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Are There Bubbles in the Art Market? The Detection of Bubbles when Fair Value is Unobservable

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Abstract:

The purpose of this paper is to look for bubbles in the Art Market using a structure based on steady state results for TAR models and appropriate definitions of bubbles recently put forward by Knight, Satchell and Srivastava (2011). The usual method for investigating bubbles is to measure prices as deviations from fair value. We assess whether it is meaningful to define a fair value of art and conclude that it is very challenging empirically to implement any definition. We then treat fair value as zero in one instance and unobservable in the other case and in both cases provide evidence of bubbles in the art market.

Keywords: Bubbles, Asset prices, Steady state, Non-linear time series, TAR models, Art markets

JEL classifications: G120
1. Introduction

Art pieces not only hold value for the beholder, but are fast becoming a sound investment option. World wide, the size of the art market was reported to be around 40 billion dollars in 2008. And it has been growing fast. The European Fine Art Foundation (TEFAF) reported\(^2\) that size was estimated at 30 billion dollars in 2001. Christie’s and Sotheby’s reported collective sales of around 12 billion in 2007. And this is only auction sales. There is a big private market, dealer market and gallery sales, of which only estimates are available as the information regarding transactions and prices is quite limited. Thus there are resultant asymmetries of information regarding the price and quality of art. Sellers’ may be willing to reveal the true value of the painting to the buyer. Mc Andrew’s report for TEFAF reports that the art dealer sales in 2006 reached a record $28.6 billion, accounting for slightly more than half of the estimated 54.9 billion dollars global total. In 2006 about 1 million transactions involving dealers took place globally\(^3\). It is also reported that in 2007 the private art market was worth approximately 30 billion dollars\(^4\).

Investment in art is fast becoming a mainstream asset class as the universe of collectors has grown enormously and the vast amounts of wealth that have been accumulated. Large gains can be realised by matching prospective wealthy buyers or collectors and specialised works of established artists. The number of funds dealing with art investments is growing indicative of this development.

Given the state of the art market, it seems possible that prices and the rather nebulous concept of true value may not be perfectly aligned at all times. Our paper explores potential discrepancies between the two. These discrepancies are often described as bubbles. Our goal in this paper is to introduce and investigate a model of art prices focusing on the creation of bubbles. The process we use, due to Blanchard (1979, 1982)

\(^2\) As reported in Artnewsletter, May 2008
\(^3\) Report is in a series of major studies commissioned by TEFAF that act as a focus for the launch of their annual Maastricht fair
\(^4\) Artnewsletter, May 2008
is a threshold autoregressive model with different regimes and we define the presence of bubbles as corresponding to non-stationary behavior in the deviation from fair value in at least one regime. More recent formal justification for such a structure comes from an agent based model by Ahn, Sandford and Sheax (2011) who derive an equilibrium between rational agents who regard art as worthless and uninformed investors who attribute value to it (albeit erroneously). This leads to a two state model where switching is endogenous. Another justification for a two state world with explosive and stationary states for prices or deviations of prices from value is presented in Abel (1988) who finds multiple equilibria in his asset pricing model which can be stationary or explosive. Kurz and Motelese (2010) use heterogeneity in beliefs to motivate simple autoregressive structures that may be stationary or explosive. They are careful to use the word ‘illustrative’ to describe the simple version of their model. We regard our model as ‘illustrative’ in the same sense.

Defining or quantifying the value of art is a rather difficult exercise. Baumol (1986) has famously described the investment in art as a ‘crap game’ explaining in economic terms why the value of art is hard to define. Section 2 discusses the art market and how it is organised. There are different hierarchies in the art market consisting of the primary market, the secondary market and at the top is the international market. Most calculations or analysis ignore the dealer market, mainly because of lack of data and this section also touches upon how that could affect price measurement and certain other difficulties which arise due to the nature of art markets. Section 3 contains the discussion on the value of art, there is both an economic and a cultural value associated with art work and how that two could be related, which makes any sort of valuation very complicated. Despite this, there an increasing belief in the value of art as a sound investment option which this section assesses. It reviews the literature on other features such as galleries’ and artists’ reputations, role of consumption capital, hedonic factors that influence the value of art. Section 4 explains the art market index used in our estimation. Following the discussion from previous sections about the value of art, it also points out problems with the data and factors that can cause potential biases. The empirical application of fitting a generalized Blanchard model (Knight, Satchell and Srivastava (2011)) using art data is
presented in section 5. We have treated the fair value of art as being zero in section 5 and then, in section 6, we explore the implications of the value of art being stochastic, namely AR (1) but unobservable. We present a model for the fair value of art using the above. This leads to two calculations, the first is when we take it into account and the second is when we ignore it and compute the biases involved in running ordinary least squares (OLS) (omitted variables).

2. The Art Market

The art market is characterised by a hierarchy of submarkets according to the analysis in Gerard-Varet (1995). The first stage consists of a primary market of individual artists who supply to galleries, exhibitions or to customers directly. This is where artists’ work gets recognised. This signals their ability to the secondary (dealer) market which consists of galleries, dealers or art museums. It is here that the market is more concentrated both on the buyer and seller side. On the seller side, this is because very few artists can make a successful transition from primary to secondary markets and establish themselves. This is exacerbated by the fact that works of famous dead artists who are still very reputable also circulate in this market. On the buyer side, it consists of mostly private or public collectors. Schonfeld and Reinstaller (2007) discuss that this interaction between these two economic agents, the sellers (art galleries mainly) and the buyers, determines the nature of competition and framework of exchange in the primary art market.

At the top of hierarchy is the international market consisting of mainly the big auction houses in cities like London, New York, Paris who are the major players and handle most of the sales. Here the buyers include individual wealthy collectors, museums, and private foundations. An essential feature of the market is informational asymmetry as the sellers make profits by exploiting information on the willingness of buyers to pay for specific art works.
However, an issue with any discussion regarding the classification of art markets is that the dealer market and transactions in this market are very often ignored. This is largely because of the lack of availability of data to estimate what share of the market is composed of dealers. Campbell (2008) discusses the impact of this on valuation of art market and the rate of return reflected by the indices. Anderson (1974) estimated that dealers sale prices are about 20-50 per cent higher than the price at which they obtain the stock, which is mainly from auctions. This reduces the rate of return for investors as dealers at a higher price than the auction which reflects high transaction costs too. Campbell (2008) points out that there is a high probability that art funds who also act more like private dealers than auction houses, adopt a very similar strategy to take advantage of the inefficiencies in the market through their insider knowledge and expertise. These features are likely to produce art market returns which could be greater than the benchmarks used in this analysis.

Another aspect of art markets is the nature of the markets itself. The goods ‘produced’ in art markets are each unique, hence inherently heterogeneous. Each seller is a monopolist of a particular painting and each artist is like a monopsonist. The other issue is that sales are not continuous and objects may not get resold very frequently. Mei and Moses (2002) discuss both these aspects as major difficulties with respect to art markets.

3. Value in Art

As mentioned in the previous section, value of art is not just determined by its aesthetic value but there is also an intrinsic economic value attached to art works as they become an investment option. We argue in this section, looking at the literature on this debate, that indeed, different characteristics of art works have a definitive affect on the economic value and the transactions in the market are not just ‘symbolic interchanges’.

The main contention with determining the value of art is that it is associated with both an economic value or financial or commercial value and a cultural or aesthetic value in order
to capture the distinction in when measured in a standard economic model as compared to in cultural terms (including characteristics like status symbol, aesthetic properties, symbolic, spiritual or historical significance, uniqueness, influence on artistic trends amongst others). There are various issues that must be considered while doing any economic analysis of art objects and markets.

Throsby (2003) contends that the cultural value of art may sometimes render the economic value irrelevant. He argues that there is a dual market to which artists’ supply; a physical market which determines its economic price, and a market for ideas, which determines its cultural price. Transactions in this market can be deemed as economic according to him, in so far as ‘when a cultural good is made available to the public, consumers absorb, interpret and evaluate the ideas contained in the work, discussing and exchanging their assessments with others’. If a consensus is reached, then this assessed value can be thought of as an exchange value reached by negotiation amongst parties to a market transaction, where the ‘market’ is that for the cultural content of the work.

Another important point he notes is the pure public good properties of art. When art work is available for the public to view in museums or public spaces, there are positive externalities and it becomes a part of ‘cultural heritage’. Frey and Pommerehne (1989) also recognise the ‘psychic returns’ from the enjoyment of viewing and looking at art work.

Values in both markets may not be independent of each other as attributes that determine both might be very similar, but keep changing with reassessments of the work’s economic and cultural worth. Velthuis (2005) believes that there is an association between the aesthetic judgment of museum experts and the commercial assessment of markets and argues that prices both create and reflect value. Economic value is often also driven by lifestyle choices and cultural norms and that in turn, cultural and social customs reflect economic interests as Valsan (2006) argues. There is an increasing strand of literature that is focused on how to integrate the two values (Koerner and Koerner (1996), Benedikt (1997)).
Of course any valuation is complicated by the existence of different conditioning factors that influence the process. To the extent that price of an art work may rise over time, it has an investment asset characteristic to it and can be considered as a store of value. Many studies in literature like Anderson (1974), Stein (1977), Baumol (1986) have contributed to examine the rates of return on paintings.

**Velthuis (2005)** examines the relationship between art prices and artwork size, which according to him, provides a stable way of pricing. He also notes that a reason for price dispersion in art markets is the allocation mechanisms along with transaction and search costs. In auction circuits, it is a market mechanism whereas administrative pricing is preferred in primary markets. More recently, auction data is generally relied upon for econometric studies.

Valsan (2006) draws a comparison between value of art and that of financial assets. A very simple interpretation of the efficient market hypothesis (EMH) is that prices are not different from their value which is determined by what investors agree upon. In a similar vein, he compares that prices not only measure the value of art but also define our concept of what art value is. He says, ‘more symbolic the asset in question, the more apparent it is that economic values are contingent on what we culturally accept as legitimate determinants of value’.

Frey (1997) discusses the nature of art as an investment and details the literature under three categories of studies that compare art investment with other traditional investment options (like bonds, stocks, real estate), studies that evaluate art auctions and studies on objects and all forms of collectibles. Singer and Lynch (1997) examine whether it pays to buy art from a financial perspective. They note that wealthy collectors of the highest quality of art benefit the most as they can take advantage of the informational asymmetry in the market. That there may be connections between art and wealth is not really a surprise. Certainly, one of the constraints limiting the price of art works in the wealth of agents willing to purchase them. As compared to them, buyers of lower quality art face a much higher (monetary) opportunity cost. Thus for high quality buyers, yields are
as high as would be in any other financial market. Frey and Eichenberger (1995) also note how high quality art is always a sound investment option from a financial point of view and that such buyers are even charged lower commissions and transaction costs by auction houses and thus earn a higher return in the market.

Reputation, interactions between galleries and interaction between galleries and artists

Here, the ‘commercial reputation’ of galleries and dealers is mixed with the ‘aesthetic reputation’ of artistic works. Given the high degree of uncertainty about the quality of art work, reputation of both galleries and their artists become important proxies or even determinants of value and quality (Adler (1985), Shubik (2003)). By signaling the gallery’s competence in choosing high potential artists, reputations are an important indicator and outcome of a functioning primary market. This feature helps alleviate intrinsic risk of buying low quality work. Featuring an art piece in the gallery is a signal of quality to potential customers decreasing the asymmetry of art specific knowledge. Thus, in many ways the reputation of the gallery and artist are intertwined. By organizing exhibitions, shows, reviewing artist’s work through experts they direct attention of public and important collectors whose purchase supports the quality judgment of the gallery (Velthuis (2005)). This helps galleries establish a long term relationship with artists and customers, and not only boost artists’ reputation, but also raise prices. These are important parameters that the gallery can influence through its own efforts.

Value of art is also dependent on the consumption capital of customers or collectors who are able to ascertain the inherent value associated with a piece of art work. This also depends on their consumption capital that they have acquired. Thus, just as sellers can be classified according to their reputation and the reputation of their featured artists, buyers can be categorized accordingly to their knowledge. This is explained in Stigler and Becker (1977) who elaborate on how people’s tastes and preferences, and hence the utility they derive from consuming art are different according to various factors like the social environment they grow up in, education amongst others. Thus the more consumption capital they have, the more knowledgeable they are, the higher the pay offs they derive from consuming art. This also generates a kind of specificity in their
consumption, in that they are more likely to buy more from a group of particular artists. As a result of this ‘path dependent’ nature of consumption capital, buyers incur switching costs if they change between different kinds of arts or artists.

Bonus and Ronne (1997) study evidence of how galleries interact and react to each other which gets reflected in their pricing decisions. Schonfeld and Reinstaller (2007) explain that under the presence of switching costs, as they also lower the risk of losing market share, galleries can charge prices slightly above the ‘competitive level’. On the other hand, by lowering prices to subsidise switching costs of customers with a different consumption capital, they can gain market share. Thus, prices are an effective tool used by galleries to increase profits or increase market share. The cost of the resultant uncertainty is borne by buyers and they thus have to rely on signals like reputation.

4. Art Data

Due to the growing art market, there have been many attempts to create art price indices to help compare art with other investment options. Campbell (2008) notes that there are four main methodologies used for producing art price indices which include geometric means, average prices, repeat sales, and hedonic regressions which are closely related over long periods (Chanel, Gerard-Varet and Ginsburgh (1990)). Candola and Scorcu (1997) warn of the potential of misleading evaluation with indexes as they involve narrow data set and might reflect experts’ opinions, heterogeneity and therefore difficulty in aggregation.

Biases arise from use of both repeat sales and average prices in a highly heterogeneous market. Repeat sales regressions require artworks to be offered for sale at auction more than once to be included as a repeat sale. These biases in repeat sales are also quite well known in the literature on house prices (Case, Pollakowski and Wachter (1991, 1997), Gatzlaff and Haurin (1997)). An important problem with repeat sales regressions is the
possibility of sample selection bias. The problem is that some types of art or paintings may trade more frequently on the market than other types so that they will be over-represented in the repeat sales sample. When these types of paintings exhibit different price changes, then the repeat sales index tends to be biased. For example, if low quality paintings sell more frequently than high quality ones but high quality ones rise in price at a slower rate, a repeat sales index will tend to have an upward bias.

Another aspect of this is the holding duration of these assets which can be quite unevenly distributed due to differences in prices and transaction costs. Zanola (2007) address the potential problem of sample selection bias by applying the Heckman two-stage procedure. The probit model predicts the probability of whether the object is sold only once or is a repeat sales object. These estimates are then used to construct an inverse Mills ratio which is used in the repeat sales regression as an explanatory variable. This is done to obtain consistent estimates and test for sample selection bias by using data on single and repeat sales to construct the price index rather than restricting it to the transactions which actually occur twice.

Models that combine information on repeat sales with hedonic approach are the new direction in construction of indices. Characteristics like reputation of the artist, artistic merit of the particular work like degree of conformity with the artist’s style, period over which it was painted, history of ownership, play an important role in realized prices. Condition, subject matter and size also affect and to some extent determine ownership of the work. However, this methodology fails to directly solve the sample selection problem. In particular, in the case of repeat sales model a double selection problem emerges (Zanola (2007)). The sample size is quite small as the repeat sales are infrequent and cannot represent the population. As second sales are omitted from the first sale data, it can create selection biases in the sample.

Here we use data from the Art100 Index from Art Market Research (AMR). The data is monthly available from 1970 onwards.
5. Estimation

Using data from the Art 100 Index, we estimate results for the framework outlined in Knight and Satchell (2011) which we call the generalized Blanchard model (Knight et al(2011)). These require the use of an exogenous switching variable. We present a brief analysis based on this structure.

Let $p_t$ be the price and $p^*$ be the fundamentals.

We can write

$$p_t - p^* = \beta_1 (p_{t-1} - p^*) + \varepsilon_t \quad \text{if } I_{t-1} = 0$$
$$p_t - p^* = \beta_2 (p_{t-1} - p^*) + \varepsilon_t \quad \text{if } I_{t-1} = 1$$

Let $\beta_1 = 0; \beta = \beta_2; X_t = p_t - p^*$

Therefore,

$$X_t = \beta I_{t-1} X_{t-1} + \varepsilon_t \quad (1)$$

where $I_{t-1} = 0$ with probability $1 - \pi$ and $I_{t-1} = 1$ with probability $\pi$.

In the Blanchard model, $\beta = 1/\pi \alpha$ where $\alpha = \frac{1}{1+r}$ where $r$ is the constant rate of return on the riskless asset. This particular value of $\beta$ corresponds to the bubble as defined very precisely in Blanchard (1979). Following Knight et al (2011), we call (1) the generalized Blanchard model. In this case, our interpretation changes slightly; essentially we are proposing a system of possibly explosive dynamics for deviations from fair value.

Theorem 2 discusses the case of the Blanchard model, and it is noted that the steady state distribution always exist; if $\beta = 1/\pi \alpha$, the mean does not exist and hence the variance does not exist. If $\beta \neq 1/\pi \alpha$, that is the general case, then the mean exists if $|\beta| \pi < 1$ and is equal to $p^*$ and the variance exists if $0 < \beta^2 \pi < 1$.
VIX and University of Michigan Sentiment Index are used to generate an exogenous threshold. For the results following Knight and Satchell (2011) in table 1, we see that in the case of VIX, the threshold value above which prices are in a high regime is 29.97. It corresponds to periods of market panic and captures the bubbles as the coefficient is greater than 1 in the high regime (which is likely to occur 5 per cent of the time). Similarly, the results for the consumer sentiment index in table 2 indicate that when the sentiment is low, there is turmoil in the market. The threshold level for the sentiment index is around 83.7 and we do observe that in the high regime, the coefficient is greater than 1 (which occurs with a probability of 30 per cent).

Insert table 1, 2, 3, 4 here

The results are equally encouraging for the generalised Blanchard model where we impose that the coefficient in the low regime is zero. The results do indicate the presence of bubbles in art market data as the coefficient of interest, $\beta$ is greater than 1 in the volatile regime as shown in table 3 and table 4.

The results above show the values of $\beta$ for the three indices based on the average price of all sales, the average of the top 10 percent and the average price of the top 2 per cent. We also report the proportion of data points in the bubble regime. In all six cases, we have value of $\beta$ greater than 1. In all cases, the steady state means and variances exist. It should be noted that high levels of volatility lead to art market bubbles whilst low levels of sentiment lead to art market bubbles. The interpretation of art as a ‘safe haven’ asset is complicated by the fact that prices are rising explosively in the bubble regime 60 per cent of the time but 40 per cent of the time they are also falling explosively.

We do not provide a rigorous interpretation of the t statistics or test for significance as it becomes very complicated owing to the difficulty in defining the problem. For instance, our null hypothesis of the existence of a bubble would be
6. Modeling the Fair Value of Art

In a previous paper, we treated the value of art as being 0 (Knight et al. (2011)). In this section, we explore the implications of the value of art being autoregressive (AR(1)) but unobservable. This leads to two calculations, the first is when we take it into account and the second is when we ignore it and compute the biases involved in running OLS (omitted variables). Initially, we shall justify the AR(1) assumption for fair value.

Using a dividend discount model or net present value argument, we can broadly say that

\[ p \approx \frac{c}{1 + D} \approx \frac{c}{d} \]

\[ D = r_j + RP \]

where \( r_j \) is the risk free rate and \( RP \) is the risk premium.

We are interested in whether prices can be stationary or not. Assuming prices follow a random walk is tantamount to saying that a steady state probability distribution of prices does not exist. In the context of cointegration, if \( c \) was I(1) and \( d \) was I(1) and they were cointegrated or both were I(0), then we would have a stationary outcome. Poterba and Summers (1988) note that to get sufficient power to discriminate between whether the price series of equity are stationary or cointegrated, a very long time series is required. It is clear that with conventional data sets, being able to say with certainty, that prices are I(1) or I(0) seems difficult.

Evidence on dividends being I(0) is vast. Bansal and Lundblad (2002) assume that the risk free rate and the time varying price of risk are both stationary, hence \( d \) is stationary. They do also assume that the cash flow is I(1). Gurkaynak (2005) takes the order of the
AR process that governs dividends to be 1 for simplicity and assumes that dividends are exogenous and follow a stationary AR(1) process. Fukuta (2002) also works under the assumption that D is time-varying but stationary. Goyal and Welch (2002) also find that dividends have remained stationary over time. Perron (1988) also notes that the real dividend and earning series are stationary around a linear deterministic trend. Kleidon (1986) show that nominal aggregate dividends and earnings are non stationary with a unit root. However, real dividend and real earnings are stationary around a significant linear trend when the deflator used is the producer price index.

Barberis, Huang and Santos (2001) assume the price-dividend ratio and risky asset risk premium are stationary. Engel, Wang and Wu (2010) look at present value asset pricing models to find existence of unobservable stationary fundamentals which could be a result of risk premium. Nilsson and Hansson (2004) note that earlier empirical applications of CAPM (Sharpe (1964)) that assumed an unconditional model, risk premium as well as asset “betas” were taken as stationary over a fixed period. Risk premia in the term structure literature are typically treated as stationary variables both in the theoretical (Backus, Gregory and Telmer (1989)) and empirical investigations. Empirically, Meese and Singleton (1982) and Bollerslev and Baillie (1988, 1994) find that the forward premium is stationary (by finding the forward and spot exchange rates to be cointegrated with zero intercept and slope coefficient close to 1). Karlsson and Schoultz (2003) find a risk premium significantly parted from zero that is stationary and time varying. Corbae, Lim and Ouliaris (1992) also note that conventionally risk premium is assumed to be stationary. Additionally, they show that when forward exchange rates are unbiased predictors of future spot rates, the risk premium is stationary. Carriero (2006) finds evidence of a stationary but time varying risk premium between the UK and the US when testing uncovered interest rate parity. A similar result is obtained in Byrne and Nagayasu (2008) who present evidence of a stationary risk premium for certain emerging European economies. Concluding, our assumption of a stationary AR(1) model for fair value is reasonable and consistent with a large literature. We could extend it to a more general ARMA structure but only at the cost of much greater complexity.
We now present an analysis of the time series properties of deviations from fair value in the generalised Blanchard model.

Let \( c_t \) be our fundamental and \( y_t \) be the price. Using the structure of (1), we can write our model as:

\[
y_t - c_t = \beta I_{t-1} (y_{t-1} - c_{t-1}) + v_t \\
c_t = \gamma + \phi c_{t-1} + e_t
\]

\[(2)\]

\[
\begin{pmatrix}
1 & -1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
y_t \\
c_t
\end{pmatrix}
= \begin{pmatrix}
0 \\
\gamma
\end{pmatrix}
+ \begin{pmatrix}
\beta I_{t-1} & 0 \\
-\beta I_{t-1} & \phi
\end{pmatrix}
\begin{pmatrix}
y_{t-1} \\
c_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
v_t \\
e_t
\end{pmatrix}
\]

\[(3)\]

where

\[
\begin{pmatrix}
v_t \\
e_t
\end{pmatrix}
\sim iid N\left[ \begin{pmatrix}
0 \\
0
\end{pmatrix}, \begin{pmatrix}
\sigma_v^2 & \rho \sigma_v \sigma_e \\
\rho \sigma_v \sigma_e & \sigma_e^2
\end{pmatrix} \right]
\]

Solving for (3), we see that

\[
\begin{pmatrix}
y_t \\
c_t
\end{pmatrix}
= \begin{pmatrix}
0 \\
\gamma
\end{pmatrix}
+ \begin{pmatrix}
1 & 1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\beta I_{t-1} & 0 \\
-\beta I_{t-1} & \phi
\end{pmatrix}
\begin{pmatrix}
y_{t-1} \\
c_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
1 & 1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
v_t \\
e_t
\end{pmatrix}
\]

\[
= \begin{pmatrix}
0 \\
\gamma
\end{pmatrix}
+ \begin{pmatrix}
\beta I_{t-1} & 0 \\
-\beta I_{t-1} \phi & \phi
\end{pmatrix}
\begin{pmatrix}
y_{t-1} \\
c_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
v_t + e_t \\
e_t
\end{pmatrix}
\]

where

\[
w_{1t} = v_t + e_t
\]

\[
w_{2t} = e_t
\]
\[
\begin{pmatrix}
    w_{1t} \\
    w_{2t}
\end{pmatrix}
\sim N\left(\theta\begin{pmatrix}
    \sigma_v^2 + \sigma_e^2 + 2\rho\sigma_v\sigma_e & \sigma_e^2 + \rho\sigma_v\sigma_e \\
    \sigma_e^2 + \rho\sigma_v\sigma_e & \sigma_e^2
\end{pmatrix}
\right)
\]

\[y_t = \beta I_{t-1}y_{t-1} + (v_t + c_t - \beta I_{t-1}c_{t-1}) \quad (4)\]

It follows that, in regime 1,

\[y_t = \beta y_{t-1} + (v_t + c_t - \beta c_{t-1})\]

\[(1 - \beta L)y_t = v_t + \frac{(1 - \beta L)}{(1 - \phi L)}e_t\]

\[(1 - \beta L)(1 - \phi L)y_t = (1 - \phi L)v_t + (1 - \beta L)e_t\]

We can identify this as an ARMA(2,1). In regime 2, this is an ARMA(1,1). Whilst it may be possible to estimate some of the parameters via methods of moments, it is not at all clear that we can identify all the parameters in the model. We present some further calculations in Appendix 1.

We now show that the assumption of fair value of 0 when in fact it follows an AR(1) leads to an downward bias in our estimation of \(\beta\). Thus our empirical estimates of \(\beta\) are likely to be less than the true value.

**Proposition 1.** Assuming the structure given by equation (4), and 
\[\frac{1}{\pi} - 1 > \beta - \phi > 0\]

and stationary fundamentals, it follows that \(\text{plim} \ \hat{\beta} < \beta\). If \(\gamma = 0\), we only require that \(\beta - \phi > 0\).

**Proof:**
We first look at the covariance between the right-hand side variable and the error term in (4).

\[ y_t - c_t = \beta_{i-1} (y_{i-1} - c_{i-1}) + v_t \]

\[ c_t = \gamma + \phi c_{i-1} + e_t \]

\[ y_t = \beta_{i-1} y_{i-1} + v_t + c_t - \beta_{i-1} c_{i-1} \]

(i)

\[ \text{Cov} (i_{i-1}, y_{i-1}, v_t + c_t - \beta_{i-1} c_{i-1}) \]

\[ E(y_t, i_{i-1}) = \pi E(y_t) \]

\[ E(y_t) = \beta \pi E(y_{i-1}) + \frac{\gamma}{1 - \phi} (1 - \beta \pi) \]

\[ E() = \frac{\gamma}{1 - \phi} (1 - \beta \pi) \]

\[ \text{Cov}(i_{i-1}, y_{i-1}, v_t + c_t - \beta_{i-1} c_{i-1}) = E(i_{i-1}, y_{i-1}, c_t) - \beta E(i_{i-1}, y_{i-1}, c_{i-1}) \]

\[ = E(i_{i-1}, y_{i-1}, (\gamma + \phi c_{i-1} + e_t)) - \beta \pi E(y_{i-1}, c_{i-1}) \]

\[ = \gamma \pi E(y_{i-1}) + (\pi \phi - \pi \beta) E(y_{i-1}, c_{i-1}) \]

\[ \text{Cov} = \gamma \pi \left( \frac{\gamma}{1 - \phi} + \pi (\phi - \beta) E(y_{i-1}, c_{i-1}) \right) - \left( \frac{\gamma}{1 - \phi} \right)^2 (1 - \beta \pi) \]

\[ = \pi (\phi - \beta) E(y_{i-1}, c_{i-1}) - \frac{\gamma^2 (1 - \phi)^2 (1 - \beta \pi)}{(1 - \phi)^2} + \frac{\gamma^2 ((\pi^2 - 1) - \phi \pi (\phi - \beta))}{(1 - \phi)^2} \]

\[ = \pi (\phi - \beta) E(y_{i-1}, c_{i-1}) - \frac{\gamma^2 (1 - \pi) + \pi (\phi - \beta)}{(1 - \phi)^2} \]

Thus, it holds if \( \gamma \neq 0 \), \( \phi > \beta \), \( (1 - \pi) + \pi (\phi - \beta) > 0 \) or \( 1 > \pi (1 + (\beta - \phi)) \) or

\[ \frac{1}{\pi} - 1 > \beta - \phi > 0 . \]

If \( \gamma = 0 \), then the following holds.
\[
\text{cov}(I_{t-1}, y_{t-1}, v_t + c_t - \beta I_{t-1} c_{t-1}) \\
= \text{cov}(I_{t-1}, y_{t-1}, v_t + (\phi - \beta I_{t-1}) c_{t-1} + e_t) \\
= \text{cov}(I_{t-1}, y_{t-1}, (\phi - \beta I_{t-1}) c_{t-1} | I_{t-1}) \\
= (\phi I_{t-1} - \beta I_{t-1}) \text{var}(y_{t-1}, c_{t-1} | I_{t-1}) \\
= (\phi I_{t-1} - \beta I_{t-1}) \frac{\sigma^2}{1 - \phi^2}
\]

So, \(\text{cov}(I_{t-1}, y_{t-1}, v_t + c_t - \beta I_{t-1} c_{t-1}) = (\phi - \beta)\pi \frac{\sigma^2}{1 - \phi^2} < 0\)

\[
\text{plim} \hat{\beta} = \beta + \frac{(\phi - \beta)\pi \frac{\sigma^2}{1 - \phi^2}}{\text{var}(y_t)} < \beta\quad \text{QED.}
\]

This shows that the estimates are downward biased. The assumption that \(\beta\) is greater than \(\phi\) assumes that expected relative price of art in the up state is larger than the overall AR(1) coefficient in the fundamentals equation and does not seem implausible at all. In effect, if fair value in art is a stationary random variable and the true value of \(\beta\) is greater than 1, we are assured that the estimated value will be less than the true value. So an estimated value greater than 1 for \(\beta\) and assumed value of \(\phi\) less than 1 lend support to the presence of a bubble. We need to also note that \(\pi\), the steady state probability of a bubble varies between 5 per cent and 30 per cent and so it seems quite plausible that proposition 1 holds in the art markets investigated if they have the structure which we assumed.

### 7. Conclusion

We have investigated whether there are bubbles in the art market over approximately the last thirty years. Using several measures of sentiment as trigger variables, we have found evidence of bubbles over both longer and shorter periods depending upon our choice of indicator. In common with many other researchers, we have struggled to define a fair
value of art. Our contribution however is to show that under plausible assumptions, ignoring the fair value of art but only assuming that it is a stationary AR(1) still allows one to investigate the presence of bubbles in a meaningful way. Our results support the presence of bubbles in the overall category as measured by the Art Market Research Index. We note a number of caveats. Firstly, given our definition of bubbles, testing for a bubble is a formidable analytical challenge. Secondly, the index data in common with other data that we considered does not include the dealer market. And this market may have a more rational pricing structure. Thirdly, we have deliberately finessed one of the deep issues of our culture by either not defining the value of art or defining it to follow a simple statistical process. Fourthly, our model is extremely simplistic involving only autoregressive and threshold relationships of order 1. This is not because we believe them to be true, but we need to able to carry out some analysis by proving that the bias in our estimation techniques can be signed. Even in our simple case, our price distribution switches between an ARMA(2,1) and an ARMA(1,1). Such switching processes have complicated moments and in our context at least, unidentifiable parameters. It was Baumol (1986) who told us how difficult it was to compute value for art; we have tried to extend his analysis by demonstrating how one can still detect bubbles even without observing value.
References


Appendix 1

Statistical properties of the price process

We examine the error term in regime 1 where

\[ w_t = v_t + e_t - \phi v_{t-1} - \beta e_{t-1} \]

\[ \text{var}(w_t) = \sigma_v^2 (1 + \phi^2) + \sigma_e^2 (1 + \beta^2) + 2 \rho \sigma_v \sigma_e (1 + \phi \beta) \]

\[ \text{cov}(w_t, w_{t-1}) = \text{cov}(v_t + e_t - \phi v_{t-1} - \beta e_{t-1}, v_{t-1} + e_{t-1} - \phi v_{t-2} - \beta e_{t-2}) \]

\[ = -\phi \sigma_v^2 - \beta \sigma_e^2 (1 + \beta^2) - (\beta + \phi) \rho \sigma_v \sigma_e \]

\[ \rho_w = \frac{-\phi \sigma_v^2 - \beta \sigma_e^2 (1 + \beta^2) - (\beta + \phi) \rho \sigma_v \sigma_e}{\sigma_v^2 (1 + \phi^2) + \sigma_e^2 (1 + \beta^2) + 2 \rho \sigma_v \sigma_e (1 + \phi \beta)} \]

\[ \text{cov}(w_t, w_{t-2}) = 0 \]

If \( \rho = 0 \), then it can be proved that \( |\rho_w| \leq \frac{1}{2} \)

In regime 2,

\[ y_t = v_t + c_t \]

\[ = v_t + \frac{e_t}{1 - \phi L} \]

\[ (1 - \phi L) y_t = (1 - \phi L) v_t + e_t = w_t \]
\[ \text{Var}(y_t) = \sigma_v^2 + \frac{\sigma_e^2}{(1 - \phi^2)} \]

\[ \text{Cov}(y_t, y_{t-1}) = \frac{\phi \sigma_e^2}{1 - \phi^2} \]

\[ \text{Cov}(y_t, y_{t-\tau}) = 0 \quad \text{for} \quad \tau \geq 2 \]

This is an ARMA(1,1) if \( |\rho| \leq \frac{1}{2 \left( 1 + \frac{\sigma_v^2}{\sigma_e^2} \right)^{1/2}} \)
Table 1 PARAMETER ESTIMATES USING VIX AS THRESHOLD (MONTHLY, 1990-2008)

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Table 2 PARAMETER ESTIMATES USING MICHIGAN SENTIMENT INDEX AS THRESHOLD (MONTHLY, 1978-2008)

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Table 3 PARAMETER ESTIMATES USING VIX AS THRESHOLD (MONTHLY, 1990-2008)

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Table 4 PARAMETER ESTIMATES USING MICHIGAN SENTIMENT INDEX AS THRESHOLD (MONTHLY, 1978-2008)

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