Meritocracy, public-sector pay and human capital accumulation*

Andri Chassamboulli — University of Cyprus[†]

Pedro Gomes — Universidad Carlos III de Madrid[‡], Birkbeck, University of London[§], Center For Macroeconomics

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Abstract

We set up a model with search and matching frictions to understand the effects of employment and wage policies, as well as non-meritocratic hiring in the public sector, on unemployment, rent seeking and education decisions. Wages and employment of skilled and unskilled public-sector workers affect educational attainment; the extent of that effect depends on the structure of the labor market and how non-meritocratic public-sector hiring is. Conditional on inefficiently high public-sector wages, less-meritocratic hiring in the public sector lowers the unemployment rate and might raise welfare because it limits the size of queues for public-sector jobs. Public-sector wage and employment policies impose an endogenous constraint on the number of workers the government can hire through connections.

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[†]Department of Economics, University of Cyprus, CY-1678 Nicosia, Cyprus. Email: andricha@ucy.ac.cy.

[‡]Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe, Spain. E-mail: pgomes@eco.uc3m.es.

[§]Department of Economics, Mathematics and Statistics, Birkbeck, University of London, Malet Street, WC1E 7HX London, United Kingdom, E-mail: p.gomes@bbk.ac.uk.

1 Introduction

Governments hire workers to produce public goods, but they do not face the same competitive forces as private firms. As a result, governments use their employment and wage policies to accomplish a multitude of goals: to attain budgetary targets [Gyourko and Tracy (1989)]; to implement a macroeconomic stabilization policy [Keynes (1936)]; to redistribute resources [Alesina, Baqir, and Easterly (2000)]; or to satisfy interest groups for electoral gains [Gelb, Knight, and Sabot (1991)]. This paper builds on the observation that, in several countries, government hiring practices are not always meritocratic.

Moreover, the government is the economy's main employer of skilled workers. On average, a European public sector hires close to 40 percent of the country's college graduates. However, in the public sector, the returns of education are typically lower. Using micro level data, several papers find that, on average, the public sector pays higher wages than the private sector but that the premium is higher for workers without a college degree.[1]

Our objective is to study how public-sector employment and wage policies, heterogeneous across skill groups, influence the incentives to accumulate human capital and affect labor market outcomes of different workers. We show that the effects of government policies on educational attainment crucially depend on the presence of more or less meritocracy in public-sector hiring. Furthermore, we find a silver lining to non-meritocracy. Conditional on inefficiently high public-sector wages, less-meritocratic public-sector hiring lowers the unemployment rate and might raise welfare.

We define non-meritocratic hiring as the restriction that some jobs in the public sector are reserved for a subset of workers that have political or personal connections. One dimension that is common to all countries is political appointments. Whenever there is a change in government, there is a subsequent turnover of jobs. The report Government at a Glance by OECD (2017) highlights the cross-country differences in staff turnover following a change of government. In countries such as Germany and the UK, there is little turnover, and the changes are mainly in advisory posts. In countries such as Greece and Spain, the turnover extends to layers of senior and middle management. A second dimension is the influence that politicians or civil servants use to hire friends or family members. Given the amount of anecdotal evidence of such practices, it is perhaps surprising the limited research documenting evidence of nepotism or cronyism in the public sector.[2] Scoppa (2009) finds that

[1]Examples include: Katz and Krueger (1991) for the United States; Postel-Vinay and Turon (2007) or Disney and Gosling (1998) for the United Kingdom; and Christofides and Michael (2013), Castro, Salto, and Steiner (2013) and Giordano et al. (2011) for several European countries. The notable exceptions to this fact are Nordic countries, such as Finland.

[2]The anecdotal evidence is particularly widespread in Southern European or developing countries, but not exclusively. Recently, the United States president hired his daughter and son-in-law, and a leading

the probability of working in the public sector in Italy is 44 percent higher for individuals whose parent also works in the public sector. Martins (2010) finds that in Portugal, between 1980 and 2008, over the months preceding an election, appointments in state-owned firms increased significantly compared to private-sector firms. Hiring also increased after elections, but only if a new government took office. Fafchamps and Labonne (2014) find that, following the 2007 and 2010 municipal elections in Philippines, individuals who shared one or more family names with a local elected official were more likely to be employed in better-paying occupations, compared to individuals with the loosing candidates' family names. The magnitude of the effect is consistent with preferential treatment of relatives as managers in the public sector. Although these papers provide suggestive evidence of nepotism and cronyism in the public sector, they do not give an unequivocal answer. Given the nature of this activity, it is difficult to collect reliable data and design an empirical strategy that identifies non-meritocratic hiring in the public sector, let alone to measure its effects.

Given the limitations of doing empirical work, we study the consequences of non-meritocratic hiring in the public sector from a theoretical angle. We set up a search model in which workers can search for jobs in either the private or the public sector. Employment and wages in the private sector are determined through the usual channels of free entry and Nash bargaining. This ensures that more-productive (educated/skilled) workers have a better chance of finding a job and a better bargaining position in wage setting and that they receive higher wages. In the public sector, by contrast, employment and wages are exogenous. We account for the possibility of nepotism or cronyism in hiring workers in the public sector by assuming that job seekers can use their personal relationships and connections to find a public-sector job. We assume that, prior to entering the labor market, workers can pay a cost to get "connections" that is drawn from an exogenous distribution across workers. In our setting, non-meritocracy means that the government reserves some of its jobs for workers with those connections. Under such practices, in equilibrium, workers with connections can more easily find public-sector jobs.

In this setup, we incorporate a human capital accumulation decision. Prior to entering the labor market, individuals also decide whether to invest in education. Doing so is costly but yields returns, as highly-educated workers benefit from higher job-finding rates and wages in both sectors. Workers are heterogeneous with respect to their education costs, reflecting either different learning abilities or financial constraints. Thus, only a fraction of individuals – those whose benefits exceed the costs – invest in education. An interesting feature of our model is that it endogenously generates substitutability or complementarity between

French presidential candidate was found to have put his wife, son and daughter employed on the public payroll.

investments in education and connections, and that these depend on the government's pay and hiring policies of skilled and unskilled workers. The possibility of finding jobs in the public sector through connections may induce individual to either substitute education for connections or invest more in education depending on how the returns in education differ between the public and private sector.

This paper contributes to the recent labor market search literature that analyzes the role and effects of public-sector employment and wages. Burdett (2012) includes the public sector in a job-ladder framework where firms post wages. Bradley, Postel-Vinay, and Turon (2017) further introduce on-the-job search and transitions between the two sectors to study the effects of public-sector policies on the distribution of private-sector wages. Albrecht, Robayo-Abril, and Vroman (2017) consider heterogeneous human capital and match specific productivity in a Diamond-Mortensen-Pissarides model. Michaillat (2014) shows that the crowding-out effect of public-sector employment is lower during recessions, giving rise to higher government spending multipliers. These papers' objective is to determine how public-sector employment and wage policies affect private employment, the unemployment rate and private wages. They assume that the unemployed randomly search across sectors, and, hence, public-sector policies affect the equilibrium only by affecting the outside option of the unemployed and their reservation wage.

Hörner, Ngai, and Olivetti (2007) study the effect of turbulence on unemployment when wages in the public sector are insulated from this volatility. Quadrini and Trigari (2007) analyze the effects of exogenous business cycle rules on unemployment volatility. Gomes (2015) emphasizes the role of public-sector wage policy in achieving the efficient allocation, while Afonso and Gomes (2014) highlight the interactions between private and public wages. These papers assume that the two sectors's labor markets are segmented, and that the unemployed choose which of the sectors to search in, depending on the government's hiring, separation and wage policies.

We add to this literature by also considering the choice of finding a public-sector job through connections and by analyzing how government policies affect this rent-seeking activity. Moreover, while, in our benchmark model, we assume segmented markets, we also contrast the transmission mechanism and our results with those from a model with random search across sectors.

With the exception of Albrecht, Robayo-Abril, and Vroman (2017), none of the cited papers explicitly consider heterogeneity in terms of education. This is quite an oversight, given that the government hires predominantly workers with college degrees and that the public-sector wage premium varies substantially with education. Other papers that consider heterogeneity include Gomes (2017), who examines the effects of a public-sector wage re-form that eliminates the wage premium for all types of public-sector workers. Domeij and Ljungqvist (2016) study, in a frictionless labor market, how the public employment hiring of skilled and unskilled workers in Sweden and the US can explain the different evolutions of the skill premium in the two countries. In a model of occupational choice, Gomes and Kuehn (2017) analyze the effects of skill-biased hiring in the public sector on the occupational choice of entrepreneurs and on firm size. All of these papers take the education endowment as exogenous. We contribute to the literature by endogenizing the choice of education and showing how government policies affect it.[3]

Our first main result shows that policies that raise the value of working for skilled workers – such as increasing wages or hiring skilled workers – tend to increase educational attainment. However, the size of the effect depends on the structure of the labor market - segmented markets or random search - and the extent of non-meritocratic hiring. Only in the unique scenario with segmented markets and no government hiring due to "connections" do government employment, wages and separation rates have no effect on the educational composition of the labor force. This is the case because any improvement in the value of working in the public sector is fully neutralized by an increase in the unemployed queueing for public-sector jobs and the consequent reduction in the job-finding rate. However, if a fraction of jobs are reserved for connected workers, this limited mobility reduces the flow from the private to the public sector, and, hence, government policies affect the incentives for education. In particular, any government policy that raises the value of working in the public sector for unskilled workers, relative to educated workers, reduces the proportion of educated workers in the labor force. This shows how the interaction between non-meritocratic hiring and government policies is crucial to the education decision.

Our second main finding is perhaps surprising. Conditional on inefficiently high public-sector wages, less meritocratic hiring in the public sector lowers the unemployment rate and might raise welfare. When the value of a public-sector job is higher than that of a private-sector job (because of either high wages or a low separation rate), more of the unemployed queue for these jobs, moving away from the private sector. If most of these jobs are available only through connections, the unconnected unemployed are not going to queue for public-sector jobs and will search for private-sector jobs instead. Although it fosters an inefficient rent-seeking activity, the recruitment through connections mitigates the negative effects of high public-sector wages.

Although the mechanism is different, this result is similar to that found in papers studying

[3]This feature is present in only one other paper. Wilson (1982), in a neoclassical model, argues that the government might use its employment policy to change equilibrium prices and redistribute income among different groups.

referrals – e.g., Horvath (2014) Galenianos (2014) or Bello and Morchio (2017) – which have focused exclusively on the private sector. These papers find that social networks can improve the matching process by working as an information channel. We do not take a stand on whether hiring through connections is more important in the public or the private sector, but we argue that they differ. In the private sector, free entry of firms ensures that the gains of alternative hiring channels translate into job creation, and wage bargaining guarantees that the surplus generated is shared between firms and workers. On the contrary, we view the public sector as having a fixed number of jobs that are safer and better paid, which induces workers to find alternative ways to get in. The mechanism does not involve better information about vacancies, but the knowledge that some vacancies are reserved for a subset of workers.

Focusing on the public rather than the private sector allows us to understand how policies affect non-meritocratic hiring. In our setting, the government can hire through connections, provided that it pays high enough wages to attract enough searchers. In other words, government employment and wage policies impose an endogenous limit on how many workers it can hire through connections. The first-best allocation can be achieved with an optimal public-sector wage that simultaneously limits the queues for public-sector jobs and makes it impossible to hire through connections. This third main result can rationalize why anecdotical evidence of non-meritocratic hiring is common in Southern European countries, in which public sectors pay substantial premia relative to the private sector, while it is absent in Nordic countries, which tend to pay a negative public-sector wage premium.

The paper is structured as follows. Section 2 presents the model economy with search and matching frictions. Section 3 describes the main results of the paper. Section 4 examines the constrained-efficient allocation and how the social planner can achieve the first best. Section 5 analyzes the robustness of the results to four alternative settings, with particular emphasis on the assumption of random search between the private and public sectors. In Section 6, we parameterize the model to the Spanish economy and perform some numerical exercises. Section 7 concludes.

2 General setup

We consider a search and matching model with private-sector firms and a public sector. Workers can be either employed and producing or unemployed and searching for a job. Each private-sector firm is endowed with a single vacancy that can be vacant or filled (job). At each instant, τ individuals are born (enter the labor market) and die (retire) so that the working population is constant and normalized to unity. All agents are risk-neutral and discount the future at a common rate $r > 0$, and time is continuous.

An agent can be either low- or high-educated. All individuals are born low-educated, but prior to entering the labor market, they can become high-educated by paying a schooling cost ϵ. The schooling cost is distributed across individuals according to the cumulative distribution function $\Xi^{\epsilon}(\cdot)$ on $[0, \bar{\epsilon}]$. Heterogeneity with respect to schooling cost reflects either different learning abilities or the existence of financial constraints.

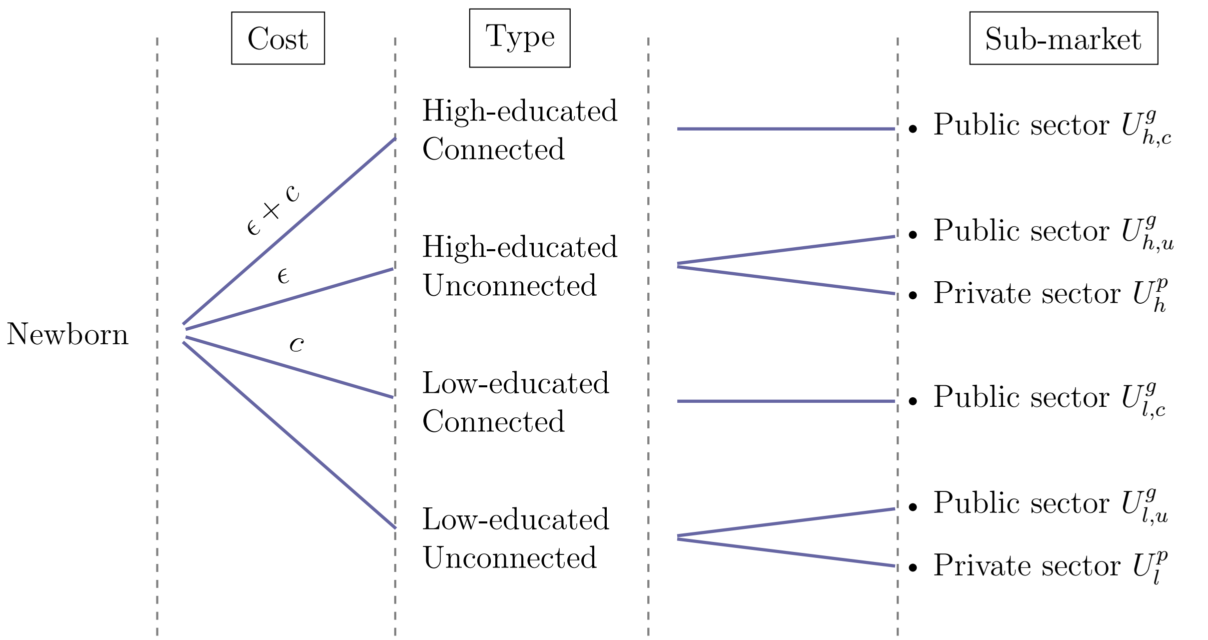
In parallel, all workers can become "connected", by paying a cost c. The cost is distributed across individuals according to the cumulative distribution function $\Xi^{c}(\cdot)$ on $[0, \bar{c}]$.[4] If a family member works in the public sector, the cost of connections is low. If getting connections requires the affiliation with a political party, it is more costly. Connected workers are as productive as workers with no connections, but might have priority – a higher job-finding rate – for public-sector jobs.

An endogenous proportion of the population (those whose schooling cost is sufficiently low) become high-educated; another fraction (those whose connection cost is sufficiently low) become connected. If both costs are low, workers become connected and high-educated, while the rest remain low-educated and "unconnected." Variables are, therefore, indexed by the superscript $x = [g, p]$, where g refers to the public (government) sector and p to the private sector, and two subscripts $i = [l, h]$ and $j = [c, u]$, where c refers to "connected," u to "unconnected", h to high- and l to low-educated. Figure 1 depicts these four choices. If the newborn remains unconnected, then she has a further decision of whether to search for private-sector jobs or for public-sector unconnected jobs.[5] The two markets are segmented. In Section 5.1, we consider the case in which the search between private- and unconnected public-sector jobs is random. If the newborn acquires connections, she will search only for connected public-sector jobs that strictly dominate the other sectors. In each sector, there are two labor markets segmented by education. In the "high-education" market, both firms and government open vacancies suited for high-educated workers, whereas in the "low-education" market, vacancies are suited to low-educated workers; high-educated individuals direct their search towards type-h jobs, whereas low-educated workers direct their search towards type-l jobs. In total, there are six active submarkets. A searching (unemployed) worker of type-i receives a flow of income b_i, which can be considered the opportunity cost of employment.

[4]We assume that the distributions of education and connections are independent. As we will show, the model endogenously generates complementarity or substitutability between education and connections. Assuming exogenously a relation between the two distributions would simply tilt the equilibrium towards one or the other.

[5]Throughout the paper, we use the terms "connected jobs/vacancies" to refer to the jobs that the government reserves for job seekers with connections. We use the term "unconnected jobs/vacancies" to refer to the vacancies that the government seek to fill through standard search in the market.

Figure 1: Decision of newborn



2.1 The Private sector

The private and public sectors differ in two aspects: hiring practices and wage-setting. The rate at which high- and low-educated workers are hired into private-sector jobs is endogenous and depends on firm profits and job entry. In particular, firms in each of the two labor markets of the private sector open vacancies and search for suitable workers until all rents are exhausted. The rate at which type-i workers find private-sector jobs of type i depends positively on the tightness, $\theta_i = \frac{v_i^p}{u_i^p}$, where v_i^p is the measure of private-sector vacancies of type i, and u_i^p is the number of type i workers that are unemployed and searching in the private sector. Workers of type i are hired into private-sector jobs (of type i) at Poisson rate $m(\theta_i)$, and private-sector firms fill type i vacancies at rate $q(\theta_i) = \frac{m(\theta_i)}{\theta_i}$.

The output y_i of a match between a worker and a firm depends on the worker's education: high-educated individuals (jobs) are more productive than low-educated individuals (jobs) ($y_h > y_l$), which is independent of the "connections" status. Wages in the private sector, denoted as w_i^p, depend on match surplus, so they also differ by education level. They are determined by Nash bargaining, such that the worker gets a share β of match surplus while the rest goes to the firm. With higher match surplus, firms expect to generate larger profits from creating jobs; firm entry is higher; and workers can more easily find jobs and also earn higher wages. Hence, the private-sector hiring and wage-setting procedures are, in a sense, meritocratic. Any differences in wage and job offer rates between the two types of workers reflect nothing but differences in their productivity (match surplus).

A vacant firm bears a recruitment cost κ_i specific to education, related to the expenses of keeping a vacancy open and looking for a worker. When a vacancy and a worker are matched, they bargain over the division of the produced surplus. The surplus that results from a match is known to both parties. After an agreement has been reached, production commences immediately. Matches in the private sector with type i workers dissolve at the rate s_i^p. Following a job destruction, the worker and the vacancy enter the corresponding sector/market and search for a new match.

2.2 Government

In the public sector, by contrast, policies are taken to be exogenous. To produce some government services, the government hires an exogenous number of workers, with and without education (e_h^g, e_l^g). In each period, the government has to hire enough workers to compensate the workers that exogenously separate or retire. That means hiring $(s_h^g + \tau)e_h^g$ skilled and $(s_l^g + \tau)e_l^g$ unskilled workers, where s_i^g is the separation rate. A fraction μ_i of these workers will be hired through connections. The matching function in the public sector is $M_{i,j}^g = \min\{v_{i,j}^g, u_{i,j}^g\}$. We assume that the number of searchers in each segment of the public sector, $u_{i,j}^g$, is at least equal to the number of job openings, $v_{i,j}^g$, meaning that $M_{i,j}^g = v_{i,j}^g$ and that no vacancies in the public sector remain unfilled.[6] As we will show in Lemma 2, this imposes a condition on public-sector wages to be high enough to attract at least the same number of searchers as of vacancies. We assume that the recruitment is part of the role of the government and is done by its workforce. Since the government's objective is to maintain employment levels (e_h^g, e_l^g) by hiring enough workers to replace those that separate or retire, it follows that $v_{i,u}^g = (1 - \mu_i)(s_i^g + \tau)e_i^g$ and $v_{i,c}^g = \mu_i(s_i^g + \tau)e_i^g$. Connected and unconnected workers of type-i find public-sector jobs at rate $m_{i,c}^g = \frac{\mu_i(s_i^g+\tau)e_i^g}{u_{i,c}^g}$ and $m_{i,u}^g = \frac{(1-\mu_i)(s_i^g+\tau)e_i^g}{u_{i,u}^g}$, respectively. For the moment, we do not assume any bias in terms of education and set $\mu_h = \mu_l = \bar{\mu}$, where $\bar{\mu}$ is an exogenous parameter reflecting the target fraction of jobs the government aims to fill through connections. In Section 3.3, we analyze the case in which the government cannot reach its target because there are not enough workers with connections.

Notice that in this setting, where the government has a fixed employment level, the separation rates s_i^g play a double role: they reflect the expected duration of the match but also determine the number of new hires. Higher separations reduce the value of employment

[6] Nothing substantial would change in the model if the matching function in the public sector were Cobb Douglas: $M_{i,j}^g = (v_{i,j}^g)^\eta (u_{i,j}^g)^{1-\eta}$. In this case, the vacancy filling probability of the government would no longer be 1, and it would need to set the vacancies to target exactly the number of workers that retire or separate – that is, $M_{i,j}^g = e_i^g(s_i^g + \tau)$. Solving for $v_{i,j}^g$ we would obtain $v_{i,j}^g = (e_i^g(s_i^g + \tau))^{\frac{1}{\eta}} / (u_{i,j}^g)^{\frac{1-\eta}{\eta}}$.

in the public sector but, at the same time, increase the vacancies and make an unemployed worker more likely to find a job there. Also, we assume that the separation rates, as well as other labor market friction parameters, are exogenous.

Finally, the public-sector wages, (w_h^g, w_l^g), are the other exogenous policy variables. We ignore the issue of how the government finances its wage bill and assume that it can tax its citizens in a non-distortionary lump-sum tax.

2.3 Value functions, Free entry, Wages

Let U_i^p and E_i^p be the values (expected discounted lifetime incomes) associated with unemployment (searching for a job) and employment, respectively, in the private sector of a worker of education level $i = [h, l]$. These are defined by:

$$(r+\tau)U_i^p = b_i + m(\theta_i)\left[E_i^p - U_i^p\right], \tag{1}$$
$$(r+\tau)E_i^p = w_i^p - s_i^p\left[E_i^p - U_i^p\right]. \tag{2}$$

The values associated with unemployment in the public sector of a worker of education level $i = [h, l]$ with and without connections are given, respectively, by:

$$(r+\tau)U_{i,u}^g = b_i + m_{i,u}^g\left[E_{i,u}^g - U_{i,u}^g\right], \tag{3}$$
$$(r+\tau)U_{i,c}^g = b_i + m_{i,c}^g\left[E_{i,c}^g - U_{i,c}^g\right]. \tag{4}$$

While the wage in the public sector does not depend on connections, the values of being employed are different for workers with and without connections:

$$(r+\tau)E_{i,u}^g = w_i^g - s_i^g\left[E_{i,u}^g - U_{i,u}^g\right], \tag{5}$$
$$(r+\tau)E_{i,c}^g = w_i^g - s_i^g\left[E_{i,c}^g - U_{i,c}^g\right]. \tag{6}$$

On the private-sector firm side, let J_i^p be the value associated with a job by a worker of type i and V_i^p be the value associated with posting a private-sector vacancy and searching for a type i worker to fill it. These values are given by

$$rJ_i^p = y_i - w_i^p - (s_i^p + \tau)\left[J_i^p - V_i^p\right], \tag{7}$$
$$rV_i^p = -\kappa_i + q(\theta_i)\left[J_i^p - V_i^p\right]. \tag{8}$$

In equilibrium, free entry drives the value of a private vacancy to zero:

$$V_i^p = 0, \quad i = [h, l]. \tag{9}$$

Wages are determined by Nash bargaining between the matched firm and worker. The outside options of the firm and the worker are the value of a vacancy and the value of being unemployed, respectively. Let $S_i^p \equiv J_i^p - V_i^p + E_i^p - U_i^p$ denote the surplus of a match with a type i worker. With Nash bargaining, the wage w_i^p is set to a level such that the worker gets a share β of the surplus, and the share $(1-\beta)$ goes to the firm. This implies two equilibrium conditions of the following form:

$$\beta S_i^p = E_i^p - U_i^p \qquad (1-\beta) S_i^p = J_i^p - V_i^p \text{ for } i = [h, l]. \tag{10}$$

Setting $V_i^p = 0$ in (8) and imposing the Nash bargaining condition in (10) gives:

$$\frac{\kappa_i}{q(\theta_i)} = (1-\beta) S_i^p \text{ for } i = [h, l]. \tag{11}$$

Using (1)-(7) together with (10) and the free-entry condition $V_i^p = 0$, we can write:

$$S_i^p = \frac{y_i - b_i}{r + \tau + s_i^p + \beta m(\theta_i)}, \tag{12}$$

and the free-entry condition as

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(y_i - b_i)(1-\beta)}{r + \tau + s_i^p + \beta m(\theta_i)} \text{ for } i = [h, l]. \tag{13}$$

The equation in (13) gives the two job-creation conditions. The job-creation condition sets the expected costs of having a vacancy (left-hand-side) equal to the expected gain from a job (right-hand-side). It can be used to determine the equilibrium market tightness θ_i and, in turn, the rates at which workers find jobs in the private sector, $m(\theta_i)$.

Imposing the free-entry condition (11) for private-sector vacancy creation, the Nash bargaining solution implies that

$$w_i^p = b_i + \beta(y_i - b_i + \kappa_i \theta_i), \quad i = [h, l]. \tag{14}$$

Lemma 1 *Tightness and wages in the private sector, in both low- and high-educated sub-markets, are independent of the government employment and wage policies (e_i^g, w_i^g and s_i^g).*

This lemma is a useful intermediate result and follows directly from equations (13) and

(14). It implies that government employment and wage policies affect the equilibrium only by affecting the education and connection decisions of the newborn or the scale of the private sector through the number of unemployed searching for a private-sector job. Given a constant tightness, policies that make the public sector more attractive will drain the unemployed from the private sector and reduce, one-to-one, the number of vacancies, leaving private wages unchanged.

2.4 Decisions and Allocations

We can summarize the six options of the newborn as

$$(r+\tau)U_i^p = b_i + \frac{m(\theta_i)}{r+\tau+s_i^p+m(\theta_i)}[w_i^p - b_i], \quad i=[h,l], \tag{15}$$

$$(r+\tau)U_{i,u}^g = b_i + \frac{m_{i,u}^g}{r+\tau+s_i^g+m_{i,u}^g}[w_i^g - b_i], \quad i=[h,l], \tag{16}$$

$$(r+\tau)U_{i,c}^g = b_i + \frac{m_{i,c}^g}{r+\tau+s_i^g+m_{i,c}^g}[w_i^g - b_i], \quad i=[h,l]. \tag{17}$$

These six options are depicted in Figure 1. Workers, with high or low education, can search without connections in either the public or the private sector. In equilibrium, the values of these two options have to equate:

$$U_i = U_{i,u}^g = U_i^p, \quad i=[h,l]. \tag{18}$$

This condition determines the number of unconnected searchers in the public sector, $u_{i,u}^g$, which is the variable that compensates any asymmetry in the value of the job in the two sectors. An increase of the value of a public-sector job, $E_{i,u}^g$, (driven by either higher wages or lower separations) raises the number of unemployed searching in the public sector and lowers their job-finding probability ($m_{i,u}^g$)), such that its effect on $U_{i,u}^g$ is neutralized.

Alternatively, workers can use connections to find jobs only in the public sector. In what follows, we drop the superscript g in $U_{i,c}^g$ and set $U_{i,c} \equiv U_{i,c}^g$. The newborn will choose the option that, given her ϵ and c, gives the highest value between:

$$Max\{U_l, U_h - \epsilon, U_{l,c} - c, U_{h,c} - c - \epsilon\}. \tag{19}$$

A worker of type $i=[h,l]$ and connections cost c will choose to obtain connections only if the benefit, $U_{i,c} - U_i$, exceeds the cost – that is, only if $c \le U_{i,c} - U_i$. The threshold level of c at which a worker of type i is indifferent between using and not using connections to find a public-sector job is, therefore, given by

$$\tilde{c}_i = U_{i,c} - U_i. \tag{20}$$

Lemma 2 *There exists a public-sector "unconnected" market for workers of type i, where no vacancy is left unfilled, provided that the public sector pays a sufficiently high wage* $w_i^g \geq \underline{w}_{i,u}^g$*. There exists a public-sector "connected" market for workers of type i, where* $\mu_i = \bar{\mu}$ *and no vacancy is left unfilled, provided that the public sector pays a sufficiently high wage* $w_i^g \geq \underline{\underline{w}}_{i,c}^g > \underline{w}_{i,u}^g$

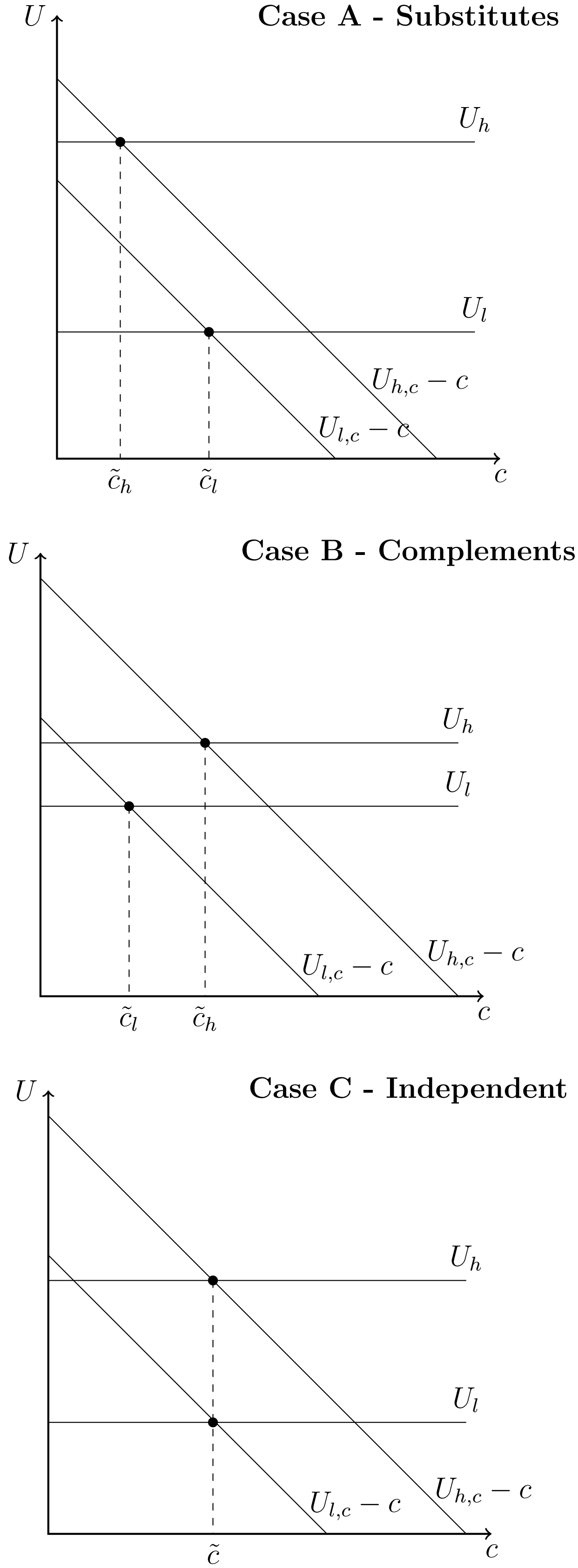
The exact expressions for $\underline{w}_{i,u}^g$ and $\underline{\underline{w}}_{i,c}^g$ are in Appendix A. This lemma states that the public sector needs to pay a sufficiently high wage in order to attract enough job seekers to fill all its vacancies and maintain a constant employment level. This threshold, $\underline{w}_{i,u}^g$, depends positively on private-sector wages, w_i^p, and unemployment benefits, b_i. However, with a connection sector, this wage has to be higher, to compensate the costs of acquiring connections. This second threshold wage, $\underline{\underline{w}}_{i,c}^g$, depends positively on $\underline{w}_{i,u}^g$ and on the size of the connection sector $\bar{\mu}$. In what follows, we assume that the public-sector wages are always above $\underline{\underline{w}}_{i,c}^g$, meaning that the government can fill any target fraction $\bar{\mu}$ of its vacancies through connections. We analyze the case in which it can not in Section 3.3 and again in the numerical exercise in Section 6.

As shown in Figure 2, we can have three different cases, each having different implications for how the existence of public-sector hiring through connections alters workers' incentives to invest in education.

Case A in Figure 2 describes a scenario in which education and connections are substitutes. The benefit from investing in education is smaller if the worker uses connections to find a public-sector job than if not. That is, $U_h - U_l > U_{h,c} - U_{l,c}$, and low-educated workers have more incentive than high-educated workers to use connections ($\tilde{c}_h < \tilde{c}_l$). Case A could reflect a situation in which public-sector wages are relatively flat across worker qualifications (i.e., public-sector wages are compressed), whereas in the private sector, wages increase steeply with workers' qualifications. In such cases, those seeking to use their connections to attain jobs in the public sector have less incentive to invest in education, while those lacking connections (i.e., those whose connection cost is high) have more incentive to opt for education. In other words, education "substitutes" for the lack of connections.

More specifically, the two thresholds $\tilde{c}_h$ and $\tilde{c}_l$ can be used to divide workers into three groups that differ in their incentives to obtain higher education. In the first group are workers whose connection cost c is low: $c < \tilde{c}_h (< \tilde{c}_l)$. For these workers, $U_{i,c} - c > U_i$ for $i = [h, l]$, and regardless of their education, using connections to find employment in the public sector

Figure 2: Decision Thresholds



always yields a higher payoff than not using them. For these workers, the net benefit from investing in education is given by $\tilde{\epsilon}_c = U_{h,c} - U_{l,c}$. Next is the group of workers whose connection cost c lies between $\tilde{c}_h$ and $\tilde{c}_l$. For these workers, using connections to find a job in the public sector is worthwhile only if they remain low-educated. That is, $U_{l,c} - c > U_l$, but $U_{h,c} - c < U_h$. If they invest in eduction, they are better off not using connections, thereby saving the connection cost. Thus, their benefit from education is $\tilde{\epsilon}_m(c) \equiv U_h - (U_{l,c} - c)$, which is increasing in c. In the last group are the workers with $c > \tilde{c}_l (> \tilde{c}_h)$, who never get connections because the cost is too high. For these workers, $U_i > U_{i,c} - c$ for $i = [h, l]$. If they choose to become high-educated, they obtain a payoff of $\tilde{\epsilon}_u \equiv U_h - U_l$.

In the opposite case – case B in Figure 2 – education "complements" the use of connections. Workers using connections to seek public-sector jobs have more incentive to become high-educated ($U_{h,c} - U_{l,c} \geq U_h - U_l$), which also implies that high-educated workers have more incentive than the low-educated to use connections to find a job in the public sector ($\tilde{c}_h > \tilde{c}_l$). Case B could arise when the public sector has many jobs available for skilled workers – jobs that are easier to get through connections.

As above, the low-connection-cost workers, those with $c < \tilde{c}_l (< \tilde{c}_h)$, always choose to target public-sector jobs through connections and have a payoff $\tilde{\epsilon}_c$ from becoming high-educated. Conversely, there are workers with $c > \tilde{c}_h (> \tilde{c}_l)$, who never use connections and have a payoff $\tilde{\epsilon}_u$ from investing in education. In between are the workers with $\tilde{c}_l < c < \tilde{c}_h$. As Figure 2 shows, for these workers, $U_{l,c} - c < U_l$ and $U_{h,c} - c > U_h$; thus, they will use connections if they become high-educated but will not if they remain low-educated. Investing in education brings them a benefit of $\tilde{\epsilon}_m(c) \equiv U_{h,c} - c - U_l$, which is decreasing in c.

Finally, case C is the knife-edge case in which the payoff from being high-educated is the same in both sectors. Having connections does not alter a worker's payoff from investing in education ($U_{h,c} - U_{l,c} = U_h - U_l$), and high- and low-educated workers both have equal incentives to use connections ($\tilde{c}_h = \tilde{c}_l$). In this case, all (connected or unconnected) workers obtain a payoff of $\tilde{\epsilon} = U_{h,c} - U_{l,c} = U_h - U_l$ from investing in education.

A worker invests in education only if the benefit exceeds the cost (ϵ). The education benefit can be either $\tilde{\epsilon}_u, \tilde{\epsilon}_m$ or $\tilde{\epsilon}_c$, depending on the worker's connection cost (c):

$$\tilde{\epsilon}_c = \tilde{\epsilon}_u + \tilde{c}_h - \tilde{c}_l, \tag{21}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_u + c - \tilde{c}_l \quad c \in [\tilde{c}_h, \tilde{c}_l], \text{ if } \tilde{c}_h < \tilde{c}_l \text{ (case A)}, \tag{22}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_u + \tilde{c}_h - c \quad c \in [\tilde{c}_l, \tilde{c}_h], \text{ if } \tilde{c}_h > \tilde{c}_l \text{ (case B)}. \tag{23}$$

Figure 3 illustrates how the education and connection cutoffs relate under the three cases. In case A, education substitutes for connections, and those most likely to invest in education have a high connection cost: $\tilde{\epsilon}_c \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_u$ and $\tilde{c}_h < \tilde{c}_l$. In this case, $\tilde{\epsilon}_m$ is increasing one-to-one with c. In case B, education complements connections, and the benefit from education is higher for those belonging in the low-connection-cost group: $\tilde{\epsilon}_c \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_u$ and $\tilde{c}_h > \tilde{c}_l$. In this case, $\tilde{\epsilon}_m$ is decreasing one-to-one with c. In case C, incentives to invest in education are independent of workers' connection cost, and equal fractions of connected and unconnected workers invest in education: $\tilde{\epsilon}_c = \tilde{\epsilon}_m = \tilde{\epsilon}_u$ and $\tilde{c}_h = \tilde{c}_l = \tilde{c}$. Workers' cutoffs determine their selection into four groups: the high- and low-educated who use connections to find public-sector jobs ($L^g_{h,c}$ and $L^g_{l,c}$), and the high- and low-educated who do not use connections ($L_{h,u}$ and $L_{l,u}$), as depicted in Figure 3. For each of the cases A,B and C, we can measure each of these four groups' share in the labor force as:

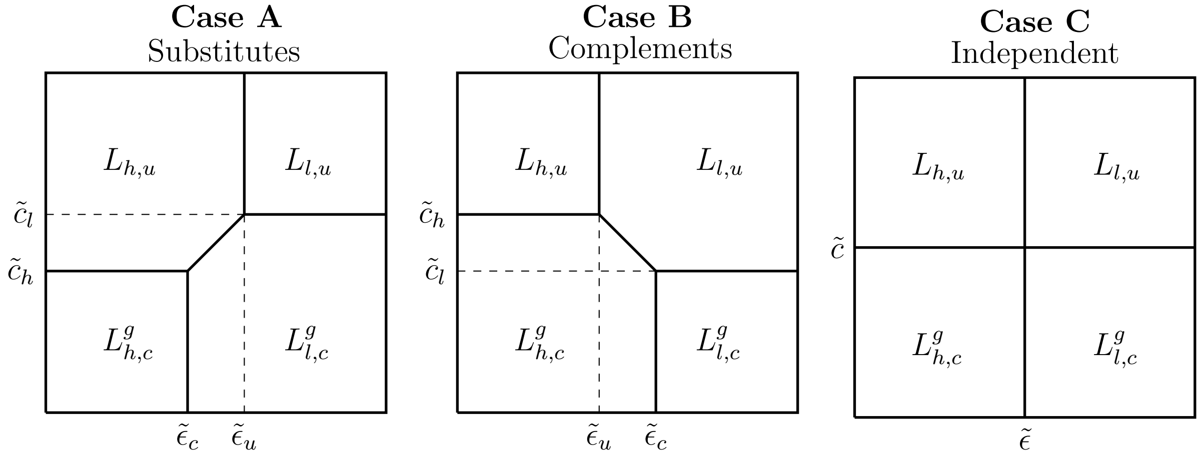
$$\text{Case A, } \tilde{c}_h < \tilde{c}_l \quad \begin{cases} L^g_{h,c} = \Xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h) \\ L^g_{l,c} = (1-\Xi^\epsilon(\tilde{\epsilon}_c))\Xi^c(\tilde{c}_h) + \int_{\tilde{c}_h}^{\tilde{c}_l}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c) \\ L_{h,u} = \int_{\tilde{c}_h}^{\tilde{c}_l}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_u) \\ L_{l,u} = (1-\Xi^\epsilon(\tilde{\epsilon}_u))(1-\Xi^c(\tilde{c}_l)) \end{cases} \tag{24}$$

$$\text{Case B, } \tilde{c}_h > \tilde{c}_l \quad \begin{cases} L^g_{h,c} = \Xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l) + \int_{\tilde{c}_l}^{\tilde{c}_h}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) \\ L^g_{l,c} = (1-\Xi^\epsilon(\tilde{\epsilon}_c))\Xi^c(\tilde{c}_l) \\ L_{h,u} = (1-\Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_u) \\ L_{l,u} = (1-\Xi^\epsilon(\tilde{\epsilon}_u))(1-\Xi^c(\tilde{c}_h)) + \int_{\tilde{c}_l}^{\tilde{c}_h}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c) \end{cases} \tag{25}$$

$$\text{Case C, } \tilde{c}_h = \tilde{c}_l \quad \begin{cases} L^g_{h,c} = \Xi^\epsilon(\tilde{\epsilon}_u)\Xi^c(\tilde{c}) \\ L^g_{l,c} = (1-\Xi^\epsilon(\tilde{\epsilon}_u))\Xi^c(\tilde{c}) \\ L_{h,u} = (1-\Xi^c(\tilde{c}))\,\Xi^\epsilon(\tilde{\epsilon}_u) \\ L_{l,u} = (1-\Xi^c(\tilde{c}))(1-\Xi^\epsilon(\tilde{\epsilon}_u)) \end{cases} \tag{26}$$

$L_h = L^g_{h,c} + L_{h,u}$ gives the share of high-educated in the labor force and $L_l = 1 - L_h = L^g_{l,c} + L_{l,u}$ the share of low-educated. Among the workers (low- or high-educated) who choose not to use connections, some will be attached to the private sector ($L^p_{i,u}$) and some to the public sector ($L^g_{i,u}$). Hence, $L_{i,u} = L^p_{i,u} + L^g_{i,u}$.

Figure 3: Cutoffs and allocations



Using (10)-(13) and (15)-(17), we can write the cutoffs as:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[\frac{\frac{\mu(s_i^g+\tau)e_i^g}{u_{i,c}^g}}{r+\tau+s_i^g+\frac{\mu(s_i^g+\tau)e_i^g}{u_{i,c}^g}}[w_i^g - b_i] - \frac{\beta\kappa_i\theta_i}{(1-\beta)}\right], \quad i=[h,l], \tag{27}$$

$$\tilde{\epsilon}_u = \frac{1}{r+\tau}\left[b_h - b_l + \frac{\beta\kappa_h\theta_h}{(1-\beta)} - \frac{\beta\kappa_l\theta_l}{(1-\beta)}\right]. \tag{28}$$

Definition 1 *A steady-state equilibrium consists of a set of cut-off costs* $\{\tilde{c}_h, \tilde{c}_l, \tilde{\epsilon}_u, \tilde{\epsilon}_c\}$*, private sector tightness* $\{\theta_h, \theta_l\}$*, and unemployed searching in each market* $\{u_h^p, u_l^p, u_{h,c}^g, u_{l,c}^g, u_{h,u}^g, u_{l,u}^g\}$*, such that, given some exogenous government policies* $\{w_h^g, w_l^g, e_h^g, e_l^g, \bar{\mu}\}$*, the following apply.*

1. *Private-sector firms satisfy the free-entry condition (13)* $i=[h,l]$*.*

2. *Private-sector wages are the outcome of Nash Bargaining (14)* $i=[h,l]$*.*

3. *Newborns decide optimally their investments in education and connections (equation 19), and the population shares are determined by the equations (24), (25) or (26), depending on the case.*

4. *The search between the public and private sectors by the unconnected unemployed satisfies equation (18).*

5. *Flows between private employment and unemployment are constant:*

$$(s_h^p+\tau)e_h^p = m(\theta_h)u_h^p, \tag{29}$$

$$(s_l^p+\tau)e_l^p = m(\theta_l)u_l^p. \tag{30}$$

6. Population add up constraints are satisfied:

$$L_{h,u} = e_h^p + (1-\mu_h)e_h^g + u_h^p + u_{h,u}^g, \tag{31}$$

$$L_{l,u} = e_l^p + (1-\mu_l)e_l^g + u_l^p + u_{l,u}^g, \tag{32}$$

$$L_{h,c}^g = \mu_h e_h^g + u_{h,c}^g, \tag{33}$$

$$L_{l,c}^g = \mu_l e_l^g + u_{l,c}^g, \tag{34}$$

$$L_{h,u} + L_{l,u} + L_{h,c}^g + L_{l,c}^g = 1. \tag{35}$$

7. The government fills its target fraction of vacancies through connections $\mu_h = \mu_l = \bar{\mu}$.

3 Main results

This section details the main results, under four propositions. All the derivations and proofs are shown in Appendix A, including the proof that the equilibrium exists and is unique.

3.1 Effects of policies on educational composition

Proposition 1 *If $\bar{\mu} = 0$, then public-sector wages and employment have no impact on the the composition of the labor force in terms of education – the educational composition of the labor force is independent of public-sector policies.*

If $\bar{\mu} > 0$, then an increase in w_h^g, e_h^g (decrease in w_l^g, e_l^g) – or, in general, any policy that improves public- sector value of the high- relative to the low-educated (i.e., anything that increases $\tilde{\epsilon}_u$ and $\tilde{c}_h$ relative to $\tilde{c}_l$) raises L_h and decreases L_l – raises the proportion of the high-educated in the labor force.

If $\bar{\mu} = 0$, then free mobility between the public and private sectors keeps the education premium fixed. Consider, for instance, an increase in w_h^g that raises the value of a high-education job in the public sector. Consequently, some high-educated workers quit searching for private-sector jobs and direct their search towards the public sector. The increased congestion due to the arrival of additional job seekers in the public sector lowers the job-finding rate, and pushes the value of searching for a high-education job in the public sector back to its initial level. Incentives to invest in education remain intact.

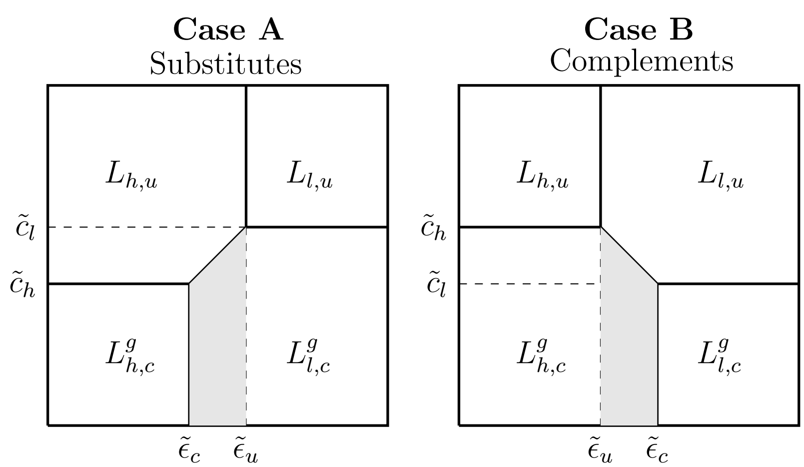
On the other hand, if $\bar{\mu} > 0$, mobility between the two sectors (public, private) is more difficult (costly). A fraction $\bar{\mu}$ of public-sector jobs are reserved for connected workers. But in order to obtain connections, a worker needs to pay a cost. Limited mobility now reduces the inflow of searchers from the private towards the public sector. As a result, the same increase in w_h^g now raises the value of searching for a high-education job in the connected public sector and, hence, the benefit from investing in education. Such an increase in w_h^g now induces a higher fraction of the labor force to become high-educated.

Proposition 2 *If $\tilde{c}_h < \tilde{c}_l$ ($\tilde{c}_l < \tilde{c}_h$) – Case A (Case B)– the existence of public-sector hiring through connections ($\bar{\mu} > 0$) worsens (improves) the educational composition of the labor force: decreases (increases) L_h. If $\tilde{c}_h = \tilde{c}_l$ – Case C– the existence of public-sector hiring through connections ($\bar{\mu} > 0$) has no impact on the educational composition of the labor force.*

If it pays relatively more to be high-educated in the private than in the public sector (Case A, where education and connections are substitutes), then the low-educated can benefit the most from using connections. For workers whose connection cost is low, it is worthwhile to substitute education for connection and use them to target low-education jobs in the public sector. As a result, the fraction of high-educated workers falls. In the opposite case, when education and connections are complements (case B), if the use of connections to find jobs in the public sector is allowed, then workers with connections have more incentive to become educated.

Figure 4 illustrates the increase and decrease, respectively, in the fraction of high-educated workers in the labor force once public-sector hiring through connections is introduced. Under Case A, the shaded area represents the fraction of people that would have become educated when $\bar{\mu} = 0$, but now prefer to remain uneducated, but use their connections to find a public-sector job. Under Case B, the shaded area represents the fraction of people that would have

Figure 4: Lack of Meritocracy and the Educational Composition



Note: Under $\bar{\mu} = 0$, $\tilde{c}_h = \tilde{c}_l = 0$, and the box is divided vertically in two, by $\tilde{\epsilon}_u$. Under $\bar{\mu} > 0$, the grey area in Case A represents the fraction of low-educated and connected people that would otherwise be educated. The grey area in Case B represents the fraction of high-educated and connected people that would otherwise have low education.

remained uneducated when $\bar{\mu} = 0$, but now prefer to get connections and education.

3.2 Effects of non-meritocratic hiring on unemployment

Proposition 3 *An increase in $\bar{\mu}$ raises the number of (low- and high-educated) workers who are both searching and are employed in the private sector (i.e., increases $L^p = 1 - L^g$, where $L^g = L^g_{h,u} + L^g_{l,u} + L^g_{h,c} + L^g_{l,c}$ is the total number of workers who are either employed or are searching in the public sector). At the same time, it lowers the total number of workers of each type who are searching in the public sector ($u^g_i = u^g_{i,u} + u^g_{i,c}$), but leaves the number of workers employed in the public sector (e^g_i) intact.*

This result is perhaps surprising but is quite logical. As shown in Lemma 2, the existence of a connection sector requires that the public-sector wage is high enough. Under this condition, there are large queues for unconnected public-sector jobs. With a higher fraction of public-sector jobs being reserved for workers with connections, the value of trying to find (searching for) a public-sector job without connections decreases. Workers have more incentive to direct their search towards the private sector or to obtain connections. Since it is costly to obtain connections, some of them – those whose connection cost is high – abandon search in the public sector and search for private-sector jobs instead. With a fixed tightness in the private sector, job creation goes up one-to-one as the number of searchers and overall employment increases. Following this result, in section 4.2, we investigate whether an increase of $\bar{\mu}$ can also raise welfare.

3.3 When the lack of meritocracy is bounded: a limit to μ

We now relax the assumption that μ_h and μ_l are isolated from labor market conditions. We show that in situations in which the public-sector wage premium is not large enough to generate queues, changes in the supply of connected job searchers can influence the size of the connected sector.

We interpreted $\bar{\mu}$ as the government's target fraction of type-i vacancies to be filled through connections. The government is able to use connections at the maximum – fill a fraction $\mu_i = \bar{\mu}$ of type-i jobs through connections – provided that it pays a sufficiently high wage to attract enough connected job searchers. According to Lemma 2, there exists a wage, $\underline{\underline{w}}^g_{i,c}$, at which the government is able to attract exactly $u^g_{i,c} = \bar{\mu}(s^g_i + \tau)e^g_i$ connected job searchers. Hence, for any wage $w^g_i \geq \underline{\underline{w}}^g_{i,c}$, the government is able to fill a fraction $\bar{\mu}$ of jobs through connections.

If the government wage is lower, but still high enough to attract some connected job searchers, $\underline{\underline{w}}^g_{i,c} > w^g_i > \underline{w}^g_{i,u}$, the number of connected job searchers is lower, but still positive:

$0 < u^g_{i,c} < \bar{\mu}(s^g_i + \tau)e^g_i$. In this case also, a connected sector exists ($\mu_i > 0$), but the government is restricted to fill only a fraction $\mu_i < \bar{\mu}$ of type-i vacancies through connections, where μ_i is such that $u^g_{i,c} = \mu_i(s^g_i + \tau)e^g_i$. The remaining vacancies $(1 - \mu_i)$ are filled by unconnected workers. Using (33) and (34), we can solve for μ_i and write:

$$\mu_i = \frac{L^g_{i,c}}{e^g_i(s^g_i + \tau + 1)}. \tag{36}$$

This equation states that there are no connected workers queuing for jobs: the total number of connected workers $L^g_{i,c}$ equals the new hires $\mu_i(s^g_i + \tau)e^g_i$ plus those already employed in the public sector $\mu_i e^g_i$.

In the limiting case, where $w^g_i = \underline{w}^g_{i,u}$, the wage is not high enough to compensate for the connection cost, and no worker has the incentive to use connections to find a public-sector job; hence, $u^g_{i,c} = 0$, which means that $\mu_i = 0$.

To sum up, we generalise Condition 7 in Definition 1, by replacing it with

$$\mu_i = \begin{cases} \bar{\mu} & \text{if } w^g_i \geq \underline{\underline{w}}^g_{i,c} \\ \frac{L^g_{i,c}}{e^g_i(s^g_i+\tau+1)} & \text{if } \underline{\underline{w}}^g_{i,c} > w^g_i > \underline{w}^g_{i,u} \\ 0 & \text{if } w^g_i = \underline{w}^g_{i,u}. \end{cases} \tag{37}$$

Proposition 4 *Provided that the public-sector wage is high enough to attract some type-i connected job searchers, but not high enough to generate queues, the fraction of vacancies of type-i that the government fills through connections, μ_i, is smaller, the smaller the public-sector wage w^g_i and the larger the size of public-sector employment, e^g_i.*

The government can fill a higher fraction of jobs through connections when the public-sector wage is higher because the supply of connected job searchers is larger. Larger public-sector employment means that the number of workers that the government needs to hire each period, to replace those that separate due to retirement or other reasons, is also larger, while the number of connected workers searching for jobs is smaller. Hence, the proportion of government jobs filled by connected job searchers is smaller.

This proposition tells us how government policies place a constraint on the level of non-meritocracy. Governments that have large employment levels but offer low premia to their workers – such as those in Nordic countries – will have endogenous limits on hiring through connections.

We examine the effects of government policies in this generalized framework in the numerical exercise in Section 6.

4 Efficiency

4.1 Constraint efficient allocation

The social planner's problem and the first-order conditions are shown in Appendix B. There are four types of inefficiencies in this model: i) the existence of a "connections" sector that propels newborns to take on rent-seeking activities; ii) the existence of queues for public-sector jobs (given the assumption of the min matching function in the public sector); iii) the usual thick-market and congestion externalities in both high- and low-education markets; and iv) the fact that the newborn might not internalize the returns of education.

Inefficiencies i) and ii) are both solved by setting the optimal wage. To avoid queues, the government should set a public-sector wage for high and low ability such that $u^g_{i,u} = (s^g_i + \tau)e^g_i$. In other words, at any instant the job-finding rate for government jobs should be 1, which implies setting $\underline{w}^g_{i,u}$. These same wages, according to equation (37), eliminate the connections sector.

We then show that inefficiencies iii) and iv) are both solved with the Hosios condition. The Hosios condition in private-sector bargaining guarantees that the thick market and the congestion externalities are internalized, and it allows the unemployed to internalize the returns of education.

4.2 Optimal μ conditional on inefficient public-sector wage

Suppose, now, that the public-sector wage is high enough so that the government can fill its target fraction $\bar{\mu}$ of vacancies through connections; that is, $w^g_i > \underline{\underline{w}}^g_{i,c}$. In this case, a connections sector exists, as some workers find it optimal to invest in connections. The question that arises is whether or not the existence of a connections sector, under inefficient government policies, improves welfare. To address this question, we discuss the impact of increasing $\bar{\mu}$ ($\mu_h = \mu_l = \bar{\mu}$) on net surplus. Net surplus is total output net of vacancy posting costs, plus unemployment income, minus the total amount that workers invest in connections and education. Since public-sector employment is fixed, an increase in total output can be the achieved by an increase in private-sector employment, which ultimately requires shorter public-sector queues.

The impact of $\bar{\mu}$ on net surplus involves the following effects. First, as summarized in Proposition 3, an increase in $\bar{\mu}$ raises employment in the private sector and, thus, increases output and net surplus. Second, it affects the educational composition of the labor force, shifting it from low- to high-educated in case A and vice versa under case B, as shown in Proposition 2. If one accepts that high-educated workers are more productive than low-educated workers, then this effect on net surplus may be positive or negative. However, an increase in the proportion of educated workers caused by an increase in $\bar{\mu}$ also raises the total costs for education, with a negative impact on net surplus. As discussed above, if $\tilde{c}_h = \tilde{c}_l$ (case C), changes in $\bar{\mu}$ leave the educational composition of labor force intact, and only the first positive effect is present in this case, implying a positive impact on net surplus overall.

However, even in this case, where changes in $\bar{\mu}$ have no impact on education incentives, we cannot conclude that a larger connections sector means higher net surplus, because an increase in $\bar{\mu}$ also induces some workers to invest in connections, thus increasing the total resources wasted on getting connections. If obtaining connections is difficult and costly for most workers, relative to the benefit of being employed in the public sector, then an increase in $\bar{\mu}$ is more likely to drive workers away from the public sector and cause a large shift in workers' search towards the private sector, resulting in a large increase in private employment. If, on the other hand, obtaining connections is easy and the benefit of a public-sector job large, then an increase in $\bar{\mu}$ will have a small impact on private employment and will, instead, cause a larger shift towards forming connections.

In general, it is difficult to establish that an increase of $\bar{\mu}$ is optimal, given an inefficient public-sector wage policy. As discussed above, the role of connections costs, the size of public-sector wages, and other public-sector benefits are important. However, the interesting point here is that we cannot rule out that non-meritocracy in the public sector can be optimal because it raises output production and shortens public-sector queues. We address this question again in the numerical exercise in Section 6.

5 Extensions

In this section, we discuss and compare the effects of non-meritocracy and government policies on employment and human capital under four alternative model assumptions: i) random search in the unconnected market; ii) competitive search in the private sector; iii) endogenous public-sector employment; and iv) the existence of a "connections premium." We further compare the alternative models introduced here in the quantitative exercise in Section 6.

5.1 Random search between the private sector and the unconnected public sector

We start by analyzing the case in which the workers without connections cannot direct their job search exclusively towards the public or the private sector. We assume that these workers search randomly for jobs that suit their skill type in the two sectors. A matching function $m(v_{i,u}, u_{i,u})$ determines the total number of matches between unconnected workers and jobs and $m(\theta_i)$, where $\theta_i = \frac{v_{i,u}}{u_{i,u}}$, gives the rate at which unconnected workers match with (either private or government) vacancies. Since they search randomly for jobs, the total number of vacancies available to them, consists of both private-sector v_i^p and government $v_{i,u}^g$ ($v_{i,u} = v_i^p + v_{i,u}^g$) vacancies, where $v_{i,u}^g$ is the number of public-sector vacancies available to unconnected workers. They find jobs in the private sector at rate $m(\theta_i)\nu_i^p$ and in the public sector at rate $m(\theta_i)(1-\nu_i^p)$, where $\nu_i^p = \frac{v_i^p}{v_{i,u}}$ is the fraction of private-sector vacancies in the total number of type i vacancies available to workers without connections.

The key difference between the model with random search and segmented markets is the value of unemployment for unconnected workers. It changes because they now randomly search for jobs in both sectors. Specifically,

$$(r+\tau)U_{i,u} = b_i + m(\theta_i)\nu_i^p \left[E_i^p - U_{i,u}\right] + m(\theta_i)(1-\nu_i^p)\left[E_{i,u}^g - U_{i,u}\right]. \tag{38}$$

Under segmented markets, tightness (job creation) in the private sector is independent of any government policy (see Lemma 1) because the outside option (unemployment value) of workers searching for private-sector jobs – and, thus, the private-sector wage – is independent of government policy. Under random search, by contrast, the outside option of searching workers also includes the possibility of finding a public-sector job. As can be seen by equation 38, the outside option of unconnected workers is a convex combination of the value a public sector job ($E_{i,u}^g$) and the value of a private sector sector (E_i^p) with weights reflecting the relative number of vacancies in the two sectors. Thus, public-sector wages, employment opportunities, and separation probabilities (as well as the lack of meritocracy in the public sector) affect private-sector wages. More specifically, the private-sector wage of a worker of skill type i is given by

$$w_i^p = b_i + \beta\left[y_i - b_i + \nu_i^p\theta_i\kappa_i\right] + (1-\beta)D_i(w_i^g - b_i), \tag{39}$$

where $D_i = \frac{(1-\nu_i^p)m(\theta_i)}{r+\tau+s_i^g+(1-\nu_i^p)m(\theta_i)}$ measures how much public-sector wages influence private-sector wage bargaining. A free-entry condition as in (11) determines the number of vacancies in the private sector. But now the match surplus, $S_i^p = \frac{p_i - w_i^p}{r+s_i^p+\tau}$, which decreases as the wage increases, depends also on public-sector policy and non-meritocracy. In addition, the cutoff connection costs, $\tilde{c}_i = U_{i,c}^g - U_{i,u}$, $i = [h,l]$, change to reflect that the value of unemployment to unconnected workers is now given by (38). The full set of equations describing the model with random search, a formal definition of a steady-state equilibrium and conditions for existence of a steady-state equilibrium are in Appendix C.

In general, under random search, the effects of government policies work through: i) the selection into connected and non-connected workers (as in segmented markets); and ii) the outside option of unconnected workers and its impact on private-sector wages. If $\mu_h = \mu_l = 0$, meaning that no connections sector exists, then all effects work only through the outside option. In the other extreme case, where $\mu_h = \mu_l = 1$ (meaning that $v_{i,u}^g = 0$, $\nu_i^p = 1$, $D_i = 0$, and all government vacancies are for connected workers), tightness and wages in the private sector become identical to those obtained under segmented markets, and all effects work through the selection into connected and non-connected workers, as in segmented markets. We isolate each of these two channels of effects by looking at these two cases separately, and we show in Appendix C that:

Proposition 5 *If $\mu_h = \mu_l = 0$, then for $i = [h, l]$, an increase in w_i^g or e_i^g (i.e., a higher wage or more public-sector jobs) raises the value of workers' outside option; increases private-sector wages (w_h^p and w_l^p); and lowers the surplus of firms in the private sector (lower S_h^p and S_l^p), leading to lower job creation (lower θ_h and lower θ_l).*

Proposition 6 *If $\mu_h = \mu_l = 1$, an increase in w_i^g or e_i^g (i.e., a higher wage or more public-sector jobs) or, in general, anything that improves a type i worker's payoff from searching for a job in the public sector, induces more workers of type i to obtain connections (i.e., higher $L_{i,c}^g$ and lower $L_{i,u}$); fewer workers to search in the private sector; and more workers to queue for public-sector jobs. The employment rate of type i workers decreases.*

In the general case, where $0 < \mu_i < 1$, both channels are present, suggesting a negative impact of more generous government policies on job creation and employment.

5.1.1 The effect of non-meritocracy on job creation and employment

As discussed above, under random search, public-sector policies work not only through the selection into connected and unconnected workers, but also through their impact on private-sector wages and in turn, job creation (tightness). For this reason, the effect of non-meritocracy on employment can be either positive or negative. With a higher fraction of public-sector jobs being retained for workers with connections, a larger fraction of workers who do not have connections end up in private- instead of public-sector jobs (i.e., ν_i^p increases, shifting weights in (38) from $E_{i,u}^g$ to E_i^p). Assuming that government jobs are more valuable to workers than private jobs are (that is, $E_{i,u}^g > E_i^p$), non-meritocracy worsens the outside option of unconnected workers; private wages decrease; and job creation in the private sector increases with a positive impact on employment.[7] In addition to this job-

[7]We get the opposite result on job creation if $E_{i,u}^g < E_i^p$, while if $E_{i,u}^g = E_i^p$, non-meritocracy has no impact on the outside option, and this job creation effect is no longer present. However, as shown in Appendix C, the sufficient condition for the existence of a steady-state equilibrium under random search is $E_{i,u}^g \geq E_i^p$.

creating effect, a decrease in the fraction of public-sector jobs available to non-connected workers makes the option of investing in connections more attractive. More workers seek public-sector jobs through their connections, and the number of connected workers queuing up for public-sector jobs increases, with a negative impact on employment.[8]

5.1.2 The effect of government policies on human capital

As shown in Proposition 1, under segmented markets, any increase in the education premium due to more generous government policies is offset by a decrease in the job-finding rate because more workers search in the public sector. Thus, government policies will have an effect on the educational composition of the labor force, only through the selection into connected and unconnected workers.[9]

However, when search is random, two additional effects are involved. First, because workers cannot direct their search, such an offsetting decrease in the job-finding rate is not possible. For this reason, policies that increase the payoff from being educated, such as increasing the public-sector wage premium or increasing public-sector employment of the high- vs. low-educated, encourage workers to become educated and put positive pressure on the proportion of high-educated workets in the labor force (L_h). Second is the effect on workers' outside option, which acts in the opposite direction. Public-sector policies that improve the education premium, also improve the outside option of high-educated workers. They are able to bargain for a higher wage in the private sector and, thus, reduce firms' profits from hiring them, leading to a decrease in job creation in the private sector for educated workers and a negative impact on L_h.

In general, we cannot tell which of the two effects dominates. Therefore, in the model with random search, it is difficult to analytically establish that policies improving the education premium have a positive impact on L_h. However, as we increase non-meritocracy in the public sector, and we further restrict access of workers without connections to public-sector jobs, the public sector becomes more isolated and the second effect (outside option effect) weaker. At $\bar{\mu} = 1$, the second effect is completely eliminated and only the first effect prevails. As already mentioned, in this case, the model resembles the model with segmented markets, and the effects of government policies on education are as summarized in Proposition 1 (when $\bar{\mu} > 0$): policies that improve the education premium also increase the number of

[8]In segmented markets a decrease in the fraction of government jobs available to non-connected workers has a positive impact on employment because some workers, those whose cost of obtaining connections is large, will direct there search towards the private sector. Under the assumption of random search this positive effect is not present, because workers cannot direct their search towards the private sector.

[9]Hence, as shown in Proposition 1, with $\bar{\mu} = 0$, meaning that no such selection exists, public-sector policies have no impact on education incentives.

high-educated workers in the labor force.

5.2 Competitive search in the private sector

Suppose now that, as in the benchmark model, the two sectors, private and public, are segmented, and markets in each sector are also segmented by skill; there are, therefore, two markets in each sector, the skilled and the unskilled. However, we depart from the assumptions of Nash bargaining and random search in the private sector. Instead, as in Moen (1997), we introduce a competitive search equilibrium in the private sector. To this end, we assume that each of the two private-sector markets consists of submarkets with different posted wages and equilibrium tightness.

In each submarket, there is a subset of unemployed workers and firms with vacant jobs that are searching for each other. A matching function determines the number of matches in each submarket. Unemployed workers of skill type i are free to move between the submarkets of market i. They choose to search for a job in the submarket that yields the highest expected income. Since workers of the same skill type are ex-ante identical, and movement across submarkets is free, in equilibrium, the value of search is equal across submarkets of the same skill type. A market maker determines the number of submarkets in each market and the wage in each submarket. The wage is chosen to maximize the value of a vacancy, and since all vacancies in the same submarket are identical, they offer the same wage. There is free entry of vacancies in each submarket, which drives the value of a vacancy to zero, and determines the number of vacancies posted in each submarket. In Appendix D, we present the full set of Bellman equations describing the optimal behavior of workers and firms in each submarket, the equilibrium conditions and the model solution.

We show, in Appendix D, that the equilibrium conditions determining job creation and the Nash bargaining wage in this alternative setup are identical to those obtained in the benchmark model when the Hosios condition holds. Hence, the results discussed in Sections 3 and 4 carry over to this alternative assumption of competitive search in the private sector.

5.3 Endogenous public-sector employment

Next, we consider an alternative modeling approach for the government's behavior. In particular, we endogenize the number of public-sector workers. We assume that the government needs to produce a minimum number of services, $\bar{g}$. To produce these services, the government hires different types of workers, with and without education (e_h^g, e_l^g). These are determined endogenously, to minimize the cost of producing these services.

$$\min_{e_h^g, e_l^g} w_h^g e_h^g + w_l^g e_l^g$$
$$s.t.$$
$$\bar{g} = (e_h^g)^{\alpha} (e_l^g)^{1-\alpha}.$$

where α is a parameter governing the importance of skilled employment in the production of public goods. Given the level of public wages, the government has to guarantee that it hires enough workers to maintain an employment level capable of providing its services. Using the production function and the two first-order conditions, we find the optimal level of employment.

$$e_h^g = \frac{\bar{g}}{\left(\frac{w_h^g}{w_l^g}\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha}}, \tag{40}$$

$$e_l^g = \frac{\bar{g}}{\left(\frac{w_l^g}{w_h^g}\frac{\alpha}{1-\alpha}\right)^{\alpha}}. \tag{41}$$

In this case, the composition of government employment depends on public-sector wages. An increase in public-sector unskilled wages makes the government hire more skilled workers and fewer unskilled workers, which can mitigate or amplify the effects of wages. In Section 6, we present quantitative results in this alternative model and compare them with results in the benchmark model, in which public-sector employment is a purely an exogenous policy variable.

5.4 Connections premium

In the benchmark model, we consider that connected and unconnected workers enjoy the same benefits of working in the public sector. We also assume that the costs incurred by the newborns to get connections were wasted. We now assume that newborn pay connections costs to current connected public-sector workers so that current workers will help fast-track them into the public sector. These payments are the "connections premium",Υ_i, that will further raise the value of working in the public sector for connected workers.

$$(r+\tau)E_{i,c}^g = w_i^g + \Upsilon_i - s_i^g \left[E_{i,c}^g - U_{i,c}^g\right]. \tag{42}$$

In equilibrium, this connections premium depends on the threshold of connections costs, $\Upsilon_i = \Upsilon_i(\tilde{c}_i)$. The total connections cost paid by newborns of education type i is $\tau \int_0^{\tilde{c}_i} c\xi(c)dc$. To avoid creating further interactions between sectors, we assume that newborns' total connections cost is divided equally among connected workers of the same education group:

$$\Upsilon_i(\tilde{c}_i) = \frac{\tau \int_0^{\tilde{c}_i} c\xi(c)dc}{\bar{\mu} e_i^g}. \tag{43}$$

In principle, this extension could create multiple equilibria, with people expecting high returns of connections investing in connections (creating a lot of side payments) or people expecting low returns of connections not investing in connections (generating few side payments). We show, in Appendix E, that provided some regularity conditions on the distribution of connections costs are satisfied, there are no multiple equilibria.

In Section 6, we also compare quantitative results in this alternative setup to those obtained in the benchmark model, in which no such connections premium exists.

6 Numerical exercise

The objective of our numerical exercise is threefold. First, we want to verify whether, conditional on an inefficient wage policy, hiring through connections increases or decreases welfare. Second, we want to investigate the results in the full model where changes in meritocracy or in government policy may switch scenarios. More specifically, as discussed earlier, the effects of non-meritocracy on education depend on whether education and connections are substitutes (Case A with $\tilde{c}_l > \tilde{c}_h$) or complements (Case B with $\tilde{c}_h > \tilde{c}_l$). In addition, given the endogenous limits that government policies place on μ_h and μ_l, discussed in Section 3.3, given a set of parameters, we might be in a region where: i) μ_h and μ_l are not constrained and are equal to $\bar{\mu}$; ii) μ_h is constrained but μ_l is not; iii) μ_l is constrained but μ_h is not; or iv) μ_h and μ_l are both constrained. As such, in the full model, there are eight possible scenarios. Changes in government policy may switch scenarios making it difficult to solve for their effect in the full model analytically. Finally, we want to compare the benchmark model with the alternative models proposed in Section 5 – in particular, to compare the transmission mechanisms under the assumptions of segmented markets and random search.

6.1 Parametrization

We parameterize our benchmark model with segmented markets to match the Spanish economy at a quarterly frequency, drawing largely on the *Spanish Labour Force Survey (SLFS)* and the *Structure of Earnings Survey (SES)* microdata for the pre-crisis period 2000-2006. A set of parameters is directly fixed to values taken from the data, while a second set of parameters targets steady-state values. We chose Spain for two reasons. First, according to the report Government at a Glance by OECD (2017), Spain is one of the countries with larger fraction of staff turnover following a change of government, reaching layers of senior and middle management. Second, there is widespread anecdotal evidence of nepotism and chronism in Spain.[10] Table 1 lists all the parameters, their values and the data sources.

From the *Spanish Labour Force Survey*, we calculate the stocks and flows of public- and private-sector workers and the unemployed, by education, for the pre-crisis period of 2005-2007. These are shown in Appendix F. Around 31.6 percent of the population have a college degree, of which 30.4 percent work in the public sector ($e_h^g = 0.097$). Out of the remaining population with no college degree, 9.4 percent work in the public sector ($e_l^g = 0.064$). These numbers reflect the fact that the government predominantly hires skilled workers. Following Gomes (2012), we construct data on worker flows to calibrate the separation rates by education and sector. The numbers are $s_h^g = 0.009$, $s_h^p = 0.019$, $s_l^g = 0.022$ and $s_l^p = 0.026$. The private sector has a higher separation rate than the public sector, particularly for college graduates. The distribution of education costs is assumed to be uniformly distributed between zero and 68 to target the share of college graduates. These numbers imply that the total cost of education is close to 2.6 percent of the total consumption of private-sector goods. In Spain, according to OECD data, private and public spending in tertiary education amounted to 2.1 percent of GDP during the pre-crisis period.

We consider, in the private sector, a Cobb-Douglas matching function with matching efficiency ζ_i and matching elasticity with respect to the unemployment of η_i. As the matching efficiency and the cost of posting vacancies are not separable, we normalize the matching efficiencies $\zeta_h = \zeta_l = 1$. The costs of posting vacancies, κ_h and κ_l, are set to target the unemployment rate of 6.1 percent for college graduates and 9.9 percent for non-college graduates. The matching elasticities are set to the common value of 0.5, and the Hosios condition is assumed to hold ($\eta_h = \eta_l = \beta = 0.5$).

We use microdata from the 2006 *Structure of Earnings Survey* to calculate the college premium and the public-sector wage premium by education. We normalize $y_l = 1$ and set $y_h = 1.378$ to target a private-sector college premium of close to 40 percent found by regression of the log gross hourly earnings on a dummy for college education, using the sub-set of private-sector workers. We set the public-sector wages to target the public-sector wage premium for college and non-college workers. We run regressions of the log gross hourly earnings on a dummy for the public sector, controlling for region, gender, age, occupation, manager status, part-time, tenure and its square. The public-sector wages of the two types

[10] Recently the press exposed that in the"Tribunal de Cuentas", the Spanish institution in charge of invigilating economic and financial irregularities in the public sector, close to 100 of its 700 workers were family members of friends of the directors or of important politicians in Spain.

Table 1: Parametrization of segmented markets model

Fixed parameters	**Source**	**Values**
Government employment	*Spanish LFS*	$e_h^g = 0.097$, $e_l^g = 0.064$
Job-separation rates (private)	*Spanish LFS*	$s_h^p = 0.0185$, $s_l^p = 0.0255$
Job-separation rates (public)	*Spanish LFS*	$s_h^g = 0.0086$, $s_l^g = 0.0220$
Matching elasticities	Standard	$\eta_h = \eta_l = 0.5$
Bargaining power of workers	Hosios Condition	$\beta = 0.5$
Discount rate	Standard	$r = 0.012$
Retirement rate	Standard	$\tau = 0.006$
Matching efficiencies	Normalization	$\zeta_h = \zeta_l = 1$
Productivity unskilled	Normalization	$y_l = 1$
Fraction of non-meritocratic hiring	Corruption Perception Index	$\bar{\mu} = 0.43$
Connections costs upper bound	Set exogenously	$\bar{c} = 34$
Other parameters	**Target (Source)**	**Values**
Public-sector wages	Public-sector wage premium (*SES*)	$w_h^g = 1.460$, $w_l^g = 1.098$
Cost of posting vacancies	Unemployment rates (*LFS*)	$\kappa_h = 2.90$, $\kappa_l = 3.65$
Unemployment benefits	Replacement rate (*EC*)	$b_h = 0.435$, $b_l = 0.425$
Education costs lower bound	Share of college graduates (*LFS*)	$\bar{\epsilon} = 68$
Productivity skilled	College premium (*SES*)	$y_l = 1.378$

are set such that $\frac{w_h^g}{w_h^p} = 1.105$ and $\frac{w_l^g}{w_l^p} = 1.164$. A recent paper by Dickson, Postel-Vinay, and Turon (2014) argues that the lifetime premium in the public sector is lower than that measured by standard cross-section methods. They report that, in Spain, ranges from 4.75 percent in the top 10th percentile of the distribution to 8.45 in the bottom 10th percentile. We report exercises using their numbers. We also report the equilibrium under the efficient public-sector wage premium: $\frac{w_h^g}{w_h^p} = 0.981$ and $\frac{w_l^g}{w_l^p} = 0.961$. These numbers for the optimal wage are interesting, as they are very close to the estimated premium in Finland. Finland is an exception, as its public sector pays a negative premium, which is larger for unskilled workers.

Salomäki and Munzi (1999) find that the net replacement rate is 45 percent for low-educated workers and 33 percent for high-educated workers in Spain. We set $b_l = 0.425$ and $b_h = 0.435$ to target these numbers. Additionally, $r = 0.012$ and $\tau = 0.006$ target a yearly interest rate of about four percent and an average working life of 40 years.

The most relevant parameters are the fraction of jobs reserved for people with connections and the distribution of connections costs, but identifying them is subject to the difficulties that prompted us to approach this question from a theoretical angle. Regarding μ, we proxy it with data from the Corruption Perception Index for 2006. The index varies from 1 (more corrupt) to 10 (least corrupt). We select 30 European countries and normalize the least corrupt country (Finland, with 9.6 points) to a $\bar{\mu} = 0$ and the most corrupt country (Romania, with 3.1) to $\bar{\mu} = 1$. As shown in the figure in Appendix F, we calculate $\bar{\mu}$ of the remaining countries by mapping their index in the relative position within the maximum and the minimum. Spain, with an index of 6.8, is attributed a $\bar{\mu}$ equal to 0.43. The distribution of connections costs is assumed to be uniformly distributed between 0 and 34, half the distribution of education costs. These numbers imply that the deadweight cost of corruption is 0.1 percent of the total consumption of private-sector goods. Most of the exercises consist of varying these parameters. We vary the parameter $\bar{\mu}$ from 0 to 1 and consider high and low values for the upper bound of the distribution of connections costs of $\bar{c} = 10$ and $\bar{c} = 80$.

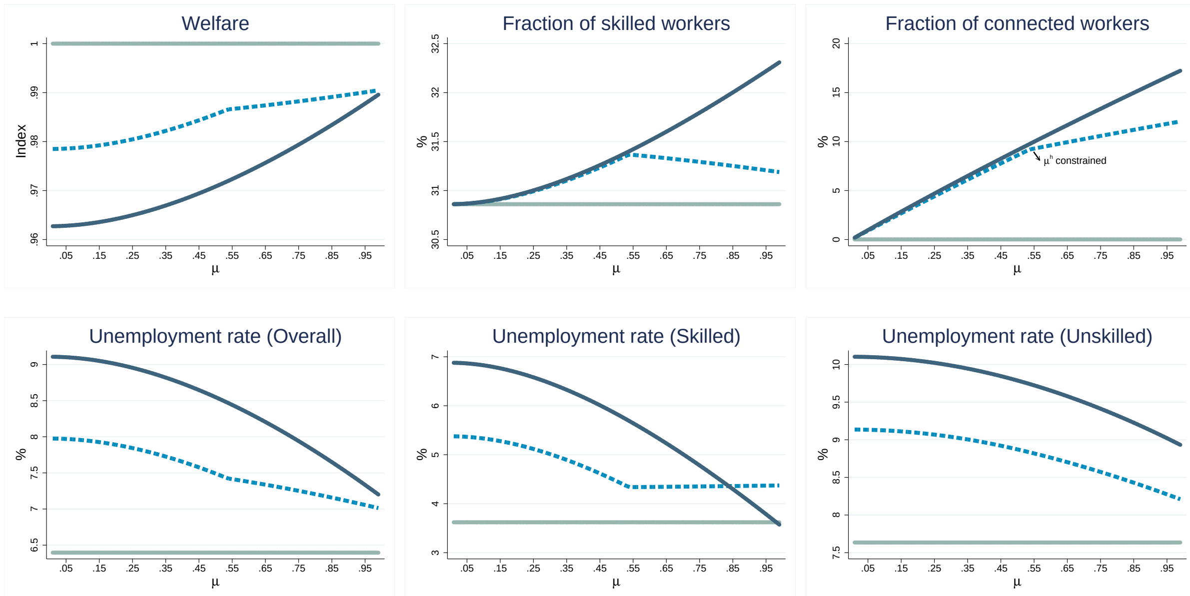
In the baseline steady-state, education and connections are complements, meaning the economy is in Case B. Despite the public-sector premium being higher for workers without a college degree, there are many more jobs available in the public sector for college graduates. Furthermore, because the public-sector premium is so high for skilled and unskilled workers, the government can target the fraction $\bar{\mu}$ of jobs through connections and both μ_h and μ_l are unconstrained.

6.2 Effects of non-meritocracy

We start by analyzing the effects of non-meritocratic hiring for different combinations of public-sector wages and connections costs. We take in account that changes in policies or parameters might trigger the endogenous limit of μ_h or μ_l to bind, as determined by equation 37. Sometimes the government might not achieve its target $\bar{\mu}$ on the skilled or unskilled market, or in both. Figure 5 shows how different variables vary with $\bar{\mu}$ for three different wage policies: the benchmark policy with premia of 11 and 16 percent; an intermediate wage policy with premia of five and eight percent; and the efficient wage policy consisting of premia of -2 and -4 percent. We examine the effects on unemployment rates, the fractions of educated and connected workers, and welfare, calculated as private-sector production net of education and connections costs (as in Section 4), relative to the efficient allocation. As in Gomes (2015), the optimal policy is a negative public-sector wage premium, that is large for unskilled workers in order to compensate for the higher relative job security.

Under the efficient wage policy, μ_h and μ_l are constrained to be zero. There are no queues for public-sector jobs and no connections sector. Unemployment rate is roughly 3 percentage points lower for both skilled and unskilled workers. The higher public-sector

Figure 5: Effects of non-meritocracy, role of public-sector wages



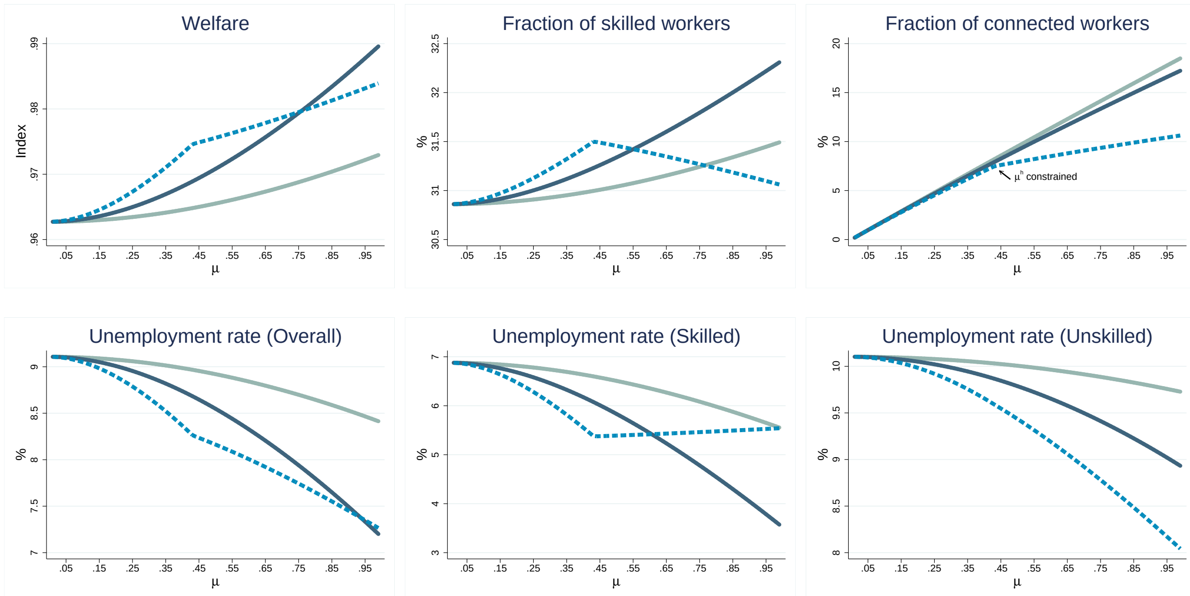
Note: The ***dark blue line*** *is the benchmark calibration ($w_h^g/w_h^p = 1.105$ and $w_l^g/w_l^p = 1.164$). The* *light green line* *is the scenario with efficient public-sector wages ($w_h^g/w_h^p = 0.981$ and $w_l^g/w_l^p = 0.961$). The* ***bright blue dashed line*** *is the scenario with an intermediate public-sector wage premium ($w_h^g/w_h^p = 1.048$ and $w_l^g/w_l^p = 1.085$). Welfare is expressed as a fraction of the efficient steady state. The economy is always in Case B. In the scenario with efficient public-sector wages, both μ_h and μ_l are constrained to zero. In the scenario with intermediate wages, μ_h becomes constrained. Tightness and wages in the private sector are constant and independent of public-sector wages or non-meritocratic hiring ($\theta_h = 0.27$, $\theta_l = 0.13$, $w_h^p = 1.32$,$w_l^p = 0.94$).*

wages are responsible for the higher unemployment and a 3 percent lower welfare relative to the efficient scenario.

The graphs reveal that the effects of non-meritocracy seem to be larger the more inefficient the public-sector wage is. In this numerical exercise, hiring through connections indeed raises welfare. As shown in Proposition 3, it lowers the unemployment rate for both types of workers. By restricting access to public-sector jobs to those with connections, workers are discouraged from searching for unconnected vacancies and turn to the private sector. As tightness is constant, there is a one-to-one effect on private vacancies. While, indeed, the fraction of connected workers increases - with the respective increase in deadweight loss - this is outweighed by the increase in private-sector employment. Thus, welfare increases.

Figure 6 reproduces the same exercise for three levels of connections costs. The increase in welfare from hiring through connections is larger for high levels of connection costs. When the connections costs are higher, the connections market becomes more exclusive. When increasing $\bar{\mu}$, more workers are pushed into the private sector, which implies larger decreases in unemployment and larger increases in welfare.

Figure 6: Effects of non-meritocracy, role of connections costs



*Note: The **dark blue line** is the benchmark calibration ($\bar{c} = 34$). The **light green line** is the scenario with low connections costs ($\bar{c} = 10$). The **bright blue dashed line** is the scenario with high connections costs ($\bar{c} = 80$). Welfare is expressed as a fraction of the efficient steady state. The economy is always in Case B. In the scenario with high connections costs, μ_h becomes constrained. Tightness and wages in the private sector are constant and independent of public-sector wages or non-meritocratic hiring ($\theta_h = 0.27$, $\theta_l = 0.13$, $w_h^p = 1.32$,$w_l^p = 0.94$).*

In all of the scenarios in the two figures, the economy is in Case B, with $\tilde{c}_h > \tilde{c}_l$. Despite a higher public-sector premium for unskilled workers, there are more public-sector job opportunities for skilled workers, making education and connections complements. As shown in Proposition 2, in Case B, education attainment is increasing with $\bar{\mu}$. Notice that when $\bar{\mu}$ is close to 0, the public-sector wage policy does not affect the fraction of educated workers in the economy, as shown in Proposition 1.

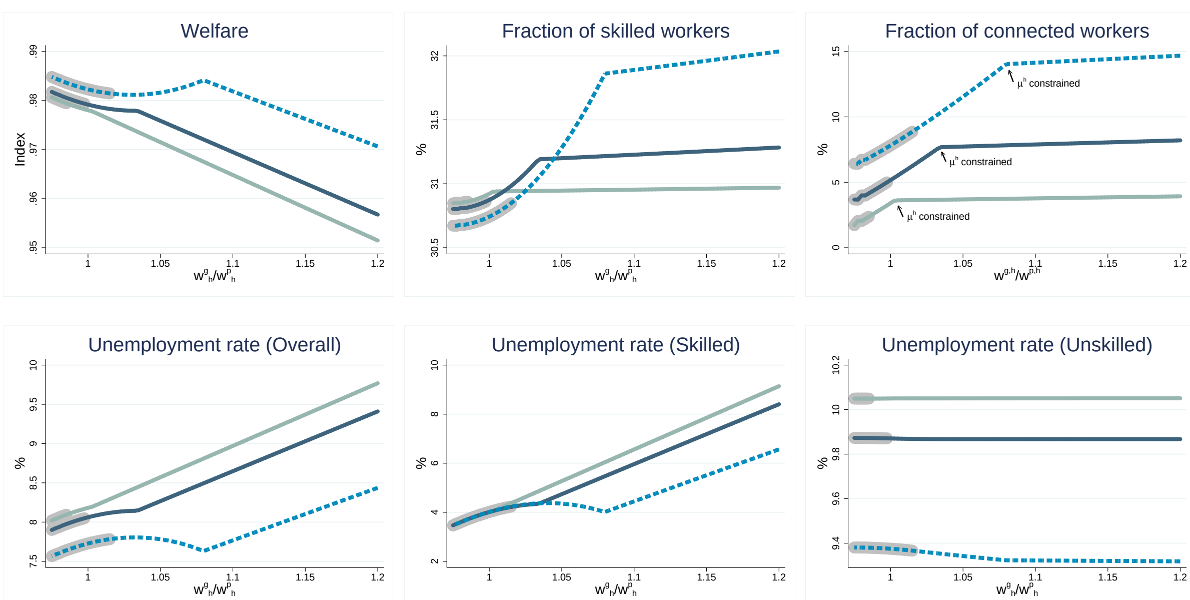
The endogenous limit to non-meritocracy poses non-trivial interactions between skilled and unskilled markets. In Figure 6, the kink observed for high connections costs reflects the fact that, because it is so costly to get connections, the endogenous limit binds for μ_h. The same happens in Figure 5 for the intermediate public-sector wage premium. As shown in Lemma 2, the minimum wage for the government to be able to fill a fraction $\bar{\mu}$ of jobs through connections – $\underline{\underline{w}}^g_{i,c}$ – is increasing in $\bar{\mu}$. If the public-sector wage is not high enough to sustain a large connections sector (that is $w_i^g < \underline{\underline{w}}^g_{i,c}$), the endogenous limits bind and μ_i is determined by equation (36). In this particular numerical example, because the public-sector wage premium is lower and public-sector employment is larger for the skilled, it is the connections market of the skilled worker that binds. Increases of $\bar{\mu}$ beyond 0.55 (Figure 5) or 0.44 (Figure 6) affect mainly the connections sector of the unskilled segment. Actually, μ_h, as determined by equation (36), decreases with $\bar{\mu}$, which explains why the unemployment rate of the skilled goes up, marginally, when μ_h is constrained.

6.3 Effects of skill-biased policies

Figures 7 and 8 show the effect of skilled and unskilled public-sector wages, for three levels of target $\bar{\mu}$: 0.2, 0.43 and 0.8. Decreasing the public-sector wages of skilled workers lowers education attainment. The effect is particularly strong when $\bar{\mu}$ is high, but μ_h is constrained. In that case, a 20 percent decrease in wages reduces the fraction of educated workers by one percentage point. Again, this large effect is driven by the combination of the wage cuts and the consequent decrease in μ_h. Given the complementarity between education and connections, the decrease in μ_h further reduces education incentives.

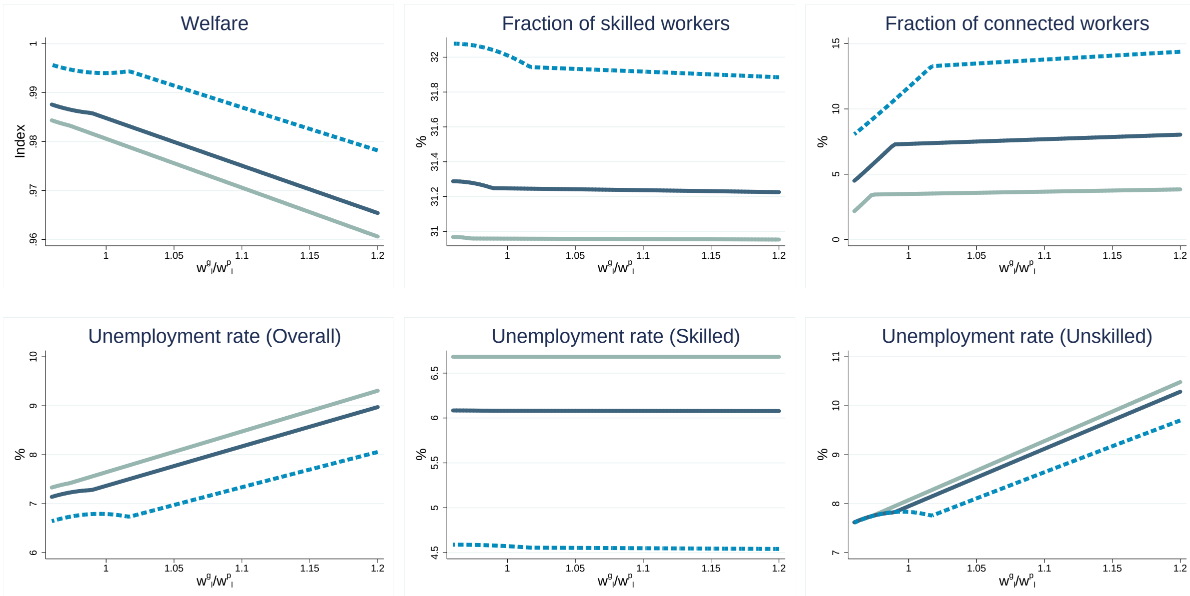
In general, decreasing skilled public-sector wages raises welfare. However, for some combination of parameters (high $\bar{\mu}$ but constrained μ_h), there is a region in which welfare declines with skilled-wage cuts. This happens because, as shown in Proposition 4, μ_h decreases with wage cuts, pushing more unemployed workers to switch from the private sector to search for the newly available unconnected public-sector jobs, thus driving up the skilled and overall

Figure 7: Effects of skilled public-sector wages



Note: The ***dark blue line*** *is the benchmark calibration ($\bar{\mu} = 0.43$). The* ***light green line*** *is the scenario with low non-meritocracy ($\bar{\mu} = 0.2$). The* ***bright blue dashed line*** *is the scenario with high non-meritocracy ($\bar{\mu} = 0.8$). Welfare is expressed as a fraction of the efficient steady state. Under the grey area, the economy is in Case A. Otherwise, it is in Case B. In all scenarios, when skilled public-sector wages are low, μ_h becomes constrained. Tightness and wages in the private sector are constant and independent of public-sector wages or non-meritocratic hiring ($\theta_h = 0.27$, $\theta_l = 0.13$, $w_h^p = 1.32$,$w_l^p = 0.94$).*

Figure 8: Effects of unskilled public-sector wages



*Note: The **dark blue line** is the benchmark calibration ($\bar{\mu} = 0.43$). The **light green line** is the scenario with low non-meritocracy ($\bar{\mu} = 0.2$). The **bright blue dashed line** is the scenario with high non-meritocracy ($\bar{\mu} = 0.8$). Welfare is expressed as a fraction of the efficient steady state. The economy is always in Case B. In all scenarios, when unskilled public-sector wages are low, μ_l becomes constrained. Tightness and wages in the private sector are constant and independent of public-sector wages or non-meritocratic hiring ($\theta_h = 0.27$, $\theta_l = 0.13$, $w_h^p = 1.32$,$w_l^p = 0.94$).*

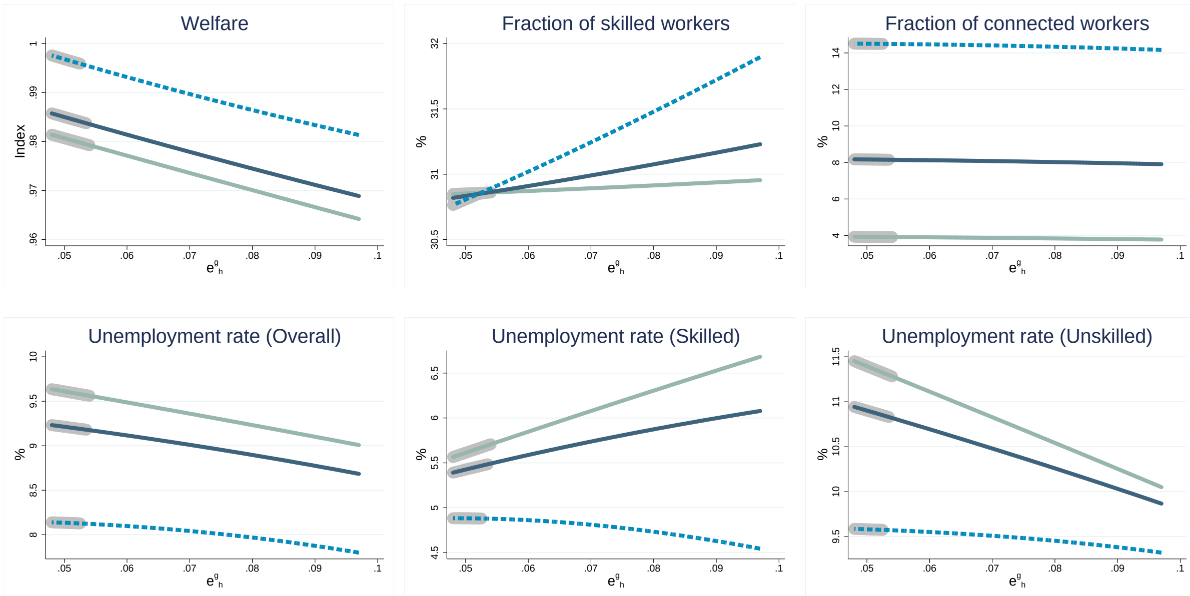
unemployment rates. When the skilled-wage cuts are too extreme, this effect is weakened.

Decreasing the public-sector wages of skilled workers lowers education attainment. The effect is particularly strong when $\bar{\mu}$ is high, but μ_h is constrained. In that case, a 20 percent decrease in wages reduces the fraction of educated workers by one percentage point. Again, this large effect is driven by the combination of the wage cuts and the consequent decrease in μ_h. Given the complementarity between education and connections, the decrease in μ_h further reduces education incentives.

At low levels of skilled public-sector wages, the economy moves from Case B to Case A, as highlighted in the grey shaded area, making education and connections substitutes. We see an implication of this when looking at the fraction of skilled workers. Education is increasing in $\bar{\mu}$ for high wage levels where the economy is in Case B, but decreasing in $\bar{\mu}$ for low wage levels where the economy is in Case A.

Cutting unskilled public-sector wages also has a positive effect on welfare and a negative effect on the unemployment rate, particularly that of the unskilled. A ten percent cut in the wages of unskilled public-sector workers lowers their unemployment rate by one percentage point. However, the effect of unskilled wage cuts on the skilled unemployment rate is minimal. The effects of unskilled wage cuts on educational attainment are quantitatively

Figure 9: Effects of skilled public-sector employment



*Note: The **dark blue line** is the benchmark calibration ($\bar{\mu} = 0.43$). The **light green line** is the scenario with low non-meritocracy ($\bar{\mu} = 0.2$). The **bright blue dashed line** is the scenario with high non-meritocracy ($\bar{\mu} = 0.8$). Welfare is expressed as a fraction of the efficient steady state. Under the grey area, the economy is in Case A. Otherwise, it is in Case B. In all scenarios, μ_h is never constrained. Tightness and wages in the private sector are constant and independent of public-sector wages or non-meritocratic hiring ($\theta_h = 0.27$, $\theta_l = 0.13$, $w^p_h = 1.32$,$w^p_l = 0.94$).*

small, suggesting that the wages that government pays skilled workers are a more important determinant of education than the wages it pays to unskilled workers. Finally, as in the previous exercise, if wage cuts are too severe, then μ_l becomes constrained.

Figure 9 shows the effects on the economy of increasing skilled public-sector employment. We keep total public-sector employment constant by decreasing the number of unskilled public-sector workers. As expected, hiring more skilled workers induces newborn to become educated, especially when hiring is less meritocratic. Substituting unskilled for skilled workers by four percentage points of the population can raise education by one percentage point. The point at which the three lines cross reflects the change in case. To the right of that point, we are in Case B, where education increases with $\bar{\mu}$, whereas to the left of that point, we are in Case A, where education decreases with $\bar{\mu}$.

The effect on unemployment might seem surprising. Hiring more skilled rather than unskilled workers raises the unemployment rate of skilled workers and lowers that of the unskilled. Many skilled public-sector jobs induce workers to queue for these high-value jobs, whether in the connected or the unconnected market. On the other hand, having few unskilled public-sector jobs reduces the queues of workers that now search for private-sector jobs.

6.4 Comparing different models

We now compare the results from the baseline segmented market model with those from the alternative models discussed in Section 5. For the model in which search in the public and private sectors is random, we reparameterize the cost of posting vacancies to target the steady-state unemployment rate ($\kappa_h = 4.45$, $\kappa_l = 4.25$). We follow the same procedure for the model with a "connections premium" ($\kappa_h = 2.80$, $\kappa_l = 3.65$). Once recalibrated, the steady state of the remaining variables is very close to that of the benchmark model. In the model with endogenous public-sector employment, we set $\alpha = 0.668$ and $\bar{g} = 0.0845$ to get the exact same steady state as in the benchmark model.

Table 2 shows the effects of four different policies: i) a decrease in $\bar{\mu}$ from 0.43 to 0.2; ii) a ten-percent decrease in skilled public-sector wages; iii) a ten-percent decrease in unskilled public-sector wages; and iv) a decrease in skilled public-sector employment by one percent of the population, compensated by an increase of one percent in unskilled public-sector employment.

We start by comparing the model with segmented markets with the model of random search. Graphs with a more detailed comparison are shown in Appendix G. We can see in the table that random search in the labor market weakens the effects of policies on unemployment but amplifies the effects on educational attainment. Although the effects go in the same direction, the mechanisms at work are different. Under random search, non-meritocracy affects job creation (tightness) positively and private wages negatively. By having fewer unconnected vacancies, the outside option of an unemployed worker bargaining with a firm is weaker, pushing wages down and raising job creation. This effect on private wages raises the public-sector wage premium endogenously.

Under random search, as discussed in Section 5.1, the effects of $\bar{\mu}$ on welfare are ambiguous. As Figure A4 in Appendix G shows, it is negative for low levels of $\bar{\mu}$ but positive for high levels of $\bar{\mu}$. This is consistent with the fact that under $\bar{\mu} = 1$, the model resembles the model with segmented markets, proven in Proposition 6. When we move from $\bar{\mu} = 0.43$ to $\bar{\mu} = 0.20$, the effect on welfare is marginally positive.

The other main result from the paper - the effects of government policies on education - is amplified in the random search model. In Section 5.1.2, we could not prove that policies that raise the value of working in the public sector for skilled workers raise educational attainment. However, in the numerical exercise, the effect is not only positive but quantitatively much larger than in segmented markets. When cutting skilled wages by ten percent, education attainment goes down by almost two percentage points, as opposed to 0.4 percentage points in the segmented markets model. When cutting unskilled wages by ten percent, education attainment goes up by 0.4 percentage points, as opposed to 0.01 percentage points in the

Table 2: Effects of policies under different models

Policy	**Segmented markets**	**Random search**	**Endogenous e_h^g and e_l^g**	**Connections premium**
Reduction on non-meritocracy to $\bar{\mu} = 0.20$				
%Δ welfare	-0.48%	0.01%	-0.48%	-0.24%
Δ fraction of skilled	-0.28 p.p.	0.01 p.p.	-0.28 p.p.	-0.28 p.p.
Δ fraction of connected	-4.13 p.p.	-4.14 p.p.	-4.13 p.p.	-4.17 p.p.
Δ unemployment rate	0.32 p.p.	0.04 p.p.	0.32 p.p.	0.28 p.p.
Reduction on skilled wages by 10 percent				
%Δ welfare	1.30%	-0.16%	1.07%	1.72%
Δ fraction of skilled	-0.35 p.p.	-1.91 p.p.	-0.32 p.p.	-0.29 p.p.
Δ fraction of connected	-1.92 p.p.	-2.16 p.p.	-2.61 p.p.	-2.06 p.p.
Δ unemployment rate	-0.43 p.p.	-0.33 p.p.	-0.67 p.p.	-0.65 p.p.
Reduction on unskilled wages by 10 percent				
%Δ welfare	1.00%	0.74%	0.99%	1.03%
Δ fraction of skilled	0.01 p.p.	0.41 p.p.	-0.02 p.p.	0.01 p.p.
Δ fraction of connected	-0.35 p.p.	-0.38 p.p.	-0.29 p.p.	-0.36 p.p.
Δ unemployment rate	-0.80 p.p.	-0.62 p.p.	-0.80 p.p.	-0.81 p.p.
Reduction on skilled employment and increase in unskilled employment by 1 p.p. §				
%Δ welfare	0.34%	0.27%	-0.48%	0.58%
Δ fraction of skilled	-0.09 p.p.	-0.38 p.p.	-0.10 p.p.	-0.09 p.p.
Δ fraction of connected	0.07 p.p.	0.07 p.p.	0.36 p.p.	0.07 p.p.
Δ unemployment rate	0.13 p.p.	0.10 p.p.	0.22 p.p.	0.13 p.p.

Note: The random search and connections premium models are recalibrated ($\kappa_h = 4.45$, $\kappa_l = 4.25$) and ($\kappa_h = 2.80$, $\kappa_l = 3.65$). In the public sector employment model, $\alpha = 0.668$ and $\bar{g} = 0.0845$, to get the $e_h^g = 0.097$ and $e_l^g = 0.064$ in the initial steady state. §: for the endogenous public-sector employment model, we reduce skilled employment by 1 p.p. but increase unskilled employment to maintain the same level of government services (requires an increase of 1.56 p.p.). In all scenarios, before and after the policy, μ_h and μ_l are unconstrained except for the skilled public-sector wage cuts that end up constraining μ_h.

segmented markets model.

Finally, under random search, reductions in the wages of unskilled public-sector workers are more welfare-increasing, than reductions in the wages of skilled public-sector workers, contrary to the model with segmented markets.

Turning, now, to the model with endogenous public-sector employment, the effects of changes in wages tend to be mitigated, compared to the benchmark model. When there is a wage cut for one type of worker, the government readjusts its hiring and increases the number of workers of that particular type. When cutting ten percent of skilled wages, the government hires 0.3 p.p. more of skilled workers and 0.4 p.p. less of unskilled workers. When cutting ten percent of unskilled wages, the government hires 0.3 p.p. less of skilled workers and

0.4 p.p. more of unskilled workers. This effect works in the opposite direction, partially compensating for the fall in the value of searching in the public sector. Still, the effects are quantitatively very similar. In the experiment of reducing skilled public-sector employment, we compensate for the reduction with an increase in the unskilled workers needed to maintain the production of government services, which amounted to 1.56 percentage points (versus one percentage point in the other models). Given this, the policy of hiring fewer skilled public-sector workers reduces welfare. The results for changes in $\bar{\mu}$ are exactly the same as in the benchmark model.

Finally, the presence of a "connections premium" tends to amplify the effects of policies on the number of connected workers, but because the premium represents only one percent of public-sector wages for skilled workers and 0.6 percent for unskilled workers, the effects are quantitatively similar to those in the benchmark model.

To sum up, we can draw five main conclusions from this section. First, under the baseline model, parameterized to a country with a large public-sector wage premia, welfare is increasing in $\bar{\mu}$, and so is educational attainment. Second, changes in skilled public-sector wages have larger impacts on educational attainment than changes in unskilled wages. Third, public-sector wage cuts have a large quantitative effect on reducing the unemployment rate. Fourth, there are non-trivial interactions when combinations of policies place an endogenous limit, typically in μ_h, creating asymmetries between the skilled and unskilled labor markets. Fifth, in the random search model, the effects of policies on unemployment are quantitatively smaller than in the model with segmented markets, but have a much larger quantitative effect on educational attainment.

7 Conclusion

This paper provides a benchmark model to understand how public-sector hiring and wage policies affect education decisions and employment. The model takes in account one pervasive characteristic in many public sectors - that not all hiring is meritocratic. Our results provide insights that can explain several European cross-country facts.

The previous literature highlights the problems of setting high public-sector wages. For example, Gomes (2015) and Afonso and Gomes (2014) shown that they generate higher unemployment. Cavalcanti and Santos (2017) argue that higher wages might lead to misallocation of resources with a lower entrepreneurship rate. We highlight two additional negative effects. First, higher public-sector wages might lead workers to pursue rent-seeking activities. Second, higher public-sector wages for unskilled workers lead to lower incentives for education.

We have shown that the existence of a "connections" market for public jobs requires that public-sector wages are very high compared to those in the private sector. This result is consistent with evidence that Southern European countries known for having non-meritocratic hiring have a higher public-sector wage premium, while Nordic countries, in which the government follows more meritocratic hiring, tend to have a lower or a negative public-sector wage premium. The results also suggest why these practices are so accepted in Souther European countries and why governments might maintain the status quo of the hiring process. Conditional on high public-sector wages and long queues for public-sector jobs, non-meritocratic hiring actually lowers unemployment. Given the high public-sector wage premium, the presence of non-meritocratic hiring might be constrained efficient.

We have also shown that government policies affect education decisions, particularly when a "connections" sector exists. In this case, a smaller education premium in the public sector reduces the incentives of the newborn to become high-educated. This finding can perhaps explain, in part, why Southern European countries have lower education attainment than Nordic Countries.

Although we have emphasized the role of non-meritocratic hiring, our model is very general, and some of the results can be extrapolated to other country-specific public-sector characteristics. Dickson, Postel-Vinay, and Turon (2014) find that countries with a positive lifetime premium of the public sector, France and Spain in their sample, are also the countries that require costly entry procedures, such as national exams. We could reinterpreted the model, considering the cost of connections as the monetary cost of preparing for an exam, and μ the fraction of civil servants hired through an exam. We would conclude that, although this channel would be inefficient, conditional on an inefficient wage policy, it might be one way to minimize the effects on unemployment.

In this paper, we have compared two benchmark models that have been used in the literature: segmented market versus random search. While the main results are consistent across the two models, the quantitative implications of policies are different. If we want to develop a model that can be used for policy analysis and forecasting, it is imperative that we realistically model the labor market. In this regard, it is important to design an empirical test to determine which model is more suitable to use.

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COMPANION APPENDIX

Meritocracy, public-sector pay and human capital accumulation

Andri Chassamboulli and Pedro Gomes

A Proofs of propositions

A.1 Lemma 2

We consider that the public-sector unconnected labour market for workers of type i breaks down if the government is not able to hire enough workers to replace the workers that have lost their job. At the limit, it means the government needs to post a wage, defined as $\underline{w}^g_{i,u}$, such that it attracts at least $(1-\mu_i)(s^g_i+\tau)e^g_i$ job searches. This means $u^g_{i,u} = (1-\mu_i)(s^g_i+\tau)e^g_i$ and the job-finding rate is 1 ($m^g_{i,u} = 1$). Applying this to (16) and then setting $U^p_i = U^g_{i,u}$ gives

$$b_i + \frac{1}{r+\tau+s^g_i+1}[\underline{w}^g_{i,u} - b_i] = (r+\tau)U^p_i$$

Substituting the $(r+\tau)U^p_i$ by equation (15) we get

$$\underline{w}^g_{i,u} = \frac{(r+\tau+s^g_i+1)m(\theta^*_i)}{r+\tau+s^p_i+m(\theta^*_i)}[w^{p,*}_i - b_i] + b_i$$

where θ^*_i and $w^{p,*}_i$ are the equilibrium tightness and wages in the private sector.

If $\mu_i = 0$ then no connections sector exists and all workers hired into the public sector are unconnected. If, on the other hand, a connections sector exists then a share μ_i of public-sector workers are hired through connections. For the existence of a connections sector, through which the government is able to hire a fraction μ_i of its employees the government needs to attract at least $\mu_i(s^g_i+\tau)e^g_i$ connected job searchers. This means that it has to pay a higher wage, $\underline{w}^g_{i,c}$, which compensates connected workers for the cost of getting connections.

$$\underline{w}^g_{i,c} = \underline{w}^g_{i,u} + \Xi^{c,-1}(\mu_i(s^g_i+\tau)e^g_i)(r+\tau+s^g_i+1)$$

where $\Xi^{c,-1}$ is the inverse of the distribution of "connection" cost. What it means is that, at the margin, the government has to pay high enough wages such that a sufficient high mass of unemployed decide to pay the cost.

Notice that $\underline{w}^g_{i,c}$ is increasing in μ_i, while $\underline{w}^g_{i,u}$ is independent of μ_i. If $\mu_i = 0$ then we get $\underline{w}^g_{i,c} = \underline{w}^g_{i,u}$, whereas if $\mu_i = \bar{\mu}$ then $\underline{w}^g_{i,c} = \underline{\underline{w}}^g_{i,c}$ where

$$\underline{\underline{w}}^g_{i,c} = \underline{w}^g_{i,u} + \Xi^{c,-1}(\bar{\mu}(s^g_i+\tau)e^g_i)(r+\tau+s^g_i+1)$$

A.2 Proof of Existence and Uniqueness of a Steady-State Equilibrium

Proof. It can be easily verified that the two free-entry conditions in (13) pin down a unique set of equilibrium values for θ_h and θ_l. Substituting these values into (28) we get the unique equilibrium value for $\tilde{\epsilon}_u$. To complete the proof of existence and uniqueness we need to show that with the equilibrium values of θ_h and θ_l and $\tilde{\epsilon}_u$ substituted in, the two threshold conditions in (27), $\tilde{c}_h = U_{h,c} - U_h$ and $\tilde{c}_l = U_{l,c} - U_l$ only cross once in the $[\tilde{c}_h, \tilde{c}_l]$ plane giving a unique set of equilibrium values for $\tilde{c}_h$ and $\tilde{c}_l$.

Let as write (27) as:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[A_{i,c} - \frac{\beta\kappa_i\theta_i}{(1-\beta)}\right], \quad i = [h,l] \tag{44}$$

where

$$A_{i,c} \equiv \frac{\frac{\mu(s_i^g+\tau)e_i^g}{L_{i,c}^g-\mu e_i^g}}{r+\tau+s_i^g+\frac{\mu(s_i^g+\tau)e_i^g}{L_{i,c}^g-\mu e_i^g}}(w_i^g - b_i) \tag{45}$$

By total differentiation of (44) we can derive their slopes:

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h=U_{h,c}-U_h} = \frac{\frac{\partial A_{h,c}}{\partial L_{h,c}^g}\frac{\partial L_{h,c}^g}{\partial \tilde{c}_l}\frac{1}{r+\tau}}{1-\frac{\partial A_{h,c}}{\partial L_{h,c}^g}\frac{\partial L_{h,c}^g}{\partial \tilde{c}_h}\frac{1}{r+\tau}} > 0 \tag{46}$$

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l=U_{l,c}-U_l} = \frac{1-\frac{\partial A_{l,c}}{\partial L_{l,c}^g}\frac{\partial L_{l,c}^g}{\partial \tilde{c}_l}\frac{1}{r+\tau}}{\frac{\partial A_{l,c}}{\partial L_{l,c}^g}\frac{\partial L_{l,c}^g}{\partial \tilde{c}_h}\frac{1}{r+\tau}} > 0 \tag{47}$$

Both slopes are positive, since, as can be easily verified from (45) and (24)-(**??**), $\frac{\partial A_{i,c}}{\partial L_{i,c}^g} < 0$, $\frac{\partial L_{i,c}^g}{\partial \tilde{c}_i} > 0$ and $\frac{\partial L_{i,c}^g}{\partial \tilde{c}_j} < 0$. But it can also be shown that $\frac{\partial L_{i,c}^g}{\partial \tilde{c}_i} > -\frac{\partial L_{i,c}^g}{\partial \tilde{c}_j} > 0$ so that:

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h=U_{h,c}-U_h} < 1$$
$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l=U_{l,c}-U_l} > 1$$

This completes the proof of existence and uniqueness. The two loci only cross once in the $[\tilde{c}_h, \tilde{c}_l]$ plane giving a unique set of equilibrium values for $\tilde{c}_h$ and $\tilde{c}_l$. ■

A.3 Proof of Proposition 1

Proof. Consider first case $\mu = 0$. If $\mu = 0$ then $\tilde{c}_h = \tilde{c}_l = 0$. Adding up $L_{h,u}$ and $L_{h,c}^g$ using equations (24) and (25) (case C) we get $L_h = \Xi^\epsilon(\tilde{\epsilon}_u)$ and $L_l = 1 - \Xi^\epsilon(\tilde{\epsilon}_u)$. Both depend only on $\tilde{\epsilon}_u$ which as can be verified from (28) it is independent of public-sector wages, employment or separation rates and depends only on private sector parameters.

Consider now $\mu > 0$. Using (27) we can derive for $x_i = [w_i^g, s_i^g, e_i^g]$, $i = [h,l]$, $j = [h,l]$

and $j \neq i$ that

$$\frac{d\tilde{c}_i}{dx_i} = \frac{\frac{\partial A_{i,c}}{\partial x_i}}{r+\tau-\frac{\partial A_{i,c}}{\partial L^g_{i,c}}\left[\frac{\partial L^g_{i,c}}{\partial \tilde{c}_i}+B_j\frac{\partial L^g_{i,c}}{\partial \tilde{c}_j}\right]} \quad (48)$$

$$\frac{d\tilde{c}_j}{dx_i} = B_j\frac{d\tilde{c}_i}{dx_i} \quad (49)$$

where where $A_{j,c}$ is as defined in (45) above and

$$B_j = \frac{\frac{\partial A_{j,c}}{\partial L^g_{j,c}}\frac{\partial L^g_{j,c}}{\partial \tilde{c}_i}}{r+\tau-\frac{\partial A_{j,c}}{\partial L^g_{j,c}}\frac{\partial L^g_{j,c}}{\partial \tilde{c}_j}} \quad (50)$$

It can be easily verified from (45) that $\frac{\partial A_{j,c}}{\partial L^g_{j,c}} < 0 \left(\frac{\partial A_{i,c}}{\partial L^g_{i,c}} < 0\right)$ and from (24)-(**??**) that $\frac{\partial L^g_{j,c}}{\partial \tilde{c}_j} > -\frac{\partial L^g_{j,c}}{\partial \tilde{c}_i} > 0 \left(\frac{\partial L^g_{i,c}}{\partial \tilde{c}_i} > -\frac{\partial L^g_{i,c}}{\partial \tilde{c}_j} > 0\right)$ so that $1 > B_j > 0$ ($1 > B_i > 0$). These imply that the denominator of (48) is positive. From (45) we also know that the numerator of (48) is also positive since $\frac{\partial A_{i,c}}{\partial x_i} > 0$. We can therefore conclude that:

$$\frac{d\tilde{c}_i}{dx_i} > 0 \text{ and } \frac{d\tilde{c}_j}{dx_i} > 0 \text{ for } x_i = [w^g_i, s^g_i, e^g_i],\ i = [h,l],\ j = [h,l] \text{ and } j \neq i \quad (51)$$

With (21)-(23) substituted in (24) and (25) we can derive an expression for $L_h (= L_{h,u} + L^g_{h,c})$ in terms of only $\tilde{c}_h$, $\tilde{c}_l$ and model parameters so that:

$$\frac{dL_h}{dx_i} = \frac{\partial L_h}{\partial \tilde{c}_h}\frac{d\tilde{c}_h}{dx_i} + \frac{\partial L_h}{\partial \tilde{c}_l}\frac{d\tilde{c}_l}{dx_i} \quad (52)$$

Using (49) we can write:

$$\frac{dL_h}{dx_h} = \frac{d\tilde{c}_h}{dx_h}\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l\frac{\partial L_h}{\partial \tilde{c}_l}\right] \quad (53)$$

$$\frac{dL_h}{dx_l} = \frac{d\tilde{c}_l}{dx_l}\left[\frac{\partial L_h}{\partial \tilde{c}_h}B_h + \frac{\partial L_h}{\partial \tilde{c}_l}\right] \quad (54)$$

Next we derive expressions for the terms in the brackets:

$$\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l \frac{\partial L_h}{\partial \tilde{c}_l}\right] = \begin{cases} \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h)\left[\frac{r+\tau-\frac{\partial A_{l,c}}{\partial L^g_{l,c}}\xi^c(\tilde{c}_l)(1-\Xi^\epsilon(\tilde{\epsilon}_u))}{r+\tau-\frac{\partial A_{l,c}}{\partial L^g_{l,c}}\frac{\partial L^g_{l,c}}{\partial \tilde{c}_l}}\right], & \text{if } \tilde{c}_h < \tilde{c}_l \\ \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l)\left[1-B_l\right] + \int_{\tilde{c}_l}^{\tilde{c}_h}\xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_l < \tilde{c}_h, \end{cases} \tag{55}$$

$$\left[\frac{\partial L_h}{\partial \tilde{c}_l} + B_h \frac{\partial L_h}{\partial \tilde{c}_h}\right] = \begin{cases} -\xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h)\left[1-B_h\right] - \int_{\tilde{c}_h}^{\tilde{c}_l}\xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_h < \tilde{c}_l \\ -\xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l)\left[\frac{r+\tau-\frac{\partial A_{h,c}}{\partial L^g_{h,c}}\xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_u)}{r+\tau-\frac{\partial A_{h,c}}{\partial L^g_{h,c}}\frac{\partial L^g_{h,c}}{\partial \tilde{c}_h}}\right], & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases} \tag{56}$$

The terms in (55) are positive while the terms in (56) negative. Using (51) we can therefore conclude that:

$$\frac{dL_i}{dx_i} > 0, \frac{dL_i}{dx_j} < 0,\ x_i = [w^g_i, s^g_i, e^g_i],\ i = [h,l],\ j = [h,l] \text{ and } j \neq i \tag{57}$$

■

A.4 Proof of Proposition 2

Proof. If $\mu = 0$ then $\tilde{c_h} = \tilde{c_h} = 0$ and from (24) and (25) we obtain

$$L_h = \Xi^\epsilon(\tilde{\epsilon}_u) \tag{58}$$

while if $\mu > 0$ we get

$$L_h = \Xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_c) + \int_{\tilde{c}_h}^{\tilde{c}_l}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_u),\ \text{if } \tilde{c}_h < \tilde{c}_l \tag{59}$$

$$L_h = \Xi^c(\tilde{c}_l)\Xi^\epsilon(\tilde{\epsilon}_c) + \int_{\tilde{c}_l}^{\tilde{c}_h}\Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_u),\ \text{if } \tilde{c}_l < \tilde{c}_h \tag{60}$$

Subtracting (59) from (58) we obtain

$$L^{\mu=0}_h - L^{\mu>0}_h = \int_{\tilde{c}_h}^{\tilde{c}_l}\left[\Xi^\epsilon(\tilde{\epsilon}_u) - \Xi^\epsilon(\tilde{\epsilon}_m(c))\right]d\Xi^c(c) + \Xi^c(\tilde{c}_h)\left[\Xi^\epsilon(\tilde{\epsilon}_u) - \Xi^\epsilon(\tilde{\epsilon}_c)\right] > 0 \tag{61}$$

As shown above, $\tilde{\epsilon}_c \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_u$, if $\tilde{c}_h < \tilde{c}_l$, implying that the terms in the brackets are positive. Subtracting (60) from (58) we obtain

$$L^{\mu=0}_h - L^{\mu>0}_h = -\int_{\tilde{c}_l}^{\tilde{c}_h}\left[\Xi^\epsilon(\tilde{\epsilon}_m(c)) - \Xi^\epsilon(\tilde{\epsilon}_u)\right]d\Xi^c(c) - \Xi^c(\tilde{c}_l)\left[\Xi^\epsilon(\tilde{\epsilon}_c) - \Xi^\epsilon(\tilde{\epsilon}_u)\right] < 0 \tag{62}$$

As shown above, $\tilde{\epsilon}_c \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_u$, if $\tilde{c}_h > \tilde{c}_l$, implying that the terms in the brackets are positive.

■

A.5 Proof of Proposition 3

Proof.
Fist, we show that $\frac{dL^p}{d\mu} > 0$:
Let $L_i^g = L_{i,u}^g + L_{i,c}^g$ denote the total number of workers of skill type i that are either employed or are searching in the public sector. Using the conditions (18) and (20) to solve, respectively, for $L_{i,u}^g$ and $L_{i,c}^g$, and then adding them up gives:

$$L_i^g = e_i^g \left[\lambda + (1-\lambda) \left(\frac{(w_i^g - b_i)(1-\beta)}{\beta \kappa_i \theta_i} \right) \left(\frac{\beta \kappa_i \theta_i + (1-\mu)(1-\beta)\tilde{c}_i(r+\tau)}{\beta \kappa_i \theta_i + (1-\beta)\tilde{c}_i(r+\tau)} \right) \right] \tag{63}$$

where $\lambda = \frac{r}{r+s_i^g+\tau}$. Note that equation (13) can be solved for the equilibrium value of θ_i which is independent of μ; thus $\frac{d\theta_i}{d\mu} = 0$. Given this, we can write:

$$\frac{dL_i^g}{d\mu} = \frac{\partial L_i^g}{\partial \mu} + \frac{\partial L_i^g}{\partial \tilde{c}_i} \frac{d\tilde{c}_i}{d\mu} < 0 \tag{64}$$

and using (27) we can derive that for $i = [h, l]$, $j = [h, l]$ and $j \neq i$:

$$\frac{d\tilde{c}_i}{d\mu} = \frac{\Delta_i + B_i \Delta_j}{1 - B_i B_j} > 0 \tag{65}$$

where B_i and B_j are as defined in (50) above, and

$$\Delta_i = \frac{\frac{\partial A_{i,c}}{\partial \mu}}{r + \tau - \frac{\partial A_{i,c}}{\partial L_{i,c}^g} \frac{\partial L_{i,c}^g}{\partial \tilde{c}_i}}$$

As explained above, $1 > B_i > 0$ and $1 > B_j > 0$, meaning that the denominator in (65) is positive. The numerator is also positive since, from the expression for $A_{i,c}$ in (45) we get that $\frac{\partial A_{i,c}}{\partial L_{i,c}^g} < 0$ $\left(\frac{\partial A_{j,c}}{\partial L_{j,c}^g} < 0\right)$, $\frac{\partial A_{i,c}}{\partial \mu} > 0$ $\left(\frac{\partial A_{j,c}}{\partial \mu} > 0\right)$ and from (24)-(**??**) that $\frac{\partial L_{i,c}^g}{\partial \tilde{c}_i} > 0$ $\left(\frac{\partial L_{j,c}^g}{\partial \tilde{c}_j} > 0\right)$ so that $\Delta_i > 0$ ($\Delta_j > 0$). Therefore, it must be the case that $\frac{d\tilde{c}_i}{d\mu} > 0$. Moreover, it can be easily verified from (63) that $\frac{\partial L_i^g}{\partial \mu} < 0$ and $\frac{\partial L_i^g}{\partial \tilde{c}_i} < 0$, implying from (64) that $\frac{dL_i^g}{d\mu} < 0$. Given that $L^p = 1 - L_h^g - L_u^g$, it follows that $\frac{dL^p}{d\mu} > 0$.

Next we show that $\frac{du_i^g}{d\mu} < 0$ while $\frac{de_i^g}{d\mu} = 0$. The number of workers searching in the public sector with and without connections are given, respectively, by $u_{i,c}^g = L_{i,c}^g - \mu e_i^g$ and $u_{i,u}^g = L_{i,u}^g - (1-\mu)e_i^g$. By adding them up we get $u_i^g = u_{i,u}^g + u_{i,c}^g = L_i^g - e_i^g$. The number of type i workers employed in the public sector, e_i^g, is exogenous and independent of μ, while, as shown above, $\frac{dL_i^g}{d\mu} < 0$. It follows that $\frac{du_i^g}{d\mu} < 0$. ■

B Efficiency

As also mentioned in the text, the existence of a connections sector and of queues for public-sector jobs are both inefficient. These two types of inefficiencies can be eliminated by setting $\mu = 0$, which implies $L^g_{h,c} = L^g_{l,c} = 0$, and $w^g_i = \underline{w}^g_i$, which ensures that $u^g_{i,u} = (s^g_i + \tau)e^g_i$ and the job-finding rate for government jobs is 1. We next compare the central planner's solution to the decentralized one, described in the text, and show that the two remaining inefficiencies, the congestion externalities and the failure to internalize the returns of education, can be eliminated with the Hosios condition.

We follow Hosios (1990), Charlot and Decreause (2005), among others, and set $r = 0$, so that the central planner maximizes the steady-state surplus. The planner's problem is to choose $\theta_h, \theta_l, \tilde{\epsilon}_u, u^p_h, u^p_l$ to maximize total output, plus unemployment income, minus job creation and education costs. Given that public sector employment is fixed. The planner's objective is to

$$\max \left[(L_h - L^g_h) \left[(1-u^p_h)y_h + u^p_h b_h - \theta_h \kappa_h u^p_h \right] + (L_l - L^g_l) \left[(1-u^p_l)y_l + u^p_l b_l - \theta_l \kappa_l u^p_l \right] - \tau \int_0^{\tilde{\epsilon}_u} \epsilon \xi^\epsilon(\epsilon) d\epsilon \right]$$

s.t

$$\begin{aligned} u^p_h &= \frac{s^p_h + \tau}{s^p_h + \tau + m(\theta_h)} \\ u^p_l &= \frac{s^p_l + \tau}{s^p_l + \tau + m(\theta_l)} \\ L_h &= \Xi^\epsilon(\tilde{\epsilon}_u) \\ L_l &= 1 - \Xi^\epsilon(\tilde{\epsilon}_u) \end{aligned}$$

If we set the Langrangian

$$\begin{aligned} \mathcal{L} &= (\Xi^\epsilon(\tilde{\epsilon}_u) - L^g_h) \left[(1-u^p_h)y_h + u^p_h b_h - \theta_h \kappa_h u^p_h \right] + (1 - \Xi^\epsilon(\tilde{\epsilon}_u) - L^g_l) \left[(1-u^p_l)y_l + u^p_l b_l - \theta_l \kappa_l u^p_l \right] \\ &\quad -\tau \int_0^{\tilde{\epsilon}_u} \epsilon \xi^\epsilon(\epsilon) d\epsilon + \phi_1 \left[u^p_h - \frac{s^p_h + \tau}{s^p_h + \tau + m(\theta_h)} \right] + \phi_2 \left[u^p_l - \frac{s^p_l + \tau}{s^p_l + \tau + m(\theta_l)} \right] \end{aligned} \quad (66)$$

The seven optimality conditions are

$$\frac{\partial \mathcal{L}}{\partial \theta_h} = 0 \quad \Rightarrow \quad \phi_1 \frac{m'(\theta_h)}{s_h^p + \tau + m(\theta_h)} = (\Xi^\epsilon(\tilde{\epsilon}_u) - L_h^g)\kappa_h \tag{67}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_l} = 0 \quad \Rightarrow \quad \phi_2 \frac{m'(\theta_l)}{s_l^p + \tau + m(\theta_l)} = (1 - \Xi^\epsilon(\tilde{\epsilon}_u) - L_l^g)\kappa_l \tag{68}$$

$$\frac{\partial \mathcal{L}}{\partial u_h^p} = 0 \quad \Rightarrow \quad \phi_1 = (\Xi^\epsilon(\tilde{\epsilon}_u) - L_h^g)\left[y_h - b_h + \kappa_h \theta_h\right] \tag{69}$$

$$\frac{\partial \mathcal{L}}{\partial u_l^p} = 0 \quad \Rightarrow \quad \phi_2 = (1 - \Xi^\epsilon(\tilde{\epsilon}_u) - L_l^g)\left[y_l - b_l + \kappa_l \theta_l\right] \tag{70}$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{\epsilon}_u} = 0 \quad \Rightarrow \quad \tau \tilde{\epsilon}_u = \left[((1 - u_h^p)y_h + u_h^p b_h - \theta_h \kappa_h u_h^p) - ((1 - u_l^p)y_l + u_l^p b_l - \theta_l \kappa_l u_l^p)\right] \tag{71}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_1} = 0 \quad \Rightarrow \quad u_h^p = \frac{s_h^p + \tau}{s_h^p + \tau + m(\theta_h)} \tag{72}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_2} = 0 \quad \Rightarrow \quad u_l^p = \frac{s_l^p + \tau}{s_l^p + \tau + m(\theta_l)} \tag{73}$$

Substituting (69) into (67) and (70) into (68) gives:

$$\frac{\kappa_h}{q(\theta_h)} = \frac{\eta(y_h - b_h)}{s_h^p + \tau + m(\theta_h)(1 - \eta)} \tag{74}$$

$$\frac{\kappa_l}{q(\theta_l)} = \frac{\eta(y_l - b_l)}{s_l^p + \tau + m(\theta_l)(1 - \eta)} \tag{75}$$

where it may be useful to recall that $m(\theta_i) = \theta_i^\eta$ and $m'(\theta_i) = \eta q(\theta_i)$.

Using (72) and (73) we can substitute for u_h^p and u_l^p in (71) and then use (74) and (75) to substitute for y_h and y_l. Simplifying and rearranging terms equation (71) gives:

$$\tilde{\epsilon}_u = \frac{1}{\tau}\left[b_h - b_l + \frac{1 - \eta}{\eta}(\kappa_h \theta_h - \kappa_l \theta_l)\right] \tag{76}$$

It is easy to verify by comparing (76) to (28) and (74)-(75) to (13), that given $r = 0$, if $\beta = (1 - \eta)$, then the decentralized equilibrium is identical to the central planner's solution.

C Random search

C.1 Setup

In this appendix we give the full set of equations of the model with random search and characterize it's steady-state equilibrium. Further, we show that in the limiting case where $\mu_h = \mu_l = 1$ the model with random search becomes identical to the model with segmented markets and we provide proofs of Propositions 5 and 6.

The values of being employed or unemployed for connected workers remain as in the Benchmark model; given by (4) and (6). The same holds for the values of being employed in either the private or the public sector for unconnected workers (equations 2 and 5), and the values of a private-sector filled jobs and vacancies (equations 7 and 8). The cutoff education costs $\tilde{\epsilon}_c$, $\tilde{\epsilon}_m(c)$ and $\tilde{\epsilon}_u$ as well as the selection of workers into the four groups, $L^g_{l,c}, L^g_{h,c}, L_{l,u}, L_{h,u}$, also remain as given in equations (21) to (23), and (24) to (26), respectively. As discussed in the text, only the value of unemployment for unconnected workers changes. It is now given by equation (38). The Nash bargaining wage of the private sector changes accordingly and is as given in (39).

Both government and private firms that seek to hire workers through regular search in the market meet with workers at rate $q(\theta_i) = \frac{m(\theta_i)}{\theta_i}$, where $\theta_i = \frac{v^p_i + v^g_{i,u}}{u_{i,u}}$. The number of vacancies in the private sector is determined endogenously by free entry that drives the value of a private-sector vacancy to zero, $V^p_i = 0$. The government needs to post enough vacancies for workers without connections to ensure that the total number of matches with such workers, $q(\theta_i)v^g_{i,u}$, equals the number of unconnected workers that it needs to hire. Hence, the government posts $v^g_{i,u}$ vacancies to ensure $q(\theta_i)v^g_{i,u} = (1-\mu_i)\left(s^g_i + \tau\right)e^g_i$.

Setting $V^p_i = 0$ and using the Nash bargaining conditions in (10), we can write the surplus of a private-sector match with a type i worker as

$$S^p_i = \frac{y_i - b_i - D_i(w^g_i - b_i)}{r + \tau + s^p_i + (1-D_i)\beta m(\theta_i)\nu^p_i} \tag{77}$$

and the zero-profit condition that determines job creation in the private sector becomes:

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(1-\beta)(y_i - b_i - D_i(w^g_i - b_i))}{r + \tau + s^p_i + (1-D_i)\beta m(\theta_i)\nu^p_i} \tag{78}$$

We can write the threshold levels of connection costs, $\tilde{c}_i = U^g_{i,c} - U_{i,u}$, $i = [h, l]$, one for each skill type, and the threshold levels of education cost for unconnected workers as:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[\frac{\frac{\mu(s^g_i+\tau)e^g_i}{u^g_{i,c}}(w^g_i - b_i)}{r + \tau + s^g_i + \frac{\mu(s^g_i+\tau)e^g_i}{u^g_{i,c}}} - D_i(w^g_i - b_i) - (1-D_i)\frac{\beta\kappa_i\theta_i}{(1-\beta)}\right] \tag{79}$$

$$\tilde{\epsilon}_u = \frac{1}{r+\tau}\left[b_h - b_l + D_h(w^g_h - b_h) - D_l(w^g_l - b_l) + (1-D_h)\frac{\beta\kappa_h\theta_h}{(1-\beta)} - (1-D_l)\frac{\beta\kappa_l\theta_l}{(1-\beta)}\right] \tag{80}$$

As in the benchmark model we treat public sector employment as an exogenous policy variable. There are e^g_i workers of each skill type employed in the public sector. Among these workers, $\mu_i e_i^g$ are workers who were hired through connections ($e_{i,c}^g$) and the remaining $(1-\mu_i)e_i^g$ are workers hired through regular search in the market ($e_{i,u}^g$). The number of workers employed in the private sector is endogenous and depends on job creation in the private sector as well as conditions in the public sector. The labor force of workers without connections consists of those employed in the public sector, those employed in the private sector ($e_{i,u}^p$), and the unemployed ($u_{i,u}$). Hence, $u_{i,u} = L_{i,u} - (1-\mu_i)e_i^g - e_{i,u}^p$. By equating the flows in, $m(\theta_i)\nu_i^p u_{i,u}$, to the flows out of the state where a worker is employed in the private sector, $e_{i,u}^p(s_i^p + \tau)$ we obtain:

$$e_{i,u}^p = \frac{m(\theta_i)\nu_i^p\left[L_{i,u} - (1-\mu_i)e_i^g\right]}{m(\theta_i)\nu_i^p + \tau + s_i^p} \tag{81}$$

$$u_{i,u} = \frac{(\tau + s_i^p)\left[L_{i,u} - (1-\mu_i)e_i^g\right]}{m(\theta_i)\nu_i^p + \tau + s_i^p} \tag{82}$$

Given $\theta_i = \frac{v_i^p + v_{i,u}^g}{u_{i,u}}$ and $q(\theta_i)v_{i,u}^g = (s_i^g + \tau)\, e_i^g$, we can use (82) to write:

$$\nu_i^p = \frac{s_i^p + \tau}{m(\theta_i)}\left[\frac{m(\theta_i)\left[L_{i,u} - (1-\mu_i)e_i^p\right] - (1-\mu_i)(s_i^g + \tau)e_i^g}{(s_i^p + \tau)\left[L_{i,u} - (1-\mu_i)e_i^p\right] + (1-\mu_i)e_i^g(s_i^g + \tau)}\right] \tag{83}$$

Using (81) and (83) we can write the total employment of workers without connections, $e_{i,u} = e_{i,u}^p + (1-\mu_i)e_i^g$ as:

$$e_{i,u} = \frac{m(\theta_i)L_{i,u} + (1-\mu_i)e_i^p(s_i^p - s_i^g)}{s_i^p + \tau + m(\theta_i)} \tag{84}$$

C.2 The case $\mu_h = \mu_l = 1$

If $\mu_h = \mu_l = 1$, then, as can be seen from (83), $\nu_i^p = 1$, which implies $D_i = 0$. Setting $\nu_i^p = 1$ and $D_i = 0$ in (39), (78), (79) and (80) gives:

$$w_i^p = b_i + \beta\left[y_i - b_i + \theta_i\kappa_i\right] \tag{85}$$

$$\frac{\kappa_i}{q(\theta_i)} = (1-\beta)\left(\frac{y_i - b_i}{r + s_i^p + \tau + \beta m(\theta_i)}\right) \tag{86}$$

$$\tilde{c}_i = \frac{1}{r+\tau}\left[A_{i,c} - \frac{\beta\kappa_i\theta_i}{(1-\beta)}\right] \tag{87}$$

$$\tilde{\epsilon}_u = \frac{1}{r+\tau}\left[b_h - b_l + \frac{\beta\kappa_h\theta_h}{(1-\beta)} - \frac{\beta\kappa_l\theta_l}{(1-\beta)}\right] \tag{88}$$

where again, $A_{i,c}$ is as defined in (45) and $i = [h, l]$ denotes the education type. The zero-profit condition in (86) gives a unique set of equilibrium values of θ_h and θ_l which is independent of government policy or conditions in the government sector.

Moreover, the zero-profit condition in (86), private-sector wages in (85) and the cut-off costs in (87) and (88) are identical to those obtained under segmented markets (equations 13, 14, 27 and 28, respectively). Hence, if $\mu_h = \mu_l = 1$, private-sector job creation and tightness, wages, as well as the composition of the labor force in terms of connections and education in the model with random search are identical to those obtained under segmented markets.

C.3 Definition of Equilibrium

A steady state equilibrium consists of a set of cut-off costs $\{\tilde{c}_h, \tilde{c}_l, \tilde{\epsilon}_u, \tilde{\epsilon}_c\}$, tightness $\{\theta_h, \theta_l\}$, and unemployed $\{u_{h,u}, u_{l,u}, u^g_{h,c}, u^g_{l,c}\}$, such that, given some exogenous government policies $\{w^g_h, w^g_l, e^g_h, e^g_l, \mu_h, \mu_l\}$, the following apply.

1. Private-sector firms satisfy the free-entry condition (78) $i = [h, l]$.
2. Private-sector wages are the outcome of Nash Bargaining (39) $i = [h, l]$.
3. Newborns decide optimally their investments in education and connections and the population shares are determined by equations (24)-(26).
4. Flows between private employment and unemployment are constant

$$\begin{aligned}(s^p_h + \tau)e^p_h &= m(\theta_h)\nu^h u^p_h, \\ (s^p_l + \tau)e^p_l &= m(\theta_l)\nu^l u^p_l.\end{aligned}$$

5. Population add up constraints are satisfied:

$$\begin{aligned}L_{h,u} &= e^p_h + (1-\mu)e^g_h + u_{h,u} \\ L_{l,u} &= e^p_l + (1-\mu)e^g_l + u_{l,u} \\ L^g_{h,c} &= \mu e^g_h + u^g_{h,c} && (89) \\ L^g_{l,c} &= \mu e^g_l + u^g_{l,c} && (90) \\ L_{h,u} + L_{l,u} + L^g_{h,c} + L^g_{l,c} &= 1\end{aligned}$$

C.4 Proof of Existence and Uniqueness

Proof. To prove the existence and uniqueness of a steady state equilibrium under random search we show below that the two free-entry conditions in (78) cross only once in the $[\theta_h, \theta_l]$ plane, giving a unique set of equilibrium values for θ_h and θ_l. The equilibrium values of the cut-off costs can then be determined by substituting the equilibrium values of the theta's in equations (79) and (80). Then using (21) to (26) we can determine $L_{l,u}, L_{h,u}, L^g_{l,c}$ and $L^g_{h,c}$, which in turn, together with the equilibrium values of theta's, can be substituted in equations (39), (81), (82), (89) and (90) to determine wages and employment in the private sector as well as the number of workers with and without connections that are unemployed.

The two job creation conditions in (78), the cut-off connection costs in (79) and education cost in (80) can be written as:

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(y_i - b_i - OO_i)}{r + \tau + s^p_i} \qquad (91)$$

$$\tilde{c}_i = \frac{1}{r+\tau}\left[A_{i,c} - OO_i\right] \tag{92}$$

$$\tilde{e}_u = \frac{1}{r+\tau}\left[b_h - b_l + OO_h - OO_l\right] \tag{93}$$

where

$$OO_i = D_i(w_i^g - b_i) + (1-D_i)\frac{\beta}{1-\beta}\nu_i^p \kappa_i \theta_i \tag{94}$$

is the expression for the outside option of workers of skill type $i = [h, l]$, $A_{i,c}$ is as defined in (45) and D_i is as defined in the text (subsection 5.1).

In what follows let $j = [h, l]$, $i = [h, l]$ and $j \neq i$. Taking the derivative with respect to θ_j of (92), (93) and of (94), after we substitute in for ν_i^p using (83), we obtain:

$$\frac{d\tilde{c}_j}{d\theta_j} = \frac{1}{r+\tau}\left[\frac{\partial A_{j,c}}{\partial L_{j,c}^g}\frac{dL_{j,c}^g}{d\theta_j} - \frac{dOO_j}{d\theta_j}\right] \tag{95}$$

$$\frac{d\tilde{c}_i}{d\theta_j} = \frac{1}{r+\tau}\left[\frac{\partial A_{i,c}}{\partial L_{i,c}^g}\frac{dL_{i,c}^g}{d\theta_j} - \frac{dOO_i}{d\theta_j}\right] \tag{96}$$

$$\frac{d\tilde{e}_u}{d\theta_j} = \frac{1}{r+\tau}\left[\frac{dOO_h}{d\theta_j} - \frac{dOO_l}{d\theta_j}\right] \tag{97}$$

$$\frac{dOO_j}{d\theta_j} = -K_j\frac{dL_{j,u}}{d\theta_j} + \Sigma_j \tag{98}$$

$$\frac{dOO_i}{d\theta_j} = -Z_i\frac{dL_{i,u}}{d\theta_j} \tag{99}$$

where

$$K_j = \frac{(1-D_j)m(\theta_j)\left(E_{j,u}^g - E_{j,u}^p\right)(s_j^p+\tau)(1-\nu_j^p)}{(s_j^p+\tau)\left[L_{j,u} - (1-\mu_j)e_j^p\right] + (1-\mu_j)e_j^g(s_j^g+\tau)} \tag{100}$$

$$\Sigma_j = (1-D_j)q(\theta_j)\left[(1-\nu_j^p)\eta\left(\frac{E_{j,u}^g m(\theta_j) + E_j^P(s_j^p+\tau)}{s_j^p+\tau+m(\theta_j)} - U_j\right) + \nu_j^p\left(E_j^p - U_j\right)\right] \tag{101}$$

$$Z_i = \frac{(1-D_i)m(\theta_i)\left(E_{i,u}^g - U_i\right)(s_i^p+\tau)(1-\nu_i^p)}{(s_i^p+\tau)\left[L_{i,u} - (1-\mu_i)e_i^p\right] + (1-\mu_i)e_i^g(s_i^g+\tau)} \tag{102}$$

Using (24) and (**??**) we can substitute for $\frac{dL_{i,c}^g}{d\theta_j}$ and $\frac{dL_{j,c}^g}{d\theta_j}$ in (95) and (96) then solve for $\frac{d\tilde{c}_j}{d\theta_j}$ and $\frac{d\tilde{c}_i}{d\theta_j}$:

$$\frac{d\tilde{c}_j}{d\theta_j} = \Phi_j\frac{d\tilde{c}_i}{d\theta_j} + \Psi_j\frac{d\tilde{e}_u}{d\theta_j} - T_j\frac{dOO_j}{d\theta_j} \tag{103}$$

$$\frac{d\tilde{c}_i}{d\theta_j} = \Phi_i\frac{d\tilde{c}_j}{d\theta_j} + \Psi_i\frac{d\tilde{e}_u}{d\theta_j} - T_i\frac{dOO_i}{d\theta_j} \tag{104}$$

where

$$T_j = \frac{\frac{1}{r+\tau}}{1 - \frac{1}{r+\tau}\frac{dA_{j,c}}{dL^g_{j,c}}\frac{dL^g_{j,c}}{d\tilde{c}_j}} \tag{105}$$

$$\Phi_j = \frac{\frac{1}{r+\tau}\frac{dA_{j,c}}{dL^g_{j,c}}\frac{dL^g_{j,c}}{d\tilde{c}_i}}{1 - \frac{1}{r+\tau}\frac{dA_{j,c}}{dL^g_{j,c}}\frac{dL^g_{j,c}}{d\tilde{c}_j}} \tag{106}$$

$$\Psi_j = \frac{\frac{1}{r+\tau}\frac{dA_{j,c}}{dL^g_{j,c}}\frac{dL^g_{j,c}}{d\tilde{e}_u}}{1 - \frac{1}{r+\tau}\frac{dA_{j,c}}{dL^g_{j,c}}\frac{dL^g_{j,c}}{d\tilde{c}_j}} \tag{107}$$

The two equations in (103) and (104) can be used to solve for $\frac{d\tilde{c}_j}{d\theta_j}$ and $\frac{d\tilde{c}_i}{d\theta_j}$ in terms of $\frac{dOO_j}{d\theta_j}$, $\frac{dOO_i}{d\theta_j}$ and $\frac{d\tilde{e}_u}{d\theta_j}$. The resulting expressions for $\frac{d\tilde{c}_j}{d\theta_j}$, $\frac{d\tilde{c}_i}{d\theta_j}$ can then be used to derive expressions for $\frac{dL_{j,u}}{d\theta_j}$ and $\frac{dL_{i,u}}{d\theta_j}$ in terms of $\frac{dOO_j}{d\theta_j}$, $\frac{dOO_i}{d\theta_j}$ and $\frac{d\tilde{e}_u}{d\theta_j}$ using (25) and (26). The resulting expressions are:

$$\begin{aligned}\frac{dL_{j,u}}{d\theta_j} &= \frac{d\tilde{e}_u}{d\theta_j}\left[\frac{\frac{\partial L_{j,u}}{\partial \tilde{c}_j}(\Phi_j\Psi_i + \Psi_j) + \frac{\partial L_{j,u}}{\partial \tilde{c}_i}(\Phi_i\Psi_j + \Psi_i)}{1 - \Phi_j\Phi_i} + \frac{\partial L_{j,u}}{\partial \tilde{e}_u}\right] \\ &\quad - T_j\frac{dOO_j}{d\theta_j}\left[\frac{\Phi_i\frac{dL_{j,u}}{d\tilde{c}_i} + \frac{dL_{j,u}}{d\tilde{c}_j}}{1 - \Phi_j\Phi_i}\right] - T_i\frac{dOO_i}{d\theta_j}\left[\frac{\Phi_j\frac{dL_{j,u}}{d\tilde{c}_j} + \frac{dL_{j,u}}{d\tilde{c}_i}}{1 - \Phi_j\Phi_i}\right] \end{aligned} \tag{108}$$

$$\begin{aligned}\frac{dL_{i,u}}{d\theta_j} &= \frac{d\tilde{e}_u}{d\theta_j}\left[\frac{\frac{\partial L_{i,u}}{\partial \tilde{c}_j}(\Phi_j\Psi_i + \Psi_j) + \frac{\partial L_{i,u}}{\partial \tilde{c}_i}(\Phi_i\Psi_j + \Psi_i)}{1 - \Phi_j\Phi_i} + \frac{\partial L_{i,u}}{\partial \tilde{e}_u}\right] \\ &\quad - T_j\frac{dOO_j}{d\theta_j}\left[\frac{\Phi_i\frac{dL_{i,u}}{d\tilde{c}_i} + \frac{dL_{i,u}}{d\tilde{c}_j}}{1 - \Phi_j\Phi_i}\right] - T_i\frac{dOO_i}{d\theta_j}\left[\frac{\Phi_j\frac{dL_{i,u}}{d\tilde{c}_j} + \frac{dL_{i,u}}{d\tilde{c}_i}}{1 - \Phi_j\Phi_i}\right] \end{aligned} \tag{109}$$

Using these expressions we can substitute for $\frac{dL_{j,u}}{d\theta_j}$ and $\frac{dL_{i,u}}{d\theta_j}$ in (98) and (99) and obtain a system of two equations which, with $\frac{d\tilde{e}_u}{d\theta_j}$ from (97) substituted in, can be solved for $\frac{dOO_j}{d\theta_j}$ and $\frac{dOO_i}{d\theta_j}$. The system is:

$$\frac{dOO_j}{d\theta_j} = -K_j\left[\frac{d\tilde{e}_u}{d\theta_j}\Gamma^1_{ji} - \frac{dOO_i}{d\theta_j}\Gamma^2_{ji} - \frac{dOO_j}{d\theta_j}\Gamma^3_{ji}\right] + \Sigma_j \tag{110}$$

$$\frac{dOO_i}{d\theta_j} = -Z_i\left[\frac{d\tilde{e}_u}{d\theta_j}\Gamma^1_{ij} - \frac{dOO_j}{d\theta_j}\Gamma^2_{ij} - \frac{dOO_i}{d\theta_j}\Gamma^3_{ij}\right] \tag{111}$$

were

$$\Gamma^1_{ji} = \frac{\frac{\partial L_{j,u}}{\partial \tilde{c}_j}(\Phi_j\Psi_i + \Psi_j) + \frac{\partial L_{j,u}}{\partial \tilde{c}_i}(\Phi_i\Psi_j + \Psi_i)}{1 - \Phi_j\Phi_i} + \frac{\partial L_{j,u}}{\partial \tilde{e}_u}$$

$$\Gamma^2_{ji} = T_i\left[\frac{\frac{\partial L_{j,u}}{\partial \tilde{c}_j}\Phi_j + \frac{\partial L_{j,u}}{\partial \tilde{c}_i}}{1 - \Phi_j\Phi_i}\right]$$

$$\Gamma^3_{ji} = T_j\left[\frac{\frac{\partial L_{j,u}}{\partial \tilde{c}_i}\Phi_i + \frac{\partial L_{j,u}}{\partial \tilde{c}_j}}{1 - \Phi_j\Phi_i}\right]$$

By solving the system with $\frac{d\tilde{e}_u}{d\theta_j}$ from (97) substituted in, we obtain

$$\frac{dOO_h}{d\theta_h} = \frac{\Sigma_h}{1 + K_h\left[\left(\frac{\Gamma^1_{hl}}{r+\tau} - \Gamma^3_{hl}\right) + \left(\frac{\Gamma^1_{hl}}{r+\tau} + \Gamma^2_{hl}\right)\left(\frac{Z_l\left(\frac{\Gamma^1_{lh}}{r+\tau} - \Gamma^2_{lh}\right)}{1 - Z_l\left(\frac{\Gamma^1_{lh}}{r+\tau} + \Gamma^3_{lh}\right)}\right)\right]} > 0 \qquad (112)$$

$$\frac{dOO_l}{d\theta_h} = -\left[\frac{Z_l\left(\frac{\Gamma^1_{lh}}{r+\tau} - \Gamma^2_{lh}\right)}{1 - Z_l\left(\frac{\Gamma^1_{lh}}{r+\tau} + \Gamma^3_{lh}\right)}\right]\frac{dOO_h}{d\theta_h} < 0 \qquad (113)$$

$$\frac{dOO_l}{d\theta_l} = \frac{\Sigma_l}{1 - K_l\left[\left(\frac{\Gamma^1_{lh}}{r+\tau} + \Gamma^3_{lh}\right) + \left(\frac{\Gamma^1_{lh}}{r+\tau} - \Gamma^2_{lh}\right)\left(\frac{Z_h\left(\frac{\Gamma^1_{hl}}{r+\tau} + \Gamma^2_{hl}\right)}{1 + Z_h\left(\frac{\Gamma^1_{hl}}{r+\tau} - \Gamma^3_{hl}\right)}\right)\right]} > 0 \qquad (114)$$

$$\frac{dOO_h}{d\theta_l} = \left[\frac{Z_h\left(\frac{\Gamma^1_{hl}}{r+\tau} + \Gamma^2_{hl}\right)}{1 + Z_h\left(\frac{\Gamma^1_{hl}}{r+\tau} - \Gamma^3_{hl}\right)}\right]\frac{dOO_l}{d\theta_l} < 0 \qquad (115)$$

Using (24) to (26) it can be shown that: $\frac{\partial L_{i,u}}{\partial \tilde{c}_j} \le 0$, $\frac{\partial L_{j,u}}{\partial \tilde{c}_j} < 0$, $\frac{\partial L^g_{j,c}}{\partial \tilde{c}_j} > 0$, $\frac{\partial L^g_{j,c}}{\partial \tilde{c}_i} < 0$, $\frac{\partial L^g_{h,c}}{\partial \tilde{e}_u} > 0$, $\frac{\partial L^g_{l,c}}{\partial \tilde{e}_u} < 0$, $\frac{\partial L_{l,u}}{\partial \tilde{e}_u} < 0$, $\frac{\partial L_{j,c}}{\partial \tilde{c}_j} > -\frac{\partial L_{j,c}}{\partial \tilde{c}_i}$, $\frac{\partial L^g_{l,c}}{\partial \tilde{c}_l} > -\frac{\partial L^g_{l,c}}{\partial \tilde{e}_u}$, $\frac{\partial L^g_{h,c}}{\partial \tilde{c}_h} > \frac{\partial L^g_{h,c}}{\partial \tilde{e}_u}$, $-\frac{\partial L^g_{l,c}}{\partial \tilde{e}_u} \ge \frac{\partial L^g_{l,c}}{\partial \tilde{c}_h}$, $\frac{\partial L^g_{h,c}}{\partial \tilde{e}_u} \ge -\frac{\partial L^g_{h,c}}{\partial \tilde{c}_l}$, $\frac{\partial L_{h,u}}{\partial \tilde{e}_u} > -\frac{\partial L_{h,u}}{\partial \tilde{c}_l}$, $-\frac{\partial L_{l,u}}{\partial \tilde{e}_u} > -\frac{\partial L_{l,u}}{\partial \tilde{c}_h}$. In turn, we can use these results to shown that $\frac{\Gamma^1_{hl}}{r+\tau} - \Gamma^3_{hl} > \frac{\Gamma^1_{hl}}{r+\tau} + \Gamma^2_{hl} > 0$, $\frac{\Gamma^1_{lh}}{r+\tau} + \Gamma^3_{lh} < \frac{\Gamma^1_{lh}}{r+\tau} - \Gamma^2_{lh} < 0$. The latter ensure that the terms in the brackets of the denominators of (112) and (114) are positive and negative, respectively, and that the terms in bracket of (113) and (115) are negative and positive, respectively. Further, as can be easily verified from (101) and (102) $\Sigma_h > 0$, $\Sigma_l > 0$, $Z_h > 0$, $Z_l > 0$, while, as can be seen from (100) sufficient condition to ensure $K_j \ge 0$, $j = [h,l]$ is $E^g_{j,u} \ge E^p_{j,u}$. It follows that if $E^g_{j,u} \ge E^p_{j,u}$, then $\frac{dOO_j}{d\theta_j} > 0$, while $\frac{dOO_j}{d\theta_i} < 0$, $i = [h,l]$, $j = [h,l]$, $j \neq i$.

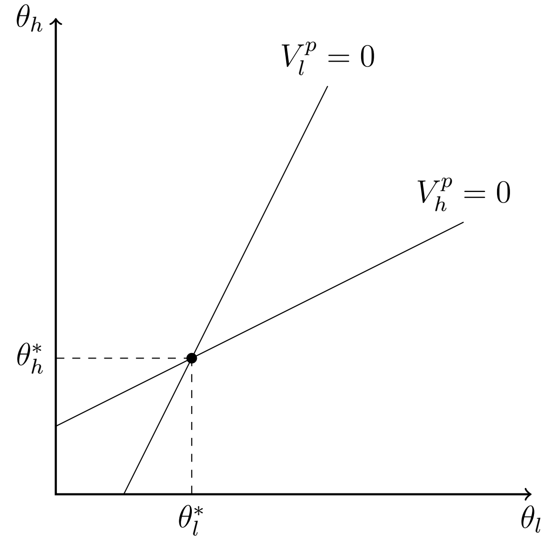
By total differentiation of (91) we can derive the slopes of the two job creation conditions in the $[\theta_h, \theta_l]$ plane:

$$\text{high-education market: } \left.\frac{d\theta_h}{d\theta_l}\right|_{V_h^p=0} = \frac{-\frac{dOO_h}{d\theta_l}}{-\frac{q'(\theta_h)\kappa_h}{(q(\theta_h))^2}(r+\tau+s_h^p) + \frac{dOO_h}{d\theta_h}} > 0 \qquad (116)$$

$$\text{Low-education market: } \left.\frac{d\theta_h}{d\theta_l}\right|_{V_l^p=0} = \frac{-\frac{q'(\theta_l)\kappa_l}{(q(\theta_l))^2}(r+\tau+s_l^p) + \frac{dOO_l}{d\theta_l}}{-\frac{dOO_l}{d\theta_h}} > 0 \qquad (117)$$

Both slopes are positive, since $q'(\theta_i) < 0$, $\frac{dOO_i}{d\theta_i} > 0$, $\frac{dOO_i}{d\theta_j} < 0$ but, as can be easily verified using the results above, $\left.\frac{d\theta_h}{d\theta_l}\right|_{V_h^p=0} < 1$ and $\left.\frac{d\theta_h}{d\theta_l}\right|_{V_l^p=0} > 1$. It follows that the two job creation conditions only cross once in the $[\theta_h, \theta_l]$ plane, as shown in Figure 10. This completes the proof of existence and uniqueness.

Figure 10: Steady State Equilibrium under Random Search



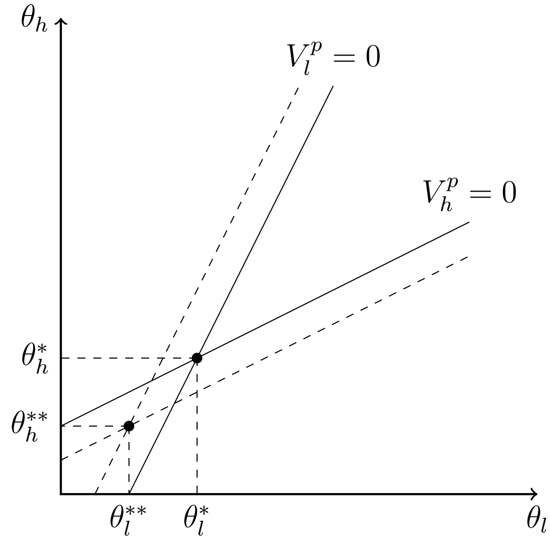
■

C.5 Proof of Proposition 5

Proof. If $\mu_h = \mu_l = 0$ it can be shown that an increase in either w_i^g or e_i^g will lower the surplus of private-sector jobs (right-hand-side of 78) of both skill types, thereby lowering job creation in both sectors. The $V_l^p = 0$ and $V_h^p = 0$ loci will shift to the left and right, respectively, as illustrated in Figure 11. Both θ_h^* and θ_l^* will decrease.

Let $x_i = [w_i^g, e_i^g]$. In what follows we show that $\frac{dOO_i}{dx_i} > 0$ and $\frac{dOO_j}{dx_i} > 0$ (for $j \neq i$), which as can be inferred from (91) imply the shifts depicted in Figure 11.

Figure 11: Effects of more generous government policies under random search



From (94) we get:

$$\frac{dOO_i}{dx_i} = -K_i \frac{dL_{i,u}}{dx_i} + \Lambda_i \tag{118}$$

$$\frac{dOO_j}{dx_i} = -K_j \frac{dL_{j,u}}{dx_i} \; j \neq i \tag{119}$$

where K_i and K_j are as defined in (100), while

$$\Lambda_i = \begin{cases} (1-\mu_i)\left[K_i + \frac{(1-D_i)(E^g_{i,u}-E^p_{i,u})(s^g_i+\tau)(s^p_i+\tau+m(\theta_i)\nu^p_i)}{(s^p_i+\tau)\left[L_{i,u}-(1-\mu_i)e^p_i\right]+(1-\mu_i)e^g_i(s^g_i+\tau)}\right] > 0 \text{ if } x_i = e^g_i \\ D_i > 0 \text{ if } x_i = w^g_i \end{cases}$$

Given $\mu_h = \mu_l = 0$ (which ultimately implies $\tilde{c}_h = \tilde{c}_l = 0$ and $L^g_{l,c} = L^g_{h,c} = 0$), it follows from (25) and (26) that $L_{h,u} = L_h = \Xi^\epsilon(\tilde{\epsilon}_u)$ and $L_{l,u} = 1 - L_h = 1 - \Xi^\epsilon(\tilde{\epsilon}_u)$. Using (93) we can write:

$$\frac{dL_{l,u}}{dx_i} = -\frac{\xi(\tilde{\epsilon}_u)}{r+\tau}\left[\frac{dOO_h}{dx_i} - \frac{dOO_l}{dx_i}\right]$$

$$\frac{dL_{h,u}}{dx_i} = \frac{\xi(\tilde{\epsilon}_u)}{r+\tau}\left[\frac{dOO_h}{dx_i} - \frac{dOO_l}{dx_i}\right] \tag{120}$$

By combining (118)-(119) and (120) we obtain that for $i = [h, l]$ and $j = [h, l]$:

$$\frac{dOO_j}{dx_i} = \left(\frac{K_j \xi(\tilde{\epsilon}_u)}{r+\tau+K_j\xi(\tilde{\epsilon}_u)}\right)\frac{dOO_i}{dx_i} \text{ if } j \neq i$$

$$\frac{dOO_i}{dx_i} = \frac{\Lambda_i(r+\tau)}{r+\tau+K_i\xi(\tilde{\epsilon}_u)\left(1-\frac{K_j\xi(\tilde{\epsilon}_u)}{r+\tau+K_j\xi(\tilde{\epsilon}_u)}\right)} \tag{121}$$

Given $\Lambda_i > 0$, it can be easily verified from (121) that $\frac{dOO_i}{dx_i} > 0$, $\frac{dOO_j}{dx_i} > 0$. ■

C.6 Proof of Proposition 6

Proof. Let $x_i = [w_i^g, e_i^g]$. As shown above, if $\mu_h = \mu_l = 1$ then the cut-off costs, $\tilde{c}_i$ and $\tilde{\epsilon}_u$ are as given in (87) and (88) and are identical to those obtained under segmented markets. The allocation of workers into connections ($L_{i,c}^g$) and no connections ($L_{i,u}$) is also identical to that obtained when markets re segmented (given in 24 to 26). Moreover, as shown above in random search with $\mu_h = \mu_l = 1$ the job creation conditions are identical to those of segmented markets, and independent of public-sector policies (i.e. $\frac{d\theta_i}{dx_i} = \frac{d\theta_j}{dx_i} = 0$). The results obtained in Appendix A (Proof of Proposition 1) and are summarised in (48), (49) and (51) therefore carry over when search is random and $\mu_h = \mu_l = 1$. Since, $\frac{d\theta_i}{dx_i} = \frac{d\theta_j}{dx_i} = 0$, it follows that $\frac{d\tilde{\epsilon}_u}{dx_i} = 0$. Using (24)-(26) and (49) we can write:

$$\frac{dL_{i,u}}{dx_i} = \frac{d\tilde{c}_i}{dx_i}\left[\frac{\partial L_{i,u}}{\partial \tilde{c}_i} + B_j \frac{\partial L_{i,u}}{\partial \tilde{c}_j}\right] < 0 \tag{122}$$

$$\frac{L_{i,c}^g}{dx_i} = \frac{d\tilde{c}_i}{dx_i}\left[\frac{\partial L_{i,c}^g}{\partial \tilde{c}_i} + B_j \frac{\partial L_{i,c}^g}{\partial \tilde{c}_j}\right] > 0 \tag{123}$$

$i = [h, l], j = [h, l]$ and $j \neq i$. As shown above, $\frac{d\tilde{c}_i}{dx_i} > 0$, and $0 < B_j < 1$, , while, as can be easily verified from (24) to (26) $\frac{dL_{i,c}^g}{d\tilde{c}_i} > -\frac{dL_{i,c}^g}{d\tilde{c}_j} > 0$, $\frac{dL_{i,u}}{d\tilde{c}_i} < 0$, $\frac{dL_{i,u}}{d\tilde{c}_j} < 0$. It follows from the equations above, that $\frac{dL_{i,u}}{dx_i} < 0$ and $\frac{dL_{i,c}^g}{dx_i} > 0$. ■

D Competitive Search in the Private Sector

As mentioned in the text, under competitive search in the private sector, each of the two markets (skilled and unskilled) of the private sector consists of submarkets. In each submarket there is a subset of unemployed workers and firms with vacant jobs that are searching for each other. The number of matches in submarket n of skill type i is $m(v_{i,n}, u_{i,n}) = (v_{i,n})^{\eta}(u^p_{i,n})^{(1-\eta)}$, $m(\theta_{i,n})$ is the job finding rate and $q(\theta_{i,n})$ the job filling rate. For a worker of skill type i in submarket n

$$
\begin{aligned}
(r+\tau)U^p_{i,n} &= b_i + m(\theta_{i,n})\left[E^p_{i,n} - U^p_{i,n}\right] \qquad (124)\\
(r+\tau)E^p_{i,n} &= w^p_{i,n} - s^p_i\left[E^p_{i,n} - U^p_{i,n}\right] \qquad (125)
\end{aligned}
$$

Unemployed workers of skill type i are free to move between the submarkets of market i. They will choose to search for a job in the submarket that yields the highest expected income. Since workers of the same skill type are ex-ante identical and movement across submarkets is free, this means that $U^p_{i,n} = U^p_i$. Using (124) and (125) we can write:

$$
m(\theta_{i,n}) = \left(\frac{(r+\tau)U^p_i - b_i}{w^p_{i,n} - (r+\tau)U^p_i}\right)(r+\tau+s^p_i) \qquad (126)
$$

The values of vacancies and filled jobs in submarket n of market i satisfy

$$
\begin{aligned}
rV^p_{i,n} &= -\kappa_i + q(\theta_{i,n})\left[J^p(w^p_{i,n}) - V^p_{i,n}\right] \qquad (127)\\
rJ^p(w^p_{i,n}) &= y_{i,n} - w^p_{i,n} + (s^p_i + \tau)\left[V^p_{i,n} - J^p(w^p_{i,n})\right] \qquad (128)
\end{aligned}
$$

Using (127) and (128) to solve for $V^p_{i,n}$ gives

$$
rV^p_{i,n} = \frac{-\kappa_i(r+s^p_i+\tau) + q(\theta_{i,n})(y_{i,n} - w^p_i)}{r + q(\theta_{i,n}) + s^p_i + \tau} \qquad (129)
$$

In a competitive search equilibrium a market maker determines the number of submarkets in each market and the wage in each submarket. The wage is chosen to maximize the value of a vacancy. All vacancies in the same submarket offer the same wage. Setting the derivative of (129) with respect to $w^p_{i,n}$ equal to 0 we get the first order condition for optimal wages:

$$
-(1-\eta)(r+s^p_i+\tau)\frac{d\theta_{i,n}}{dw^p_{i,n}}\left[y_{i,n} - w^p_{i,n} + \kappa_i\right] = \theta_{i,n}(r+s^p_i+\tau) + m(\theta_{i,n}) \qquad (130)
$$

There is free entry of vacancies in each submarket, which drives the value of a vacancy to zero. Setting $V^p_{i,n} = 0$ in (129) gives:

$$
\frac{\kappa_i}{q(\theta_{i,n})} = \frac{y_{i,n} - w^p_i}{r + s^p_i + \tau} \qquad (131)
$$

Taking the derivative of (126) with respect to $w^p_{i,n}$ we obtain

$$\frac{d\theta_{i,n}}{dw^p_{i,n}} = -\left(\frac{\theta_{i,n}}{w^p_{i,n} - (r+\tau)U^p_i}\right)\frac{1}{\eta} \tag{132}$$

Using (131) and (132) to substitute for κ_i and $\frac{d\theta_{i,n}}{dw^p_{i,n}}$, respectively, in (130) and then solving for $w^p_{i.n}$ we get

$$w^p_{i,n} = (1-\eta)y_{i,n} + \eta(r+\tau)U^p_i \tag{133}$$

Using (126) and (131) we can substitute for $(r+\tau)U^p_i$ in (133) and obtain

$$w^p_{i,n} = b_i + (1-\eta)\left(y_{i,n} - b_i + \theta_{i,n}\kappa_i\right) \tag{134}$$

Substituting $w^p_{i,n}$ from (134) into (131) we get the job creation condition in each submarket

$$\frac{\kappa_i}{q(\theta_{i,n})} = \frac{\eta(y_{i,n} - b_i)}{r + s^p_i + \tau + (1-\eta)m(\theta_{i,n})} \tag{135}$$

Notice that if $y_{i,n} = y_i$, meaning that productivity is the same across all submarkets of market i then $\theta_{i,n} = \theta_i$ and $w^p_{i,n} = w^p_i$. All submarkets in market i offer the same wage and job finding rate. If in addition the Hosios condition holds, i.e. $1 - \eta = \beta$, then job creation, market tightness and the Nash bargaining wage in the Benchmark model described in the text (see equations 13 and 14) are identical to those derived under competitive search.

E Connections Premium

With the introduction of a connection premium all other Bellman equations but the value of being employed in the public sector for a connected worker ($E^g_{i,c}$) remain as in the Benchmark model described in Section 2. It follows that all equilibrium conditions remain the same, but equations (27) that determine the cut-off connection costs. The cut-off connection cost now change to take into account that the existence of a connection premium increases the value of being a connected and employed public employee. In particular, equation (27) becomes:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[\tilde{A}_{i,c} - \frac{\beta\kappa_i\theta_i}{(1-\beta)}\right], \quad i = [h,l] \tag{136}$$

where

$$\tilde{A}_{i,c} \equiv \frac{\frac{\mu(s^g_i+\tau)e^g_i}{L^g_{i,c}-\mu e^g_i}}{r+\tau+s^g_i+\frac{\mu(s^g_i+\tau)e^g_i}{L^g_{i,c}-\mu e^g_i}}\left[w^g_i - b_i + \frac{\tau\int_0^{\tilde{c}_i} c\xi(c)dc}{\mu e^g_i}\right] \tag{137}$$

As shown in Appendix A.2, equations (13) and (28) give unique equilibrium values for θ_h, θ_l and $\tilde{\epsilon_u}$. To guarantee the existence and uniqueness of a steady-state condition we need to show that with the equilibrium values of θ_h, θ_l and $\tilde{\epsilon_u}$ substituted in, equations (136) cross only once in the $[\tilde{c}_h, \tilde{c}_l]$ plane. As in Appendix A.2, we can derive the slopes and show that:

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h=U_{h,c}-U_h} = \frac{\frac{\partial\tilde{A}_{h,c}}{\partial L^g_{h,c}}\frac{\partial L^g_{h,c}}{\partial\tilde{c}_l}\frac{1}{r+\tau}}{1-\frac{\partial\tilde{A}_{h,c}}{\partial L^g_{h,c}}\frac{\partial L^g_{h,c}}{\partial\tilde{c}_h}\frac{1}{r+\tau}-\frac{m^g_{h,c}}{r+\tau+s^g_h+m^g_{h,c}}\frac{\tau}{r+\tau}\frac{\tilde{c}_h\xi(\tilde{c}_h)}{\mu e^g_h}} \tag{138}$$

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l=U_{l,c}-U_l} = \frac{1-\frac{\partial\tilde{A}_{l,c}}{\partial L^g_{l,c}}\frac{\partial L^g_{l,c}}{\partial\tilde{c}_l}\frac{1}{r+\tau}-\frac{m^g_{l,c}}{r+\tau+s^g_l+m^g_{l,c}}\frac{\tau}{r+\tau}\frac{\tilde{c}_l\xi(\tilde{c}_l)}{\mu e^g_l}}{\frac{\partial\tilde{A}_{l,c}}{\partial L^g_{l,c}}\frac{\partial L^g_{l,c}}{\partial\tilde{c}_h}\frac{1}{r+\tau}} \tag{139}$$

From (137) it can be shown that $\frac{\partial\tilde{A}_{i,c}}{\partial L^g_{i,c}} < 0$, while, as already mentioned, it can be verified from (24)-(**??**) that $\frac{\partial L^g_{i,c}}{\partial\tilde{c}_i} > -\frac{\partial L^g_{i,c}}{\partial\tilde{c}_j} > 0$. Given these, sufficient condition to ensure that

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_h=U_{h,c}-U_h} < 1$$

$$\left.\frac{d\tilde{c}_h}{d\tilde{c}_l}\right|_{\tilde{c}_l=U_{l,c}-U_l} > 1$$

meaning that the two conditions only cross once in the $[\tilde{c}_h, \tilde{c}_l]$ plane is

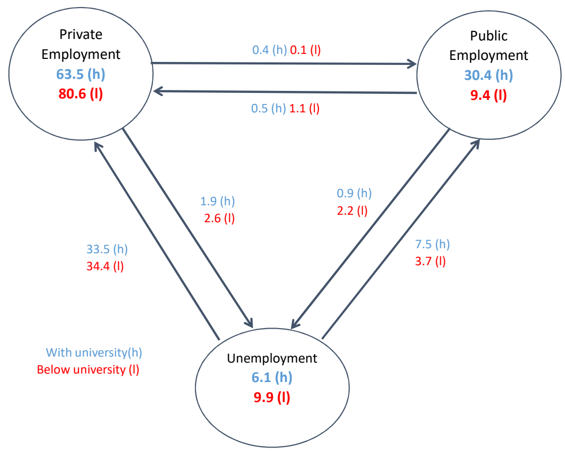
$$1-\frac{m^g_{i,c}}{r+\tau+s^g_i+m^g_{i,c}}\frac{\tau}{r+\tau}\frac{\tilde{c}_i\xi(\tilde{c}_i)}{\mu e^g_i} > 0$$

Sufficient but not necessary condition for the above inequality to be always satisfied is

$$\mu\bar{c}\xi(\bar{c}) \leq \min[e^g_h, e^g_l]$$

F Data for Calibration

Figure A1: 3-state stocks and flows, Spain

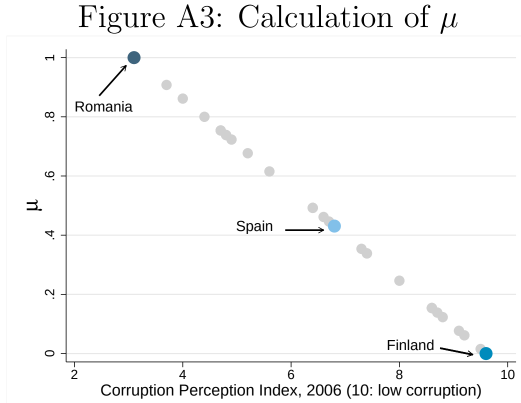


Source: Spanish Labour Force Survey, average 2005-2007.

Figure A2: Estimated public-sector wage premium, Spain and Finland

Source: Structure of Earnings Survey, 2006.

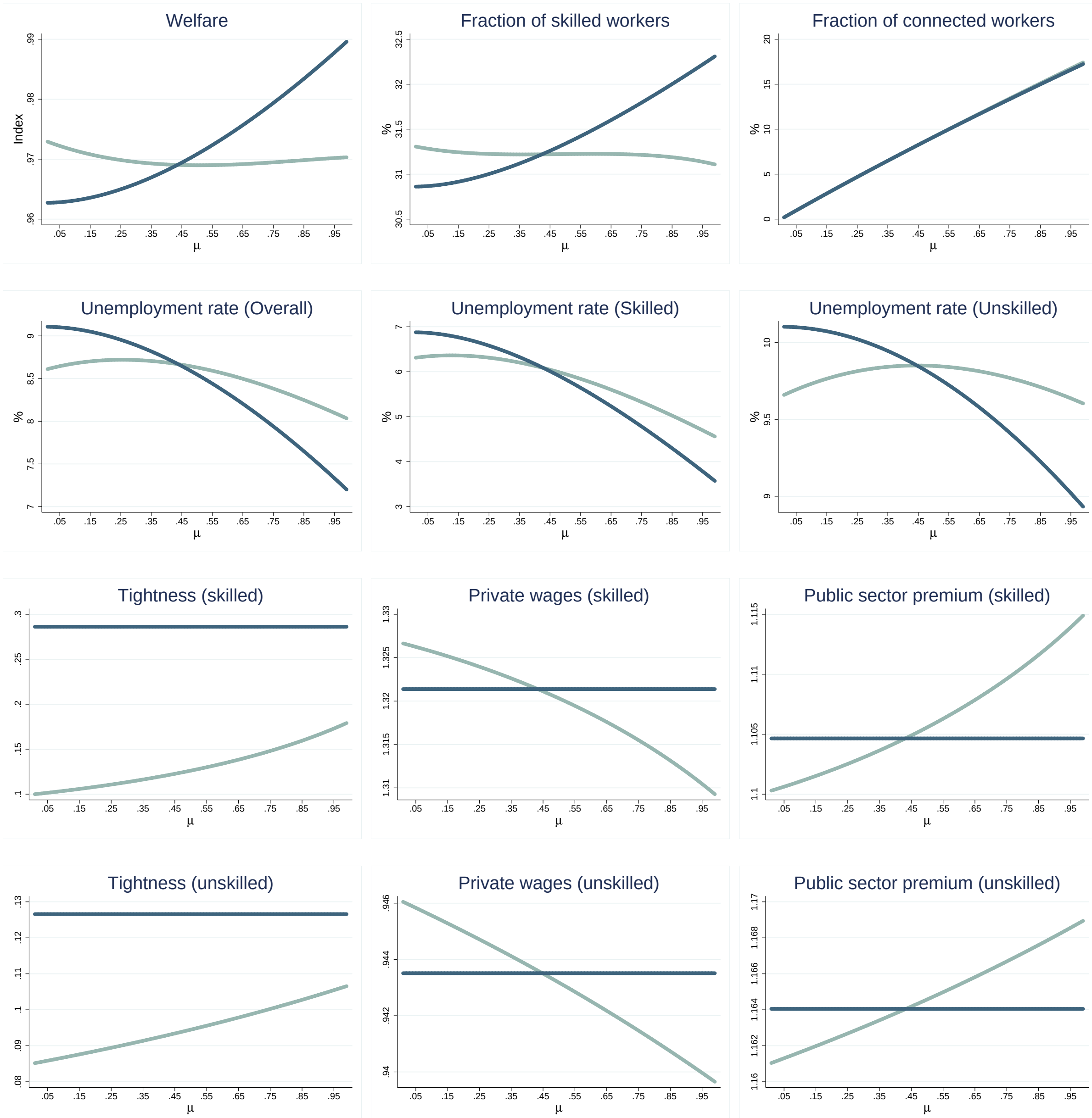
Figure A3: Calculation of μ



Source: Corruption Perception Index, 2006, Own calculations.

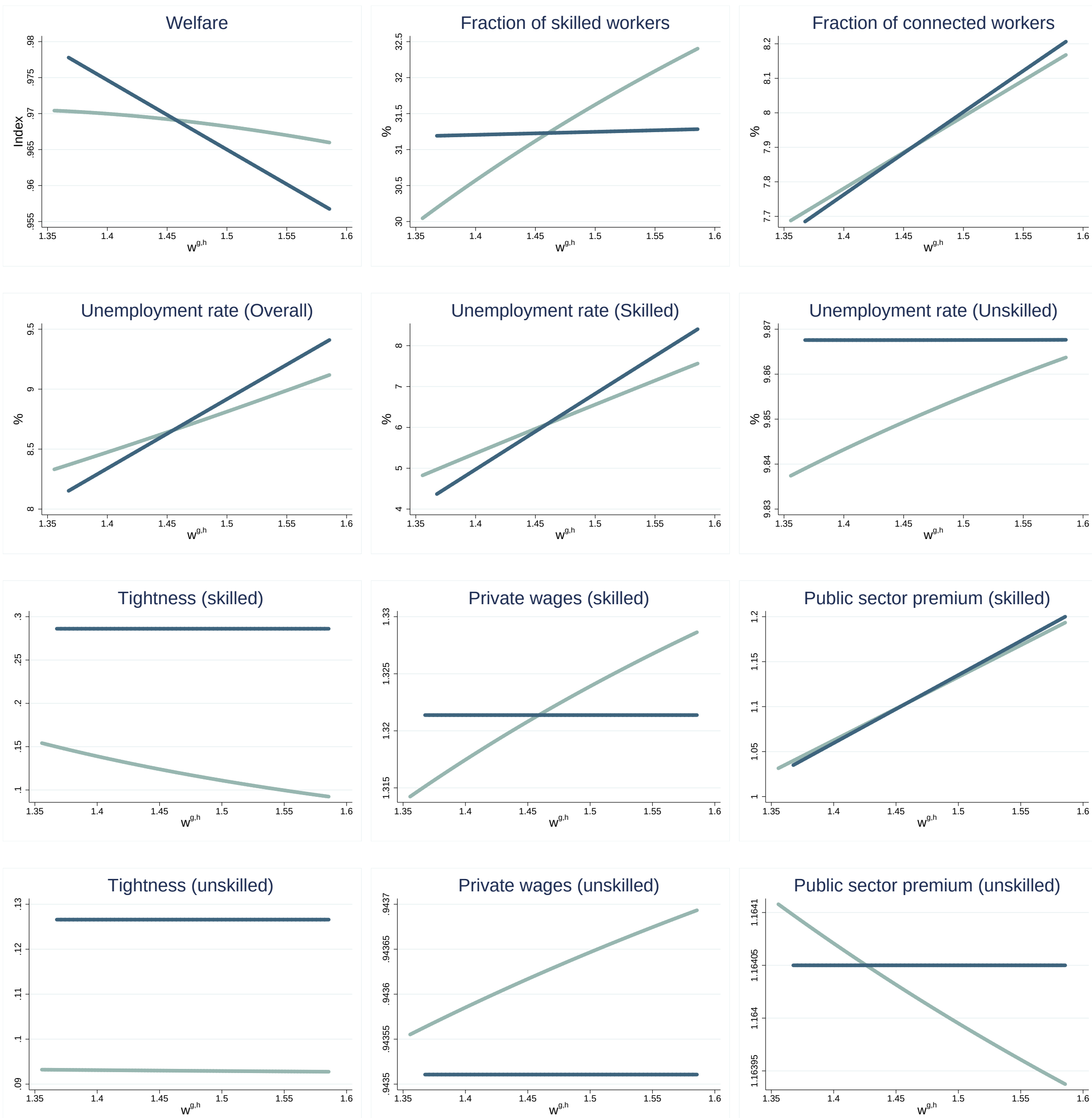
G Segmented Markets Vs Random Search

Figure A4: Effects of non-meritocracy



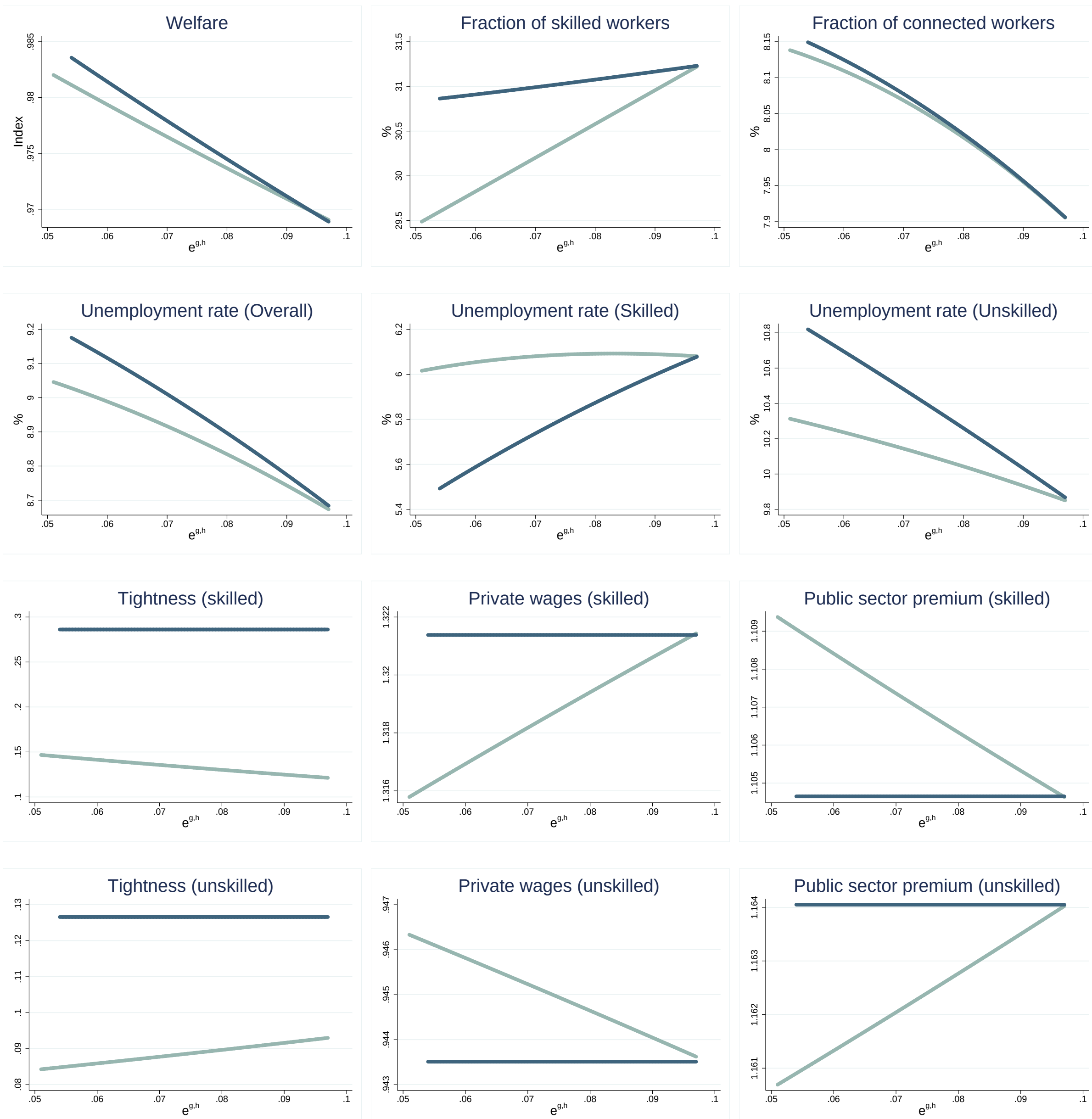
Note: The ***dark blue line*** *is the economy with segmented markets. The* ***light green line*** *is the economy with random search. In all scenarios the economy is in Case B with both* μ_h *and* μ_l *unconstrained.*

Figure A5: Effects of skilled public-sector wages



Note: The ***dark blue line*** *is the economy with segmented markets. The light green line is the economy with random search. We restrict our attention to scenarios where the economy is in Case B with both* μ_h *and* μ_l *unconstrained.*

Figure A6: Effects of skilled public-sector employment



*Note: The **dark blue line** is the economy with segmented markets. The **light green line** is the economy with random search. We restrict our attention to scenarios where the economy is in Case B with both μ_h and μ_l unconstrained.*