A Bayesian learning model of hedge fund performance

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We provide a Bayesian learning model in hedge fund performance. Our modelling provides a novel Bayesian aspirational model for panel data that is stable across different priors as reported from the mapping of the prior to the posterior of the Bayesian baseline model with the adoption of different priors. The parameters of our learning equation are time-varying which, to the best of our knowledge, is only addressed in Hu et al. (2017) who assumed that the parameters have time and individual effects and depend on observed covariates. Our data set comes from the Lipper Trading Advisor Selection System (TASS) database which includes data on performance and types of assets under management. Results reveal that a higher initial share price, management fee, leveraged and redemption notice period had a negative effect on learning. The learning curve has a U-shaped relationship, specifically, learning improves over the first three years, and gradually declines to zero by the eight-year. The second stage of analysis shows that though mean levels of learning do not directly influence performance, a higher standard deviation in learning lowers the decline in performance with higher mean learning. But we report variability in results across various models that we test for robustness.

Keywords: Bayesian panel learning model, Fund performance, time-varying covariance.

JEL codes: C11; G23; G32

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Abstract

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1. Introduction

This paper uses a novel Bayesian aspiration model to estimate learning in hedge funds, including the level of skill and returns to scale. To the best of our knowledge, this is the first time that such modelling is used in studying the hedge fund industry. Our data set comes from Lipper Trading Advisor Selection System database (hereafter, TASS) that includes data on performance and types of assets under management.

There are several issues in the hedge fund literature that have not been resolved. Hedge fund performance has received considerable attention since the seminal paper of Jensen (1968) with mixed findings, as it is openly challenged whether funds would outperform their passive benchmark (Gruber 1996, Carhart 1997; Lunde, et al. 1999; Fama and French 2010; Basak and Makarov, 2014; Cullen et al., 2012; Cabello et al. 2014; Utz et al. 2015; Vidal-García et al. 2018; Giuzio Kay et al. 2018). At the core of the debate is accurately measuring the performance of funds. Traditional performance measures compare the returns of the examined portfolio to the returns of an unmanaged portfolio of comparable risk. Several measures of funds' performance such as the net return ratio, and the abnormal return use a panel data set (Khorana and Servaes 2012). The abnormal return is the difference between a fund's return and the return of a portfolio that shares the same risk characteristics as the fund in consideration. Other measures include a dummy that equals 1 if a particular family of funds has at least one fund operating in the top 5% best performing funds of a given category in a given year (Khorana and Servaes 2012); the Sharpe ratio (Daraio and Simar 2006), whereas risk is computed as the standard deviation of monthly returns (Huang et al. 2007).[1] (Khorana and Servaes 2012) using panel data focus on alphas and argue that there should be some bounds that depend on cross-sectional investor heterogeneity with the flow response to past fund alphas. This strand of research picks earlier findings (see Clode, 2011; Busse 2001) arguing

[1]However, several drawbacks of these metrics such as their inability to incorporate funds' transaction costs or the issue of selecting the proper benchmark have fuelled the introduction of performance measures that rely on frontier analysis in the spirit of Koopmans (1951) and Farrell (1957).

that alphas might not be without issues when it comes to selecting a fund. The underlying autocorrelation could explain the results whist the hypothesis of funds being cross-sectionally independent might not be valid (Goriaev et al. 2005).[2]

Beyond issues related to accurately measuring finds' performance, there is an open discussion regarding what are the important covariates of funds' performance. As expected, the focus has been for some time on the role of risk. Most studies show that, indeed, the risk is important for funds' performance (Giuzio Kay et al. 2018; Vidal-García et al. 2018; Utz et al. 2015; Basak and Makarov, 2014; Brown et al., 1996; Cullen et al., 2012; Goriaev et al., 2005; Koski and Pontiff, 1999).[3] Brown et al. (2001) examine both competition and risk in the hedge fund, reporting similar results as in Brown et al., (1996). Busse (2001) shows that poorly performing fund managers alter their risk to be able to catch up with interim winners at the end of the year. Basak and Makarov (2014) focus on the manager's portfolio choice with respect to the strategic interactions among managers competing for fund flows. Their model builds on the strategic behaviours of two risk-averse managers, revealing that a manager either wins or loses, and never opts for a draw.[4]

Other studies (Prather et al., 2004; Vidal-García et al. 2018; Giuzio Kay et al. 2018) report the link between fund performance and various operational characteristics such as expenses, size, and past performance. This type of information could be rather beneficial to investors who decide among offered funds should a reliable relation exists between a fund's performance and some of its observable characteristics. Ferson and Mo (2016) argue that a well-specified performance measure should be based on the sum of covariance between the portfolio holdings and the subsequent abnormal, or risk-adjusted returns, with an underlying stochastic discount factor (see also Cabello et al. 2014). Their modelling has a certain appeal, but it still does not address issues related to time-varying covariance where the evidence shows that indeed this is the case (Cabello et al. 2014; Utz et al. 2015; Ferson and Mo 2016; Basak and Makarov 2014; Blake, et al. 2014, 2017).

Vidal et al. (2021) focus on understanding of mutual fund behavior in response to market volatility across international borders. Leveraging a comprehensive dataset of daily returns from diverse countries, the authors investigate how mutual fund managers strategically navigate fluctuations in market volatility. Addressing a void in the literature that often confines analyses to specific countries or short time frames, to underscore a universal trend: mutual funds reduce market exposure during times of elevated volatility, underscoring the heightened sensitivity of systemic risk to global market fluctuations. Vidal et al. (2015) challenge existing studies of mutual fund market timing that have found little evidence to support timing ability. Using a sample of daily returns for 35 countries, the authors find that more than a third of mutual funds show significantly positive market timing ability across all countries and demonstrate that using daily returns rather than monthly returns increases the number of significant estimates of timing ability, observation frequency is relevant when examining fund performance. The authors also show the importance of market timing in recessions and document that the effect of the business cycle on market timing is much stronger for extremely successful fund managers. Idiosyncratic risk is negatively related to returns for all funds investment style categories (Vidal-García et al., 2019), and assessing portfolio optimization

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[2]Kempf and Ruenzi (2008) study the competition between fund managers across funds' family. They argue that an optimal policy of fund managers is to alter their risk-taking. Studying US equity mutual funds between 1993 and 2001, they report the presence of the family tournament, which is more pronounced in large families.

[3]Brown et al. (1996) identify that interim losers who underperform the benchmark in the first half of the year are likely to increase their risk relative to mid-year winners. Funds are ranked according to their cumulative return, while risk is measured by the ratio of fund's standard deviation after the interim performance assessment to its standard deviation before that date. Another proxy for risk is the tracking error variance, which is the variance of the difference between fund's return and the value-weighted market index (Chevalier and Ellison, 1997).

[4]Basak and Makarov (2014) show that, even when a manager is significantly ahead in the tournament, her investment behaviour and thus portfolio volatility is still influenced by the tournament incentives. In addition, Sato (2015) show the importance of flow-performance relationship and asset bubbles.

htrough rebalancing for ETFs portfolios in time-varying volatility environment is an important consideration (Xidonas et al. 2020). Mamatzakis and Tsionas (2021); (Vidal-García et al. 2019) introduce a Bayesian panel model designed to address the issue of persistence in the performance of US funds, concurrently addressing the intricate challenge of errors in variables. Their approach stands in stark contrast to conventional paradigms that often rely on restrictive assumptions, particularly the assumption of independent error terms across funds. Presenting an innovative and versatile Bayesian model for dynamic panel data, characterized by its stability and adaptability across various prior specifications the authors map different priors to the posterior of our Bayesian baseline model.

From the above literature becomes apparent that to date there is no silver bullet regarding an appropriate modelling of hedge fund performance and its underlying determinants across funds and over time. In some detail, we deal with survivorship bias, backfill bias, and selection bias (see Fung and Hsieh 2000 or Lhabitant 2009 for an overview). Survivorship bias arises when a database only includes funds that are currently operating. The evidence in Edelman, Fung, and Hsieh (2013) and Agarwal, Fos, and Jiang (2014) indicate that various types of survivorship biases cancel each other out. We require funds to follow equity-oriented strategies because we evaluate the returns to scale effect at both the fund- and strategy-level. For hedge funds with equity-oriented strategies, the total value of the securities that they invest in can be accurately measured (e.g., the size of all firms listed on the NYSE, Amex, and NASDAQ). Therefore, we limit the sample to hedge funds that follow one of the following strategies: equity market neutral, event-driven, and long-short equity. Finally, funds are required to have at least 24 months of performance and assets under management data available over a 3-year period (see Cao and Velthuis, 2017).

This paper bridges a gap in the literature by providing a novel way modelling hedge funds' performance, relaxing some of the strong assumptions in the literature. Moreover, we argue that learning is of importance for measuring fund performance without resorting to strong assumptions regarding the unobservable underlying idiosyncratic characteristics of fund managers. To this end, the purpose of our study is fourfold. First, we propose a new Bayesian panel model of learning. Second, this model allows measuring various parameters of the learning process. Moreover, the parameters of our learning equation are time-varying which, to the best of our knowledge, is only addressed in Hu et al. (2017) who assumed that the parameters have time and individual effects and depend on observed covariates. Third, as the estimation of this new model is cumbersome, we apply Bayesian techniques that facilitate the robustness of the estimation (Annaert et al. 2003; Barber 2012). Bayesian analysis is implemented using state-of-the-art Sequential Monte Carlo / Particle-Filtering (SMC/PF) techniques. Fourth, we broaden the findings of the relatively few studies measuring fund learnings for an up-to-date set concerning US hedge funds for which we demonstrate that results remain stable across different priors as reported from the mapping of the before the posterior.

Our analysis follows the following path. To derive the measure of learning, we employ a model of equations and report the parameter estimates for learning. The results indicate strong statistical significance for the measure of learning (*q_it*),* and we observe that certain fund variables have an impact on learning. Specifically, variables such as initial share price, management fee, leverage, and redemption notice period negatively impact learning, while initial net asset value (NAV), minimum investment, incentive fee, high water mark, leveraged funds, lock-up period, payout period, and graveyard positively influence learning.

We present the marginal effects of the learning measure to provide insights into how different fund variables affect learning behavior. We find that management fees, incentive fees, and certain time-related variables have notable impacts on learning. The learning curve over the fund's duration resembles an inverse U shape, suggesting that learning tends to increase over a medium-term period of around three years but then gradually declines to zero by year 8. This indicates that learning is more prevalent in the medium term rather than being a long-term activity. Moving forward, we explore the task difficulty variable and its relationship with fund characteristics. Similar to the learning measure, we present the parameter estimates and marginal effects for task difficulty. Here again, we find significant associations with fund variables, and some variables have negative impacts on task difficulty. We complement these results by depicting the task difficulty over the fund's duration. As expected, task difficulty increases as the fund's duration extends.

Shifting our focus to the progress ratio, we investigate its marginal effects and associations with fund variables 4. The results suggest that certain fund characteristics have significant impacts on the progress ratio. The progress ratio over time demonstrates a rising trend in the short run (around two years), followed by a gradual decline and stabilization for the remaining period. This pattern aligns with the observations from the learning curve in in the main analysis.

In the subsequent sections, we delve into the return equation and examine its associations with fund variables. Parameter estimates for the return equation reveal significant relationships between certain fund characteristics and returns. Notably, variables such as initial share price, management fee, leverage, and redemption notice period have negative impacts on returns, while other variables positively affect returns. Our analysis further extends to second-stage panel regressions, where we incorporate additional factors such as industry variables and fund fixed effects. We introduce interaction terms to explore the moderation effects of different variables on returns. These interactions highlight the complex relationships between learning, fund characteristics, and performance outcomes. To validate our findings, we conduct robustness checks across different fund types, leveraged and non-leveraged funds, and funds with and without their own capital invested. These checks strengthen the reliability of our results and provide insights into the consistency of associations across various fund categories.

In summary, the results reveals that a higher initial share price, management fee, leveraged and redemption notice period had a negative effect on learning. The learning curve has a U-shaped relationship, specifically, learning improves over the first three years, and gradually declines to zero by the eighth year. In the second stage of analysis, we test the effects of mean, standard deviation, and kurtosis of learning to predict next period performance outcomes. We find that though mean levels of learning do not directly influence performance, a higher standard deviation in learning lowers the decline in performance with higher mean learning. Perhaps implying that hedge funds that improve learning with higher levels of variability in learning can mitigate performance decline.

The rest of the paper is organized as follows: Section 2 presents the aspirational model for funds, whilst section 3 reports the data set. Section 4 discusses empirical results. Finally, Section 5 provides some concluding remarks and policy implications.

2. Methodology

2.1 A Bayesian learning model for hedge funds

Our model follows from the standard return equation of a fund where we measure the log of the rate of return of the corresponding fund as the monthly returns at t + 1 which is the outcome variable. Our predictor variables are mean, standard deviation, and kurtosis of learning which we note as $q*$.

In detail, suppose $q *_t$ denotes the learning level of the fund industry at time t. The demand for hedge funds is d_t and we start our modelling by linking demand to learning as follows:

$$d_t = q *_t + v_t, v_t \sim \text{i.i.d. } \mathcal{N}(0, \sigma_v^2). \quad (1)$$

Let $D_{1:t-1}$ denote the data up to period $t-1$, that is $D_{1:t-1} = [d_1, \dots, d_{t-1}]$.

By Bayes' theorem we have

$$p(q *_t \mid D_{1:t-1}) \propto p(D_{1:t-1}; q *_t) p(q *_t), \quad (2)$$

where $p(D_{1:t-1}; q *_t)$ is the likelihood and $p(q *_t)$ is the prior. Mapping the prior to the posterior is essential in learning modeling and the behavioral theory of the firm (Cyert and March, 1963, 1992).

Given that $D_0 =$ we have

$$p(q *_1 \mid D_{0:1}) = p(q *_1 \mid D_{1:1}) \propto \sigma_v^{-1} \exp\left[-\frac{1}{2\sigma_v^2}(d_1 - q *_1)^2\right] p(q *_1),$$

where $p(q *_1) \propto \text{const.}$, i.e., the prior on initial learning is flat.

Similarly, we have

$$p(q *_2 \mid D_{1:2}) \propto \sigma_v^{-1} \exp\left[-\tfrac{1}{2\sigma_v^2}\{(d_1 - q *_1)^2 + (d_2 - q *_2)^2\}\right] \quad (3)$$

More generally

$$p(q *_t \mid D_{1:t}) \propto \sigma_v^{-t} \exp\left[-\frac{1}{2\sigma_v^2}\sum_{\tau=1}^{t}(d_t - q *_t)^2\right]. \quad (4)$$

Note that we have

$$q *_t \mid D_{1:t}, \sigma_v \sim \mathcal{N}(\bar{d}_t, t^{-1}\sigma_v^2), \quad (5)$$

where $\mu_t \equiv \bar{d}_t = \frac{1}{t}\sum_{\tau=1}^{t} d_t$ is the posterior mean, which is also equal to the sample mean.

Since $\mu_t = \frac{t-1}{t}\mu_{t-1} + \frac{1}{t}d_t$ we see that the posterior mean cannot be put into the form:

$$\mu_t = \alpha\mu_{t-1} + (1-\alpha)d_{t-1}, \quad (6)$$

for some positive value of α.

To have a similar form we need a change in the timing so that the posterior is $p(q *_t \mid D_{1:t-1})$ which also seems more appropriate in revising beliefs about learning levels. To accomplish this, we need to revise (1) as follows.

$$d_{t-1} = q *_t + v_t, v_t \sim \text{i.i.d. } \mathcal{N}(0, \sigma_v^2). \quad (7)$$

Under this assumption we have

$$q *_t \mid D_{1:t-1}, \sigma_v \sim \mathcal{N}(\bar{d}_{t-1}, (t-1)^{-1}\sigma_v^2), \quad (8)$$

and we can write the posterior mean $\mu_t \equiv \bar{d}_{t-1}$ as

$$\mu_t = \frac{t-2}{t-1}\mu_{t-1} + \frac{1}{t-1}d_{t-1}. \quad (9)$$

In turn, this can be written as

$$\mu_t = \alpha_t\mu_{t-1} + (1-\alpha_t)d_{t-1}, \quad (10)$$

where $\alpha_t = \frac{t-2}{t-1}$. This coefficient is positive, less than one, but depends on time and converges to one (as t increases to infinity). Based on the most common aspiration models (Washburn and Bromiley, 2012, Table I) this model does not fit any of the suggested models. The same is the case compared to Hu et al. (2017).

To examine a different model, suppose we keep (7) but also relax the no excess demand assumption as the market participants in hedge funds market should be also allowed to assert excess demand (Hu et al., 2017):

$$d_{t-1} = q *_t + w_t - u_t, \quad (11)$$

where $w_t \sim \text{i.i.d. } \mathcal{N}(0, \sigma_v^2/h)$, where h is a prior parameter, and $u_t \geq 0$ has a half-normal distribution, viz. $u_t \sim \text{i.i.d. } \mathcal{N}_+(0, \sigma_u^2)$.

The no excess demand assumption means that $d_t \leq q*_t$ apart from measurement error captured by the error component w_t. Conditional on excess supply (u_t) we have

$$q*_t \,|\sigma_v, u_{1:t-1}, \sigma_v, D_{1:t-1} \sim \mathcal{N}(\mu_t, \sigma_t^2), \quad (12)$$

where

$$\mu_t = \frac{(t-1)\bar{d}_{t-1} + h(d_{t-1} + u_t)}{t+h-1}, \quad (13)$$

$$\sigma_t^2 = \frac{\sigma_v^2}{t-1+h}. \quad (14)$$

It can be shown that the conditional posterior of excess supply is

$$u_t | q*_t, \sigma_v, D_{1:t-1} \sim \mathcal{N}_+\left(\hat{u}_t, \frac{\sigma_v^2}{\lambda + h}\right), \quad (15)$$

where $\hat{u}_t = \frac{h(q*_t - d_{t-1})}{\lambda+h}$, and $\lambda = \frac{\sigma_v^2}{\sigma_u}$ is the noise-to-signal ratio for excess supply.
This expression follows because of the timing assumption in (11). If, instead, we have $d_t = q*_t + v_t$ then $\hat{u}_t = \frac{h(q*_t - d_t)}{\lambda+h}$, from which we see that $\hat{u}_t$ depends on d_t rather than d_{t-1}. The expected value of the truncated normal distribution in (15) is

$$\tilde{u}_t \equiv E(u_t | q*_t, \sigma_v, D_{1:t-1}) = \hat{u}_t + \frac{\sigma_v}{\sqrt{\lambda+h}} \frac{\phi\left(\frac{\hat{u}_t(\lambda+h)}{\sigma_v^2}\right)}{\Phi\left(\frac{\hat{u}_t(\lambda+h)}{\sigma_v^2}\right)},$$

(16)
where $\phi(\cdot)$ and $\Phi(\cdot)$ denote, respectively, the density and distribution function of the standard normal distribution.
From (13) we have

$$\mu_t = \frac{(t-1)\bar{d}_{t-1} + h(d_{t-1} + \tilde{u}_t)}{t+h-1}, \quad (17)$$

and we can get a final expression by substituting (16).
The resulting expression is nonlinear, but we can use a linear approximation for $\frac{\phi(z)}{\Phi(z)} \simeq A + Bz$ to show[5] that, approximately,

$$\mu_t \simeq \frac{(t-1)\bar{d}_{t-1} + h\left\{1 - \frac{h}{\lambda+h}\left(1 + B\frac{\lambda+h}{\sigma_v^2}\right)\right\}d_{t-1} + \frac{h}{\sqrt{\lambda+h}}A\sigma_v}{h\frac{t+h}{\lambda+h}[1 + B(\lambda+h)]}. \quad (18)$$

Therefore, at least approximately, we still obtain a linear relationship between $\mu_t, \bar{d}_{t-1}$ and d_{t-1}, where the parameters are related to the fundamentals of the learning process.

Notice that an intercept now appears in the equation (for a discussion see Washburn and Bromiley, 2012, p. 909). However, the important message is that the parameters of the learning equation are time-varying which, to the best of our knowledge, is only addressed in Hu et al. (2017) who assumed that the parameters have time and individual effects and depend on observed covariates (see their equation (5), p. 1444).

3. Data

To test for the effect of the nature of Bayesian learning at the fund-period level we draw on the Lipper Trading Advisor Selection System database (hereafter TASS), a widely used data in the hedge fund literature. The data allows us to assess the level of learning and helps control for survival bias, backfill bias, and selection bias (refer to Fung and Hsieh (2000) and Lhabitant (2009) for an overview). To lower survivor bias we use the earliest available year, 1994). We require that the funds at least have 24 months of performance reports. The final sample contains

[5]Using 40 points in the interval $[-3,3]$ and simple least squares regression we obtain $A = 0.95$ and $B = -0.598$ with $R^2 = 0.97$.

976,417 product periods from 1994 to 2020, representing 12,778 products from 3720 hedge funds.

We use the log of monthly returns at t + 1 as the outcome variable. The predictor variables are mean, standard deviation, and kurtosis of learning (q*) rolling five-month windows from (t-4) to (t) from learning curve values. We control for a management fee, incentive fee, high watermark (0=no; 1=yes), leveraged (0=no; 1=yes), Redemption Notice Period (days), Lock-Up Period (months), Pay Out Period (days), tracking frequency (monthly or quarterly).

We also include dummies for sector focus, investment approach, geographic focus, investment focus, legal structure, subscription frequency, redemption frequency, and country of domicile. Table B1 in Appendix B provides variable definitions and data source for each variable.

In Table 1, regarding fund fee structures, the average (sd) management fee is 1.41% (0.65%), and the incentive fee is 12.33% (8.73%). About 57% of the funds are high watermark funds, and 46% are leveraged funds. The average redemption period is 37 days (35 days) and the mean lock-up period is 2.31 months (6.18 months).

[Insert Table 1 about here]

4. Empirical Results

4.1 Estimating the learning equation parameters

Having derived the measure of learning of hedge funds (see Appendix I for details overestimation) we present next its association with main fund variables like management fees. Note that we first apply the model of equations (7) to (8). Table 2 reports the parameter estimates for learning q^*_{it}. From a statistical point of view, we get a strong statistical significance, while variables such as initial share price, management fee, leveraged, and redemption notice period assert a negative impact on learning. On the other hand, initial NAV, minimum investment, incentive fee, high water mark, leveraged funds, lock-up period, pay out period, and graveyard all would enhance learning.

[Insert Table 1 about here]

Figure 1, in addition, shows the learning curve over the fund's duration. It resembles an inverse U shape, implying that learning would increase over three years period but then will decline gradually to zero in year 8. Based on this diagram learning takes place up to medium term and it is not a long-term kind of activity.

[Insert Figure 1 about here]

Table 3 reports the parameter estimates of task difficulty $\boldsymbol{N_{it}}$. From a statistical point of view, we get strong significance. In terms of signs, results are broadly similar to learning.

[Insert Table 3 about here]

Figure 2 shows the task difficulty over a fund time duration. Clearly, and as expected, the task difficulty is an increasing function of fund duration.

Next, we present marginal effects on progress ratio, that is $(\frac{\sigma_w}{\sigma_\theta})$, in Table 4. For completeness of the presentation of results, Appendix B reports results for the underlying variances of progress ratio. Once more the statistical significance of our estimations is strong across all parameter estimates while the signs and magnitudes of marginal effects are in line with the learning equation.

[Insert Figures 2 and 5 and Table 4 about here]

Figure 3 shows that there is rising progress in the short-run of fund duration of two years and thereafter falls and remains stable for the remaining period. This seems to agree with Figure 1 of learning.

4.2 The Return equation

Table 5 reports the parameter estimates of the return equation. From a statistical point of view, we get a strong statistical significance, while variables such as initial share price, management fee, leveraged, and redemption notice period assert a negative impact on learning. On the other hand, initial NAV, minimum investment, incentive fee, high water mark, leveraged funds, lock-up period, payout period, and graveyard would enhance learning.

[Insert Table 5 about here]

4.3 Second Stage Panel Regressions

Based on the parameter estimate of the learning equation such as mean learning (q*_mean), standard deviation of learning (q*_sd) and kurtosis (q*_kurtosis) we proceed next with second stage panel regression analysis. In the model, we include the five Fung-Hsieh factors on industry variables and fund fixed effects:

$$\ln\_RateofReturn_{fi,t+1} = \beta_0 + \beta_1 \text{q}*\_\text{mean}_{fi,t+1} + \beta_2 \text{q}*\_\text{sd}_{fi,t+1} + \beta_3 \text{q}*\_\text{kurtosis}_{fi,t+1} + \beta_4(\text{q}*\_\text{mean x q}*\_\text{kurtosis})_{fi,t+1} + \beta_5(\text{q}*\_\text{sd x q}*\_\text{kurtosis})_{fi,t+1} + \beta_6(\text{q}*\_\text{sd x q}*\_\text{kurtosis})_{fi,t+1} + \beta_7(\text{q}*\_\text{mean x q}*\_\text{sd x q}*\_\text{kurtosis})_{fi,t+1} + \beta_c X_C + \sigma + \iota + \gamma + \kappa + \nu + \zeta + \varphi + \psi + \epsilon$$

We use the log of monthly returns at *t + 1* as the outcome variable for the fund *f* in hedge fund *i* in period *t*. The predictor variables are mean, standard deviation and kurtosis of learning (q*). We control (X_C) for the management fee, incentive fee, high watermark (0=no; 1=yes), leveraged (0=no; 1=yes), Redemption Notice Period (days), Lock-Up Period (months), Pay Out Period (days), tracking frequency (monthly or quarterly). In addition, we control for the five factors from Bond lookback straddle (PTFSBD), Currency lookback straddle (PTFSFX), Commodity lookback straddle (PTFSCOM), and Short-term interest rate lookback straddle (PTFSIR) (Fung and Hsieh, 2001).[6] We also include dummies for sector focus (σ), investment approach (ι), geographic focus (γ), investment focus (κ), legal structure (ν), subscription frequency (ζ), redemption frequency (φ), and country of domicile (ψ).

Table 6 presents the estimates. In Model 2, the direct effect of learning (qstar_mean) is not significant. Implying that improved learning has no association with the next period return. In Model 3, higher variation in learning (qstar_sd) has a negative association with returns in the next period. With increasing learning higher variability in learning lowers the decline in returns (Model 4). Finally, with increasing learning, higher standard deviation and lower kurtosis improve the performance in the next period (Table 6, Model 5). However, higher kurtosis irrespective of the level of standard deviation of learning lowers performance.

[Insert Table 6 about here]

Figure 4 reports the moderation effect of standard deviation and kurtosis of learnings. Figure 4 (a) confirms the results of Table 6 as the standard deviation of learning is negatively associated with the return. Interestingly, the kurtosis learning would improve return (see Figure 4(b)).

[Insert Figure 4 about here]

4.3.3. Robustness checks

We further check the validity of findings across different fund types. In Table 7, we report results for both leveraged and non-leveraged funds. Interestingly we get significance for both funds. In detail, Models 1 and 2 report that the direct effect of higher variation in learning (qstar_sd) is highly significant as well for models 3 and 4, implying that uncertainty over learning would assert a negative impact on the next period's return. The direct effect of mean learning (qstar_mean) also asserts a negative impact on the next period's return. However, the interaction terms between mean learning and variation in learning carry a positive sign across all models while it is highly significant. These results are of some interest because they show that the net effect of learning on returns is positive whether for leveraged or non-leveraged

[6] The data is available at: http://faculty.fuqua.duke.edu/~dah7/DataLibrary/TF-Fac.xls

funds. When we include the kurtosis results do not alter and report that lower kurtosis improves the performance in the next period.

[Insert Tables 7 and 8 about here]

In Table 8 based on subgroup analysis by funds with and without own capital invested in the fund the effects of the two-way interaction are stronger for funds with own capital (Model 2 vs. Model 3), however, the three-way interaction is stronger for own capital invested in the firm (Model 4 vs. Model 2).

[Insert Tables 9 and 10 about here]

Table 9, explore the effects by fund type. For convertible arbitrage funds (models 1 and 2) we do not find support for effects. For the dedicated short bias funds, we find that a higher mean and standard deviation of learning lower performance (model 3), however, the three-way interaction is consistent with the main effects and the effect size is stronger (model 4). For the emerging market focus, higher mean and standard deviation improve performance (model 5), and higher kurtosis along with higher standard deviation and mean of learning has a small positive effect on performance (model 6). For equity market neutral focus (models 7 and 8), event-driven focus (models 9 and 10), fixed income arbitrage (models 11 and 12), managed futures (models 19 and 20), or options strategy (models 23 and 24) we do not find strong effects. For the fund of funds focus we find effects like the main effects (models 13 and 14). The global macro strategy (models 15 and 16) did not have support two-way interaction. However, the effects of three-way interaction were small and positive. The multi-strategy focus related two-way interaction is consistent with the main effect (model 21), however, the three-way interaction has a small positive effect (model 22) on performance.

Using gender information, we classify the fund owners by gender. In Table 10 we find consistent effects for males and females for the two-way interaction (models 1 and 3), However, the effects were consistent with the main effects for males (model 2) but not for females (model 4).

5. Conclusion

The study aimed to understand the nature of learning in hedge funds (the first-stage analysis) and the effect of such learning on future performance. Based on the first-stage analysis of Bayesian learning a higher initial share price, management fee, leveraged and redemption notice period had a negative effect on learning, however, higher initial NAV, minimum investment, incentive fee, high water mark, leveraged funds, lock-up period, pay out period had a positive effect on learning. It is important to note that, the learning curve has a U-shaped relationship, specifically, learning improves over first three years, and gradually declines to zero by the eighth year.

In the second stage of analysis, we test the effects of mean, standard deviation, and kurtosis of learning to predict next period performance outcomes. We find that though mean levels of learning do not directly influence performance, higher standard deviation in learning lowers the decline in performance with higher mean learning. Perhaps implying that hedge funds that improve learning with higher levels of variability in learning can mitigate performance decline. To assess the effects of extremeness in learning, a three-way interaction among mean, standard deviation, and kurtosis shows that hedge funds with higher mean, higher standard deviations, and lower kurtosis (i.e., avoiding the extremes of learning) have a positive return. Our results show that these effects are generally supported.

One potential limitation of our study is the selection of the study period from 1994 to 2020. While this time frame allows us to comprehensively analyze the trends, it may lead to a perception of truncation, and we acknowledge that the use of a different time range could yield different insights. Our choice of initiating the study in 1994 was due to data limitations as this was the earliest available year. However, we recognize that this argument may benefit from additional clarity. Despite this limitation, we believe that our analysis of the available data provides meaningful and insightful implications for practitioners and policymakers alike.

In conclusion, while Hu et al. (2017) explored the time-varying nature of parameters in a learning equation and considered time and individual effects dependent on observed covariates, our research significantly advances this understanding by introducing a novel Bayesian aspirational model for hedge fund performance and learning. Our model not only accommodates time-varying parameters but also offers a comprehensive framework enabling the measurement of various parameters associated with the learning process. The incorporation of Bayesian techniques, particularly Sequential Monte Carlo / Particle-Filtering (SMC/PF) methods, contributes to the robustness of our parameter estimation process. This methodological enhancement ensures the reliability and validity of our findings and distinguishes our study from the prior literature. In terms of contributions to the field, our research not only introduces Bayesian panel model for hedge fund learning but also offers insights into the intricate interplay between learning and performance outcomes. By investigating the effects of mean, standard deviation, and kurtosis of learning on predicting future performance, we advance the understanding of how hedge fund learning trajectories relate to fund performance over time. This nuanced analysis stands in contrast to the more limited focus of previous studies, including Hu et al. (2017), which primarily addressed specific aspects of time-varying parameters and did not explore the multifaceted relationship between learning dynamics and performance outcomes.

In summary, while Hu et al. (2017) laid a foundation by exploring time-varying parameters and their dependence on observed covariates, our research extends beyond this scope by introducing a comprehensive Bayesian aspirational model for hedge fund performance and learning. By encompassing a broader range of factors and utilizing advanced Bayesian techniques, our study provides a more holistic view of hedge fund learning dynamics and their implications for performance outcomes. This distinctive approach establishes a substantial contribution to the literature, enhancing the understanding of the complex relationship between learning and performance within the hedge fund industry.”

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Table 1. Sample descriptive statistics

The table provides a comprehensive overview of key descriptive statistics for the variables in the dataset. These variables include the logarithm of the rate of return, learning metrics such as standard deviation, mean, and kurtosis, management and incentive fees, high watermark, leveraged status, redemption, lock-up, and pay out periods, and tracking frequency. Each variable's significance is elucidated in its caption. Table 1(b). Pearson Correlations showcases the Pearson correlation coefficients among the same variables. These coefficients shed light on the relationships between the variables, with positive, negative, or no correlations indicated. The definition of each variable are in Appendix table B1.

Table 1(a). Sample mean, S.D., min, and max estimates

		Mean	S.D.	Min	Max
1	Log of rate of return	0.5250	0.3867	-10.2738	2.5885
2	Learning (standard deviation)	0.0574	0.0180	0.0000	0.2789
3	Learning (mean)	0.1792	0.0207	0.0832	0.3953
4	Learning (kurtosis)	2.7272	1.2325	1	18.2746
5	Management fee	1.4102	0.6506	0	22
6	Incentive fee	12.3336	8.7388	0	200
7	High watermark	0.5739	0.4945	0	1
8	Leveraged	0.4684	0.4990	0	1
9	Redemption period	36.9702	34.9887	0	365
10	Lock up period	2.3134	6.1878	0	90
11	Pay out period	14.2997	17.2946	0	90
12	Tracking frequency	1.0025	0.0502	1	2

Table 1(b). Pearson correlations

		1	2	3	4	5	6	7	8	9	10	11
1	Log of rate of return	1										
2	Learning (sd)	-0.0414*	1									
3	Learning (mean)	-0.0016	0.5470*	1								
4	Learning (kurtosis)	-0.1289*	0.1875*	-0.0480*	1							
5	Management fee	0.0606*	-0.0015	0	0.0153*	1						
6	Incentive fee	0.2121*	-0.0366*	0.0018	-0.1639*	0.2254*	1					
7	High watermark	0.0987*	-0.0401*	-0.0038*	-0.1767*	0.0529*	0.4760*	1				
8	Leveraged	0.1041*	-0.0125*	0.0030*	-0.0517*	0.0794*	0.2542*	0.1772*	1			
9	Redemption period	0.0488*	-0.0170*	-0.0053*	-0.0910*	0.0033*	0.1471*	0.2946*	0.0827*	1		
10	Lock up period	0.1130*	-0.0232*	0.0013	-0.1170*	-0.0006	0.1621*	0.1788*	0.0554*	0.2644*	1	
11	Pay out period	0.0883*	-0.0253*	-0.0045*	-0.1088*	0.0156*	0.1584*	0.2951*	0.0938*	0.3811*	0.2078*	1
12	Tracking frequency	0.0216*	0.0046*	0.0011	0.0017	-0.0148*	-0.0343*	-0.0389*	0.0023	0.0321*	0.0046*	-0.0222*

N = 454,898; *p<0.05 (two-tailed)

Table 2. The marginal effects of q_{it}^{*} equation

Variable	post.mean	post.s.d.	z
Intercept	0.1123	0.0843	1.3332
ln_NAV (X1)	-0.0007	0.0104	-0.0671
ln_Initial NAV (X2)	5.9823	0.8930	6.6993
ln_Initial Share Price (X3)	-5.9887	0.8882	-6.7428
ln_Minimum Investment (X4)	0.0160	0.0040	4.0230
Management Fee (X5)	-0.0416	0.0219	-1.9024
Incentive Fee (X6)	0.0234	0.0021	11.2390
High Water Mark (X7)	0.0997	0.0279	3.5730
Leveraged (X8)	-0.0516	0.0224	-2.3088
Redemptio Notice Period (X9)	-0.0031	0.0005	-6.1550
Lock Up Period (X10)	0.0190	0.0029	6.5119
Pay Out Period (X11)	0.0033	0.0008	4.2816
Live or Graveyard (X12)	0.0747	0.0287	2.6046
Time (X13)	0.0009	0.0001	6.4937

Adjusted R-Squared 0.106.

Note: The table presents the results of the regression analysis investigating the impact of various fund variables on the measure of learning (q_{it}^{*}), or represents the cumulative improvement or progress achieved through experience or repetition of a task or activity. It quantifies the relationship between the cumulative production quantity (or experience) and the corresponding reduction in the average time or cost per unit of output. The table includes estimated coefficients for each variable along with their associated post-mean, post-standard deviation, and z-values. The variables under consideration encompass a range of fund characteristics, such as ln_NAV, ln_Initial NAV, ln_Initial Share Price, ln_Minimum Investment, Management Fee, Incentive Fee, High Water Mark, Leveraged status, Redemption Notice Period, Lock Up Period, Pay Out Period, Live or Graveyard status, and Time. These coefficients provide insights into the direction and magnitude of their effects on learning. The significance of each variable's impact is indicated by the z-values. The definition of each variable are in Appendix table B1.

Table 3. Marginal effects on 'task difficulty' N_{it}

Variable	post.mean	post.s.d.	z
Intercept	0.9589	0.1343	7.1392
ln_NAV (X1)	0.0098	0.0103	0.9501
ln_Initial NAV (X2)	0.0333	0.0153	2.1763
ln_Initial Share Price (X3)	-0.0426	0.0105	-4.0446
ln_Minimum Investment (X4)	-0.0775	0.0106	-7.3082
Management Fee (X5)	0.0956	0.0231	4.1318
Incentive Fee (X6)	0.0173	0.0015	11.2120
High Water Mark (X7)	-0.0479	0.0276	-1.7316
Leveraged (X8)	0.0310	0.0237	1.3082
Redemption Notice Period (X9)	0.0009	0.0004	2.2663
Lock Up Period (X10)	0.0114	0.0023	5.0384
Pay Out Period (X11)	-0.0067	0.0006	-12.1740
Live or Graveyard (X12)	0.3676	0.0319	11.5270
Time (X13)	0.0003	0.0001	2.4448

Adjusted R-Squared 0.0416.

Note: The table presents the outcomes of a regression analysis investigating the influence of various fund attributes on 'task difficulty'. The learning curve model assumes that as individuals or entities repeat a task, they become more skilled and efficient, leading to improvements in performance. Tasks with higher difficulty levels will generally exhibit slower rates of improvement, resulting in shallower learning curves, while tasks with lower difficulty levels may show more rapid improvement and steeper learning curves. 'Task difficulty' refers to the level of complexity, effort, and skill required to perform a specific task or activity. Specifically, it is the relationship between cumulative production or experience and the average time or cost per unit of output. Task difficulty is often quantified using mathematical functions or formulas that consider various factors such as the number of steps involved in the task, the learning rate of the individual or organization performing the task, the amount of time or resources required, and the potential for errors or mistakes. In mathematical terms, task difficulty can be represented as a parameter or variable within the learning curve equation.

The table showcases estimated coefficients for each variable, accompanied by their respective post-mean, post-standard deviation, and z-values. The variables encompass diverse fund characteristics, including ln_NAV, ln_Initial NAV, ln_Initial Share Price, ln_Minimum Investment, Management Fee, Incentive Fee, High Water Mark, Leveraged status, Redemption Notice Period, Lock Up Period, Pay Out Period, Live or Graveyard status, and Time. These coefficients offer insights into the magnitude and direction of their effects on task difficulty. The significance of each variable's impact is denoted by the z-values. The definition of each variable are in Appendix table B1.

Table 4. The marginal effects of progress ratio

Variable	post.mean	post.s.d.	z
Intercept	1.1105	0.1986	5.5909
ln_NAV (X1)	-0.0126	0.0132	-0.9531
ln_Initial NAV (X2)	-6.0522	1.8728	-3.2316
ln_Initial Share Price (X3)	5.9739	1.8718	3.1915
ln_Minimum Investment (X4)	-0.0280	0.0150	-1.8656
Management Fee (X5)	0.3283	0.0358	9.1627
Incentive Fee (X6)	0.0266	0.0023	11.7120
High Water Mark (X7)	-0.3469	0.0440	-7.8802
Leveraged (X8)	-0.2149	0.0454	-4.7361
Redemptio Notice Period (X9)	0.0011	0.0005	2.2142
Lock Up Period (X10)	0.0086	0.0033	2.6340
Pay Out Period (X11)	-0.0075	0.0014	-5.5848
Live or Graveyard (X12)	0.0842	0.0474	1.7772
Time (X13)	0.0005	0.0002	2.2544

Adjusted R-Squared 0.0637.

Note: The table presents the outcomes of a regression analysis investigating the influence of various fund attributes on progress ratio. The progress ratio (also known as the learning curve ratio or learning curve index) is a key parameter used to quantify the rate of improvement or learning over time. It represents the ratio of the time required to perform a task or produce a unit of output at a given point on the learning curve to the time required to perform the same task or produce the same unit of output at a previous point on the learning curve. The table showcases estimated coefficients for each variable, accompanied by their respective post-mean, post-standard deviation, and z-values. The variables encompass diverse fund characteristics, including ln_NAV, ln_Initial NAV, ln_Initial Share Price, ln_Minimum Investment, Management Fee, Incentive Fee, High Water Mark, Leveraged status, Redemption Notice Period, Lock Up Period, Pay Out Period, Live or Graveyard status, and Time. These coefficients offer insights into the magnitude and direction of their effects on task difficulty. The significance of each variable's impact is denoted by the z-values. The definition of each variable are in Appendix table B1.

Table 5. Marginal effects of return equation

Variable	Post Mean	Post S.D.	z
Intercept	0.2386	0.0070	33.9490
ln_NAV (X1)	-0.0082	0.0009	-8.6855
ln_Initial NAV (X2)	0.0275	0.0054	5.0608
ln_Initial Share Price (X3)	-0.0094	0.0053	-1.7588
ln_Minimum Investment (X4)	0.0070	0.0004	17.4520
Management Fee (X5)	0.0105	0.0018	5.8692
Incentive Fee (X6)	0.0103	0.0002	65.4960
High Water Mark (X7)	-0.0145	0.0027	-5.3001
Leveraged (X8)	0.0496	0.0023	21.3100
Redemptio Notice Period (X9)	-0.0003	0.0000	-8.8836
Lock Up Period (X10)	0.0044	0.0002	23.5710
Pay Out Period (X11)	0.0013	0.0001	18.0520
Live or Graveyard (X12)	-0.0053	0.0029	-1.8633
Time (X13)	0.0004	0.0000	21.0670

Number of observations: 734,308, Adjusted R-Squared 0.0163.

Note: This table presents the estimated marginal effects of various fund characteristics on the rate of return. The variables included in the analysis reflect different attributes of the funds and their operational features. The Intercept represents the baseline effect, while the other variables, such as ln_NAV (X1), ln_Initial NAV (X2), ln_Initial Share Price (X3), and so on, represent specific fund attributes. The post mean and post S.D. columns provide the post-estimation means and standard deviations of the respective variables, reflecting their impact on the rate of return. The z values indicate the calculated z-scores, which are measures of statistical significance. The number of observations for this analysis is 734,308, and the adjusted R-squared value of 0.0163 suggests the proportion of variance in the dependent variable explained by the independent variables. This table allows readers to comprehend the relationship between fund characteristics and the rate of return without needing to refer to the main text, offering a self-contained overview of the findings. The definition of each variable is in Appendix table B1.

Table 6. Panel estimations of the impact of learning on return

	log of Rate of Return (t+1)				
	(1)	(2)	(3)	(4)	(5)
qstar_sd			-0.311***	-3.200***	-4.487***
			(0.0291)	(0.178)	(0.419)
qstar_mean		-0.0168		-0.584***	-0.00271
		(0.0247)		(0.0598)	(0.148)
c.qstar_sd*c.qstar_mean				14.67***	16.38***
				(0.942)	(2.256)
qstar_kurtosis					-0.0158
					(0.0125)
c.qstar_sd*c.qstar_kurtosis					1.286***
					(0.198)
c.qstar_mean*c.qstar_kurtosis					-0.225***
					(0.0714)
c.qstar_sd*c.qstar_mean*c.qstar_kurtosis					-3.613***
					(1.091)
Bond lookback straddle (PTFSBD)	0.00251	0.00251	0.00251	0.00241	0.00229
	(0.00273)	(0.00273)	(0.00273)	(0.00273)	(0.00272)
Currency lookback straddle (PTFSFX)	0.00319	0.00319	0.00315	0.00318	0.00296
	(0.00228)	(0.00228)	(0.00228)	(0.00228)	(0.00228)
Commodity lookback straddle (PTFSCOM)	-0.00164	-0.00163	-0.00161	-0.00177	-0.00171
	(0.00287)	(0.00287)	(0.00287)	(0.00287)	(0.00286)
Short-term interest rate lookback straddle (PTFSIR)	0.00876***	0.00875***	0.00878***	0.00882***	0.00889***
	(0.00138)	(0.00138)	(0.00138)	(0.00138)	(0.00138)
Stock index lookback straddle (PTFSSTK)	-0.00852***	-0.00852***	-0.00849***	-0.00851***	-0.00840***
	(0.00287)	(0.00287)	(0.00287)	(0.00287)	(0.00286)
ManagementFee	0.00653***	0.00653***	0.00656***	0.00670***	0.00716***
	(0.000603)	(0.000603)	(0.000603)	(0.000603)	(0.000601)
IncentiveFee	0.00286***	0.00286***	0.00286***	0.00283***	0.00276***
	(6.78e-05)	(6.78e-05)	(6.78e-05)	(6.77e-05)	(6.75e-05)
HighWaterMark	-0.0147***	-0.0147***	-0.0148***	-0.0149***	-0.0149***
	(0.00103)	(0.00103)	(0.00103)	(0.00103)	(0.00102)
Leveraged	0.0238***	0.0238***	0.0238***	0.0238***	0.0243***
	(0.000886)	(0.000886)	(0.000886)	(0.000885)	(0.000884)
RedemptionNoticePeriod	-0.000146***	-0.000146***	-0.000145***	-0.000140***	-0.000122***
	(1.57e-05)	(1.57e-05)	(1.57e-05)	(1.57e-05)	(1.56e-05)
LockUpPeriod	0.000161**	0.000161**	0.000164**	0.000156**	0.000137*
	(7.96e-05)	(7.96e-05)	(7.96e-05)	(7.94e-05)	(7.92e-05)
PayOutPeriod	0.00102***	0.00102***	0.00102***	0.00102***	0.00101***
	(3.15e-05)	(3.15e-05)	(3.15e-05)	(3.15e-05)	(3.15e-05)
Various FEs	included	included	included	included	included
Constant	0.518***	0.521***	0.536***	0.653***	0.656***
	(0.00169)	(0.00475)	(0.00240)	(0.0106)	(0.0259)
Observations	976,417	976,417	976,417	976,417	976,417
R-squared	0.240	0.240	0.240	0.241	0.243

Robust standard errors in parentheses. Fixed effects for sector focus (σ), investment approach (ι), geographic focus (γ) investment focus (κ), legal structure (ν), subscription frequency (ζ), redemption frequency (φ), and country of domicile (ψ),

Note: This table presents panel estimations exploring the impact of learning on the log of rate of return (t+1) for various fund characteristics. The columns (1) through (5) represent different model specifications. The variable qstar_sd is the measure of learning standard deviation, while qstar_mean is the mean of the learning measure. The interaction terms c.qstar_sdc.qstar_mean, c.qstar_sdc.qstar_kurtosis, c.qstar_meanc.qstar_kurtosis, and .qstar_sdc.qstar_meanc.qstar_kurtosis capture combined effects. The subsequent variables (PTFSBD, PTFSFX, PTFSCOM, PTFSIR, PTFSSTK) represent different types of straddle options. ManagementFee, IncentiveFee, HighWaterMark, Leveraged, RedemptionNoticePeriod, LockUpPeriod, and PayOutPeriod are fund-specific attributes. The table also includes fixed effects for sector focus, investment approach, geographic focus, investment focus, legal structure, subscription frequency, redemption frequency, and country of domicile. The table provides comprehensive information on the relationship between learning and return, along with other relevant variables, allowing readers to understand the results independently. The definition of each variable is in Appendix table B1.

*** p<0.01, ** p<0.05, * p<0.1

Table 7. Panel regressions for leveraged vs. non-leveraged funds

VARIABLES	(1) Leveraged = 0	(2) Leveraged = 0	(3) Leveraged = 1	(4) Leveraged =1
qstar_sd	-3.016***	-5.048***	-3.160***	-3.613***
	(0.240)	(0.574)	(0.259)	(0.599)
qstar_mean	-0.598***	-0.408**	-0.553***	0.434**
	(0.0821)	(0.203)	(0.0853)	(0.210)
qstar_sd*qstar_mean	14.02***	21.15***	14.34***	10.25***
	(1.278)	(3.089)	(1.365)	(3.227)
qstar_kurtosis		-0.0426**		0.0180
		(0.0171)		(0.0181)
qstar_sd*qstar_kurtosis		1.626***		0.804***
		(0.270)		(0.285)
qstar_mean*qstar_kurtosis		-0.00374		-0.478***
		(0.0971)		(0.103)
qstar_sd*qstar_mean*qstar_kurtosis		-6.331***		-0.199
		(1.486)		(1.571)
Constant	0.639***	0.689***	0.704***	0.655***
	(0.0145)	(0.0356)	(0.0152)	(0.0368)
Observations	512,451	512,451	463,966	463,966
R-squared	0.272	0.274	0.221	0.224

Robust standard errors in parentheses. Controls and fixed effects for sector focus (σ), investment approach (ι), geographic focus (γ) investment focus (κ), legal structure (ν), subscription frequency (ζ), redemption frequency (φ), and country of domicile (ψ)

*** p<0.01, ** p<0.05, * p<0.1

Note: This table presents panel regressions comparing the impact of learning measures on leveraged (Leveraged = 1) and non-leveraged (Leveraged = 0) funds. The four columns represent different model specifications. The variable qstar_sd represents the measure of learning (q_it^) standard deviation, while qstar_mean is the mean of the learning measure. The interaction terms qstar_sdqstar_mean, qstar_sdqstar_kurtosis, qstar_meanqstar_kurtosis, and qstar_sdqstar_meanqstar_kurtosis capture combined effects. The qstar_kurtosis variable is the kurtosis of the learning measure. The table also includes a constant term. Fixed effects for sector focus, investment approach, geographic focus, investment focus, legal structure, subscription frequency, redemption frequency, and country of domicile are controlled for. The definition of each variable are in Appendix table B1.

Table 8. Random effects-estimates for funds with and without own capital

VARIABLES	(1) yes_own_capital == 0	(2) yes_own_capital == 0	(3) yes_own_capital == 1	(4) yes_own_capital == 1
qstar_sd	-3.508***	-4.340***	-1.474***	-4.069***
	(0.213)	(0.496)	(0.322)	(0.781)
qstar_mean	-0.710***	0.0692	-0.107	-0.365
	(0.0718)	(0.175)	(0.106)	(0.276)
qstar_sd*qstar_mean	16.53***	15.81***	5.719***	15.60***
	(1.131)	(2.673)	(1.686)	(4.204)
qstar_kurtosis		0.00502		-0.0877***
		(0.0144)		(0.0254)
qstar_sd*qstar_kurtosis		1.005***		1.950***
		(0.228)		(0.398)
qstar_mean*qstar_kurtosis		-0.331***		0.224
		(0.0824)		(0.145)
qstar_sd*qstar_mean*qstar_kurtosis		-2.158*		-7.848***
		(1.260)		(2.184)
Constant	0.674***	0.639***	0.563***	0.694***
	(0.0127)	(0.0307)	(0.0191)	(0.0486)
Observations	803,589	803,589	172,828	172,828
R-squared	0.246	0.248	0.239	0.240

Robust standard errors in parentheses. Controls and fixed effects for sector focus (σ), investment approach (ι), geographic focus (γ) investment focus (κ), legal structure (ν), subscription frequency (ζ), redemption frequency (φ), and country of domicile (ψ)

*** p<0.01, ** p<0.05, * p<0.1.

Note: Table presents the random effects estimates comparing funds with and without own capital. The purpose of this table is to investigate how the ownership structure, specifically whether funds have their own capital, affects various fund characteristics. The table displays four different specifications (columns), each examining different variables and their interactions. The key variable of interest is the measure of learning (q_it^*) represented by qstar_sd (standard deviation) and qstar_mean (mean). The interactions between these learning variables and the presence of own capital are analyzed. Additionally, qstar_kurtosis and its interactions are explored. The coefficients and statistical significance of these variables provide insights into the relationship between own capital and learning, as well as their combined effects on fund characteristics. Robust standard errors are reported in parentheses, and the table controls for sector focus, investment approach, geographic focus, legal structure, subscription frequency, redemption frequency, and country of domicile. The results shed light on the role of own capital in shaping the learning curve dynamics within funds. The definition of each variable are in Appendix table B1.

Table 9: Panel regressions for various fund categories

	Convertible Arbitrage		Dedicated Short Bias		Emerging Markets		Equity Market Neutral		Event driven		Fixed Income Arbitrage		Fund of Funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
qstar_sd	-1.677	-0.609	10.13***	-13.78**	-3.292***	0.968	1.307*	2.628	-1.538**	-1.868	0.994	-0.887	-1.230***	-4.167***
	(1.165)	(2.971)	(2.785)	(5.442)	(0.582)	(1.427)	(0.773)	(1.919)	(0.602)	(1.492)	(0.763)	(1.778)	(0.292)	(0.672)
qstar_mean	-0.621*	0.395	3.018***	-0.672	-0.570***	1.640***	0.520*	0.208	-0.402**	0.511	0.532**	-0.236	-0.252**	-1.136***
	(0.375)	(1.025)	(0.884)	(2.133)	(0.201)	(0.515)	(0.267)	(0.709)	(0.200)	(0.533)	(0.251)	(0.629)	(0.0995)	(0.237)
qstar_sd*qstar_mean	9.577	2.058	-48.08***	72.77**	15.40***	-15.25**	-5.817	-9.809	6.039*	-0.0272	-7.180*	2.547	6.409***	23.00***
	(6.086)	(15.80)	(14.66)	(30.27)	(3.009)	(7.507)	(4.119)	(10.24)	(3.190)	(8.085)	(3.939)	(9.489)	(1.579)	(3.647)
qstar_kurtosis		0.0886		-0.457**		0.161***		-0.00927		0.0249		-0.0674		-0.0971***
		(0.0801)		(0.181)		(0.0480)		(0.0601)		(0.0457)		(0.0512)		(0.0185)
qstar_sd*qstar_kurtosis		-0.707		12.25***		-2.146***		-0.653		0.308		0.855		1.785***
		(1.237)		(2.805)		(0.730)		(0.911)		(0.717)		(0.816)		(0.293)
qstar_mean*qstar_kurtosis		-0.551		2.431**		-1.362***		0.167		-0.499*		0.399		0.525***
		(0.450)		(1.102)		(0.274)		(0.342)		(0.263)		(0.289)		(0.105)
qstar_sd*qstar_mean*qstar_kurtosis		4.679		-64.30***		16.85***		1.923		2.694		-4.653		-9.878***
		(6.712)		(16.38)		(3.976)		(4.943)		(3.967)		(4.452)		(1.619)
Constant	0.725***	0.563***	0.145	0.795*	1.098***	0.858***	0.372***	0.389***	0.539***	0.499***	0.284***	0.415***	0.483***	0.646***
	(0.0844)	(0.187)	(0.302)	(0.422)	(0.0368)	(0.0907)	(0.0479)	(0.124)	(0.0383)	(0.0933)	(0.0474)	(0.112)	(0.0175)	(0.0414)
Observations	12,924	12,924	2,658	2,658	59,670	59,670	31,287	31,287	50,127	50,127	23,640	23,640	387,654	387,654
R-squared	0.540	0.540	0.423	0.432	0.330	0.333	0.222	0.222	0.266	0.267	0.379	0.380	0.195	0.195

	Global Macro		Long/Short Equity Hedge		Managed Futures		Multi-Strategy		Options Strategy	
	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
qstar_sd	-1.677*	1.161	-2.762***	-2.884***	-0.990	1.864	-3.332***	-2.432*	-0.616	-7.497
	(0.894)	(2.215)	(0.341)	(0.783)	(5.388)	(10.76)	(0.552)	(1.370)	(0)	(0)
qstar_mean	-0.215	2.321***	-0.347***	0.835***	-1.426	3.176	-0.863***	1.035**	-1.658	-1.698
	(0.298)	(0.785)	(0.111)	(0.276)	(1.629)	(4.064)	(0.186)	(0.487)	(0)	(0)
qstar_sd*qstar_mean	7.212	-15.27	11.28***	4.313	1.786	-36.26	17.54***	3.227	6.297	35.60
	(4.702)	(12.07)	(1.778)	(4.185)	(28.70)	(61.55)	(2.903)	(7.436)	(0)	(0)
qstar_kurtosis		0.192***		0.0244		0.352		0.114***		-0.0804
		(0.0681)		(0.0255)		(0.386)		(0.0425)		(0)
qstar_sd*c.qstar_kurtosis		-1.439		0.708*		-2.638		-0.415		3.694
		(1.075)		(0.397)		(6.528)		(0.682)		(0)
qstar_mean*qstar_kurtosis		-1.456***		-0.619***		-2.811		-1.053***		0.180
		(0.393)		(0.146)		(2.310)		(0.245)		(0)
qstar_sd*qstar_mean*qstar_kurtosis		12.24**		1.281		26.98		7.587**		-16.39
		(5.975)		(2.180)		(37.87)		(3.799)		(0)
Constant	0.572***	0.255*	0.743***	0.688***	1.643***	1.104*	0.617***	0.428***	7.381	7.161
	(0.0549)	(0.136)	(0.0207)	(0.0487)	(0.300)	(0.666)	(0.0332)	(0.0846)	(0)	(0)
Observations	32,675	32,675	213,393	213,393	1,247	1,247	99,298	99,298	3,845	3,845
R-squared	0.234	0.237	0.166	0.170	0.148	0.152	0.257	0.259	0.379	0.381

Robust standard errors in parentheses. Controls and fixed effects for sector focus (σ), investment approach (ι), geographic focus (γ) investment focus (κ), legal structure (ν), subscription frequency (ζ), redemption frequency (φ), and country of domicile (ψ)

*** p<0.01, ** p<0.05, * p<0.1.

Note: Table presents panel regressions analyzing different fund categories, each represented by a separate specification (columns). The purpose of this table is to examine how the measure of learning and its interactions with other variables vary across different fund categories. The fund categories analyzed include Convertible Arbitrage, Dedicated Short Bias, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Fund of Funds, Global Macro, Long/Short Equity Hedge, Managed Futures, Multi-Strategy, and Options Strategy. The table displays coefficients and statistical significance for various variables, including qstar_sd (standard deviation), qstar_mean (mean), and their interactions. Additionally, qstar_kurtosis and its interactions are explored. The results provide insights into how learning dynamics, measured by q_it^*, are influenced by different fund categories and their characteristics. Robust standard errors are reported in parentheses, and the table controls for sector focus, investment approach, geographic focus, legal structure, subscription frequency, redemption frequency, and country of domicile. The coefficients' significance helps in understanding the relationship between fund categories, learning dynamics, and other relevant variables. The results contribute to a deeper understanding of how learning curves vary across different types of funds and shed light on the unique dynamics within each fund category. The definition of each variable are in Appendix table B1.

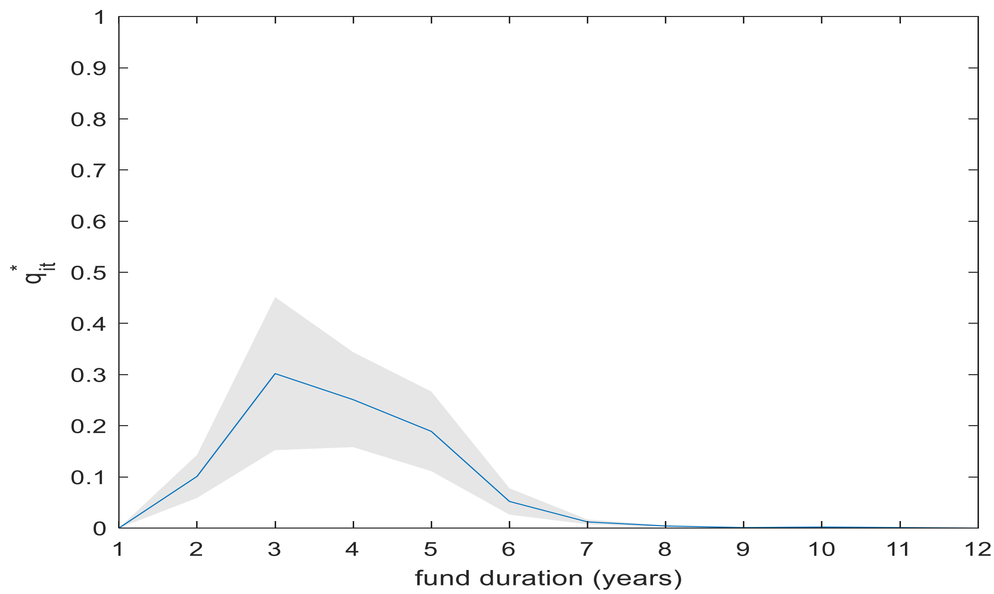
Table 10. Panel estimations for funds male and female owner

VARIABLES	(1) male	(2) male	(3) female	(4) female
qstar_sd	-2.658***	-4.073***	-4.048***	-3.782***
	(0.221)	(0.520)	(0.350)	(0.824)
qstar_mean	-0.425***	0.0960	-0.893***	0.539*
	(0.0739)	(0.185)	(0.117)	(0.288)
qstar_sd*qstar_mean	12.05***	14.56***	19.02***	10.60**
	(1.162)	(2.789)	(1.889)	(4.499)
qstar_kurtosis		-0.0196		0.0594**
		(0.0160)		(0.0232)
qstar_sd*qstar_kurtosis		1.328***		0.360
		(0.250)		(0.371)
qstar_mean*qstar_kurtosis		-0.209**		-0.692***
		(0.0913)		(0.133)
qstar_sd*qstar_mean*qstar_kurtosis		-3.862***		2.241
		(1.377)		(2.060)
Constant	0.609***	0.621***	0.675***	0.542***
	(0.0132)	(0.0324)	(0.0208)	(0.0501)
Observations	521,546	521,546	303,527	303,527
R-squared	0.244	0.246	0.301	0.304

Robust standard errors in parentheses. Controls and fixed effects for sector focus (σ), investment approach (ι), geographic focus (γ) investment focus (κ), legal structure (ν), subscription frequency (ζ), redemption frequency (φ), and country of domicile (ψ)
*** p<0.01, ** p<0.05, * p<0.1
Note: Table presents panel estimations that explore the impact of fund ownership gender (male and female) on the measure of learning and its interactions with other variables. The purpose of this table is to investigate whether the learning dynamics in funds differ based on the gender of the fund owners. The table contains four specifications (columns), with each specification representing a different ownership gender: male (columns 1 and 2) and female (columns 3 and 4). The coefficients and statistical significance of various variables, including qstar_sd (standard deviation) and qstar_mean (mean), are displayed. Additionally, interactions between qstar_sd and qstar_mean are examined, along with qstar_kurtosis and its interactions. The table controls for sector focus, investment approach, geographic focus, legal structure, subscription frequency, redemption frequency, and country of domicile. The coefficients' significance provides insights into how the

gender of fund owners may influence the learning dynamics and performance of funds. The results contribute to the understanding of potential gender-related differences in learning curves and their implications for fund performance. It sheds light on whether the gender of fund owners has an impact on the learning dynamics and subsequent outcomes of their respective funds. The definition of each variable are in Appendix table B1.

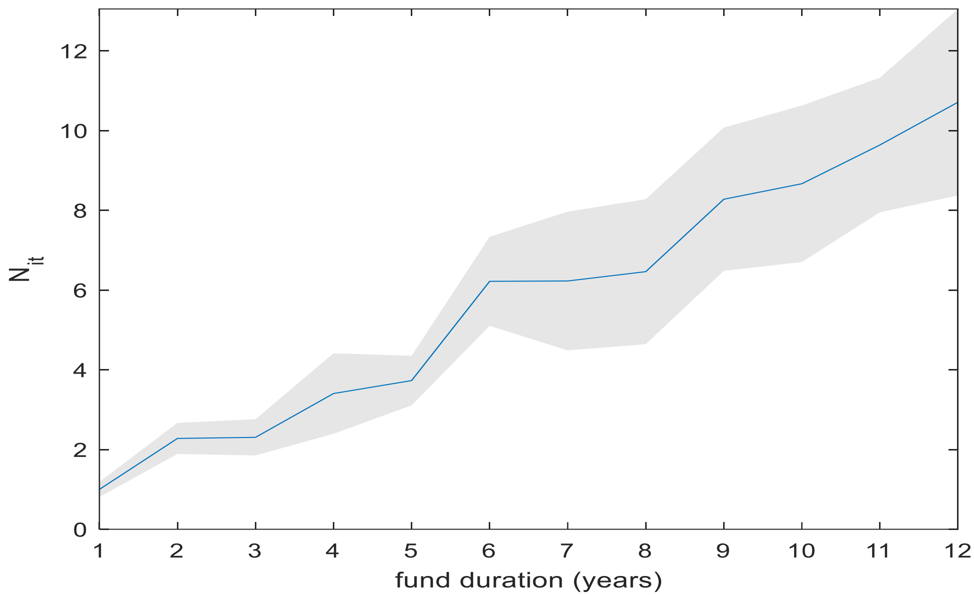
Figure 1. Normalized learning curves (q_{it}^{*}) (along with 95% HPDI bands)



Note: Figure illustrates the normalized learning curves along with 95% Highest Posterior Density Interval (HPDI) bands. The purpose of this figure is to visually represent the learning dynamics of the studied funds while accounting for uncertainty in the estimates. The x-axis of the figure represents time, and the y-axis represents the values of the normalized learning measure.

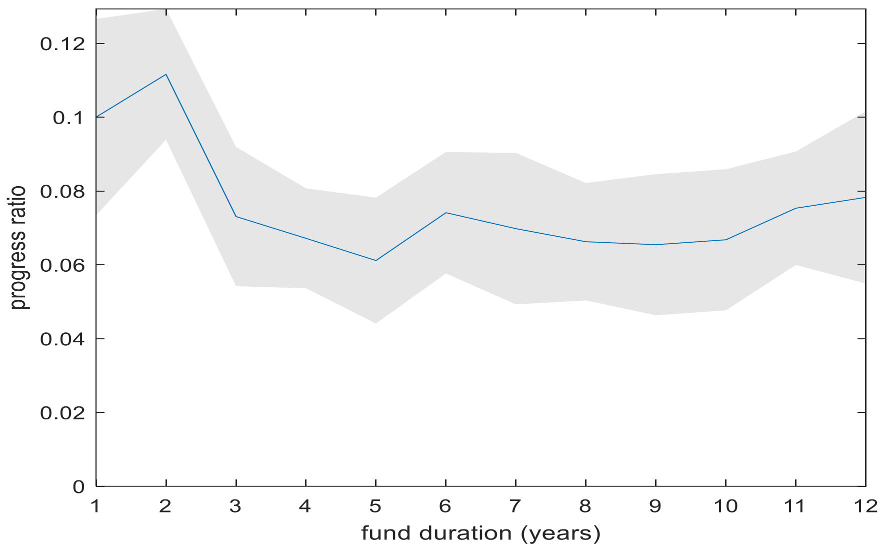
The main focus of the figure is to showcase the trend in learning, while the shaded bands around each curve indicate the uncertainty or variability in the estimates. The 95% HPDI bands capture the range within which the true curve is likely to fall with 95% confidence, accounting for the inherent uncertainty in the data and the estimation process.

Figure 2. Posterior mean estimates of "task difficulty" N_{it} (along with 95% HPDI bands)



Note: Figure illustrates the task difficulty along with 95% Highest Posterior Density Interval (HPDI) bands. The purpose of this figure is to visually represent the learning dynamics of the studied funds while accounting for uncertainty in the estimates. The x-axis of the figure represents time, and the y-axis represents the values of the task difficulty. The main focus of the figure is to showcase the trend in task difficulty, while the shaded bands around each curve indicate the uncertainty or variability in the estimates. The 95% HPDI bands capture the range within which the true curve is likely to fall with 95% confidence, accounting for the inherent uncertainty in the data and the estimation process.

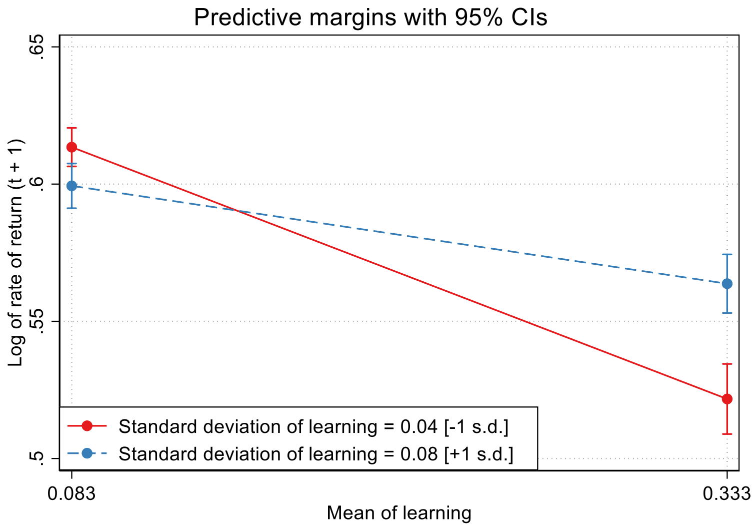
Figure 3. Posterior mean estimates of progress ratio ($\frac{\sigma_w}{\sigma_\theta}$) (along with 95% HPDI bands)



Note: Figure illustrates the progress ratio along with 95% Highest Posterior Density Interval (HPDI) bands. The purpose of this figure is to visually represent the learning dynamics of the studied funds while accounting for uncertainty in the estimates. The x-axis of the figure represents time, and the y-axis represents the values of the progress ratio. The main focus of the figure is to showcase the trend in task difficulty, while the shaded bands around each curve indicate the uncertainty or variability in the estimates. The 95% HPDI bands capture the range within which the true curve is likely to fall with 95% confidence, accounting for the inherent uncertainty in the data and the estimation process.

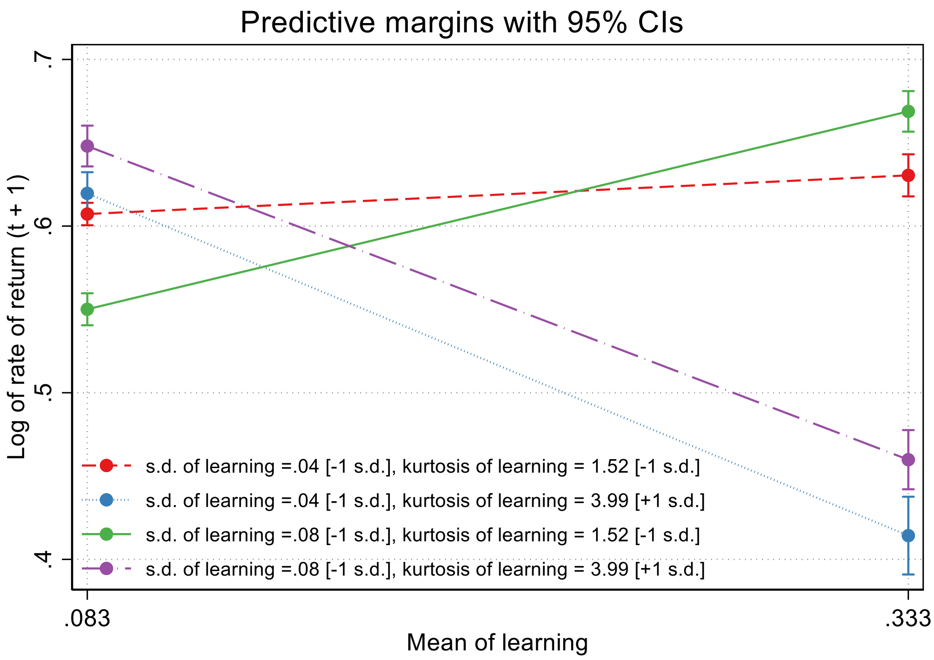
Figure 4. Margins plots

Figure 4(a): Margins effects of standard deviation of learning



Note: Figure presents the marginal effects of the mean of learning. The x-axis of the plot represents the minimum and maximum values of mean of learning, the y-axis represents the corresponding impact or effect on rate of return. Solid (dashed) line represents standard deviation of learning below (above) the mean. Each point or line on the plot represents a specific fund variable, and its position along the y-axis illustrates how changes in the standard deviation of learning influence the outcome.

Figure 4(b): Three-way margins effect of the standard deviation of learning and kurtosis of learning



Note: Figure presents the three-way marginal effects of the mean of learning. The x-axis of the plot represents the minimum and maximum values of mean of learning, the y-axis represents the corresponding impact or effect on rate of return. Solid line represents standard deviation of learning above and kurtosis of learning below the mean. Dashed line represents both standard deviation of learning and kurtosis of learning below the mean.

Dash-dot line represents standard deviation of learning above and kurtosis of learning above the mean. Dotted line represents standard deviation of learning below and kurtosis of learning above the mean.

Appendix A

A.1. Fast Markov Chain Monte Carlo

The posterior distribution of the model, augmented with the unobserved parameters $\{\Theta_i\}_{i\in\mathbb{I}}$ is as follows.

$$p(\{\Theta_i\}_{i\in\mathbb{I}}, \bar{\Theta}, \Omega, \sigma_u, \boldsymbol{\gamma}; \boldsymbol{y}) \propto \sigma_u^{-(n+1)} \exp\left\{-\frac{1}{2\sigma_u^2}\sum_{i\in\mathbb{I}}\left[\ln\, q_i^* - N_i \ln\left[1 - \frac{\sigma_{\theta,i}^2}{(1+T_i\pi_i)(1-\sigma_{w,i}^2)}\right] - \boldsymbol{z}'_i\boldsymbol{\gamma}\right]^2\right\} \quad \text{(A.1)}$$
$$\left\{\prod_{i\in\mathbb{I}} p\left(\Theta_i|\bar{\Theta}, \Omega\right)\right\} \cdot |\Omega|^{-5/2}.$$

The posterior can be integrated analytically with respect to σ_u to obtain

$$p(\{\Theta_i\}_{i\in\mathbb{I}}, \bar{\Theta}, \Omega, \boldsymbol{\gamma}; \boldsymbol{y}) = \int_0^\infty L\left(\{\Theta_i\}_{i\in\mathbb{I}}, \bar{\Theta}, \Omega, \sigma_u, \boldsymbol{\gamma}; \boldsymbol{y}\right) \mathrm{d}\sigma_u \propto$$
$$\prod_{i\in\mathbb{I}} p\left(\Theta_i|\bar{\Theta}, \Omega\right) \cdot |\Omega|^{-5/2} \cdot \left\{\sum_{i\in\mathbb{I}}\left[\ln\, q_i^* - N_i \ln\left[1 - \frac{\sigma_{\theta,i}^2}{(1+T_i\pi_i)(1-\sigma_{w,i}^2)}\right] - \boldsymbol{z}'_i\boldsymbol{ma}\right]^2\right\}^{-n/2}, \quad \text{(A.2)}$$

where $p(\Theta_i|\bar{\Theta}, \Omega) \propto |\Omega|^{-1/2} \exp\left\{-\frac{1}{2}(\Theta_i - \bar{\Theta})'\Omega^{-1}(\Theta_i - \bar{\Theta})\right\}$. Therefore, we can write (A.2) as follows.

$$p(\{\Theta_i\}_{i\in\mathbb{I}}, \bar{\Theta}, \Omega, \boldsymbol{\gamma}; \boldsymbol{y}) = \int_0^\infty L\left(\{\Theta_i\}_{i\in\mathbb{I}}, \sigma_u, \boldsymbol{\gamma}; \boldsymbol{y}\right) \mathrm{d}\sigma_u \propto$$
$$|\Omega|^{-5n/2} \cdot \exp\left\{-\frac{1}{2}\left[tr\Omega^{-1}\sum\nolimits_{i\in\mathbb{I}}(\Theta_i - \bar{\Theta})\,(\Theta_i - \bar{\Theta})'\right]\right\}\left\{\sum_{i\in\mathbb{I}}\left[\ln\, q_i^* - N_i \ln\left[1 - \frac{\sigma_{\theta,i}^2}{(1+T_i\pi_i)(1-\sigma_{w,i}^2)}\right] - \boldsymbol{z}'_i\boldsymbol{\gamma}\right]^2\right\}^{-n/2}, \quad \text{(A.3)}$$

where $tr(\cdot)$ denotes the trace operator (sum of diagonal elements of a square matrix). Integration with respect to the different elements of matrix Ω (Zellner, 1971, p. 229) yields:

$$p(\{\Theta_i\}_{i\in\mathbb{I}}, \bar{\Theta}, \boldsymbol{\gamma}; \boldsymbol{y}) = \int_0^\infty L\left(\{\Theta_i\}_{i\in\mathbb{I}}, \bar{\Theta}, \Omega, \sigma_u, \boldsymbol{\gamma}; \boldsymbol{y}\right) \mathrm{d}\Omega \propto$$
$$\left\{\sum_{i\in\mathbb{I}}(\Theta_i - \bar{\Theta})\,(\Theta_i - \bar{\Theta})'\right\}^{-n/2} \cdot \left\{\sum_{i\in\mathbb{I}}\left[\ln\, q_i^* - N_i \ln\left[1 - \frac{\sigma_{\theta,i}^2}{(1+T_i\pi_i)(1-\sigma_{w,i}^2)}\right] - \boldsymbol{z}'_i\boldsymbol{\gamma}\right]^2\right\}^{-n/2}, \quad \text{(A.4)}$$

We use a variation of the Metropolis Adjusted Langevin Algorithm (MALA) called fast MALA (fMALA), see Durmus et al. (2017). Suppose the parameter vector is $\boldsymbol{\theta} \in \Re^d$, and we target $\pi(\boldsymbol{\theta})$ which represents the posterior, omitting the dependence on data to ease notation. We consider the Langevin diffusion:

$$\mathrm{d}\boldsymbol{\theta}_t = \frac{1}{2}\Sigma \cdot \nabla \ln\, \pi\left(\boldsymbol{\theta}_t\right) + \Sigma^{1/2}\mathrm{d}\boldsymbol{W}_t, \quad \text{(A.5)}$$

where $\{\boldsymbol{W}_t, t \geq 0\}$ is a standard d-dimensional Brownian motion, and Σ is a given positive definite self-adjoint matrix. Under appropriate assumptions on π one can show that the dynamics generated by (A.5) are ergodic and result in $\pi(\boldsymbol{\theta})$ as unique invariant distribution. A standard approach is to discretize (A.5) using a one-step integrator, and sample using the averages over the numerical trajectories. This approach introduces a bias because the posterior does not coincide in general with the exact π.

An alternative way of sampling from π exactly, i.e. that is not biased by discretizing (A.5), is by using the Metropolis-Hastings algorithm (Hastings, 1970). The idea is to construct a Markov chain $\{\boldsymbol{\theta}_j\}$, where at each step j, given $\boldsymbol{\theta}_j$, a new sample proposal $\boldsymbol{\theta}^c$ is generated from the Markov chain with transition kernel $q(\boldsymbol{\theta},\cdot)$. This proposal is then accepted ($\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}^c$) with probability $\alpha(\boldsymbol{\theta}_j, \boldsymbol{\theta}^c)$ and rejected ($\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j$) otherwise. If we have

$$\alpha(\boldsymbol{\theta}, \boldsymbol{\theta}^c) = \min\left\{1, \frac{\pi(\boldsymbol{\theta}^c)q(\boldsymbol{\theta}^c, \boldsymbol{\theta})}{\pi(\boldsymbol{\theta})q(\boldsymbol{\theta}, \boldsymbol{\theta}^c)}\right\}, \tag{A.6}$$

then the resulting Markov chain $\{\theta_j\}$ is π-invariant and will, for large j generate samples from π under mild ergodicity assumptions (Liu, 2008, Robert and Casella, 2004). Specifically, a candidate is generated as:

$$\boldsymbol{\theta}^c = \boldsymbol{\mu}(\boldsymbol{\theta}, h) + \boldsymbol{S}(\boldsymbol{\theta}, h)\boldsymbol{\zeta}, \tag{A.7}$$

where $\boldsymbol{\zeta} \sim \mathcal{N}_d(\mathbf{0}, \boldsymbol{I}_d)$. The fMALA proposal has

$$\boldsymbol{\mu}(\boldsymbol{\theta}, h) = x + \frac{h}{2}f(\boldsymbol{\theta}) - \frac{h^2}{24}\nabla f(\boldsymbol{\theta}) \cdot f(\boldsymbol{\theta}) + \{\Sigma : \nabla^2 f(\boldsymbol{\theta})\}, \tag{A.8}$$

$$\boldsymbol{S}(\boldsymbol{\theta}, h) = \left(h^{1/2}\boldsymbol{I}_d + \frac{h^{3/2}}{12}\nabla f(\boldsymbol{\theta})\right)\Sigma^{1/2}, \tag{A.9}$$

where $f(\boldsymbol{\theta}) =^{\triangle} \Sigma \cdot \nabla \ln\, \pi\,(\boldsymbol{\theta})$, $\nabla f(\boldsymbol{\theta})$ and $\nabla^2 f(\boldsymbol{\theta})$ are the $d \times d$ Jacobian and $d \times d^2$ Hessian of $f(\boldsymbol{\theta})$, respectively, and $\Sigma = \boldsymbol{S}(\boldsymbol{\theta}, h)$. Let $\nabla^2 f(\boldsymbol{\theta}) = [\boldsymbol{H}_1(\boldsymbol{\theta}), \ldots, \boldsymbol{H}_d(\boldsymbol{\theta})]$ where $[\boldsymbol{H}_i(\boldsymbol{\theta})]_{jk} = \frac{\partial f_i(\theta)}{\partial\theta_k \partial\theta_j}$.

In turn, $\{\Sigma : \nabla^2 f(\boldsymbol{\theta})\}_i =^{\triangle} tr[\Sigma' \boldsymbol{H}_i(\boldsymbol{\theta})]$. The selection of the scaling constant has been discussed in Durmus et al. (2017) and it is related to the discretization of (A.5). Specifically, Durmus et al. (2017) recommend $h = \varepsilon d^{-1/5}$ for some $\varepsilon > 0$. The acceptance rate that maximizes first-order efficiency is very close to 0.704 (see Theorem 3.2 of Durmus et al., 2017). Therefore, we calibrate the constant ε (during the burn-in phase) so that the acceptance rate is close to 0.70.

This approach has been found to perform excellently once ε is and h are calibrated correctly during the burn-in phase. All derivatives are computed numerically[7] during the burn-in phase, and they are interpolated[8] in the main phase of the MCMC algorithm. This results in dramatic computational savings and, as a matter of fact, different chains can be run in parallel in computers with multiple nodes. We run ten different chains starting from randomly selected initial conditions and we compare the chains after 150,000 iterations with a burn-in phase consisting of 50,000 iterations. Our transition density $q(\theta, \theta^c)$ is a d-dimensional Student-t distribution with five degrees of freedom, and we monitor convergence using the standard diagnostics of Geweke (1992).

A.1. Prior sensitivity analysis

Here, we examine the sensitivity of our posterior results to changes in the prior assumptions in (16)

$$p(\bar{\Theta}, \Omega) = p(\bar{\Theta}|\Omega) \cdot p(\Omega) \propto |\Omega|^{-5/2}. \tag{A.10}$$

In the interest of brevity we report results only for the full sample for entrepreneurs with at least three businesses. This sample has the largest number of unique entrepreneurs (175,578) and observations (855,069).

Instead of assuming that $p(\bar{\Theta}|\Omega)$ is flat and $p(\Omega) \propto |\Omega|^{-5/2}$ we assume

$$\bar{\Theta}|\Omega \sim \mathcal{N}(\Theta_o, \Sigma_o), \tag{A.11}$$

and

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[7]We use the Fortran77 subroutines in package NDL of Voglis et al. (2009). Specifically we use version 2.0 of Hadjidoukas et al. (2014), https://data.mendeley.com/datasets/j2fhmszg85/1, see also http://cpc.cs.qub.ac.uk/summaries/AEDG_v1_0.html

[8]We use the Fortran subrourines in finterp by Jacob Williams inhttps://github.com/jacobwilliams/finterp/blob/master/README.md Alternatively, we use for comparison RBF_INTERP_ND in https://people.sc.fsu.edu/~jburkardt/f_src/rbf_interp_nd/rbf_interp_nd.html. RBF_INTERP_ND is a Fortan90 library by John Burkardt which defines and evaluates radial basis function (RBF) interpolants to multidimensional data.

$$p(\Omega) \propto |\Omega|^{-(d+\nu_o+1)} \exp\left\{-\frac{1}{2} tr\boldsymbol{A}_o\Omega^{-1}\right\}. \tag{A.12}$$

So, the prior for $\bar{\Theta}|\Omega$ is normal with a mean vector Θ_o and covariance matrix Σ_o. The prior for Ω is inverted Wishart with degrees of freedom parameter $\nu_o \geq 0$ and scale matrix $\boldsymbol{A}_o$ (Zellner, 1971, p. 395). The prior in (16) may be recovered when $\Theta_o = \boldsymbol{0}$, Σ_o is diagonal and its elements along the main diagonal diverge ton infinity, $\nu_o = 0$ and $\boldsymbol{A}_o = \boldsymbol{O}$, a zero matrix. We intend to assign different values to the hyper-parameters of the prior, repeat posterior analysis, and examine whether the results reported in the main section become different. We intend to consider 1,000 different prior configurations so we have to streamline a proper prior that implies, however, considerable differences among the configurations.

We assume $\Sigma_o = \bar{\sigma}_o^2\boldsymbol{I}$, i.e. a diagonal matrix and parameter $\bar{\sigma}_o$ is chosen so that it is a draw from a lognormal distribution:

$$\ln\, \bar{\sigma}_o \sim \mathcal{N}(a, b^2), \tag{A.13}$$

where we specify that a is uniformly distributed in the interval (–3, 3) and $b = 10$. For ν_o we assume that it is drawn from a uniform distribution in the interval (0, 10), and the elements of $\boldsymbol{A}_o$ are chosen as follows. Since $\boldsymbol{A}_o$ must be positive semi-definite we set $\boldsymbol{A}_o = \bar{n}^{-1}\sum_{i=1}^{\bar{n}}(\boldsymbol{x}_i - \bar{x})(\boldsymbol{x}_i - \bar{x})'$ where $\bar{n}$ is drawn from a uniform distribution in the interval $(1, T/10)$, $\{\boldsymbol{x}_i\}_{i=1}^{\bar{n}}|\bar{n} \sim \mathcal{N}(\boldsymbol{0}, \bar{\sigma}_A^2)$, i.e. a random sample of size $\bar{n}$ from a normal distribution with zero mean vector, $\bar{\sigma}_A = 10$, the zero mean is without loss of generality and we set the upper bound of $\bar{n}$ to $T/10$ so that the prior is still relatively uninformative relative to the likelihood (T is the actual number of observations in the sample).

To produce new MCMC samples corresponding to each of the 1,000 different priors we use the method of Sampling-Importance-Resampling (Rubin, 1987, 1988, see also Smith and Gelfand, 1992). MCMC convergence diagnosis is performed using the statistics in Geweke (1992).

APPENDIX B

Table B1. Variable definitions and data source

		Variable definition	Source
1	Log of rate of return (t+1)	Log of monthly hedge-fund return	TASS
2	Learning (standard deviation) (t)	Standard deviation of the learning curve values based on rolling five-month windows from (t-4) to (t)	Calculations from the first-stage based on TASS data
3	Learning (mean)	Mean of the learning curve values based on rolling five-month windows from (t-4) to (t)	Calculations from the first-stage based on TASS data
4	Learning (kurtosis)	Kurtosis of the learning curve values based on rolling five-month windows from (t-4) to (t)	
5	Management fee	Hedgefund management fee in percentage	TASS
6	Incentive fee	Hedgefund incentive fee in percentage	TASS
7	High watermark	Whether the fund has a high watermark provision, that is, pays performance fees are paid when the value of current performance is higher than its previous highest value (1-yes; 0-no).	TASS
8	Leveraged	Hedge fund is leveraged (1-yes; 0-no)	TASS
9	Redemption period	Redemption period in days	TASS
10	Lock up period	Lock up period in months	TASS
11	Pay out period	Payout period in days	TASS
12	Tracking frequency	Tracking frequency (1-monthly or 2-quarterly)	TASS
13	Bond lookback straddle (PTFSBD)	Bond lookback straddle; Fung and Hsieh (2001) hedge fun risk factor	Fung and Hsieh (2001)
14	Currency lookback straddle (PTFSFX)	Currency lookback straddle; Fung and Hsieh (2001) hedge fun risk factor	Fung and Hsieh (2001)
15	Commodity lookback straddle (PTFSCOM)	Commodity lookback straddle; Fung and Hsieh (2001) hedge fun risk factor	Fung and Hsieh (2001)
16	Short-term interest rate lookback straddle	Short-term interest rate lookback straddle; Fung and Hsieh (2001) hedge fun risk factor	Fung and Hsieh (2001)
17	Sector dummies	Sector dummies[9]	TASS
18	Investment approach	Investment approach[10]	TASS
19	Geographic focus	Geographic focus of the fund[11]	TASS
20	Investment focus	Investment focus of the fund[12]	TASS
21	Legal structure	Legal structure dummies[13]	TASS
22	Subscription frequency	Subscription frequency of the fund: Annual; Continuous; Daily; Fortnightly; Monthly; Not Defined; Quarterly; Semi-Annually; Semi-Monthly; Variable; Weekly	TASS
23	Redemption frequency	Redemption frequency of the fund: Annual; Biennial; Continuous; Daily; Fortnightly; Monthly; Not Defined Quarterly; Semi-Annually; Semi-Monthly; Triennial; Variable; Weekly	TASS
24	Country of domicile	Country of domicile dummies[14]	TASS

[9] Bio Technology; Close Ended Funds; Corporate Bonds; Diversified; Emerging Market Bonds; Emerging Market Equities; Financial; Gold; Government Bonds; Growth Stocks; Health Care; Large Cap; Media Communications; Medium Cap; Micro Cap; Money Markets; Natural Resources; New Issues; Oil Energy; Other; Private Equity; Pure Currency; Pure Emerging Market; Pure Managed Futures; Real Estate Property; Shipping; Small Cap; Sovereign Debt; Technology; Turnarounds Spin Offs; Utilities; Value Stocks

[10] Arbitrage; Bottom Up; Contrarian; Directional; Discretionary; Diversified; Fundamental; Long Bias; Market Neutral; Non Directional; Opportunistic; Other; Relative Value; Short Bias; Systematic Quant; Technical; Top Down Macro; Trend Follower

[11] Africa; Asia Pacific; Asia Pacific Excluding Japan; Eastern Europe; Global; India; Japan; Latin America; North America; North America Excluding USA; Other; Russia; UK; USA; Western Europe; Western Europe Excluding

[12] Bankruptcy; Capital Structure; Distressed Bonds; Distressed Markets; Equity Derivative; High Yield Bonds; Merger Arbitrage Risk; Mortgage Backed; Multi Strategy; Pairs Trading; Reg D; Shareholder Activist; Socially Responsible; Special Situations; Statistical Arbitrage

[13] AD - Fons d'Inversi; AE - Open-ended Fund; AT - Alternative Investments; AT - Publikumfonds; BR - Condominio Aberto; BR - Condominio fechado; CA - Corporation; CA - Ltd Partnership; CA - Trust; CH - Uebrige Fds altern. Anl.; Closed Ended Investment Co; Commodity Pool Operator; DE-DachSonderverm.zus.Risiken; DE-Sonderverm.zus.Risiken; DK - Hedgeforening; ES & PT - FI; ES & PT - FIL; ES - IICIICIL; Exempted Company; Exempted Limited Liability Co; Exempted Limited Partnership; Exempted Unit Trust; FI - Erikoissijoitusrahasto; FL - InvestUntern andere Werte; FL - InvestUntern qual Anleger; FR - FCIMT; FR - FCP; ICVC; IT - Fondi Speculativi; Individual Managed Accounts; International Business Company; KY - Collective Invt Scheme; LU - FCP - Part 2; LU - SICAF/other - Part 2; LU - SICAV - Part 2; LU - SIF - FCP; LU - SIF - SICAF/other; LU - SIF - SICAV; Limited Corporation; Limited Liability Company; Limited Partnership; NL-Beleggingsfonds(Semi-open); NL-Beleggingsfonds(open); Not Defined; OEIC; OEIC - B scheme; OEIC - unclassified; Open Ended Investment Scheme; Open Ended Investment Trust; Open-ended fund; Open-ended investment fund; Protected Cell Company; SE - Specialfond; SICAV; Segregated Portfolio Company; UK - Investment Trust; Unit trust; Unit trust - B scheme

[14] Andorra; Anguilla; Argentina; Australia; Austria; Bahamas; Bahrain; Barbados; Belgium; Bermuda; Botswana; Brazil; Canada; Cayman Islands; China; Colombia; Curacao; Cyprus; Czech Republic; Denmark; Estonia; Finland; France; Germany; Gibraltar; Guernsey; Hong Kong; India; Ireland; Isle of Man; Israel; Italy; Japan; Jersey; Liechtenstein; Lithuania; Luxembourg; Malaysia; Malta; Mauritius; Netherlands; New Zealand; None; Poland; Portugal; Saint Kitts And Nevis; Saint Martin; Saint Vincent And The Grenadin; Samoa; Singapore; South Africa; Spain; Sweden; Switzerland; United Arab Emirates; United Kingdom; United States; Vanuatu; Virgin Islands (British)

Table B2. Marginal effects of $\sigma_{\{w,it\}}$

Variable	post.mean	post.s.d.	z
Intercept	1.106	0.12155	9.099
ln_NAV (X1)	-0.013121	0.010123	-1.2962
ln_Initial NAV (X2)	0.024175	0.040069	0.60333
ln_Initial Share Price (X3)	-0.082607	0.038938	-2.1215
ln_Minimum Investment (X4)	-0.035103	0.0086282	-4.0684
Management Fee (X5)	0.11609	0.028175	4.1204
Incentive Fee (X6)	0.011328	0.0017505	6.4714
High Water Mark (X7)	-0.025695	0.02699	-0.95201
Leveraged (X8)	0.092298	0.029734	3.1042
Redemptio Notice Period (X9)	-0.0009949	0.000378	-2.632
Lock Up Period (X10)	0.013499	0.0013588	9.9347
Pay Out Period (X11)	-0.0029875	0.00070105	-4.2614
Live or Graveyard (X12)	0.188	0.030248	6.2151
Time (X13)	0.00021363	0.00015557	1.3732

Adjusted R-Squared 0.0268

Table B3. Marginal effects of $\boldsymbol{\sigma_{\{\theta,it\}}}$

Variable	post.mean	post.s.d.	z
Intercept	-0.10652	0.25039	-0.42543
ln_NAV (X1)	-0.02189	0.017674	-1.2385
ln_Initial NAV (X2)	0.06667	0.025817	2.5824
ln_Initial Share Price (X3)	0.01357	0.18745	0.07239
ln_Minimum Investment (X4)	0.020136	0.02365	0.85139
Management Fee (X5)	-0.0064869	0.042875	-0.1513
Incentive Fee (X6)	-0.0007618	0.0039832	-0.19126
High Water Mark (X7)	0.062676	0.072708	0.86203
Leveraged (X8)	0.094588	0.043703	2.1643
Redemption Notice Period (X9)	-5.44E-05	0.0010197	-0.053364
Lock Up Period (X10)	-0.0028098	0.0060096	-0.46756
Pay Out Period (X11)	-0.0040144	0.00083483	-4.8087
Live or Graveyard (X12)	0.15396	0.049558	3.1066
Time (X13)	4.57E-05	0.00020911	0.21839

Adjusted R-Squared 0.0149

Figure B1. Posterior mean estimates of σ_w (along with 95% HPDI bands)

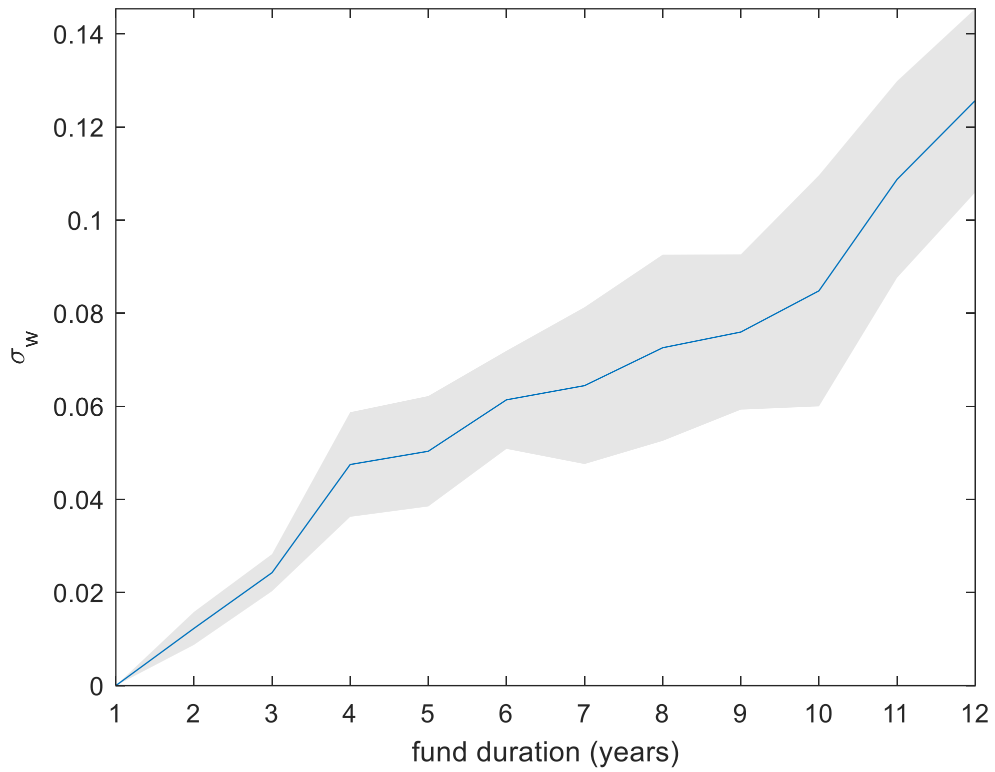

Figure B2. Posterior mean estimates of σ_θ (along with 95% HPDI bands)