l = 1 - \Xi^{\epsilon}(\tilde{\epsilon}_p), \tag{33}$$

where the cutoff education costs $\tilde{\epsilon}_p$ is as given in (25).

Lemma 4 *With segmented markets and no costs to entering the public sector, the presence of a public sector does not alter the educational composition of the labor force. Moreover, the educational composition of the labor force is independent of policies.*

In the absence of barriers, free entry of searchers into the public sector equalizes any differences in the education premium across the two sectors.[9] Now, let $e_i = e_i^g + e_i^p$ be the employed of type i. Without a public sector (e_i^g=0) then $e_i = e_i^p$ and all workers are employed in the private sector. The presence of a public sector drains some of these workers away from the private sector. In this special case, where the public sector does not affect the educational composition of the labor force, this may increase or decrease the total employment of workers of type i (e_i), depending on their wage and lay-off policies. Specifically, as shown in Appendix A, we can write the total employment of type i workers as

$$e_i = \frac{m(\theta_i)L_i}{s_i^p + \tau + m(\theta_i)} - \frac{m(\theta_i)Q_i}{s_i^p + \tau + m(\theta_i)} + \frac{m(\theta_i)e_i^g \left(\frac{s_i^p + \tau}{m(\theta_i)} - s_i^g + \tau\right)}{s_i^p + \tau + m(\theta_i)} \tag{34}$$

[9]Throughout the paper we use the terms "education premium" or "public-sector premium" to refer to the difference in the unemployment value between high- and low-educated workers and between public and private public employees, respectively. These premia are not restricted to wage differences, but reflect also differences in job finding probabilities and job security.

The first term captures the effect of changes in labor force shares due to changes in the education decision. The employment of type i workers is larger when their share in the labor force (L_i) is larger. The second and third terms capture the role of queues and job security in the public sector. In particular, $Q_i = L_i^g - e_i^g - e_i^g(s_i^g + \tau)$ denotes the size of the "queue" in public-sector submarket i. $Q_i > 0$ means that it attracts a larger number of unemployed than that needed to fill its vacancies. If $w_i^g = \underline{w}_i^g$ then $Q_i = 0$ meaning that there are as many unemployed workers seeking jobs in the public sector as vacancies, whereas, if $w_i^g > \underline{w}_i^g$ then $Q_i > 0$ and the existence of a public sector involves an additional negative effect on employment. On the other hand, the public sector increases total employment if it offers sufficient greater job security than the private sector, ($s_i^g + \tau < \frac{s_i^p+\tau}{m(\theta_i)}$).[10] As shown in Lemma 4, in the absence of entry barriers, the education shares (L_i) are independent of the public policies. In this case, the public sector affects employment only through job security and queues. But, if entry barriers exist, they prevent the education premium from being equalized across the two sectors, so the public sector influence newborns' decision to invest in education and the labor force shares, L_i. Consequently, changes in policies affect total employment, not only through job stability and job queues, but also through their impact on education decisions. We discuss this general case in the next section.

The key to how policies affect employment and education under segmented markets is that some workers queue for public jobs, waiting for new vacancies instead of searching in the private sector. This leads to lower employment, as public-sector vacancies (and employment) are unresponsive to the supply side, while in the private sector, a constant tightness implies that vacancies respond one-to-one with the number of unemployed searching there. This mechanism does not depend on the assumption that the initial choice of a career in the public sector is an absorbing state. In Section 6 we allow for the public sector entry cost to be redrawn each period a worker is unemployed, so that they can choose where to search for a job each period. This modification does not alter the size of public-sector queues. In fact, it does not alter the expression for total employment in the model - it remains as in equation (34). Some workers chose to pay the entry cost and queue for public jobs simply because they are more valuable. The key feature behind the creation of public-sector queues is not the restriction of movements across the two sectors but: (i) that the supply of job searchers in the public sector does not drive job creation (i.e. public employment is fixed); and (ii) the payoff of having a public job is larger so that some workers search for these jobs even if their chances of finding a job in the private sector are larger.

[10]Note that $\frac{s_i^p+\tau}{m(\theta_i)} = \frac{u^p}{e^p}$ and is a measure of job security in the private sector: the larger this term is the smaller the chances a worker attached to the private sector is employed.

4 Main results

This section details five propositions about the benchmark model where entry into the public sector is costly. The first two summarize the effects of policies on educational composition and the next two the effects on employment. The last proposition explains how the public sector affects the educational composition of the private-sector labor force and employment. All the derivations and proofs are shown in Appendix A, including the proof that the equilibrium exists and is unique.

4.1 Effects on educational composition

When entry into the public sector is costly, differences in the education premium between the two sectors (or equivalently, differences in the public-sector premium for high- and low-educated workers) affect the choice of education and the composition of the labor force.

Proposition 1 *If entry into the public sector is costly, the existence of the public-sector*

- *decreases L_h (increases L_l) if $\tilde{c}_h < \tilde{c}_l$ – Case A/substitutes*
- *increases L_h (decreases L_l) if $\tilde{c}_h > \tilde{c}_l$ – Case B/complements*
- *does not affect L_h (and L_l) if $\tilde{c}_h = \tilde{c}_l$ – Case C/independent.*

In Case A, where for some workers it is worthwhile to substitute education for a low-education job in the public sector, a career in the public sector deters obtaining education. In the opposite case, where education complements search for government jobs (case B), in the presence of the public sector, some workers have more incentive to become educated and the share of high-educated is higher.

The shaded areas in Figure 4 illustrate the decrease and increase, respectively, in the fraction of high-educated workers in the labor force once a public-sector is introduced. Under Case A, the shaded area represents the fraction of people that would have become educated in the absence of the public sector, but prefer to remain uneducated and search for low-education jobs in the public-sector. Under Case B, the shaded area represents the fraction of people that would have remained uneducated, when only search in the private sector was allowed, but now prefer to get education and search for government jobs.

Let us next consider the effect of government policies.

Proposition 2 *If entry into the public sector is costly, an increase in w_h^g, e_h^g or a decrease in w_l^g, e_l^g raises the proportion of the high-educated in the labor force (raises L_h and decreases L_l).*

Any improvement in the value of working in the public sector cannot be fully neutralised since the presence of entry costs reduces the inflow of searchers into the public sector. As a result, government policies that increase the benefit from investing in education now induce a higher fraction of the labor force to become high-educated.[11]

4.2 Effects on employment

Besides job stability and job queues, the public sector also affects total employment though its impact on the education decision. As explained above, this occurs when barriers to entry into the public sector prevent the education premium from being equalized across the two sectors. To see this consider the case where $s_i^g + \tau = \frac{s_i^p + \tau}{m(\theta_i)}$ and $w_i^g = \underline{w}_i^g$ for $i = [h, l]$, which implies equal job security in both sectors and that there are no queues for jobs in the public sector. In terms of equation (34) this means setting the last two terms equal to zero so that only changes in the labor force shares L_i matter for employment changes. The proposition that follows shows that even in this case the public sector affects the employment of high- and low-educated by changing their labour force shares.

Proposition 3 *The existence of a public sector with entry costs,* $s_i^g + \tau = \frac{s_i^p + \tau}{m(\theta_i)}$*, and* $w_i^g = \underline{w}_i^g$ *for* $i = [h, l]$*,*

- *decreases* e_h *and increases* e_l *if* $\tilde{c}_h < \tilde{c}_l$ *– Case A/substitutes*
- *increases* e_h *and decreases* e_l *if* $\tilde{c}_h > \tilde{c}_h$ *– Case B/complements*
- *does not affect* e_h *and* e_l *if* $\tilde{c}_h = \tilde{c}_l$ *– Case C/independent.*

As summarised in Proposition 1, the public sector may increase, decrease or leave the proportion of high-educated in the labor force unchanged, depending on how the education premium differs between the two sectors (whether in Case A, B or C). The employment of high- and low-educated workers changes accordingly: with a higher share of high-educated workers the number of employed high educated workers increases while that of low-educated decreases and vice versa. If, on the other hand, the public sector does not distort the education decision (e.g., if we have Case C: $\tilde{c}_h = \tilde{c}_l$), then its consequences on employment

[11] Note that in our framework the public-sector separation rate plays a double role. It lowers the expected duration of a match with a negative impact on the value of public-sector employment, on one hand, but increases the number of vacancies, on the other hand, with a positive impact on the value of searching in the public sector. For this reason, it is not clear cut that decreasing the public-sector separation rate increases the public sector premium. But one can think of increasing the value of public sector job security, due to either higher separation rate or higher unemployment in the private sector as having similar effects as an increase in public sector wages. We discuss these effects in Subsection 7.2.

reflect only differences in job stability and the creation of job queues, as shown in equation 34. If public-sector jobs are more stable than private-sector jobs, the existence of the public sector will have a more positive (or less negative impact) on the employment of each of the two education types. If public-sector wages are high ($w_i^g > \underline{w}_i^g$), the effects on employment will be more negative (or less positive). It follows that more generous public-sector wage policies have a negative impact on employment.

Proposition 4 *An increase in* w_i^g *increases* L_i^g *and decreases* L_j^g, L_i^p, L_j^p. *It also decreases* e_i *and* e_j *and the total employment rate* ($e = e_i + e_j$). $i = [h, l]$, $j = [h, l]$ *and* $i \neq j$.

A higher wage in the public sector market i raises the value of searching for a job there and shifts workers away from all other submarkets. By shifting workers away from the private sector (where job creation adjusts) and into the public sector (where job creation does not adjust), it lowers the employment of both types of workers, as well as the total employment rate. Notice that Propositions 2 and 4 together imply that increasing the public-sector education premium (through e.g. higher w_h^g or lower w_l^g) increases the fraction of high-educated workers, not the employment of high-educated workers, because it generates longer queues. The additional workers moving away from other sub-markets and into the skilled market of the public sector queue up waiting for jobs.

An increase in the size of the public sector i, e_i^g, has similar effects. It drains workers away from all other sub-markets, thus it decreases e_j and increases L_i^g. However, the impact of such an increase on the employment of type i workers is not necessarily negative because of the direct employment effect.

4.3 Effects on the private sector

Proposition 5 *The existence of the public-sector improves (worsens) the educational composition of the private-sector labor force and employment: increases (decreases)* $\frac{L_h^p}{L_l^p}$ *and* $\frac{e_h^p}{e_l^p}$, *if* $\tilde{c}_h < \tilde{c}_l$ ($\tilde{c}_l < \tilde{c}_h$) *– Case A (Case B). The existence of the public-sector has no impact on the educational composition of the private-sector if* $\tilde{c}_h = \tilde{c}_l$ *– Case C.*

As summarised in Proposition 1 the public sector may decrease or increase the share of high-educated in the labor force overall, but its effects on private-sector composition go in the opposite direction. If the payoff from having a job in the public sector is relatively higher for low-educated workers (Case A) then those more likely to stay attached to the private sector are those whose education cost is low. Allowing for the option to enter the public sector therefore increases the share of low-educated in the labor force overall, but lowers the proportion of low-educated in the private sector. In this case, the ratio of high-to low-educated is higher in the private than in the public sector. On the other hand, if a government job is relatively more attractive to workers that have high-education (case B) then with the introduction of a public sector, a relatively higher fraction of workers with low education cost opt for government jobs. The share of high-educated in the labor force overall increases, but among those that stay attached to the private sector a higher fraction is low-educated.

5 Random search between the two sectors

We now compare the effects of government policies on employment and education under the alternative model assumption that search between the two sectors is random. We assume that workers search randomly for jobs that suit their skill type in the two sectors, which ultimately implies that there are no costs to entering the public sector. As shown in Lemma 3, if a public sector with entry barriers exists, then those choosing to pay in order to enter it must be better off searching exclusively there, while those choosing not to pay the entry cost can only get a job in the private sector. In other words, the existence of entry costs rules out the possibility of random search between the two sectors.

A matching function $m(v_i, u_i)$ determines the total number of matches between workers and jobs and $m(\theta_i)$, where $\theta_i = \frac{v_i}{u_i}$, gives the rate at which workers match with (either private or government) vacancies. Since they search randomly for jobs, the total number of vacancies available to them, consists of both private-sector v_i^p and government v_i^g vacancies. They find jobs in the private sector at rate $m(\theta_i)\nu_i^p$ and in the public sector at rate $m(\theta_i)(1-\nu_i^p)$, where $\nu_i^p = \frac{v_i^p}{v_i}$ is the fraction of private-sector vacancies in the total number of type i vacancies ($v_i = v_i^p + v_i^g$). As above, the number of vacancies in the private sector is determined endogenously by free entry ($V_i^p = 0$). However, to maintain its employment constant at e_i^g we must assume that the government posts enough vacancies such that the number of matches, $q(\theta_i)v_i^g$, equals the number of workers that it needs to hire ($q(\theta_i)v_i^g = (s_i^g + \tau)\, e_i^g$). These assumptions also imply that there are no public sector queues, because all workers meeting a public job are hired $m(\theta_i)\nu_i^g u = (s_i^g + \tau)\, e_i^g$ (since, $q(\theta_i)v_i^g = m(\theta_i)\nu_i^g u$). The existence of the public sector does not "directly" reduce the chances of an unemployed worker finding a job.

Another key difference between the model with random search and our benchmark model with segmented markets is the value of unemployment. It changes to take into account that workers now can match randomly with either private or government jobs.

$$(r+\tau)U_i = b_i + m(\theta_i)\nu_i^p \left[E_i^p - U_i\right] + m(\theta_i)(1-\nu_i^p) \left[E_i^g - U_i\right]. \tag{35}$$

The outside option of all workers is a convex combination of the value a public-sector job (E_i^g) and the value of a private-sector job (E_i^p) with weights reflecting the relative number of vacancies in the two sectors. The option value of finding a public job strengthens the bargaining position of workers holding private jobs. Thus, public-sector wages, employment opportunities, and separation probabilities affect private-sector wages, that are given by

$$w_i^p = b_i + \beta \left[y_i - b_i + \nu_i^p \theta_i \kappa_i \right] + (1-\beta) D_i (w_i^g - b_i), \tag{36}$$

where $D_i = \frac{(1-\nu_i^p)m(\theta_i)}{r+\tau+s_i^g+(1-\nu_i^p)m(\theta_i)}$ measures how much public-sector wages influence private-sector wage bargaining. A free-entry condition as in (9) determines the number of vacancies in the private sector. But now the match surplus, $S_i^p = \frac{y_i - w_i^p}{r+s_i^p+\tau}$, which decreases as the wage increases, depends also on policies. The full set of equations describing the model with random search, a formal definition of a steady-state equilibrium conditions and its existence are in Appendix B .

Under random search, and given constant public-sector employment, the effects of government policies work only through the outside option of workers and its impact on private-sector wages. We show in Appendix B that:

Proposition 6 *If search between the private and public sector is random, then for* $i = [h, l]$*, an increase in* w_i^g *or* e_i^g *increases private-sector wages (*w_h^p *and* w_l^p*) and lowers market tightness (*θ_h *and* θ_l*).*

Under segmented markets, more generous policies lower total employment by decreasing the size of private-sector labor force and increasing the queues for public-sector jobs. Under random search, the public sector hurts job creation in the private sector by putting upward pressure on wages thereby lowering firm profits and inducing firms to open fewer vacancies per unemployed worker. In both cases, negative effects on total employment arise when the value of employment in the public sector is relatively high.

The effect of government policies on education

As discussed above, when the two sectors are segmented, an increase in the education premium generates offsetting decreases in the job-finding rate by inducing more workers to search in the public sector. The lower the public-sector entry cost, the larger these offsetting decreases are. However, under random search, such offsetting decreases in the job-finding rate do not exist, so policies that increase the payoff from being educated encourage workers to become educated and raise the proportion of high-educated workers in the labor force (L_h). More formally, we show in Appendix B that:

Proposition 7 *If search between the private and public sector is random, an increase in* w_i^g *or* e_i^g *increases* L_i *and decreases* L_j*.* $i = [h, l]$, $j = [h, l]$ *and* $i \neq j$*.*

Notice that although the mechanisms are different, the qualitative effects of policies are similar in both labour market structures - segmented markets (with entry barriers) and random search (no entry barriers). Under both structures, policies that increase the value of public sector employment for high-educated workers raises the fraction of high-educated in the labor force and generate a negative employment effect. In segmented markets by generating longer queues and under random search by lowering private-sector tightness. One notable difference, however, between the two structures is that under random search, despite the decrease in tightness, the increase in share of high-educated may also increase their employment in the private sector, since some of the additional high-educated workers may end up employed in the private sector.

6 Model with segmented markets and non-absorbing states

In this section we incorporate the central feature of the random search model into our benchmark model with segmented markets, by allowing individuals to redraw the public sector entry cost each time they are found unemployed. This implies that the choice of either of the two sectors is not an absorbing state. Workers can move, through unemployment, from one sector to the other, depending on their draw of entry costs. The rest of the assumptions are as in the benchmark model in Section 3. The full set of equations describing this version of the model are in Appendix C.

The key difference with the benchmark model is the value of unemployment, which changes to take into account that all the unemployed can now draw a new entry costs and can chose to search for either a job in the public or in the private sector. Let $\tilde{c}_i$, denote the threshold cost at which a worker is indifferent between the two. The value of unemployment is given by:

$$(r+\tau)U_i = b_i - \hat{c}_i + (1-\Xi(\tilde{c}_i))m(\theta_i)\left[E_i^p - U_i\right] + \Xi(\tilde{c}_i)m_i^g\left[E_i^g - U_i\right], \quad i = [h, l], \tag{37}$$

where $\hat{c}_i = \int_0^{\tilde{c}_i} c d\Xi(c)$. As in the model with random search, in this model also, the outside option of workers is a convex combination of the value a public-sector job (E_i^g) and the value of a private-sector job (E_i^p). This means that here also, and in contrast to the benchmark model, public-sector wages, employment opportunities, and separation probabilities affect private-sector wages, through workers' outside options. In particular,

$$w_i^p = b_i + \beta\left[y_i - b_i + \hat{c}_i + (1-\Xi(\tilde{c}_i))(1-F_i)\theta_i\kappa_i\right] + (1-\beta)F_i(w_i^g - b_i + \hat{c}_i). \tag{38}$$

where $F_i = \frac{\Xi(\tilde{c}_i)m_i^g}{r+\tau+s_i^g+\Xi(\tilde{c}_i)m_i^g}$. As above, a free-entry condition (9) determines tightness in the private sector. As in the random-search model, the surplus of private-sector jobs (S_i^p) decreases as the value of searching for public-sector jobs increases, as this improves the worker's outside option and thus wage. In this version of the model also, more generous policies put upward pressure on wages, thereby lowering firm profits and tightness in the private sector (as summarized in Proposition 6).

At this point it should be clear that this version of the model looks very similar to the random search model in terms of equations. In fact, one can easily verify by inspecting the equations in Section 5 and Appendix B (and comparing them to those here and in Appendix C) that the two versions of the model are identical in all respects, but the matching probabilities. In the random search model, workers match *randomly* with either a private- or a public-sector vacancy at rate $m(\theta_i)\nu^p$ and $m(\theta_i)(1-\nu^p)$, respectively, where $\theta_i = \frac{v_i^p+v_i^g}{u_i}$. The corresponding probabilities in this version of the model are $(1-\Xi(\tilde{c}_i))m(\theta_i)$ and $\Xi(\tilde{c}_i)m_i^g$ where $\theta_i = \frac{v^p}{u^p}$ and $m_i^g = \frac{(s^g+\tau)e^g}{u^g}$, as in the benchmark model, and arise as workers choose to *direct their search* towards one sector or the other. This is an important difference, however, since it implies that workers choosing to search for public jobs will not necessarily find one (i.e. they may have to queue for such jobs), whereas, in the random-search model all workers randomly matching with a public job are hired. We can measure the size of the queue in public-sector i as $Q_i = L_i^g - e_i^g - e_i^g(s_i^g+\tau)$, since, as in the benchmark model, there are $L_i^g - e_i^g$ workers searching for jobs in public-sector i and only $e_i^g(s_i^g+\tau)$ vacancies. So only $e_i^g(s_i^g+\tau)$ will be hired. Hence, we can decompose total employment of type i workers into three parts, capturing the role of education decisions, public-sector queues and of public-sector job security, respectively, exactly as in (34).

This version introduces, in a sense, the key aspect of the random search model, into the benchmark model. It allows for movements between sectors so that more generous public sector policies place an upward pressure on private-sector wages, through increasing workers' outside option. It is important to note, however, that the main mechanism we highlight in the benchmark model, the role of public-sector queues, is still present. The existence of public sector queues does not rest on the assumption that workers cannot move away from the public sector. Even when workers move between sectors, as in the current version, queues exist. The key feature behind queues is that some workers choose to direct their search towards the public sector, where the chances of a finding a job are smaller. As discussed in Section 3, in the benchmark model, any negative employment effects of policies are due to increases in the queues for public-sector jobs. In this version of the model, with movements between sectors, the negative employment effects of more generous public sector policies work through both channels: increasing workers' outside option and generating longer public-sector queues. Notice, however, that the latter offsets the former, as longer queues for public jobs reduce the effects of policies increasing the value of these jobs, on the workers' outside option. So even if effects work through both channels, we cannot conclude that more generous public sector policies will have more negative employment effects in this version, than in the random search model.

7 Quantitative analysis

The objective of our numerical exercise is threefold. First, we inspect the quantitative effects of changes in government policies on the educational composition and employment. As discussed earlier, the effects of policies on the education choice (and thus on the employment of the high- and low-educated workers) depend on whether education and public-sector jobs are substitutes (Case A with $\tilde{c}_l > \tilde{c}_h$) or complements (Case B with $\tilde{c}_h > \tilde{c}_l$). Their effects on employment depend also on differences in job security between the two sectors and their wages. The model is parsimonious enough to be calibrated for four countries – United States, United Kingdom, France and Spain – that have different but sizable public sectors. Second, we compare the quantitative results in the benchmark model, where the choice of sector is permanent, with those in the random search model proposed in Section 5 and those in the model proposed in Section 6, where the choice of sector is a non-absorbing state. Third, we quantify the value of job security in the public-sector, for high- and low-educated workers.

7.1 Parameterization

We parameterize the three versions of the model (segmented markets, random search and non-absorbing state) for the four countries. We match the US economy at a monthly frequency, drawing largely on the *CPS* for the period 2006-2016. We also calibrate the version for UK, French and Spain at a quarterly frequency, drawing from their LFS and the SES. The data were discussed in Section 2 and shown in Table 1. A set of parameters is directly fixed to values taken from the data, while a second set of parameters targets steady-state values. The tables in Appendix F list all the parameters and their values, as well as the targeted and non-targeted steady-state variables for the three model. In here we describe the targets used to identify each parameter, common to the three models, in the case of the

US.

In the US, around 26.6 per cent of the population have a college degree, of which 24.6 per cent work in the public sector ($e_h^g = 0.066$). Out of the remaining population with no college degree, 10.8 per cent work in the public sector ($e_l^g = 0.079$). We set the separation rates by education and sector: $s_h^g = 0.005$, $s_h^p = 0.007$, $s_l^g = 0.016$ and $s_l^p = 0.028$. We consider, in the private sector, a Cobb-Douglas matching function with efficiency ζ_i and elasticity with respect to the unemployment of η_i. As the matching efficiency and the cost of posting vacancies are not separable, we normalize the efficiencies $\zeta_h = \zeta_l = 1$. The costs of posting vacancies, κ_h and κ_l, target the unemployment rate of 3.2 per cent for college graduates and 7.3 per cent for non-college graduates. The matching elasticities are set to the common value of 0.5, and the Hosios condition is assumed to hold ($\eta_h = \eta_l = \beta = 0.5$). The unemployment benefits, b_h and b_l, match a replacement rate of 0.29 and 0.43, an average of estimates for the replacement rate of workers with 150 and 67 per cent of the average wage, according to OECD. Additionally, $r = 0.004$ and $\tau = 0.002$ target a yearly interest rate of about four per cent and an average working life of 40 years.

We use CPS data from the 2002 to calculate the college premium and the public-sector wage premium by education. We normalize $y_l = 1$ and set y_h to target a private-sector college premium of 72 per cent found by regression of the log gross hourly earnings on a dummy for college education, using the sub-set of private-sector workers. Public-sector wages target the public-private wage differentials for college and non-college workers, $\frac{w_h^g}{w_h^p} = 1.027$ and $\frac{w_l^g}{w_l^p} = 1.064$, shown in Section 2. We allow the minimization routine to deviate from these for values within the 95 per cent confidence interval of the regression, to improve the fit in other dimensions.

The distribution of education costs is assumed to be log-normal. The mean and the standard deviation target the fraction of the labour force with a college degree and the total cost of tertiary education, which according to the OECD represents 3.8 per cent of US private consumption. The distribution of costs of joining the public sector is also a log-normal. The parameters of this distribution determine the size of queues in the public sector. However, the searching behaviour of the unemployed is in general unobservable. We calculate the average unemployment durations of workers who have found a job in the private sector and of those who have found a job in the public sector. Under segmented markets, the ratio of the two is equivalent to the inverse ratio of the average conditional job-finding rates, which is 1.25 for workers with college and 1.01 for workers without college. This means than in the US it is faster to find a job in the public sector than the private, particularly for college graduates.

We use the same procedure to calibrate the model for the remaining three countries. The targets are matched perfectly with the exception of the unemployment duration ratio for unskilled workers that is lower than the one found in the data for France and Spain. In the baseline steady-state, education complements a career in the public sector in the US, UK and Spain, meaning that these economies are in Case B. Despite the public-sector premium being higher for workers without a college degree, there are many more jobs available in the public sector for college graduates. In France, the difference between the two cutoffs c_h and c_l is very small, meaning that it is close to Case C. Given the calibration, we calculate the minimum wage required for the existence of the public sector relative to the actual wage. The gap between the minimum wage and the actual wage is higher for unskilled workers in all countries. It is particularly high in Spain where the minimum wage is 15 to 24 per cent lower than the actual wage. In France this gap is only 4 and 8 per cent.

We repeat this procedure for the model with public-sector as a non-absorbing state (NAS), with the exact same targets. For the random search model (RS), the distribution of public-sector entry cost is nonexistent, so we drop the ratio of the duration of unemployment which, by definition, must equal 1. As in the segmented markets model (SM), the value of employment is always higher in the public sector than in the private.

7.2 Job-security premium

We now demonstrate the potential of our model to inform policymakers. The argument that jobs are safer is often used in policy discussions over public-sector pay. However, there have been no attempts to quantify this value and it is not clear whether it is taken in account by pay-review bodies determining the public-sector pay scale. Moreover, the value of job security, as it varies across education, affects workers' selection into the two sectors and education decisions and is important for how public-sector wage and employment policies affect these decisions. Fontaine et al. (2020) illustrate, using a two-equation model, the principle of how we can use a model with search and matching frictions to evaluate the value of job security. We can use the same principle in our fully structural model.

We ask private-sector workers what fraction of their wage they would be willing to pay to have the same job-separation rate as in the public sector (Ψ_i^p). Alternatively, we can ask public-sector workers how much they would need to be compensated to be given the same job-separation rate as in the private sector (Ψ_i^g). To calculate both values, we maintain the value of employment and unemployment at the baseline value. Using (2) and (4), respectively, we can derive the following expressions for the value of job security:

$$\Psi_i^p = \frac{(s_i^g - s_i^p)[E_i^p - U_i^p]}{w_i^p} \times 100, \quad i = [h, l] \tag{39}$$

$$\Psi_i^g = \frac{(s_i^p - s_i^g)[E_i^g - U_i^g]}{w_i^g} \times 100, \quad i = [h, l]. \tag{40}$$

With linear utility, safer jobs raise the expected duration of a job and reduce the expected time spent in unemployment. The monetary value of job-security reflects how painful unemployment is, both in terms of payoff, b_i, but also persistence in the form of job-finding rates in each sector, so the values of Ψ_i^p and Ψ_i^g should be different. We can interpret these values as lower bounds of the job-security premia, because they do not incorporate that risk-averse agents would find unemployment more damaging.

Table 2 shows our estimates for job-security premium. Across countries, the value of job-security is related to how dynamic the labour market is, with the US and UK being the countries with lower premia and France and Spain the countries with higher premia. With the exception of Spain, the job-security premium is higher for workers without college. In the US, for instance, private-sector workers with college would sacrifice 0.5 per cent of their wage, while workers without college would sacrifice more than 1.5 per cent. In Spain, the difference between job-separation rates across sectors is also quite high for college graduates so they would be willing to sacrifice 3 to 4 per cent of their wage, while no-college workers would only pay 2.5 per cent. When measured by the compensation needed to give to public-sector workers to reduce their job-security, the numbers are larger, in particular to unskilled

Table 2: Job-security premium different models/calibrations

	% wage change private-sector workers would accept for the public-sector job security			*% wage change public-sector workers would accept for the private-sector job security*		
	SM	RS	NAS	SM	RS	NAS
United States						
Skilled	-0.60	-0.51	-0.46	0.49	1.13	1.08
Unskilled	-1.81	-1.66	-1.52	1.91	5.91	5.72
United Kingdom						
Skilled	-1.01	-0.76	-0.61	1.38	2.89	2.70
Unskilled	-1.52	-1.34	-1.15	2.20	4.93	4.69
France						
Skilled	-1.72	-1.69	-1.74	1.80	1.85	1.93
Unskilled	-1.86	-1.77	-1.66	3.28	4.64	4.48
Spain						
Skilled	-3.83	-3.35	-3.01	4.19	7.85	7.36
Unskilled	-2.75	-2.48	-2.27	4.42	6.94	6.71

workers. The values would vary from 2 per cent in the US to close to 7 per cent in Spain. The values for UK and France are in between. When measured by the private-sector workers, the premium is slightly lower in the random search and non-absorbing state models, but it is higher when measured by the public-sector workers.

These estimates reflect the average levels of job security in the two sectors. In France and Spain, countries with significant labour market duality, even in the public sector, these numbers might mask some heterogeneity. For some public-sector workers with temporary contracts, the job-security premium is lower than the average. For the ones with permanent contracts the job-security premium is even higher. Such contract guarantees no involuntary spell on unemployment until retirement, unlike private-sector workers with permanent contracts whose firms might go bankrupt.

7.3 The effects of skilled-biased policies

Table 3 compares the effect for the three models, following: i) a ten-percent increase in skilled public wages and ii) a ten-percent increase in unskilled public wages. It shows the percentage change in the number of skilled workers and the percentage points difference in skilled and unskilled unemployment rate.

In line with Propositions 2 and 7, under all three labor market structures, policies that improve the public-sector value of the high- relative to the low-educated increase the proportion of high-educated in the labor force and vice versa. But our quantitative results show that allowing for movements between the two sectors amplifies the effects of policies on educational composition. When increasing skilled wages by ten per cent, the share of high-educated in the labor force goes up by at most 0.2 per cent in the segmented markets model, as opposed to 2.5 to 7 per cent when transitions between sectors are allowed. When increasing unskilled wages by ten per cent, the share of high-educated decreases by less than 0.1 per cent in segmented markets, as opposed to 1 to 2.3 per cent in the other two models. Education responds less in the segmented markets model, since only a fraction of workers, those whose public-sector entry cost is sufficiently low, are affected by the policies.

Under the three labor market structures, wage policies that increase the public-sector value for high- or low-educated workers, involve a negative effect on total employment. In the segmented market model by attracting too many job seekers into the public sector and creating longer queues of unemployed. A ten per cent increase in the wages of high-educated public-sector employees in the US, decreases the employment of high-educated workers by 2 per cent and of low-educated workers by 0.06 per cent. In the random search model they spillover to private-sector pay, lower firm profits, and decrease their job creation. However, in

Table 3: Effects of wage policies under different models

	Increase on skilled wages by 10 percent					
	United States			**United Kingdom**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	0.18	6.85	7.16	0.02	2.49	3.02
%Δ employment skilled	-2.03	5.54	5.88	-2.43	0.76	1.49
%Δ employment unskilled	-0.06	-2.49	-2.58	-0.01	-2.13	-2.53
Δ u. rate skilled	2.14 p.p.	1.19 p.p.	1.15 p.p.	2.36 p.p.	1.63 p.p.	1.44 p.p.
Δ u. rate unskilled	0.00 p.p.	0.01 p.p.	0.01 p.p.	0.00 p.p.	0.01 p.p.	0.01 p.p.
	France			**Spain**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	0.00	4.00	4.12	0.01	2.67	2.75
%Δ employment skilled	-2.87	2.69	2.87	-3.13	1.09	1.25
%Δ employment unskilled	-0.00	-1.88	-1.92	-0.01	-1.39	-1.42
Δ u. rate skilled	2.70 p.p.	1.19 p.p.	1.14 p.p.	2.79 p.p.	1.37 p.p.	1.31 p.p.
Δ u. rate unskilled	0.00 p.p.	0.00 p.p.	-0.01 p.p.	0.00 p.p.	0.01 p.p.	0.01 p.p.

	Increase on unskilled wages by 10 percent					
	United States			**United Kingdom**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	-0.07	-2.34	-2.37	-0.01	-1.47	-1.55
%Δ employment skilled	-0.07	-2.34	-2.37	-0.01	-1.48	-1.55
%Δ employment unskilled	-1.49	-0.03	-0.01	-1.72	0.20	0.30
Δ u. rate skilled	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.01 p.p.	0.00 p.p.
Δ u. rate unskilled	1.41 p.p.	0.81 p.p.	0.79 p.p.	1.63 p.p.	0.98 p.p.	0.93 p.p.
	France			**Spain**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	-0.00	-2.08	-2.13	-0.01	-1.02	-1.02
%Δ employment skilled	-0.00	-2.05	-2.11	-0.01	-1.01	-1.01
%Δ employment unskilled	-3.23	-0.62	-0.51	-2.15	-0.64	-0.59
Δ u. rate skilled	0.00 p.p.	-0.02 p.p.	-0.02 p.p.	0.00 p.p.	-0.01 p.p.	-0.01 p.p.
Δ u. rate unskilled	2.89 p.p.	1.42 p.p.	1.34 p.p.	1.69 p.p.	0.92 p.p.	0.88 p.p.

the random search model the changes in educational composition overshadow these negative job creation effects. Hence, changes in the employment of the two education groups reflect changes in their proportions in the labor force. A ten per cent increase in the wages of high-educated public-sector employees in the US increases their employment by about 5.5 per cent, because it increases their proportion in the labor force by about 7 per cent. The employment of the low-educated falls by 2.5 per cent. In the third model, with public sector entry costs and movements across the sectors, the negative employment effects work through both of these channels. But in line with our previous discussion, since workers are not attached to a specific sector, education and employment responses are of the same magnitude as those of the random search model.

Table 4: Effects of employment policies under different models

	Increase on skilled employment by 10 percent					
	United States			**United Kingdom**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	0.20	0.28	0.21	0.02	0.26	0.19
%Δ employment skilled	0.27	0.26	0.21	0.06	0.20	0.18
%Δ employment unskilled	-0.07	-0.10	-0.08	-0.02	-0.23	-0.16
Δ u. rate skilled	-0.07 p.p.	0.02 p.p.	0.01 p.p.	-0.03 p.p.	0.06 p.p.	0.01 p.p.
Δ u. rate unskilled	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.
	France			**Spain**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	0.00	0.02	-0.07	0.01	0.29	0.22
%Δ employment skilled	0.10	0.12	0.05	0.23	0.35	0.32
%Δ employment unskilled	0.00	-0.01	0.03	-0.01	-0.15	-0.12
Δ u. rate skilled	-0.10 p.p.	-0.10 p.p.	-0.11 p.p.	-0.19 p.p.	-0.05 p.p.	-0.08 p.p.
Δ u. rate unskilled	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.
	Increase on unskilled employment by 10 percent					
	United States			**United Kingdom**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	-0.05	-0.19	-0.18	-0.01	-0.16	-0.14
%Δ employment skilled	-0.05	-0.19	-0.18	-0.01	-0.16	-0.14
%Δ employment unskilled	0.09	0.04	0.04	0.05	0.10	0.10
Δ u. rate skilled	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.
Δ u. rate unskilled	-0.06 p.p.	0.03 p.p.	0.02 p.p.	-0.04 p.p.	0.04 p.p.	0.02 p.p.
	France			**Spain**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	0.00	-0.15	-0.10	0.00	-0.18	-0.16
%Δ employment skilled	0.00	-0.15	-0.10	0.00	-0.18	-0.15
%Δ employment unskilled	0.01	0.08	0.09	-0.02	-0.02	0.00
Δ u. rate skilled	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.	0.00 p.p.
Δ u. rate unskilled	-0.01 p.p.	-0.01 p.p.	-0.04 p.p.	0.01 p.p.	0.09 p.p.	0.06 p.p.
	No public employment					
	United States			**United Kingdom**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	-0.86	-1.15	-0.49	-0.09	-1.63	-0.68
%Δ employment skilled	-1.22	-0.96	-0.41	-0.33	-1.00	-0.52
%Δ employment unskilled	-0.07	0.71	0.41	-0.25	1.80	0.80
Δ u. rate skilled	0.35 p.p.	-0.18 p.p.	-0.08 p.p.	0.23 p.p.	-0.62 p.p.	-0.16 p.p.
Δ u. rate unskilled	0.35 p.p.	-0.27 p.p.	-0.21 p.p.	0.31 p.p.	-0.38 p.p.	-0.21 p.p.
	France			**Spain**		
	SM	RS	NAS	SM	RS	NAS
%Δ fraction of skilled	0.00	1.44	1.69	-0.05	-1.21	-0.76
%Δ employment skilled	0.00	0.42	0.48	-2.12	-1.64	-1.57
%Δ employment unskilled	-0.13	-0.73	-1.21	0.27	1.85	1.24
Δ u. rate skilled	0.00 p.p.	0.95 p.p.	1.13 p.p.	1.83 p.p.	0.39 p.p.	0.73 p.p.
Δ u. rate unskilled	0.12 p.p.	0.05 p.p.	0.37 p.p.	-0.19 p.p.	-0.97 p.p.	-0.68 p.p.

Table 4 compares the effects of employment policies: i) a ten-percent increase in skilled public employment; ii) a ten-percent increase in unskilled public employment; iii) no public sector. France is close to Case C ($\tilde{c}_h = \tilde{c}_l$ and $\tilde{\epsilon}_p = \tilde{\epsilon}_g$) meaning that the public sector premium in France is the same for both types of workers. All other countries are in case B ($\tilde{c}_h > \tilde{c}_l$ and $\tilde{\epsilon}_p < \tilde{\epsilon}_g$), where education complements search in the public sector, but for different reasons. In the UK, the public sector is a major source of job opportunities for high-educated workers. In the US, on the other hand, high-educated workers can find jobs in the public sector faster than in the private sector, which also acts to increase the benefit from investing in education. In Spain, the public sector strengthens incentives to invest in education especially under random search, by offering to high-educated workers access to more secure and better paying jobs in the public sector. This explains why eliminating the public sector lowers the proportion of high-educated in all countries but France. Because France is in Case C, in the segmented markets model eliminating the French public sector leaves the educational composition intact, in line with Proposition 1. However, in the other two models, it increases the proportion of educated suggesting that the public sector in France improves by relatively more the outside option of low-educated workers. This is consistent with a much larger portion of government employment in France being low-educated – 60 per cent of government employees in France are low-educated as opposed to about 40 per cent in the UK and Spain and 54 per cent in the US.

Public-sector job security is a feature that seems to boost employment in all countries. While in the two models with movements between sectors changes in employment follow the changes in the labor force composition, in the segmented markets model, we see strong negative employment effects from eliminating the public sector. In Spain eliminating the public sector decreases the proportion of high-educated in the labor force by only 0.05 per cent. Nevertheless, it decreases their employment by more than 2 per cent. The excess decrease is due to restricting access of workers to government jobs that are safer and last longer. Although in the absence of a public-sector the proportion of low-educated increases in all countries (except France where it remains the same), the employment of low-educated falls, reflecting the impact of lower job security in the private sector. The only exception is Spain, where actual public-sector wages, especially of low-educated workers, are much larger than the minimum wage, suggesting the existence of long queues. This explains, why in the case of Spain, eliminating the public sector, increases the employment of low-educated by 0.27 per cent, despite lowering their proportion in the labor force.

8 Conclusion

Earlier literature has highlighted the problems of setting high public-sector wages. Gomes (2015) shows that they generate higher unemployment. Cavalcanti and Santos (2020) argue that higher wages might lead to misallocation of resources with a lower entrepreneurship rate. Chassamboulli and Gomes (2021) argue that they might foster rent-seeking activities of unemployed trying to get a public-sector job through political or personal connections. Garibaldi et al. (2018) demonstrate that high wages for workers with low qualifications might generate under-employment. We highlight another potential negative effect. Higher public-sector wages for unskilled workers lead to lower incentives for education.

We have compared three labour market structures to understand how public-sector hiring and wage policies affect education decisions and employment. While the main results are consistent across the three models, the mechanisms and the quantitative implications of policies are different. In segmented markets public-sectors policies that increase the value of education generate offsetting decreases in job finding rates by inducing workers to move away from the private and into the public sector, where they will queue up waiting for jobs. Thus such policies have a relatively small effect on education and larger effects on unemployment. However, when workers search randomly for jobs in both sectors, such offsetting decreases in job finding rates are not possible. Instead, such policies place a downward pressure on firm profits and job creation in the private sector, by improving workers' outside options. In this case, incentives to obtain education increase by more, while effects on unemployment are relatively small. A model with segmented markets but mobility between the sectors encapsulates both effects.

If we want to develop a model that can be used for policy analysis and forecasting, it is imperative that we realistically model the labor market. In reality, the search structure might be different across countries and in different occupations. In this regard, it is important to develop the empirical research on public-sector hiring procedures and the unemployed search behaviour to determine which model is more suitable to use.

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COMPANION APPENDIX

Public-sector employment and wages and education decisions or education

Andri Chassamboulli and Pedro Gomes

A Proofs of propositions

A.1 Lemma 2

We consider that the public-sector labour market for workers of type i breaks down if the government is not able to hire enough workers to replace the workers that have lost their job. At the limit, it means the government needs to post a wage, defined as $\underline{w}_i^g$, such that it attracts at least $(s_i^g + \tau)e_i^g$ job searches. This means that at $w_i^g = \underline{w}_i^g$ the cutoffs $\tilde{c}_h, \tilde{c}_l$ are such that, using equations (21)-(23), we get $L_i^g - e_i^g \equiv u_i^g = (s_i^g + \tau)e_i^g$ and the job-finding rate is 1 ($m_i^g = 1$). Let $\underline{\tilde{c}}_h, \underline{\tilde{c}}_l$ be that cutoffs that satisfy this condition. Applying $m_i^g = 1$ to (15) and then setting $U_i^g - U_i^p = \underline{\tilde{c}}_i$ gives

$$b_i + \frac{1}{r + \tau + s_i^g + 1}[\underline{w}_i^g - b_i] = (r + \tau)(U_i^p + \underline{\tilde{c}}_i)$$

Substituting the $(r + \tau)U_i^p$ by equation (14) we get

$$\underline{w}_i^g = (r + \tau + s_i^g + 1)\left(\frac{m(\theta_i^*)}{r + \tau + s_i^p + m(\theta_i^*)}[w_i^{p,*} - b_i] + b_i + (r + \tau)\underline{\tilde{c}}_i\right)$$

where θ_i^* and $w_i^{p,*}$ are the equilibrium tightness and wages in the private sector.

A.2 Lemma 3

Proof. Based on Lemma 2 (see also subsection A.1) if $w_i^g \geq \underline{w}_i^g$ then $L_i^g - e_i^g \geq (s_i^g + \tau)e_i^g$ which means that $L_i^g > 0$. From (22)-(23) we can verify that this implies $\tilde{c}_i > 0$ and from (17) that this, in turn, implies $U_i^g > U_i^p$. ■

A.3 Proof of Existence and Uniqueness of a Steady-State Equilibrium

Proof. It can be easily verified that the two free-entry conditions in (11) pin down a unique set of equilibrium values for θ_h and θ_l. Substituting these values into (25) we get the unique equilibrium value for $\tilde{\epsilon}_p$. To complete the proof of existence and uniqueness we need to show that with the equilibrium values of θ_h, θ_l and $\tilde{\epsilon}_p$ substituted in, the two threshold conditions in (24), $\tilde{c}_h = U_h^g - U_h^p$ and $\tilde{c}_l = U_l^g - U_l^p$ only cross once in the $[\tilde{c}_h, \tilde{c}_l]$ plane giving a unique set of equilibrium values for $\tilde{c}_h$ and $\tilde{c}_l$.

Let as write (24) as:

$$\tilde{c}_i = \frac{1}{r + \tau}\left[A_i - \frac{\beta \kappa_i \theta_i}{(1 - \beta)}\right], \quad i = [h, l] \tag{41}$$

where

$$A_i \equiv \frac{\frac{(s_i^g + \tau)e_i^g}{L_i^g - e_i^g}}{r + \tau + s_i^g + \frac{(s_i^g + \tau)e_i^g}{L_i^g - e_i^g}}(w_i^g - b_i) \tag{42}$$

By total differentiation of (41) we can derive their slopes:

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h = U_h^g - U_h^p} = \frac{\frac{\partial A_h}{\partial L_h^g}\frac{\partial L_h^g}{\partial \tilde{c}_l}\frac{1}{r+\tau}}{1 - \frac{\partial A_h}{\partial L_h^g}\frac{\partial L_h^g}{\partial \tilde{c}_h}\frac{1}{r+\tau}} > 0 \tag{43}$$

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l = U_l^g - U_l^p} = \frac{1 - \frac{\partial A_l}{\partial L_l^g}\frac{\partial L_l^g}{\partial \tilde{c}_l}\frac{1}{r+\tau}}{\frac{\partial A_l}{\partial L_l^g}\frac{\partial L_l^g}{\partial \tilde{c}_h}\frac{1}{r+\tau}} > 0 \tag{44}$$

Both slopes are positive, since, as can be easily verified from (42) and (21)-(23), $\frac{\partial A_i}{\partial L_i^g} < 0$, $\frac{\partial L_i^g}{\partial \tilde{c}_i} > 0$ and $\frac{\partial L_i^g}{\partial \tilde{c}_j} < 0$. But it can also be shown that $\frac{\partial L_i^g}{\partial \tilde{c}_i} > -\frac{\partial L_i^g}{\partial \tilde{c}_j} > 0$ so that:

$$\begin{aligned} \left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h = U_h^g - U_h^p} &< 1 \\ \left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l = U_l^g - U_l^p} &> 1 \end{aligned}$$

This completes the proof of existence and uniqueness. The two loci only cross once in the $[\tilde{c}_h, \tilde{c}_l]$ plane giving a unique set of equilibrium values for $\tilde{c}_h$ and $\tilde{c}_l$. ■

A.4 Proof of Lemma 4

Proof. We see from (25) that the cutoff e^p depends only on private sector tightness θ_h and θ_l, which as shown in Lemma 1 (see subsection A.1) are independent of public-sector policies.

■

A.5 The impact of public sector on total employment

Proof. Using (26), (27), and (28) we can solve for e_i^p. Then setting $e_i = e_i^p + e_i^g$ gives:

$$e_i = e_i^g + \frac{m(\theta_i) L_i^p}{s_i^p + \tau + m(\theta_i)} \tag{45}$$

Note that $L_i^p = L_i - L_i^g$ and let $Q_i = L_i^g - e_i^g - e_i^g(s_i^g + \tau)$ denote the size of the "queue" in public sector i. There are $L_i^g - e_i^g$ workers searching for jobs in sector in public-sector i and only $e_i^g(s_i^g + \tau)$ vacancies. So only $e_i^g(s_i^g + \tau)$ will be hired, the rest will queue up waiting for new vacancies. We can write the total employment rate as

$$e_i = \frac{m(\theta_i) L_i}{s_i^p + \tau + m(\theta_i)} - \frac{m(\theta_i) Q_i}{s_i^p + \tau + m(\theta_i)} + \frac{m(\theta_i) e_i^g \left(\frac{s_i^p + \tau}{m(\theta_i)} - s_i^g + \tau\right)}{s_i^p + \tau + m(\theta_i)} \tag{46}$$

In the absence of a public sector total employment equals total employment in the private sector. Let the superscript "ng" denote the absence of a public-sector. Total employment in that case is given by

$$e_i^{ng} = \frac{m(\theta_i) L_i^{ng}}{s_i^p + \tau + m(\theta_i)} \tag{47}$$

Subtracting one from the other we get

$$e_i - e_i^{ng} = \frac{m(\theta_i)(L_i - L_i^{ng})}{s_i^p + \tau + m(\theta_i)} - \frac{m(\theta_i) Q_i}{s_i^p + \tau + m(\theta_i)} + \frac{m(\theta_i) e_i^g \left(\frac{s_i^p + \tau}{m(\theta_i)} - s_i^g + \tau\right)}{s_i^p + \tau + m(\theta_i)} \tag{48}$$

The first term in the above expression captures the impact of changes in the educational composition of the labor force. The second term shows the impact of public-sector queues, while the last term shows the impact of higher public-sector job security. As shown in Lemma 4, when entry into the public sector is free, the educational composition of the labor force is independent of the public sector: $L_h = L_h^{ng} = \Xi^\epsilon(\tilde{\epsilon}_p)$, $L_l = L_l^{ng} = 1 - \Xi^\epsilon(\tilde{\epsilon}_p)$. In the case, the existence of a public sector will affect total employment only through job security and queues. As shown in Section A.1, if $w_i^g = \underline{w}_i^g$ then $Q_i = 0$, i.e., there are no queues for public-sector jobs. In this particular case, of free entry into the public sector and no public-sector queues the existence of a public sector will alter employment only if it differs from the private sector in terms of job security. If $s_i^g + \tau < \frac{s_i^p + \tau}{m(\theta_i)}$ then $e_i < e_i^{ng}$, if $s_i^g + \tau > \frac{s_i^p + \tau}{m(\theta_i)}$ then $e_i > e_i^{ng}$ and if $s_i^g + \tau = \frac{s_i^p + \tau}{m(\theta_i)}$ then

$e_i = e_i^{ng}$. Note that $\frac{s_i^p+\tau}{m(\theta_i)} = \frac{u^p}{e^p}$ and is a measure of job security in the private sector: the larger this term is the smaller the chances a worker attached to the private sector is employed. If, on the other hand, entry into the public sector is free and $w_i^g > \underline{w}_i^g$, then $Q_i > 0$ and the existence of the public sector generates an additional negative effect on total employment. In this case, it is apparent from the expression above that $e_i < e_i^{ng}$ if $s_i^g + \tau \geq \frac{s_i^p+\tau}{m(\theta_i)}$, while if $s_i^g + \tau < \frac{s_i^p+\tau}{m(\theta_i)}$ then $e_i < e_i^{ng}$ or $e_i > e_i^{ng}$ depending on which of the two effects dominates: the negative effect of longer queues or the positive effect of job security.

■

A.6 Proof of Proposition 1

Proof. From the expressions for L_h^p and L_h^g in (21)-(23) we obtain

$$L_h = \Xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_g) + \int_{\tilde{c}_h}^{\tilde{c}_l} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1 - \Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_p), \text{ if } \tilde{c}_h < \tilde{c}_l \tag{49}$$

$$L_h = \Xi^c(\tilde{c}_l)\Xi^\epsilon(\tilde{\epsilon}_g) + \int_{\tilde{c}_l}^{\tilde{c}_h} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1 - \Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_p), \text{ if } \tilde{c}_l < \tilde{c}_h \tag{50}$$

$$L_h = \Xi^\epsilon(\tilde{\epsilon}_p), \text{ if } \tilde{c}_l = \tilde{c}_h \tag{51}$$

While if a public sector does not exist (denoted by the superscript "ng"), then $L_h^{ng} = \Xi^\epsilon(\tilde{\epsilon}_p)$. Subtracting, L_h^{ng} from (49), (50) and (51), respectively, we obtain

$$L_h - L_h^{ng} = \int_{\tilde{c}_h}^{\tilde{c}_l} \left[\Xi^\epsilon(\tilde{\epsilon}_p) - \Xi^\epsilon(\tilde{\epsilon}_m(c))\right] d\Xi^c(c) + \Xi^c(\tilde{c}_h)\left[\Xi^\epsilon(\tilde{\epsilon}_p) - \Xi^\epsilon(\tilde{\epsilon}_g)\right] > 0, \text{ if } \tilde{c}_h < \tilde{c}_l \tag{52}$$

$$L_h - L_h^{ng} = -\int_{\tilde{c}_l}^{\tilde{c}_h} \left[\Xi^\epsilon(\tilde{\epsilon}_m(c)) - \Xi^\epsilon(\tilde{\epsilon}_p)\right] d\Xi^c(c) - \Xi^c(\tilde{c}_l)\left[\Xi^\epsilon(\tilde{\epsilon}_g) - \Xi^\epsilon(\tilde{\epsilon}_p)\right] < 0, \text{ if } \tilde{c}_h > \tilde{c}_l \tag{53}$$

$$L_h - L_h^{ng} = 0, \text{ if } \tilde{c}_h = \tilde{c}_l. \tag{54}$$

As shown above, $\tilde{\epsilon}_g \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_p$, if $\tilde{c}_h > \tilde{c}_l$, while $\tilde{\epsilon}_g \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_p$, if $\tilde{c}_h < \tilde{c}_l$, implying that the terms in the brackets of (52) and (53) are positive. ■

A.7 Proof of Proposition 2

Proof. Using (24) we can derive for $x_i = [w_i^g, s_i^g, e_i^g]$, $i = [h, l]$, $j = [h, l]$ and $j \neq i$ that

$$\frac{d\tilde{c}_i}{dx_i} = \frac{\frac{\partial A_i}{\partial x_i}}{r + \tau - \frac{\partial A_i}{\partial L_i^g}\left[\frac{\partial L_i^g}{\partial \tilde{c}_i} + B_j \frac{\partial L_i^g}{\partial \tilde{c}_j}\right]} \tag{55}$$

$$\frac{d\tilde{c}_j}{dx_i} = B_j \frac{d\tilde{c}_i}{dx_i} \tag{56}$$

where A_j is as defined in (42) above and

$$B_j = \frac{\frac{\partial A_j}{\partial L_j^g}\frac{\partial L_j^g}{\partial \tilde{c}_i}}{r + \tau - \frac{\partial A_j}{\partial L_j^g}\frac{\partial L_j^g}{\partial \tilde{c}_j}} \tag{57}$$

It can be easily verified from (42) that $\frac{\partial A_j}{\partial L_j^g} < 0$ $\left(\frac{\partial A_i}{\partial L_i^g} < 0\right)$ and from (21)-(23) that $\frac{\partial L_j^g}{\partial \tilde{c}_j} > -\frac{\partial L_j^g}{\partial \tilde{c}_i} > 0$ $\left(\frac{\partial L_i^g}{\partial \tilde{c}_i} > -\frac{\partial L_i^g}{\partial \tilde{c}_j} > 0\right)$ so that $1 > B_j > 0$ $(1 > B_i > 0)$. These imply that the denominator of (55) is positive.

From (42) we also know that the numerator of (55) is also positive since $\frac{\partial A_i}{\partial x_i} > 0$. We can therefore conclude that:

$$\frac{d\tilde{c}_i}{dx_i} > 0 \text{ and } \frac{d\tilde{c}_j}{dx_i} > 0 \text{ for } x_i = [w_i^g, s_i^g, e_i^g],\ i = [h, l],\ j = [h, l] \text{ and } j \neq i \tag{58}$$

With (18)-(20) substituted in the expressions for L_h^p and L_h^g from (21)-(23) we can derive an expression for $L_h (= L_h^p + L_h^g)$ in terms of only $\tilde{c}_h$, $\tilde{c}_l$ and model parameters so that:

$$\frac{dL_h}{dx_i} = \frac{\partial L_h}{\partial \tilde{c}_h}\frac{d\tilde{c}_h}{dx_i} + \frac{\partial L_h}{\partial \tilde{c}_l}\frac{d\tilde{c}_l}{dx_i} \tag{59}$$

Using (56) we can write:

$$\frac{dL_h}{dx_h} = \frac{d\tilde{c}_h}{dx_h}\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l \frac{\partial L_h}{\partial \tilde{c}_l}\right] \tag{60}$$

$$\frac{dL_h}{dx_l} = \frac{d\tilde{c}_l}{dx_l}\left[\frac{\partial L_h}{\partial \tilde{c}_h} B_h + \frac{\partial L_h}{\partial \tilde{c}_l}\right] \tag{61}$$

Next we derive expressions for the terms in the brackets:

$$\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l \frac{\partial L_h}{\partial \tilde{c}_l}\right] = \begin{cases} \xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_h)\left[\frac{r+\tau-\frac{\partial A_l}{\partial L_l^g}\xi^c(\tilde{c}_l)(1-\Xi^\epsilon(\tilde{\epsilon}_p))}{r+\tau-\frac{\partial A_l}{\partial L_l^g}\frac{\partial L_l^g}{\partial \tilde{c}_l}}\right], & \text{if } \tilde{c}_h < \tilde{c}_l \\ \xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_l)\left[1 - B_l\right] + \int_{\tilde{c}_l}^{\tilde{c}_h} \xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_l < \tilde{c}_h, \end{cases} \tag{62}$$

$$\left[\frac{\partial L_h}{\partial \tilde{c}_l} + B_h \frac{\partial L_h}{\partial \tilde{c}_h}\right] = \begin{cases} -\xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_h)\left[1 - B_h\right] - \int_{\tilde{c}_h}^{\tilde{c}_l} \xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_h < \tilde{c}_l \\ -\xi^\epsilon(\tilde{\epsilon}_g)\Xi^c(\tilde{c}_l)\left[\frac{r+\tau-\frac{\partial A_h}{\partial L_h^g}\xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_p)}{r+\tau-\frac{\partial A_h}{\partial L_h^g}\frac{\partial L_h^g}{\partial \tilde{c}_h}}\right], & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases} \tag{63}$$

The terms in (62) are positive while the terms in (63) negative. Using (58) we can therefore conclude that:

$$\frac{dL_i}{dx_i} > 0, \frac{dL_i}{dx_j} < 0,\ x_i = [w_i^g, s_i^g, e_i^g],\ i = [h, l],\ j = [h, l] \text{ and } j \neq i \tag{64}$$

■

A.8 Proof of Proposition 3

Proof. From (48), for $s_i^p = s_i^g$ and $w_i^g = \underline{w}_i^g$ (which means $Q_i = 0$) we obtain that the introduction of a public sector whose entry is costly will cause a change in total employment of workers of type i of

$$e_i - e_i^{ng} = \frac{m(\theta_i)(L_i - L_i^{ng})}{s_i^p + \tau + m(\theta_i)} \tag{65}$$

From the results in Proposition 1 it is easy to verify that, since the share of high-educated labor force decreases (increases) in Case A (case B), while it remains unchanged in case C, then $e_h - e_h^{ng} < 0$ ($e_l - e_l^{ng} > 0$) in Case A, $e_h - e_h^{ng} > 0$ ($e_l - e_l^{ng} < 0$) in Case B, and $e_h - e_h^{ng} = 0$ ($e_l - e_l^{ng} = 0$) in Case C. ■

A.9 Proof of Proposition 4

Proof. Using the labor force shares in (21)-(23) and the result in (56) we can write

$$\frac{dL_i^p}{dx_i} = \frac{\partial L_i^p}{\partial \tilde{c}_i}\frac{\partial \tilde{c}_i}{\partial x_i} + \frac{\partial L_i^p}{\partial \tilde{c}_j}\frac{\partial \tilde{c}_j}{\partial x_i} \tag{66}$$

$$\frac{dL_j^p}{dx_i} = \frac{\partial L_j^p}{\partial \tilde{c}_i}\frac{\partial \tilde{c}_i}{\partial x_i} + \frac{\partial L_j^p}{\partial \tilde{c}_j}\frac{\partial \tilde{c}_j}{\partial x_i} \tag{67}$$

$$\frac{dL_i^g}{dx_i} = \frac{\partial \tilde{c}_i}{\partial x_i}\left[\frac{\partial L_i^g}{\partial \tilde{c}_i} + B_j \frac{\partial L_i^g}{\partial \tilde{c}_j}\right] \tag{68}$$

$$\frac{dL_j^g}{dx_i} = \frac{\partial \tilde{c}_i}{\partial x_i}\left[\frac{\partial L_j^g}{\partial \tilde{c}_i} + B_j \frac{\partial L_j^g}{\partial \tilde{c}_j}\right] \tag{69}$$

where $i = [h, l]$, $j = [h, l]$ and $i \neq j$. Using (21)-(23) one can easily verify that $\frac{\partial L_i^p}{\partial \tilde{c}_i} < 0$, $\frac{\partial L_j^p}{\partial \tilde{c}_j} < 0$, $\frac{\partial L_i^p}{\partial \tilde{c}_j} \leq 0$, $\frac{\partial L_j^p}{\partial \tilde{c}_i} \leq 0$, while, as shown above (see 58), for $x_i = [w_i^g, s_i^g, e_i^g]$, $\frac{d\tilde{c}_i}{dx_i} > 0$ and $\frac{d\tilde{c}_j}{dx_i} > 0$. It follows that

$$\begin{aligned} \frac{dL_i^p}{dx_i} &< 0 \\ \frac{dL_j^p}{dx_i} &< 0 \end{aligned} \tag{70}$$

Further, $\frac{\partial L_i^g}{\partial \tilde{c}_i} > -\frac{\partial L_i^g}{\partial \tilde{c}_j} > 0$, $\frac{\partial L_j^g}{\partial \tilde{c}_j} > -\frac{\partial L_j^g}{\partial \tilde{c}_i} > 0$, meaning that $1 > B_j$ and the term in the bracket of (68) is positive. Using the expression for B_j in (57) we obtain

$$\left[\frac{\partial L_j^g}{\partial \tilde{c}_i} + B_j \frac{\partial L_j^g}{\partial \tilde{c}_j}\right] = \frac{\partial L_j^g}{\partial \tilde{c}_i}\left[\frac{r+\tau}{r+\tau-\frac{\partial A_j}{\partial L_j^g}\frac{\partial L_j^g}{\partial \tilde{c}_j}}\right] < 0 \tag{71}$$

It follows that

$$\begin{aligned} \frac{dL_i^g}{dx_i} &> 0 \\ \frac{dL_j^g}{dx_i} &< 0 \end{aligned} \tag{72}$$

Since market tightness in the private sector is independent of public sector policies (see Lemma 1), from (45) we can write:

$$\frac{de_i}{dw_i^g} = \frac{m(\theta_i)}{s_i^p + \tau + m(\theta_i)}\frac{dL_i^p}{dw_i^g} < 0 \tag{73}$$

$$\frac{de_j}{dw_i^g} = \frac{m(\theta_j)}{s_j^p + \tau + m(\theta_j)}\frac{dL_j^p}{dw_i^g} < 0 \tag{74}$$

■

A.10 Proof of Proposition 5

Proof. Let $L_i^{p,ng}$ denote the size of the labor force of type i when there is no government sector, and thus all labor forced is attached to the private sector. When no public-sector exists, the cutoff education cost is

$\tilde{\epsilon}_p$, $L_h^{p,ng} = \Xi^\epsilon(\tilde{\epsilon}_p)$ and $L_l^{p,ng} = 1 - \Xi^\epsilon(\tilde{\epsilon}_p)$ so that:

$$\frac{L_h^{p,ng}}{L_l^{p,ng}} = \frac{\Xi^\epsilon(\tilde{\epsilon}_p)}{1 - \Xi^\epsilon(\tilde{\epsilon}_p)}$$

Using (21)-(23), we get:

$$\frac{L_h^p}{L_l^p} - \frac{L_h^{p,ng}}{L_l^{p,ng}} = \begin{cases} \frac{\int_{\tilde{c}_h}^{\tilde{c}_l} \Xi^\epsilon(\tilde{\epsilon}_m(c)) d\Xi^c(c)}{(1-\Xi^c(\tilde{c}_l))(1-\Xi^\epsilon(\tilde{\epsilon}_p))} > 0, & \text{if } \tilde{c}_h < \tilde{c}_l \\ -\frac{\Xi^\epsilon(\tilde{\epsilon}_p)}{1-\Xi^\epsilon(\tilde{\epsilon}_p)} \left[1 - \frac{1}{1+\frac{\int_{\tilde{c}_l}^{\tilde{c}_h} (1-\Xi^\epsilon(\tilde{\epsilon}_m(c))) d\Xi^c(c)}{(1-\Xi^c(\tilde{c}_h))(1-\Xi^\epsilon(\tilde{\epsilon}_p))}} \right] < 0, & \text{if } \tilde{c}_l = \tilde{c}_h \\ 0, & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases} \tag{75}$$

From (45) and (47) we obtain

$$\frac{e_h^p}{e_l^p} - \frac{e_h^{p,ng}}{e_l^{p,ng}} = \frac{\frac{m(\theta_h)}{s_h^p+\tau+m(\theta_h)}}{\frac{m(\theta_l)}{s_l^p+\tau+m(\theta_l)}} \left[\frac{L_h^p}{L_l^p} - \frac{L_h^{p,ng}}{L_l^{p,ng}} \right]$$

where as above the superscript "ng" is used to denote the case of no government sector. Hence, from (75), it follows that

$$\frac{e_h^p}{e_l^p} - \frac{e_h^{p,ng}}{e_l^{p,ng}} = \begin{cases} > 0, & \text{if } \tilde{c}_h < \tilde{c}_l \\ < 0, & \text{if } \tilde{c}_l = \tilde{c}_h \\ 0, & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases}$$

■

B Random search

B.1 Setup

In this appendix we give the full set of equations of the model with random search and characterize it's steady-state equilibrium. Further, we provide proofs of Propositions 8 and 9.

The values of being employed in either the private or the public sector for workers (equations 2 and 4), and the values of private-sector filled jobs and vacancies (equations 5 and 6) remain as in the Benchmark model. As discussed in the text, only the value of unemployment changes. It is now given by equation (35). The Nash bargaining wage of the private sector changes accordingly and is as given in (36).

Both government and private firms that seek to hire workers meet with them at rate $q(\theta_i) = \frac{m(\theta_i)}{\theta_i}$, where $\theta_i = \frac{v_i^p + v_i^g}{u_i}$. The number of vacancies in the private sector is determined endogenously by free entry that drives the value of a private-sector vacancy to zero, $V_i^p = 0$. The government needs to post enough vacancies to ensure that the total number of matches, $q(\theta_i)v_i^g$, equals the number of workers that it needs to hire. Hence, the government posts v_i^g vacancies to ensure $q(\theta_i)v_i^g = (s_i^g + \tau) e_i^g$.

Setting $V_i^p = 0$ and using the Nash bargaining conditions in (8), we can write the surplus of a private-sector match with a type i worker as

$$S_i^p = \frac{y_i - b_i - D_i(w_i^g - b_i)}{r + \tau + s_i^p + (1 - D_i)\beta m(\theta_i)\nu_i^p} \tag{76}$$

and the zero-profit condition that determines job creation in the private sector becomes:

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(1-\beta)(y_i - b_i - D_i(w_i^g - b_i))}{r + \tau + s_i^p + (1 - D_i)\beta m(\theta_i)\nu_i^p} \tag{77}$$

We can write the threshold level of education cost:

$$\tilde{\epsilon} = \frac{1}{r+\tau}\left[b_h - b_l + D_h(w_h^g - b_h) - D_l(w_l^g - b_l) + (1-D_h)\frac{\beta\nu_h^p\kappa_h\theta_h}{(1-\beta)} - (1-D_l)\frac{\beta\nu_l^p\kappa_l\theta_l}{(1-\beta)}\right] \tag{78}$$

where it may be recalled that $D_i = \frac{(1-\nu_i^p)m(\theta_i)}{r+\tau+s_i^g+(1-\nu_i^p)m(\theta_i)}$.

As in the benchmark model we treat public sector employment as an exogenous policy variable. There are e_i^g workers of each skill type employed in the public sector. The number of workers employed in the private sector is endogenous and depends on job creation in the private sector as well as conditions in the public sector. The labor force consists of those employed in the public sector, those employed in the private sector (e_i^p), and the unemployed (u_i). Hence, $u_i = L_i - e_i^g - e_i^p$. By equating the flows in, $m(\theta_i)\nu_i^p u_i$, to the flows out of the state where a worker is employed in the private sector, $e_i^p(s_i^p + \tau)$ we obtain:

$$e_i^p = \frac{m(\theta_i)\nu_i^p\left[L_i - e_i^g\right]}{m(\theta_i)\nu_i^p + \tau + s_i^p} \tag{79}$$

$$u_i = \frac{(\tau + s_i^p)\left[L_i - e_i^g\right]}{m(\theta_i)\nu_i^p + \tau + s_i^p} \tag{80}$$

Given $\theta_i = \frac{v_i^p + v_i^g}{u_i}$ and $q(\theta_i)v_i^g = (s_i^g + \tau)\, e_i^g$, we can use (80) to write:

$$\nu_i^p = \frac{s_i^p + \tau}{m(\theta_i)}\left[\frac{m(\theta_i)\left[L_i - e_i^g\right] - (s_i^g + \tau)e_i^g}{(s_i^p + \tau)\left[L_i - e_i^g\right] + e_i^g(s_i^g + \tau)}\right] \tag{81}$$

Using (79) and (81) we can write the total employment of type-i workers, $e_i = e_i^p + e_i^g$ as:

$$e_i = \frac{m(\theta_i)L_i + e_i^g(s_i^p - s_i^g)}{s_i^p + \tau + m(\theta_i)} \tag{82}$$

B.2 Definition of Equilibrium

A steady state equilibrium consists of a cut-off $\tilde{\epsilon}$, tightness $\{\theta_h, \theta_l\}$, and unemployed $\{u_h, u_l\}$, such that, given some exogenous government policies $\{w_h^g, w_l^g, e_h^g, e_l^g\}$, the following apply.

1. Private-sector firms satisfy the free-entry condition (77) $i = [h, l]$.
2. Private-sector wages are the outcome of Nash Bargaining (36) $i = [h, l]$.
3. Newborns decide optimally their investments in education and population shares are determined by $L_h = \Xi^\epsilon(\tilde{\epsilon})$ and $L_l = 1 - \Xi^\epsilon(\tilde{\epsilon})$.
4. Flows between private employment and unemployment are constant

$$(s_h^p + \tau)e_h^p = m(\theta_h)\nu^h u_h^p \tag{83}$$

$$(s_l^p + \tau)e_l^p = m(\theta_l)\nu^l u_l^p \tag{84}$$

5. Population add up constraints are satisfied:

$$L_h = e_h^p + e_h^g + u_h$$
$$L_l = e_l^p + e_l^g + u_l$$
$$L_h + L_l = 1$$

B.3 Proof of Existence and Uniqueness

Proof. To prove the existence and uniqueness of a steady state equilibrium under random search we show below that the two free-entry conditions in (77) cross only once in the $[\theta_h, \theta_l]$ plane, giving a unique set of equilibrium values for θ_h and θ_l. The equilibrium value of the cut-off education cost can then be determined by substituting the equilibrium values of the theta's in equation (107). Then we can determine $L_h = 1-\Xi(\tilde{\epsilon})$, $L_l = 1-\Xi(\tilde{\epsilon})$, which in turn, together with the equilibrium values of theta's, can be substituted in equations (36), (79), (80) to determine wages, employment in the private sector and unemployment.

The two job creation conditions in (77), and the cut-off education cost in (79) can be written as:

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(y_i - b_i - OO_i)}{r+\tau+s_i^p} \tag{85}$$

$$\tilde{\epsilon} = \frac{1}{r+\tau}\left[b_h - b_l + OO_h - OO_l\right] \tag{86}$$

where

$$OO_i = D_i(w_i^g - b_i) + (1-D_i)\frac{\beta}{1-\beta}\nu_i^p \kappa_i \theta_i \tag{87}$$

is the expression for the outside option of workers of skill type $i = [h, l]$ and A_i is as defined in (42).

In what follows let $j = [h, l]$, $i = [h, l]$ and $j \neq i$. Taking the derivative with respect to θ_j of (86) and of (87), after we substitute in for ν_i^p using (81), we obtain:

$$\frac{d\tilde{\epsilon}}{d\theta_j} = \frac{1}{r+\tau}\left[\frac{dOO_h}{d\theta_j} - \frac{dOO_l}{d\theta_j}\right] \tag{88}$$

$$\frac{dOO_j}{d\theta_j} = -K_j \frac{dL_j}{d\tilde{\epsilon}}\frac{d\tilde{\epsilon}}{d\theta_j} + \Sigma_j \tag{89}$$

$$\frac{dOO_i}{d\theta_j} = -Z_i \frac{dL_i}{d\tilde{\epsilon}}\frac{d\tilde{\epsilon}}{d\theta_j} \tag{90}$$

where

$$K_j = \frac{(1-D_j)m(\theta_j)\left(E_j^g - E_j^p\right)(s_j^p+\tau)(1-\nu_j^p)}{(s_j^p+\tau)\left[L_j - e_j^p\right] + e_j^g(s_j^g+\tau)} \tag{91}$$

$$\Sigma_j = (1-D_j)q(\theta_j)\left[(1-\nu_j^p)\eta\left(\frac{E_j^g m(\theta_j) + E_j^P(s_j^p+\tau)}{s_j^p+\tau+m(\theta_j)} - U_j\right) + \nu_j^p\left(E_j^p - U_j\right)\right] \tag{92}$$

$$Z_i = \frac{(1-D_i)m(\theta_i)\left(E_i^g - U_i\right)(s_i^p+\tau)(1-\nu_i^p)}{(s_i^p+\tau)\left[L_i - e_i^p\right] + e_i^g(s_i^g+\tau)} \tag{93}$$

Using these expressions we can solve for $\frac{dOO_j}{d\theta_j}$ and $\frac{dOO_i}{d\theta_j}$.

$$\frac{dOO_h}{d\theta_h} = \frac{\Sigma_h}{1+\frac{K_h \frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1-Z_l\frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}} > 0 \tag{94}$$

$$\frac{dOO_l}{d\theta_l} = \frac{\Sigma_l}{1-\frac{K_l \frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1+Z_h\frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}} > 0 \tag{95}$$

$$\frac{dOO_l}{d\theta_h} = \left[\frac{-Z_l\frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1-Z_l\frac{dL_l}{d\tilde{\epsilon}}\frac{1}{r+\tau}}\right]\frac{dOO_h}{d\theta_h} > 0 \tag{96}$$

$$\frac{dOO_h}{d\theta_l} = \left[\frac{Z_h\frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}{1+Z_h\frac{dL_h}{d\tilde{\epsilon}}\frac{1}{r+\tau}}\right]\frac{dOO_l}{d\theta_l} > 0 \tag{97}$$

Since $L_h = \Xi(\tilde{\epsilon})$ and $L_l = 1-\Xi(\tilde{\epsilon})$ then, evidently, $\frac{dL_l}{d\tilde{\epsilon}} < 0$ and $\frac{dL_h}{d\tilde{\epsilon}} > 0$. Further, as can be easily verified from (92) and (93), $\Sigma_h > 0$, $\Sigma_l > 0$, $Z_h > 0$, $Z_l > 0$, while, as can be seen from (91), sufficient condition to ensure $K_j \geq 0$, $j = [h, l]$ is $E_j^g \geq E_j^p$. Hence, if $E_j^g \geq E_j^p$, then $\frac{dOO_j}{d\theta_j} > 0$ and $\frac{dOO_j}{d\theta_i} > 0$,

$i = [h, l]$, $j = [h, l]$, $j \neq i$.

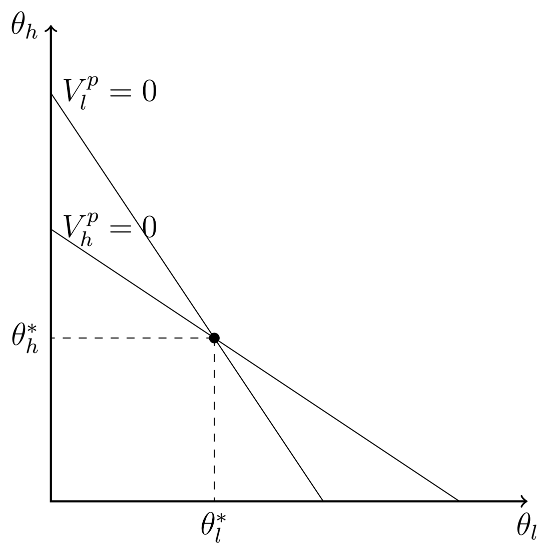
By total differentiation of (85) we can derive the slopes of the two job creation conditions in the $[\theta_h, \theta_l]$ plane:

$$\text{high-education market: } \left.\frac{d\theta_h}{d\theta_l}\right|_{V_h^p=0} = \frac{-\frac{dOO_h}{d\theta_l}}{-\frac{q'(\theta_h)\kappa_h}{(q(\theta_h))^2}(r+\tau+s_h^p)+\frac{dOO_h}{d\theta_h}} < 0 \quad (98)$$

$$\text{Low-education market: } \left.\frac{d\theta_h}{d\theta_l}\right|_{V_l^p=0} = \frac{-\frac{q'(\theta_l)\kappa_l}{(q(\theta_l))^2}(r+\tau+s_l^p)+\frac{dOO_l}{d\theta_l}}{-\frac{dOO_l}{d\theta_h}} < 0 \quad (99)$$

Both slopes are negative, since $q'(\theta_i) < 0$, $\frac{dOO_i}{d\theta_i} > 0$, $\frac{dOO_i}{d\theta_j} > 0$ but, as can be easily verified from (96) and (97), $\frac{dOO_h}{d\theta_l} < \frac{dOO_l}{d\theta_l}$ and $\frac{dOO_l}{d\theta_h} < \frac{dOO_h}{d\theta_h}$, which ensures $\left.\frac{d\theta_h}{d\theta_l}\right|_{V_h^p=0} > \left.\frac{d\theta_h}{d\theta_l}\right|_{V_l^p=0}$ and the two job creation conditions cross once once in the $[\theta_h, \theta_l]$ plane, as shown in Figure 5. This completes the proof of existence and uniqueness.

Figure 5: Steady State Equilibrium under Random Search



■

B.4 Proof of Proposition 6

Proof. It can be shown that an increase in either w_i^g or e_i^g will lower the surplus of private-sector jobs (right-hand-side of 77) of both skill types, thereby lowering job creation in both sectors. The $V_l^p = 0$ and $V_h^p = 0$ loci will shift inwards as illustrated in Figure 6. Both θ_h^* and θ_l^* will decrease.

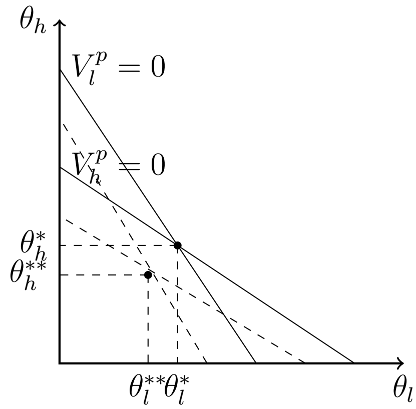
Let $x_i = [w_i^g, e_i^g]$. In what follows we show that $\frac{dOO_i}{dx_i} > 0$ and $\frac{dOO_j}{dx_i} > 0$ (for $j \neq i$), which as can be inferred from (85) imply the shifts depicted in Figure 6.

From (87) we get:

$$\frac{dOO_i}{dx_i} = -K_i \frac{dL_i}{dx_i} + \Lambda_i \quad (100)$$

$$\frac{dOO_j}{dx_i} = -K_j \frac{dL_j}{dx_i} \;\; j \neq i \quad (101)$$

Figure 6: Effects of more generous government policies under random search



where K_i and K_j, as defined in (91), are both greater than zero when $E_i^g - E_i^P > 0$ and $E_j^g - E_j^P > 0$, while

$$\Lambda_i = \begin{cases} \left[K_i + \frac{(1-D_i)(E_i^g - E_i^p)(s_i^g+\tau)(s_i^p+\tau+m(\theta_i)\nu_i^p)}{(s_i^p+\tau)\left[L_i - e_i^p\right]+e_i^g(s_i^g+\tau)}\right] > 0 \text{ if } x_i = e_i^g \\ D_i > 0 \text{ if } x_i = w_i^g \end{cases}$$

Since $L_h = \Xi^\epsilon(\tilde{\epsilon})$ and $L_l = 1 - L_h = 1 - \Xi^\epsilon(\tilde{\epsilon})$, using (86) we can write:

$$\frac{dL_l}{dx_i} = -\frac{\xi(\tilde{\epsilon})}{r+\tau}\left[\frac{dOO_h}{dx_i} - \frac{dOO_l}{dx_i}\right]$$
$$\frac{dL_h}{dx_i} = \frac{\xi(\tilde{\epsilon})}{r+\tau}\left[\frac{dOO_h}{dx_i} - \frac{dOO_l}{dx_i}\right] \quad (102)$$

By combining (100)-(101) and (102) we obtain that for $i = [h, l]$ and $j = [h, l]$:

$$\frac{dOO_j}{dx_i} = \left(\frac{K_j\xi(\tilde{\epsilon})}{r+\tau+K_j\xi(\tilde{\epsilon})}\right)\frac{dOO_i}{dx_i} \text{ if } j \neq i$$
$$\frac{dOO_i}{dx_i} = \frac{\Lambda_i(r+\tau)}{r+\tau+K_i\xi(\tilde{\epsilon})\left(1 - \frac{K_j\xi(\tilde{\epsilon})}{r+\tau+K_j\xi(\tilde{\epsilon})}\right)} \quad (103)$$

Given $\Lambda_i > 0$, $K_i > 0$ and $K_j > 0$ it can be easily verified from (103) that $\frac{dOO_i}{dx_i} > 0$, $\frac{dOO_j}{dx_i} > 0$. ■

B.5 Proof of Proposition 7

As above (see section B.4) let $x_i = [w_i^g, e_i^g]$. By substituting for $\frac{dOO_i}{dx_i}$ and $\frac{dOO_j}{dx_i}$ into (102), using (103), we get that for $i = [h, l]$, $j = [h, l]$ and $i \neq j$:

$$\frac{dL_i}{dx_i} = \left[\frac{\xi(\tilde{\epsilon})\frac{dOO_i}{dx_i}}{r+\tau+K_j\xi(\tilde{\epsilon})}\right] > 0$$
$$\frac{dL_j}{dx_i} = -\left[\frac{\xi(\tilde{\epsilon})\frac{dOO_j}{dx_j}}{r+\tau+K_i\xi(\tilde{\epsilon})}\right] < 0 \quad (104)$$

C Model extension: the choice of sector is a non-absorbing state

In this appendix we give the full set of equations of the model extension where individuals can redraw the public sector entry cost, each time they are found unemployed. This implies that the choice of either of the two sectors is not an absorbing state and workers can move, trough unemployment, from one sector to the other, depending on their draw of entry costs.

The values of employment in private and public sectors, and the values of jobs and vacancies in the private sector remain as in equations (2), (4), (5) and (6) in the text. As discussed in the text, the value of unemployment changes to take into account that all the unemployed can now draw a new entry costs and can chose to search for either a job in the public or in the private sector. It is now given by (37). The Nash bargaining wage changes accordingly and is as given in (38).

Setting $V_i^p = 0$ and using the Nash bargaining conditions in (8), we can write the surplus of a private-sector match with a type i worker as

$$S_i^p = \frac{y_i - b_i + \hat{c}_i - F_i(w_i^g - b_i + \bar{c})}{r + \tau + s_i^p + (1 - F_i)\beta m(\theta_i)(1 - \Xi(\tilde{c}_i))} \tag{105}$$

and the zero-profit condition that determines job creation in the private sector becomes:

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(1-\beta)(y_i - b_i + \hat{c}_i - F_i(w_i^g - b_i + \hat{c}_i))}{r + \tau + s_i^p + (1 - F_i)\beta m(\theta_i)(1 - \Xi(\tilde{c}_i))} \tag{106}$$

where it may be recalled that $F_i = \frac{\Xi(\tilde{c}_i)m_i^g}{r+\tau+s_i^g+\Xi(\tilde{c}_i)m_i^g}$.

The threshold level of education cost, which satisfies $\tilde{\epsilon} = U_h - U_l$, can be written as:

$$\tilde{\epsilon} = \frac{1}{r+\tau}\left[\begin{matrix} b_h - b_l + \hat{c}_l - \hat{c}_h + F_h(w_h^g - b_h) - F_l(w_l^g - b_l) + \\ (1 - F_h)\dfrac{\beta(1 - \Xi(\tilde{c}_h))\kappa_h\theta_h}{(1-\beta)} - (1 - F_l)\dfrac{\beta(1 - \Xi(\tilde{c}_l))\kappa_l\theta_l}{(1-\beta)} \end{matrix}\right] \tag{107}$$

The threshold level of public-sector entry cost $\tilde{c}_i$ at which the worker is indifferent between searching for a job in the public or in the private sector satisfies:

$$m_i^g[E_i^g - U] - \tilde{c}_i = m(\theta_i)[E_i^p - U]. \tag{108}$$

Using the free entry condition (106) and the Bellman equations for the values of unemployment and employment in the two sectors we can write:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[\frac{m_i^g[w_i^g - b_i + \hat{c}_i]}{r + \tau + s_i^g + \Xi(\tilde{c}_h)m_i^g} - \frac{\beta\kappa_i\theta_i}{(1-\beta)}\frac{r + \tau + s_i^g + m_i^g}{r + \tau + s_i^g + \Xi(\tilde{c}_h)m_i^g}\right] \tag{109}$$

The labor force consists of those employed in the public sector, those employed in the private sector (e_i^p), and the unemployed (u_i). Hence, $u_i = L_i - e_i^g - e_i^p$. By equating the flows in, $(1 - \Xi(\tilde{c}_i))m(\theta_i)u_i$, to the flows out of the state where a worker is employed in the private sector, $e_i^p(s_i^p + \tau)$, we obtain:

$$e_i^p = \frac{m(\theta_i)(1 - \Xi(\tilde{c}_i))\left[L_i - e_i^g\right]}{(1 - \Xi(\tilde{c}_i))m(\theta_i) + \tau + s_i^p} \tag{110}$$

$$u_i = \frac{(\tau + s_i^p)\left[L_i - e_i^g\right]}{(1 - \Xi(\tilde{c}_i))m(\theta_i) + \tau + s_i^p} \tag{111}$$

$\Xi(\tilde{c}_i)$ of the unemployed search for jobs in the public sector and the rest, $(1 - \Xi(\tilde{c}_i))$, search for jobs in the private sector. Hence: $u_i^g = \Xi(\tilde{c}_i)u_i$, $u_i^p = (1 - \Xi(\tilde{c}_i))u_i$.

As in the benchmark model (see equation 45), total employment in sector i, $e_i = e_i^g + e_i^p$, is given by:

$$e_i = e_i^g + \frac{m(\theta_i)L_i^p}{s_i^p + \tau + m(\theta_i)} \tag{112}$$

The private-sector labor force of type i, L_i^p is composed of workers employed or searching for jobs in the private sector: $L_i^p = e_i^p + u_i^p$. Given than $u_i^p = (1 - \Xi(\tilde{c}_i))u_i$, we get:

$$L_i^p = \frac{[L_i - e_i^g](s_i^p + \tau + m(\theta_i))(1 - \Xi(\tilde{c}_i))}{s_i^p + \tau + (1 - \Xi(\tilde{c}_i))m(\theta_i)} \tag{113}$$

Notice that here, the choice of entering the public sector cannot substitute or complement investments in education as in the benchmark model, since neither of the two sectors is an absorbing state. Workers choose education taking into account conditions in both sectors, as they might switch from one sector to the other. Hence, $L_h = \Xi^\epsilon(\tilde{\epsilon})$, $L_l = 1 - \Xi^\epsilon(\tilde{\epsilon})$. Still, the education composition is going to depend on the education premium of the public sector and the private sector through $\tilde{\epsilon}$.

We can measure the size of the queue in public-sector i as $Q_i = L_i^g - e_i^g - e_i^g(s_i^g + \tau)$. As in the benchmark model there are $L_i^g - e_i^g$ workers searching for jobs in public-sector i and only $e_i^g(s_i^g + \tau)$ vacancies. So only $e_i^g(s_i^g + \tau)$ will be hired. We can write the total employment rate as

$$e_i = \frac{m(\theta_i)L_i}{s_i^p + \tau + m(\theta_i)} - \frac{m(\theta_i)Q_i}{s_i^p + \tau + m(\theta_i)} + \frac{m(\theta_i)e_i^g\left(\frac{s_i^p + \tau}{m(\theta_i)} - s_i^g + \tau\right)}{s_i^p + \tau + m(\theta_i)} \tag{114}$$

We get exactly the same expression here as in the benchmark model (see eq. 34). The only difference is that here the educational composition of the labor force L_i does not depend on whether the public sector complements or substitutes for investments in education.

Given that the public sector is not an absorbing state in this version of the model, we can calculate transitions from one sector to the other. Workers of type i switch from employment in the private to employment in the public sector at rate $s^p\Xi(\tilde{c}_i)m_i^g$ and from employment in the public to employment in the private sector at rate $s^g(1-\Xi(\tilde{c}_i))m(\theta_i)$. Notice that the latter is likely to be small, meaning that movements from the public to the private are rare, when the public sector separation rate is small. Likewise, movements from the private to the public are rare, when entry barriers into the public sector are large ($\Xi(\tilde{c}_i)$ is small), or when the public sector job finding rate (m_i^g) is small, which implies long public sector queues.

D Endogenizing public-sector employment and wages

We can provide microeconomic foundations for the public-sector employment and wage policies that are taken as exogenous in the baseline model. Consider a government that is limited in its amount of spending to $\bar{\omega}$, exogenous, that arises from budgetary constraints. The government has an objective function with two components. The first, is the production of government services, g that use a Cobb-Douglas production function in skilled and unskilled public employment, with weight γ on the skilled workers, $g = (e_h^g)^\gamma(e_l^g)^{1-\gamma}$. The second, is the preferences of a union represented by $\varsigma\mu(a_h) + (1-\varsigma)\mu(a_l)$. Here ς represents the weight of skilled workers in the union's preferences and a_h and a_l are the extra payment to public-sector workers on top of the minimum required wage for the existence of the public sector ($w_h^g = \underline{w}_h^g + a_h$ and $w_l^g = \underline{w}_l^g + a_l$). The union knows what this minimum required wage is and tries to push the wages above. $\mu(a_i)$ is a function expressing the utility of the extra payment to type i workers, which for convenience we assume it is $\log(a_i)$.

The government's problem can be written as:

$$\max_{e_h^g, e_l^g, a_h, a_l} (e_h^g)^\gamma(e_l^g)^{1-\gamma} + \varphi(\varsigma\log(a_h) + (1-\varsigma)\log(a_l))$$

$$s.t.$$

$$(\underline{w}_h^g + a_h)e_h^g + (\underline{w}_l^g + a_l)e_l^g = \bar{\omega}.$$

where φ is the weight of the unions in the government's maximization problem. The first-order conditions determining employment and wages of skilled and unskilled workers are given by:

$$e_g^g = \bar{\omega}\left(\frac{\gamma}{w_h^g}\right), \tag{115}$$

$$e_l^g = \bar{\omega}\left(\frac{1-\gamma}{w_l^g}\right), \tag{116}$$

$$\frac{\varphi\varsigma}{a_h} = \frac{g e_h^g}{\bar{\omega}} \tag{117}$$

$$\frac{\varphi(1-\varsigma)}{a_l} = \frac{g e_l^g}{\bar{\omega}} \tag{118}$$

The first two conditions pin down the employment level of the government. Given technology and a certain wage, the government spends a constant fraction γ of its budget on skilled workers and $1-\gamma$ on unskilled workers. In this setting, the reason why the government hires more skilled workers is due to technology – skilled workers are more important inputs in the production functions. We can rearrange the last two conditions to pin down government wages:

$$w_h^g = \frac{g\gamma}{g\gamma - \varphi\varsigma}\underline{w}_h^g \tag{119}$$

$$w_l^g = \frac{g(1-\gamma)}{g(1-\gamma)-\varphi(1-\varsigma)}\underline{w}_l^g \tag{120}$$

If the weight of the unions in the government objective function tends to zero, the government would set the wages equal to the minimum required for the government to hire its workers. However, if this weight is higher, the government raises public wages. Whether it raises more the skilled or unskilled wages, depends on the relative weight on the union preference. Note, however, that government wages and employment are independent of productivity; they only reflect budgetary constraints, union preferences and technology.

Summarizing, we can endogenize the four policy parameters, that now depend on four exogenous parameters reflecting technology (γ), budgetary pressures ($\bar{\omega}$), union power (φ) and union relative preferences (ς). This is one model of government behaviour, but there could certainly be others. We think however, when studying the effects of public-sector employment and wages, it is a clearer exercise to take them as exogenous.

E Evidence on direct flows between sectors

Table E.1 shows the fraction of entries and exits from public employment, from and to private employment. In France and Spain, that we think are more representative of segmented markets, only 15 and 10 percent of the flows in and out of public employment are directly from and to the private sector. In the US and UK, there are between 20 to 30 per cent. These flows include everyone that had a spell of unemployment and inactivity within the period, so the true direct transitions should be lower. They are not negligible, but they are clearly the minority. These transitions are also small relative to the overall job-to-job transitions. For instance, in the UK, the direct flows from public to private employment represent only 4.7 per cent of all job-to-job inflows in the private sector, much less than the 23 per cent share of public employment in total employment. Also, 9.4 per cent of all job-to-job transitions are direct transitions between the two sectors. If search was completely random, we would expect them to represent 35.3 per cent ($2 \times 0.23 \times 0.77$).

For the US, we have calculated the job-finding rate in the public and private sectors conditional on the labour market status on the previous month, shown in E.2. These rates support the conclusion that the choice of sector is persistent, even after an unemployment spell. The unconditional job-finding rate in the public sector is only 1.8 per cent, but conditional on being in the public sector in the month preceding unemployment it is close to 30 per cent. Curiously, the job-finding rate conditional on being previously employed in the private is 1.4 per cent, roughly equal to the rates conditional on previously being unemployed or inactive.

Table E.1: Size of entries and exits in public sector

	US[§]	**UK**	**France**	**Spain**
Entries into public employment				
As a fraction of public employment	0.029	0.029	0.028	0.054
Share from				
Private emp.	0.223	0.325	0.154	0.116
Exits out of public employment				
As a fraction of public employment	0.030	0.029	0.029	0.050
Share to:				
Private emp.	0.189	0.253	0.143	0.098

Note: Data are extracted from the FLFS, UKLFS, SLFS and the CPS. Sample: US (1996-2018), UK (1996-2018), France (2003-2017), Spain (2005-2018).

For the private sector, again we see the attachment of workers with a conditional job-finding rate of more than 40 per cent. Being previously employed in the public sector does not raise the job-finding rate in the private sector relative to the ones that were unemployed or inactive (with job-finding rates of around 16 per cent).

Table E.2: Conditional job-finding rates, US (Average 1996-2018)

	U-E rate		*I-E rate*	
	Public	Private	Public	Private
Unconditional	1.83	20.38	0.85	5.90
Conditional				
G	29.48	16.85	26.35	11.19
P	1.39	40.48	1.53	31.91
U	1.48	17.14	1.12	10.63
I	1.53	15.79	0.38	2.89

Note: The table reports the unconditional transition rates from unemployment (inactivity) to employment in a given sector, conditional on the state prior to unemployment (inactivity). Source: CPS.

F Parametrization of the three models for quantitative exercise

Table F.1: Parametrization of segmented markets model

Fixed parameters	**United States**	**United Kingdom**	**France**	**Spain**
Public-sector employment				
Skilled	$e_h^g = 0.066$	$e_h^g = 0.170$	$e_h^g = 0.095$	$e_h^g = 0.105$
Unskilled	$e_l^g = 0.079$	$e_l^g = 0.110$	$e_l^g = 0.137$	$e_l^g = 0.079$
Job-separation rates (private)				
Skilled	$s_h^p = 0.007$	$s_h^p = 0.012$	$s_h^p = 0.016$	$s_h^p = 0.031$
Unskilled	$s_l^p = 0.028$	$s_l^p = 0.014$	$s_l^p = 0.023$	$s_l^p = 0.051$
Job-separation rates (public)				
Skilled	$s_h^g = 0.005$	$s_h^g = 0.005$	$s_h^g = 0.005$	$s_h^g = 0.014$
Unskilled	$s_l^g = 0.016$	$s_l^g = 0.005$	$s_l^g = 0.010$	$s_l^g = 0.036$
Matching elasticities		$\eta_h = \eta_l = 0.5$		
Matching efficiencies		$\zeta_h = \zeta_l = 1$		
Bargaining power of workers		$\beta = 0.5$		
Discount rate	$r = 0.004$		$r = 0.0120$	
Retirement rate	$\tau = 0.002$		$\tau = 0.0056$	
Other parameters				
Cost of posting vacancies				
Skilled	$\kappa_h = 17.250$	$\kappa_h = 3.901$	$\kappa_h = 7.146$	$\kappa_h = 11.821$
Unskilled	$\kappa_l = 3.781$	$\kappa_l = 5.117$	$\kappa_l = 5.599$	$\kappa_l = 7.612$
Unemployment benefits				
Skilled	$b_h = 0.475$	$b_h = 0.330$	$b_h = 0.659$	$b_h = 0.430$
Unskilled	$b_l = 0.405$	$b_l = 0.397$	$b_l = 0.562$	$b_l = 0.429$
Productivity skilled	$y_h = 1.697$	$y_h = 1.387$	$y_h = 1.475$	$y_h = 1.442$
Public sector entrance cost distribution				
Mean	$\mu^c = 2.911$	$\mu^c = 1.537$	$\mu^c = 0.223$	$\mu^c = 1.774$
Variance	$\sigma^c = 0.466$	$\sigma^c = 0.054$	$\sigma^c = 0.001$	$\sigma^c = 0.035$
Education cost distribution				
Mean	$\mu^\epsilon = 5.133$	$\mu^\epsilon = 3.204$	$\mu^\epsilon = 4.275$	$\mu^\epsilon = 4.242$
Variance	$\sigma^\epsilon = 0.622$	$\sigma^\epsilon = 1.656$	$\sigma^\epsilon = 2.312$	$\sigma^\epsilon = 2.830$
Public-private wages ratio				
Skilled	$\frac{w_h^g}{w_h^p} = 1.038$	$\frac{w_h^g}{w_h^p} = 1.067$	$\frac{w_h^g}{w_h^p} = 1.001$	$\frac{w_h^g}{w_h^p} = 1.053$
Unskilled	$\frac{w_l^g}{w_l^p} = 1.050$	$\frac{w_l^g}{w_l^p} = 1.092$	$\frac{w_l^g}{w_l^p} = 1.038$	$\frac{w_l^g}{w_l^p} = 1.176$

Table F.2: Steady-state values of variables in segmented markets model

Targets (data value)				
Unemployment rate				
Skilled	0.032 (0.032)	0.033 (0.033)	0.057 (0.057)	0.112 (0.110)
Unskilled	0.073 (0.073)	0.057 (0.057)	0.106 (0.103)	0.213 (0.208)
Replacement rates				
Skilled	0.290 (0.290)	0.248 (0.248)	0.470 (0.474)	0.331 (0.331)
Unskilled	0.425 (0.425)	0.418 (0.418)	0.595 (0.598)	0.483 (0.498)
Unemp. duration - private over public				
Skilled	1.248 (1.248)	0.744 (0.744)	0.985 (0.948)	0.994 (0.988)
Unskilled	1.009 (1.009)	0.735 (0.735)	0.585 (0.767)	0.683 (0.794)
Public-private wage ratio				
Skilled	1.038 (1.027)	1.067 (1.059)	1.001 (0.985)	1.053 (1.060)
Unskilled	1.050 (1.064)	1.092 (1.096)	1.038 (1.045)	1.176 (1.179)
Skilled-unskilled wage premium	1.720 (1.720)	1.401 (1.401)	1.483 (1.474)	1.463 (1.434)
Share of college graduates	0.266 (0.266)	0.460 (0.460)	0.320 (0.320)	0.337 (0.340)
Tertiary education costs % of consumption	0.047 (0.474)	0.031 (0.031)	0.022 (0.026)	0.018 (0.019)
Other SS variables				
Value of employment				
Private - Skilled	272.3	72.57	75.87	67.10
Public - Skilled	154.9	51.46	50.75	44.72
Private - Unskilled	285.0	78.03	77.23	73.35
Public - Unskilled	165.5	56.85	53.12	51.86
Total employment				
Skilled	0.258	0.445	0.301	0.299
Unskilled	0.680	0.509	0.608	0.522
Private wages				
Skilled	1.639	1.329	1.401	1.299
Unskilled	0.953	0.949	0.945	0.888
Public wages				
Skilled	1.701	1.419	1.402	1.367
Unskilled	1.001	1.036	0.981	1.044
Treshholds (ϵ_x)				
Private	114.4	20.79	24.29	21.08
Public	117.3	20.91	24.29	21.20
Treshholds (c_i)				
Skilled	13.315	4.573	1.249	5.806
Unskilled	10.491	4.453	1.249	5.684

Table F.3: Parametrization of random search model and steady-state values

	United States	United Kingdom	France	Spain
Other parameters				
Cost of posting vacancies				
Skilled	$\kappa_h = 12.486$	$\kappa_h = 2.228$	$\kappa_h = 6.990$	$\kappa_h = 8.819$
Unskilled	$\kappa_l = 3.177$	$\kappa_l = 3.989$	$\kappa_l = 5.145$	$\kappa_l = 6.451$
Unemployment benefits				
Skilled	$b_h = 0.479$	$b_h = 0.334$	$b_h = 0.665$	$b_h = 0.431$
Unskilled	$b_l = 0.408$	$b_l = 0.402$	$b_l = 0.569$	$b_l = 0.452$
Productivity skilled	$y_h = 1.696$	$y_h = 1.378$	$y_h = 1.476$	$y_h = 1.410$
Education cost distribution				
Mean	$\mu^\epsilon = 5.152$	$\mu^\epsilon = 3.260$	$\mu^\epsilon = 3.939$	$\mu^\epsilon = 4.176$
Variance	$\sigma^\epsilon = 0.631$	$\sigma^\epsilon = 1.738$	$\sigma^\epsilon = 1.655$	$\sigma^\epsilon = 2.741$
Public-private wages ratio				
Skilled	$\frac{w_h^g}{w_h^p} = 1.027$	$\frac{w_h^g}{w_h^p} = 1.069$	$\frac{w_h^g}{w_h^p} = 0.986$	$\frac{w_h^g}{w_h^p} = 1.060$
Unskilled	$\frac{w_l^g}{w_l^p} = 1.064$	$\frac{w_l^g}{w_l^p} = 1.090$	$\frac{w_l^g}{w_l^p} = 1.049$	$\frac{w_l^g}{w_l^p} = 1.181$
Targets (data value)				
Unemployment rate				
Skilled	0.032 (0.032)	0.033 (0.033)	0.057 (0.057)	0.110 (0.110)
Unskilled	0.073 (0.073)	0.057 (0.057)	0.103 (0.103)	0.208 (0.208)
Replacement rates				
Skilled	0.290 (0.290)	0.248 (0.248)	0.474 (0.474)	0.331 (0.331)
Unskilled	0.425 (0.425)	0.418 (0.418)	0.598 (0.598)	0.498 (0.498)
Public-private wage ratio				
Skilled	1.027 (1.027)	1.069 (1.059)	0.986 (0.985)	1.060 (1.060)
Unskilled	1.064 (1.064)	1.090 (1.096)	1.049 (1.045)	1.181 (1.179)
Skilled-unskilled wage premium	1.720 (1.720)	1.401 (1.401)	1.474 (1.474)	1.435 (1.434)
Share of college graduates	0.266 (0.266)	0.460 (0.460)	0.320 (0.320)	0.340 (0.340)
Tertiary education costs % of consumption	0.047 (0.474)	0.031 (0.031)	0.026 (0.026)	0.018 (0.019)
Other SS variables				
Value of employment				
Private - Skilled	275.9	74.21	76.07	68.47
Public - Skilled	157.2	52.51	51.40	46.60
Private - Unskilled	280.6	78.53	76.22	72.09
Public - Unskilled	160.7	56.73	53.62	50.01
Total employment				
Skilled	0.258	0.445	0.302	0.302
Unskilled	0.680	0.509	0.610	0.523
Private wages				
Skilled	1.652	1.348	1.404	1.301
Unskilled	0.960	0.962	0.952	0.907
Public wages				
Skilled	1.697	1.440	1.383	1.379
Unskilled	1.022	1.048	0.999	1.071
Treshholds (ϵ_x)	116.5	21.89	23.70	21.00

Table F.4: Parametrization of non-absorbing state model and steady-state values

	United States	United Kingdom	France	Spain
Other parameters				
Cost of posting vacancies				
Skilled	$\kappa_h = 24.489$	$\kappa_h = 9.025$	$\kappa_h = 10.601$	$\kappa_h = 16.823$
Unskilled	$\kappa_l = 4.302$	$\kappa_l = 7.438$	$\kappa_l = 8.544$	$\kappa_l = 9.792$
Unemployment benefits				
Skilled	$b_h = 0.475$	$b_h = 0.326$	$b_h = 0.653$	$b_h = 0.419$
Unskilled	$b_l = 0.404$	$b_l = 0.393$	$b_l = 0.559$	$b_l = 0.440$
Productivity skilled	$y_h = 1.706$	$y_h = 1.401$	$y_h = 1.465$	$y_h = 1.419$
Public sector entrance cost distribution				
Mean	$\mu^c = 0.723$	$\mu^c = -0.286$	$\mu^c = -1.399$	$\mu^c = 2.539$
Variance	$\sigma^c = 0.327$	$\sigma^c = 0.014$	$\sigma^c = 1.018$	$\sigma^c = 3.943$
Education cost distribution				
Mean	$\mu^\epsilon = 5.059$	$\mu^\epsilon = 3.115$	$\mu^\epsilon = 3.884$	$\mu^\epsilon = 3.906$
Variance	$\sigma^\epsilon = 0.523$	$\sigma^\epsilon = 1.422$	$\sigma^\epsilon = 1.566$	$\sigma^\epsilon = 2.286$
Public-private wages ratio				
Skilled	$\frac{w_h^g}{w_h^p} = 1.026$	$\frac{w_h^g}{w_h^p} = 1.058$	$\frac{w_h^g}{w_h^p} = 0.987$	$\frac{w_h^g}{w_h^p} = 1.060$
Unskilled	$\frac{w_l^g}{w_l^p} = 1.067$	$\frac{w_l^g}{w_l^p} = 1.095$	$\frac{w_l^g}{w_l^p} = 1.043$	$\frac{w_l^g}{w_l^p} = 1.173$
Targets (data value)				
Unemployment rate				
Skilled	0.032 (0.032)	0.033 (0.033)	0.057 (0.057)	0.110 (0.110)
Unskilled	0.073 (0.073)	0.057 (0.057)	0.103 (0.103)	0.208 (0.208)
Replacement rates				
Skilled	0.290 (0.290)	0.248 (0.248)	0.474 (0.474)	0.331 (0.331)
Unskilled	0.425 (0.425)	0.418 (0.418)	0.598 (0.598)	0.498 (0.498)
Unemp. duration - private over public				
Skilled	1.248 (1.248)	0.744 (0.744)	0.985 (0.948)	0.994 (0.988)
Unskilled	1.009 (1.009)	0.735 (0.735)	0.585 (0.767)	0.683 (0.794)
Public-private wage ratio				
Skilled	1.026 (1.027)	1.058 (1.059)	0.999 (0.985)	1.060 (1.060)
Unskilled	1.067 (1.064)	1.095 (1.096)	1.041 (1.045)	1.173 (1.179)
Skilled-unskilled wage premium	1.724 (1.720)	1.401 (1.401)	1.475 (1.474)	1.434 (1.434)
Share of college graduates	0.266 (0.266)	0.460 (0.460)	0.320 (0.320)	0.340 (0.340)
Tertiary education costs % of consumption	0.047 (0.474)	0.031 (0.031)	0.026 (0.026)	0.018 (0.019)
Other SS variables				
Value of employment				
Private - Skilled	271.4	71.20	74.23	65.07
Public - Skilled	276.2	75.35	74.69	69.10
Private - Unskilled	154.2	50.70	49.86	44.30
Public - Unskilled	157.9	55.34	52.03	47.60
Total employment				
Skilled	0.258	0.445	0.302	0.303
Unskilled	0.680	0.509	0.610	0.523
Private wages				
Skilled	1.639	1.316	1.377	1.267
Unskilled	0.951	0.939	0.934	0.883
Public wages				
Skilled	1.681	1.391	1.360	1.343
Unskilled	1.014	1.029	0.974	1.037
Treshholds (ϵ_x)	113.6	19.54	23.39	19.36
Treshholds (c_i)				
Skilled	1.514	0.747	0.102	0.733
Unskilled	1.276	0.742	0.113	0.297